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Charmless final state interaction in $B \to \pi\pi$ decays

S. Fajfer,[1,2,3] T. N. Pham,[4] and A. Prapotnik-Brdnik[1,5]

[1]*Department of Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia*
[2]*J. Stefan Institute, Jamova 39, P. O. Box 300, 1001 Ljubljana, Slovenia*
[3]*Physik Department, Technische Universität München, D-85748, Garching, Germany*
[4]*Centre de Physique Teorique, Centre National de la Recherche Scientifique, UMR 7644, Ecole Polytechnique, 91128 Palaiseau Cedex, France*
[5]*Faculty of Civil Engineering, University of Maribor, Smetanova ul. 17, 2000 Maribor, Slovenia*

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We estimate effects of the final state interactions in $B \to \pi\pi$ decays coming from rescattering of $\pi\pi$ via exchange of ρ, σ, f_0 mesons. Then we include the $\rho\rho$ rescattering via exchange of π, ω, a_1 mesons and finally we consider contributions of the $a_1\pi$ rescattering via exchange of ρ. The absorptive parts of amplitudes for these processes are determined. In the case of $\pi^+\pi^-$ decay mode, due to model uncertainties, the calculated contribution is $|\mathcal{M}_A| \leq 1.7 \times 10^{-8}$ GeV. This produces a small relative strong phase for the tree and color-suppressed $B \to \pi\pi$ amplitudes consistent with the result of a recent phenomenological analysis based on the *BABAR* and Belle results for the $B \to \pi\pi$ branching ratios and *CP* asymmetries.

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I. INTRODUCTION

The experimental results on B decays coming from Belle and *BABAR* offer many puzzles for theoretical studies. Among them the $B \to \pi\pi$ decays are particularly interesting [1,2]. Many theoretical frameworks such as perturbative QCD approach of Beneke, Buchalla, Neubert and Sachrajda (BBNS) [3] and the approach of [4], Soft Collinear Effective Theory (SCET) [5–9] and many others [10–24] have attempted to understand the observed decay rates. Within QCD factorization charmless two-body decays of B mesons have amplitudes which factorize at lowest order in $1/m_b$. It means that in this approach, in neglecting the next-to-leading terms in $1/m_b$ expansion, one ends up with the naive factorization ansatz. The naive factorization (e.g. [10,11]) gave the rate of $\bar{B}^0 \to \pi^+\pi^-$ too large in comparison with the observed rate while the $\bar{B}^0 \to \pi^0\pi^0$ decay rate came out too small within this simple framework. Agreement with experimental data on $B \to \pi\pi$ has been found within both BBNS and SCET frameworks. The improved $\bar{B}^0 \to \pi^0\pi^0$ decay rate was obtained recently within BBNS [3] with the presence of parameter λ_b whose precise value is unknown [21]. Within SCET the agreement with the experimental data is achieved [9] with the presence of non negligible long-distance charming penguin contributions. It has been pointed out in Ref. [25] that in B weak decays one cannot neglect the effects of final state interactions due to the growth of forward scattering of the final state with the squared center off-mass energy, as required by the optical theorem and cross section data. This indicates that "soft scattering does not decrease for large m_B" [25].

Recently the authors of [24] considered two-body decay modes by including final state interactions (FSI). Contributions of the $c\bar{c}$ state, which in the literature very often called charming penguins were considered in [9,26]. The charm meson rescattering due to charm meson exchange has been considered in Refs. [13,23] and more recently in [24]. It was found the largest contribution appears in the $B \to K\pi$ mode [13], but is much smaller in the case of $\pi\pi$ final state [23]. The authors of [24] found that the absorptive part of the rescattering cannot explain the observed enhancement of the $\pi^0\pi^0$ branching ratio and cannot produce a small branching ratio of the $\pi^+\pi^-$ rate.

Motivated by this study [24] we reexamine final state interactions in $\bar{B}^0 \to \pi^+\pi^-$ and $\bar{B}^0 \to \pi^0\pi^0$ modes which result from the light mesons rescattering. We use mainly the same framework as described in [24], but we point out that there are more intermediate states which contribute to both amplitudes and give important contributions. As in [24] we take into account only dominant contributions proportional to the effective Wilson coefficient a_1. In this approach for the charmless final state interactions only the contributions of $\pi\pi$ and $\rho\rho$ intermediate states were used in [24]. Since in B decays, resonant FSI is expected to be suppressed due to the absence of resonances at energies close to the mass of the B meson, we consider only t-channel FSI. However, in the case of $\pi\pi \to \pi\pi$ rescattering we include possibility that in addition to the ρ meson exchange there are contributions coming from σ and f_0 exchange. In the case of $\rho\rho \to \pi\pi$ rescattering we find that there is a contribution of the ω meson for the $\pi^+\pi^-$ final state as well as contributions of the $a_1(1260)$ axial meson. We determine contributions coming from $a_1(1260)\pi$ intermediate states, inspired by the recent *BABAR* measurement of the very large rate for $\bar{B}^0 \to a_1^-\pi^+$ state with the branching ratio $\mathrm{BR}(B^0 \to a_1^+(1260)\pi^-) = (40.2 \pm 3.9 \pm 3.9) \times 10^{-6}$ [27]. In our approach the $a_1^-(1269)$ rescatter via ρ^0 exchange into the $\pi^+\pi^-$ final state. Although the $\bar{B}^0 \to a_1^+\pi^-$ decay rate has not been observed yet, we estimate this contribution assuming the naive factorization for the amplitude. The paper is organized as follows: in Sec. 2 we give basic formulas for the two-body B amplitudes and the Lagrangian describing the strong interactions of the light mesons used in our calculations, in Sec. 3 we present results of our calculations for the absorptive part of the amplitude, in Sec. 4 we discuss our results and we summarize them in Sec. 5.

II. THE FRAMEWORK

In the studies of the $B \to \pi\pi$ and $B \to K\pi$ branching ratios and CP asymmetries it was found that the decay amplitudes arise from tree, color-suppressed, penguin and the electroweak penguin diagrams (see e.g. [19,24]). In our approach we consider only leading contributions in charmless FSI and therefore we only use the effective weak Lagrangian for the process $b \to \bar{u}du$ at the tree level in the following form:

$$\mathcal{L}_w = -\frac{G}{\sqrt{2}} V_{ub} V_{ud}^* a_1 (\bar{u}b)_{V-A} (\bar{d}u)_{V-A}. \tag{1}$$

Here a_1 is the Wilson coefficient and we use the same value as given in [24] ($a_1(\mu) = 0.991 + i0.0369$; the scale $\mu = 2.1$ GeV), which includes short-distance nonfactorizable corrections such as vertex corrections and the hard spectator interactions determined within QCD factorization approach [3]. In our further study we use naive factorization approximation [10], in which the B meson decay amplitude can be written as a product of two weak current matrix elements. The standard decomposition of the weak current matrix elements is:

$$\begin{aligned}
\langle V(k, \varepsilon, m_V)|\bar{q}\Gamma^\mu q|P(p, M)\rangle
&= \epsilon^{\mu\nu\alpha\beta}\varepsilon_\nu p_\alpha k_\beta \frac{2V(q^2)}{M + m_V} + 2im_V \frac{\varepsilon \cdot q}{q^2} q^\mu A_0(q^2) \\
&\quad + i(M + m_V)\left[\varepsilon^\mu - \frac{\varepsilon \cdot q}{q^2} q^\mu\right] A_1(q^2) \\
&\quad - i\frac{\varepsilon \cdot q}{M + m_V}\left[P^\mu - \frac{M^2 - m_V^2}{q^2} q^\mu\right] A_2(q^2).
\end{aligned} \tag{2}$$

Similarly, heavy pseudoscalar to light pseudoscalar transition is described by the matrix element:

$$\langle P(k, m_P)|\bar{q}\Gamma^\mu q|P(p, M)\rangle = \left[P^\mu - \frac{(M^2 - m_P^2)}{q^2} q^\mu\right] F_+(q^2) + \frac{(M^2 - m_P^2)}{q^2} q^\mu F_0(q^2), \tag{3}$$

while for the heavy pseudoscalar to light axial vector transition, we use the expression given in [28]:

$$\begin{aligned}
\langle A(k, \varepsilon, m_A)|\bar{q}\Gamma^\mu q|P(p, M)\rangle
&= i\Big[(M + m_A)\varepsilon^\mu V_1(q^2) - \frac{\varepsilon \cdot q}{M + m_A} P^\mu V_2(q^2) \\
&\quad - 2m\frac{\varepsilon \cdot q}{q^2} q^\mu (V_3(q^2) - V_0(q^2))\Big] \\
&\quad - \epsilon^{\mu\nu\alpha\beta}\varepsilon_\nu p_\alpha k_\beta \frac{2A(q^2)}{M + m_A},
\end{aligned} \tag{4}$$

with $V_3(q^2) = (M + m_A)/(2m_A)V_1(q^2) - (M - m_A)/(2m_A)V_2(q^2)$. In above equations $q^\mu = p^\mu - k^\mu$ and $P^\mu = p^\mu + k^\mu$. The light meson creation (annihilation) is described by the matrix elements:

$$\begin{aligned}
\langle P(p)|\bar{q}\gamma^\mu(1 - \gamma_5)q|0\rangle &= if_P p^\mu, \\
\langle V(p, \varepsilon)|\bar{q}\gamma^\mu(1 - \gamma_5)q|0\rangle &= f_V m_V \varepsilon^\mu, \\
\langle A(p, \varepsilon)|\bar{q}\gamma^\mu(1 - \gamma_5)q|0\rangle &= f_A m_A \varepsilon^\mu.
\end{aligned} \tag{5}$$

In our numerical calculations we use the following values of relevant parameters as given in [24]: $f_\pi = 0.132$ GeV, $f_\rho = 0.21$ GeV, $f_{a_1} = 0.205$ GeV, $F_0^{B\pi}(0) \simeq F_0^{B\pi}(m_\pi^2) = 0.25 \simeq F_+^{B\pi}(m_{a_1}^2)$, $A_1^{B\rho}(0) \simeq A_1^{B\rho}(m_\rho^2) = 0.27$, $A_2^{B\rho}(0) \simeq A_2^{B\rho}(m_\rho^2) = 0.26$. We use: $V_0^{Ba1}(0) \simeq V_0^{Ba1}(m_\pi^2) = 0.13$ [28].

Using above expressions, the leading contribution to the amplitude for $\bar{B}^0 \to \pi^-\pi^+$ was found to be (e.g. [11]).

$$\begin{aligned}
\mathcal{A}(\bar{B}^0 \to \pi^+\pi^-) &= i\mathcal{A}_\pi \\
&= -i\frac{G}{\sqrt{2}} V_{ub} V_{ud}^* a_1 [F_0^{B\pi}(m_\pi^2)(m_B^2 - m_\pi^2)] f_\pi.
\end{aligned} \tag{6}$$

In [24] the value $a_1 = 0.9921 + i0.036$ led to the amplitude $\mathcal{A}_\pi(\bar{B}^0 \to \pi^+\pi^-)_{\mathrm{SD}} = 3.2 \times 10^{-8} + i1.2 \times 10^{-9}$ GeV (we took the $V_{ub} = 0.00439$ [29]). Without color-suppressed and penguin contributions this gives the branching ratio $BR(\bar{B}^0 \to \pi^+\pi^-)_{\mathrm{SD}} = 9 \times 10^{-6}$, too large in comparison with the average experimental value $(4.6 \pm 0.4) \times 10^{-6}$ as given in [24]. The inclusion of color-suppressed and penguin amplitudes decreases the rate [11,24], but it is still too large in comparison with experimental result.

The amplitude for $\bar{B}^0 \to \rho^+\rho^-$ is

$$\begin{aligned}
\mathcal{A}(\bar{B}^0(p_B) \to \rho^+(q_1, \epsilon_1)\rho^-(q_2, \epsilon_2))
&= i\mathcal{A}_\rho\Big(\epsilon^{\mu\nu\alpha\beta}\epsilon_{1\mu}\epsilon_{2\nu}q_{1\alpha}q_{2\beta}\frac{-2iV(m_\rho^2)}{M_B + m_\rho} \\
&\quad + A_1(m_\rho^2)(M_B + m_\rho)\epsilon_1 \cdot \epsilon_2 \\
&\quad - 2A_2(m_\rho^2)\frac{\epsilon_1 \cdot p_B p_B \cdot \epsilon_2}{M_B + m_\rho}\Big),
\end{aligned} \tag{7}$$

with $A_\rho = -\frac{G}{\sqrt{2}} V_{ub} V_{ud}^* a_1 f_\rho m_\rho$. The amplitudes for $\bar{B}^0 \to a_1^-\pi^+$ and $\bar{B}^0 \to a_1^+\pi^-$ are:

$$\mathcal{A}\,(\bar{B}^0(p) \to a_1^-(q_2, \epsilon)\pi^+(q_1)) = i\mathcal{A}_{a_1,1}(p + q_1)\cdot\epsilon,$$
$$\mathcal{A}(\bar{B}^0(p) \to a_1^+(q_1, \epsilon)\pi^-(q_2)) = i\mathcal{A}_{a_1,2}(p + q_1)\cdot\epsilon, \tag{8}$$

with $\mathcal{A}_{a_1,1} = -\frac{G}{\sqrt{2}}V_{ub}V_{ud}^* a_1 f_{a_1} m_{a_1} F_+^{B\pi}(m_{a1}^2)$ and $\mathcal{A}_{a_1,2} = -\frac{G}{\sqrt{2}}V_{ub}V_{ud}^* a_1 f_\pi 2m_{a_1} V_0^{Ba1}(m_\pi^2)$.

The light mesons' strong interactions are described by

$$\begin{aligned}\mathcal{L}_{\text{strong}} = {}& i\frac{g_{\rho\pi\pi}}{\sqrt{2}}Tr(\rho^\mu[\Pi, \partial_\mu\Pi]) \\ & - 4\frac{C_{VVP}}{f}\epsilon^{\mu\nu\alpha\beta}Tr(\partial_\mu\rho_\nu\partial_\alpha\rho_\beta\Pi) \\ & + G_{AVP}Tr(A_\mu[\rho^\mu, \Pi]) + iG_s\sqrt{2}Tr(\Pi\Pi S) \\ & + iG_{s'}\sqrt{2}Tr(\Pi\Pi S').\end{aligned} \tag{9}$$

In these equations Π is the 3×3 matrix containing pseudoscalar mesons, ρ is the 3×3 matrix describing light vector mesons, and S, S' are matrices describing scalar mesons. In our numerical calculations we use $g_{\rho\pi\pi} = 5.9$ and $C_{VVP} = 0.33$ (see [30–32]). The coupling $|G_{AVP}| = 3.12$ GeV is obtained from the experimental results for $a_1^0 \to \rho^-\pi^+$ decay width $\Gamma_A = 0.2$ GeV. Finally, the couplings G_s and G_s' are obtained by using PDG data [1] on σ (or $f_0(600)$) and $f_0(980)$ meson: $m_\sigma \approx (0.4 - 1.2)$ GeV, $\Gamma_\sigma \approx (0.6 - 1)$ GeV, $m_f = 0.98$ GeV and $\Gamma_f \approx (0.04 - 0.1)$ GeV. In the numerical calculation we take the average values $m_\sigma = 0.8$ GeV, $\Gamma(\sigma \to \pi\pi) = 0.8$ GeV, $m_f = 0.98$ GeV and $\Gamma(f_0(980) \to \pi\pi) = 0.07$ GeV and we determine $G_s = 4.24$ GeV, and $G_s' = 1.37$ GeV.

Using naive factorization we obtain for the branching ratio $\text{BR}(\bar{B}^0 \to a_1^-\pi^+) = 1.8\times 10^{-5}$ about 2 times smaller than the experimental result given in [27]. Using above mentioned data we predict that $\text{BR}(\bar{B}^0 \to a_1^+\pi^-) = 8.2\times 10^{-6}$.

III. THE ABSORPTIVE PARTS OF THE AMPLITUDES

In our calculation of the absorptive parts of amplitudes we include the contributions coming from the graphs presented in Fig. 1. The absorptive parts of amplitudes are obtained when the cut is done over the intermediate states $\pi\pi$, $\rho\rho$ and $a_1\pi$ as schematically given in Fig. 2. In our formulas below we denote momenta of particles as given in Fig. 2. The couplings describing the strong interactions of light mesons in these diagrams are all far of mass shell. In the approach of [24,33] the additional form factor was included. Its role is to take care of the off-mass shell effects [34]:

$$F(y, M_3) = \left(\frac{\Lambda^2 - M_3^2}{\Lambda^2 - t(y)}\right), \tag{10}$$

where $t(y) = (q_1 - k_1)^2$, $\Lambda = M_3 + \Lambda_{QCD}$ and M_3 is the mass of exchanged particle A_3 (see Fig. 2). We take $\Lambda_{\text{QCD}} = 0.3 \pm 0.05$ GeV. Following the contributions given in Fig. 1, we determine the absorptive parts of the amplitudes

$$\begin{aligned}\mathcal{M}_A^{\pi\pi\rho} = {}& -\mathcal{A}_\pi g_{\rho\pi\pi}^2\frac{\lambda^{1/2}(m_B^2, m_\pi^2, m_\pi^2)}{32\pi m_B^2}\int_{-1}^1 dy C_\pi(y) \\ & \times\frac{F^2(y, m_\rho)}{2m_\pi^2 - 2S(y) - m_\rho^2},\end{aligned} \tag{11}$$

$$\begin{aligned}\mathcal{M}_A^{\pi\pi\sigma} = {}& -\mathcal{A}_\pi 4G_s^2\frac{\lambda^{1/2}(m_B^2, m_\pi^2, m_\pi^2)}{32\pi m_B^2} \\ & \times\int_{-1}^1 dy\frac{F^2(y, m_\sigma)}{2m_\pi^2 - 2S(y) - m_\sigma^2},\end{aligned} \tag{12}$$

$$\begin{aligned}\mathcal{M}_A^{\pi\pi f} = {}& -\mathcal{A}_\pi 4G_s'^2\frac{\lambda^{1/2}(m_B^2, m_\pi^2, m_\pi^2)}{32\pi m_B^2} \\ & \times\int_{-1}^1 dy\frac{F^2(y, m_f)}{2m_\pi^2 - 2S(y) - m_f^2},\end{aligned} \tag{13}$$

$$\begin{aligned}\mathcal{M}_A^{\rho\rho\pi} = {}& -\mathcal{A}_\rho g_{\rho\pi\pi}^2\frac{\lambda^{1/2}(m_B^2, m_\rho^2, m_\rho^2)}{32\pi m_B^2} \\ & \times\int_{-1}^1 dy\frac{F^2(y, m_\pi)}{m_\pi^2 + m_\rho^2 - 2S(y) - m_\pi^2} \\ & \times((m_B + m_\rho)A_1(m_\rho^2)C_{\rho,1}(y) \\ & - 2A_2(m_\rho^2)/(m_B + m_\rho)C_{\rho,2}(y)),\end{aligned} \tag{14}$$

$$\begin{aligned}\mathcal{M}_A^{\rho\rho\omega} = {}& -2\mathcal{A}_\rho\left(\frac{4C_{VVP}}{f_\pi}\right)^2\frac{\lambda^{1/2}(m_B^2, m_\rho^2, m_\rho^2)}{32\pi m_B^2} \\ & \times\int_{-1}^1 dy\frac{F^2(y, m_\omega)}{m_\pi^2 + m_\rho^2 - 2S(y) - m_\omega^2} \\ & \times((m_B + m_\rho)A_1(m_\rho^2)C_{\rho,3}(y) \\ & - 2A_2(m_\rho^2)/(m_B + m_\rho)C_{\rho,4}(y)),\end{aligned} \tag{15}$$

$$\begin{aligned}\mathcal{M}_A^{\rho\rho a_1} = {}& -2\mathcal{A}_\rho G_{AVP}^2\frac{\lambda^{1/2}(m_B^2, m_\rho^2, m_\rho^2)}{32\pi m_B^2} \\ & \times\int_{-1}^1 dy\frac{F^2(y, m_{a_1})}{m_\pi^2 + m_\rho^2 - 2S(y) - m_{a_1}^2} \\ & \times((m_B + m_\rho)A_1(m_\rho^2)C_{\rho,5}(y) \\ & - 2A_2(m_\rho^2)/(m_B + m_\rho)C_{\rho,6}(y)),\end{aligned} \tag{16}$$

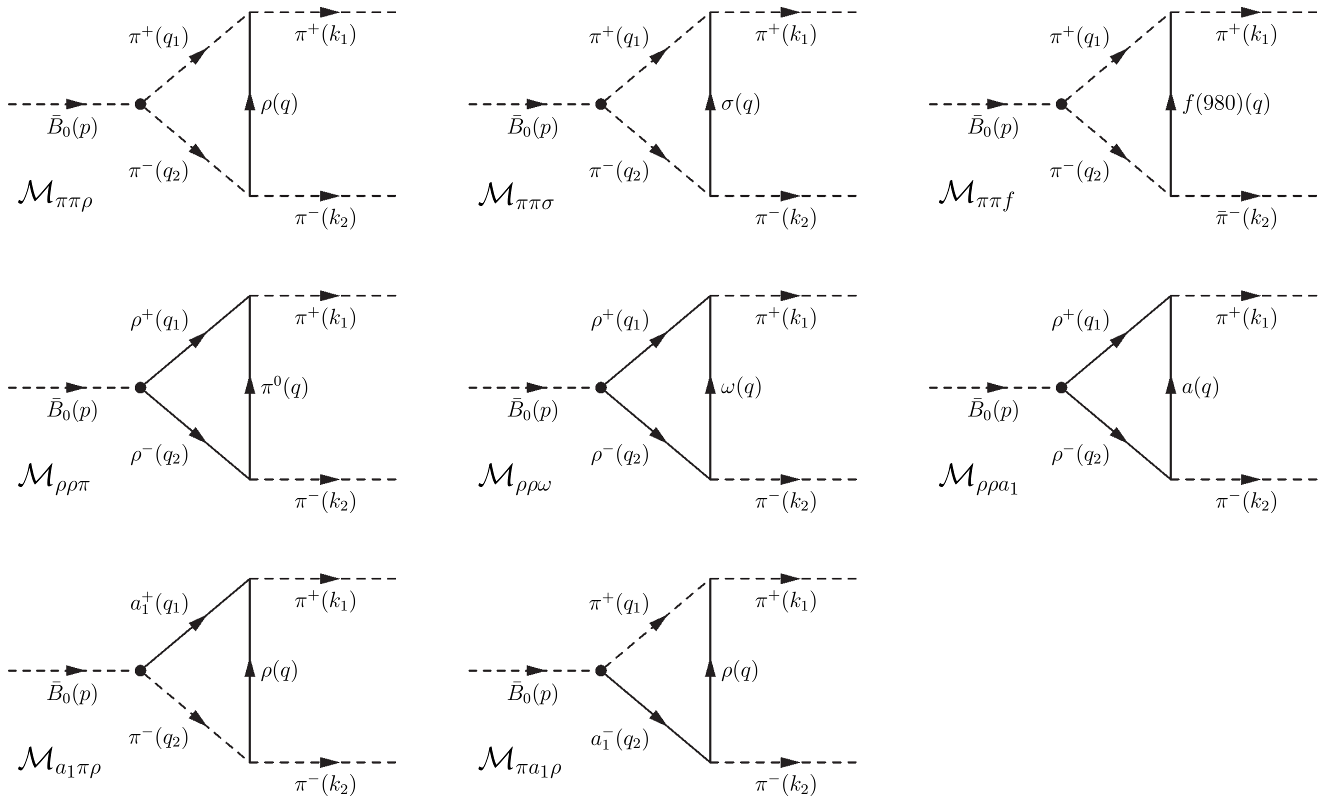

FIG. 1. Feynman diagrams for $\bar{B}_0 \to \pi^+\pi^-$ decay coming from rescattering of $\pi\pi$ via exchanges of ρ, σ, f_0 , $\rho\rho$ rescattering via exchanges of π, ω, a_1 and $a_1\pi$ rescattering via exchange of ρ.

$$\mathcal{M}_A^{a_1\pi\rho} = -\mathcal{A}_{a_1,2}\sqrt{2}G_{AVP}g_{\rho\pi\pi}\frac{\lambda^{1/2}(M_B^2, m_\pi^2, m_{a_1}^2)}{32\pi m_B^2} \times \int_{-1}^{1} dy C_{a_1,2}(y)\frac{F^2(y, m_\rho)}{m_\pi^2 + m_{a_1}^2 - 2S(y) - m_\rho^2}, \tag{17}$$

$$\mathcal{M}_A^{\pi a_1\rho} = -\mathcal{A}_{a_1,1}\sqrt{2}G_{AVP}g_{\rho\pi\pi}\frac{\lambda^{1/2}(m_B^2, m_\pi^2, m_{a_1}^2)}{32\pi m_B^2} \times \int_{-1}^{1} dy C_{a_1,1}(y)\frac{F^2(y, m_\rho)}{2m_\pi^2 - 2S(y) - m_\rho^2}, \tag{18}$$

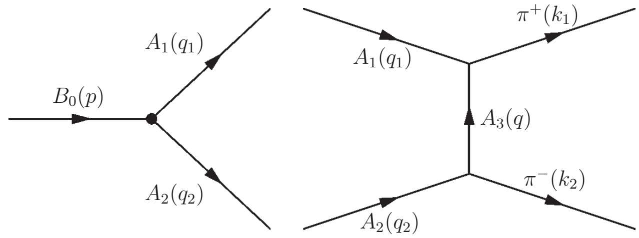

FIG. 2. The absorptive parts of amplitudes are obtained when the cut is done over the intermediate states A_1 and A_2.

where C stands for the functions of momenta defined in Appendix A, while $S(y)$ is the scalar product:

$$S(y) = k_1 \cdot q_1 = k_{10}E_1 - |\vec{k}_1||\vec{q}_1|y \tag{19}$$

and $y = cos(\vec{k}_1, \vec{q}_1)$. We use $|\vec{q}_1|^2 = \frac{1}{4m_B^2}\lambda(m_B^2, M_1^2, M_2^2)$, $|\vec{k}_1|^2 = \frac{1}{4m_B^2}\lambda(m_B^2, m_\pi^2, m_\pi^2)$ and $E_1^2 = |\vec{q}_1|^2 + M_1^2$ and $k_{10}^2 = |\vec{k}_1|^2 + m_\pi^2$. Here M_i stands for the masses of intermediate particles A_i and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2cb - 2ac$ as usual.

IV. DISCUSSION

After numerical evaluation[1] of these integrals we present our results in Table I. We give values of the absorptive parts of the amplitude for three different values of the scale $\Lambda = 0.25; 0.3; 0.35$ GeV. As seen from the table these amplitudes are sensitive to the choice of this parameter. It is important to note that the relative sign of these contributions cannot be completely determined [33]. By assuming that strong couplings do not have any phases, the sum of contributions coming from the $\pi\pi \to \pi\pi$ rescat-

[1]Numerical results were obtained with the help of the computer program FeynCalc [35].

TABLE I. The absorptive parts of amplitudes coming from the diagrams $\mathcal{M}_A^i \times 10^{-7} V_{ub}$[GeV] given in Fig. 1.

	$\Lambda = 0.25$ GeV	$\Lambda = 0.30$ GeV	$\Lambda = 0.35$ GeV
$\pi\pi(\rho)$	$13.3 + 0.5i$	$17.3 + 0.6i$	$21.8 + 0.8i$
$\pi\pi(\sigma)$	$-0.5 - 0.02i$	$-0.6 - 0.02i$	$-0.8 - 0.03i$
$\pi\pi(f_0)$	$-0.03 - 0.001i$	$-0.05 - 0.002i$	$-0.06 - 0.002i$
$\Sigma_{\pi\pi}$	$12.5 + 0.5i$	$16.7 + 0.6i$	$21 + 0.8i$
$\rho\rho(\pi)$	$-1.7 - 0.06i$	$-2.2 - 0.08i$	$-2.8 - 0.1i$
$\rho\rho(\omega)$	$5.5 + 0.2i$	$7.7 + 0.3i$	$10.3 + 0.4i$
$\rho\rho(a_1)$	$-0.9 - 0.03i$	$-1.4 - 0.05i$	$-1.6 - 0.06i$
$\Sigma_{\rho\rho}$	$2.8 + 0.1i$	$4.3 + 0.2i$	$5.9 + 0.2i$
$a_1^- \pi^+(\rho^0)$	$5.6 + 0.2i$	$7.5 + 0.3i$	$9.5 + 0.3i$
$a_1^+ \pi^-(\rho^0)$	$1.9 + 0.1i$	$2.5 + 0.1i$	$3.2 + 0.1i$

tering is then $\Sigma_{\pi\pi} = (1.7 + 0.06i) \times 10^{-6} V_{ub}$ GeV, which for $|V_{ub}| = 0.00439$ gives $|\Sigma_{\pi\pi}| = 7.5 \times 10^{-9}$ GeV (for $\Lambda = 0.3$ GeV). It is interesting that the exchanges of scalar mesons give very small contributions. The contribution of $\rho\rho$ intermediate states with the exchanges of π^0, ω and a_1 is about 4 times smaller than the total $\pi^+\pi^-$ intermediate state contribution. Among these the effect of the ω exchange is important. This contribution was not considered in [24]. The contributions of $a_1\pi$ intermediate states might be significant, close in size to the leading $\pi^+\pi^-$ elastic-rescattering effect. Then in the best case (by summing the contributions given in Table I, all with the positive signs) we can give an upper value for the absorptive part of the amplitude ($\Lambda = 0.3$ GeV):

$$|\mathcal{M}_A(\bar{B}^0 \to \pi^+\pi^-)| \leq 1.7 \times 10^{-8} \text{ GeV}. \tag{20}$$

This value is very close in size to the short-distance amplitude discussed in [24] (Eqs. (5.14)). On the other hand, for the certain choice of the strong couplings phases, the calculated contributions might almost cancel each other, leading to the disappearance of the absorptive part of FSI amplitude.

In the case of $\bar{B}^0 \to \pi^0\pi^0$ the absorptive part of amplitude comes from the same FSI and the upper bound is $|\mathcal{M}_A(\bar{B}^0 \to \pi^0\pi^0)| \leq 1.4 \times 10^{-8}$ GeV, ($\Lambda = 0.3$ GeV). Note that there are no contributions coming from the exchanges of neutral mesons as σ, f_0 in the case of $\pi^+\pi^- \to \pi^0\pi^0$ and ω in $\rho^+\rho^- \to \pi^0\pi^0$ mode. Comparing this result with short-distance amplitude given in [24] (Eqs. (5.14)) we see that the effect we discuss might enhance the amplitude by a factor of 2. However, the corresponding branching ratio is still too small in comparison with the experimental result.

[2]Note that we have used the Feynman diagram convention for the $\pi^0\pi^0$ amplitude as in [36].

In order to estimate the effects of this leading FSI contribution in $B^- \to \pi^-\pi^0$ decay amplitudes one can rely on the isospin relation[2]

$$A(\bar{B}^0 \to \pi^+\pi^-) - A(\bar{B}^0 \to \pi^0\pi^0) = -\sqrt{2} A(B^- \to \pi^0\pi^-). \tag{21}$$

We find that the absorptive part from $\pi\pi$ (elastic rescattering) and quasielastic FSI $\rho\rho$ via the t-channel π, a_1, ω-exchange contributions might be important for $B \to \pi\pi$ amplitudes. Here we point out that the absorptive part of the $\bar{B}^0 \to \pi^+\pi^-$ amplitude produces the phase of the tree amplitude of [37,38] while the absorptive part of the $\bar{B}^0 \to \pi^0\pi^0$ amplitude determines the color-suppressed phase of the amplitude in [37,38]. In a recent paper [37] it was shown that it is possible to determine the strong phase separately for the tree, color-suppressed, and penguin amplitudes from the current *BABAR* and Belle measurements on $B \to \pi\pi$ branching ratios and CP asymmetries. The results show that the relative phase between the tree and color-suppressed amplitudes $\delta_T - \delta_C$ is rather small. Since we found the strong phase coming from calculated FSI effect for $\pi^+\pi^-$ (tree amplitude) and $\pi^0\pi^0$ (color-suppressed amplitude) to be almost of the same size, we can confirm the results of the phenomenological study given in Ref. [37].

Our calculations contain only information on the absorptive part of amplitudes indicating sources of uncertainties. One can in principle determine the dispersive parts of amplitudes, but due to many uncertainties we do not pursue in calculating these effects. As noticed in [12,13] these contributions are expected to be of similar size as the absorptive parts of amplitudes for both $\pi^+\pi^-$ and $\pi^0\pi^0$ decay modes.

Recently the authors of Ref. [20] estimated the effects of final state interactions using the Regge model. This analysis shows that the long-distance charming penguins do not play important role. However, the long-distance effects due to the light meson rescattering are very important in obtaining correct rates for $B \to \pi\pi$ decays [20], in agreement with the result of our calculation.

In Ref. [38], using the $SU(3)$ symmetry relations, it was found that in $B \to \pi\pi$ decays the ratio of the color-suppressed and tree amplitudes is very large. Our calculations, obtained within a very different framework, confirm this finding.

V. SUMMARY

We can briefly summarize our results:

(i) The absorptive parts of amplitudes in $B \to \pi\pi$ decays are calculated using the rescattering of $\pi\pi$ via exchange of ρ, σ, f_0; $\rho\rho$ rescattering via exchange of π, ω, a_1 and contributions of the $a_1\pi$ rescattering via exchange of ρ.

(ii) Although our results suffer from many uncertainties due to unknown relative phases and the dependence on the parameter Λ, we can say that our study shows the importance of the charmless final state interactions in $B \to \pi\pi$ decays. Both the $\bar{B}^0 \to \pi^+\pi^-$ and $\bar{B}^0 \to \pi^0\pi^0$ amplitudes might get significant contributions from absorptive parts of the FSI amplitudes.

(iii) Our result shows that the relative phase between the tree and color-suppressed amplitude $\delta_T - \delta_C$ is rather small and in agreement with the results of previous phenomenological studies.

APPENDIX

The functions of momenta C are (momenta q_i, k_i and q are defined in Fig. 2):

$$\begin{aligned} C_\pi(y) &= (q_1 + k_1)^\alpha (q_2 + k_2)^\beta (-g_{\alpha\beta} + q_\alpha q_\beta/m_\rho^2) \\ &= 2(m_\pi^2 - m_b^2 + S(y)), \end{aligned} \tag{A1}$$

$$\begin{aligned} C_{\rho,1}(y) &= (-g_{\alpha\beta} + q_{1\alpha}q_{1\beta}/m_\rho^2)(-g^{\alpha\delta} + q_2^\alpha q_2^\delta/m_\rho^2) \\ &\quad \times (2k_1 - q_1)^\beta (2k_2 - q_2)_\delta \\ &= \frac{2}{m_\rho^4}(-2m_\pi^2 m_\rho^4 + m_B^2 m_\rho^4 + 2S(y)(S(y) - m_B^2)m_\rho^2 \\ &\quad + m_B^2 S(y)^2), \end{aligned} \tag{A2}$$

$$\begin{aligned} C_{\rho,2}(y) &= (k_1 + k_2)_\alpha (k_1 + k_2)_\gamma (-g^{\alpha\beta} + q_1^\alpha q_1^\beta/m_\rho^2) \\ &\quad \times (2k_1 - q_1)_\beta (-g^{\gamma\delta} + q_2^\gamma q_2^\delta/m_\rho^2)(2k_2 - q_2)_\delta \\ &= \frac{m_B^4}{m_\rho^4}(m_\rho^2 - S(y))^2, \end{aligned} \tag{A3}$$

$$\begin{aligned} C_{\rho,3}(y) &= (-g_{\alpha\sigma} + q_{2\alpha}q_{2\sigma}/m_\rho^2)(-g_{\alpha\sigma'} + q_{1\alpha}q_{1\sigma'}/m_\rho^2) \\ &\quad \times (-g_{\gamma\gamma'} + (k_1 - q_1)_\gamma (k_1 - q_1)_{\gamma'}/m_\omega^2) \\ &\quad \times \epsilon^{\kappa\sigma\rho\gamma}\epsilon^{\kappa'\sigma'\rho'\gamma'} q_{2\kappa} q_{1\kappa'} (k_1 - q_1)_\rho (k_1 - q_1)_{\rho'} \\ &= m_\pi^2(m_B^2 - 2m_\rho^2) + m_\rho^2 m_B^2 + 2S(y)(S(y) - m_B^2), \end{aligned} \tag{A4}$$

$$\begin{aligned} C_{\rho,4}(y) &= (k_1 + k_2)^{\alpha'}(-g_{\alpha'\sigma} + q_{2\alpha'}q_{2\sigma}/m_\rho^2)(k_1 + k_2)^\alpha \\ &\quad \times (-g_{\alpha\sigma'} + q_{1\alpha}q_{1\sigma'}/m_\rho^2)(-g_{\gamma\gamma'} + (k_1 - q_1)_\gamma \\ &\quad \times (k_1 - q_1)_{\gamma'}/m_\omega^2)\epsilon^{\kappa\sigma\rho\gamma} \\ &\quad \times \epsilon^{\kappa'\sigma'\rho'\gamma'} q_{2\kappa} q_{1\kappa'} (k_1 - q_1)_\rho (k_1 - q_1)_{\rho'} \\ &= \frac{m_B^2}{4}((m_B^2 - 4m_\rho^2)m_\pi^2 + m_\rho^2 m_B^2 \\ &\quad - 2S(y)(m_B^2 - 2S(y))), \end{aligned} \tag{A5}$$

$$\begin{aligned} C_{\rho,5(y)} &= (-g_{\alpha\gamma} + q_{2\alpha}q_{2\gamma}/m_\rho^2)(-g^{\gamma\rho} + q^\rho q^\gamma/m_{a1}^2) \\ &\quad \times (-g_\rho^\alpha + q_{1\rho}q_1^\alpha/m_\rho^2) \\ &= -\frac{1}{4m_\rho^4 m_{a1}^2}(2m_\rho^4(6m_{a1}^2 + m_B^2) - 4m_\pi^2 m_\rho^4 \\ &\quad - 4m_\rho^2(m_{a1}^2 m_B^2 + S(y)(m_B^2 - S(y))) + m_{a1}^2 m_B^4 \\ &\quad + 2m_B^2 S(y)^2), \end{aligned} \tag{A6}$$

$$\begin{aligned} C_{\rho,6(y)} &= (k_1 + k_2)_{2\beta}(k_1 + k_2)^\alpha (-g_{\alpha\gamma} + q_{2\alpha}q_{2\gamma}/m_\rho^2) \\ &\quad \times (-g^{\gamma\rho} + q^\rho q^\gamma/m_{a1}^2)(-g_\rho^\beta + q_{1\rho}q_1^\beta/m_\rho^2) \\ &= -\frac{m_B^2}{8m_\rho^4 m_{a1}^2}(2m_\rho^4(4m_{a1}^2 + m_B^2) - 2m_B^2 m_\rho^2(3m_{a1}^2 \\ &\quad + 2S(y)) + m_{a1}^2 m_B^4 + 2m_B^2 S(y)^2). \end{aligned} \tag{A7}$$

$$\begin{aligned} C_{a_1,1}(y) &= (2q_1 + q_2)^\alpha (-g_{\alpha\beta} + q_{2\alpha}q_{2\beta}/m_{a_1}^2)(-g^{\beta\delta} \\ &\quad + q_\beta q_\delta/m_\rho^2)(q_1 + k_1)_\delta \\ &= \frac{-1}{2m_{a1}^2}(m_\pi^4 + m_\pi^2(2S(y) - 3m_B^2 - 2m_{a1}^2) + m_{a1}^4 \\ &\quad + 2m_B^2(m_B^2 - S(y)) - m_{a1}^2(3m_B^2 + 2S(y))), \end{aligned} \tag{A8}$$

$$\begin{aligned} C_{a_1,2}(y) &= (2q_1 + q_2)^\alpha (-g_{\alpha\beta} + q_{1\alpha}q_{1\beta}/m_{a_1}^2) \\ &\quad \times (-g^{\beta\delta} + q_\beta q_\delta/m_\rho^2)(q_2 + k_2)_\delta \\ &= \frac{-1}{2m_{a1}^2}(m_\pi^4 + m_\pi^2(S(y) - 2m_B^2 - 2m_{a1}^2) \\ &\quad + m_{a1}^4 + m_B^4 - m_B^2 S(y) - m_{a1}^2(m_B^2 + S(y))). \end{aligned} \tag{A9}$$

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[1] S. Eidelman, Phys. Lett. B **592**, 1 (2004).
[2] Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag.
[3] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000); **B606**, 245 (2001).
[4] Y. Y. Keum, T. Kurimoto, H. N. Li, C. D. Lü, and A. I. Sanda, Phys. Rev. D **69**, 094018 (2004).
[5] C. W. Bauer, S. Fleming, and M. Luke, Phys. Rev. D **63**, 014006 (2001).
[6] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001).
[7] C. W. Bauer and I. W. Stewart, Phys. Lett. B **516**, 134 (2001).
[8] C. W. Bauer, D. Pirjol, and I. W. Stewart, Phys. Rev. D **65**, 054022 (2002).
[9] C. W. Bauer, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D **70**, 054015 (2004).
[10] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985).
[11] A. Ali, G. Kramer, and C. D. Lü, Phys. Rev. D **58**, 094009 (1998).
[12] C. Isola, M. Ladisa, G. Nardulli, T. N. Pham, and P. Santorelli, Phys. Rev. D **64**, 014029 (2001).
[13] C. Isola, M. Ladisa, G. Nardulli, T. N. Pham, and P. Santorelli, Phys. Rev. D **65**, 094005 (2002).
[14] D. Delepine, J. M. Gerard, J. Pestieau, and J. Weyers, Phys. Lett. B **429**, 106 (1998).
[15] P. Zenczykowski, Acta Phys. Polon. B **28**, 1605 (1997).
[16] C. K. Chua, W. S. Hou, and K. C. Yang, Mod. Phys. Lett. A **18**, 1763 (2003).
[17] S. Barshay, L. M. Sehgal, and J. van Leusen, Phys. Lett. B **591**, 97 (2004).
[18] T. Feldmann and T. Hurth, J. High Energy Phys. 11 (2004) 037.
[19] C. W. Chiang, M. Gronau, J. L. Rosner, and D. A. Suprun, Phys. Rev. D **70**, 034020 (2004).
[20] A. Deandrea, M. Ladisa, V. Laporta, G. Nardulli, and P. Santorelli, hep-ph/0508083.
[21] W. N. Cottingham, I. B. Whittingham, and F. F. Wilson, Phys. Rev. D **71**, 077301 (2005).
[22] P. Colangelo, F. De Fazio, and T. N. Pham, Phys. Rev. D **69**, 054023 (2004).
[23] C. Isola, M. Ladisa, G. Nardulli, and P. Santorelli, Phys. Rev. D **68**, 114001 (2003).
[24] H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D **71**, 014030 (2005).
[25] J. F. Donoghue, E. Golowich, A. A. Petrov, and J. M. Soares, Phys. Rev. Lett. **77**, 2178 (1996).
[26] M. Ciuchini, E. Franco, G. Martinelli, and L. Silvestrini, Nucl. Phys. **B501**, 271 (1997).
[27] *BABAR* Collaboration, hep-ex/0507029.
[28] H. Y. Cheng, Phys. Rev. D **68**, 094005 (2003).
[29] U. Nierste, hep-ph/0511125.
[30] M. Bando, T. Kugo, S. Uehara, K. Yamawaki, and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985); M. Bando, T. Kugo, and K. Yamawaki, Nucl. Phys. **B259**, 493 (1985); Phys. Rep. **164**, 217 (1988).
[31] T. Fujiwara, T. Kugo, H. Terao, S. Uehara, and K. Yamawaki, Prog. Theor. Phys. **73**, 926 (1985).
[32] A. Bramon, A. Grau, and G. Pancheri, Phys. Lett. B **344**, 240 (1995).
[33] M. Ablikim, D. S. Du, and M. Z. Yang, Phys. Lett. B **536**, 34 (2002).
[34] O. Gortchakov, M. P. Locher, V. E. Markushin, and S. von. Rotz, Z. Phys. A **353**, 447 (1996).
[35] J. Küblbeck, M. Böhm, and A. Denner, Comput. Phys. Commun. **60**, 165 (1990).
[36] A. N. Kamal and T. N. Pham, Phys. Rev. D **50**, R1832 (1994).
[37] L. Wolfenstein and F. Wu, hep-ph/0506224.
[38] Y. L. Wu and Y. F. Zhou, Phys. Rev. D **72**, 034037 (2005).