$X(3872)$ boson: Molecule or charmonium

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It has been argued that the mystery boson $X(3872)$ is a molecule state consisting of primarily $D^0\overline{D}^{*0} + \overline{D}^0 D^{*0}$. In contrast, apparent puzzles and potential difficulties have been pointed out for the charmonium assignment of $X(3872)$. We examine several aspects of these alternatives by semiquantitative methods since quantitatively accurate results are often hard to reach on them. We point out that some of the observed properties of $X(3872)$, in particular the binding and the production rates, are incompatible with the molecule interpretation. Despite puzzles and obstacles, $X(3872)$ may fit more likely to the excited $^3P_1$ charmonium than to the molecule after the mixing of $c\overline{c}$ with $D\overline{D}^* + \overline{D}D^*$ is taken into account.

I. INTRODUCTION

The narrow state $X(3872)$ was discovered by the Belle collaboration [1] in B decay and subsequently confirmed by the *BABAR* collaboration [2]. Both the CDF [3] and the D0 collaborations [4] saw a clear signal of inclusive $X(3872)$ production in pp collision. Since $X(3872)$ decays into $\pi^+\pi^- J/\psi$, it was the most natural to assign $X(3872)$ to one of the excited charmonia [5,6]. In fact, the experimental study of the $\pi\pi J/\psi$ mode in B decay had been encouraged by theorists, notably Eichten, Lane, and Quigg [7], as a means to explore the excited charmonia. However, experiment has revealed unexpected properties of $X(3872)$.

The very narrow width rules out assignment of $X(3872)$ to a natural spin-parity state $[P = (-1)^J$ with $C = P]$ since it would quickly decay into $D\overline{D}$. Among the unnatural spin-parity states, the $2^1P_1(h_c')$ charmonium of 1^{+-} was ruled out by Belle [8] at an early stage through the angular distribution of the final $\pi^+\pi^-$ [9]. If we trust the potential-model calculations of the charmonium mass spectrum from the past, there is no suitable candidate for $X(3872)$ in the close neighborhood of 3872 MeV. Meanwhile, the coincidence of the $X(3872)$ mass ($3871.9 \pm 0.5 \pm 0.5$ MeV [1]) with the $D^0\overline{D}^{*0}$ threshold (3871.3 ± 0.5 MeV) prompted many theorists [10] to revive the idea of molecular states [11] and to speculate that $X(3872)$ may be a loosely bound state of $D\overline{D}^* + \overline{D}D^*$ through a color-neutral force.

Then the Belle collaboration [12] reported the startling discovery that $X(3872)$ decays into $\omega J/\psi$ as well, actually $\pi^+\pi^-\pi^0$ off the ω resonance peak, with roughly the same branching fraction as $\pi^+\pi^- J/\psi$. Since any hadron must have a definite charge parity, the dipion $\pi^+\pi^-$ in $\pi^+\pi^- J/\psi$ ought to be in p-wave $[C = (-1)^l = -1]$, namely, the ρ and its resonance tail in $I = 1$. In the same analysis they saw a signal of the radiative decay mode $\gamma J/\psi$ [13], reinforcing the $C = +1$ assignment for $X(3872)$. With the mass difference between $D^0\overline{D}^{*0}$ and $D^+\overline{D}^{*-}$ being as large as 8 MeV, isospin violation may be enhanced if a very loosely bound moleculelike state should be formed. Then it would nicely explain the large isospin mixing in the $X(3872)$ decay. The most recent analysis of the decay angular distributions [14,15] favors $J^{PC} = 1^{++}$ for $X(3872)$ among the positive C states. Since $D\overline{D}^* + \overline{D}D^*$ would be most likely bound in s-wave, if at all, 1^{++} fits well to the molecule. The molecule interpretation has gained steam among some theorists for this reason. If $X(3872)$ is indeed a molecule state, it would open a large new field in hadron spectroscopy. Most recently, however, a few pieces of crucial experimental information [16], though yet preliminary, have appeared against the molecule assignment. We are now in a rapidly moving state of confusion.

In this paper we cast more doubt on the molecule interpretation of $X(3872)$. We first show that, unlike the proton and the neutron inside a deuteron, no long-range potential arises from one-pion exchange between D and $\overline{D}^*$. Though it may sound strange, it is a simple consequence of the numerical accident, $m_{D^*} - m_D - m_\pi \simeq 0$, and of the derivative $D^*D\pi$ coupling. This makes the "deuteronlike molecular binding" of $D\overline{D}^* + \overline{D}D^*$ highly unlikely. Secondly, we argue that the observed decay branching of $B^+ \to K^+ X(3872)$ in B decay and the inclusive production cross section of $X(3872)$ in high-energy proton-proton collision are both too large for a loosely bound object of binding energy 1 MeV or less.

We then turn to the charmonium option. We cannot offer a quantitative resolution as for the mass spectrum computed in the potential model. It has been acknowledged that the computation involves large uncertainties near and above the open charm thresholds. In the 1960's the elaborate coupled-channel N/D method was developed in the S-matrix theory to compute the spectrum of light hadrons, which were actually the molecule states in the present-day language. The N/D method in its simple version iterates the Born amplitudes in s-channel. The computation is very close to the coupled-channel potential problem [17]. But we realized at the end how difficult it was to obtain reliable quantitative results and could not go much beyond semi-quantitative analysis in most cases. The charmonia above the open charm threshold share similar uncertainties though the ambiguity coming from the high-energy tail is

more contained. We leave the difficulty of the $X(3872)$ mass with the potential-model calculation of charmonia to future study. Instead we focus on the production rate and cross section of $X(3872)$. Our estimate shows that because of the very loose binding of the molecule, the production rates of the molecule $X(3872)$ should be at least an order of magnitude smaller than what we see in experiment. In comparison, the charmonium is more easily produced because of a little tighter binding. One obvious obstacle to the charmonium option is the large isospin breaking in the decay $X(3872) \to \rho(\omega)J/\psi$. However, the ω resonance peak is nearly two full widths outside the phase space boundary. We ask if there is any chance that this severe kinematic suppression of the isospin-allowed decay into $\omega J/\psi$ can bring $B(X \to \rho J/\psi)/B(X \to \omega J/\psi)$ up close to $O(1)$. Our finding is that such chance is not ruled out within the small range of experimental uncertainty in the mass of $X(3872)$. Equally or even more serious is the absolute magnitude of the $X(3872) \to \rho(\omega)J/\psi$ decay rate; the decay through $c\bar{c}$ annihilation should not dominate over the mode $X(3872) \to \rho(\omega)J/\psi$ to hide them. If the charmonium mixes with a fair amount of the $D\overline{D}{}^* + D^*\overline{D}$ component, it could generate measurable branching fractions into $\rho(\omega)J/\psi$. The best scenario that emerges is as follows: The $X(3872)$ state is the 1^{++} charmonium that is bound primarily with the gluon-exchange force but mixed to $D\overline{D}{}^* + D^*\overline{D}$ of $I = 0$ through a light quark exchange. This scenario will survive even if $X(3872)$ is above the $D^0\overline{D}{}^{*0}$ threshold and the decay $X(3872) \to D^0\overline{D}{}^{*0} + \overline{D}{}^0D^{*0}$ indeed occurs. We must admit that many of the following analyses are only semiquantitative. This is an inevitable shortcoming due to the relevant long-distance dynamics that we do not really know in details.

II. MOLECULE STATE

A. One-pion exchange potential

We first show that one-pion exchange produces practically no force between D and $\overline{D}{}^*$ contrary to intuition. In the limit of $m_{D^*} = m_D + m_\pi$, there exists only a δ-function potential in the s-wave channel. Therefore it is incapable of binding D and $\overline{D}{}^*$.

Let us define the $D^*D\pi$ coupling by

$$L_{\text{int}} = -ig\overline{D}{}^{*\mu}\boldsymbol{\tau}D \cdot \partial_\mu \boldsymbol{\pi} + \text{H.c.} \qquad (1)$$

The value of g can be fixed by the $D^{*+} \to D^+\pi^0$ decay rate to $g^2/4\pi = 12.8 \pm 3.1$. A single pion can be exchanged only in the t-channel of the $D\overline{D}{}^* \to D^*\overline{D}$ scattering. (See Fig. 1.)

For concreteness, let us examine the π^0 exchange potential in $D^0\overline{D}{}^{*0} \to D^{*0}\overline{D}{}^0$. The one-pion exchange would be normally the primary source of the binding force. In the rest of this paper we shall write $D\overline{D}{}^* + \overline{D}D^*$ simply as $D\overline{D}{}^*$ unless it may cause a confusion. The potential is extracted from the Born amplitude near the threshold:
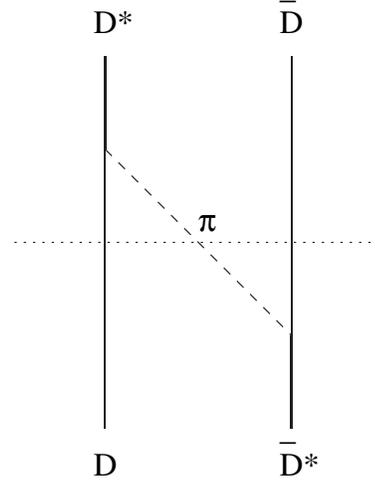

FIG. 1. The π-exchange in the scattering $D\overline{D}{}^* \to D^*\overline{D}$. The intermediate state at the horizontal dotted line occurs on mass shell for π^0 exchange.

$$T_B = (\boldsymbol{\epsilon}'^* \cdot \mathbf{q}) \frac{g^2}{m_\pi^2 - q^2 - i\varepsilon}(\boldsymbol{\epsilon} \cdot \mathbf{q}), \qquad (2)$$

where $\mathbf{q}$ is the pion momentum, and $\boldsymbol{\epsilon}$ and $\boldsymbol{\epsilon}'$ are the polarizations of $\overline{D}{}^{*0}$ and D^{*0}, respectively. It is important to notice here a peculiarity of kinematics in the pion propagator. Since the decay $\overline{D}{}^{*0} \to \overline{D}{}^0\pi^0$ actually occurs with a tiny Q-value, the denominator of the propagator is practically zero as compared with m_π in the nonrelativistic limit;

$$m_\pi^2 - q^2 \simeq m_\pi^2 - \left(m_{D^*} - m_D + \frac{\mathbf{q}^2}{8m_{D^*}} - \frac{\mathbf{q}^2}{8m_D}\right)^2 + \mathbf{q}^2,$$

$$\simeq -2m_\pi\Delta + \left(1 + \frac{m_\pi^2}{4m_Dm_{D^*}}\right)\mathbf{q}^2$$

$$+ O\left(\Delta^2, \frac{\mathbf{q}^2\Delta}{m_D}, \left(\frac{\mathbf{q}^2}{m_D}\right)^2\right), \qquad (3)$$

where $\Delta \equiv m_{D^*} - m_D - m_\pi$ is very small ($\simeq 7$ MeV). Let us denote the static limit of the denominator by

$$\mu^2 = 2m_\pi\Delta(\simeq 0.1m_\pi^2). \qquad (4)$$

With the two powers of $\mathbf{q}$ from the derivative $DD^*\pi$ coupling in the numerator, the Born amplitude T_B survives only at $\mathbf{q}^2 \gg 2m_\pi\Delta$ and varies slowly there. The Fourier transform of T_B gives the one-pion exchange potential. Since $\overline{D}{}^{*0} \to \overline{D}{}^0\pi^0$ can occur on mass shell, the principal part of the pion denominator is relevant to the potential:

$$4m_D m_{D^*} V(\mathbf{r}) = \int \mathrm{Re}\, T_B(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}} \frac{d^3\mathbf{q}}{(2\pi)^3}$$

$$\simeq \int (\boldsymbol{\epsilon}'^* \cdot \mathbf{q})(\boldsymbol{\epsilon} \cdot \mathbf{q}) \frac{g^2 e^{i\mathbf{q}\cdot\mathbf{r}}}{\mu^2 - \mathbf{q}^2} \frac{d^3\mathbf{q}}{(2\pi)^3}$$

$$\simeq -\frac{1}{3} g^2 (\boldsymbol{\epsilon}'^* \cdot \boldsymbol{\epsilon})\delta(\mathbf{r}) + \frac{g^2}{4\pi}(\boldsymbol{\epsilon}'^* \cdot \hat{\mathbf{r}})(\boldsymbol{\epsilon} \cdot \hat{\mathbf{r}})$$

$$\times \frac{\mu^2 \cos\mu r}{r} + \frac{g^2}{4\pi}((\boldsymbol{\epsilon}'^* \cdot \boldsymbol{\epsilon}) - 3(\boldsymbol{\epsilon}'^* \cdot \hat{\mathbf{r}})$$

$$\times (\boldsymbol{\epsilon}'^* \cdot \hat{\mathbf{r}}))\left(\frac{\cos\mu r}{r^3} + \frac{\mu \sin\mu r}{r^2}\right). \tag{5}$$

For the s-wave potential relevant to binding of $D\overline{D}^*$ into a molecule of 1^{++}, only a tiny contribution of $O(\mu^2)$ survives aside from the δ-function term since the last term (valid off $r=0$) in Eq. (5) goes away up to $O(\mu^2)$ after partial-wave projection.[1] Contrary to the naive expectation, therefore, the one-pion exchange potential is a δ-function in good approximation between $D(\overline{D})$ and $\overline{D}^*(D^*)$. Unlike the one-dimensional δ-function potential, the three-dimensional δ-function potential cannot generate a bound state when one puts it in the Schrödinger equation. The Yukawa potential can arise only from "crossed" multipion exchange. Although the dipion-exchange potential of an intermediate range plays an important role in tightly bound nuclei [18] and even in the deuteron by providing a spin-isospin independent force, lack of the long-range force would not lead to binding of the deuteron. Therefore the speculation fails that $X(3872)$ may be an analog of the deuteron on the basis of a long-range force [19]. Swanson [20] "appended" the one-pion exchange potential with a quark-model potential which he calls the "quark Born diagram potential" [21]. He turned his quark potential into a meson-meson interaction potential and added to it the standard one-pion exchange potential of range $1/m_\pi$, which should be absent between D and $\overline{D}^*$ according to our argument. For a spatially extended composite state, it is the force of the longest range that is most sensitive to binding. It is not clear in his paper how much his quark-model potential is responsible for binding of $X(3872)$. While we are unable to read quantitative details of calculation in his short paper, it appears that some fundamental revision is needed in his calculation of binding, at least in the one-pion exchange part.

The imaginary part of the Born amplitude T_B due to the on-shell intermediate state gives an absorptive potential: Its s-wave contribution is small: $\mathrm{Im}\, V(r) \sim (g^2\mu^2/12\pi r) \times (\boldsymbol{\epsilon}' \cdot \boldsymbol{\epsilon})\sin\mu r$. This absorptive potential is nothing other than a tiny channel coupling to $\pi^0 D^0\overline{D}^0$ by pion emission

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[1]For the π^+-exchange in $D^0\overline{D}^{*0} \to D^{*+}D^-$, the Yukawa potential appears rather than the oscillatory potential since the on-shell transition does not occur with $m_{\overline{D}^{*0}} < m_{D^-} + m_{\pi^+}$. However, the leading long-range s-wave potential is $\sim\mu^2 e^{-\mu r}/r$ instead of $\sim\mu^2 \cos\mu r/r$ and still totally negligible.

from $\overline{D}^{*0}$. The derivative pion coupling or the p-wave emission of the pion in $D^* \to D\pi$ is the source of the behavior $\mathrm{Im}\, V(r) \sim \mu^3$ as $\mu \to 0$.

It is obvious why the one-pion exchange does not lead to the familiar Yukawa potential of $e^{-m_\pi r}/r$: The pion emission in $D^* \to D + \pi$ can occur without violating energy conservation so that the emitted pion can travel over long distances. This would otherwise produce a long-range potential of $\sim 1/r$ and $1/r^3$, but the derivative coupling erases them leaving only the δ-function in the small μ limit.

Is it possible to bind D and $\overline{D}^*$ through coupling to the $\omega J/\psi$ channel? In general, a channel coupling can enhance the effective potential for the channel of the strongest force. For this channel coupling to occur, D (or D^*) must be exchanged. (See Fig. 2.) Since the denominator of the $D(D^*)$ propagator is $\simeq 2m_{D/D^*} m_\omega$ or larger, the range of this off-diagonal Yukawa potential is very short ($\simeq 0.1$ F). Consequently, no strong coupling occurs between the $D\overline{D}^*$ and the $\omega J/\psi$ channel at distances large enough to be relevant to a loosely bound molecule. As for the diagonal potential of $\omega J/\psi$, the elastic scattering process is the so-called "disconnected" quark diagram (Fig. 3) or multigluon-exchange processes. Just as $\phi(s\bar{s})$ interacts only weakly with π, ρ, ω and so forth even at low energies, elastic $\omega J/\psi$ scattering is expected to be weak and hardly a source of binding. To summarize, we see no chance of generating a bound state in the s-wave $D\overline{D}^*$ channels, no matter how the Born diagrams are iterated, by solving the Schrödinger equation or the Bethe-Salpeter equation with or without channel coupling, or by approximating the N-function of the matrix N/D with the Born amplitudes.

If there is any relevant channel coupling, it would be to the $c\bar{c}$ channel. The confining potential between c and $\bar{c}$ is stronger than the hadron exchange potentials. The coupling between $D\overline{D}^*$ and $c\bar{c}$ occurs through the $u(d)$-quark exchange. (See Fig. 4.) If this channel coupling is strong enough, $D\overline{D}^*$ and $c\bar{c}$ can mix substantially. Even when such a mixing occurs, the primary source of binding is the
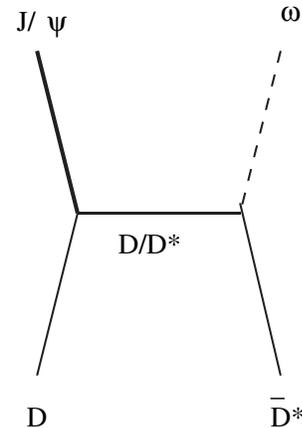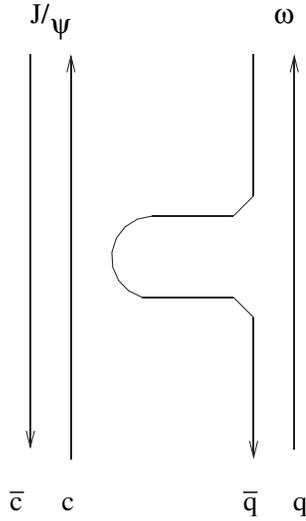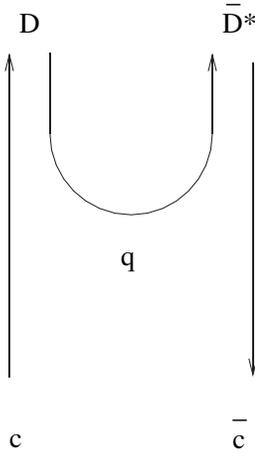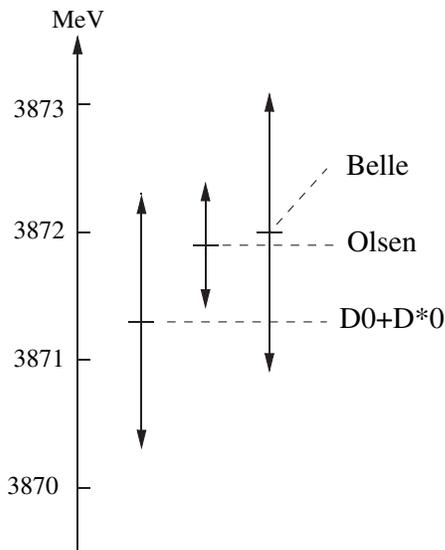

FIG. 2. The D/D^* exchange in $D\overline{D}^* \to \omega J/\psi$.

FIG. 3.   The disconnected quark diagram of $\omega J/\psi \to \omega J/\psi$.

FIG. 4.   The light quark exchange in $c\bar{c} \to D\overline{D}^*$.

FIG. 5.   The ranges of $m_{X(3872)}$ vs the value of $m_{D^0} + m_{D^{*0}}$ (denoted as $D0 + D^*0$).

confining force of gluon exchange between c and $\bar{c}$, not the van der Waals force of QCD, i.e. not the hadron exchange force in the $D\overline{D}^*$ channel. Such a bound state is conceptually not the molecule and dynamically different from it . We will later discuss this possibility in more detail.

B. Decay rate and production cross section

We compare the observed production rates of $X(3872)$ with the theoretical expectation for the molecule state. We are limited to semiquantitative discussion here since accurate calculation would require much more information of long-distance dynamics and experimental input than we have at present. Nonetheless we see a serious difficulty.

The $X(3872)$ mass has been given by the four experimental groups as follows [1–4]:

$$m_{X(3872)} = \begin{cases} 3872.0 \pm 0.6 \pm 0.5 \text{ MeV} & \text{(Belle)} \\ 3873.4 \pm 1.4 \text{ MeV} & (BABAR) \\ 3871.3 \pm 0.7 \pm 0.4 \text{ MeV} & \text{(CDFII)} \\ 3871.8 \pm 3.1 \pm 3.0 \text{ MeV} & \text{(D0).} \end{cases} \quad (6)$$

Olsen [8] quotes 3871.9 ± 0.5 MeV as the weighted average from all groups. We should compare these numbers with $m_{D^0} + m_{D^{*0}} = 3871.3 \pm 1.0$ MeV and $m_{D^+} + m_{D^{*+}} = 3879.4 \pm 1.0$ MeV [22]. For m_D and m_{D^*}, we have added the quoted errors ± 0.5 MeV of the *Review of Particle Physics* [22] since the errors are correlated between m_{D^0} and $m_{D^{*0}}$ and between m_{D^+} and $m_{D^{*+}}$. Although the central values are on the side of $m_{X(3872)} \geq m_{D^0} + m_{D^{*0}}$, even the number from Belle that has the highest accuracy is not conclusive as to whether $m_{X(3872)}$ is really above the $D^0\overline{D}^{*0}$ threshold or not. We have shown the current ranges of $m_{X(3872)}$ and $m_{D^0} + m_{D^{*0}}$ in Fig. 5.

In order for $X(3872)$ to be a molecular bound state, $X(3872)$ must be below the $D^0\overline{D}^{*0}$ threshold; $m_{X(3872)} < m_{D0} + m_{D^{*0}}$. However, Belle apparently saw a large decay branching to $D^0\overline{D}^{0*}$ with the preliminary result [16],

$$B(X(3872) \to D^0\overline{D}^0\pi^0)/B(X(3872) \to \pi^+\pi^- J/\psi) \approx 10. \quad (7)$$

If this result holds, the molecule model ought to be modified into the ''virtual state'' model [23]. For our discussion of the molecule, we proceed with

$$m_{X(3872)} < m_{D^0} + m_{D^{*0}} = 3871.3 \pm 1.0 \text{ MeV}. \quad (8)$$

This places the binding energy of $D^0\overline{D}^{*0}$ in $X(3872)$ at less than 1 MeV.

The production rate of a loosely bound state in decay and scattering is obtained generally by convoluting the bound-state wave function with a production amplitude of its constituents. When the range of production dynamics is much shorter than the size of the bound state, the production of the constituents and the formation of a bound state factorize. If $X(3872)$ is a loosely bound state of $D^0\overline{D}^{*0}$, the constituent momenta in the $X(3872)$ rest frame

is $\simeq \sqrt{E_B m_{D/D^*}}(\simeq 30 \text{ MeV})$. The constituents are streaming in parallel with practically no relative motion. Even after boosting to the overall c.m. frame in B decay or in pp collision at small rapidity, the relative momentum is still much smaller than the characteristic momentum over which the production amplitude $A(\mathbf{p}, \mathbf{k})$ of $D(\frac{1}{2}\mathbf{p}, \mathbf{k})\overline{D}^*(\frac{1}{2}\mathbf{p} - \mathbf{k}, s)$ varies significantly. In this case the invariant amplitude of production may be approximated as

$$A(\mathbf{p}, \mathbf{k}) \simeq A(\mathbf{p}, \mathbf{0}), \tag{9}$$

where we have suppressed dependence of $A(\mathbf{p}, \mathbf{k})$ on all other variables. Let us express the s-wave bound state of $\overline{D}D^*$ in the rest frame with the wave function $\tilde{\Psi}(\mathbf{k})$ in momentum space and the creation operators $a^\dagger_{\mathbf{k}s}$ and $a^\dagger_{-\mathbf{k}}$ of $\overline{D}^*$ and D, respectively:

$$|X(\mathbf{p} = 0, s)\rangle = \int \Psi(\mathbf{k}) \frac{a^\dagger_{\mathbf{k}s} a^\dagger_{-\mathbf{k}} + \text{conj.}}{\sqrt{2}} |0\rangle \frac{d^3 k}{(2\pi)^3}. \tag{10}$$

For production of $X(3872)$ with momentum $\mathbf{p}$ and a given helicity, start with the production amplitude of $D\overline{D}^*$ with small relative momentum $\mathbf{k}$:

$$B/pp \rightarrow D + \overline{D}^* + \text{anything}. \tag{11}$$

Superpose the production amplitude $A(\mathbf{p}, \mathbf{k})$ with the bound-state wave function first in the c.m. frame of $\overline{D}D^*$ to get the X production amplitude in the molecule rest frame:

$$\int \frac{1}{\sqrt{4E_D E_{D^*}}} \tilde{\Psi}(\mathbf{k}) A(\mathbf{0}, \mathbf{k}) \frac{d^3 k}{(2\pi)^3}. \tag{12}$$

When $|\mathbf{k}|$ is so small that $A(\mathbf{0}, \mathbf{k}) \simeq A(\mathbf{0}, \mathbf{0})$, we factor out $A(\mathbf{0}, \mathbf{0})$ and integrate over $\mathbf{k}$ to obtain $(4m_D m_{D^*})^{-1/2}\Psi(\mathbf{0})A(\mathbf{0}, \mathbf{0})$, where $\Psi(\mathbf{0})$ is the wave function at the origin of the relative position. Moving back to the overall c.m. frame, one obtains for the production rate of X with momentum $\mathbf{p}$,

$$(2\pi)^3 E_p \frac{d\Gamma}{d^3 p} \simeq \frac{|\Psi(\mathbf{0})|^2}{4m_D m_{D^*}} \sum_i (2\pi)^4$$
$$\times \delta^4\Big(P - p - \sum_j p_j\Big)|A(\mathbf{p}, \mathbf{0})|^2, \tag{13}$$

where P^μ is the initial four-momentum and the first $\sum$ denotes integration and summation over degrees of freedom of all final particles other than $X(3872)$. Because of the very loose binding, $|\Psi(\mathbf{0})|^2$ appears even when D and D^* are produced by nonlocal interactions.

The wave function at origin square $|\Psi(\mathbf{0})|^2$ in Eq. (13) gives us a clue about the production rates. The value of $|\Psi(\mathbf{0})|^2$ is fairly insensitive to details of dynamics for a very loosely bound state. We estimate as follows: Note first model independently $\tilde{\Psi}(\mathbf{k}) \propto 1/(\kappa^2 + \mathbf{k}^2)$ near the $X(3872)$ pole ($\kappa^2 = 2E_B m_D m_{D^*}/(m_D + m_{D^*})$). The $\tilde{\Psi}(\mathbf{k})$

of this form is not suitable for determining $\Psi(\mathbf{0})$ since its Fourier transform $\Psi(\mathbf{r}) \sim e^{-\kappa r}/4\pi r$ is infinite at $r = 0$. It means that the behavior above $|\mathbf{k}| \simeq \kappa$ matters in determining $\Psi(\mathbf{0})$. $\tilde{\Psi}(\mathbf{k})$ must fall off faster at large $|\mathbf{k}|$ than the simple pole form.[2] We regularize $\tilde{\Psi}(\mathbf{k})$ into the double pole form with parameter M,

$$\tilde{\Psi}(\mathbf{k}) = N\Big(\frac{1}{\mathbf{k}^2 + \kappa^2} - \frac{1}{\mathbf{k}^2 + M^2}\Big), \tag{14}$$

where $N = \sqrt{8\pi\kappa M(M + \kappa)}/(M - \kappa)$ is the normalization. Then we obtain

$$|\Psi(\mathbf{0})|^2 = \frac{\kappa M(M + \kappa)}{2\pi}. \tag{15}$$

For our estimate we choose $M \simeq 3\kappa$ for which the regulator term contributes only about 5% at $r = 1/\kappa$ in Eq. (14). Then in the ratio to $|\Psi(\mathbf{0})_{\psi(2S)}|^2 \simeq 0.024 \text{ GeV}^3$ of $\psi(2S)$, we obtain

$$\left|\frac{\Psi(\mathbf{0})_{X(3872)}}{\Psi(\mathbf{0})_{\psi(2S)}}\right|^2 \simeq 2.5 \times 10^{-3} \Big(\frac{E_B}{0.5 \text{ MeV}}\Big)^{3/2}. \tag{16}$$

Since the wave function spreads far out in space, the value at the origin $|\Psi(\mathbf{0})_{X(3872)}|^2$ is quite small. Consequently, production of $X(3872)$ is highly suppressed in B decay and in pp collision. We proceed with Eq. (16) to compare with experiment.

$B \rightarrow K^+ X(3872)$ *decay.*—For the charmonium production through the decay $\overline{b} \rightarrow \overline{c}c\overline{s}$, the color-suppressed tree interaction is important and the factorization calculation appears to be qualitatively in line with experiment. The two-body charmonium decay $B^+ \rightarrow K^+(c\overline{c})$ has been observed for η_c, J/ψ, χ_{c1}, and $\psi(2S)$ [22,24]:

$$B(B^+ \rightarrow K^+ \eta_c) = 9.0 \pm 2.7 \times 10^{-4},$$
$$B(B^+ \rightarrow K^+ J/\psi) = 10.61 \pm 0.15 \pm 0.48 \times 10^{-4},$$
$$B(B^+ \rightarrow K^+ \chi_{c1}) = 5.79 \pm 0.26 \pm 0.65 \times 10^{-4}, \tag{17}$$
$$B(B^+ \rightarrow K^+ \psi(2S)) = 6.49 \pm 0.59 \pm 0.97 \times 10^{-4}.$$

In comparison, only the upper bounds have been obtained for two-body production of χ_{c0} and χ_{c2} that would not occur in the simple factorization limit:

$$B(B^+ \rightarrow K^+ \chi_{c0}) < 4.4 \pm 3.3 \pm 0.7 \times 10^{-4}$$
$$B(B^+ \rightarrow K^+ \chi_{c2}) < 0.09 \pm 0.10 \pm 0.11 \times 10^{-4}. \tag{18}$$

The Belle collaboration [25] has determined the product of the branching fractions as

$$B(B^+ \rightarrow K^+ X(3872)) \times B(X(3872) \rightarrow \pi^+ \pi^- J/\psi)$$
$$= 1.31 \pm 0.24 \pm 0.13 \times 10^{-5}. \tag{19}$$

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[2]If one proceeds to compute Eq. (12) with the simple pole form for $\tilde{\Psi}(\mathbf{k})$, there is a danger to overestimate the production rate.

The experimental lower bound can be set on $B(B^+ \to K^+ X(3872))$ with Eq. (19) after taking account of the presence of $B(X(3872) \to \pi^+ \pi^- \pi^0 J/\psi)$:

$$B(B^+ \to K^+ X(3872)) > 2.6 \times 10^{-5}. \qquad (20)$$

However, if the preliminary result of Belle [Eq. (7)] is used, the lower bound rises to

$$B(B^+ \to K^+ X(3872)) > 1.6 \times 10^{-4}. \qquad (21)$$

Computation of the decay amplitude for $B^+ \to K^+ X(3872)$ in the case of the molecular $X(3872)$ has been attempted, but without a definite numerical result for the branching fraction [26]. It is not surprising when one considers uncertainties involved in such computations. Nonetheless, if we dare to make a very crude estimate, we would proceed with Eq. (13) and compare $B(B^+ \to K^+ X(3872))$ with $B(B^+ \to K^+ \psi(2S))$:

$$\frac{B(B^+ \to K^+ X(3872))}{B(B^+ \to K^+ \psi(2S))} \approx \left| \frac{A(B^+ \to K^+ D\overline{D}^*)}{A(B^+ \to K^+ c\overline{c})} \right|^2$$
$$\times \left| \frac{\Psi(\mathbf{0})_{X(3872)}}{\Psi(\mathbf{0})_{\psi(2S)}} \right|^2, \qquad (22)$$

where the $c\overline{c}$ is in 1^{--} and the $D\overline{D}^*$ is in 1^{++}, both near their production thresholds. We estimate the first factor in the right-hand side Eq. (22) with the following reasoning. Imagine that a $c\overline{c}$ pair is produced near its threshold, most likely in relative s-wave. When the $c\overline{c}$ invariant mass is below the open charm threshold, the s-wave $c\overline{c}$ forms a charmonium, J/ψ, $\psi(2S)$, η_c, or η_c'. Above the threshold, they form a pair of $D^{(*)}$ and $\overline{D}^{(*)}$ by picking up light quarks. Since by assumption the $c\overline{c}$ pair production amplitude is insensitive to its invariant mass, we expect

$$A(B^+ \to K^+ (D^0 \overline{D}^{*0})_{C=+1}) \approx 0.5 \times A(B^+ \to K^+ (c\overline{c})), \qquad (23)$$

where the energy $E_{c\overline{c}}$ is a little below the open charm threshold while $E_{D\overline{D}^*}$ is a little above it. The factor 0.5 comes from counting of the relevant charmonia and the multiplicity of spin, charge, charge parity states of charmed meson pairs after $u\overline{u}$ or $d\overline{d}$ are picked up. We use Eq. (16) for the second factor in Eq. (22). Combining the two factors together, we reach for the molecule $X(3872)$

$$B(B^+ \to K^+ X(3872)) \approx 1.3 \times 10^{-3} \left(\frac{E_B}{0.5\,\text{MeV}} \right)^{3/2}$$
$$\times B(B^+ \to K^+ \psi(2S)), \qquad (24)$$
$$\approx 0.8 \times 10^{-6} \left(\frac{E_B}{0.5\,\text{MeV}} \right)^{3/2}.$$

The number in the last line is more than 1 order of magnitude smaller than the experimental lower bound $\simeq 2.6 \times 10^{-5}$ in Eq. (20). It is more than 2 orders of magnitude smaller if the preliminary result on the $D^0\overline{D}^{*0}$ mode is taken into account. Physically speaking, it is not easy to produce a large composite object like the molecule in the short-distance B decay since $|\Psi(\mathbf{0})|^2$ requires the constituents to come close to each other in the position space. To enhance the production rate to the level of the experimental lower bound, $X(3872)$ must be an object of stronger binding. Ironically from this viewpoint, if $X(3872)$ were a bound state primarily of $D^+ D^{*-}$ instead of $D^0 \overline{D}^{*0}$, the decay branching fraction would be a little closer to experiment.

$pp \to X(3872) +$ anything.—The CDF collaboration observed 580 ± 100 events of X production in the region of rapidity $|\eta| < 1$ at $\sqrt{s} = 1.96\,\text{TeV}$ when $X(3872)$ is identified with $\pi^+ \pi^- J/\psi$. They are about 10% of $\psi(2S)$ production events in the same kinematical region [3]:

$$\frac{\sigma(pp \to X(3982) + \cdots)B(X(3872) \to \pi^+ \pi^- J/\psi)}{\sigma(pp \to \psi(2S) + \cdots)B(\psi(2S) \to \pi^+ \pi^- J/\psi)}$$
$$= \frac{580 \pm 100}{5790 \pm 140}(\simeq 0.10), \qquad (25)$$

where "$\cdots$" denotes "anything." The cross section for $\psi(2S)$ above is that of the primary production ignoring the $\sim$10%–20% contribution of the feed-down from the $B(\overline{B})$ decay [27]. The ratio of the decay branching fractions is 1.6 if $X(3872) \not\to D^0\overline{D}^{*0}$ and 0.27 if $B(X(3872) \to D^0\overline{D}^0\pi^0) = 10 \times B(X(3872) \to \pi^+\pi^- J/\psi)$ [cf. Eq. (21)]. Therefore,

$$\sigma(pp \to X(3982) + \cdots) = (0.06 \sim 0.36)$$
$$\times \sigma(pp \to \psi(2S) + \cdots). \qquad (26)$$

We compare the yields for $X(3872)$ and $\psi(2S)$ production using the same argument on the amplitude ratio as described for the B decay.

$$\sigma(p\overline{p} \to X(3872) + \cdots)$$
$$\approx \left| \frac{A(p\overline{p} \to D\overline{D}^* + \cdots)}{A(p\overline{p} \to c\overline{c} + \cdots)} \right|^2 \left| \frac{\Psi(\mathbf{0})_{X(3872)}}{\Psi(\mathbf{0})_{\psi(2S)}} \right|^2$$
$$\times \sigma(p\overline{p} \to \psi(2S) + \cdots). \qquad (27)$$

This leads us to the estimate

$$\sigma(pp \to X(3982) + \cdots) \approx 1.3 \times 10^{-3} \left(\frac{E_B}{0.5\,\text{MeV}} \right)^{3/2}$$
$$\times \sigma(pp \to \psi(2S) + \cdots). \qquad (28)$$

The ratio of $X(3872)$ and $\psi(2S)$ production is one and a half to two orders of magnitude smaller than the CDF observation, depending on whether $X(3872)$ indeed decays into $D^0\overline{D}^{*0}$ or not. Strong suppression of the $D\overline{D}^*$ molecule production should not be a surprise. Such suppression was well known experimentally in similar situations: The production cross section of a deuteron ($E_B = 2.2\,\text{MeV}$) in $pp \to d\pi^+$ at the c.m. energy $\sqrt{s} = 2.98\,\text{GeV}$ [28], for instance, is only $0.11 \pm 0.06\,\text{mb}$, one hundredth of the

continuum $pp \to pn\pi^+$ production cross section, 11.44 ± 0.65 mb, at the same energy.

To summarize the case of the molecule, lack of the binding force is a serious shortcoming on a theoretical side. Despite the numerical uncertainties involved in our theoretical estimate, experiment shows that production of $X(3872)$ is much stronger than we expect for a very loosely bound state. Our conclusion on the molecule applies to the $D\bar{D}^*$ virtual state as well since the wave function of the virtual state spreads even more than that of the bound state.

III. CHARMONIUM

If $X(3872)$ is a charmonium, the most likely candidate is the radially excited $^3P_1$ state with 1^{++}. While $^1D_2(2^{-+})$ may look promising, the angular distribution analysis disfavors it [14,15]. The charmonium interpretation of $X(3872)$ encounters two immediate problems. One is the discrepancy with the potential-model calculation of the mass spectrum [5,6,29] and the other is the large decay branching into the $I = 1$ channel, $X(3872) \to \rho J/\psi$. We do not attempt to propose any resolution for the potential-model calculation. Soon after the first discovery of charmonia, the mass spectrum below the open charm threshold was fitted and reproduced well with the Cornell potential model [30]. Above the open charm threshold, calculation must include coupling to the charm-meson-pair channels. While such computation was already undertaken even in the early paper of the Cornell model, the results involved much larger uncertainties than those below the threshold. A decade before the charmonium, strenuous efforts had been made in multichannel computation of the light hadron mass spectrum. However, we were unsuccessful in producing quantitatively accurate results and content with the semiquantitative predictions in the *strongest-attractive-force channel* argument.[3] With this excuse, we will not go into the question as to whether or not the channel coupling to $\bar{D}D^*$ and $D^*\bar{D}^*$ can indeed lower the 2^3P_1 mass to 3872 MeV from the existing predictions of the potential-model calculation. As for the production in B-decay and pp-collision, the larger wave function overlap of the 1^{++} charmonium than that of the molecule enhances significantly the production rate. However, we are unable to produce quantitatively accurate results, since the charmonium wave function is just as sensitive to the channel coupling as the mass spectrum is. Furthermore, the approximation of the wave function at origin is less reliable for the charmonium production in B decay.

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[3]In retrospect, of course, we were barking up the wrong tree in those days by trying to reproduce the quark-antiquark bound states as the "molecular states." Nonetheless, lack of accuracy or magnitude of uncertainty in the multichannel calculation of the bound-state spectrum was clearly appreciated. The N/D method was the most commonly used technique in those computations. It is more sophisticated than the naive nonrelativistic potential model and its extensions [17].

Leaving the mass spectrum aside, we instead focus on the issue of the large $\rho J/\psi$ branching fraction.

A. Kinematical suppression of $X(3872) \to \rho(\omega)J/\psi$

Coexistence of the $\omega J/\psi(I = 0)$ and $\rho J/\psi(I = 1)$ decay modes [12] appears as a strong argument in favor of the molecule model:

$$\frac{B(X \to \pi^+\pi^-\pi^0 J/\psi)}{B(X \to \pi^+\pi^- J/\psi)} = 1.0 \pm 0.4 \pm 0.3. \quad (29)$$

However, this relative branching fraction can be misleading with respect to the magnitude of isospin mixing in $X(3872)$. The $\pi^+\pi^-\pi^0$ in the final state $\pi^+\pi^-\pi^0 J/\psi$ comes from the far tail of the ω resonance since $\omega J/\psi$ is outside the phase space boundary. In contrast, $\rho J/\psi$ is right on the phase space boundary so that $\pi^+\pi^-$ can come from the lower half of the ρ resonance region. Consequently, $\pi^+\pi^-\pi^0$ receives much stronger suppression than $\pi^+\pi^-$. Magnitude of this relative suppression is purely kinematical and sensitive to the $X(3872)$ mass even within the small uncertainty of the $X(3872)$ mass.

The mass difference $m_X - m_{J/\psi}$ is smaller than the peak value of the ω resonance (782.6 ± 0.1) by several MeV:

$$m_X - m_{J/\psi} - m_\omega = \begin{cases} -7.5 \pm 0.6 \pm 0.5 \, \text{MeV} & \text{(Belle)} \\ -6.1 \pm 1.4 \, \text{MeV} & (BABAR) \\ -8.2 \pm 0.7 \, \text{MeV} & \text{(CDF)}. \end{cases} \quad (30)$$

Recall furthermore that $m_{D^0} + m_{D^{*0}} \geq m_{X(3872)}$ is required to prevent the open charm decay, that is,

$$m_{X(3872)} \leq m_{D^0} + m_{D^{*0}} = 3871.3 \pm 1.0 \, \text{MeV}, \quad (31)$$

which leads to $m_X - m_{J/\psi} - m_\omega \leq -8.3 \pm 1.0$ MeV. The $\pi^+\pi^-\pi^0$ resonates about twice the half width ($\frac{1}{2}\Gamma_\omega = 4.25 \pm 0.05$ MeV) below the ω peak. Even at the high-mass end of the $\pi^+\pi^-\pi^0$ invariant mass, the height of the Breit-Wigner resonance is ~ 0.19–0.23 of its peak value for $m_{X(3872)} = 3871.3$ MeV. Another phase space suppression occurs by the relative motion between the "off-shell" ω and J/ψ. The s-wave threshold factor $|\mathbf{p}_{J/\psi - \pi\pi\pi}|$ skews the Breit-Wigner shape and suppresses the high-mass end of $\pi^+\pi^-\pi^0$. The combined suppression from the two sources is quite severe. (See Fig. 6.) According to one computation [31] in the molecule model; the observed ratio of Eq. (29) would be reproduced if the effective $XJ/\psi V$ coupling ratio $|g(X\omega J/\psi)/g(X\rho J/\psi)|^2 \simeq 11.5 \pm 5.5$.

We explore here whether there is a chance to explain the large isospin breaking of Eq. (29) by the charmonium decay process, not by the isospin mixing composition of $X(3872)$ that is advocated in the molecule model. Assuming that ω is entirely responsible for $\pi^+\pi^-\pi^0$ in line with Belle's invariant mass plot, we write out the sensitive part of the decay rate as
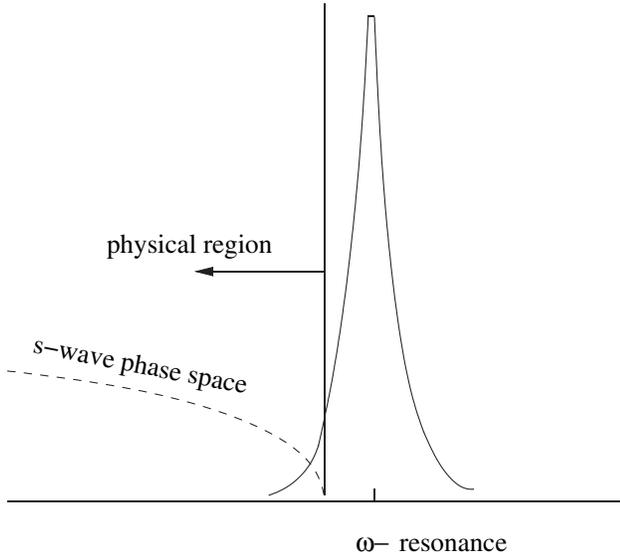
FIG. 6.   A schematic drawing of the kinematical suppression near the phase space boundary for $X(3872) \to \omega J/\psi$. The ρ peak sits right on the phase space boundary with the height $\simeq$ 1/18th of the ω peak. In this scale the ρ resonance profile is almost flat.

$$\Gamma(X(3872) \to \pi^+\pi^-\pi^0 J/\psi)$$
$$= \Gamma_0 \int_{3m_\pi}^{m_\omega - \delta m} \frac{|\mathbf{p}(W)|\Gamma_\omega}{[(W - m_\omega)^2 + \Gamma_\omega^2/4]} dW,$$
(32)

where W is the $\pi^+\pi^-\pi^0$ invariant mass, $\mathbf{p}(W)$ denotes the three-pion momentum $\mathbf{p}_{\pi\pi\pi}$ in the $X(3872)$ rest frame, and $\delta m \equiv m_\omega + m_{J/\psi} - m_{X(3872)}$ is the distance from the phase space boundary to the ω peak. We have computed numerically the integral in Eq. (32) in ratio to the corresponding quantity for ρ by varying m_X over 3870 to 3872 MeV. In carrying this calculation, we have cut off the $\pi^+\pi^-\pi^0$ invariant mass at 750 MeV, as chosen by Belle, and the $\pi^+\pi^-$ invariant mass at 450 MeV. The resulting kinematical suppression K is put in ratio:

$$\frac{K(\rho J/\psi)}{K(\omega J/\psi)} = 13.3 \pm 1.7 (m_{X(3872)} = 3871 \pm 1 \text{ MeV}).$$
(33)

In order to account for the near equality of the observed $\rho J/\psi$ and $\omega J/\psi$ rates, the production amplitude ratio must be such that

$$\left| \frac{A(\rho J/\psi)}{A(\omega J/\psi)} \right| = 0.27 \pm 0.02.$$
(34)

Being kinematical in origin, this is in general agreement with Ref. [31] (0.27^2 vs $1/11.3$). The required number of Eq. (34) is an order of magnitude larger than the electromagnetic isospin breaking. However, it may not be totally out of line for the isospin breaking due to the u-d quark

mass difference, which is after all the origin of the 8 MeV mass difference between $D^0(D^{*0})$ and $D^+(D^{*+})$. Therefore we next explore the magnitude of isospin breaking to see whether the amplitude ratio of Eq. (34) can be realized by the isospin mass splitting.

B. Isospin breaking due to mass difference

The molecule $X(3872)$ state is predominantly made of $D^0\overline{D}^{*0}$ and mixed with $D^+\overline{D}^{*-}$, $\omega J/\psi$, and $\rho J/\psi$. The large $\rho J/\psi$ branching is attributed to the isospin mixed composition of $D\overline{D}^*$. In the charmonium $X(3872)$, the particle composition is almost purely in $I = 0$ and the isospin breaking occurs during the decay process. Although experiment cannot distinguish between them, the two cases are fundamentally different in hadron dynamics. They are not different pictures of the same physics related by the quark-hadron duality or the like.

We expect that the main source of the large isospin violation is in the $\overline{D}D^*$ intermediate states near the thresholds where the splitting between $m_{D^0} + m_{D^{*0}}$ and $m_{D^+} + m_{D^{*-}}$ has the prominent effect. We can make some quantitative discussion of its magnitude with the dispersion relation for the $X \to \rho(\omega)J/\psi$ decay amplitudes by dispersing the variable $s = p_X^2$. In this way we can at least quantify the magnitude within our limited knowledge. Since $X(3872)$ is below the $D^0\overline{D}^{*0}$ threshold, only the dispersive part exists at $s = m_{X(3872)}^2$. Define the invariant decay amplitude $A(s)$ by

$$\langle V(q)J/\psi(p)^{\text{out}}|(\Box + m_X^2)X^\mu(0)|0\rangle$$
$$= \sqrt{\frac{1}{4E_pE_q}}\varepsilon^{\mu\nu\kappa\lambda}\epsilon(p)_\nu^*\epsilon(q)_\kappa^* P_\lambda A(s)$$
(35)

for the decay $X(P) \to \rho/\omega(q) + J/\psi(p)$ with $P = p + q$ and $s = P^2$. We keep only the s-wave term near the $D\overline{D}^*$ threshold ignoring the d-wave contribution for our semiquantitative analysis. We write the unsubtracted dispersion relation[4] for $A(s)$ integrating along the cut which runs on the right-hand side:

$$A(s) = \frac{1}{\pi} \int_{s_{\text{th}}}^\infty \frac{\text{Im} A(s')}{s' - s} ds'.$$
(36)

The lowest intermediate state is $\pi^+\pi^- J/\psi$ for $\rho J/\psi$ and $\pi^+\pi^-\pi^0 J/\psi$ for $\omega J/\psi$. The dispersion integral of such intermediate states represents elastic rescattering of $\rho(\omega)J/\psi$ in the final state. In the quark diagrams they are the "disconnected processes" (Fig. 3) that are not the most favored processes. The dominant low-energy absorptive parts come from the open charm intermediate states $D\overline{D}^* + \overline{D}D^*$ which couple more strongly with $X(3872)$ above the threshold [$s > (m_D + m_{D^*})^2$].

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[4]We obviously write an unsubtracted dispersion relation since a subtracted one would have no predictive power on the quantity of our interest.

Since our primary interest is in the contributions of the $D^0\overline{D}{}^{*0}$ and $D^+\overline{D}{}^{*-}$ intermediate states of $C = +$ to the dispersion integrals, we separate them out as

$$
\begin{aligned}
A_{\omega/\rho}(s) =\ & \frac{1}{\pi} \int_{(m_{D^0}+m_{D^{*0}})^2}^{\infty} \frac{\operatorname{Im} A(s')_{00}}{s' - s}\, ds' \\
& \pm \frac{1}{\pi} \int_{(m_{D^+}+m_{D^{*+}})^2}^{\infty} \frac{\operatorname{Im} A(s')_{-+}}{s' - s}\, ds' \\
& + \frac{1}{\pi} \int_{4m_{D^*}^2}^{\infty} \frac{\operatorname{Im} A(s')_{**}}{s' - s}\, ds',
\end{aligned}
\tag{37}
$$

where the dual sign in front of the second integral is $+$ for $\omega J/\psi$ and $-$ for $\rho J/\psi$. Define s-wave XDD^* coupling as approximately isospin invariant as

$$
L_{\mathrm{int}} = -m_X f_{XDD^*}\overline{D}D^{*\mu}X_\mu,
\tag{38}
$$

and $D\overline{D}{}^* \to \rho(\omega)J/\psi$ scattering amplitudes as

$$
\langle V(q)J/\psi(p)^{\mathrm{out}}|\overline{D}(p_1)D^*(p_2)^{\mathrm{in}}\rangle
\tag{39}
$$

$$
\begin{aligned}
=\ & \sqrt{\frac{1}{16E_qE_pE_1E_2}}i(2\pi)^4\delta(q + p - p_1 - p_2) \\
& \times \frac{\varepsilon^{\mu\nu\kappa\lambda}}{m_X}\epsilon_\mu(q)^*\epsilon_\nu(p)^*\epsilon_\kappa(p_2)[(p + q)_\lambda M_1(s, t) \\
& + (p - q)_\lambda M_2(s, t)].
\end{aligned}
\tag{40}
$$

The absorptive part for the $\overline{D}D^*$ intermediate state is

$$
\operatorname{Im} A_{00,-+}(s) = \frac{|\mathbf{p}(s)_{00,-+}|}{8\pi\sqrt{s}} f_{XDD^*} M(s)_{00,-+}(s)
\tag{41}
$$

with $|\mathbf{p}(s)| = \sqrt{(s - (m_D + m_{D^*})^2)(s - (m_D - m_{D^*})^2)}/2\sqrt{s}$

and $M(s)_{00,-+}$ is the s-wave projection of $M_1(s, t)$. The ratio of the $I = 1$ to the $I = 0$ decay amplitude is given by

$$
\frac{A(m_X^2)_\rho}{A(m_X^2)_\omega} = \frac{I_{00} - I_{-+}}{I_{00} + I_{-+} + I_{**}},
\tag{42}
$$

where

$$
\begin{aligned}
I_{00} &= \frac{1}{\pi} \int_{(m_{D^0}+m_{D^{*0}})^2} \frac{\operatorname{Im} A(s')_{00}}{s' - m_X^2}\, ds', \\
I_{-+} &= \frac{1}{\pi} \int_{(m_{D^+}+m_{D^{*+}})^2} \frac{\operatorname{Im} A(s')_{-+}}{s' - m_X^2}\, ds', \\
I_{**} &= \frac{1}{\pi} \int_{4m_{D^*}^2} \frac{\operatorname{Im} A(s')_{00} + A(s')_{-+}}{s' - m_X^2}\, ds'.
\end{aligned}
\tag{43}
$$

Since the decay $X_{(3872)} \to \rho(\omega)J/\psi$ is a long-distance process, high intermediate states are less important. That is, the absorptive part $\operatorname{Im} A(s)$ falls off with increasing s. In order to make a numerical estimate, we need to know how far the integrals over s' should be extended. Let us choose here the effective cutoff $s_{\max}$ no higher than $\sqrt{s'} = 2m_{D^*}(\simeq m_D + m_{D^*} + m_\pi)$ [i.e., $\overline{s}_{\max} \equiv s_{\max} - (m_D + m_{D^*})^2 \simeq 1\ \mathrm{GeV}^2$ and $I_{**} \simeq 0$] and see how large the isospin breaking can be. In this energy region, the most important s'-dependence is in $|\mathbf{p}(s')|$. We approximate the rest of $\operatorname{Im} A(s')$ to be constant. In this crude approximation which is almost independent of dynamics except for the value of the cutoff of integral, the ratio of Eq. (43) takes a simple form particularly when we take the limit of $m_D - m_{D^*} \ll m_D + m_{D^*}$ and $m_D + m_{D^*} - m_X \ll m_{D^*} - m_D \simeq m_\pi$,

$$
\left|\frac{A(m_X^2)_\rho}{A(m_X^2)_\omega}\right| \approx \frac{\pi\sqrt{m_X/2}[(m_{D^-} + m_{D^{*+}} - m_X)^{1/2} - (m_{D^0} + m_{D^{*0}} - m_X)^{1/2}]}{2\sqrt{\overline{s}_{\max}} - \pi\sqrt{m_X/2}[(m_{D^-} + m_{D^{*+}} - m_X)^{1/2} + (m_{D^0} + m_{D^{*0}} - m_X)^{1/2}]}.
\tag{44}
$$

For $\sqrt{\overline{s}_{\max}} \simeq 1$ GeV, the right-hand side varies from $\simeq 0.23$ at $m_{X(3872)} = m_D + m_{\overline{D}^*}$ to 0.18 at $m_{X(3872)} = m_D + m_{\overline{D}^*} - 0.8$ MeV. These numbers are larger than $(m_d - m_u)/m_\pi \approx 0.05$. The enhancement arises from the fact that the small number $m_D + m_{D^*} - m_X$ is made less small by the square root threshold factor. Combining this isospin breaking with the preceding estimate of the $\omega J/\psi$ suppression Eq. (34), we obtain for $m_X = 3871$ MeV [cf. Eq. (31)]

$$
\frac{B(X \to \pi^+\pi^-\pi^0 J/\psi)}{B(X \to \pi^+\pi^- J/\psi)} \approx \left(\frac{1}{0.2}\right)^2 \frac{1}{13.3}
$$

$$
\simeq 2 (\text{vs } 1.0 \pm 0.4 \pm 0.3).
\tag{45}
$$

We are short of the central value of experiment by a factor of 2 with large uncertainties. The ratio $|A(m_X^2)_\rho/A(m_X^2)_\omega|$ is sensitive to the value of the cutoff $s_{\max}$. If $\sqrt{\overline{s}_{\max}} =$

$\sqrt{s_{\max} - (m_D + m_{D^*})^2}$ is lowered from 1 GeV to 700 MeV, the ratio of Eq. (45) would become unity. Although choosing the cutoff $\sqrt{\overline{s}_{\max}}$ below $2m_{D^*}$ would be unrealistic, tapering off $\operatorname{Im} A_{00,-+}(s')$ towards $\sqrt{s_{\max}} = 2m_{D^*}$ has the same effect.

We have computed above only the kinematical effect due to the mass splitting between the charged and neutral charmed mesons. We must admit that we have stretched the numbers to the limit in this estimate. Since we have used the isospin symmetric XDD^* coupling, however, our $(I_{00} - I_{-+})$ term does not include the final state rescattering of $D\overline{D}{}^*$ in $I = 1$ channel at $m_{X(3872)}^2 < s' < s_{\max}$. It can generate additional enhancement or suppression to the integrals. But we do not have enough knowledge to analyze such dynamical effects.

We should learn from the exercise above that, although the problem of the large isospin breaking is serious, it is too

early to reject the charmonium interpretation of $X(3872)$ on the basis of the large $\rho J/\psi$ decay branching alone.

C. Magnitude of $\rho(\omega)J/\psi$ rates

We should be equally or even more concerned with the magnitude of the branching fractions themselves than their ratio. In order to observe the decay mode $\rho(\omega)J/\psi$ in experiment at all, it must have a branching fraction large enough to stand out of the annihilation process $^3P_1 \rightarrow ggg(q\overline{q}g) \rightarrow$ hadrons. With the kinematical suppression being so strong, would the decays into $\rho(\omega)J/\psi$ be still visible? This problem made some theorists suspicious about the charmonium interpretation of $X(3872)$ already at an early stage.[5] We have no means to estimate reliably the magnitude of the coupling f_{XDD^*} nor the amplitude $\mathrm{Im}\,A(s)_{00,-+}$ in Eq. (41). Nonetheless we must address this question.

Since we mean by charmonium the $c\overline{c}$ state that is bound primarily by the confining force, mixing of $c\overline{c}$ with $D\overline{D}^*$ is not large in a "clean" charmonium by definition. Therefore $|\Psi'(0)_{X(3872)}|^2$ should not be very far from the value of the simplest potential model that ignores the open charm channels. When we take the ratio of the branching fractions in B decay to that of the production cross sections in pp collision for $X(3872)$ and $\psi(2S)$, the wave functions $|\Psi(0)_{\psi(2S)}|^2$ and $|\Psi'(0)_{X(3872)}|^2$ cancel out. It is a reasonable assumption that the ratio of the $c\overline{c}$ production in s-wave to p-wave in B decay is similar to the same ratio in pp collision at low rapidity. If so, we obtain

$$\frac{B(B \rightarrow K^+X(3872))}{B(B \rightarrow K^+\psi(2S))} \approx \frac{\sigma(pp \rightarrow X(3872) + \text{anything})}{\sigma(pp \rightarrow \psi(2S) + \text{anything})}. \tag{46}$$

Denoting the branching fraction $B(X(3872) \rightarrow \pi^+\pi^-J/\psi)$ by $b_{\pi^+\pi^-}$, we have $0.02/b_{\pi^+\pi^-}$ for the left-hand side with Eqs. (17) and (19) while $0.03/b_{\pi^+\pi^-}$ for the right-hand side with Eq. (25). Both sides are in fair agreement with each other, which may indicate that our crude reasoning on the long-distance physics is not out of line. However, we should be concerned with the annihilation decay $X(3872) \rightarrow ggg + q\overline{q}g$. This mode, which is insensitive to the D and D^* masses, is expected to be two and half orders of magnitude stronger than $2^3P_1 \rightarrow \gamma J/\psi$ when the mass is adjusted to 3872 MeV in the estimate by Barnes and Godfrey [5]. Since $B(X(3872) \rightarrow \gamma J/\psi)$ is about 10% of $B(X(3872) \rightarrow \omega J/\psi)$ according to Belle, $b_{\pi^+\pi^-}$ should be quite small:

$$b_{\pi^+\pi^-} \simeq 0.015 \tag{47}$$

with the annihilation mode even in the absence of the

---

[5]See for instance Ref. [5].

$D^0\overline{D}^{*0}$ mode. For the pure charmonium $X(3872)$, Eq. (47) requires with Eq. (19) that $\psi(2S)$ and $X(3872)$ are equally copiously produced in B-decay in the case of $X(3872) \nrightarrow D^0\overline{D}^{*0}$. $X(3872)$ could be more strongly produced than $\psi(2S)$ if $X(3872)$ indeed decays into $D^0\overline{D}^{*0}$. This is a serious problem: $|\Psi'(0)|^2$ must be large enough for sufficient production of $X(3872)$ in B decay and pp collision while a large value of $|\Psi'(0)|^2$ potentially makes the annihilation decay too strong and the decay $X(3872) \rightarrow \pi^+\pi^-(\pi^0)J/\psi$ invisible. One escape from this problem is that $D\overline{D}^*$ is already present in $X(3872)$ and easily decays into $X(3872) \rightarrow \pi^+\pi^-(\pi^0)J/\psi$ by the quark-rearrangement process. We now look into the $D\overline{D}^*$ content of the charmonium, namely, mixing with $D\overline{D}^*$.

IV. LARGE MIXING BETWEEN CHARMONIUM AND CHARMED MESONS

The channel coupling between $c\overline{c}$ and $D^{(*)}\overline{D}^{(*)}$ was studied in the potential model. The s-wave channels of $c\overline{c}$ were studied numerically even in the expanded paper of the Cornell model [30]. However, it is not clear how much numerical uncertainty should be attached to the mass spectrum involving $c\overline{c}$ at and above the open charm threshold. Diagonalization must be made not simply for the multichannel amplitudes at the energy of a bound state, but for certain integrals of them. In the multichannel N/D method, the tractable approximation close to the potential model is to represent the N-function of given J^{PC} by the Born amplitudes and to set to $D(\infty) = 1$ for the D-function. One advantage of the N/D method over the potential model is its notational simplicity in discussing channel coupling. Normalize the partial-wave amplitude such that unitarity holds as $\mathrm{Im}\,a_J(s) = a_J^\dagger(s)\rho(s)^{-1}a_J(s)$ and $\mathrm{Im}[a_J(s)^{-1}] = -\rho(s)$ where $\rho(s)$ is the diagonal phase space matrix [17,32]. Introduce the N and D functions by $a_J(s) = N_J(s)D_J(s)^{-1}$. Suppressing the subscript J hereafter,

$$N(s) = \begin{pmatrix} B_{11}(s) & B_{12}(s) \\ B_{21}(s) & B_{22}(s) \end{pmatrix}, \tag{48}$$

where the rows and columns refer to $c\overline{c}$ and $D\overline{D}^*$ of $I = 0$. We do not include other channels such as $\omega J/\psi$ and ggg as constituents of $X(3872)$ here since coupling of $c\overline{c}$ to these channels is expected to be much weaker than to $D\overline{D}^*$. We treat the coupling to the weakly coupled channels as decay. Such separation would become less clear if their channel coupling were stronger. The (2×2) D-function is

$$D(s) = I - \frac{1}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\rho(s')N(s')ds'}{s' - s}. \tag{49}$$

The zero of det $D(s)$ gives the mass square of a bound state and the diagonalization matrix of $D(s)$ at the zero deter-

mines the composition of the bound state.[6] Although it is hard to get numerical results in our case, we can make one simple observation about the $c\bar{c}$-$D\overline{D}^*$ mixing.

We are interested in the possibility that a large off-diagonal element $B_{12}(s)(= B_{21}(s))$ for $c\bar{c} \leftrightarrow D\overline{D}^*$ causes a strong mixing. The Born diagram for $c\bar{c} \leftrightarrow D\overline{D}^*$ is a light quark exchange (Fig. 4). We have already shown in Sec. II that there is practically no force between $D(D^*)$ and $\overline{D}^*(\overline{D})$ in the 1^{++} channel. Therefore $B_{22}(s) \simeq 0$ in the low-energy region and $D(s)$ has the pattern of

$$D(s) \simeq \begin{pmatrix} 1 + \overline{D}_{11}(s) & \overline{D}_{12}(s) \\ \overline{D}_{21}(s) & 1 \end{pmatrix}, \qquad (50)$$

where $\overline{D}_{ij}(s) = -\int[\rho N/(s'-s)]_{ij}ds'$. It is diagonalized by the orthogonal rotation of angle θ that is given by

$$\tan 2\theta = \frac{2\overline{D}'_{12}(m_X^2)}{\overline{D}'_{11}(m_X^2)}, \qquad (51)$$

where the prime on $\overline{D}_{ij}$ denotes the first derivative in s. The confining potential of gluon exchange for $c\bar{c} \leftrightarrow c\bar{c}$ is no weaker than the quark exchange for $c\bar{c} \leftrightarrow D\overline{D}^*$. We can thus set a bound on the mixing,

$$|\tan 2\theta| < 2 \rightarrow |\theta| < 32°. \qquad (52)$$

If the mixing of $\theta \simeq 32°$ really occurs, the binding force would be enhanced by about 60% as a channel coupling effect.

We make the following observation from this exercise: It is possible for the 1^{++} charmonium to contain a $D\overline{D}^*$ component up to one third $[\simeq (\tan 32°)^2]$. A stronger mixing is possible only if the force of shorter distances in the elastic $D\overline{D}^*$ channel should play a role in binding. The mass splitting between $D^{(*)0}$ and $D^{(*)+}$ should not generate a very large departure from isospin symmetry in the $D\overline{D}^*$ content of X(3872) since the effective binding force is the integral of the Born amplitudes smoothed out over energy, not the Born amplitudes themselves at or near $m_{X(3872)}$. The $D\overline{D}^*$ content of X(3872) is approximately in $I = 0$. It should also be pointed out that the binding force due to the channel coupling, determined by $D_{12}(m_X^2)$ is insensitive whether X(3872) is above or below the $D^0\overline{D}^{*0}$ threshold.

In the presence of a large mixing, production of X(3872) occurs mainly through the dominant $c\bar{c}$ component. The production is robust since $|\Psi'(0)|^2$ is large for the p-wave charmonia. On the other hand, the $D\overline{D}^*$ component is unimportant for production since its $|\Psi(0)|^2$ is small. However, the $D\overline{D}^*$ component plays the major role in the decay into $\pi^+\pi^- J/\psi$ and $\pi^+\pi^-\pi^0 J/\psi$ since the virtual $D\overline{D}^*$ component can decay more easily into those channels

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[6]In the case of a single channel, Eqs. (48) and (49) combined reduce to solving in effect the corresponding Bethe-Salpeter equation where the kernel is the Born amplitude. In the non-relativistic limit, therefore, it is equivalent to the potential calculation.

than $c\bar{c}$ does. The decay $D\overline{D}^* \rightarrow \rho(\omega)J/\psi$ is a quark-rearrangement process and the strength of the $D\overline{D}^*$ binding is unimportant. Since the $D\overline{D}^*$ component can make up to one third, X(3872) $\rightarrow \rho(\omega)J/\psi$ is more competitive with X(3872) $\rightarrow ggg + q\bar{q}g$ than in the case of the unmixed pure charmonium. In our dispersion relation of the tree-point function in Sec. III, a large $D\overline{D}^*$ component is present when the coupling f_{XDD^*} is strong and the transition to the $D\overline{D}^*$ intermediate state is easy. The 2^3P_1 charmonium mixed with $D\overline{D}^*$ was discussed by Meng, Gao, and Chao [33] in the case that binding is entirely due to the confining force.

V. CONCLUDING REMARKS

We have examined the molecule model and the charmonium model for X(3872). The main motivations of the molecule idea are the coincidence of the X(3872) mass with $m_{D^0} + m_{D^{*0}}$ and the large isospin violation in the decay modes. However, there is no long-range force to bind D and $\overline{D}^*$ into a deuteronlike state. The observed production rates of X(3872) in B decay and pp collision are too large for a very loosely bound state. On the other hand, the charmonium has its share of difficulties; The mass does not agree with the potential-model prediction of the 2^3P_1 state and the large decay branching for $c\bar{c} \rightarrow ggg + q\bar{q}g$ could make the experimental signal of $\pi^+\pi^-(\pi^0)J/\psi$ hardly visible. One resolution appears to be as follows: X(3872) is bound primarily by the confining force between c and $\bar{c}$ which is boosted by the channel coupling to $D\overline{D}^*$. Production of X(3872) occurs mostly through its $c\bar{c}$ component. The $D\overline{D}^*$ component is in $I = 0$ in good approximation. The large isospin breaking of the decay mode $\rho J/\psi$ relative to $\omega J/\psi$ results from a normal magnitude of isospin breaking due to $D^{(*)+} - D^{(0)*}$ mass difference that is enhanced by the severe kinematical suppression of the $\omega J/\psi$ mode. This picture is very different conceptually from the molecule model or the multiquark model. In this picture, the binding force comes primarily from $c\bar{c}$ and secondarily from the channel coupling. The elastic $D\overline{D}^*$ channel provides practically no binding force. This is an important distinction from the viewpoint of hadron spectroscopy because, if our picture is right, X(3872) is not an opening of a flood gate for multiquark or molecule states.

How can we distinguish among the different models and pictures by experiment? We should test the particle content of X(3872). In the case of the charmonium and the charmonium mixed with $D\overline{D}^*$ of $I = 0$, the production rate is the same for B^+ and B^0 by isospin symmetry as long as it occurs through the dominant interaction $\bar{b} \rightarrow \bar{c}c\bar{s}$. Independent of dynamics, therefore, we expect for the charmonium X(3872),

$$\Gamma(B^+ \rightarrow K^+ X(3872)) = \Gamma(B^0 \rightarrow K^0 + X(3872)). \quad (53)$$

This equality should hold equally well in the case of the

large mixing between $c\bar{c}$ and $D\overline{D}^*$ since $X(3872)$ is produced primarily through its $c\bar{c}$ component whose $|\Psi'(0)|^2$ is large. For the molecule $X(3872)$ where the $D\overline{D}^*$ component has a sizable isospin breaking, the equality would be violated since the decay amplitudes into $I = 0$ and 1, or $D^0\overline{D}^{0-}$ and $D^+\overline{D}^{*-}$, are dynamical independent. If the molecule $X(3872)$ is made as

$$X(3872) = D^0\overline{D}^{*0}\cos\alpha + D^+\overline{D}^{*-}\sin\alpha, \quad (54)$$

we can express the decay amplitudes for $B^+ \to K^+X(3872)$ as

$$A(B^+ \to K^+X(3872)) = A_{00}\cos\alpha + A_{+-}\sin\alpha. \quad (55)$$

Then the decay amplitude for $B^0 \to K^0X(3872)$ is obtained by isospin rotation:

$$A(B^0 \to K^0X(3872)) = -A_{+-}\cos\alpha - A_{00}\sin\alpha. \quad (56)$$

The color of the spectator quark (u/d) matches that of the $\bar{c}$-quark from $\bar{b} \to \bar{c}c\bar{s}$ in $B^+ \to K^+D^0\overline{D}^{*0}$ and $B^0 \to K^0D^+\overline{D}^{*-}$, but not in $B^+ \to K^+D^+\overline{D}^{*-}$ nor $B^0 \to K^0D^0\overline{D}^{*0}$. In the $1/N_c$ expansion, therefore, $A_{+-} = O(1/N_c) \times A_{00}$. Consequently, for the molecule $X(3872)$ we have

$$A(B^0 \to K^0X(3872)) = -(\tan\alpha + O(1/N_c))$$
$$\times A(B^+ \to K^+X(3872)). \quad (57)$$

Since $D^0\overline{D}^{*0}$ dominates over $D^+\overline{D}^{*-}$ in the molecule $(\tan^2\alpha < 1)$, we predict for the molecule

$$\Gamma(B^0 \to K^0X(3872)) \simeq \tan^2\alpha\,\Gamma(B^+ \to K^+X(3872))$$
$$< \Gamma(B^+ \to K^+X(3872)). \quad (58)$$

Comparison of the B^0 decay with the B^+ decay clearly distinguishes between the deuteronlike bound state of $D^0\overline{D}^{*0}$ ($\tan^2\alpha \ll 1$) and the charmonium. For more general molecules, it will determine how much asymmetry exists between the $D^0\overline{D}^{*0}$ and the $D^+\overline{D}^{*-}$ components inside the molecule $X(3872)$. The *BABAR* collaboration recently provided [34]

$$B(B^0 \to K^0X(3872))/B(B^+ \to K^+X(3871))$$
$$= 0.50 \pm 0.30 \pm 0.05, \quad (59)$$

which is still inconclusive in distinguishing between the molecule and the charmonium. A smaller statistical error will decisively rule out the molecule.

Although we definitely favor the charmonium over the molecule, even the charmonium with mixing still has potential difficulties. In addition to the molecule and the charmonium, many other models have been proposed [35]. However, there is less handle to extend our semiquantitative discussion to those models. A common difficulty for them is that the production rate is likely much lower than the experimental observation since they are generally objects of large spatial spread. The branching fraction ratio of Eq. (59) is useful in distinguishing among them. This ratio is unity for the hybrid ($c\bar{c}g$) and the glueball (gg). It is far away from unity for the $cu\overline{cu}$.

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