Color ferromagnetism of quark matter: A possible origin of a strong magnetic field in magnetars

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We show a possibility that a strong "magnetic field" $\sim 10^{15}$ G is produced by color ferromagnetic quark matter in neutron stars. In the quark matter a color magnetic field is generated spontaneously owing to the Savvidy mechanism and a gluon condensate arises for the stabilization of the field. Since the quark matter is electrically charged in the neutron stars, the rotation of the quarks around the color magnetic field produces the strong "magnetic field."

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Observations of soft gamma-ray repeaters [1] and anomalous X-ray pulsars [2] reveal an intriguing phenomenon associated with neutron star physics, that is, the existence of an extremely strong magnetic field $\geq 10^{15}$ G. The standard neutron stars [3] also possess strong magnetic fields but their strength is about 10^{12} G, less by 3 orders of magnitude than the one above. Such neutron stars with the extremely strong magnetic field are called magnetars. Both the soft gamma-ray repeaters and the anomalous X-ray pulsars have been considered to be magnetars. The strength of the magnetic field is estimated by spin down rates of pulsars and there is a direct measurement [4] of the strength. Furthermore, there are some other reasons [5] supporting the strong magnetic field in the compact stars.

Since the observed value $\sim 10^{15}$ G of the magnetic field is the one around surfaces of the stars, its strength reaches $\sim 10^{18}$ G in the cores, if the magnetic field is generated in the stellar cores $R_c \sim 0.1R$ where R is the neutron star radius. Then, a naive question may arise: How is such a strong magnetic field produced? Conventionally, a dynamo mechanism [6] and an accretion-induced collapse of strongly magnetized white dwarfs [7] are believed for the production of the field. We may, however, speculate from a simple energetical argument that quark matter causes the field. That is, the typical neutron stars with a surface magnetic field, 10^{12} G, would involve a nuclear or hadronic process generating the field whose energy scale is of the order of 10 MeV. This energy scale comes from that of the magnetic field 10^{15} G present inside of the stars: $\sqrt{10^{15}\mathrm{G}} \simeq 8$ MeV. Similar consideration for the magnetar field leads to a typical energy scale of QCD: $\sqrt{10^{18}\mathrm{G}} \simeq 260$ MeV. This indicates that the process generating the magnetar field would be associated with quark matter [8], not hadronic matter.

In this paper we propose a possible mechanism for producing the strong magnetic field in the magnetars. We assume that inside the stars there exists a phase transition between dense hadronic matter and quark matter in a color ferromagnetic phase (CF phase). Namely, the stars involve the quark matter with a color magnetic field. The CF phase has been discussed in our previous papers [9,10]. The point of the mechanism is that since a gas of quarks is both colored and electrically charged in the CF phase, an observable magnetic field is produced because of its rotation around the color magnetic field.

We first give a brief review of the CF phase of the quark matter. In the CF quark matter, the color magnetic field, B, is generated spontaneously, not by the alignment of quark color spins, but by the dynamics of gluons. Namely, one loop effective potential of the color magnetic field has a nontrivial minimum at $B \neq 0$. This comes from the quantum effects of gluons. Thus, there is a possible CF ground state of gluons. This is the original analysis by Savvidy [11]. Since the loop approximation is valid at large baryon chemical potentials, namely, at a small gauge coupling constant, the CF phase may arise in the dense quark matter. Usually, di-quark condensation is taken into account and only the color superconducting phase (CS phase) is discussed [12] in the dense quark matter. But, in order to find possible phases of the dense quark matter, the CF state should be taken into account. Obviously, these two phases are incompatible. Hence we have compared [10] free energies of quarks in each phase in order to find which phase is favored. We have found that the CF phase is more favored than the CS phase at lower baryon chemical potentials. On the other hand, the CS phase is more favored than the CF phase at higher baryon chemical potentials. This result holds only at the extremely large chemical potential so as for the loop approximation to be valid. In this paper we assume that the result may hold even at smaller chemical potentials at which the phase transition occurs from hadronic matter to the quark matter.

Roughly speaking, the hadronic phase is realized due to the condensation of color magnetic monopoles. At a large gauge coupling constant, g, interactions between the monopoles, $\sim 1/g^2$, are much smaller so that almost-free gas of the monopoles may condense. Thus, the quark confinement arises due to the realization of the color magnetic superconducting state [13,14]. By decreasing the gauge coupling constant, the interactions between the monopoles increase. Consequently, the dipoles of the monopoles are formed and the condensation melts down. Their dipole moments are probably aligned so that the color magnetic field is produced. This is a naive physical picture of the CF phase.

It seems apparently that the quarks do not play any roles for the realization of the CF phase in the above argument except for yielding a small gauge coupling constant. We have shown [9] that the quarks play an important role for the stabilization of the color magnetic field. Namely, unstable modes of gluons, which are present [15] under the color magnetic field, have been shown to be stabilized with their condensation, just as scalar fields in Higgs potentials. The condensation leads to a fractional quantum Hall state of the gluons [9,16] with a color charge density. This color charge density of the gluons is supplied by quarks. That is, the quarks give color charges for the gluon sector necessary for the formation of the quantum Hall state. This is a role of the quarks for the realization of the CF phase. Consequently, the gas of the quarks in the quark matter is charged in color, which results in the production of the observable magnetic field.

Now, we explain the production mechanism of the strong magnetic fields observed in magnetars. For this purpose, we show in more detail how the unstable gluons under the color magnetic field are stabilized. We consider SU(2) gauge theory with massless quarks of the two flavors for simplicity.

Assuming $B \propto \lambda_3 = \sigma_3/2$, we decompose the gluon's Lagrangian with the use of the variables "U(1) gauge field" $A_\mu = A_\mu^3$, and "charged vector field" $\Phi_\mu = (A_\mu^1 + iA_\mu^2)/\sqrt{2}$, where indices 1–3 denote color components,

$$\begin{aligned} L &= -\frac{1}{4}\vec{F}_{\mu\nu}^2 \\ &= -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}|D_\mu \Phi_\nu - D_\nu \Phi_\mu|^2 \\ &\quad + ig(\partial^\mu A^\nu - \partial^\nu A^\mu)\Phi_\mu^\dagger \Phi_\nu + \frac{g^2}{4}(\Phi_\mu \Phi_\nu^\dagger - \Phi_\nu \Phi_\mu^\dagger)^2 \end{aligned} \tag{1}$$

with $D_\mu = \partial_\mu + igA_\mu$. We have omitted a gauge term $D_\mu \Phi^\mu = 0$. Using the Lagrangian we can derive that the energy E of the charged vector field $\Phi_\mu \propto e^{iEt - ikx_3}$ in the magnetic field, $A_\mu = A_\mu^B$, is given by $E^2 = k^2 + 2gB(n + 1/2) \pm 2gB$ with a gauge choice, $A_j^B = (0, x_1 B, 0)$ and $(\partial_\mu + igA_\mu^B)\Phi^\mu = 0$, where we have taken the spatial direction of $\vec{B}$ along the x_3 axis. $\pm 2gB$ (the integer $n \geq 0$) denotes the contribution from spin components of Φ_μ (Landau levels) and k denotes momentum in the direction parallel to the magnetic field.

We find that unstable modes of gluons are given by $\Phi = (\Phi_1 - i\Phi_2)\sqrt{1/2}$ occupying the lowest Landau level, $n = 0$, and that their spectra are given by $E^2 = k^2 - gB$, which are negative for $k^2 < gB$. Thus, their amplitudes increase rapidly in time: $\Phi \sim e^{|E|t}$. In other words, they are excited spontaneously and eventually form a stable ground state owing to the self interactions, $g^2\Phi^4$. Nielsen *et al.* [17] tried to find a stable configuration of the mode Φ with $k = 0$, which minimizes a classical potential energy of Φ, $V(\Phi) = -2gB\Phi^2 + g^2\Phi^4/2$; the form of $V(\Phi)$ can be derived by taking only the unstable modes, Φ in the Lagrangian. The configuration they obtained is a "flux lattice" of Φ, and the resultant magnetic field is given by $F_{12} = B - g^2|\Phi^2|$. Namely, they found that $|\Phi(x_1, x_2)|$ is periodic in two spatial coordinates, x_1 and x_2. Their periodicity is approximately given by the magnetic length, $l_B = 1/\sqrt{gB}$. Since fractional quantum Hall states [16] were not known before 1982, it was a reasonable solution for them to obtain. At present we know that even bosons such as the unstable gluons, Φ with $k = 0$, may form stable fractional quantum Hall states [18] when they occupy the lowest Laundau level, interacting repulsively with each other. The states are characterized by the following filling factor, ν,

$$\nu = \frac{2\pi\rho_2}{gB} = \frac{1}{2 \times \text{positive integer}} \tag{2}$$

with two-dimensional color charge density, ρ_2 of the gluons. The filling factor is defined by the ratio of the number (charge) density of gluons to the degeneracy per unit area of each Landau level: $\nu =$ "density"$/(gB/2\pi)$. We note that even integers in the denominator appear because of the bosons; odd integers appear in the case of fermions (electrons). Here we consider only the case of $\nu = 1/2$. The details of how the unstable gluons form the quantum Hall state are addressed in our previous papers [9,10]. (We have demonstrated the formation of the gluons' quantum Hall states by using Chern-Simons gauge theory, which has originally been applied [19] by us for understanding Laughlin states of electrons.) We have shown [9] that this quantum Hall state with $\nu = 1/2$ is energetically more favored than the "flux lattice," which may be regarded as a Wigner crystal [16] of the gluons in the modern point of view. In this way the unstable gluons under the color magnetic field are stabilized by forming the quantum Hall states.

Then, we must ask how large the coherent length of the color magnetic field in the x_3 direction is. Namely, we must ask how large the width of the two-dimensional "quantum well" [16] is. The quantum Hall states are formed in the well. Here we give only a plausible argument about the width, although more rigorous treatment is necessary. Since the unstable modes of gluons disturb the coherence of the magnetic field, their excitations with momenta $k < \sqrt{gB}$ make the coherent length diminish to be $l_B = 1/\sqrt{gB}$. (The interpretation is consistent with the fact [9] that the condensation of the gluons gives a mass $\sim\sqrt{gB}$ to the magnetic field.) This implies that the quantum Hall state is realized effectively in a quantum well with its width, $1/\sqrt{gB}$. This well is extending infinitely in x_1 and x_2 directions in quark matter. There exist many of these wells perpendicular to $\vec{B}$ in the quark matter. The direction of each color magnetic field in the wells is aligned to a direction of e.g. x_3. Consequently, the color magnetic field may exist globally in the quark matter, although the real coherence of the field is restricted within a well. Since the two-dimensional color charge density ρ_2 necessary for the formation of the quantum Hall state is localized in the well, the three-dimensional color charge density, ρ_3, is given by

$$\rho_3 \simeq \rho_2/l_B = \rho_2\sqrt{gB} = \frac{(gB)^{3/2}}{4\pi}. \tag{3}$$

This charge density is carried by the gluons. Since the quark matter in compact stars is color neutral, the color charge of the gluons is compensated by quarks. Thus, the gas of the quarks comes to possess a color charge density of $-\rho_3$. Since the quark matter is not electrically neutral, its rotation around the color magnetic field can produce the observable magnetic field. This is the physical origin in our model for the generation of the observable strong magnetic field.

Now, we calculate the strength of the observable magnetic field produced spontaneously in the quark matter, that is, spontaneous magnetization of the matter. The point is that the number of negative color charged quarks is different from the number of positive color charged quarks. The difference induces an electric current around the color magnetic field. We consider only u and d quarks. Number densities and energy densities of the quarks are given by

$$n^{\pm}_{u,d}(gB) = \frac{gB\mu^{\pm}_{u,d}}{4\pi^2} \quad \text{and} \quad \epsilon^{\pm}_{u,d}(gB) = \frac{gB(\mu^{\pm}_{u,d})^2}{8\pi^2} \tag{4}$$

where $\mu^{\pm}_{u,d}$ are chemical potentials of each flavor of quarks with $\pm$ color charges associated with the generator λ_3 of an SU(2) gauge group. We assume for simplicity that all of the quarks occupy only the lowest Landau level; that is, Fermi energy μ is less than $\sqrt{gB}$. In order to obtain free energies, we notice three conditions [10,20] which must be satisfied in the neutron stars: the conditions of color and electric neutralities, and of beta equilibrium ($u \leftrightarrow d + e^-$).

$$\begin{aligned} \rho_3 &= \frac{(gB)^{3/2}}{4\pi} = (n^-_u + n^-_d - n^+_u - n^+_d)/2, \\ 0 &= 2(n^+_u + n^-_u)/3 - (n^+_d + n^-_d)/3 - n_e, \\ \mu^{\pm}_d &= \mu^{\pm}_u + \mu_e, \end{aligned} \tag{5}$$

where $n_e = \mu_e^4/(4\pi^2)$ is the number density of electrons with the chemical potential denoted by μ_e.

In order to obtain the magnetization, M, we need to estimate the free energy, $G(H)$: $M = -\partial_H G(H)$ for $H \to 0$, where H is an external magnetic field coupled with electric charges. Assuming that the direction of $\vec{H}$ points to the one of the color magnetic field, $\vec{B}$, we obtain

$$\begin{aligned} G(H) &= \epsilon^+_u(gB + 2eH/3) + \epsilon^-_u(gB - 2eH/3) \\ &\quad + \epsilon^+_d(gB - eH/3) + \epsilon^-_d(gB + eH/3) \\ &\quad - (\mu^+_u n^+_u(gB + 2eH/3) + \mu^-_u n^-_u(gB - 2eH/3) \\ &\quad + \mu^+_d n^+_d(gB - eH/3) + \mu^-_d n^-_d(gB + eH/3)) \\ &\simeq -\frac{gB((\mu^+_u)^2 + (\mu^-_u)^2 + (\mu^+_d)^2 + (\mu^-_d)^2)}{8\pi^2} \\ &\quad + \frac{5eH((\mu^+_u)^2 - (\mu^-_u)^2)}{12\pi^2}, \end{aligned} \tag{6}$$

where we have neglected the contribution of electrons because they do not produce any magnetization in the limit of $H \to 0$. Thus, the magnetization is

$$M = -5e\frac{(\mu^+_u + \mu^-_u)(\mu^+_u - \mu^-_u)}{12\pi^2} \tag{7}$$

where we have used the beta-equilibrium condition. Since the difference of $\mu^+_u - \mu^-_u$ can be represented by the color charge density of the quarks, $-\rho_3$, we find that

$$\begin{aligned} M &= 5e\frac{\sqrt{gB}(\mu^+_u + \mu^-_u)}{12\pi} \\ &\simeq \frac{5e\pi n_B}{2\sqrt{gB}} \sim 10^{18}\ \text{G}\frac{400\ \text{MeV}}{\sqrt{gB}}\frac{n_B}{1/\text{fm}^3} \end{aligned} \tag{8}$$

where n_B denotes the baryon number density of the quarks: $n_B = (n^+_u + n^-_u + n^+_d + n^-_d)/3 \simeq gB(\mu^+_u + \mu^-_u)/6\pi^2$. We have neglected the negligible contribution of electrons to M. It turns out that the magnetic field (magnetization) obtained is sufficiently large to explain the strong magnetic field of the magnetars.

We have not yet determined the value of gB. The value should be obtained by using the phenomena associated with the color ferromagnetism of the quark matter; there are still no such phenomena. But we may expect that it takes about a typical QCD scale such as several hundred MeV; it does neither several 10 MeV or several GeV. Therefore, if the radius of the quark matter in compact stars is about 2 km, the strength of the surface magnetic field reaches 10^{15} G. In this way, we explain that the extremely strong magnetic field of the magnetars is caused by the quark matter in the CF phase. Thus, the magnetars may involve the quark matter inside their cores. (We have presented [10] a structure of a neutron star involving both the CF quark matter and nuclear matter by using appropriate equations of state in both matters.)

We have argued that the kinds of quantum wells in which quantum Hall states of gluons are present are formed effectively. The argument is based on the fact that the unstable modes, $\Phi(k)$ with momenta, $k < \sqrt{gB}$, make the coherent length of the color magnetic field, $F_{12} = B - g^2|\Phi(k)|^2$, diminish. As a result, a quantum well may be formed in which the color magnetic field with the diminished coherent length $\simeq l_B$ is present. This formation of the well is essential to obtain the observable strong magnetic field, since the color charge density $\rho_3 \simeq \rho_2/l_B$ of the quark matter is used to derive the magnetization. The argument is plausible, but never rigorous. We wish to make the point more clearly in the future.

We have argued the neutron stars in which the quark matter occupies only their cores. There is a possibility that the quark matter entirely makes a star, i.e. a quark star. If such quark stars in the ferromagnetic phase exist, the strength of the magnetic field at their surfaces reaches 10^{17} G $\sim 10^{18}$ G. Then, the frequencies of their rotations decrease extremely rapidly due to magnetic dipole radiations. Consequently, such stars would not show any pulsation except for in a period just after their births.

We have estimated the spontaneous magnetization of the CF quark matter in SU(2) gauge theory. We have neglected the effects of the strange quarks and of higher Landau levels occupied by quarks. It is straightforward to include their effects in SU(3) gauge theory. It is expected that even if we include all of the effects, the strength of the magnetic field will be of the same order of magnitude as the ones estimated in this paper. We will report it in the near future.

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[1] E. P. Mazets *et al.*, Nature (London) **282**, 587 (1979); K. Hurley *et al.*, Nature (London) **397**, 41 (1999); N. Gehrels *et al.*, Astrophys. J. **611**, 1005 (2004).
[2] J. van Paradijs *et al.*, Astron. Astrophys. **299**, L41 (1995).
[3] S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars* (John Wiley & Sons, New York, 1983); J. M. Lattimer and M. Prakash, astro-ph/0405262.
[4] A. I. Ibrahim, J. H. Swank, and W. Parke, Astrophys. J. Lett. **584**, L17 (2003).
[5] C. Thompson and R. C. Duncan, Astrophys. J. **473**, 322 (1996).
[6] C. Thompson and R. C. Duncan, Astrophys. J. **408**, 194 (1993).
[7] V. V. Usov, Nature (London) **357**, 472 (1992).
[8] C. Alcock, E. Farhi, and A. Olinto, Phys. Rev. Lett. **57**, 2088 (1986); V. V. Usov, Phys. Rev. Lett. **87**, 021101 (2001); R. Ouyed *et al.*, Astron. Astrophys. **420**, 1025 (2004).
[9] A. Iwazaki and O. Morimatsu, Phys. Lett. B **571**, 61 (2003); A. Iwazaki, O. Morimatsu, T. Nishikawa, and M. Ohtani, Phys. Lett. B **579**, 347 (2004); Phys. Rev. D **71**, 034014 (2005).
[10] A. Iwazaki, O. Morimatsu, T. Nishikawa, and M. Ohtani, hep-ph/0507151.
[11] G. K. Savvidy, Phys. Lett. B **71**, 133 (1977); H. Pagels, report, 1978 (unpublished).
[12] K. Rajagopal and F. Wilczek, hep-ph/0011333.
[13] S. Mandelstam, Phys. Lett. **53B**, 476 (1975); G. t'Hooft, Nucl. Phys. **B190**, 455 (1981).
[14] Z. F. Ezawa and A. Iwazaki, Phys. Rev. D **25**, 2681 (1982).
[15] N. K. Nielsen and P. Olesen, Nucl. Phys. **B144**, 376 (1978); Phys. Lett. **79B**, 304 (1978).
[16] *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvan (Springer-Verlag, New York, 1990), 2nd ed.; *Perspectives in Quantum Hall Effects*, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1997).
[17] H. B. Nielsen and M. Ninomiya, Nucl. Phys. **B156**, 1 (1979); H. B. Nielsen and P. Olesen, Nucl. Phys. **B160**, 330 (1979).
[18] T. Nakajima and M. Ueda, Phys. Rev. Lett. **91**, 140401 (2003).
[19] Z. F. Ezawa, M. Hotta, and A. Iwazaki, Phys. Rev. B **46**, 7765 (1992); Z. F. Ezawa and A. Iwazaki, J. Phys. Soc. Jpn. **61**, 4133 (1992).
[20] M. Alford and K. Rajagopal, J. High Energy Phys. 06 (2002) 031.