Transverse momentum resummation in soft collinear effective theory

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We present a universal formalism for transverse momentum resummation in the view of soft-collinear effective theory (SCET), and establish the relation between our SCET formula and the well known Collins-Soper-Sterman's pQCD formula at the next-to-leading logarithmic order (NLLO). We also briefly discuss the reformulation of joint resummation in SCET.

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I. INTRODUCTION

Recently, the soft-collinear effective theory (SCET) has made great simplifications on the proof of factorization in B meson decays [1] and high energy hard scattering processes [2,3], including resummation of large logarithms in certain regions of phase space, for example, e^+e^- annihilation into two jets of thrust $T \to 1$ [3], the deep inelastic scattering (DIS) in the threshold region $x \to 1$ [4] and Drell-Yan (DY) process in the case of $z \to 1$ [5]. The reason for these facts is that SCET can be viewed as an operator realization of the pQCD analysis when the modes participating the interactions of interest are soft and collinear, just like chiral dynamics vs. QCD at low energy region. This effective field theory (EFT) provides a simple and systematic method for factorization of hard, collinear and usoft or soft degrees of freedom at operator level, especially usoft modes can be decoupled from collinear modes in the Lagrangian at leading order by making a field redefinition, and the large double logarithms such as $(\alpha_s \log^2 \frac{Q^2}{\Lambda^2})^n$, where Q, Λ are two typical scales that characterize a process, can be resummed naturally through the running of renormalization group equation (RGE).

However, all the above works have not discussed the transverse momentum (Q_T) distributions of high energy hard scattering processes. In this paper, we will investigate the resummed Q_T distributions [6], taking the Higgs-boson production via gluon fusion in small Q_T region [7,8] as an example, within the framework of SCET. It can be seen that in SCET the Q_T resummation formula automatically separates the process-dependent Wilson coefficient and universal anomalous dimension of the effective operator in a process, which once has been studied by the authors of [9] within the Collins-Soper-Sterman (CSS) frame.

The paper is organized as follows. In Sec. II we start by reviewing the basic steps for factorization and resummation in SCET. In Sec. III we apply it to derive the Q_T distribution at small Q_T region directly, which confirms the CSS formula. In Sec. IV, we also discuss a similar formula for joint resummation. Section V contains our concluding remarks. The details of calculation are given in the Appendix.

II. PRELIMINARIES

SCET is appropriate for the kinematic regions of collinear and usoft (soft) modes with momenta scaling: $p_c = (p^+, p^-, p_\perp) = (n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$ and $p_{us} \sim Q(\lambda^2, \lambda^2, \lambda^2)$ or $p_s \sim Q(\lambda, \lambda, \lambda)$, where $\lambda \ll 1$ is the scaling parameter, and the lightlike vectors $n = (1, 0, 0, 1)$, $\bar{n} = (1, 0, 0, -1)$ satisfy $n \cdot \bar{n} = 2$, and the perpendicular components of any four vector V are defined by $V_\perp^\mu = V^\mu - (n \cdot V)\bar{n}^\mu/2 - (\bar{n} \cdot V)n^\mu/2$.

In constructing SCET, one should first identify the scaling of all possible modes of initial and final states with soft and collinear degrees of freedom, then integrate other degrees of freedom, and the remaining modes must reproduce all the infrared physics of the full theory in the region where SCET is valid, which is ensured by the method of regions for Feynman integrals with massless quarks and gluons[1] [10]. The EFT describing the usoft (soft) and collinear modes is known as $\mathrm{SCET_{I(II)}}$, and to distinguish the two theories, the scaling parameter corresponding to $\mathrm{SCET_{II}}$ is denoted by $\eta \sim \lambda^2$, i.e., $p_c \sim Q(\eta^2, 1, \eta)$ and $p_s \sim Q(\eta, \eta, \eta)$.

The elements of $\mathrm{SCET_I}$ consist of usoft sectors $\{q_{us}, A_{us}\}$ and collinear sectors $\{\xi_n, A_n\}$ moving in the n direction, which are expanded as

$$\phi_n(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x} \phi_{n,p}(x), \qquad p = \tilde{p} + k, \tag{1}$$

where $k \sim Q\lambda^2$ resides in the space-time dependence of $\phi_{n,p}(x)$, i.e., $\partial \phi_{n,p}(x) \sim (Q\lambda^2)\phi_{n,p}(x)$, and $\tilde{p} \sim Q(0, 1, \lambda)$ is called label momentum, and the label operators $\bar{\mathcal{P}}$, $\mathcal{P}_\perp$ are defined by picking out $\tilde{p}^-$, $\tilde{p}_\perp$ momenta for collinear fields $\phi_n(x)$,[2] respectively. The Wilson line for n-collinear fields has the form of

[1]In the presence of masses, the regions analysis is very complicated, and we only discuss the massless case.

[2]The convention $\phi_n(x) = \phi_{n,p}(x)$ for collinear fields will be used for convenience.

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$$W_n(x) = \left[\sum_{\text{perms}} \exp\left(\frac{-g}{\bar{\mathcal{P}}}\bar{n}\cdot A_n(x)\right)\right],$$

which is required to ensure collinear gauge invariance. The Lagrangian of collinear sectors, which is invariant under the usoft and collinear gauge transformation, at leading order[3] (LO) in λ is [1],

$$\begin{aligned}\mathcal{L}_c &= \mathcal{L}_{cg} + \mathcal{L}_{cq},\\ \mathcal{L}_{cg} &= \frac{1}{2g^2}\mathrm{Tr}\{[i\mathcal{D}^\mu + gA_n^\mu, i\mathcal{D}^\nu + gA_n^\nu]\}^2\\ &\quad + 2\,\mathrm{Tr}\{\bar{c}_n[i\mathcal{D}_\mu, [i\mathcal{D}^\mu + A_n^\mu, c_n]]\}\\ &\quad + \frac{1}{\alpha}\mathrm{Tr}\{[i\mathcal{D}_\mu, A_n^\mu]\}^2,\end{aligned} \tag{2}$$

$$\mathcal{L}_{cq} = \bar{\xi}_n\left[in\cdot D + i\not{D}_\perp^c\frac{1}{i\bar{n}\cdot D^c}i\not{D}_\perp^c\right]\frac{\not{\bar{n}}}{2}\xi_n. \tag{3}$$

Here the third line are the gauge fixing terms with parameter α and c_n denotes collinear ghost field, and

$$\begin{aligned}i\mathcal{D}^\mu &= \bar{\mathcal{P}}\frac{n^\mu}{2} + \mathcal{P}_\perp^\mu + (in\cdot\partial + gn\cdot A_{us})\frac{\bar{n}^\mu}{2},\\ in\cdot D &= in\cdot D_{us} + gn\cdot A_n, \qquad iD_{us} = i\partial + gA_{us},\\ i\bar{n}\cdot D^c &= \bar{\mathcal{P}} + g\bar{n}\cdot A_n, \qquad iD_\perp^c = \mathcal{P}_\perp + gA_n^\perp.\end{aligned} \tag{4}$$

The Lagrangian of soft sectors in SCET is identical to that of QCD.

As for $\mathrm{SCET_{II}}$, it was emphasized that it can also be viewed as the EFT of $\mathrm{SCET_I}$, and is the final theory [11]. This suggests a short path to go into $\mathrm{SCET_{II}}$ from $\mathrm{SCET_I}$, if the following matching and running steps are taken [11]:

(1) Matching QCD onto $\mathrm{SCET_I}$ at a scale $\mu^2 \sim Q^2$ with $p_c^2 \sim Q^2\lambda^2$;

(2) Decoupling the usoft-collinear interactions with the field redefinitions, $\xi_n = Y_n^\dagger \xi_n^{(0)}$ and $A_n = Y_n^\dagger A_n^{(0)} Y_n$. Here $Y_n(x) = \mathrm{P}\exp(ig\int ds n\cdot A_{us}(ns + x))$ is the usoft Wilson line of usoft gluons in n direction from $s = 0$ to $s = \infty$ for final state particles, and P means path-ordered product, while for initial state particles, Y_n is from $s = -\infty$ to $s = 0$ and the daggers are reversed. This step leads to $\mathcal{L}_c(\xi_n, A_n, n\cdot A_{us}) = \mathcal{L}_c(\xi_n^{(0)}, A_n^{(0)}, 0)$;

(3) Matching $\mathrm{SCET_I}$ onto $\mathrm{SCET_{II}}$ at a scale $\mu^2 \sim Q^2\lambda^2$ with $p_c^2 \sim Q^2\eta^2$. Thus, the soft and collinear modes are decoupled in the Lagrangian of $\mathrm{SCET_{II}}$.

Next, we extend SCET to include the possibility of collinear fields moving in different light-cone directions $n_1, n_2, n_3, \ldots$. These directions defined by n_i and n_j satisfy $n_i\cdot n_j \gg \lambda^2$ for $i \neq j$. For simplicity we will only consider the case of head-on jets corresponding to collinear particles moving in the n and $\bar{n}$ directions. Since the effective theory only takes account the interactions of the modes in the local way, the Lagrangian of the effective theory contains no direct coupling of collinear particles moving in the two separate directions, however the usoft gluons can mediate between them in $\mathrm{SCET_I}$. Hence the Lagrangian in this case can be written by

$$\mathcal{L}^c_{\{n,\bar{n}\}} = \mathcal{L}^c_n + \mathcal{L}^c_{\bar{n}},$$

and the soft parts are unchanged, so the decoupling transformations are also valid here.

To illustrate the application of SCET and warm up, we consider the Sudakov effect of quark electromagnetic form factor in QCD [12], i.e., the double logarithmic asymptotic of conservative current $j^\mu = \bar{\psi}\gamma^\mu\psi$ in the following kinematics:

(a) nearly on-shell case

$$\begin{aligned}&Q^2 = -(p_1 - p_2)^2 \gg -p_1^2 = -p_2^2 \sim \Lambda_{\mathrm{QCD}}^2,\\ &p_1 \sim Q(\eta^2, 1, \eta), \qquad p_2 \sim Q(1, \eta^2, \eta),\\ &\eta \sim \Lambda_{\mathrm{QCD}}/Q.\end{aligned}$$

(b) off-shell case

$$\begin{aligned}&Q^2 = -(p_1 - p_2)^2 \gg -p_1^2 = -p_2^2 \sim Q\Lambda_{\mathrm{QCD}},\\ &p_1 \sim Q(\lambda^2, 1, \lambda), \qquad p_2 \sim Q(1, \lambda^2, \lambda),\\ &\lambda \sim \sqrt{\Lambda_{\mathrm{QCD}}/Q}.\end{aligned}$$

Here p_1, p_2 are the momenta of the initial and final quarks. In the above two cases we have omitted quark mass effects.

Following the treatment of heavy-to-collinear current discussed in [11,13] for case (a), we first match the full current onto the corresponding operator in $\mathrm{SCET_I}$. At LO in λ, it gives [3]

$$j^\mu = [\bar{\xi}_{\bar{n}}W_{\bar{n}}]\gamma^\mu C_q(\mathcal{P}^\dagger, \bar{\mathcal{P}}, \mu^2)[W_n^\dagger\xi_n]. \tag{5}$$

By the requirement of collinear and usoft gauge invariance, the LO effective operator is determined uniquely, and the reparametrization invariance (RPI) [11] implies that the Wilson coefficient satisfies $C_q(\mathcal{P}^\dagger, \bar{\mathcal{P}}, \mu^2) = C_q(\mathcal{P}^\dagger\cdot\bar{\mathcal{P}}, \mu^2)$.

Obviously, the tree level matching condition for j^μ leads to $C_q(Q^2, \mu^2) = 1 + \mathcal{O}(\alpha_s)$. We certainly can determine $\mathcal{O}(\alpha_s)$ correction by adopting dimensional regularization[4] (DR) to regulate UV and IR divergences to compute on-shell matrix elements on both sides of Eq. (5), for which is valid at the operator level and the matching calculation is independent of regularization method. With this choice, the fact $\mathrm{IR_{QCD}} = \mathrm{IR_{SCET}}$ provides us a direct matching calculation to read off the Wilson coefficients and anoma-

[3] We will restrict our discussion only at this order through the paper.

[4] The $\overline{\mathrm{MS}}$ scheme, i.e., $d = 4 - 2\epsilon$ and $\mu^2 \to \mu^2 e^{\gamma_E}/4\pi$ is used through this paper, where γ_E is Euler's constant.

lous dimensions of the operators in SCET. As all the on-shell loop integrals in SCET are scaleless and vanish, $\mathrm{IR}_{\mathrm{SCET}} = -\mathrm{UV}_{\mathrm{SCET}}$, and then $\mathrm{IR}_{\mathrm{QCD}} = -\mathrm{UV}_{\mathrm{SCET}}$. Furthermore, the self energy diagrams in full QCD with massless quarks are also vanish, and all the wave function renormalization constants and the residues of the related propagators are equal to unity, and we also note that the conserved current in full QCD needs not to be renormalized. Thus, what we need to calculate is an one particle irreducible diagram, Fig. 1, in the full theory, which is given by

$$\begin{aligned}\langle p_2|j^\mu|p_1\rangle &= \langle p_2|j^\mu|p_1\rangle^{\mathrm{tree}} + \langle p_2|j^\mu|p_1\rangle^{\mathrm{one-loop}}\\ &= \bar{u}(p_2)\gamma^\mu(1+V_q)u(p_1),\end{aligned} \tag{6}$$

$$\begin{aligned}V_q &= \frac{\alpha_s C_F}{4\pi}\left(\frac{\mu^2 e^{\gamma_E}}{Q^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-8-\frac{\pi^2}{3}\right)\\ &= \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon^2}-\frac{1}{\epsilon}\left(2\log\frac{\mu^2}{Q^2}+3\right)-\log^2\frac{\mu^2}{Q^2}\right.\\ &\quad\left.-3\log\frac{\mu^2}{Q^2}-8+\frac{\pi^2}{6}\right].\end{aligned} \tag{7}$$

Here $C_F = (N_c^2-1)/2N_c$ for $SU(N_c)$, and $N_c = 3$ for QCD. The ϵ-poles in Eq. (7) are of IR character, whose opposition are just the UV poles in SCET. Thus the matching calculation at one-loop level gives

$$C_q(Q^2, Q^2) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-8+\frac{\pi^2}{6}\right), \tag{8}$$

$$Z_V \equiv \sum_n \frac{Z_V^{(n)}}{\epsilon^n} = 1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2}+\frac{1}{\epsilon}\left(2\log\frac{\mu^2}{Q^2}+3\right)\right]. \tag{9}$$

Here Z_V is defined as $\overline{\mathrm{MS}}$ renormalization constant of the effective operator, and μ has been set to Q to minimize the logarithms in the Wilson coefficient. It was pointed out [1] that the anomalous dimension of the effective operator is independent of its spin structure, for which can be factorized out from loop integrals. This means the evolution equation in SCET is universal, and only the Wilson coefficient is process-dependent.

From Eq. (9), we obtain the RGE of $C_q(Q^2, \mu^2)$,

$$\frac{d\log C_q(Q^2,\mu^2)}{d\log(\mu)} = \gamma_1(\mu) = -g\frac{\partial Z_V^{(1)}}{\partial g}, \tag{10}$$

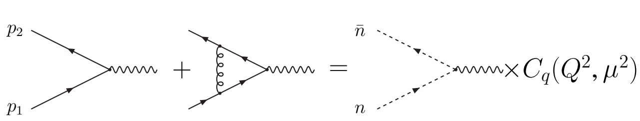

FIG. 1. Graphical representation for quark current matching.

$$\begin{aligned}\gamma_1(\mu) &\equiv \mathbf{A}_q(\alpha_s)\log\frac{Q^2}{\mu^2} + \mathbf{B}_q(\alpha_s)\\ &= -\frac{\alpha_s C_F}{4\pi}\left(4\log\frac{\mu^2}{Q^2}+6\right).\end{aligned} \tag{11}$$

Here $\mathbf{A}_q^{(1)} = C_F$ and $\mathbf{B}_q^{(1)} = -\frac{3}{2}C_F$.[5] With Eq. (10), we can resum the terms such as double logarithms from the scale $\sim Q^2$ down to the scale $\sim Q^2\lambda^2$, we abbreviate this matching step as a chain $\mathrm{QCD}|_{Q^2} \longrightarrow \mathrm{SCET_I}|_{Q^2\lambda^2}$.

Next, we decouple the usoft and collinear modes by the field redefinitions, which results in

$$\begin{aligned}\langle p_2|[\bar{\xi}_{\bar{n}}W_{\bar{n}}]\gamma^\mu[W_n^\dagger\xi_n]|p_1\rangle \longrightarrow &\langle p_2|[\bar{\xi}_{\bar{n}}^{(0)}W_{\bar{n}}^{(0)}]|\Omega\rangle\\ &\times\gamma^\mu\langle\Omega|\mathrm{T}[Y_{\bar{n}}Y_n]|\Omega\rangle\\ &\times\langle\Omega|[W_n^{(0)\dagger}\xi_n^{(0)}]|p_1\rangle,\end{aligned} \tag{12}$$

where T means time ordering operator.

For the final step we integrate out all the off-shell modes of order $\sqrt{Q\Lambda_{\mathrm{QCD}}}$ and go into $\mathrm{SCET_{II}}$. We can rename the usoft fields as soft fields for the usoft degrees of freedom scaling as soft ones, and then lower the the off-shellness of the collinear fields that would be matched onto $\mathrm{SCET_{II}}$. Since the leading collinear Lagrangians in $\mathrm{SCET_I}$ and $\mathrm{SCET_{II}}$ are the same and (u)soft and collinear fields are decoupled at LO in $(\lambda)\eta$, all possible time-ordered products involve collinear fields agree exactly and we can simply replace $\bar{\xi}_{\bar{n}}^{(0)}W_{\bar{n}}^{(0)} \to \bar{\xi}_{\bar{n}}^{\mathrm{II}}W_{\bar{n}}^{\mathrm{II}}$ and $W_n^{(0)\dagger}\xi_n^{(0)} \to W_n^{\mathrm{II}\dagger}\xi_n^{\mathrm{II}}$, where the superscript II denotes $\mathrm{SCET_{II}}$ will be dropped from now on.

Because p_1, p_2 of (a) are described by the collinear modes in $\mathrm{SCET_{II}}$, the general matching structure of $\mathrm{SCET_{II}}$ diagram is shown in Fig. 2(a). The Wilson coefficient at this step is unity and anomalous dimension is the same as the first step, except that it runs from the scale $\sim Q^2\lambda^2$ to the scale $\sim Q^2\eta^2$. We abbreviate this step as $\mathrm{SCET_I}|_{Q^2\lambda^2} \Rightarrow \mathrm{SCET_{II}}|_{Q^2\eta^2}$. Collecting all the results above, we obtain the known Sudakov form factor $S_q^{(a)}(\Lambda_{\mathrm{QCD}}, Q)$, leaving other coefficients omitted,

$$S_q^{(a)}(\Lambda_{\mathrm{QCD}}, Q) = \exp\left(-\int_{\Lambda_{\mathrm{QCD}}}^{Q}\gamma_1(\mu)d\log\mu\right). \tag{13}$$

For case (b), it can be taken as a subdiagram of the on-shell case, from kinematical considerations, of which the external legs are amputated. Thus, step (1) is unchanged, and in step (2), $\langle 0|\mathrm{T}[Y_{\bar{n}}Y_n]|0\rangle$ changes into

$$-\int_0^\infty\int_0^\infty ds\,dt\,e^{iQ\lambda^2(s+t)}\langle\Omega|\mathrm{T}[Y_{\bar{n}}(0,\bar{n}s)Y_n^\dagger(0,-nt)]|\Omega\rangle, \tag{14}$$

where $1/(Q\lambda^2)$ is the effective contour length [12] and

[5]The notion $\mathbf{A} \equiv \sum_n(\alpha_s/\pi)^n\mathbf{A}^{(n)}$, we adopt $\{\mathbf{A}, \mathbf{B}\}$ to distinguish the well known coefficients $\{A, B\}$ in pQCD.

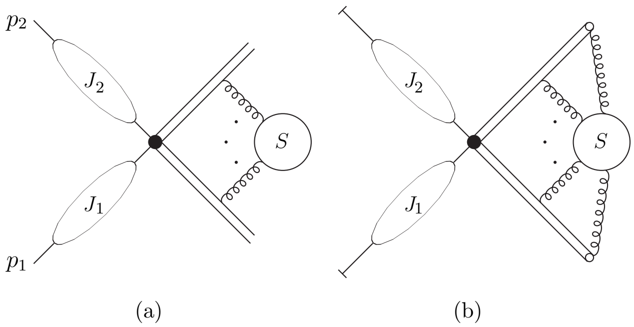

FIG. 2. Factorization of on-shell form factor (a) and off-shell form factor (b) in SCET. In (b), the soft Wilson lines are terminated and the external quarks are amputated, therefore, (b) can be taken as a subdiagram of (a). Both diagrams are depicted under the gauge $\bar{n}\cdot A_n = n\cdot A_{\bar{n}} = 0$.

$$Y_{\bar{n}}(0, \bar{n}s) \equiv \mathrm{P}\exp\left(ig\int_0^s d\beta\bar{n}\cdot A_{us}(\bar{n}\beta)\right), \tag{15}$$

$$Y_n(0, -nt) \equiv \mathrm{P}\exp\left(-ig\int_0^t d\beta n\cdot A_{us}(-n\beta)\right). \tag{16}$$

Because of the jets J_1, J_2 with fluctuations $-p_1^2 = -p_2^2 = Q\Lambda_{\mathrm{QCD}} \gg Q^2\eta^2$ in Fig. 2(b), they must be integrated out in $\mathrm{SCET_{II}}$, and only (14) is left over after step (3) associated with renaming the usoft modes in $\mathrm{SCET_I}$ as the soft modes in $\mathrm{SCET_{II}}$, of which the running behavior is the same as F_{IR} of [12]. Finally, the Sudakov factor in the off-shell case is

$$S_q^{(b)}(Q\eta, Q) = \exp\left(-\int_{Q\lambda}^{Q}\gamma_1(\mu)d\log\mu - \int_{Q\eta}^{Q\lambda}\gamma_2(\mu)d\log\mu\right), \tag{17}$$

$$\gamma_2(\mu) = \frac{\alpha_s}{\pi}C_F\log\frac{\mu^2}{Q^2\eta^2} + \mathcal{O}(\alpha_s^2), \tag{18}$$

where $\gamma_2(\mu)$ is the anomalous dimension of (14). We conclude this section with a chain for the off-shell case, $\mathrm{QCD}|_{Q^2} \longrightarrow \mathrm{SCET_I}|_{Q^2\lambda^2} \longrightarrow \mathrm{SCET_{II}}|_{Q^2\eta^2}$. Now we are ready to turn into the Q_T resummation in the following.

III. METHOD OF Q_T RESUMMATION IN SCET

Since SCET is powerful to disentangle the soft and collinear interaction, and IR power counting [14] tells that the singular terms of Q_T distribution for DY-like processes in the limit of $Q_T \to 0$ originate from soft and collinear modes, which are emitted by partons from hadrons p_1, p_2, it is not unexpected that SCET can be applied to treat them and to derive the resummed part of full transverse momentum distribution for these semi-inclusive processes, while the remaining regular terms Y [6–8] and the prescription of incorporating nonperturbative region ($Q_T \sim \Lambda_{\mathrm{QCD}}$) are neglected in this paper.

For the sake of simplicity, the process of Higgs-boson production is taken as a demonstration, but the method we used is not confined to this example. The dominant process for Higgs-boson production at the large hadron collider (LHC) in the standard model are gluon fusion through a heavy quark loop, mainly the top quark, $p_1(P_1) + p_2(P_2) \to gg \to \phi(Q) + X$ with $P_1 = (0, 2p, 0)$, $P_2 = (2p, 0, 0)$ and $S = P_1^- P_2^+$. It is convenient to start from the effective Lagrangian for one Higgs-boson and gluons coupling [15],

$$\mathcal{L}_{\phi gg} = \tau(\alpha_s)\phi G^a_{\mu\nu}G_a^{\mu\nu}, \tag{19}$$

where $\tau(\alpha_s) = [\alpha_s(Q)/12\pi](\sqrt{2}G_F)^{1/2} + \mathcal{O}(\alpha_s^2)$ and $Q = m_\phi$. Therefore, the operator for Higgs-boson production is $\mathcal{H} = G^a_{\mu\nu}G_a^{\mu\nu}$. Here the coupling α_s suffers the QCD correction, which is unlike the case of electro-charge coupling. Furthermore, because the renormalization constant of $\alpha_s G^a_{\mu\nu}G_a^{\mu\nu}$ is unity up to $\mathcal{O}(\alpha_s)$, the renormalization constant of $\mathcal{H}$ is just Z_g^{-2}, where Z_g is the renormalization constant of gauge coupling-g.

If we set $\lambda^2 \sim Q_T/Q$ with $Q \gg Q_T \gg \Lambda_{\mathrm{QCD}}$the situation is much like that of quark form factor (a) discussed in last section, and the matching and running procedure can be followed. The operator $\mathcal{H}$ can match at LO in λ onto

$$\mathcal{H} = \tfrac{1}{2}\mathcal{B}^\mu_{\bar{n}a}C_g(\mathcal{P}^\dagger\cdot\bar{\mathcal{P}}, \mu^2)\mathcal{B}^a_{n\mu}, = \tfrac{1}{2}\mathcal{B}^\mu_{\bar{n}a}C_g(Q^2, \mu^2)\mathcal{B}^a_{n\mu}, \tag{20}$$

where

$$\begin{aligned}\mathcal{B}_n^\mu &= \bar{n}_\nu\mathcal{G}_n^{\nu\mu},\\ \mathcal{G}_n^{\mu\nu} &= W_n^\dagger[i\mathcal{D}_n^\mu + gA_n^\mu, i\mathcal{D}_n^\nu + gA_n^\nu]W_n,\end{aligned} \tag{21}$$

and $n \leftrightarrow \bar{n}$ for $\mathcal{G}_{\bar{n}}^{\mu\nu}$.

The one-loop calculation, Fig. 3, is similar to quark current, except for dividing the final result by Z_g^{-2}. Finally,

$$\begin{aligned}\langle g_1|\mathcal{H}|g_2\rangle &= \langle g_1|\mathcal{H}|g_2\rangle^{\mathrm{tree}} + \langle g_1|\mathcal{H}|g_2\rangle^{\mathrm{one-loop}} + c.t.\\ &= \langle g_1|\mathcal{H}|g_2\rangle^{\mathrm{tree}}(1 + V_g),\end{aligned} \tag{22}$$

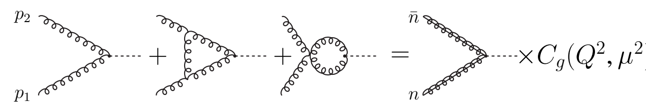

FIG. 3. Graphical representation for gluon current matching.

$$V_g = \frac{\alpha_s}{4\pi}\left(\frac{\mu^2 e^{\gamma_E}}{Q^2}\right)^{\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(-\frac{2C_A}{\epsilon^2} - \frac{2\beta_0}{\epsilon} + \mathcal{A}_g^H\right)$$
$$= \frac{\alpha_s}{4\pi}\left[-\frac{2C_A}{\epsilon^2} - \frac{1}{\epsilon}\left(2C_A \log\frac{\mu^2}{Q^2} + 2\beta_0\right) - C_A \log^2\frac{\mu^2}{Q^2} - 2\beta_0 \log\frac{\mu^2}{Q^2} + \mathcal{A}_g^H + \frac{C_A\pi^2}{2}\right] \quad (23)$$

$-2\beta_0 \log\frac{\mu^2}{Q^2}$ is obtained,[6] from which we read

$$C_g(Q^2, Q^2) = 1 + \frac{\alpha_s}{4\pi}\left(\mathcal{A}_g^H + \frac{C_A\pi^2}{2}\right), \quad (24)$$

$$Z_{\mathcal{H}} \equiv \sum_n \frac{Z_{\mathcal{H}}^{(n)}}{\epsilon^n} = 1 + \frac{\alpha_s}{4\pi}\left[\frac{2C_A}{\epsilon^2} + \frac{1}{\epsilon}\left(2C_A \log\frac{\mu^2}{Q^2} + 2\beta_0\right)\right]. \quad (25)$$

Here $C_A = N_c$, $\beta_0 = \frac{11}{6}C_A - \frac{2}{3}n_f T_R$, $\mathcal{A}_g^H = 11 + 2\pi^2$, $T_R = \frac{1}{2}$ and $n_f = 5$ is the number of active quark flavors. Then the RGE of $C_g(Q^2, \mu^2)$ is

$$\frac{d \log C_g(Q^2, \mu^2)}{d\log(\mu)} = \gamma_1(\mu) = -g\frac{\partial Z_{\mathcal{H}}^{(1)}}{\partial g}, \quad (26)$$

$$\gamma_1(\mu) \equiv \mathbf{A}_g(\alpha_s)\log\frac{Q^2}{\mu^2} + \mathbf{B}_g(\alpha_s) = -\left(\frac{\alpha_s}{\pi}\right)\left(C_A \log\frac{\mu^2}{Q^2} + \beta_0\right), \quad (27)$$

with $\mathbf{A}_g^{(1)} = C_A$ and $\mathbf{B}_g^{(1)} = -\beta_0$. Thus the evolution from the scale $\sim Q^2$ to the scale $\sim Q^2\lambda^2$ gives

$$C_g(Q^2, Q^2\lambda^2) = C_g(Q^2, Q^2)\exp\left(-\int_{Q^2\lambda^2}^{Q^2}\frac{d\mu^2}{2\mu^2}\gamma_1(\mu)\right). \quad (28)$$

As shown above, the extraction of $\mathbf{A}, \mathbf{B}$ in SCET is different from that of A, B in pQCD, i.e., there is no need to calculate real correction which is more difficult to handle. Using the virtual part of higher order calculation, such as the two loop on-shell quark and gluon form factor [16], we can find the $\mathcal{O}(\alpha_s^2)$ universal anomalous dimension. For example,

$$\mathbf{A}_a^{(2)} = \frac{1}{2}C_a K, \qquad K = C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{10}{9}T_R n_f, \quad (29)$$

where $C_q = C_F$, $C_g = C_A$.

After performing field redefinitions, Eq. (26) can be directly used to running $\mathcal{H}$ from the scale $\sim Q^2$ to the scale $\sim Q^2\eta^2$ without loss of degrees of freedom. So the relevant operator for Higgs-boson production at the scale $\sim Q^2\eta^2(Q_T^2)$ is

$$\mathcal{H} = C_g(Q^2, Q^2\eta^2)\,\mathrm{Tr}\{\mathrm{T}[Y_n^\dagger Y_{\bar{n}}\mathcal{B}_{\bar{n}}^\mu Y_{\bar{n}}^\dagger Y_n \mathcal{B}_{n\mu}]\}$$
$$= C_g(Q^2, Q^2\eta^2)\tfrac{1}{2}\mathrm{T}[\mathcal{Y}_{\bar{n}}^{ab}\mathcal{Y}_n^{ac}\mathcal{B}_{\bar{n}}^{b\mu}\mathcal{B}_{n\mu}^c]$$
$$\equiv C_g(Q^2, Q^2\eta^2)\hat{\mathcal{H}}. \quad (30)$$

Here $\mathcal{Y}_{n(\bar{n})}$ is the adjoint soft Wilson line from $-\infty$ to 0 in $n(\bar{n})$ direction for incoming fields. Now, we have completed the procedures corresponding to step (1), (2) and (3).

To obtain the differential cross section, we relate it to the composite operator $\mathcal{H}$ at the renormalization scale $\mu^2 \sim Q^2\eta^2$ in $\mathrm{SCET_{II}}$, where the cross section can be written as

$$\frac{1}{\sigma_{gg}^{(0)}}\frac{d\sigma^{\mathrm{resum}}}{dQ^2 dy dQ_T^2} = \mathbf{H}_g^\phi(Q)e^{-\mathbf{S}_g(\mu,Q)}\sigma_{\mathrm{SCET}}(Q_T, Q, \mu), \quad (31)$$

where $\mathbf{H}_g^\phi(Q) = |C_g(Q^2, Q^2)|^2$ is a function of $\alpha_s(Q)$, and

$$\sigma_{gg}^{(0)} = (\sqrt{2}G_F)\frac{\alpha_s^2(Q)m_H^2}{576S}\delta(Q^2 - m_H^2), \quad (32)$$

$$\mathbf{S}_g(\mu, Q) = \int_{\mu^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left[\mathbf{A}_g(\alpha_s)\log\frac{Q^2}{\mu^2} + \mathbf{B}_g(\alpha_s)\right], \quad (33)$$

and

$$\sigma_{\mathrm{SCET}}(Q_T, Q, \mu) = \frac{1}{\sigma_{gg}^{(0)}}\frac{d\sigma^{\mathrm{SCET}}(\mu)}{dQ^2 dy dQ_T^2} \quad (34)$$

represents the normalized differential cross section calculated in $\mathrm{SCET_{II}}$ with the composite operator $\hat{\mathcal{H}}$. The general structure of relevant diagram is shown in Fig. 4, where the soft and collinear modes are decoupled and the spin and color are summed over in the matrix element of hadron, from which the SCET cross section can be written in the form of multiple convolution,

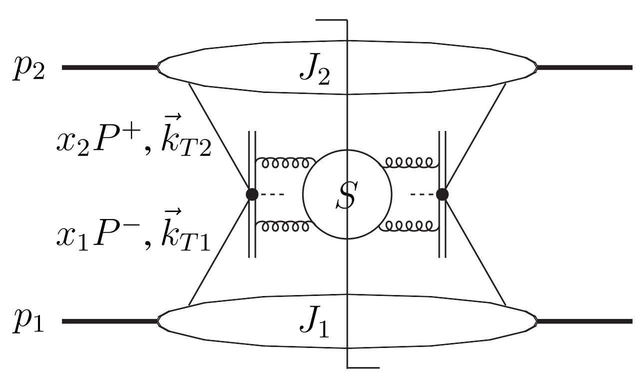

FIG. 4. General structure of SCET cross section for Higgs-boson production under the gauge $n \cdot A_{\bar{n}} = \bar{n} \cdot A_n = 0$.

[6] Here we have absorbed the scale dependence of α_s into Z_g.

$$\sigma_{\rm SCET}(Q_T, Q, \mu) = \int d^2\vec{k}_{T1} d^2\vec{k}_{T2} d^2\vec{k}_{TS} \delta^2(\vec{k}_{T1} + \vec{k}_{T2} + \vec{k}_{TS} - \vec{Q}_T) J_{p_1}(x_1, k_{T1}, \mu) \times J_{p_2}(x_2, k_{T2}, \mu) S(k_{TS}, \mu), \tag{35}$$

where $x_1 = Qe^y/\sqrt{S}$, $x_2 = Qe^{-y}/\sqrt{S}$ for $Q_T^2 \ll Q^2$, and

$$J_{p_1}(x_1, k_{T1}, \mu) = \frac{2}{x_1 P^-} \frac{1}{(2\pi)^3} \int dy^+ d^2\vec{y}_\perp e^{-i(x_1 P^- y^+ - \vec{k}_{T1}\cdot\vec{y}_\perp)} \times \langle p_1 | {\rm Tr}[\mathcal{B}_n^\alpha(y^+, 0, \vec{y}_\perp)\mathcal{B}_{n\alpha}(0)] | p_1\rangle, \tag{36}$$

$$J_{p_2}(x_2, k_{T2}, \mu) = \frac{2}{x_2 P^+} \frac{1}{(2\pi)^3} \int dy^- d^2\vec{y}_\perp e^{-i(x_2 P^+ y^- - \vec{k}_{T2}\cdot\vec{y}_\perp)} \times \langle p_2 | {\rm Tr}[\mathcal{B}_{\bar n}^\alpha(0, y^-, \vec{y}_\perp)\mathcal{B}_{\bar n\alpha}(0)] | p_2\rangle, \tag{37}$$

$$S(k_{TS}, \mu) = \frac{1}{(2\pi)^2} \int d^2\vec{y}_\perp e^{i\vec{k}_{TS}\cdot\vec{y}_\perp} \langle \Omega | \bar{\rm T}[\mathcal{Y}_n^{\dagger ec}\mathcal{Y}_{\bar n}^{\dagger eb}] \times (0, 0, \vec{y}_\perp) {\rm T}[\mathcal{Y}_{\bar n}^{ab}\mathcal{Y}_n^{ac}](0) | \Omega\rangle, \tag{38}$$

with $\bar{\rm T}$ denoting the antitime ordering operator. Obviously, in Eq. (38) the matrix element has been factorized, and the delta function is imposed by momentum conservation.

Next, to factorize the phase space, the trick of Fourier transforming to impact parameter space is significant [6],

$$\int d^2\vec{Q}_T e^{i\vec{b}\cdot\vec{Q}_T} \delta^2\left(\sum_i \vec{k}_{Ti} - \vec{Q}_T\right) = \prod_i e^{i\vec{b}\cdot\vec{k}_{Ti}}. \tag{39}$$

Then, for each transverse momentum $\vec{k}_{Ti}$, one obtains

$$\int d^2\vec{k}_{Ti} e^{i\vec{b}\cdot\vec{k}_{Ti}} f(\vec{k}_{Ti}) = \tilde{f}(b). \tag{40}$$

This produces the simple product

$$\tilde{\sigma}_{\rm SCET}(b, Q, \mu) = \tilde{J}_{p_1}(x_1, b, \mu)\tilde{J}_{p_2}(x_2, b, \mu)\tilde{S}(b, \mu). \tag{41}$$

Because of KLN theorem, the contributions from the soft modes are free of IR divergences. So only the collinear divergences are survived, therefore after matching the SCET cross section onto a product of two parton distribution functions (PDFs) given by [2], which are equivalent to the conventional PDFs $f_{a/p_i}(x_i, \mu)$ at LO in λ, the remaining IR divergences can be absorbed into these nonperturbative inputs, of which the evolutions are controlled by the DGLAP equations. This leads to [6]

$$\tilde{J}_{p_i}(x_i, b, \mu) = \sum_a (f_{a/p_i} \otimes \mathbf{c}_{ga})(x_i, b, \mu) = \sum_a \int_{x_i}^1 \frac{d\xi}{\xi} f_{a/p_i}(\xi, \mu) \mathbf{c}_{ga}\left(\frac{x_i}{\xi}, b, \mu\right), \tag{42}$$
$$\mathbf{c}_{ga} \equiv \sum_{n=0} \left(\frac{\alpha_s}{4\pi}\right)^n \mathbf{c}_{ga}^{(n)}.$$

If we define

$$\mathbf{C}_{ga}\left(z, \frac{b_0}{b}, \mu\right) = \mathbf{c}_{ga}(z, b, \mu)[\tilde{S}(b, \mu)]^{1/2}, \quad b_0 = 2e^{-\gamma_E}, \tag{43}$$

then

$$\tilde{\sigma}_{\rm SCET}(b, Q, \mu) = (f_{a/p_1} \otimes \mathbf{C}_{ga})\left(x_1, \frac{b_0}{b}, \mu\right) \times (f_{b/p_2} \otimes \mathbf{C}_{gb})\left(x_2, \frac{b_0}{b}, \mu\right). \tag{44}$$

Obviously, $\mathbf{C}_{ga}^{(0)}(z) = \delta_{ga}\delta(1-z)$, and we have derived in the Appendix that

$$\mathbf{C}_{ga}^{(1)}(z) = -2P_{ga}^\epsilon(z) - C_A \frac{\pi^2}{6} \delta_{ga}\delta(1-z), \tag{45}$$

where μ has been set to b_0/b to eliminate large constant factors in $\mathbf{C}_{ga}^{(1)}(z, \mu)$, and $P_{ga}^\epsilon(z)$ represent the $\mathcal{O}(\epsilon)$ terms of the DGLAP splitting kernels.

Combining Eq. (31)–(44) and Fourier transforming back to Q_T space, we obtain the resumed formula of transverse momentum distribution for Higgs-boson production in SCET,

$$\frac{1}{\sigma_{gg}^{(0)}} \frac{d\sigma^{\rm resum}}{dQ^2 dy dQ_T^2} = \mathbf{H}_g^\phi(Q) \int_0^\infty \frac{db}{2\pi} b J_0(bQ_T) \times \sum_{ab} e^{-\mathbf{S}_g(b_0/b, Q)} (f_{a/p_1} \otimes \mathbf{C}_{ga})\left(x_1, \frac{b_0}{b}\right) \times (f_{b/p_2} \otimes \mathbf{C}_{gb})\left(x_2, \frac{b_0}{b}\right). \tag{46}$$

The similar reasoning leads to the general form for Q_T resummation,

$$\frac{1}{\sigma^{(0)}} \frac{d\sigma^{\rm resum}}{dQ^2 dy dQ_T^2} = \mathbf{H}_c^F(Q) \int_0^\infty \frac{db}{2\pi} b J_0(bQ_T) \times \sum_{ab} e^{-\mathbf{S}_c(b_0/b, Q)} (f_{a/p_1} \otimes \mathbf{C}_{ca})\left(x_1, \frac{b_0}{b}\right) \times (f_{b/p_2} \otimes \mathbf{C}_{\bar{c}b})\left(x_2, \frac{b_0}{b}\right), \tag{47}$$

where F and c stand for the type of process and of parton participating the elementary subprocess, for example, $F = DY$ and $c = q$ for Drell-Yan process. The formula (47) is a little different from the known CSS formula [6],

$$\frac{1}{\sigma^{(0)}} \frac{d\sigma^{\rm resum}}{dQ^2 dy dQ_T^2} = \int_0^\infty \frac{db}{2\pi} b J_0(bQ_T) \times \sum_{ab} e^{-S_c(b_0/b, Q)} (f_{a/p_1} \otimes C_{ca})\left(x_1, \frac{b_0}{b}\right) \times (f_{b/p_2} \otimes C_{\bar{c}b})\left(x_2, \frac{b_0}{b}\right). \tag{48}$$

Transforming Eq. (47) into the form as Eq. (48) by the identity

$$\mathbf{H}_c^F(Q) = \exp\left[\int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\beta(\alpha_s)\frac{d\log\mathbf{H}_c^F}{d\log\alpha_s}\right]\mathbf{H}_c^F(b_0/b), \quad (49)$$

with $\beta(\alpha_s)$ denoting the QCD β-function, one finds

$$\frac{1}{\sigma^{(0)}}\frac{d\sigma^{\rm resum}}{dQ^2dydQ_T^2} = \int_0^\infty \frac{db}{2\pi}bJ_0(bQ_T) \times \sum_{ab} e^{-\bar{S}_c(b_0/b,Q)}(f_{a/p_1}\otimes\bar{C}_{ca})\left(x_1,\frac{b_0}{b}\right) \times (f_{b/p_2}\otimes\bar{C}_{\bar{c}b})\left(x_2,\frac{b_0}{b}\right). \quad (50)$$

The corresponding coefficients $\{\bar{A},\bar{B},\bar{C}\}$ are related to $\{\mathbf{A},\mathbf{B},\mathbf{C}\}$ through equation

$$\bar{A}(\alpha_s) = \mathbf{A}(\alpha_s), \qquad \bar{B}(\alpha_s) = \mathbf{B}(\alpha_s) - \beta(\alpha_s)\frac{d\log\mathbf{H}_c^F}{d\log\alpha_s},$$
$$\bar{C}_{ab}(z) = \mathbf{C}_{ab}(z)[\mathbf{H}_a^F(b_0/b)]^{1/2}, \quad (51)$$

Up to next-to-leading logarithmic order (NLLO), $\{\mathbf{A}^{(2)},\mathbf{B}^{(1)},\mathbf{C}^{(1)}\}$ and $\{A^{(2)},B^{(1)},C^{(1)}\}$ is compatible with each other, and further calculation and confirmation are required at higher order.

It can be seen that SCET provides a natural framework of Q_T resummation by conventional RGE in EFT. We have noted that the matched effective operator is determined by collinear and soft gauge invariance and is unique. In addition, the corresponding anomalous dimension is independent of its spin structure and is universal, while the process-dependent quantity resides in the Wilson coefficient. Even more, the coefficients $\mathbf{C}_{ab}$ defined in SCET are process-independent too. So the matching and running procedure in EFT naturally separated the process-dependent and universal contributions to a process, i.e., $\{\mathbf{A},\mathbf{B},\mathbf{C}\}$ are universal and only $\mathbf{H}_c^F$ is process-dependent. In pQCD, the formula and relation like Eqs. (47) and (51) have been proposed by the authors of [9].

Compared with SCET, pQCD analysis invoke gauge invariance and a new evolution equation [6] which comes from differentiating the jet-function from the factorized cross section with respect to the axial parameter in axial gauge to separate soft and collinear contributions, which is crucial to resum the double logarithms, since RGE in full theory only resums single logarithms between two scales.

We conclude this section with a chain for the Q_T resummation in this section,

$$\mathrm{QCD}\,|_{Q^2} \longrightarrow \mathrm{SCET_I}|_{Q^2\lambda^2} \Rightarrow \mathrm{SCET_{II}}|_{Q^2\eta^2} \longleftarrow \mathrm{DGLAP}|_{\mu_0^2},$$

where the last arrow indicates that the PDFs used at the scale $Q^2\eta^2$ can be obtained from those at some fixed scale μ_0^2 by the evolutions of the DGLAP equations.

IV. DISCUSSION

(I) Applying the formula (47) to the production of lepton pair via virtual photon, one can find,

$$\frac{1}{\sigma_{qq}^{(0)}}\frac{d\sigma^{\rm resum}}{dQ^2dydQ_T^2} = \mathbf{H}_q^{\rm DY}(Q)\int_0^\infty \frac{db}{2\pi}bJ_0(bQ_T) \times \sum_{ab} e^{-\mathbf{S}_q(b_0/b,Q)}(f_{a/p_1}\otimes\mathbf{C}_{qa})\left(x_1,\frac{b_0}{b}\right) \times (f_{b/p_2}\otimes\mathbf{C}_{qb})\left(x_2,\frac{b_0}{b}\right), \quad (52)$$

where

$$\sigma_{qq}^{(0)} = e_q^2\frac{4\pi^2\alpha_{em}^2}{9SQ^2},$$
$$\mathbf{H}_q^{\rm DY}(Q) = 1 + \frac{\alpha_s C_F}{2\pi}\left(-8+\frac{7}{6}\pi^2\right),$$
$$\mathbf{S}_q\left(\frac{b_0}{b},Q\right) = \int_{b_0^2/b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left[\mathbf{A}_q(\alpha_s)\log\frac{Q^2}{\mu^2}+\mathbf{B}_q(\alpha_s)\right], \quad (53)$$
$$\mathbf{C}_{qa}^{(0)} = \delta_{qa}\delta(1-z),$$
$$\mathbf{C}_{qa}^{(1)} = -2P_{qa}^{\epsilon}(z) - C_F\frac{\pi^2}{6}\delta_{qa}\delta(1-z).$$

(II) The reformulation of joint resummation can also be made straightforwardly in SCET. In fact, threshold resummation [5] under $z = Q^2/S \to 1$ for $d\sigma^{\rm resum}/dQ^2$ is performed in moment-N space, and the relevant $\lambda^2 = 1/\bar{N} \sim 1-z$ with $\bar{N} = e^{\gamma_E}N$. The conclusion of [5] can be represented by a chain,

$$\mathrm{QCD}\,|_{Q^2} \longrightarrow \mathrm{SCET_I}|_{Q^2\lambda^2} \Rightarrow \mathrm{SCET_{II}}|_{Q^2\eta^2} \longleftarrow \mathrm{DGLAP}|_{\mu_0^2}.$$

We observe that the two chains for Q_T and threshold resummation in SCET have identical structure. This suggests that we can do threshold and Q_T resummation for $d\sigma^{\rm resum}/dQ^2dQ_T^2$ simultaneously. The relevant $\lambda^2 \sim 1/\chi(\bar{N},\bar{b})$ with $\bar{b} \equiv bQ/b_0$ is an interpolation of $\lambda^2 \sim 1/\bar{N}$ and $\lambda^2 \sim 1/\bar{b}$, let us say [17]

$$\chi(\bar{N},\bar{b}) = \bar{b} + \frac{\bar{N}}{1+\rho\bar{b}/\bar{N}}, \qquad \rho = \frac{1}{4}, \quad (54)$$

which approaches to $\bar{N}$ for $\bar{b} \ll \bar{N}$ and to $\bar{b}$ for $\bar{b} \gg \bar{N}$, respectively. The matching steps for joint resummation then can be written as

$$\mathrm{QCD}\,|_{Q^2} \longrightarrow \mathrm{SCET_I}|_{Q^2/\chi} \Rightarrow \mathrm{SCET_{II}}|_{Q^2/\chi^2} \longleftarrow \mathrm{DGLAP}|_{\mu_0^2},$$

which leads to similar result as Eq. (47) corresponding to that of [17], and the Mellin transformed and jointly resummed cross section follows,

$$\frac{1}{\sigma^{(0)}}\frac{d\sigma^{\rm resum}(N)}{dQ^2dQ_T^2} = \mathbf{H}_c^F(Q)\int_0^\infty \frac{db}{2\pi} bJ_0(bQ_T) \times \sum_{ab} e^{-\mathbf{S}_c(Q/\chi,Q)}(\mathbf{C}_{ca}(N)f_{a/p_1}(N,Q/\chi)) \times (\mathbf{C}_{\bar{c}b}(N)f_{b/p_2}(N,Q/\chi)), \qquad (55)$$

where $\phi(N) \equiv \int_0^1 d\xi \xi^N \phi(\xi)$ for any function $\phi(\xi)$ with $0 \le \xi \le 1$ is used.

V. CONCLUSION

We have presented the method of Q_T resummation in the framework of SCET and given a simple correspondence between $\{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$ in SCET and the well known coefficients $\{A, B, C\}$ in pQCD, with which the available information is compatible. The equivalence of the two framework can be confirmed by higher order computation. We have also shown that the reformulation of joint resummation can be performed in SCET directly. So any process, which is confined to the soft and collinear regions by dynamics or kinematics, can be treated in SCET following the steps outlined above.

ACKNOWLEDGMENTS

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Note added.—After an expanded version of our manuscript was completed, a new paper [18] by A. Idilbi, X. D. Ji, and F. Yuan appeared, in which they also discussed the transverse momentum distribution in b space and derived a similar result to ours.

APPENDIX

In this appendix, the details of the calculation to extract $\mathbf{C}^{(1)}$ in SCET are given explicitly.

Because $\mathbf{C}^{(1)}$ is related to the emission of a soft or collinear gluon, and the phase space is already factorized in this case, we will exploit a special form at $\mathcal{O}(\alpha_s)$, for which there is no need to cover the nonperturbative region ($Q_T \sim \Lambda_{\rm QCD}$),

$$\frac{1}{\sigma^{(0)}}\frac{d\sigma^{\rm resum}}{dQ^2dydQ_T^2} = \mathbf{H}_c^F(Q)\frac{d}{dQ_T^2} \times [e^{-\mathbf{S}_c(\mu,Q)}\hat{\sigma}_{\rm SCET}(Q_T,Q,\mu)], \qquad (A1)$$

$$\hat{\sigma}_{\rm SCET}(Q_T,Q,\mu) = \int_0^{Q_T^2} dq_T^2 \frac{1}{\sigma^{(0)}}\frac{d\sigma^{\rm SCET}(\mu)}{dQ^2dydq_T^2} = J_{p_1}(x_1,Q_T,\mu)J_{p_2}(x_2,Q_T,\mu)S(Q_T,\mu). \qquad (A2)$$

The same reasoning as in Sec. III leads to

$$J_{p_1}(x_1,Q_T,\mu)J_{p_2}(x_2,Q_T,\mu)S(Q_T,\mu) = \sum_{a,b}(f_{a/p_1}\otimes\hat{\mathbf{C}}_{ca})(x_1,Q_T,\mu)(f_{b/p_2}\otimes\hat{\mathbf{C}}_{\bar{c}b})(x_2,Q_T,\mu). \qquad (A3)$$

Then we get the differential form for Q_T resummation,

$$\frac{1}{\sigma^{(0)}}\frac{d\sigma^{\rm resum}}{dQ^2dydQ_T^2} = \mathbf{H}_c^F(Q)\frac{d}{dQ_T^2}\sum_{ab}e^{-\mathbf{S}_c(Q_T,Q)}(f_{a/p_1}\otimes\hat{\mathbf{C}}_{ca}) \times (x_1,Q_T)(f_{b/p_2}\otimes\hat{\mathbf{C}}_{\bar{c}b})(x_2,Q_T). \qquad (A4)$$

Here $\mu = Q_T$ is set to minimize large factor in $\hat{\mathbf{C}}^{(1)}$. Taking the Q_T^2 integral of Eq. (47) and (A4) from 0 to Q_T^2 and then expanding them at $\mathcal{O}(\alpha_s)$, we find $\mathbf{C}^{(1)}(z,\frac{b_0}{b}) = \hat{\mathbf{C}}^{(1)}(z,Q_T)$, thus we will use Eq. (A2) and (A3) to calculate $\mathbf{C}^{(1)}$. However, it should be emphasized that this formula is valid only with $\{\mathbf{A}^{(1)},\mathbf{B}^{(1)},\hat{\mathbf{C}}^{(1)}\}$, and the two formulas must be equal at this order, which suggests a way to adjust the parameters in b^* prescription [6]. Previously, the authors (DDT) of [19] have derived a similar formula, which is corresponding to our result at leading logarithmic order (LLO). Later, the extended DDT formula in the CSS frame with $\{A^{(2)},B^{(2)},C^{(1)}\}$ was suggested in [20], whose coefficients $\{\tilde{A},\tilde{B},C\}$ at $\mathcal{O}(\alpha_s)$ are just our $\{\mathbf{A}^{(1)},\mathbf{B}^{(1)},\hat{\mathbf{C}}^{(1)}\}$. The real radiative contribution to the differential cross section at the scale Q_T^2 is

$$\frac{d\sigma_r^{\rm SCET}(\mu)}{dQ^2dydq_T^2} = \frac{1}{\Gamma(1-\epsilon)}\left(\frac{4\pi}{q_T^2}\right)^\epsilon \int\frac{du}{u}\delta\left(\frac{1}{u}(u-u_{\rm min}) \times (u-u_{\rm max})\right)\frac{|\mathcal{M}_{ab}^{(0)}(p_1,p_2,k,y)|^2}{8s(2\pi)^2}. \qquad (A5)$$

Here $\mathcal{M}_{ab}^{(0)}(p_1,p_2,k,y)$ denotes the corresponding matrix element, and the usual invariants are defined as

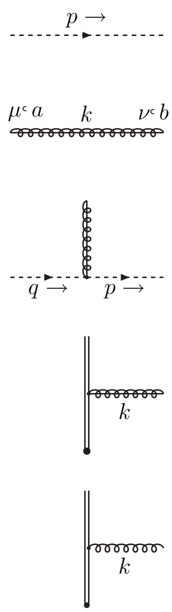
$$s = (p_1+p_2)^2, \qquad t = (p_1-k)^2,$$

$$u = (p_2-k)^2, \qquad z = \frac{Q^2}{s},$$

and the two roots of the equation $(p_1+p_2-q)^2 = 0$ are

$$u_{\rm min} = Q^2\frac{z-1-\sqrt{(1-z)^2-4zq_T^2/Q^2}}{2z},$$

$$u_{\rm max} = Q^2\frac{z-1+\sqrt{(1-z)^2-4zq_T^2/Q^2}}{2z}.$$



$$= i\frac{\not{n}}{2}\frac{\bar{n}\cdot p}{p^2}$$

$$= -ig^{\mu\nu}\delta^{ab}\frac{1}{k^2}$$

$$= igT^a\frac{\bar{\not{n}}}{2}\left(n_\mu + \frac{\not{p}_\perp\gamma_\mu^\perp}{\bar{n}\cdot p} + \frac{\gamma_\mu^\perp\not{q}_\perp}{\bar{n}\cdot q} - \frac{\not{p}_\perp\not{q}_\perp}{\bar{n}\cdot p\bar{n}\cdot q}\bar{n}_\mu\right)$$

$$= -gT^a\frac{\bar{n}_\mu}{\bar{n}\cdot k}$$

$$= -gT^a\frac{n_\mu}{n\cdot k}$$

FIG. 5. Feynman rules for n-direction collinear particles in Feynman gauge $\alpha = 1$ [1]. Here the direction of gluon momentum in the Wilson line is along the collinear particle. The rules for soft fields and collinear gluon are the same as that for QCD, and $n \leftrightarrow \bar{n}$ for $\bar{n}$-direction collinear particles.

The expression $|\mathcal{M}_{ab}^{(0)}(p_1, p_2, k, y)|^2$ in SCET can be written down by cut diagrams, and the Feynman rules we need are shown in Fig. 5. For example, in Fig. 6, the real contribution of the collinear gluon in the n direction is

$$\begin{aligned}
|\mathcal{M}_{qq}^{c1(0)}(p_1,p_2,k,y)|^2 = {}& -4\pi\alpha_s\mu^{2\epsilon}\frac{1}{36}C_F\Big\{\mathrm{Tr}\Big[s\gamma_\mu\frac{\not{n}}{2}\frac{\bar{n}\cdot(p_1-k)}{(p_1-k)^2}\frac{\bar{\not{n}}}{2}\Big(n_\nu + \frac{(\not{p}_1-\not{k})_\perp\gamma_\nu^\perp}{\bar{n}\cdot(p_1-k)} + \frac{\gamma_\nu^\perp\not{p}_{1\perp}}{\bar{n}\cdot p_1} - \frac{(\not{p}_1-\not{k})_\perp\not{p}_1}{\bar{n}\cdot(p_1-k)\bar{n}\cdot p_1}\bar{n}_\nu\Big) \\
& \times\frac{\not{n}}{2}\frac{\bar{\not{n}}}{2}\Big(n^\nu + \frac{\gamma_\perp^\nu(\not{p}_1-\not{k})_\perp}{\bar{n}\cdot p_1 - k} + \frac{\not{p}_{1\perp}\gamma_\perp^\nu}{\bar{n}\cdot p_1} - \frac{(\not{p}_1-\not{k})_\perp\not{p}_{1\perp}}{\bar{n}\cdot(p_1-k)\bar{n}\cdot p_1}\bar{n}^\nu\Big)\frac{\not{n}}{2}\frac{\bar{n}\cdot(p_1-k)}{(p_1-k)^2}\gamma^\mu\frac{\not{n}}{2}\Big] \\
& + 2\mathrm{Tr}\Big[s\gamma_\mu\frac{\bar{n}_\nu}{\bar{n}\cdot k}\frac{\not{n}}{2}\frac{\bar{\not{n}}}{2}n^\nu\frac{\not{n}}{2}\frac{\bar{n}\cdot(p_1-k)}{(p_1-k)^2}\gamma^\mu\frac{\not{n}}{2}\Big]\Big\} \\
= {}& -4\pi\alpha_s\mu^{2\epsilon}\frac{1}{36}\mathrm{Tr}\Big[s\gamma_\mu\frac{\not{n}}{2}\gamma^\mu\frac{\bar{\not{n}}}{2}\Big]2C_F\Big\{(1-\epsilon)\Big[2\frac{(p_1-k)_\perp\not{p}_{1\perp}}{\bar{n}\cdot(p_1-k)\bar{n}\cdot p_1} - \frac{(p_1-k)_\perp^2}{(\bar{n}\cdot(p_1-k))^2} - \frac{p_{1\perp}^2}{(\bar{n}\cdot p_1)^2}\Big] \\
& \times\Big[\frac{\bar{n}\cdot(p_1-k)}{(p_1-k)^2}\Big]^2 + 2\frac{\bar{n}\cdot(p_1-k)}{\bar{n}\cdot k(p_1-k)^2}\Big\}.
\end{aligned} \tag{A6}$$

If we drop the common factor

$$\mathcal{M} \equiv 4\pi\alpha_s\mu^{2\epsilon}\frac{1}{36}\mathrm{Tr}\Big[Q^2\gamma_\mu\frac{n}{2}\gamma^\mu\frac{\bar{n}}{2}\Big], \tag{A7}$$

and use the following parametrization for momenta p_1, p_2, k,

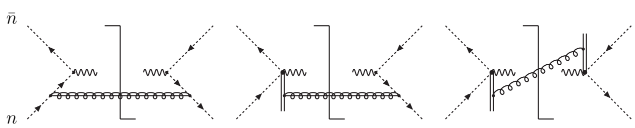


FIG. 6. Cut diagrams for the emission of gluon in $\mathrm{SCET_{II}}$, and the mirror diagrams are not shown

$$\begin{aligned} p_1 &= (0, P^-, 0), \qquad p_2 = (P^+, 0, 0), \\ k &= \Big(\frac{q_T^2}{(1-z_1)P^-}, (1-z_1)P^-, -q_T\Big), \end{aligned} \tag{A8}$$

$$\begin{aligned} s &= P^+P^-, \qquad t = \frac{-1}{1-z_1}q_T^2, \\ u &= -(1-z_1)s, \qquad tu = sq_T^2, \end{aligned} \tag{A9}$$

the above expression can be simplified to

$$|\mathcal{M}_{qq}^{c1(0)}(p_1, p_2, k, y)|^2 \rightarrow \frac{2C_F[(1-\epsilon)(1-z_1)^2 + 2z_1]}{zq_T^2}. \tag{A10}$$

The contribution of $\bar{n}$ collinear gluon is given by $n \leftrightarrow \bar{n}$ and $z_1 \leftrightarrow z_2$,

$$|\mathcal{M}_{qq}^{c2(0)}(p_1, p_2, k, y)|^2 \to \frac{2C_F[(1-\epsilon)(1-z_2)^2 + 2z_2]}{zq_T^2}, \tag{A11}$$

and the soft gluon contribution is

$$|\mathcal{M}_{qq}^{s(0)}(p_1, p_2, k, y)|^2 \to \frac{4C_F}{zq_T^2}. \tag{A12}$$

The condition that the emitted gluon is to be collinear is $(1-z_i)P^- \gg q_T$ or $z_i \to z$ for $i = 1, 2$, so only half of the phase space is covered, i.e., $u = u_{\min}$ for $u = -(1-z_i)s$, under which we could safely make the substitution $z_i \to z$. The soft gluon is guaranteed by $(1-z_i)P^- \sim q_T$ or $z \to 1$. Note that $|\mathcal{M}^{ci(0)}|^2 \to |\mathcal{M}^{s(0)}|^2$ when $z_i \to z \to 1$, which results in[7]

$$\begin{aligned}|\mathcal{M}_{qq}^{i(0)}(p_1, p_2, k, y)|^2 &= |\mathcal{M}_{qq}^{ci(0)}(p_1, p_2, k, y)|^2_{1-z_i \gg q_T/P^-} \\ &\quad + |\mathcal{M}_{qq}^{s(0)}(p_1, p_2, k, y)|^2_{1-z_i \sim q_T/P^-} \\ &\to \frac{2C_F[(1-\epsilon)(1-z)^2 + 2z]}{zq_T^2} \\ &\equiv \frac{2(1-z)P_{qq}(z, \epsilon)}{zq_T^2},\end{aligned} \tag{A13}$$

similarly,

$$|\mathcal{M}_{ab}^{i(0)}(p_1, p_2, k, y)|^2 \to \frac{2(1-z)P_{ab}(z, \epsilon)}{zq_T^2},$$

where

$$\begin{aligned}P_{qq}(z, \epsilon) &= C_F\left[\frac{1+z^2}{1-z} - \epsilon(1-z)\right], \\ P_{gq}(z, \epsilon) &= C_F\left[\frac{1+(1-z)^2}{z} - \epsilon z\right], \\ P_{gg}(z, \epsilon) &= 2C_A\left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right], \\ P_{qg}(z, \epsilon) &= T_R\left[1 - \frac{2z(1-z)}{1-\epsilon}\right].\end{aligned} \tag{A14}$$

These are the corresponding results of [8].

Next we match the Q_T integral Eq. (A2) onto PDFs after making Mellin transformation,

$$\begin{aligned}\Sigma_{q\bar{q}}(n) &\equiv \frac{\alpha_s}{2\pi}\Sigma_{q\bar{q}}^{(1)}(n) = \int_0^{1-(2q_T/Q)} dz z^n \frac{1}{\sigma^{(0)}} \frac{d\sigma_r^{\rm SCET}(\mu)}{dQ^2 dy dq_T^2}, \\ \Sigma_{q\bar{q}}^{(1)}(n) &= \frac{1}{\Gamma(1-\epsilon)} \frac{1}{q_T^2}\left(\frac{\mu^2 e^{\gamma_E}}{q_T^2}\right)^{\epsilon} I_n,\end{aligned} \tag{A15}$$

$$I_n = \int_0^{1-(2q_T/Q)} dz z^n \frac{2(1-z)P_{qq}(z, \epsilon)}{\sqrt{(1-z)^2 - 4z\frac{q_T^2}{Q^2}}}, \tag{A16}$$

where the upper limit $z = 1 - 2q_T/Q$ is to make the integrand meaningful, and the integral (A16) can be evaluated easily if we retain only the singular terms as $q_T^2 \to 0$,

$$\begin{aligned}I_n &\to \int_0^{1-(2q_T/Q)} dz \frac{2(1-z)P_{qq}(z, \epsilon)}{\sqrt{(1-z)^2 - 4z\frac{q_T^2}{Q^2}}} \\ &\quad + \int_0^1 dz (z^n - 1) 2P_{qq}(z, \epsilon) \\ &\to 2C_F \log\frac{Q^2}{q_T^2} - 3C_F + 2\gamma_{qq}(n) + 2\epsilon\gamma_{qq}^{\epsilon}(n),\end{aligned} \tag{A17}$$

where $\gamma_{qq}(n)$ and $\gamma_{qq}^{\epsilon}(n)$ are the moment of regularized splitting function and of $P_{qq}^{\epsilon}(z)$. So

$$\begin{aligned}\int_0^{Q_T^2} dq_T^2 \Sigma_{q\bar{q}}^{(1)}(n) &= \frac{1}{\Gamma(1-\epsilon)} \int_0^{Q_T^2} \frac{dq_T^2}{q_T^2}\left(\frac{\mu^2 e^{\gamma_E}}{q_T^2}\right)^{\epsilon}\left[2C_F \log\frac{Q^2}{q_T^2} - 3C_F + 2\gamma_{qq}(n) + 2\epsilon\gamma_{qq}^{\epsilon}(n)\right] \\ &= \frac{2C_F}{\epsilon^2} + \frac{1}{\epsilon}\left(3C_F + 2C_F \log\frac{\mu^2}{Q^2}\right) - \frac{2\gamma_{qq}(n)}{\epsilon} - 2\gamma_{qq}^{\epsilon}(n) - \frac{C_F\pi^2}{6} + C_F \log^2\frac{\mu^2}{Q_T^2} + 3C_F \log\frac{\mu^2}{Q_T^2} \\ &\quad - 2\gamma_{qq}(n) \log\frac{\mu^2}{Q_T^2} - 2C_F \log\frac{Q^2}{Q_T^2} \log\frac{\mu^2}{Q_T^2}.\end{aligned} \tag{A18}$$

The virtual correction comes from the UV renormalization constant,

$$2\delta Z_V = \frac{\alpha_s}{2\pi}\left[\frac{2C_F}{\epsilon^2} + \frac{1}{\epsilon}\left(3C_F + 2C_F \log\frac{\mu^2}{Q^2}\right)\right],$$

which cancels the first two ϵ poles. The remaining is cancelled by the renormalization of PDF, i.e.,

$$f_q(n) = f_{\bar{q}}(n) = 1 + \frac{\alpha_s}{4\pi}\left[2\gamma_{qq}(n)\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right)\right], \tag{A19}$$

where $\mu = \mu_F$ is implied. Finally, we get

[7]We have divided soft contribution into two parts.

$$\hat{\mathbf{C}}^{(1)}_{qq}(n, Q_T) = -2\gamma^{\epsilon}_{qq}(n) - \frac{C_F\pi^2}{6}, \tag{A20}$$

$$\mathbf{C}^{(1)}_{qq}\left(z, \frac{b_0}{b}\right) = -2P^{\epsilon}_{qq}(z) - \frac{C_F\pi^2}{6}\delta(1-z), \tag{A21}$$

Similarly,

$$\mathbf{C}^{(1)}_{ab}(z, \frac{b_0}{b}) = -2P^{\epsilon}_{ab}(z) - \frac{C_a\pi^2}{6}\delta_{ab}\delta(1-z). \tag{A22}$$

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