PHYSICAL REVIEW D **72,** 115001 (2005)

Determining neutralino parameters in left-right supersymmetric models

Nibaldo Alvarez-Moraga[1,*] and Artorix de la Cruz de Oña[1,2,†]

[1]*Autonomous Center of Theoretical Physics and Applied Mathematics, 3561 Hutchison #3, Montréal (Québec) Canada, H2X 2G9*

[2]*Department of Mathematic and Statistics, Concordia University, 1455 de Maisonneuve Boulevard West, Montréal (Québec), Canada H3G 1M8*

(Received 27 June 2005; published 1 December 2005)

We report exact analytical expressions relating the fundamental parameters describing the neutralino sector in the context of the left-right supersymmetric model. The method used for such effects is the projector formalism deduced without taking into account the Jarlskog's projector formulas. Also, expressions for the neutralino masses and the neutralino mixing matrix are determined . The results are compared with numerical and analytical ones obtained in similar scenarios in the context of the minimal supersymmetric standard model.

DOI: 10.1103/PhysRevD.72.115001 PACS numbers: 12.60.Jv

I. INTRODUCTION

In Ref. [1], based on Jarlskog's treatment of the Cabibbo-Kabayashi-Maskawa matrix, the neutralino observables, in the context of the minimal supersymmetric standard (MSSM), were described in terms of projectors. There exact analytic expressions for the neutralino masses were also obtained by diagonalizing the associated real symmetric neutralino mass matrix. Then the same formalism was applied to treat a more general case where the associated neutralino mass matrix was given by a complex symmetric matrix [2]. In this last reference, several CP-conserving and -violating possible scenarios were considered in the study of the determining parameters of the theory.

The purpose of this work is first to apply the projector formalism [1,2] to study the existing connections among the fundamental parameters describing the neutralino sector in the context of the left-right supersymmetric (LR SUSY) model [3,4], and second, to compare the results obtained to the ones obtained in the context of the MSSM [2].

In the LR SUSY model which is based on the gauge group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ [5], the masses and mixing matrices of the neutralinos and charginos are determined by M_L , M_R, the left-right gaugino mass parameters associated with the gauge group $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$, respectively, M_V, the gaugino mass parameter associated with the gauge group $\mathrm{U}(1)_{B-L}$, μ, the Higgsino mass parameter and the ratio $\tan\theta_k \equiv k_u/k_d$, where k_u and k_d are the vacuum expectation values of the Higgs fields which couple to d-type and u-type quarks, respectively [6–13].

In Sec. II, we give a brief description of the LR SUSY model and we write the Lagrangian density describing the neutralino sector in terms of the two-component fermion fields and the neutralino mass matrix expressed in terms of the fundamental parameters M_L, μ, $\tan\theta_k$, M_R, and M_V, where M_L and μ are considered, in general, as complex numbers. In Sec. III, we compute the exact analytical expressions for the neutralino masses and the corresponding diagonalizing unitary matrix. Also, we plot these masses versus the Higgsino parameter, in both the CP-conserving and CP-violating cases, and we compare the corresponding CP-conserving results with the numerical ones obtained in [12]. In Sec. IV, the projector formalism [14] for this model is revised. Based on the explicit construction of the diagonalizing neutralino mass matrix, new formulas for the so-called reduced projectors are constructed without appealing to the Jarlskog's projector formulas [14,15]. The fundamental properties of these reduced projectors as well as the projectors and the so-called pseudoprojectors [2] are proved. Also, the equivalence of these reduced projectors with those obtained using the Jarlskog's formulas is proved. In Sec. V, using the new reduced projector formulas, we express the complex parameter M_L, in terms of the so-called eigenphases [2] and the rest of the parameters. Moreover, taking advantage of the mentioned equivalence we get a novel formula expressing the norm of this complex parameter in terms of its phase and of the remaining fundamental parameters. An alternative method to disentangle these parameters is presented in the Appendix. In Sec. VI, we compare the expected values of the fundamental parameters in similar scenarios predicted by both the LR SUSY model and the MSSM. Finally, in Sec. VII, we give our conclusions and prospects.

II. A BRIEF DESCRIPTION OF THE LEFT-RIGHT SUPERSYMMETRIC MODEL

In the LR SUSY model the full Lagrangian is given by [4]

$$\mathcal{L} = \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{matter}} + \mathcal{L}_{\mathrm{Y}} - \mathcal{V} + \mathcal{L}_{\mathrm{soft}}, \qquad (2.1)$$

where $\mathcal{L}_{\mathrm{gauge}}$ contains the kinetic and self-interaction terms for the boson vector fields $(W^{\pm}, W^0)_{L,R}$ and V^0, and the

*Email address: nibaldo.alvarez.m@exa.pucv.cl
†Email address: artorde@vax2.concordia.ca

Dirac Lagrangian of their corresponding superpartners, i.e., the gaugino fields $(\lambda^{\pm}, \lambda^0)_{L,R}$ and λ_V^0; $\mathcal{L}_{\text{matter}}$ contains the kinetic terms for the fermionic and bosonic matter fields, the Higgs fields and interaction of the gauge and matter multiplets; $\mathcal{V}$ is a scalar potential, $\mathcal{L}_Y$ (Yukawa Lagrangian) contains the self-interaction terms of the matter multiplets as well as of the Higgs multiplets, e.g., it contains the self-interaction terms involving the fundamental Higgsino mass parameters $\mu_1 \equiv \mu$, μ_2, and μ_3: $\text{Tr}[\mu_1(\tau_1\tilde{\phi}_u\tau_1)^T\tilde{\phi}_d]$, $\text{Tr}[\mu_2(\tau\cdot\tilde{\Delta}_L)(\tau\cdot\tilde{\delta}_L)\phi_d]$, and $\text{Tr}[\mu_3(\tau\cdot\tilde{\Delta}_R)(\tau\cdot\tilde{\delta}_R)\phi_d]$, where τ_j, $j=1,2,3$ are the usual Pauli matrices, $\tilde{\phi}_d$, $\tilde{\Delta}_{L,R}$ and $\tilde{\delta}_{L,R}$ are the superpartners of the bidoublet field ϕ_d and the four triplet fields $\Delta_{L,R}$ and $\delta_{L,R}$, respectively, which we will define later (in the following we will consider $\mu_2=\mu_3=0$); and $\mathcal{L}_{\text{soft}}$ is the soft-breaking Lagrangian, involving the fundamental gaugino mass parameters M_L, M_R, and M_V, which gives Majorana mass to the gauginos:

$$\mathcal{L}_{\text{soft}} = M_L(\lambda_L^a\lambda_L^a + \bar{\lambda}_L^a\bar{\lambda}_L^a) + M_R(\lambda_R^a\lambda_R^a + \bar{\lambda}_R^a\bar{\lambda}_R^a) + M_V(\lambda_V^0\lambda_V^0 + \bar{\lambda}_V^0\bar{\lambda}_V^0). \tag{2.2}$$

The Higgs sector contains two bidoublet fields,

$$\phi_{u,d} = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}_{u,d} \equiv \left(\tfrac{1}{2}, \tfrac{1}{2}, 0\right), \tag{2.3}$$

and four triplet fields,

$$\Delta_{L,R} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}}\Delta^+ \end{pmatrix}_{L,R}, \tag{2.4}$$

and

$$\delta_{L,R} = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta^+ & \delta^{++} \\ \delta^0 & -\frac{1}{\sqrt{2}}\delta^+ \end{pmatrix}_{L,R}. \tag{2.5}$$

The Higgs $\Delta_{L,R}$ transform as $(1,0,2)$ and $(0,1,2)$, respectively. The triplet Higgs $\delta_{L,R}$ which transform as $(1,0,-2)$ and $(0,1,-2)$, respectively, are introduced to cancel anomalies in the fermionic sector that would otherwise occur.

In order to generate mass for the gauge bosons we can choose the vacuum expectation values of the Higgs fields in the form [13]

$$\langle\Delta_L\rangle = \langle\delta_{L,R}\rangle = 0, \qquad \langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \tag{2.6}$$

$$\langle\phi_u\rangle = \begin{pmatrix} k_u & 0 \\ 0 & 0 \end{pmatrix}, \qquad \langle\phi_d\rangle = \begin{pmatrix} 0 & 0 \\ 0 & k_d \end{pmatrix}. \tag{2.7}$$

Thus, in the first stage, the spontaneous breaking of $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$, according to the vacuum expectation value $\langle\Delta_R\rangle \neq 0$, given in Eq. (2.6), generates masses for $W_R^{\pm}$, W_R^0, and V^0. The two neutral states W_R^0 and V^0 mix yielding the physical field Z_R and the massless field B. The vacuum expectation value v_R of the triplet Higgs Δ_R has been chosen in the order of the TeV to provide large masses to gauge bosons $W_R^{\pm}$ and Z_R. Next, through the spontaneous breaking of $SU(2)_L \times U(1)_Y$ into $U(1)_{\text{em}}$, according to the chosen vacuum expectation values $\phi_{u,d}$ given in Eq. (2.6), the left weak bosons $W_L^{\pm}$ and W_L^0 as well as B_μ acquire mass. Once again, the neutral fields mix forming the massless photon A_μ and the physical gauge field Z_L. The masses of the right-handed gauge bosons are given by

$$M_{W_R} = \frac{1}{\sqrt{2}} g_R(k_u^2 + k_d^2 + v_R^2)^{1/2}, \tag{2.8}$$

$$M_{Z_R} = \frac{1}{\sqrt{2}} v_R(g_R^2 + 4g_V^2)^{1/2}, \tag{2.9}$$

whereas the masses of the left-handed ones are given by

$$M_{W_L} = \frac{1}{\sqrt{2}} g_L(k_u^2 + k_d^2)^{1/2}, \tag{2.10}$$

$$M_{Z_L} = \frac{1}{\sqrt{2}}[(k_u^2 + k_d^2)(g_L^2 + 4g'^2)]e^{1/2}, \tag{2.11}$$

where g_L, g_R, g_V, and $g' = g_R g_V/(g_R^2 + 4g_V^2)^{1/2}$ are the coupling constants of the gauge groups $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$, and $U(1)_Y$, respectively.

To find the neutralino masses we must consider the interaction terms between the gauge bosons, the Higgs, and their superpartners. The neutralino particles are produced in two stages of symmetry breaking. The first stage involving the vacuum expectation value v_R of Δ_R generates masses for three heavy neutralinos $\tilde{\chi}_k^0$, $k=5,6,7$. The second stage involving the vacuum expectation values k_u and k_d of the Higgs ϕ_u and ϕ_d generates mass for the light neutralinos $\tilde{\chi}_k^0$, $k=1,\ldots,4$. The Lagrangian for light neutralinos is given by [13]

$$\begin{aligned}\mathcal{L}_{LN} = &-\frac{i}{\sqrt{2}} g_L k_u \tilde{\phi}_{1u}^0 \lambda_L^0 + i\sqrt{2}\frac{g_R g_V}{g_1} k_u \tilde{\phi}_{1u}^0 \lambda_B^0 \\ &- i\sqrt{2}\frac{g_R g_V}{g_1} k_d \tilde{\phi}_{2d}^0 \lambda_B^0 + \frac{i}{\sqrt{2}} g_L k_d \tilde{\phi}_{2d}^0 + M_L \lambda_L^0 \lambda_L^0 \\ &+ \left[\frac{(4M_R g_V^2 + M_V g_R^2)}{g_1}\right]\lambda_B^0\lambda_B^0 + 2\mu\tilde{\phi}_{1u}^0\tilde{\phi}_{2d}^0 \\ &+ \text{H.c.},\end{aligned} \tag{2.12}$$

where $\lambda_B^0 = (g_R\lambda_V^0 + 2g_V\lambda_R^0)/g_1$ with $g_1 = (g_R^2 + 4g_V^2)^{1/2}$; $\lambda_{L,R}^0$ and λ_V^0 are the neutral gaugino fields; and $\tilde{\phi}_{1u}^0$ and $\tilde{\phi}_{2d}^0$ are the neutral Higgsino fields, i.e., the superpartner of the neutral Higgs fields ϕ_{1u}^0 and ϕ_{2d}^0, , respectively, defined in Eq. (2.3).

The above Lagrangian in matrix form can be written as follows:

$$\mathcal{L}_{LN} = -\tfrac{1}{2}(\xi^0)^T N \xi^0 + \text{H.c.}, \tag{2.13}$$

where N is in general a complex symmetric matrix given by

$$N = \begin{pmatrix} M_L & 0 & -\frac{1}{\sqrt{2}}g_L k_u & \frac{1}{\sqrt{2}}g_L k_d \\ 0 & \frac{4M_R g_V^2 + M_V g_R^2}{g_1^2} & \frac{\sqrt{2}g_R g_V k_u}{g_1} & -\frac{\sqrt{2}g_R g_V k_d}{g_1} \\ -\frac{1}{\sqrt{2}}g_L k_u & \frac{\sqrt{2}g_R g_V k_u}{g_1} & 0 & -2\mu \\ \frac{1}{\sqrt{2}}g_L k_d & -\frac{\sqrt{2}g_R g_V k_d}{g_1} & -2\mu & 0 \end{pmatrix}, \tag{2.14}$$

and the two-component fermion field is

$$(\xi^0)^T = (-i\lambda_L^0, -i\lambda_B^0, \tilde{\phi}_{1u}^0, \tilde{\phi}_{2d}^0). \tag{2.15}$$

III. THE NEUTRALINO MASSES AND THE DIAGONALIZING MATRIX IN THE LEFT-RIGHT SUPERSYMMETRIC MODEL

The two-component light neutralino mass eigenstates χ_j^0 are related to the two-component fermion fields given in Eq. (2.15) as

$$\xi_k^0 = \sum_{l=1}^{4} V_{kl}\chi_l^0, \qquad k = 1, \ldots, 4, \tag{3.1}$$

where V is a unitary matrix satisfying

$$N_D = V^T N V, \equiv \sum_{j=1}^{4} m_{\tilde{\chi}_j^0} E_j, \tag{3.2}$$

and

$$N_D^2 = V^{-1}N^\dagger N V, \tag{3.3}$$

$$\equiv \sum_{j=1}^{4} m_{\tilde{\chi}_j^0}^2 E_j, \tag{3.4}$$

where $(E_j)_{4\times 4}$ are the basic matrices defined by $(E_j)_{ik} = \delta_{ji}\delta_{jk}$, and $\tilde{\chi}_j^0$ stand for the four component Majorana neutralinos:

$$\tilde{\chi}_j^0 = \begin{pmatrix} \chi_j^0 \\ \bar{\chi}_j^0 \end{pmatrix}, \qquad j = 1, \ldots, 4. \tag{3.5}$$

Here, we suppose that the real eigenvalues of N_D are ordered in the following way:

$$m_{\tilde{\chi}_1^0} \leq m_{\tilde{\chi}_2^0} \leq m_{\tilde{\chi}_3^0} \leq m_{\tilde{\chi}_4^0}. \tag{3.6}$$

A. Exact analytical expressions for the neutralino masses

As we have seen in the above section, in the left-right supersymmetric model, the masses, the mixing parameters, and the CP-violating properties of the neutralino are determined by the fundamental complex $M_L = |M_L|e^{i\Phi_L}$ and $\mu = |\mu|e^{i\Phi_\mu}$ and real $\tan\theta_k = k_u/k_d$, M_R, and M_V parameters. To know the neutralino masses predicted by the present model, we can solve the characteristic equation associated with the Hermitian matrix $H \equiv N^\dagger N$. More precisely, the square root of the positive roots of this characteristic equation corresponds to the physical neutralino masses. The neutralino masses predicted by the present model are known only for the CP-conserving case under the limit of large $M_{L,R}$ or large $|\mu|$ [13], more precisely on the assumptions that $|M_{L,R} \pm \mu| \gg M_{Z_L}$, $M_R > M_V$, and $4g_V^2 M_R + g_R^2 M_V/g_1^2 \simeq 4g_V^2 M_R/g_1^2$. Indeed, a numerical analysis has been implemented to solve the mentioned characteristic equation [13], assuming determined values for the gauge boson masses, couplings constants and taking μ, the Higgsino mass parameter, as a free quantity. Here we put into practice a method [1,2] giving exact analytic expressions for the neutralino masses.

Starting from Eq. (3.3), we get

$$(N^\dagger N)V - VN_D^2 = 0. \tag{3.7}$$

A more explicit form of this matrix equation is

$$\begin{aligned} &(H_{11} - m_{\tilde{\chi}_j^0}^2)V_{1j} + H_{12}V_{2j} + H_{13}V_{3j} + H_{14}V_{4j} = 0, \\ &H_{21}V_{1j} + (H_{22} - m_{\tilde{\chi}_j^0}^2)V_{2j} + H_{23}V_{3j} + H_{24}V_{4j} = 0, \\ &H_{31}V_{1j} + H_{32}V_{2j} + (H_{33} - m_{\tilde{\chi}_j^0}^2)V_{3j} + H_{34}V_{4j} = 0, \\ &H_{41}V_{1j} + H_{42}V_{2j} + H_{43}V_{3j} + (H_{44} - m_{\tilde{\chi}_j^0}^2)V_{4j} = 0, \end{aligned} \tag{3.8}$$

$j = 1, \ldots, 4$, where $H_{ij} = \sum_{k=1}^4 N_{ki}^* N_{kj}$:

$$\begin{aligned} &H_{11} = M^2 + |M_L|^2, \qquad H_{22} = 4\kappa^2 M^2 + M_{RV}^2, \\ &H_{33} = 4|\mu|^2 + (1 + 4\kappa^2)M^2\sin^2\theta_k, \\ &H_{44} = 4|\mu|^2 + (1 + 4\kappa^2)M^2\cos^2\theta_k, \\ &H_{12} = H_{21}^* = -2\kappa M^2, \\ &H_{13} = H_{31}^* = -M(2|\mu|e^{i\Phi_\mu}\cos\theta_k + |M_L|e^{-i\Phi_L}\sin\theta_k), \\ &H_{14} = H_{41}^* = M(2|\mu|e^{i\Phi_\mu}\sin\theta_k + |M_L|e^{-i\Phi_L}\cos\theta_k), \\ &H_{23} = H_{32}^* = 2\kappa M(2|\mu|e^{i\Phi_\mu}\cos\theta_k + M_{RV}\sin\theta_k), \\ &H_{24} = H_{42}^* = -2\kappa M(2|\mu|e^{i\Phi_\mu}\sin\theta_k + M_{RV}\cos\theta_k), \\ &H_{34} = H_{43}^* = -\tfrac{1}{2}(1 + 4\kappa^2)M^2\sin(2\theta_k), \end{aligned}$$

where $M = g_L M_{Z_L}/\sqrt{g_L^2 + 4g'^2}$, $\kappa = g_R g_V/g_1 g_L$, and $M_{RV} = (4g_V^2 M_R + g_R^2 M_V)/g_1^2$.

For fixed j, Eq. (3.8) represents a system of homogeneous linear equations depending on only one of the neutralino masses. Thus, the neutralino masses can be determined by solving the characteristic equation associated with this system; that is

$$X^4 - aX^3 + bX^2 - cX + d = 0, \tag{3.9}$$

where

$$a = |M_L|^2 + 8|\mu|^2 + M_{RV}^2 + 2(1 + 4\kappa^2)M^2,$$

$$b = (4|\mu|^2 + (1 + 4\kappa^2)M^2)^2 + M_{RV}^2(|M_L|^2 + 8|\mu|^2 + 2M^2) + 8|M_L|^2(|\mu|^2 + \kappa^2 M^2) - 16\kappa^2 M^2|\mu|M_{RV}\sin(2\theta_k)\cos\Phi_\mu - 4M^2|\mu||M_L|\sin(2\theta_k)\cos(\Phi_\mu + \Phi_L),$$

$$c = 16|\mu|^4|M_L|^2 + 4(1 + 4\kappa^2)^2M^4|\mu|^2\sin^2(2\theta_k) + 16\kappa^2M^2|M_L|^2(2|\mu|^2 + \kappa^2M^2) + M_{RV}^2(M^4 + 8|\mu|^2(M^2 + |M_L|^2) + 16|\mu|^4) - 4M^2|\mu||M_L|(4|\mu|^2 + M_{RV}^2)\cos(\Phi_L + \Phi_\mu)\sin(2\theta_k) + 8\kappa^2M^2M_{RV}[M^2|M_L|\cos\Phi_L - 2|\mu|(4|\mu|^2 + |M_L|^2)\cos\Phi_\mu\sin(2\theta_k)],$$

and

$$d = 64\kappa^4M^4|\mu|^2|M_L|^2\sin^2(2\theta_k) + 32\kappa^2M^2|\mu|^2|M_L|M_{RV}\sin(2\theta_k)(M^2\cos\Phi_L\sin(2\theta_k) - 2|\mu||M_L|\cos\Phi_\mu) + 4M^2|\mu|^2M_{RV}^2\sin(2\theta_k)(M^2\sin(2\theta_k) - 4|\mu||M_L|\cos(\Phi_L + \Phi_\mu)) + 16|\mu|^4|M_L|^2M_{RV}^2.$$

Solving Eq. (3.9), we get the exact analytic formulas for the neutralino masses

$$m_{\tilde{\chi}_1^0}^2, m_{\tilde{\chi}_2^0}^2 = \frac{a}{4} - \frac{\alpha}{2} \mp \frac{1}{2}\sqrt{\beta - \varpi - \frac{\lambda}{4\alpha}}, \tag{3.10}$$

$$m_{\tilde{\chi}_3^0}^2, m_{\tilde{\chi}_4^0}^2 = \frac{a}{4} + \frac{\alpha}{2} \mp \frac{1}{2}\sqrt{\beta - \varpi + \frac{\lambda}{4\alpha}}, \tag{3.11}$$

where

$$\alpha = \sqrt{\frac{\beta}{2} + \varpi}, \qquad \varpi = \frac{\epsilon}{32^{1/3}} + \frac{(2^{1/3}\gamma)}{3\epsilon},$$

$$\epsilon = (\delta + \sqrt{\delta^2 - 4\gamma^3})^{1/3}, \qquad \beta = \frac{a^2}{2} - \frac{4b}{3},$$

$$\lambda = a^3 - 4ab + 8c, \qquad \gamma = b^2 - 3ac + 12d,$$

$$\delta = 2b^3 - 9abc + 27c^2 + 27a^2d - 72bd.$$

B. Neutralino masses, numerical results

Let us consider the *CP*-conserving scenarios Sc_1 and Sc_2 described in Table I. These scenarios are similar to the ones studied in Ref. [13] where they have been used to compare the predicted results for the neutralino masses in the left-right SUSY model and the MSSM. Thus, for both scenarios, we consider the coupling constant values $g_R \approx g_L \approx g_V = 0.65$, the gaugino parameters $M_L \gg M_V \approx 0.0$ GeV, and the mixing phases $\Phi_L = \Phi_\mu = 0$. Figures 1 and 2 show the behavior of the physical neutralino masses $m_{\tilde{\chi}_i^0}$, $i = 1, \ldots, 3$, versus μ, computed from Eq. (3.11), for the inputs of scenarios Sc_1 and Sc_2 with $\tan\theta_k = 1.6$, respectively. Notice that the values of the neutralino mass $m_{\tilde{\chi}_4^0}$ are so big that they cannot be seen. Both figures accurately reproduce the results of Ref. [13]. We observe the correct size ordering of the neutralino masses, such as required by Eq. (3.6). Also, in both scenarios, we find that for values of $|\mu| \sim 200$ GeV, the neutralino masses $m_{\tilde{\chi}_1^0}$ are approximately M_L and for large values of $|\mu|$, the masses of the neutralinos $m_{\tilde{\chi}_i^0}$, $i = 3, 4$, are heavier than M_R. The same analysis is true in the case of scenarios Sc_1 and Sc_2, where $\tan\theta_k = 4.0$, as can be ob-

TABLE I. Input parameters for scenarios Sc_1 and Sc_2. All mass quantities are given in GeV.

Scenario	M_R	M_L	k_u	$\tan\theta_k$
Sc_1	300	50	92.75	1.6
				4.0
Sc_2	1000	250	92.75	1.6
				4.0

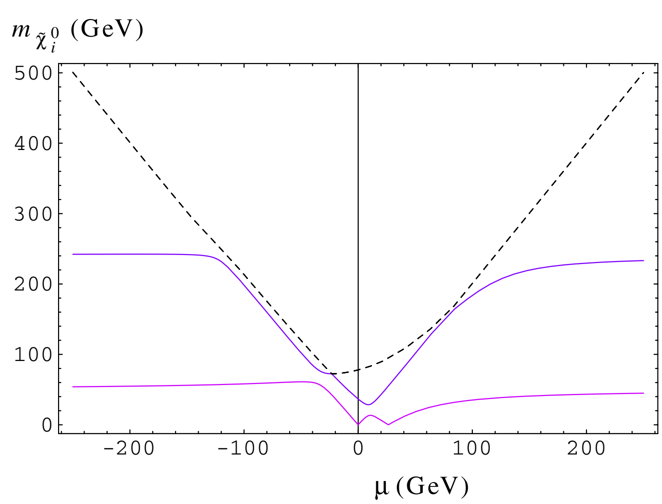

FIG. 1 (color online). Neutralino masses $m_{\tilde{\chi}_i^0}$, $i = 1, \ldots, 3$, as functions of μ for scenario Sc_1, assuming $\tan\theta_k = 1.6$.

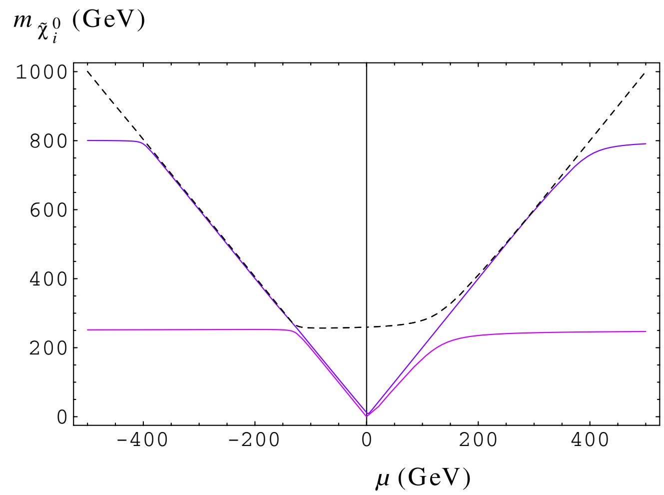

FIG. 2 (color online). Neutralino masses $m_{\tilde{\chi}_i^0}$, $i = 1, \ldots, 3$, as functions of μ for scenario Sc_2, assuming $\tan\theta_k = 1.6$.

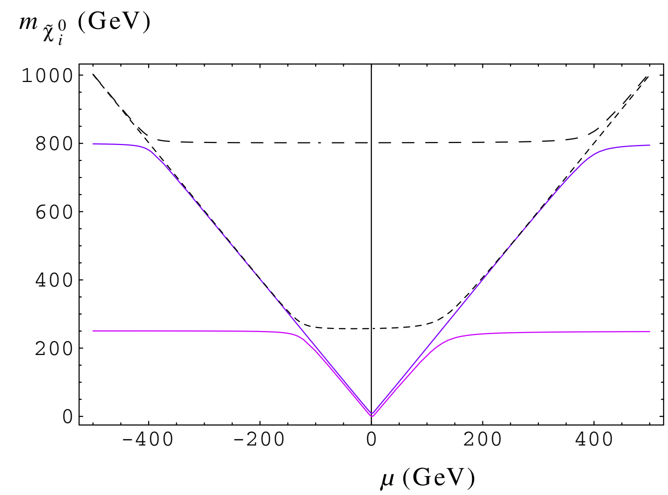

FIG. 4 (color online). Neutralino masses $m_{\tilde{\chi}_i^0}$, $i = 1, \ldots, 4$, as functions of μ for scenario Sc_2, assuming $\tan\theta_k = 4.0$.

served in Figs. 3 and 4. However, comparing Figs. 1 and 3, corresponding to scenarios Sc_1 with different values of $\tan\theta_k$, i.e., $\tan\theta_k = 1.6$ and $\tan\theta_k = 4.0$, respectively, we find that for small values of $|\mu|$, the variations of the neutralino masses with respect to μ in Fig. 3 are smoother than in Fig. 1. This is an important fact to consider when we study the inverse problem, that is, the determination of the fundamental parameters based on the knowledge of the physical neutralino masses.

Let us now study the behavior of the neutralino masses $m_{\tilde{\chi}_i^0}$, $i = 1, 2$, with respect to the variation of Φ_μ and Φ_L. Let us consider two possible CP-violating scenarios Snc_1 described in Table II, characterized by two different values of the Higgsino mass parameter, $|\mu| = 20$ GeV and $|\mu| = 248$ GeV. Figures 5 and 6 show the behavior of the neutralino masses $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$, respectively, as a function of Φ_μ and Φ_L for input parameters of scenario Snc_1 with $|\mu| = 20$ GeV. Comparing these figures we observe that the variation of the values of $m_{\tilde{\chi}_1^0}$ is bigger than the one of $m_{\tilde{\chi}_2^0}$. Superposing these figures, the corresponding surfaces do not overlap, that is, the size ordering [see Eq. (3.6)] of the masses $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$ is conserved even if in the CP-violating case. Figures 7 and 8 show the behavior of the neutralino masses $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$, respectively, as a function of Φ_μ and Φ_L for input parameters of scenario

TABLE II. Input parameters for scenario Snc_1. All mass quantities are given in GeV.

Scenario	$\|\mu\|$	M_R	M_L	k_u	$\tan\theta_k$
	20				
Snc_1		300	50	92.75	4.0
	248				

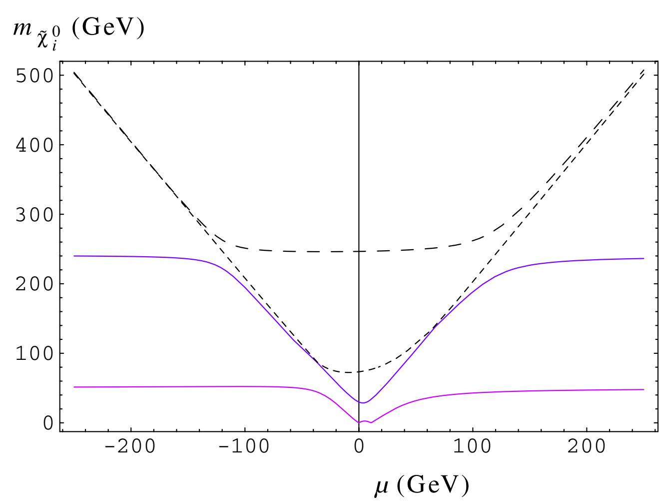

FIG. 3 (color online). Neutralino masses $m_{\tilde{\chi}_i^0}$, $i = 1, \ldots, 4$, as functions of μ for scenario Sc_1, assuming $\tan\theta_k = 4.0$.

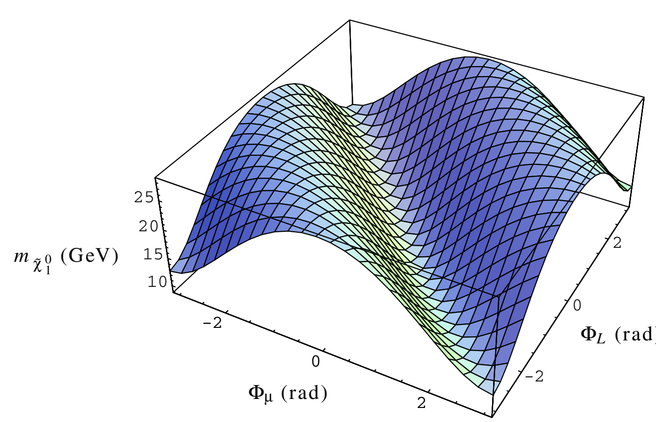

FIG. 5 (color online). Neutralino mass $m_{\tilde{\chi}_1^0}$, as a function of Φ_μ and Φ_L for input parameters according to scenario Snc_1 with $|\mu| = 20$ GeV.

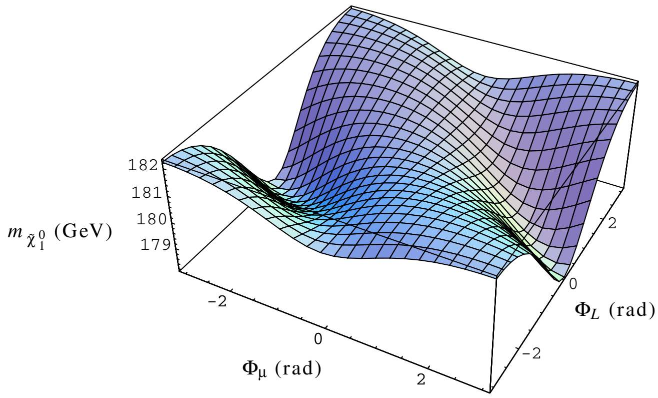

FIG. 6 (color online). Neutralino mass $m_{\tilde{\chi}_2^0}$, as a function of Φ_μ and Φ_L for input parameters according to scenario Snc_1 with $|\mu| = 20$ GeV.

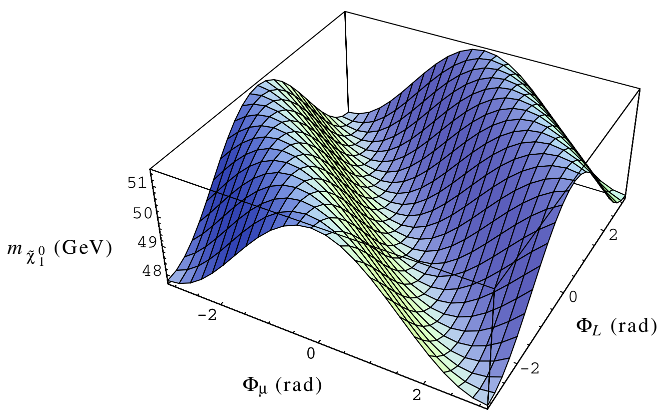

FIG. 7 (color online). Neutralino mass $m_{\tilde{\chi}_1^0}$, as a function of Φ_μ and Φ_L for input parameters according to scenario Snc_1 with $|\mu| = 248$ GeV.

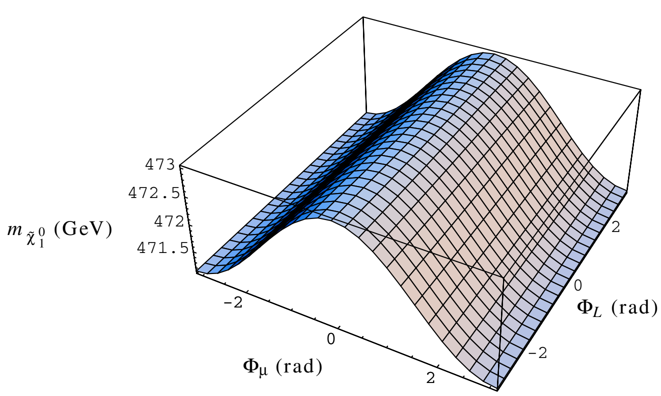

FIG. 8 (color online). Neutralino mass $m_{\tilde{\chi}_2^0}$, as a function of Φ_μ and Φ_L for input parameters according to scenario Snc_1 with $|\mu| = 248$ GeV.

Snc_1 with $|\mu| = 248$ GeV. The same considerations as in the previous analysis done for Figs. 5 and 6 are valid in this case. However, we observe that the energy gap between the surfaces in Figs. 7 and 8 is greater than in the case of the surfaces of Figs. 5 and 6.

C. The eigenvectors forming the matrix V

We have found it useful to finish this section with the computation of the matrix V. A more explicit form of this matrix will allow us to prove some important relations in the next section.

The diagonalizing matrix V can be obtained by computing the eigenvectors corresponding to the eigenvalues given in Eq. (3.11). Indeed, by inserting a generic eigenvalue $m_{\tilde{\chi}_j^0}$, into Eq. (3.8) and dividing each one of these equations by V_{1j}, where it is assumed that $V_{1j} \neq 0$, we get

$$\begin{aligned} &H_{12}\frac{V_{2j}}{V_{1j}} + H_{13}\frac{V_{3j}}{V_{1j}} + H_{14}\frac{V_{4j}}{V_{1j}} - m^2_{\tilde{\chi}_j^0} = -H_{11}, \\ &(H_{22} - m^2_{\tilde{\chi}_j^0})\frac{V_{2j}}{V_{1j}} + H_{23}\frac{V_{3j}}{V_{1j}} + H_{24}\frac{V_{4j}}{V_{1j}} = -H_{21}, \\ &H_{32}\frac{V_{2j}}{V_{1j}} + (H_{33} - m^2_{\tilde{\chi}_j^0})\frac{V_{3j}}{V_{1j}} + H_{34}\frac{V_{4j}}{V_{1j}} = -H_{31}, \\ &H_{42}\frac{V_{2j}}{V_{1j}} + H_{43}\frac{V_{3j}}{V_{1j}} + (H_{44} - m^2_{\tilde{\chi}_j^0})\frac{V_{4j}}{V_{1j}} = -H_{41}. \end{aligned} \tag{3.12}$$

Solving this system of equations, and taking into account the relation

$$|V_{1j}|^2 + |V_{2j}|^2 + |V_{3j}|^2 + |V_{4j}|^2 = 1, \tag{3.13}$$

it yields the V_{ij} matrix's component

$$V_{ij} = \frac{\Delta_{ij}}{\Delta_{1j}} \frac{|\Delta_{1j}|e^{i\theta_j}}{\sqrt{|\Delta_{1j}|^2 + |\Delta_{2j}|^2| + |\Delta_{3j}|^2| + |\Delta_{4j}|^2}}, \tag{3.14}$$

when $i = 1, \ldots, 4$. Here, the θ_j's are arbitrary phases, related to the CP eigenphases, which will be fixed by the requirement that V satisfies Eq. (3.2), as we will see in the next section,

$$\Delta_{1j} = \begin{vmatrix} H_{22} - m^2_{\tilde{\chi}_j^0} & H_{23} & H_{24} \\ H_{32} & H_{33} - m^2_{\tilde{\chi}_j^0} & H_{34} \\ H_{42} & H_{43} & H_{44} - m^2_{\tilde{\chi}_j^0} \end{vmatrix}, \tag{3.15}$$

and Δ_{ij}, $i = 2, 3, 4$, is formed from Δ_{1j} by substituting the $(i-1)$th column by $(-H_{21}, -H_{31}, -H_{41})$.

IV. THE NEUTRALINO PROJECTORS, PSEUDOPROJECTORS, AND CP EIGENPHASES

To describe the neutralino observables we can use the projector formalism [1,2]. The neutralino projector matrices can be defined as [14]

$$P_j = P_j^\dagger = V E_j V^{-1}, \tag{4.1}$$

so that

$$P_{j\alpha\beta} = V_{\alpha j} V^*_{\beta j}. \tag{4.2}$$

These projectors satisfy the relations

$$P_i P_j = P_j \delta_{ij}, \qquad \mathrm{Tr} P_j = 1, \qquad \sum_{j=1}^{4} P_j = 1, \tag{4.3}$$

where $(i, j) = (1\text{–}4)$ describe the neutralino mass-eigenstate indices. Notice that from Eqs. (3.3) and (4.1) it is possible to write

$$N^\dagger N = \sum_{j=1}^{4} m^2_{\tilde{\chi}^0_j} P_j. \tag{4.4}$$

As in the case of the study of the neutralino projector formalism for complex supersymmetry parameters based on the MSSM [2], here only the projectors are not sufficient to describe the physical observables. For a complete description of physical observables it is also necessary to know the so-called pseudoprojector matrices and CP eigenphases. In the following we implement a method based on the explicit knowledge of the diagonalizing matrix V to obtain these quantities and demonstrate some of their properties.

A. Reduced projectors

By inserting (3.14) into (4.2), we get

$$P_{j\alpha\beta} = \frac{p_{j\alpha} p^*_{j\beta}}{1 + |p_{j2}|^2 + |p_{j3}|^2 + |p_{j4}|^2}, \tag{4.5}$$

where we define the reduced projectors

$$p_{j\alpha} \equiv \frac{\Delta^*_{\alpha j}}{\Delta^*_{1j}}. \tag{4.6}$$

Notice that the expression given in (4.6) is a new version of the reduced projector formula [2]. Indeed, from this last equation, it is clear that $p_{j1} = 1$. Moreover, from Eq. (4.5) we deduce

$$P_{j11} = \frac{1}{1 + |p_{j2}|^2 + |p_{j3}|^2 + |p_{j4}|^2}. \tag{4.7}$$

Thus, inserting this last result into (4.5), we prove the ansatz used in [2]

$$P_{j\alpha\beta} = P_{j11} p^*_{j\alpha} p_{j\beta}. \tag{4.8}$$

On the other hand, using Eqs. (4.6) and (4.7), we can write the matrix elements of the diagonalizing matrix V given in Eq. (3.14) in terms of the reduced projectors; that is

$$V_{\alpha j} = \sqrt{\frac{P_{j11}}{\eta_j}} p^*_{j\alpha}, \tag{4.9}$$

where $\eta_j \equiv e^{-2i\theta_j}$ stands for the CP eigenphases. As we will see below, this last equation allows us to express the LR SUSY parameters in terms of the reduced projectors and the eigenphases.

A useful property verified by the reduced projectors $p_{j\alpha}$ is

$$P_{j11} \sum_{\beta=1}^{4} p_{i\beta} p^*_{j\beta} = \delta_{ij}, \tag{4.10}$$

which can be directly deduced from Eq. (4.9), taking into account the unitarity of V.

Let us now define other important matrices. From Eq. (3.2), we can write

$$N = \sum_{j=1}^{4} m_{\tilde{\chi}^0_j} V^* E_j V^\dagger = \sum_{j=1}^{4} m_{\tilde{\chi}^0_j} \bar{P}_j, \tag{4.11}$$

where

$$\bar{P}_j \equiv V^* E_j V^\dagger \tag{4.12}$$

are the so-called pseudoprojectors [2]. Using Eq. (4.9) and the definition of E_j, the matrix elements of these pseudoprojectors can easily be written in the form

$$\bar{P}_{j\alpha\beta} = V^*_{\alpha j} V^*_{\beta j} = P_{j11} p_{j\alpha} p_{j\beta} \eta_j. \tag{4.13}$$

From this last equation it is clear that $\bar{P}_j$ is a symmetric matrix, that is $\bar{P}^T_j = \bar{P}_j$.

Using Eq. (4.13), taking again into account the unitarity of V, and the definition (4.2), we have

$$(\bar{P}^*_j \bar{P}_k)_{\alpha\beta} = \sum_{\rho=1}^{4} \bar{P}^*_{j\alpha\rho} \bar{P}_{k\rho\beta} = \sum_{\rho=1}^{4} V_{\alpha j} V_{\rho j} V^*_{\rho k} V^*_{\beta k} = \delta_{jk} V_{\alpha j} V^*_{\beta j} = \delta_{jk} P_{j\alpha\beta}, \tag{4.14}$$

that is, the pseudoprojectors satisfy

$$\bar{P}^*_j \bar{P}_k = \delta_{jk} P_j. \tag{4.15}$$

In the same way, we can show that

$$\bar{P}^*_j P_k = P^t_k \bar{P}_j = \delta_{jk} \bar{P}_j. \tag{4.16}$$

As we have mentioned in the previous section, the eigenphases η_j must be chosen in such a way that the diagonalizing matrix V satisfies Eq. (3.2) or equivalently Eq. (4.11). Inserting Eq. (4.13) into Eq. (4.11) and using the property Eq. (4.10) we get

$$\eta_j m_{\tilde{\chi}^0_j} = \sum_{\alpha=1}^{4} N_{\alpha\beta} \frac{p^*_{j\alpha}}{p_{j\beta}} = \sum_{\alpha=1}^{4} N_{\alpha\beta} \frac{\Delta_{\alpha j}}{\Delta^*_{\beta j}}. \tag{4.17}$$

This last equation represents, for fixed j, four equivalent relations serving to determine the fundamental parameters of the model, namely, M_L, μ, M_R, M_V, and $\tan\theta_k$, in terms of the reduced projectors, the physical neutralino masses, the eigenphases, and the LR SUSY coupling constants. We notice that, starting from Eq. (3.2) and using Eq. (4.9), a more symmetric structure for the eigenphases η_j can be reached; that is

$$\eta_j m_{\tilde{\chi}_j^0} = P_{j11} \sum_{\alpha,\beta=1}^{4} p_{j\alpha}^* N_{\alpha\beta} p_{j\beta}^*. \tag{4.18}$$

This relation can also be constructed directly from the more fundamental Eq. (4.17), by means of property Eq. (4.10).

B. Explicit form of the reduced projectors

According to Eq. (4.6), to obtain the explicit form of the reduced projectors in terms of the fundamental parameters of the theory only we need to know the explicit form of quantities $\Delta_{\alpha j}^*$. For fixed j, they are given by

$$\begin{aligned}\Delta_{1j}^* = &-4\kappa^2(1+4\kappa^2)M^4(m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2\sin^2(2\theta_k)) + M^2(m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2)[m_{\tilde{\chi}_j^0}^2 - M_{RV}^2 + 8\kappa^2 m_{\tilde{\chi}_j^0}^2 \\ &+ 16\kappa^2 M_{RV}|\mu|\cos\Phi_\mu \sin(2\theta_k)] - (m_{\tilde{\chi}_j^0}^2 - M_{RV}^2)(m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2)^2,\end{aligned} \tag{4.19}$$

$$\begin{aligned}\Delta_{2j}^* = &-2\kappa(1+4\kappa^2)M^4[m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2\sin^2(2\theta_k)] + 2\kappa M^2[m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2]\{m_{\tilde{\chi}_j^0}^2 + M_{RV}|M_L|e^{-i\Phi_L} + 2|\mu|\sin(2\theta_k) \\ &\times[|M_L|e^{-i(\Phi_L+\Phi_\mu)} + M_{RV}e^{i\Phi_\mu}]\},\end{aligned} \tag{4.20}$$

$$\begin{aligned}\Delta_{3j}^* = &2M^3\{|\mu|\cos\theta_k\cos(2\theta_k)[4\kappa^2 M_{RV}|M_L|e^{i(\Phi_\mu-\Phi_L)} - (m_{\tilde{\chi}_j^0}^2(1+4\kappa^2) - M_{RV}^2)e^{i\Phi_\mu}] \\ &+ 2\kappa^2\sin\theta_k(m_{\tilde{\chi}_j^0}^2 - 8|\mu|^2\cos^2\theta_k)(M_{RV} - |M_L|e^{-i\Phi_L})\} + M(m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2)(m_{\tilde{\chi}_j^0}^2 - M_{RV}^2)(|M_L|e^{-i\Phi_L}\sin\theta_k \\ &+ 2|\mu|e^{i\Phi_\mu}\cos\theta_k),\end{aligned} \tag{4.21}$$

and

$$\begin{aligned}\Delta_{4j}^* = &2M^3\{|\mu|\sin\theta_k\cos(2\theta_k)[4\kappa^2 M_{RV}|M_L|e^{i(\Phi_\mu-\Phi_L)} - (m_{\tilde{\chi}_j^0}^2(1+4\kappa^2) - M_{RV}^2)e^{i\Phi_\mu}] \\ &- 2\kappa^2\cos\theta_k(m_{\tilde{\chi}_j^0}^2 - 8|\mu|^2\sin^2\theta_k)(M_{RV} - |M_L|e^{-i\Phi_L})\} - M(m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2)(m_{\tilde{\chi}_j^0}^2 - M_{RV}^2)(|M_L|e^{-i\Phi_L}\cos\theta_k \\ &+ 2|\mu|e^{i\Phi_\mu}\sin\theta_k).\end{aligned} \tag{4.22}$$

The formulas (4.19), (4.20), (4.21), and (4.22) allow us to express, through the reduced projectors, all the essential quantities of the model in terms of the original parameters.

C. Consistence with the Jarlskog's formula

Using the projector properties (4.3) and some associated with the coefficients of the characteristic polynomial (3.9), we can write the projectors P_j in terms of the neutralino masses and the H matrix, in the Jarlskog's form [8]:

$$\begin{aligned}P_1 &= \frac{(m_{\tilde{\chi}_4^0}^2 - H)(m_{\tilde{\chi}_3^0}^2 - H)(m_{\tilde{\chi}_2^0}^2 - H)}{(m_{\tilde{\chi}_4^0}^2 - m_{\tilde{\chi}_1^0}^2)(m_{\tilde{\chi}_3^0}^2 - m_{\tilde{\chi}_1^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2)}, \\ P_2 &= \frac{(m_{\tilde{\chi}_1^0}^2 - H)(m_{\tilde{\chi}_4^0}^2 - H)(m_{\tilde{\chi}_3^0}^2 - H)}{(m_{\tilde{\chi}_1^0}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_4^0}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_3^0}^2 - m_{\tilde{\chi}_2^0}^2)}, \\ P_3 &= \frac{(m_{\tilde{\chi}_2^0}^2 - H)(m_{\tilde{\chi}_1^0}^2 - H)(m_{\tilde{\chi}_4^0}^2 - H)}{(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_3^0}^2)(m_{\tilde{\chi}_1^0}^2 - m_{\tilde{\chi}_3^0}^2)(m_{\tilde{\chi}_4^0}^2 - m_{\tilde{\chi}_3^0}^2)}, \\ P_4 &= \frac{(m_{\tilde{\chi}_3^0}^2 - H)(m_{\tilde{\chi}_2^0}^2 - H)(m_{\tilde{\chi}_1^0}^2 - H)}{(m_{\tilde{\chi}_3^0}^2 - m_{\tilde{\chi}_4^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_4^0}^2)(m_{\tilde{\chi}_1^0}^2 - m_{\tilde{\chi}_4^0}^2)}.\end{aligned} \tag{4.23}$$

A more useful expression for these projectors is obtained if we define

$$P_j = \frac{\tilde{P}_j}{\tilde{\Delta}_j}, \tag{4.24}$$

where

$$\tilde{\Delta}_j = -3m_{\tilde{\chi}_j^0}^8 + 2am_{\tilde{\chi}_j^0}^6 - bm_{\tilde{\chi}_j^0}^4 + d. \tag{4.25}$$

Indeed, by performing some algebraic manipulations we get

$$\begin{aligned}\tilde{P}_{j\alpha\beta} = &-m_{\tilde{\chi}_j^0}^6 H_{\alpha\beta} + m_{\tilde{\chi}_j^0}^4(aH_{\alpha\beta} - H_{\alpha\beta}^2) \\ &+ m_{\tilde{\chi}_j^0}^2(aH_{\alpha\beta}^2 - bH_{\alpha\beta} - H_{\alpha\beta}^3) + d\delta_{\alpha\beta}.\end{aligned} \tag{4.26}$$

Now, combining Eqs. (4.8) and (4.24), we deduce the expression

$$p_{j\alpha} = \frac{P_{j1\alpha}}{P_{j11}} = \frac{\tilde{P}_{j1\alpha}}{\tilde{P}_{j11}}, \tag{4.27}$$

which can also be considered as a definition for the reduced projectors.

Equations (4.6) and (4.27) are equivalent expressions for the reduced projectors when we substitute into them the exact analytical masses $m_{\tilde{\chi}_j^0}$ given in (3.11). Thus, combining these equations and comparing the expressions (4.20), (4.21), and (4.22) with the corresponding $\tilde{P}_{j1\beta}$, $\beta = 2, 3, 4$, computed from Eq. (4.26), we can show that

$$\tilde{P}_{j1\alpha} = m^2_{\tilde{\chi}_j^0}\Delta^*_{\alpha j}, \quad \forall\ \alpha = 1, \ldots, 4, \tag{4.28}$$

with

$$\begin{aligned}\tilde{P}_{j11} = {} & M^4[m^2_{\tilde{\chi}_j^0} - 4|\mu|^2\sin^2(2\theta_k)][m^2_{\tilde{\chi}_j^0}(1 + 4\kappa^2) - 16\kappa^4|M_L|^2 - M^2_{RV} - 8\kappa^2|M_L|M_{RV}\cos\Phi_L] \\ & - M^2(m^2_{\tilde{\chi}_j^0} - 4|\mu|^2)\{2|\mu||M_L|\sin(2\theta_k)[2(m^2_{\tilde{\chi}_j^0} - M^2_{RV})\cos(\Phi_L + \Phi_\mu) - 8|M_L|M_{RV}\kappa^2\cos\Phi_\mu] \\ & + m^2_{\tilde{\chi}_j^0}[(m^2_{\tilde{\chi}_j^0} - M^2_{RV}) - 8\kappa^2|M_L|^2]\} - |M_L|^2(m^2_{\tilde{\chi}_j^0} - M^2_{RV})(m^2_{\tilde{\chi}_j^0} - 4|\mu|^2)^2.\end{aligned} \tag{4.29}$$

Indeed, Eqs. (4.28), for $\alpha = 1, 2, 3$, constitute an identity whereas for $\alpha = 1$ it constitutes a useful equivalence, as we will show in the next section.

V. GENERAL DISENTANGLED FORMULA OF M_L IN TERMS OF THE EIGENPHASES

From Eq. (4.17), choosing $\beta = 1$ and using (2.14), we get

$$\eta_j m_{\tilde{\chi}_j^0} = M_L - M\frac{\sin\theta_k\Delta_{3j} - \cos\theta_k\Delta_{4j}}{\Delta^*_{1j}}. \tag{5.1}$$

Inserting (4.21) and (4.22) into (5.1) and solving a linear algebraic equation for M_L, we get

$$\begin{aligned}M_L = {} & \frac{\Delta^*_{1j}m_{\tilde{\chi}_j^0}}{\mathcal{D}_j}\eta_j + \frac{2M^2}{\mathcal{D}_j}\{|\mu|e^{-i\Phi_\mu}\sin(2\theta_k)(m^2_{\tilde{\chi}_j^0} - M^2_{RV}) \\ & \times(m^2_{\tilde{\chi}_j^0} - 4|\mu|^2) \\ & + 2\kappa^2M^2M_{RV}[m^2_{\tilde{\chi}_j^0} - 4|\mu|^2\sin^2(2\theta_k)]\} \\ = {} & A_j\eta_j + B_j,\end{aligned} \tag{5.2}$$

where

$$\begin{aligned}A_j = {} & \frac{\Delta^*_{1j}m_{\tilde{\chi}_j^0}}{\mathcal{D}_j} = -\frac{m_{\tilde{\chi}_j^0}}{\mathcal{D}_j}\{(m^2_{\tilde{\chi}_j^0} - M^2_{RV})(m^2_{\tilde{\chi}_j^0} - 4|\mu|^2)^2 \\ & + 4\kappa^2(1 + 4\kappa^2)M^4[m^2_{\tilde{\chi}_j^0} - 4|\mu|^2\sin^2(2\theta_k)] \\ & - M^2(m^2_{\tilde{\chi}_j^0} - 4|\mu|^2)[8\kappa^2m^2_{\tilde{\chi}_j^0} + (m^2_{\tilde{\chi}_j^0} - M^2_{RV}) \\ & + 16\kappa^2|\mu|M_{RV}\cos\Phi_\mu\sin(2\theta_k)]\},\end{aligned} \tag{5.3}$$

$$\begin{aligned}B_j = {} & \frac{2M^2}{\mathcal{D}_j}\{|\mu|e^{-i\Phi_\mu}\sin(2\theta_k)(m^2_{\tilde{\chi}_j^0} - M^2_{RV})(m^2_{\tilde{\chi}_j^0} - 4|\mu|^2) \\ & + 2\kappa^2M^2M_{RV}[m^2_{\tilde{\chi}_j^0} - 4|\mu|^2\sin^2(2\theta_k)]\},\end{aligned} \tag{5.4}$$

and

$$\begin{aligned}\mathcal{D}_j = {} & -\{(m^2_{\tilde{\chi}_j^0} - M^2_{RV})(m^2_{\tilde{\chi}_j^0} - 4|\mu|^2)^2 \\ & - 8\kappa^2M^2(m^2_{\tilde{\chi}_j^0} - 4|\mu|^2)[m^2_{\tilde{\chi}_j^0} + 2|\mu|M_{RV} \\ & \times\cos\Phi_\mu\sin(2\theta_k)] \\ & + 16\kappa^4M^4[m^2_{\tilde{\chi}_j^0} - 4|\mu|^2\sin^2(2\theta_k)]\}.\end{aligned} \tag{5.5}$$

Equation (5.2) allows us to determinate the behavior of $|M_L|$ and Φ_L in terms of the eigenphases η_j and the physical masses $m_{\tilde{\chi}_j^0}$, when the rest of the fundamental parameters are fixed. We notice that this equation has been obtained without use of the Jarlskog's projector formula (4.23) or its equivalent (4.26). The method used to obtain it is direct and it is based essentially on the fact that Δ_{1j} is independent of $|M_L|$ and of Φ_L.

Figure 9 shows the behavior of $|M_L|$ as a function of the neutralino masses $m_{\tilde{\chi}_j^0}$, for input parameters of the

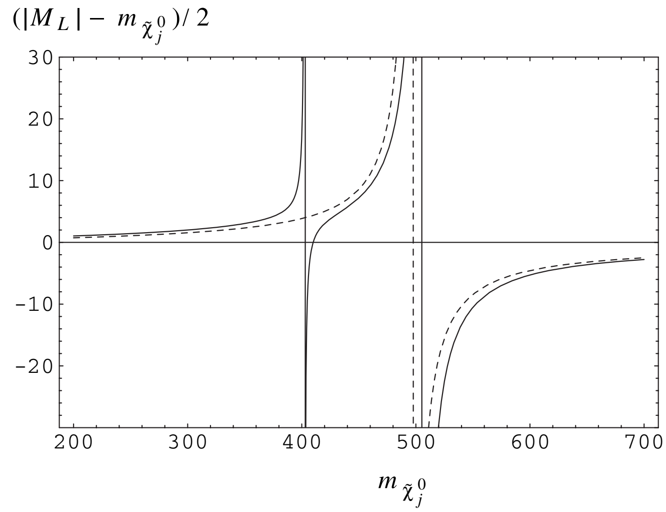

FIG. 9. Graph of $|M_L|$, using the general formula (5.2), for inputs of scenario Sc_{3a}, with $\eta_j = 1$ (solid lines) and $\eta_j = -1$ (dashed lines), as a function of the physical neutralino masses $m_{\tilde{\chi}_i^0}$.

TABLE III. Input parameters for scenarios $Sc_{3a}, \ldots, Sc_{3d}$. All mass quantities are in GeV.

Scenario	$\vert\mu\vert$	M_R	M_V	$\tan\theta_k$	η_j
Sc_{3a}	248	500	50	30	1
					-1
Sc_{3b}	248	500	500	30	1
					-1
Sc_{3c}	500	500	50	30	1
					-1
Sc_{3d}	248	500	50	10	1
				50	-1

CP-conserving scenario Sc_{3a}, given in Table III (as before, we assume $g_L = g_R = g_V = 0.65$, and $k_u = 92.75$). We observe that for small values of the neutralino masses, i.e., for masses of order 200 GeV approximately, the size of $|M_L|$ becomes the same in both cases, the scenario Sc_{3a} with $\eta_j = 1$ and the scenario Sc_{3a} with $\eta_j = -1$. Let us now consider the scenario Sc_{3b}, which is the same as the scenario Sc_{3a}, except for the value of M_V which has been increased from 50 to 500 GeV. In this case, the common value of $|M_L|$ in both scenarios, i.e., Sc_{3b} with $\eta = \pm 1$, is found to be $|M_L| \approx 300$ GeV, in the region of small physical neutralino masses of the order of 300 GeV, as we can see from Fig. 10. Figure 11 shows the behavior of $|M_L|$ as a function of the neutralino masses $m_{\tilde{\chi}_j^0}$, for input parameters of the *CP*-conserving scenario Sc_{3c}, given in Table III.

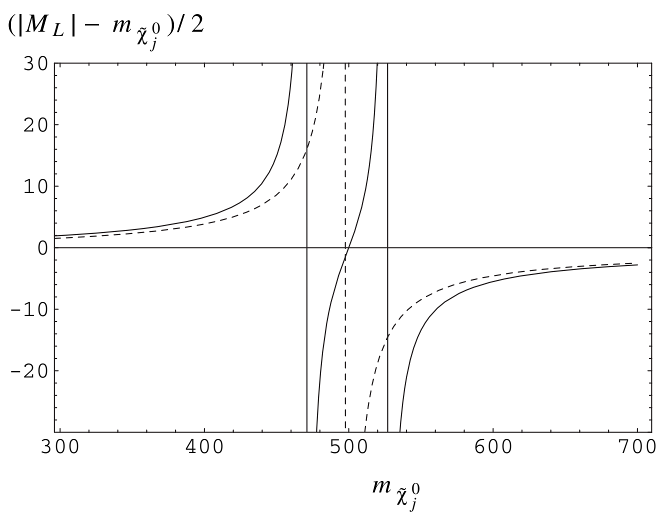

FIG. 10. Graph of $|M_L|$, using the general formula (5.2), for inputs of scenario Sc_{3b}, with $\eta_j = 1$ (solid lines) and $\eta_j = -1$ (dashed lines), as a function of the physical neutralino masses $m_{\tilde{\chi}_i^0}$.

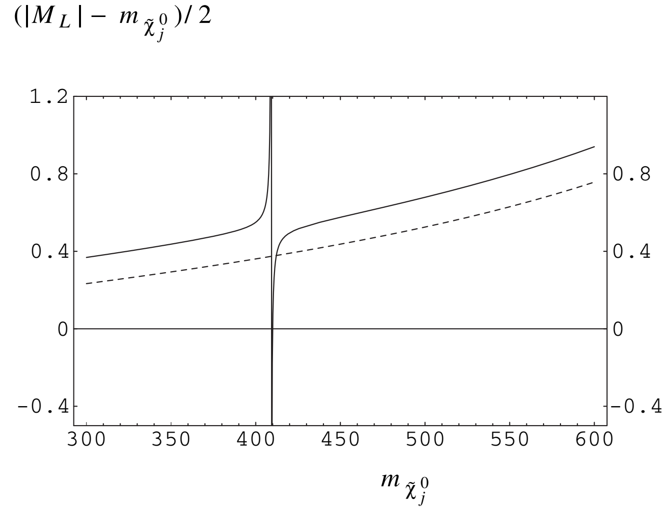

FIG. 11. Graph of $|M_L|$, using the general formula (5.2), for inputs of scenario Sc_{3c}, with $\eta_j = 1$ (solid lines) and $\eta_j = -1$ (dashed lines), as a function of the physical neutralino masses $m_{\tilde{\chi}_i^0}$.

This scenario differs from the scenario Sc_{3a} in the value of μ which has now been taken $|\mu| = 500$ GeV. We observe that the curves corresponding to the input parameters of scenario Sc_{3c} with different eigenphase values, i.e., $\eta_j = \pm 1$, intersect when $m_{\tilde{\chi}_j^0} \approx 412.17$ GeV, giving the common value of $|M_L| \approx 412.93$ GeV, which is bigger than the corresponding common values of $|M_L|$ computed in the previous scenarios. On the other hand, if we compare the curves representing the behavior of $|M_L|$ as a function of

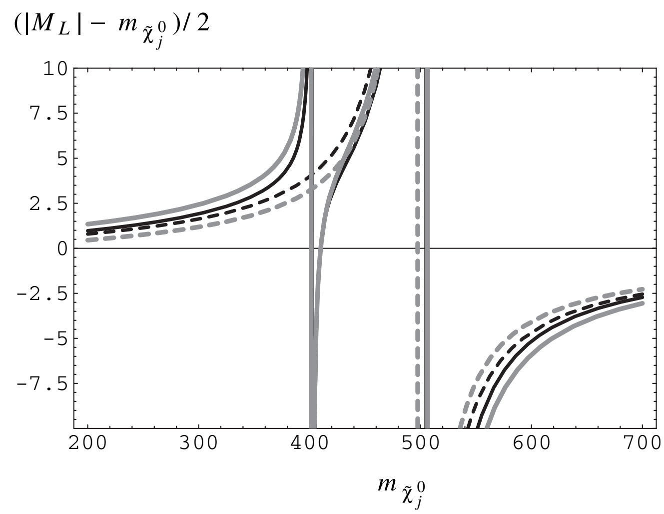

FIG. 12. Graph of $|M_L|$ as a function of the physical neutralino masses $m_{\tilde{\chi}_i^0}$, using the general formula (5.2), for inputs of scenario Sc_{3d}. The curves are $\tan\theta_k = 10$, $\eta_j = 1$ (light solid lines); $\tan\theta_k = 10$, $\eta_j = -1$ (light dashed lines); $\tan\theta_k = 50$, $\eta_j = 1$ (heavy solid lines) and $\tan\theta_k = 50$, $\eta_j = -1$ (heavy dashed lines).

the physical neutralino masses for different values of the parameter $\tan\theta_k$, we do not observe important differences between them when the values of this last parameter are chosen in the range 30–50. However, if we compare the mentioned curves for values of $\tan\theta_k$ chosen, for instance, in the ranges 10–30 or 10–50, we observe that for small neutralino masses $m_{\tilde{\chi}_i^0}$, the values of $|M_L|$ approach from the right to the given value of $m_{\tilde{\chi}_i^0}$, and this approach is more significant for big values of $\tan\theta_k$ than for small ones when $\eta_j = 1$ and vice versa. This approach is more significant for small values of $\tan\theta_k$ than for big ones when $\eta_j = -1$. This means that the value of the light neutralino mass which provides the value of $|M_L|$ which is independent of the eigenphases $\eta_j = \pm 1$ increases when $\tan\theta_k$ augments. The above mentioned behavior of the parameters is verified as seen in the plots of Fig. 12, where we plot $|M_L|$ versus $m_{\tilde{\chi}_i^0}$, for inputs of scenario Sc_{3d} with $\tan\theta_k = 10$ and $\tan\theta_k = 50$.

A. An alternative way to obtain $|M_L|$

When $\alpha = 1$, Eq. (4.28) combined with Eqs. (4.19) and (4.29), allows us to express the norm of M_L in terms of the rest of the fundamental parameters Φ_L, $|\mu|$, Φ_μ, $\tan\theta_k$, and M_{RV} and the physical masses $m_{\tilde{\chi}_j^0}$. Indeed, inserting (4.19) and (4.29) into (4.28) and solving a quadratic algebraic equation for $|M_L|$, we get

$$|M_L| = \frac{-\mathcal{B}_j \pm \sqrt{\mathcal{B}_j^2 - 4\mathcal{D}_j(C_j - m^2_{\tilde{\chi}_j^0}\Delta^*_{1j})}}{2\mathcal{D}_j}, \tag{5.6}$$

where

$$\begin{aligned}\mathcal{B}_j = &-4M^2\{|\mu|(m^2_{\tilde{\chi}_j^0} - M^2_{RV})(m^2_{\tilde{\chi}_j^0} - 4|\mu|^2) \\ &\times \cos(\Phi_L + \Phi_\mu)\sin(2\theta_k) \\ &+ 2\kappa^2 M^2 M_{RV}[m^2_{\tilde{\chi}_j^0} - 4|\mu|^2\sin^2(2\theta_k)]\cos\Phi_L\},\end{aligned} \tag{5.7}$$

$$\begin{aligned}C_j = &-M^2 m^2_{\tilde{\chi}_j^0}(m^2_{\tilde{\chi}_j^0} - M^2_{RV})(m^2_{\tilde{\chi}_j^0} - 4|\mu|^2) \\ &+ M^4[m^2_{\tilde{\chi}_j^0}(1+4\kappa^2) - M^2_{RV}][m^2_{\tilde{\chi}_j^0} - 4|\mu|^2\sin^2(2\theta_k)],\end{aligned} \tag{5.8}$$

and $\mathcal{D}_j$ is given in Eq. (5.5).

The formula for $|M_L|$ given in Eq. (5.6) constitutes an alternative to the one given in Eq. (5.2), serving to study the behavior of $|M_L|$ as a function of the phase Φ_L, the physical mass $m_{\tilde{\chi}_j^0}$, and the rest of the fundamental parameters. For instance, let us consider the possible CP-conserving scenario Sc_4 given in Table IV. In this case, the behavior of $|M_L|$ in terms of one of the physical masses $m_{\tilde{\chi}_j^0}$ is shown in Figs. 13 and 14. It is clear that superposing Figs. 13 and 14, we reconstruct Fig. 9. Comparing these figures, we also observe that, in the

TABLE IV. Input parameters for scenario Sc_4. All mass quantities are in GeV.

Scenario	$\lvert\mu\rvert$	M_R	M_V	$\tan\theta_k$	Φ_L
Sc_4	248	500	50	30	0
					π

CP-conserving case, when $\Phi_\mu = 0$, the eigenphase values $\eta_j = \pm 1$ correspond to the M_L phase values $\Phi_L = \pm\pi$, respectively. The same considerations are valid when we take $\Phi_\mu = \pi$. That is, in the CP-conserving case, when all the parameters but $m_{\tilde{\chi}_j^0}$ are fixed, the choice of the two different values $\Phi_L = 0, \pi$ in Eq. (5.6) corresponds to the

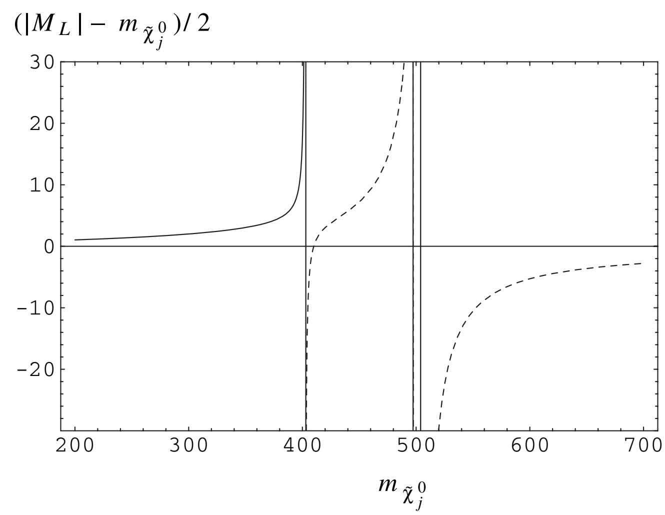

FIG. 13. Graph of $|M_L|$ as a function of the physical mass $m_{\tilde{\chi}_i^0}$ for input parameters of scenario Sc_4 with $\Phi_L = 0$. Here, according to Eq. (5.6), the graphs with the + and − signs are represented in solid and dashed lines, respectively.

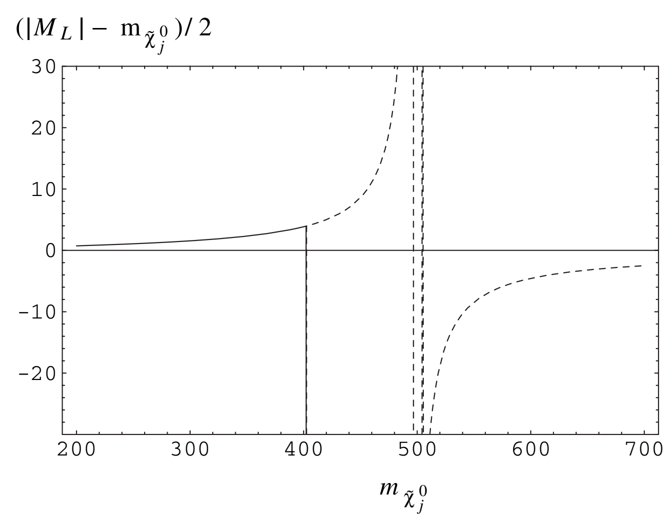

FIG. 14. The same as in Fig. 13 but with $\Phi_L = \pi$.

choice of the two possible values of the eigenphases $\eta_j = 1, -1$, in Eq. (5.2) .

VI. DETERMINING LR SUSY PARAMETERS

In this section we investigate the behavior of $|M_L|$ and Φ_L when the eigenphases η_j, $j = 1, 2$, change. We concentrate on two possible scenarios Snc_2 and Snc_3, described in Table V, for fixed input constants $g_L = g_R = g_V = 0.65$ and $k_u = 92.75$. Thus, in these cases we assume that the physical masses $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$ as well as μ and M_R are known. The M_V parameter would eventually be allowed to vary but in this case we assume that it has a fixed value in each one of the mentioned scenarios.

Figure 15 shows the behavior of the norm of M_L, calculated from Eq. (5.2), as a function of the eigenphase η_1, with input parameters of scenario Snc_2 when $m_{\tilde{\chi}_1^0} = 164.36$ GeV. This is a scenario, similar to the Sp1-type scenario used in [2] in the context of the MSSM, characterized by a big rate between k_u and k_d, i.e., $\tan\theta_k = 30$. For small values of Φ_μ, we observe small differences among the plots of $|M_L|$. For all the plots shown in this figure, the mean value of $|M_L|$ is 165.75 GeV approximately and the maximum amplitude difference of them is 0.6 GeV approximately.

Figures 16 and 17 show the dependence of the mixing phase Φ_L and the relative phase $\phi_L - \arg(\eta_1)$, respectively, calculated from Eq. (5.2), with respect to the eigenphase η_1, in the Snc_2 scenario with $m_{\tilde{\chi}_1^0} = 164.36$ GeV. We observe, for all the cases $\Phi_\mu = 0, \frac{\pi}{8}, \frac{\pi}{6}, \pi$, a linear dependence between Φ_L and $\arg(\eta_1)$. Thus, $\Phi_L \approx \arg(\eta_1)$ when $\arg(\eta_1) \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\Phi_L \approx \pi + \arg(\eta_1)$ when $\arg(\eta_1) \in (-\pi, -\frac{\pi}{2})$, and $\Phi_L \approx -\pi + \arg(\eta_1)$ when $\arg(\eta_1) \in (\frac{\pi}{2}, \pi)$.

TABLE V. Input parameters for scenarios Snc_2 and Snc_3. All mass quantities are in GeV and all angles are in radians.

Scenario	$\|\mu\|$	Φ_u	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	M_R	M_V	$\tan\theta_k$
		0					
		$\pi/8$					
Snc_2	248		164.36	241.94	300	20	30
		$\pi/6$					
		π					
		0					
		$\pi/8$					
Snc_3	150		156.24	236.79	300	50	4.0
		$\pi/6$					
		π					

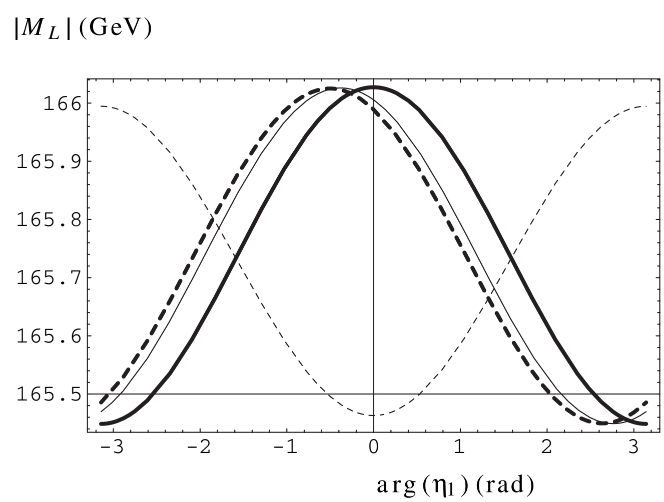

FIG. 15. Norm of M_L as a function of $\arg(\eta_1)$ according to scenario Snc_2, for $m_{\tilde{\chi}_1^0} = 164.36$ GeV, $\Phi_\mu = 0$ (heavy solid curve), $\pi/8$ (light solid curve), $\pi/6$, (heavy dashed curve), and π (light dashed curve).

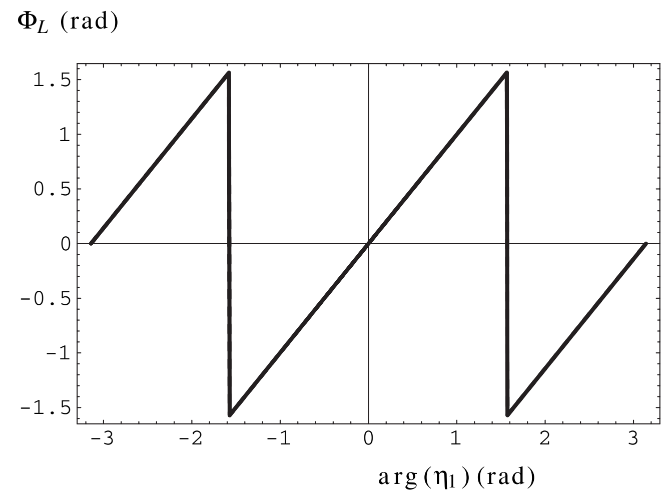

FIG. 16. Mixing parameter Φ_L as a function of $\arg(\eta_1)$ with the same set of fixed parameters used in Fig. 15.

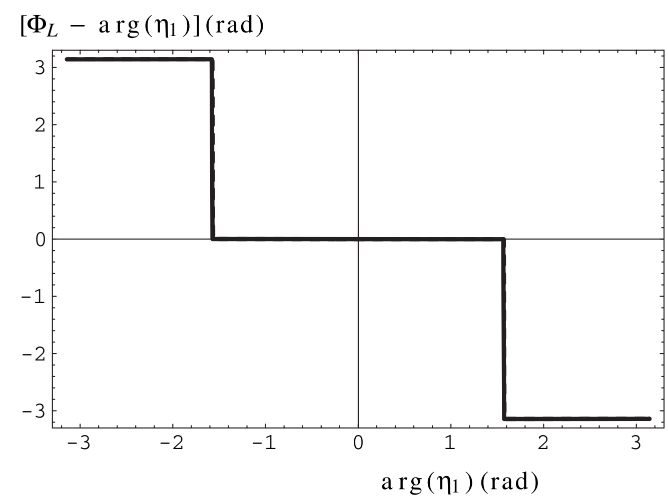

FIG. 17. Relative difference between Φ_L and $\arg(\eta_1)$ as a function of $\arg(\eta_2)$ as observed from Fig. 16.

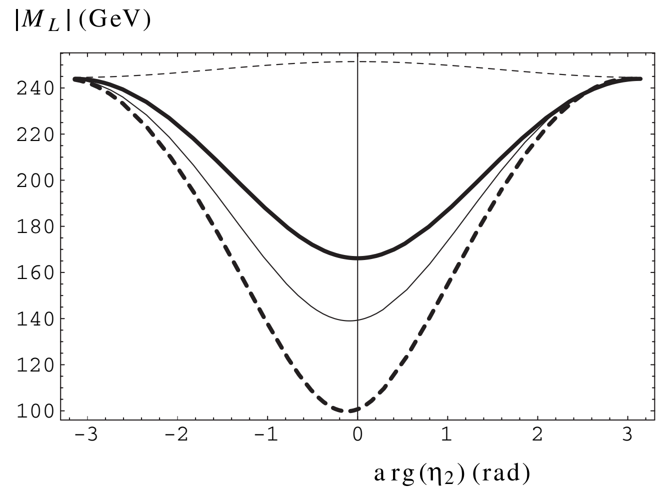

FIG. 18. Norm of $|M_L|$ as a function of η_2 in the case of scenario Snc_2, with $m_{\tilde{\chi}_2^0} = 241.94$ GeV, $\Phi_\mu = 0$ (heavy solid line), $\pi/8$ (light solid light), $\pi/6$ (heavy dashed line), and π (light dashed line).

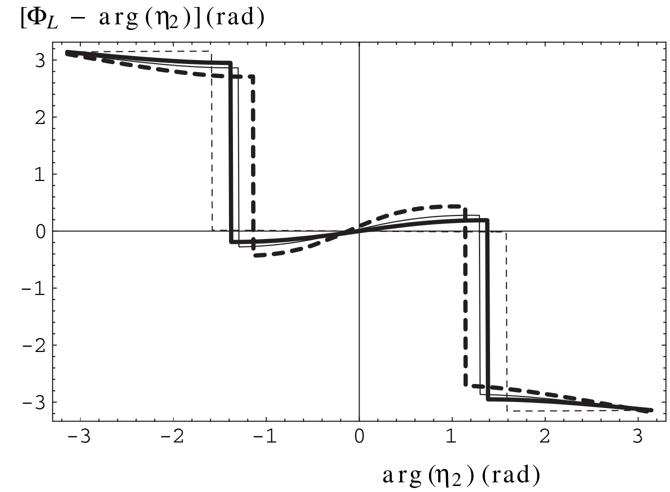

FIG. 20. Relative difference between Φ_L and arg(η_2) as a function of arg(η_2), as observed from Fig. 19.

Figure 18 shows the behavior of the norm of M_L, calculated from Eq. (5.2), as a function of the eigenphase η_2, with input parameters of scenario Snc_2 when $m_{\tilde{\chi}_2^0} = 241.94$ GeV. In this case, the mean amplitude difference of $|M_L|$ for the different plots is greater than before, 120 GeV approximately. However, in the region of small Φ_μ, and arg(η_j), $j = 1, 2$, the results are closely similar, that is, the values of $|M_L|$ concentrate in the range 150–170 GeV. Figures 19 and 20 show the dependence of the mixing phase Φ_L and of the relative phase $\phi_L - \arg(\eta_2)$, respectively, calculated from Eq. (5.2), with respect to the eigenphase η_2, for input parameters of scenario Snc_2. As before, for all the cases $\Phi_\mu = 0, \frac{\pi}{8}, \frac{\pi}{6}, \pi$, we observe the same linear dependence between Φ_L and arg(η_1) practically. However, the nonexact linearity implies differences in the behavior of $|\Phi_L|$ when it is measured with respect to arg(η_2) and Φ_L, as we can see by comparing Figs. 18 and 21.

Let us now assume another possible scenario, Snc_3, described in Table V, where $|\mu| = 150$ GeV and $\tan\theta_k = 4$. In this case, either the physical masses are given by $m_{\tilde{\chi}_1^0} = 156.238$ GeV or $m_{\tilde{\chi}_2^0} = 236.39$ GeV, the same as in the case of scenario Snc_2, there exists practically a linear dependence between the eigenphase and the mixing phase Φ_L. Thus, a description of $|M_L|$ in terms of Φ_L is similar to the one based on the eigenphases. Figure 22 shows the behavior of M_L with respect to the phase Φ_L, computed from Eq. (5.6), according to the Snc_3 scenario with

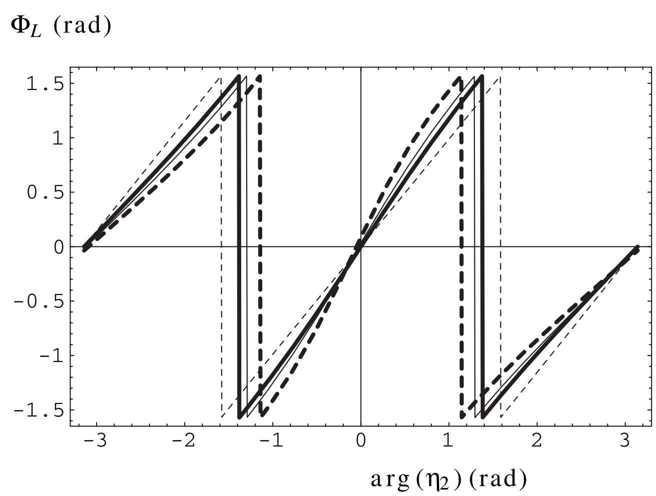

FIG. 19. Mixing parameter Φ_L as a function of arg(η_2) with the same set of fixed parameters used in Fig. 18.

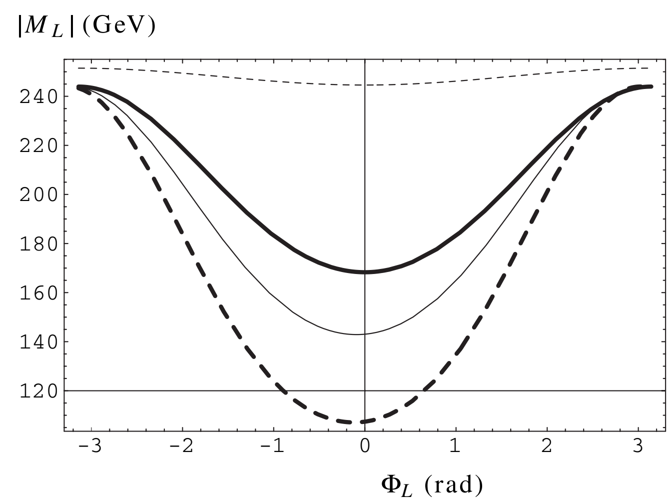

FIG. 21. Norm of $|M_L|$ as a function of Φ_L, computed from Eq. (5.6), for scenario Snc_2, with $m_{\tilde{\chi}_2^0} = 241.94$ GeV, $\Phi_\mu = 0$ (heavy solid curve), $\pi/8$ (light solid curve), $\pi/6$ (heavy dashed curve), and π (light dashed curve).

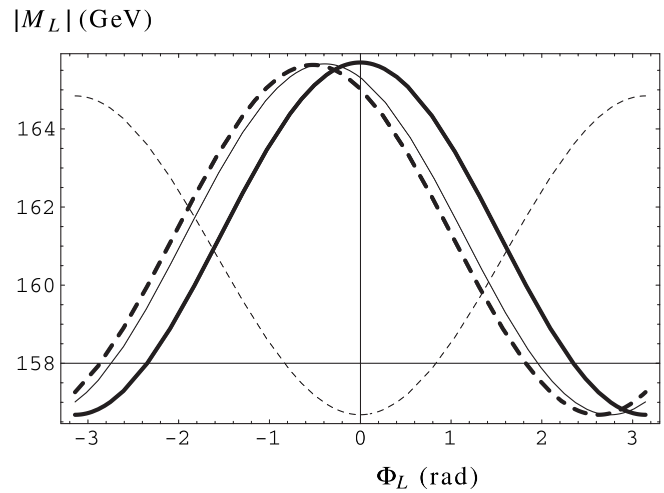

FIG. 22. Norm of $|M_L|$ as a function of Φ_L, computed from Eq. (5.6), for scenario Snc_3, with $m_{\tilde{\chi}_1^0} = 156.24$ GeV, $\Phi_\mu = 0$ (heavy solid curve), $\pi/8$ (light solid curve), $\pi/6$ (heavy dashed curve), and π (light dashed curve).

$m_{\tilde{\chi}_1^0} = 156.238$ GeV. Comparing with Fig. 15, constructed in similar conditions according to scenario Snc_2, in this case we observe a greater dispersion of the values of $|M_L|$ when Φ_μ vary. For small phases $-1 \leq \Phi_L \leq 1$ and $0 \leq \Phi_\mu \leq \frac{\pi}{8}$, the values of M_L lie in the range 163–166 GeV, approximately. Figure 23 shows the behavior of M_L with respect to the phase Φ_L, computed from Eq. (5.6) for input parameters of scenario Snc_3 with $m_{\tilde{\chi}_2^0} = 236.39$ GeV. Similarly, in this case, the values of $|M_L|$ in the mentioned lies in the range 155–185 GeV, approximately. Thus, the value of $|M_L|$ must be localized in the intersection of these regions, i.e. it is determined more accurately in the case of the scenarios where the mass $m_{\tilde{\chi}_1^0}$ is a known quantity.

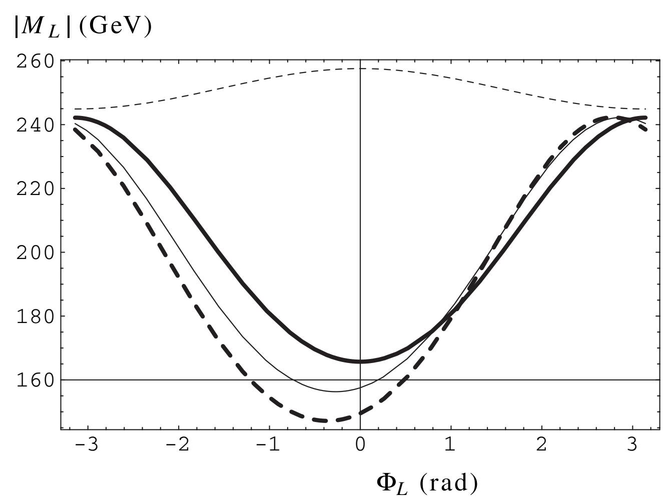

FIG. 23. The same inputs as in Fig. 22, but considering the neutralino mass $m_{\tilde{\chi}_2^0} = 236.79$ GeV.

VII. CONCLUSIONS

In this paper we have studied the implications of a complex symmetric neutralino mass matrix in the context of the left-right SUSY model. This matrix was described by seven real parameters $|M_L|$, Φ_L, $|\mu|$, Φ_μ, M_R, M_V, and $\tan\theta_k$. To find analytical expressions for the physical masses $m_{\tilde{\chi}_j^0}$, $j = 1, \ldots, 4$, of the neutralinos and some connecting relations among the parameters, at the tree level, we have diagonalized this matrix by constructing the corresponding diagonalizing unitary matrix. The masses, obtained by solving the associated characteristic polynomial to this problem, have been ordered by sizes and plotted as a function of the Higgsino parameter $|\mu|$, and also as a function of the mixing phases Φ_μ and Φ_L. In the CP-conserving case, when all except the μ parameter were fixed, according to the possible scenarios studied in Ref. [6], we observe that there is no intersection between the different curves representing the behavior of the neutralino masses as a function of μ. In the CP-violating case, considering two possible scenarios, similar to the previous ones but where $|\mu|$ was fixed and Φ_μ and Φ_L were allowed to vary, we observe that there is no overlapping between the surfaces representing the behavior of the neutralino masses $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$.

The inverse problem consisting to determine the mixing parameters $|M_L|$ and Φ_L in terms of the rest of the fundamental parameters has been solved using the projector formalism without appeal to the Jarlskog' projector formula. In this way, the so-called reduced projectors have been expressed essentially in terms of the minors of the determinant of the matrix formed from the product between the original mass matrix and its adjoint. Thus, the M_L parameter has been disentangled and expressed in terms of the eigenphases by solving a simple linear algebraic equation, in contrast to the standard treatment where you need to solve a system of six linear equations with six unknowns (see the Appendix). Moreover, combining the novel definition of the reduced projectors with the Jarlskog' formula and then solving a quadratic algebraic equation we have obtained a new formula expressing the norm of M_L in terms of the mixing parameter Φ_L and of the rest of the fundamental parameters. This last formula provides a description for the behavior of $|M_L|$ in terms of Φ_L equivalent to the one in terms of the eigenphases.

In the treatment of the inverse problem, in the CP-violating case, we have considered two scenarios, the first one similar to the $Sp1$ type considered in [2] in the context of the MSSM, characterized by a big rate between k_u and k_d and the second one characterized by a relatively small rate between k_u and k_d, with similar conditions to those studied in [13] but adapted to the CP-violating case. In both scenarios, we have observed that the value of $|M_L|$ can be determined more accurately if we know the mass of the lighter neutralino.

A similar analysis can be carried out for the chargino sector. This sector is more difficult to treat using the projector technique because the corresponding chargino mass matrix is not symmetric and requires two unitary matrices to diagonalize it. This analysis is underway and will be reported in a separate communication.

ACKNOWLEDGMENTS

N. A. M. thanks the members of the organization *Baobab Familial* of Montreal for valuable support. A. d. l. C. would like to thank Mariana Frank for enlightening discussions.

APPENDIX: THE STANDARD METHOD

In this Appendix we demonstrate the equivalence between the method implemented in the above section and the one using the Jarlskog's formula (4.23), or Eq. (4.26). The method using the Jarlskog's formula to express M_L in terms of the eigenphases and of the rest of the fundamental parameters has been used in Ref. [2], in the case of the MSSM.

Equation (4.17), for fixed j, represent a system of four complex algebraic equations serving to determine the six fundamental LR SUSY parameters and corresponding eigenphase and physical neutralino mass in terms of the reduced projectors. The explicit form of this system of equations is obtained by inserting Eq. (2.14) into Eq. (4.17), to give

$$\eta_j m_{\tilde{\chi}_j^0} = M_L - M(\sin\theta_k p_{j3}^* - \cos\theta_k p_{j4}^*) \tag{A1}$$

$$= \frac{M_{RV} p_{j2}^* + 2\kappa M(\sin\theta_k p_{j3}^* - \cos\theta_k p_{j4}^*)}{p_{j2}} \tag{A2}$$

$$= \frac{M \sin\theta_k (2\kappa p_{j2}^* - 1) - 2\mu p_{j4}^*}{p_{j3}} \tag{A3}$$

$$= \frac{M \cos\theta_k (1 - 2\kappa p_{j2}^*) - 2\mu p_{j3}^*}{p_{j4}}. \tag{A4}$$

The inverse of these equations determines the fundamental LR SUSY parameters in terms of $p_{j\alpha}$; that is

$$M_L = \eta_j m_{\tilde{\chi}_j^0} + M(\sin\theta_k p_{j3}^* - \cos\theta_k p_{j4}^*), \tag{A5}$$

$$M_{RV} = \frac{p_{j2}\eta_j m_{\tilde{\chi}_j^0} - 2\kappa M(\sin\theta_k p_{j3}^* - \cos\theta_k p_{j4}^*)}{p_{j2}^*}, \tag{A6}$$

$$\mu = \frac{M}{2} \frac{(\sin\theta_k p_{j4} + \cos\theta_k p_{j3})(1 - 2\kappa p_{j2}^*)}{|p_{j3}|^2 - |p_{j4}|^2}, \tag{A7}$$

where the complex neutralino mass of $\tilde{\chi}_j^0$ is given by

$$\eta_j m_{\tilde{\chi}_j^0} = -M \frac{(\sin\theta_k p_{j3}^* + \cos\theta_k p_{j4}^*)(1 - 2\kappa p_{j2}^*)}{|p_{j3}|^2 - |p_{j4}|^2}. \tag{A8}$$

Also, as M_{RV} is a real quantity, from (A6) we obtain

$$\tan\theta_k = -\frac{\mathrm{Im}[p_{j4}(p_{j2}^*)^2] + 2\kappa(|p_{j4}|^2 - |p_{j2}|^2 - |p_{j3}|^2)\mathrm{Im}[p_{j4}p_{j2}^*]}{\mathrm{Im}[p_{j3}(p_{j2}^*)^2] + 2\kappa(|p_{j3}|^2 - |p_{j2}|^2 - |p_{j4}|^2)\mathrm{Im}[p_{j3}p_{j2}^*]}. \tag{A9}$$

The complex reduced projectors p_{j2}, p_{j3}, and p_{j4} can be computed from (A2)–(A4) without considering an explicit dependence of $|M_L|$ and Φ_L. Solving this system, equivalent to six linear equations with six real unknowns, we get

$$p_{j2} = \frac{1}{2\kappa} + (m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2)\mathcal{Z}_j^*, \tag{A10}$$

$$p_{j3} = 2\kappa M(m_{\tilde{\chi}_j^0}\eta_j^* \sin\theta_k \mathcal{Z}_j + 2|\mu|e^{i\Phi_\mu}\cos\theta_k \mathcal{Z}_j^*), \tag{A11}$$

$$p_{j4} = -2\kappa M(m_{\tilde{\chi}_j^0}\eta_j^* \cos\theta_k \mathcal{Z}_j + 2|\mu|e^{i\Phi_\mu}\sin\theta_k \mathcal{Z}_j^*), \tag{A12}$$

where

$$\mathcal{Z}_j = \mathcal{Z}_{j1} - m_{\tilde{\chi}_j^0}\eta_j \mathcal{Z}_{j2}, \tag{A13}$$

with

$$\mathcal{Z}_{j1} = \frac{1}{2\kappa\mathcal{D}_j}\{(m_{\tilde{\chi}_j^0}^2 - M_{RV^2})(m_{\tilde{\chi}_j^0}^2 - 4|\mu|^2) - 4\kappa^2 M^2[m_{\tilde{\chi}_j^0}^2 + 2M_{RV}|\mu|e^{i\Phi_\mu}\sin(2\theta_k)]\}, \tag{A14}$$

and

$$\mathcal{Z}_{j2} = -\frac{2\kappa M^2}{\mathcal{D}_j}[M_{RV} + 2\mu e^{i\Phi_\mu}\sin(2\theta_k)]. \tag{A15}$$

Thus inserting (A11) and (A12) into (A5), we obtain

$$\begin{aligned} M_L &= m_{\tilde{\chi}_j^0}\eta_j + 2\kappa M^2(m_{\tilde{\chi}_j^0}\eta_j \mathcal{Z}_j^* + 2|\mu|e^{-i\Phi_\mu}\sin(2\theta_k)\mathcal{Z}_j) \\ &= A_j\eta_j + B_j, \end{aligned} \tag{A16}$$

where A_j and B_j are given in Eqs. (5.3) and (5.4), respectively.

[1] G. J. Gounaris, C. Le Mouël, and P. I. Porfyriadis, Phys. Rev. D **65**, 035002 (2002).
[2] G. J. Gounaris and C. Le Mouël, Phys. Rev. D **66**, 055007 (2002).
[3] M. Frank and C. S. Kalman, Phys. Rev. D **38**, 1469 (1988).
[4] R. M. Francis, M. Frank, and C. S. Kalman, Phys. Rev. D **43**, 2369 (1991).
[5] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975); **11**, 2558 (1975); G. Senjanović and R. N. Mohapatra, Phys. Rev. D **12**, 1502 (1975); R. E. Marshak and R. N. Mohapatra, Phys. Lett. B **91**, 222 (1980).
[6] H. N. Saif and C. S. Kalman, Z. Phys. C **56**, 447 (1992).
[7] H. N. Saif, Mod. Phys. Lett. A **9**, 1131 (1994).
[8] M. Cvetic and J. Paty, Phys. Lett. **135B**, 57 (1984).
[9] R. Kuchimanchi and R. N. Mohapatra, Phys. Rev. D **48**, 4352 (1993).
[10] K. Huitu, J. Maalampi, and M. Raidal, Phys. Lett. B **328**, 60 (1994).
[11] K. Huitu, J. Maalampi, and M. Raidal, Nucl. Phys. **B420**, 449 (1994).
[12] K. Huitu and J. Maalampi, Phys. Lett. B **344**, 217 (1995).
[13] M. Frank, C. S. Kalman, and H. S. Saif, Z. Phys. C **59**, 655 (1993).
[14] C. Jarlskog, Phys. Rev. D **36**, 2128 (1987); Z. Phys. C **29**, 491 (1985); Stockholm Report No. USIP 87-14.
[15] A. Bartl, H. Fraas, W. Majerotto, and N. Oshimo, Phys. Rev. D **40**, 1594 (1989); J. F. Gunion and H. Haber, *ibid.* **37**, 2515 (1988); S. P. Martin and P. Ramond, *ibid.* **48**, 5365 (1993).