PHYSICAL REVIEW D **72,** 113002 (2005)

2-3 symmetry: Flavor changing b, τ decays, and neutrino mixing

Alakabha Datta* and Patrick J. O'Donnell†

Department of Physics, University of Toronto, 60 St. George Street, Toronto, ON, Canada M5S 1A7

(Received 22 November 2005; published 15 December 2005)

The observed pattern of neutrino mixing may be the result of a 2-3($\mu - \tau$) symmetry in the leptonic sector. We consider a two Higgs doublet model with a 2-3 symmetry in the down-type quark and the charged lepton sector. The breaking of the 2-3 symmetry by the strange quark mass and the muon mass leads to flavor changing neutral currents in the quark sector and the charged lepton sector that are suppressed by m_s/m_b and m_μ/m_τ in addition to the mass of the heavy Higgs boson of the second Higgs doublet. A Higgs boson mass of $m_H \sim$ 600–900 GeV can explain the deviation from the standard model reported in several rare B decays. Predictions for other B decays are made, and a new CP phase is predicted in $B_s - \bar{B}_s$ mixing. The lepton flavor violating decays $\tau \to \mu \bar{l}(\bar{q}) l(q)$ are below the experimental limits. The breaking of 2-3 symmetry in the lepton sector can lead to deviations of the atmospheric neutrino mixing angle from the maximal value by ~2 degrees.

DOI: 10.1103/PhysRevD.72.113002 PACS numbers: 14.60.Pq, 13.35.Dx, 14.80.Cp

I. INTRODUCTION

The discovery of neutrino masses and mixing have led to many speculations about extension of the standard model (SM). In contrast to the mixing in the quark sector which is hierarchical, the neutrino mixing is large. A useful framework to understand the large neutrino mixing are models that have a leptonic $\mu - \tau$ interchange symmetry [1,2]. Several ongoing and future neutrino experiments are expected to provide us with insights into the physics behind neutrino masses and mixing.

On the other hand there is lot of data now available on CP violation in the B system. The goal of the B factories is to test the Cabibbo-Kobayashi-Maskawa (CKM) picture of CP violation and look for evidence of new physics.

For some time now the B factories have been reporting several experimental hints of new physics. First, within the SM, the measurement of the CP phase sin2β in $B_d^0(t) \to J/\psi K_S$ should be approximately equal to that in decays dominated by the quark-level penguin transition $b \to sq\bar{q}$ $(q = u, d, s)$ like $B_d^0(t) \to \phi K_s$, $B_d^0(t) \to \eta' K_s$, $B_d^0(t) \to \pi^0 K^0$, etc. However, there is a difference between the measurements of sin2β in the $b \to s$ penguin dominated modes (sin2$\beta = 0.50 \pm 0.06$) and that in $B_d^0(t) \to J/\psi K_S$(sin2$\beta = 0.685 \pm 0.032$) [3–5]. Note that the sin2β number for the $b \to s$ penguin dominated modes is the average of several modes. The effect of new physics can be different for different final hadronic states and so the individual sin2β measurements for the different modes are important. Second, the latest data on $B \to \pi K$ decays (branching ratios and various CP asymmetries) appear to be inconsistent with the SM [6,7].[1] Third, within the SM, one expects no triple-product asymmetries in $B \to \phi K^*$ [9], but *BABAR* has measured such an effect at 1.7σ level [10]. There are also polarization anomalies where the decays $\bar{B}_d^0 \to \phi K^*$ and $B^- \to \rho^- K^*$ appear to have large transverse polarization amplitudes in conflict with naive SM expectations [5,11,12].

While these deviations certainly do not unambiguously signal new physics (NP), they give reason to speculate about NP explanations of the experimental data. Furthermore, it is far more compelling to find NP scenarios that provide a single solution to all the deviations than to look for solutions to individual discrepancies. Taking all these deviations seriously one is led to certain structures of NP operators that can explain the present data [13]. The question then is what kind of NP models can generate these specific operator structures at low energies. One can be more ambitious and ask if such models may have any connection with the physics behind neutrino mixing. In fact, the large $\nu_\mu - \nu_\tau$ mixing can lead to new effects in $b \to s$ transitions through squark mixing in certain supersymmetric grand unified theories [14]. However, the dominant contributions in this scenario are new QCD penguin amplitudes to B decays involving $b \to s$ transitions and such contributions appear to be ruled out by the present $B \to K\pi$ data [6,7].

In this paper we present a simple two Higgs doublet model with a 2-3 interchange symmetry in the down quark sector like the $\mu - \tau$ interchange symmetry in the leptonic sector. This model can provide a single solution to the deviations from SM observed in B decays and has interesting implications for the leptonic sector. It is believed that one of the phenomenological problems of the 2-3 symmetry model is the predictions $m_\mu = m_\tau$ or $m_s = m_b$. However, we will assume the 2-3 symmetry in the gauge basis where the mass matrix has off-diagonal terms and is 2-3 symmetric. Diagonalizing the mass matrix will split the masses of s and b or μ and τ. In fact we will consider the ''enhanced'' 2-3 symmetry where the matrix element of the mass matrix are invariant under any 2-3 interchange. In

*Electronic address: datta@physics.utoronto.ca
†Electronic address: odonnell@physics.utoronto.ca

[1] A cleaner test of the SM could be provided by looking at the quasiexclusive decays $B \to KX$ [8] rather than the exclusive $B \to K\pi$ decays.

 © 2005 The American Physical Society

other words we assume $\langle 2|M|2\rangle = \langle 2|M|3\rangle = \langle 3|M|2\rangle = \langle 3|M|3\rangle$. Diagonalizing the mass matrix then leads to vanishing $m_s(m_\mu)$.

The breaking of the 2-3 symmetry is then introduced though the strange quark mass in the quark sector and the muon mass in the leptonic sector. The breaking of the 2-3 symmetry leads to flavor changing neutral currents (FCNC) in the quark sector and the charged lepton sector that are suppressed by m_s/m_b and m_μ/m_τ in addition to the mass of the Higgs boson of the second Higgs doublet. Additional FCNC effects of similar size can be generated from the breaking of the $s - b$ symmetry in the Yukawa coupling of the second Higgs doublet.

In this paper we will be interested in the simplest two Higgs doublet model with a 2-3 symmetric Yukawa coupling that can explain the hints of new physics observed in several rare B decays. We, therefore, make some simplifying assumptions to make our model as predictive as possible. First, we assume a discrete symmetry involving the down quark to prevent FCNC effects in $s \to d$ and $b \to d$ transitions. This allows us to satisfy constraints from measurements in the kaon system and B_d mixing. Second, we assume a simple ansatz for the Yukawa coupling of the second Higgs doublet where the Yukawa matrix is described by two real parameters and a universal weak phase. Finally, we assume there is no mixing among the neutral Higgs bosons in the model and we only consider FCNC effects generated by the lightest Higgs bosons of the second Higgs doublet. Possible FCNC effects from the other heavier neutral Higgs boson are neglected by assuming the mass of the Higgs boson to be sufficiently high.

The low energy effective Hamiltonian generated by this model can explain the general features of the deviations from the SM seen in the various rare B decays. A new physics fit to the $B \to K\pi$ data [7] allows us to fix the parameters of the model including the lightest Higgs mass of the second doublet. To extract the parameters of the model we have to calculate the ratio of the tree amplitude in the standard model relative to a new physics amplitude. We use factorization to calculate the tree amplitude and the new physics amplitudes in the $K\pi$ system. Since nonfactorizable effects in the tree and the new physics amplitudes are expected to be small [15,16] the use of factorization to extract the parameters of the model is reasonable. A more precise determination of the parameters of the model using the calculation of the nonleptonic amplitudes in the framework of QCD factorization [15] will be carried out in a future work [17].

Having fixed the parameters of the model we make predictions for the rare B decays $B \to \rho K^*$, ϕK^*, ϕK_s and $\eta' K_s$. Here again we use factorization to calculate the relevant nonleptonic amplitudes. The inclusion of nonfactorizable effects will change the predicted values of quantities such as polarization fractions, CP violation etc., but we do not expect the general pattern of new physics effects of the model in the various B decays to change. This is because our predictions are largely dependent on heavy quark theory considerations. More precise predictions for the various B decays in this model, including nonfactorizable effects, will be carried out in a future work [17].

We also make predictions for $B_{d,s} - \bar{B}_{d,s}$ mixing and for B decays with the underlying quark transitions $b \to s\bar{u}u$ and $b \to s\bar{c}c$. The implication of the 2-3 symmetry in the up quark sector and on the CKM matrix is studied. This model generates FCNC effects in the leptonic sector and predictions for the decay $\tau \to \mu\bar{l}l$ and $\tau \to \mu\bar{q}q$ are made. Finally, a deviation of about 2 degrees from the maximal value for the atmospheric mixing angle θ_{23} is predicted from the breaking of the 2-3 symmetry in the charged lepton mass matrix.

We begin in Sec. II with a description of the two Higgs doublet model with 2-3 interchange symmetry and study the effects of the model in various B decays. In Sec. III we study the implication of the model for the up quark sector and the CKM matrix. In Sec. IV we study the effects of the model in the lepton sector, concentrating on FCNC effects in the τ decays and the neutrino mixing matrix. Finally in Sec. V we conclude with a summary of the results reported in this work.

II. THE MODEL—QUARK SECTOR

Two Higgs doublet models (2HDM) have been studied widely and a particularly interesting version with FCNC was studied in detail in Ref. [18]. The model presented here also has FCNC effects but its origin and structure are different from that in Ref. [18].

We start with the discussion of our model in the quark sector. Consider a Yukawa Lagrangian of the form

$$\mathcal{L}_Y^Q = Y_{ij}^U \bar{Q}_{i,L}\tilde{\phi}_1 U_{j,R} + Y_{ij}^D \bar{Q}_{i,L}\phi_1 D_{j,R} + S_{ij}^U \bar{Q}_{i,L}\tilde{\phi}_2 U_{j,R} + S_{ij}^D \bar{Q}_{i,L}\phi_2 D_{j,R} + \text{H.c.}, \tag{1}$$

where ϕ_i, for $i = 1, 2$, are the two scalar doublets of a 2HDM, while $Y_{ij}^{U,D}$ and $S_{ij}^{U,D}$ are the nondiagonal matrices of the Yukawa couplings.

For convenience we can choose to express ϕ_1 and ϕ_2 in a suitable basis such that only the $Y_{ij}^{U,D}$ couplings generate the fermion masses. In such a basis one can write

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \qquad \langle\phi_2\rangle = 0. \tag{2}$$

The two Higgs doublets in this case are of the form,

$$\begin{aligned} \phi_1 &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix}, \\ \phi_2 &= \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}H^+ \\ H^1 + iH^2 \end{pmatrix}. \end{aligned} \tag{3}$$

In principle there can be mixing among the neutral Higgs but here we neglect such mixing. We assume the doublet ϕ_1 corresponds to the scalar doublet of the SM and H^0 to the SM Higgs field. In addition, we assume that the second Higgs doublet does not couple to the up-type quarks ($S^U \equiv 0$). For the down-type couplings in Eq. (1) we have

$$\mathcal{L}_Y^D = Y_{ij}^D \bar{Q}_{i,L} \phi_1 D_{j,R} + S_{ij}^D \bar{Q}_{i,L} \phi_2 D_{j,R} + \text{H.c.} \tag{4}$$

We assume the following symmetries for the matrices Y^D and S^D:

(i) There is a discrete symmetry under which $d_{L,R} \rightarrow -d_{L,R}$.

(ii) There is a $s - b$ interchange symmetry: $s \leftrightarrow b$.

The discrete symmetry involving the down quark is enforced to prevent $s \rightarrow d$ transition because of constraints from the kaon system. It also prevents $b \rightarrow d$ transitions since B_d mixing as well as the value of $\sin 2\beta$ measured in $B_d^0(t) \rightarrow J/\psi K_S$ are consistent with SM predictions. Although there may still be room for NP in $b \rightarrow d$ transitions, almost all deviations from the SM have been reported only in $b \rightarrow s$ transitions and so we assume no NP in $b \rightarrow d$ transitions in this work.

The above symmetries then give the following structure for the Yukawa matrices:

$$Y^D = \begin{pmatrix} y_{11} & 0 & 0 \\ 0 & y_{22} & y_{23} \\ 0 & y_{23} & y_{22} \end{pmatrix}, \qquad S^D = \begin{pmatrix} s_{11} & 0 & 0 \\ 0 & s_{22} & s_{23} \\ 0 & s_{23} & s_{22} \end{pmatrix}. \tag{5}$$

The down-type mass matrix, M^D is now given by $M^D = (v/\sqrt{2})Y^D$. The matrix Y^D is symmetric and choosing the elements in Y^D to be real the mass matrix is diagonalized by

$$M_{\text{diag}}^D = U^T M^D U = \begin{pmatrix} \frac{v}{\sqrt{2}} y_{11} & 0 & 0 \\ 0 & \frac{v}{\sqrt{2}}(y_{22} - y_{23}) & 0 \\ 0 & 0 & \frac{v}{\sqrt{2}}(y_{22} + y_{23}) \end{pmatrix},$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{6}$$

It is clear that the matrix U will also diagonalize the S^D matrix when we transform the quarks from the gauge to the mass eigenstate via $d_{L,R} \rightarrow U d_{L,R}$. Hence there are no FCNC effects involving the Higgs ϕ_2.

The down quark masses are given by

$$m_d = \pm \frac{v}{\sqrt{2}} y_{11}, \qquad m_s = \pm \frac{v}{\sqrt{2}}(y_{22} - y_{23}), \qquad m_b = \pm \frac{v}{\sqrt{2}}(y_{22} + y_{23}). \tag{7}$$

Since $m_s \ll m_b$ there has to be a fine-tuned cancellation between y_{22} and y_{23} to produce the strange quark mass. Hence, it is more natural to consider the symmetry limit $y_{22} = y_{23}$ which leads to $m_s = 0$. We then introduce the strange quark mass as a small breaking of the $s - b$ symmetry and consider the structure

$$Y_n^D = \begin{pmatrix} y_{11} & 0 & 0 \\ 0 & y_{22}(1 + 2z) & y_{22} \\ 0 & y_{22} & y_{22} \end{pmatrix}, \tag{8}$$

with $z \sim 2m_s/m_b$ being a small number. Note that we do not break the $s - b$ symmetry in the 2-3 element so that the Y_n^D matrix remains symmetric. This down quark matrix is now diagonalized by

$$M_{\text{diag}}^D = W^T M^D W = \begin{pmatrix} \pm \frac{v}{\sqrt{2}} y_{11} & 0 & 0 \\ 0 & \pm \frac{v}{\sqrt{2}} z y_{22} & 0 \\ 0 & 0 & \pm \frac{v}{\sqrt{2}}(2 + z) y_{22} \end{pmatrix},$$

$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}}(1 - \frac{1}{2}z) & \frac{1}{\sqrt{2}}(1 + \frac{1}{2}z) \\ 0 & \frac{1}{\sqrt{2}}(1 + \frac{1}{2}z) & \frac{1}{\sqrt{2}}(1 - \frac{1}{2}z) \end{pmatrix}. \tag{9}$$

In Eq. (9) we have dropped terms of $O(z^2)$. The down masses are now taken to be

$$m_d = \pm \frac{v}{\sqrt{2}} y_{11}, \qquad m_s = \pm \frac{v}{\sqrt{2}} z y_{22}, \qquad m_b = \pm \frac{v}{\sqrt{2}}(2 + z) y_{22}. \tag{10}$$

We find $z \approx \pm 2m_s/m_b$ and now the transformation to the mass eigenstate will generate FCNC effects involving ϕ_2. For definiteness we will choose the positive sign for z though both signs are allowed. The matrix S^D in the mass eigenstate basis now has the form

$$S^D \rightarrow S^{D'} = \begin{pmatrix} s_{11} & 0 & 0 \\ 0 & (s_{22} - s_{23}) & s_{23} z \\ 0 & s_{23} z & (s_{22} + s_{23}) \end{pmatrix}. \tag{11}$$

There is now FCNC involving a $b \rightarrow s$ transition which is proportional to $z \sim m_s/m_b \sim \lambda^2$ where λ is the cosine of the Cabibbo angle.

It is also possible to find a S^D that breaks the $s - b$ symmetry. For example, we could choose

$$S^D = \begin{pmatrix} s_{11} & 0 & 0 \\ 0 & s_{22} & s_{23}(1+2\epsilon) \\ 0 & s_{23} & s_{33} \end{pmatrix}, \qquad (12)$$

with ϵ being a small quantity of the same size or smaller than z. A fit to the $B \to K\pi$ data, presented later, will rule out small z/ϵ. The transformation to the mass eigenstate leads to the general parametrization

$$S^D \to S^{D'} = \begin{pmatrix} s_d e^{i\phi_{dd}} & 0 & 0 \\ 0 & s_s e^{i\phi_{ss}} & s e^{i\phi_{sb}} - p e^{i\psi_{sb}} \\ 0 & s e^{i\phi_{sb}} + p e^{i\psi_{sb}} & s_b e^{i\phi_{bb}} \end{pmatrix}, \qquad (13)$$

where

$$\begin{aligned} s_d e^{i\phi_{dd}} &= s_{11}, \\ s_s e^{i\phi_{ss}} &= (\tfrac{1}{2}s_{22} + \tfrac{1}{2}s_{33} - s_{23}) + (\tfrac{1}{2}s_{33} - \tfrac{1}{2}s_{22})z - s_{23}\epsilon, \\ s_b e^{i\phi_{bb}} &= (\tfrac{1}{2}s_{22} + \tfrac{1}{2}s_{33} + s_{23}) + (\tfrac{1}{2}s_{22} - \tfrac{1}{2}s_{33})z + s_{23}\epsilon, \\ s e^{i\phi_{sb}} &= (\tfrac{1}{2}s_{33} - \tfrac{1}{2}s_{22}) + s_{23}z, \\ p e^{i\psi_{sb}} &= s_{23}\epsilon. \end{aligned} \qquad (14)$$

There is now an additional FCNC involving $b \to s$ transitions whose source is the $s - b$ symmetry breaking in S^D. Note that the 2-3 off-diagonal elements in $S^{D'}$ contain a part that is symmetric under $s - b$ interchange and a part that is antisymmetric under the $s - b$ interchange. The parameters in $S^{D'}$ can be obtained or constrained from a fit to B decay data.

We will not consider this general case in the paper but to make our model predictive we will assume a simplified ansatz for S^D in Eq. (12). Here we will consider the following structure for the S^D matrix with a small $s - b$ symmetry breaking:

$$S^D = e^{i\phi} \begin{pmatrix} s & 0 & 0 \\ 0 & 0 & \pm s(1+2\epsilon) \\ 0 & \pm s & 0 \end{pmatrix}, \qquad (15)$$

where s and ϵ are real numbers and we have introduced a universal weak phase ϕ. On moving to the mass eigenstate we obtain

$$S^D \to S^{D'} = e^{i\phi} \begin{pmatrix} s & 0 & 0 \\ 0 & \mp s(1+\epsilon) & \pm s(z-\epsilon) \\ 0 & \pm s(z+\epsilon) & \pm s(1+\epsilon) \end{pmatrix}. \qquad (16)$$

Here the coupling of the Higgs to the d and the s quarks are the same, up to a sign, to a good approximation since ϵ is a small parameter. The FCNC $b \to s$ transition arises from two sources. The first, represented by $z \sim 2m_s/m_b$, comes from the breaking of the $s - b$ symmetry in the matrix Y_n^D [Eq. (8)] and is symmetric under $s - b$ interchange while the second represented by $\epsilon \lesssim z$, comes from the breaking of the $s - b$ symmetry in the matrix S^D [Eq. (15)] and is antisymmetric under $s - b$ interchange.

The size of the matrix elements in S^D are not known. One might argue that the size of the matrix elements should be the same as the matrix elements in the Y matrix that gives mass to the down quarks. Hence the elements in S^D are given as $S^D \sim Y \sim m_q/v$ where $q = d, s, b$ and $v \sim 246$ GeV and are small. However it is possible that the down quark masses get their masses from a different Higgs boson than the top quark [19], in which case the vacuum expectation value of ϕ_1 giving mass to the down quarks can be small, much less than v, thus allowing for larger values of Y and S^D. Note that it is not necessary for Y and S^D to be related and thus we assume no relation between them.

The Lagrangian describing the interaction of the Higgs, $H_{1,2}$, is given by

$$\begin{aligned} \mathcal{L}_1 &= \frac{1}{2\sqrt{2}} J(H_1 + iH_2), \\ J_{ij}^{H_{1,2}} &= S_{ij}\bar{d}_i(1+\gamma_5)d_j \pm S_{ij}^*\bar{d}_j(1-\gamma_5)d_i. \end{aligned} \qquad (17)$$

Specifically for $b \to s\bar{q}q$ $(q = d, s)$ transitions the relevant currents, J_{ij} are

$$\begin{aligned} J_{sb}^{H_{1,2}} + J_{bs}^{H_{1,2}} &= S_{sb}\bar{s}(1+\gamma_5)b \pm S_{bs}^*\bar{s}(1-\gamma_5)b, \\ J_{qq}^{H_{1,2}} &= S_{qq}\bar{q}(1+\gamma_5)q \pm S_{qq}^*\bar{q}(1-\gamma_5)q. \end{aligned} \qquad (18)$$

Using Eq. (16) we have

$$S_{sb(bs)} = \pm s e^{i\phi}(z \mp \epsilon), \qquad (19)$$

$$S_{dd} = -S_{ss} = \pm s e^{i\phi}. \qquad (20)$$

After integrating out the heavy Higgs boson, $H_{1,2}$, we can generate the following effective Hamiltonian with four quark operators:

$$\begin{aligned} H_{H_{1,2}}^{\text{eff}} &= H_{\text{eff}}^s + H_{\text{eff}}^a, \\ H_{\text{eff}}^s &= \eta_{qq}\frac{G_F}{\sqrt{2}}\frac{2m_s}{m_b}\frac{s^2}{g^2}\frac{m_W^2}{m_{H_{1,2}}^2}[\pm O_{RR} \pm O_{LL} + O_{RL} + O_{LR}], \\ H_{\text{eff}}^a &= \eta_{qq}\frac{G_F}{\sqrt{2}}\epsilon\frac{s^2}{g^2}\frac{m_W^2}{m_{H_{1,2}}^2}[\pm O_{LL} \mp O_{RR} + O_{LR} - O_{RL}], \\ O_{RR} &= e^{i2\phi}\bar{s}(1+\gamma_5)b\bar{q}(1+\gamma_5)q, \\ O_{LL} &= e^{-i2\phi}\bar{s}(1-\gamma_5)b\bar{q}(1-\gamma_5)q, \\ O_{RL} &= \bar{s}(1+\gamma_5)b\bar{q}(1-\gamma_5)q, \\ O_{LR} &= \bar{s}(1-\gamma_5)b\bar{q}(1+\gamma_5)q, \end{aligned} \qquad (21)$$

where $\eta_{qq} = 1\ (-1)$ for the choices in Eq. (16), g is the weak coupling and the plus (minus) signs in front of $O_{LL,RR}$ correspond to the Higgs exchange $H_{1,2}$. Note that a new weak phase is associated only with the operators $O_{LL,RR}$ for the ansatz in Eq. (16). The Higgs effective potential will be assumed to make one of the neutral Higgs lighter than the other and we will be interested in the phenomenology of the lightest Higgs only which we will generically denote as H and for the sake of simplification and clarity we choose $H = H_1$ in Eq. (21). Possible FCNC effects associated with the other neutral Higgs boson will be neglected by taking its mass to be sufficiently heavy. An effective Hamiltonian will also be generated by the exchange of the charged H^+ Higgs boson which will generate transitions of the form $b \to c\bar{q}q$. We will not consider such effects in this work.

Now from the structure of $H_H^{\rm eff}$ we can calculate nonleptonic decays where hints of deviations from the SM have been reported. We will make a semiquantitative analysis of NP effects in the various decays and a more thorough investigation will be carried out in a later work [17]. We will first fix the parameters of the model from the fit results to the $K\pi$ system obtained in Ref. [7]. We will then make predictions for several nonleptonic B decays. For the purpose of the paper we will use factorization to calculate nonleptonic decays. The impact of nonfactorizable effects will be addressed later in the paper. For the decay $B \to P_1 P_2$, where $P_{1,2}$ are any final state particles, the matrix element of any operator in Eq. (21), in factorization, has the structure $\langle O\rangle \sim \langle P_1|\bar{s}\gamma_A b|B\rangle\langle P_2|\bar{q}\gamma_B q|0\rangle$ where $\gamma_{A,B} = (1 \pm \gamma_5)$ if the meson P_2 contains the $\bar{q}q$ component in its flavor wave function. An example of such a decay is $B^- \to K^-\pi^0$ with $P_{1,2} \equiv K^-(\pi^0)$. On the other hand for the decay $B^- \to K^0\pi^-$ the operators in Eq. (21) have to be given a Fierz transformation. We can define the Fierzed operators

$$\begin{aligned}
O_{LL}^F &= -\frac{1}{2N_c} e^{-i2\phi}\bar{q}(1-\gamma_5)b\bar{s}(1-\gamma_5)q \\
&\quad - \frac{1}{8N_c} e^{-i2\phi}\bar{q}\sigma_{\mu\nu}(1-\gamma_5)b\bar{s}\sigma^{\mu\nu}(1-\gamma_5)q, \\
O_{RR}^F &= -\frac{1}{2N_c} e^{i2\phi}\bar{q}(1+\gamma_5)b\bar{s}(1+\gamma_5)q \\
&\quad - \frac{1}{8N_c} e^{i2\phi}\bar{q}\sigma_{\mu\nu}(1+\gamma_5)b\bar{s}\sigma^{\mu\nu}(1+\gamma_5)q, \\
O_{LR}^F &= -\frac{1}{2N_c}\bar{q}\gamma_\mu(1-\gamma_5)b\bar{s}\gamma^\mu(1+\gamma_5)q, \\
O_{RL}^F &= -\frac{1}{2N_c}\bar{q}\gamma_\mu(1+\gamma_5)b\bar{s}\gamma^\mu(1-\gamma_5)q, \qquad (22)
\end{aligned}$$

where we have done also a color Fierz and dropped octet operators that do not contribute in factorization.

We can now look at the various nonleptonic B decays and we start with those decays with the underlying quark transition $b \to s\bar{d}d$.

A. $B \to K\pi$ decays

Let us denote by A^{ij} the amplitude for the decay $B_d^0 \to \pi^i K^j$. In the SM they are described to a good approximation by a "tree" amplitude T', a gluonic "penguin" amplitude or P', and a color-favored electroweak (EW) penguin amplitude $P'_{\rm EW}$. In $B \to \pi K$ decays, there are four classes of NP operators, differing in their color structure: $\bar{s}_\alpha\Gamma_i b_\alpha\bar{q}_\beta\Gamma_j q_\beta$ and $\bar{s}_\alpha\Gamma_i b_\beta\bar{q}_\beta\Gamma_j q_\alpha$ $(q = u, d)$. The matrix elements of these operators are then combined into single NP amplitudes, denoted by $\mathcal{A}^{\prime,q}e^{i\Phi'_q}$ and $\mathcal{A}^{\prime C,q}e^{i\Phi'^C_q}$ respectively with $q = u, d$. In the presence of NP the most general $B \to \pi K$ amplitudes take the form [7,16],

$$\begin{aligned}
A^{+0} &= -P' + \mathcal{A}^{\prime C,d}e^{i\Phi'^C_d}, \\
\sqrt{2}A^{0+} &= P' - T'e^{i\gamma} - P'_{\rm EW} + \mathcal{A}^{\prime,{\rm comb}}e^{i\Phi'} - \mathcal{A}^{\prime C,u}e^{i\Phi'^C_u}, \\
A^{-+} &= P' - T'e^{i\gamma} - \mathcal{A}^{\prime C,u}e^{i\Phi'^C_u}, \\
\sqrt{2}A^{00} &= -P' - P'_{\rm EW} + \mathcal{A}^{\prime,{\rm comb}}e^{i\Phi'} + \mathcal{A}^{\prime C,d}e^{i\Phi'^C_d}, \qquad (23)
\end{aligned}$$

where $\mathcal{A}^{\prime,{\rm comb}}e^{i\Phi'} \equiv -\mathcal{A}^{\prime,u}e^{i\Phi'_u} + \mathcal{A}^{\prime,d}e^{i\Phi'_d}$, $\mathcal{A}^{\prime C,u}$, and $\mathcal{A}^{\prime C,d}$ are the NP amplitudes [16].

It was found in Ref. [7] that a good fit to the data can be obtained with the NP amplitudes $|\mathcal{A}^{\prime,{\rm comb}}/T'| = 1.64$, $\Phi' \sim 100^0$, $\mathcal{A}^{\prime C,u} \approx 0$, and $\mathcal{A}^{\prime C,d} \approx 0$. It is clear from the structure of Eq. (21) that $\mathcal{A}^{\prime C,u} = 0$ as the NP involves only down-type quarks.

Now one obtains, using Eq. (21) and factorization, the matrix element relations for the decay $B^- \to K^-\pi^0$,

$$\begin{aligned}
\langle O_{LL}(O_{LL}^F)\rangle &= -\langle O_{RR}(O_{RR}^F)\rangle, \\
\langle O_{LR}(O_{LR}^F)\rangle &= -\langle O_{RL}(O_{RL}^F)\rangle.
\end{aligned} \qquad (24)$$

These relations follow from the fact that in factorization,

$$\langle O_{AB}\rangle \sim \langle K^-|\bar{s}\gamma_A b|B\rangle\langle\pi^0|\bar{d}\gamma_B d|0\rangle,$$

where $\gamma_{A,B} = (1 \pm \gamma_5)$. We then have

$$\begin{aligned}
\langle O_{LL}\rangle &= \langle K^-|\bar{s}(1-\gamma_5)b|B\rangle\langle\pi^0|\bar{d}(1-\gamma_5)d|0\rangle \\
&= -\langle K^-|\bar{s}b|B\rangle\langle\pi^0|\bar{d}\gamma_5 d|0\rangle \\
&= -\langle K^-|\bar{s}(1+\gamma_5)b|B\rangle\langle\pi^0|\bar{d}(1+\gamma_5)d|0\rangle \\
&= -\langle O_{RR}\rangle.
\end{aligned}$$

Similar arguments lead to the other relations in Eq. (24). Using the matrix element relations in Eq. (24) one obtains

$$\begin{aligned}
\langle K^-\pi^0|H_H^{\rm eff}|B^-\rangle &= |\mathcal{A}^{\prime,{\rm comb}}|e^{i\Phi'} \\
&= \frac{G_F}{\sqrt{2}}A_{dd}[2i\sin 2\phi\,\chi_s \\
&\quad - 2\cos 2\phi\,\chi_a + 2\chi_a], \\
\chi_s &= \frac{2m_s}{m_b}\frac{s^2}{g^2}\frac{m_W^2}{m_H^2}, \qquad (25)\\
\chi_a &= \epsilon\frac{s^2}{g^2}\frac{m_W^2}{m_H^2}, \\
A_{dd} &= \langle K^-|\bar{s}b|B^-\rangle\langle\pi^0|\bar{d}\gamma_5 d|0\rangle.
\end{aligned}$$

To make estimates we choose $\epsilon \sim z = 2m_s/m_b$ and

$\cos 2\phi \sim -\sin 2\phi$. Choosing $2\phi = -45°$ and the fact that $\tan\Phi' \approx \cot\phi$ leads to $\Phi' \sim 113°$, which is consistent with the fit obtained in Ref. [7]. We observe that if $\chi_a/\chi_s = \epsilon/z \sim 0$ then $\Phi' \sim 90°$ which is also consistent with the fit obtained in Ref. [7]. However, with $\chi_s/\chi_a = z/\epsilon \sim 0$ we obtain $\Phi' \sim 0$ which is inconsistent with the fit in Ref. [7]. Hence only $\epsilon \lesssim z$ is allowed as indicated earlier. Turning now to the amplitude $\mathcal{A}'^{C,d}$ we have

$$\begin{aligned}\langle K^0\pi^-|H^{\rm eff}|B^-\rangle &= |\mathcal{A}'^{C,d}|e^{i\Phi'_u} = -\frac{1}{2N_c}\frac{G_F}{\sqrt{2}}[2F_{dd}(i\sin 2\phi\chi_s - \cos 2\phi\chi_a) + 2\chi_a G_{dd}],\\ F_{dd} &= \langle\pi^-|\bar{d}b|B^-\rangle\langle K^0|\bar{s}\gamma_5 d|0\rangle,\\ G_{dd} &= \langle\pi^-|\bar{d}\gamma_\mu b|B^-\rangle\langle K^0|\bar{s}\gamma^\mu\gamma_5 d|0\rangle.\end{aligned} \tag{26}$$

It is clear from Eq. (26) that $\mathcal{A}'^{C,d}$ is suppressed relative to $\mathcal{A}'^{C,\rm comb}$ by $1/(2N_c)$, and hence small, which is again consistent with the fit obtained in Ref. [7]. Now using $|\mathcal{A}'^{,\rm comb}/T'| = 1.64$ we find, using naive factorization [13],

$$1.64 = \left|\frac{2k\sin 2\phi\chi_s A_{dd}}{V_{ub}^* V_{us}\langle\pi^0|\bar{b}\gamma_\mu(1-\gamma_5)u|B^-\rangle\langle K^-|\bar{u}\gamma^\mu(1-\gamma_5)s|0\rangle}\right|, \qquad k = \sqrt{1+\tan^2\phi}. \tag{27}$$

One can then convert this to

$$2k|\sin 2\phi|\chi_s = \frac{f_K(m_B^2 - m_\pi^2)F_0^\pi/\sqrt{2}}{[(m_B^2 - m_K^2)/(m_b - m_s)]F_0^K(m_\pi^2/2m_d)f_\pi/\sqrt{2}}1.64\left(c_1 + \frac{c_2}{N_c}\right)|V_{ub}^* V_{us}|. \tag{28}$$

We take $(f_K/f_\pi)(F_0^\pi/F_0^K) \sim 1$, $|V_{ub}^* V_{us}/V_{tb}^* V_{ts}| = 1/48$ and $c_1 + c_2/N_c = 1.018$.

Taking the masses from the Particle Data Group [20], we find

$$\chi_s = \frac{1}{k|\sin 2\phi|}0.05\frac{m_d}{6\ \mathrm{MeV}}|V_{tb}^* V_{ts}|. \tag{29}$$

This then leads to

$$\frac{m_W^2}{m_H^2} = \frac{g^2}{2s^2k|\sin 2\phi|}\frac{m_b}{m_s}\begin{cases}0.033|V_{tb}^*V_{ts}| & m_d = 4\ \mathrm{MeV},\\ 0.07|V_{tb}^*V_{ts}| & m_d = 8\ \mathrm{MeV},\end{cases} \tag{30}$$

and finally to,

$$\begin{aligned} m_W\frac{\sqrt{2}s}{g}\sqrt{\frac{m_s}{m_b}\frac{k|\sin 2\phi|}{0.07|V_{tb}^*V_{ts}|}} &\le m_H\\ &\le m_W\frac{\sqrt{2}s}{g}\sqrt{\frac{m_s}{m_b}\frac{k|\sin 2\phi|}{0.033|V_{tb}^*V_{ts}|}},\\ 4.7M_W\left(\frac{s}{g}\right) \le m_H &\le 6.8M_W\left(\frac{s}{g}\right),\end{aligned} \tag{31}$$

where we have used $m_s = 100$ MeV and $m_b = 5$ GeV. For $s \sim 1$ we obtain $m_H \sim 600$–900 GeV.

Our estimate of the parameters of the model, like the new physics weak phase ϕ and the Higgs mass m_H, used the factorization assumption to calculate the nonleptonic amplitudes. It is therefore important to address the impact of nonfactorizable effects on the values of the parameters of the model. We note that to extract the parameters of the model we have to calculate the ratio of the tree amplitude in the standard model relative to a new physics amplitude. Since nonfactorizable effects in the tree and the new physics amplitudes are expected to be small [15,16] we do not expect these effects to significantly alter the values of the parameters of the model extracted using factorization. A more precise determination of the model parameters using QCD factorization [15] will be carried out in a future work [17].

B. $B \to K^*\rho$ decays

Here the final states contain two vector mesons and so from angular momentum considerations there are three amplitudes. They are usually chosen to be the longitudinal amplitude, A_0, and the two transverse amplitudes $A_\parallel$ and $A_\perp$. In the SM, A_0 is the dominant amplitude and the transverse amplitudes are $O(1/m_B)$ because of the $V-A$ structure of the weak interactions. Now it is clear from the structure of the operators in Eq. (21) that the scalar operators do not contribute to vector-vector final states in factorization. Hence we expect no new physics effect in the decay $B^- \to K^{-*}\rho^0$. On the other hand the decay $B^- \to K^{0*}\rho^-$ gets contribution from the Fierzed operators in Eq. (22). The tensor operators in $O^F_{LL,RR}$ contribute to the transverse amplitude in the leading order in the large m_B limit [13] unlike in the SM. This follows from the fact that the matrix elements of the tensor operators contain the piece $\sim\langle K^{*0}(q_\mu)|\bar{s}\sigma_{\mu\nu}d|0\rangle \sim (\epsilon_\mu q_\nu - \epsilon_\nu q_\mu)$ that contributes to the transverse amplitudes in the large m_B limit. For a longitudinally polarized state the polarization vector can be approximated as, $\epsilon_\mu \sim q_\mu$ neglecting $(m_{K^*}/m_B)^2$ effects, and hence the contribution to the longitudinal amplitude, A_0, arise only at $O(m_V/m_B)$. The operators $O^F_{LR,RL}$ contribute in the leading order in the large m_B limit to the longitudinal amplitude, A_0 while the transverse amplitudes are suppressed by $0(m_V/m_B)$ [9,13].

Hence our prediction is that the decay $B^- \to K^{-*}\rho^0$ should be dominantly longitudinally polarized as in the SM because there is no new physics effect in this decay, while the decay $B^- \to K^{0*}\rho^-$ may have a sizable transverse polarization. These predictions appear to be consistent with present experiments where *BABAR* measures the longitudinal polarization fraction, f_L, for $B^- \to K^{-*}\rho^0$ to be $0.96^{+0.04}_{-0.15} \pm 0.04$ while the *BABAR* and Belle measurements of f_L for $B^- \to K^{0*}\rho^-$ are $0.79 \pm 0.08 \pm 0.04$ and $0.43 \pm 0.11^{0.05}_{-0.02}$ respectively giving an average f_L of 0.66 ± 0.07, thereby showing sizable transverse polarization in this mode [12,13].

To calculate specific quantities we again use factorization to compute the nonleptonic amplitudes. We do not expect nonfactorizable effects to significantly alter the pattern of new physics effects in the B decays as our conclusions to a large extent rely on heavy quark theory considerations. As far as specific values of various quantities, like polarization fraction, CP violation etc., in the B decays are concerned it is possible that nonfactorizable effects will change the values of these quantities. Hence, our calculation of these quantities should be taken as rough estimates. A more precise estimate of the quantities will be carried out in a future work [17].

For the $A_\perp$ amplitude the factorized matrix element satisfies $\langle O^F_{LL}\rangle = \langle O^F_{RR}\rangle$ while for the $A_\parallel$ amplitude the factorized matrix element satisfies $\langle O^F_{LL}\rangle = -\langle O^F_{RR}\rangle$ and so one obtains

$$\begin{aligned} A_\perp &\sim [\chi_s \cos 2\phi - i\chi_a \sin 2\phi], \\ A_\parallel &\sim [-\chi_a \cos 2\phi + i\chi_s \sin 2\phi]. \end{aligned} \tag{32}$$

Note that even though the Fierzed tensor operators are color suppressed their effects for the transverse amplitudes can nonetheless be significant as the SM penguin amplitude for vector-vector final state is smaller than the pseudoscalar-pseudoscalar final states [13]. It follows from Eq. (32), for $\chi_s \sim \chi_a$, that the transverse polarization fractions $f_\perp \sim f_\parallel$ where $f_T \sim |A_T|^2$ ($T = \perp, \parallel$). We also expect no triple-product asymmetry [9] from the interference of $A_\perp$ and $A_\parallel$ because the relative weak phase difference is zero (or π). A triple-product asymmetry from the interference of the longitudinal amplitude A_0 and $A_\perp$ can be significant.

We now consider nonleptonic B decays involving the underlying $b \to s\bar{s}s$ transitions.

C. $B \to \phi K_s$, $B \to \eta' K_s$ decays

We begin with the decay $B \to \phi K_s$ where only the Fierzed operators in Eq. (22) contribute as ϕ is a vector meson. In factorization, for this mode, we have the matrix elements relations, $\langle O_{LL}\rangle = \langle O_{RR}\rangle$ and $\langle O_{LR}\rangle = \langle O_{RL}\rangle$. We can therefore write

$$\begin{aligned} A(B_d \to \phi K_s) &= A^{\mathrm{SM}} + A^{\mathrm{NP}}_{LR\pm RL} + A^{\mathrm{NP}}_{LL\pm RR}, \\ A^{\mathrm{SM}} &= -\frac{G_F}{\sqrt{2}} V_{tb} V^*_{ts} Z\bigg[a^t_3 + a^t_4 + a^t_5 - \frac{1}{2}a^t_7 \\ &\quad - \frac{1}{2}a^t_9 - \frac{1}{2}a^t_{10} - a^c_3 - a^c_4 - a^c_5 + \frac{1}{2}a^c_7 \\ &\quad + \frac{1}{2}a^c_9 + \frac{1}{2}a^c_{10}\bigg], \\ A^{\mathrm{NP}}_{LR\pm RL} &= -\frac{1}{N_c}\frac{G_F}{\sqrt{2}} Z\chi_s, \\ A^{\mathrm{NP}}_{LL\pm RR} &= -\frac{1}{N_c}\frac{m_\phi}{m_B}\frac{G_F}{\sqrt{2}} Z[\chi_s \cos 2\phi - i\chi_a \sin 2\phi], \\ \chi_s \sim \chi_a &= -\frac{1}{k|\sin 2\phi|} 0.05 |V^*_{tb} V_{ts}|, \\ Z &= 2 f_\phi m_\phi F_{BK}(m^2_\phi)\varepsilon^* \cdot p_B, \end{aligned} \tag{33}$$

where the SM contribution can be found in Ref. [21] and we have chosen $m_d = 6$ MeV.

We can now calculate $\sin(2\beta)_{\mathrm{eff}}$ from

$$\sin(2\beta)_{\mathrm{eff}} = -\frac{2\,\mathrm{Im}[\lambda_f]}{(1 + |\lambda_f|^2)}, \qquad \lambda_f = e^{-2i\beta}\frac{\bar{A}}{A}, \tag{34}$$

where $A = A^{\mathrm{SM,}}_{\phi K_S} + A^{\mathrm{NP}}_{\phi K_S}$ and $\bar{A}$ is the amplitude for the CP-conjugate process. Note that from Eq. (33), $\sin(2\beta)_{\mathrm{eff}}$ is independent of Z and hence free from uncertainties in the form factor and decay constants. We also observe that the deviation from the SM expectation of $\sin(2\beta)_{\mathrm{eff}}$ comes from $A^{\mathrm{NP}}_{LL\pm RR}$ which is however color and m_ϕ/m_B suppressed, leading to a rough estimate, $\sin(2\beta)_{\mathrm{eff}} \approx 0.43$ which agrees well with the present experimental average of $0.44 \pm 0.27 \pm 0.05$ [3–5]. Note that the fact $A^{\mathrm{NP}}_{LR\pm RL}$ does not carry any new weak phase is a consequence of the form in Eq. (16) and in the general case [Eq. (13)] there will be a new physics phase associated with this operator. Furthermore, if $\chi_a/\chi_s = \epsilon/z \sim 0$ then there is no new weak phase in this decay and the prediction of $\sin(2\beta)_{\mathrm{eff}}$ should be given by the SM.

The decay $B \to \eta' K_s$ is more complicated as both the $b \to s\bar{d}d$ and $b \to s\bar{s}s$ transitions contribute [22]. The final states are pseudoscalars and so both the operators in Eq. (21) and the Fierzed operators in Eq. (22) for $b \to s\bar{s}s$ transition contribute. Hence in this model this decay could show a significant deviation from the SM prediction for $\sin(2\beta)_{\mathrm{eff}}$. For the "standard" η' wave function $\eta' \sim (\bar{u}u + \bar{d}d - 2\bar{s}s)/\sqrt{6}$ the NP $b \to s\bar{d}d$ and $b \to s\bar{s}s$ amplitudes add or partially cancel depending on the choice in Eq. (16). In the η they would cancel or add producing a much smaller or bigger deviation from the SM in $B \to \eta K_s$ compared to $B \to \eta' K_s$. It should be noted that the decay $B \to \eta K_s$ has a much smaller branching ratio, probably because of destructive interference in the SM amplitudes [23].

D. $\bar{B}_d^0 \to \phi K^*$ decays

Here the final states contain two vector mesons like the $B^- \to \rho^- K^*$ decays and the predictions here are very similar. This decay gets contributions only from the Fierzed operators in Eq. (22). The tensor operators in $O^F_{LL,RR}$ can contribute significantly to the transverse amplitude in the large m_B limit [13] thus explaining the low longitudinal polarization fraction, $f_L = 0.45 \pm 0.05 \pm 0.02$ [5,11], measured in this decay. Given the large NP weak phase we could also observe a sizable triple-product asymmetry and/or direct CP asymmetry in this decay.

To summarize, the model can provide explanations for the deviation from SM seen in several rare B decays. It also makes specific predictions in other B decays. For example, decays going through $b \to s\bar{u}u$ transitions like $B_s \to K^+K^-$ should not be affected. The decays going through $b \to s\bar{c}c$ transitions like $B_{d,s} \to D_s^{(+*)} D_{d(s)}^{(-*)}$ should also not be affected and so this decay along with $B_d \to D^{(+*)}D^{(-*)}$ can be used to measure the angle γ [24] without NP pollution. The model also has no effect in the decays $B_{d,s} \to J/\psi K^{(*)}(\phi, \eta')$. A detail study of the predictions of this model in B decays will be presented elsewhere [17].

We now discuss the important case of $B_{d,s} - \bar{B}_{d,s}$ mixing. By construction of the model there is no effect in $B_d - \bar{B}_d$ mixing. In $B_s - \bar{B}_s$ mixing this model can produce a new CP phase through contributions associated with the operators $B_{LL} = \bar{b}(1-\gamma_5)s\bar{b}(1-\gamma_5)s$ and $B_{RR} = \bar{b}(1+\gamma_5)s\bar{b}(1+\gamma_5)s$. In the vacuum insertion approximation, the matrix element of B_{LL} and B_{RR} are the same as only the pseudoscalar currents contribute. The contribution to B_s mixing comes from the sum $\langle S_{sb}^{*2} e^{-i2\phi} B_{LL} + S_{bs}^2 e^{i2\phi} B_{RR}\rangle$ which contain a term $\sim i\chi_s\chi_a \sin 2\phi$ which is a source of new weak phase in $B_s - \bar{B}_s$ mixing. This term will be small or vanish if $\chi_a/\chi_s = \epsilon/z \sim 0$. Hence the presence of a new weak phase in the $B_s - \bar{B}_s$ mixing requires the breaking of the $s - b$ symmetry in the Yukawa coupling of the second Higgs doublet.

Finally, we note that the model will produce new effects in $b \to sl^+l^-$ and $B_s \to l^+l^-$ decays which will depend on the couplings s and s_l of the second Higgs doublet to the quarks and leptons. A detailed study of such processes in the model will be discussed elsewhere [17].

III. UP QUARK SECTOR

So far we have neglected the up sector and it is possible that there will be FCNC decays in that sector also. However the diagonalizing of the up quark mass matrix is connected to the CKM matrix via $V_{\rm CKM} = V_L^\dagger W$ where W in Eq. (9) diagonalizes the down quark matrix and $V_L^\dagger$ transform the left-handed up-type quarks from the gauge to the mass basis. If we assume the up sector has the same symmetry as the down sector then $V_L \sim W$. We can write

$$W_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}}(1-\frac{1}{2}z_u) & \frac{1}{\sqrt{2}}(1+\frac{1}{2}z_u) \\ 0 & \frac{1}{\sqrt{2}}(1+\frac{1}{2}z_u) & \frac{1}{\sqrt{2}}(1-\frac{1}{2}z_u) \end{pmatrix}, \quad W_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}}(1-\frac{1}{2}z_d) & \frac{1}{\sqrt{2}}(1+\frac{1}{2}z_d) \\ 0 & \frac{1}{\sqrt{2}}(1+\frac{1}{2}z_d) & \frac{1}{\sqrt{2}}(1-\frac{1}{2}z_d) \end{pmatrix}, \tag{35}$$

where $z_u \approx \pm 2m_c/m_t$ and $z_d \approx \pm 2m_s/m_b$ break the 2-3 symmetry in the up and the down sectors. The CKM matrix is now obtained as

$$V_{\rm CKM} = W_u^T \cdot W_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 + \frac{z_u z_d}{4} & \frac{1}{2}(z_u - z_d) \\ 0 & \frac{1}{2}(z_d - z_u) & 1 + \frac{z_u z_d}{4} \end{pmatrix}. \tag{36}$$

We see that we get the right order and sign for the CKM element V_{cb} and V_{ts}. To obtain the realistic $V_{\rm CKM}$ we have to introduce the Cabibbo angle λ as a symmetry breaking effect. Possible FCNC effects in the top sector will be discussed in a later work [17].

IV. LEPTONIC SECTOR

In this section we study the consequences of the 2-3 symmetry applied to the charged lepton sector. We assume that the structure for the charged lepton mass matrix, Y^L is given by

$$Y^L = \begin{pmatrix} l_{11} & 0 & 0 \\ 0 & l_{22}(1+2z_l) & l_{22} \\ 0 & l_{22} & l_{22} \end{pmatrix}, \tag{37}$$

where $z_l \sim 2m_\mu/m_\tau$ is a small number. This charged lepton mass matrix is now diagonalized by

$$M^L_{\rm diag} = W_l^T M^D W_l = \begin{pmatrix} \pm\frac{v}{\sqrt{2}} l_{11} & 0 & 0 \\ 0 & \pm\frac{v}{\sqrt{2}} z_l l_{22} & 0 \\ 0 & 0 & \pm\frac{v}{\sqrt{2}}(2+z_l) l_{22} \end{pmatrix},$$

$$W_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}}(1-\frac{1}{2}z_l) & \frac{1}{\sqrt{2}}(1+\frac{1}{2}z_l) \\ 0 & \frac{1}{\sqrt{2}}(1+\frac{1}{2}z_l) & \frac{1}{\sqrt{2}}(1-\frac{1}{2}z_l) \end{pmatrix}. \tag{38}$$

Just like the down quark sector we have broken the $\mu - \tau$ symmetry by the μ mass. In the symmetry limit

$$W_l \to U_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{39}$$

The Yukawa interaction associated with the second Higgs doublet is now taken similar to Eq. (16) as

$$S^L = e^{i\phi_l}\begin{pmatrix} s_l & 0 & 0 \\ 0 & 0 & \pm s_l(1+2\epsilon_l) \\ 0 & \pm s_l & 0 \end{pmatrix}, \qquad (40)$$

where s_l and ϵ_l are real numbers and we have introduced a universal weak phase ϕ_l. On moving to the mass eigenstate we obtain

$$S^L \to S^{L'} = e^{i\phi_l}\begin{pmatrix} s_l & 0 & 0 \\ 0 & \mp s_l(1+\epsilon_l) & \pm s_l(z_l-\epsilon_l) \\ 0 & \pm s_l(z_l+\epsilon_l) & \pm s_l(1+\epsilon_l) \end{pmatrix}. \qquad (41)$$

To simplify our discussion we assume $\epsilon_l \sim 0$. The model now generates FCNC interactions of the τ like $\tau \to \mu \bar{l} l$ where l are muon or electrons. Note that decays $\tau \to \mu \bar{l}_a l_b$ where $l_a \neq l_b$ are forbidden. The effective Hamiltonian for such decays is easily written down as

$$\begin{aligned} H_l^{\text{eff}} &= \eta_{ll}\frac{1}{4}\frac{m_\mu}{m_\tau}\frac{s_l^2}{m_H^2}[\pm O_{LL} \pm O_{RR} + O_{LR} + O_{RL}], \\ O_{LL} &= e^{-i2\phi_l}\bar{\mu}(1-\gamma_5)\tau\bar{l}(1-\gamma_5)l, \\ O_{RR} &= e^{i2\phi_l}\bar{\mu}(1+\gamma_5)\tau\bar{l}(1+\gamma_5)l, \\ O_{LR} &= \bar{\mu}(1-\gamma_5)\tau\bar{l}(1+\gamma_5)l, \\ O_{RL} &= \bar{\mu}(1+\gamma_5)\tau\bar{l}(1-\gamma_5)l. \end{aligned} \qquad (42)$$

To further simplify the discussion we will choose $\phi_l = 0$. In Ref. [25] the FCNC leptonic transitions were studied in an effective operator formalism where the coefficient of the four quark operators were taken to be $\sim 4\pi$. This lead to a constraint on the scale of NP, $\Lambda \sim 10$ TeV. To compare to our model we have the correspondence

$$\frac{4\pi}{\Lambda^2} \equiv \frac{1}{4}\frac{m_\mu}{m_\tau}\frac{s_l^2}{m_H^2}. \qquad (43)$$

Choosing $s_l \sim 1$ we find $m_H \sim 340$ GeV. From B decays we found $m_H \sim 600$–900 GeV and so we predict the branching ratio of $\tau \to \mu \bar{l} l$ to be below the experimental bounds [20] by about $(340/900)^4 - (340/600)^4 \sim 2 \times 10^{-2}$–$10^{-1}$. A careful analysis would include varying the parameters ϕ_l and s_l and will be presented elsewhere [17]. We can also study the decay $\tau \to \mu \bar{q} q$ which can be studied in the decays $\tau \to \mu \pi^0(\rho)$. The experimental bounds produce similar size limit on the Higgs mass as the decays $\tau \to \mu \bar{l} l$. Note that for $\phi_l = 0$ only H_2, which is a pseudoscalar, can contribute to $\tau \to \mu \pi^0$ and the contribution from H_1 exchange vanishes.

Let us now turn to the neutrino sector. The neutrino mixing, the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, arises from the lepton mass Lagrangian as follows:

$$\mathcal{L}_m = \nu_\alpha^T C^{-1}\mathcal{M}_{\nu,\alpha\beta}\nu + \bar{e}_{\alpha,L}M_{\alpha\beta}^e e_R + \text{H.c.} \qquad (44)$$

Diagonalizing the mass matrices by the transformations $U_\nu^T\mathcal{M}_\nu U_\nu = \mathcal{M}_{\text{diag}}^\nu$ and $U_\ell^\dagger M^e V = M_{\text{diag}}^e$, one defines the neutrino mixing matrix as $U_{\text{PMNS}} = U_\ell^\dagger U_\nu$. We will parametrize U_{PMNS} as follows:

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}K, \qquad (45)$$

where $K = \text{diag}(1, e^{i\phi_1}, e^{i\phi_2})$.

The 2-3 symmetry in the leptonic sector leads to a simplified form of the PMNS matrix, with $s_{13} = 0$, given by

$$U_{\text{PMNS}}^s = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}s_{12} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \end{pmatrix}. \qquad (46)$$

In the basis where the mass matrix of the charged leptons is diagonal, the left-handed flavor-based (symmetric) neutrino mass matrix m' is related to its diagonal form, $M_{\nu,\text{diag}} = \text{diag}[m_1, m_2, m_3]$ by

$$U_{\text{PMNS}}^T M_\nu' U_{\text{PMNS}} = M_{\nu,\text{diag}}. \qquad (47)$$

The form of U_{PMNS}^s just follows from the $\mu - \tau$ symmetry in M_ν' [1].

Note that we can express U_{PMNS}^s, using Eq. (39), as

$$U_{\text{PMNS}}^s = U_\ell^\dagger U_\nu, \qquad (48)$$

where

$$\begin{aligned} U_\ell^\dagger &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \\ U_\nu &= \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned} \qquad (49)$$

So the neutrino matrix, U_ν is just a combination of a simple rotation matrix and a phase matrix. Now the breaking of the 2-3 symmetry in the leptonic sector will change $U_\ell^\dagger$ and U_{PMNS} to

$$U_\ell^\dagger \to W_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}}(1-\frac{1}{2}z_l) & \frac{1}{\sqrt{2}}(1+\frac{1}{2}z_l) \\ 0 & \frac{1}{\sqrt{2}}(1+\frac{1}{2}z_l) & \frac{1}{\sqrt{2}}(1-\frac{1}{2}z_l) \end{pmatrix},$$

$$U_{\mathrm{PMNS}} \to \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}}s_{12}(1-\frac{z_l}{2}) & \frac{1}{\sqrt{2}}c_{12}(1-\frac{z_l}{2}) & \frac{1}{\sqrt{2}}(1+\frac{z_l}{2}) \\ \frac{1}{\sqrt{2}}(1+\frac{z_l}{2})s_{12} & -\frac{1}{\sqrt{2}}c_{12}(1+\frac{z_l}{2}) & \frac{1}{\sqrt{2}}(1-\frac{z_l}{2}) \end{pmatrix}, \tag{50}$$

where $z_l = \pm 2(m_\mu/m_\tau)$. This corresponds to $s_{23} = 1/\sqrt{2}[1 + (z_l/2)]$ and $c_{23} = 1/\sqrt{2}[1 - (z_l/2)]$ and to an atmospheric mixing angle of $\theta_{23} \sim 43.26°$ for the negative sign of z_l.

Finally, we point out that this model will have interesting collider signatures. The heavy Higgs, H, according to the structure in Eq. (16), couples equally to all three generations, to a very good approximation, unlike the usual SM Higgs which has couplings proportional to the mass. The effect of electroweak precision measurements and collider signatures will be explored elsewhere [17].

V. CONCLUSIONS

In summary, there are now several B decay modes in which there appear to be deviations from the SM predictions. These deviations could signal the presence of beyond the SM physics. In this work we were interested in a NP scenario that can provide a single solution to all the deviations. We considered a two Higgs doublet model with a 2-3 symmetry in the down-type quark and the charged lepton sector. The breaking of the 2-3 symmetry, introduced by the strange quark mass and the muon mass lead to FCNC in the quark sector and the charged lepton sector that are suppressed by m_s/m_b and m_μ/m_τ in addition to the mass of the heavy Higgs boson. Additional FCNC effects of similar size were generated from the breaking of the $s - b$ symmetry in the Yukawa coupling of the second Higgs doublet. From a fit to the $B \to K\pi$ data we found the mass of the lightest neutral Higgs in the second doublet to be of the order $m_H \sim 600$–900 GeV. We made several predictions in B decays listed below:

(i) A sizable transverse polarization in the decays $\bar{B}_d^0 \to \phi K^*$ and $B^- \to \rho^- K^{0*}$ was predicted but not in $B^- \to \rho^0 K^{*-}$. This is consistent with present measurements [5,12]. We also predicted the possibility of observing a sizable triple-product asymmetry and/or direct CP asymmetry in the decays $\bar{B}_d^0 \to \phi K^*$ and $B^- \to \rho^- K^{0*}$.

(ii) The $\sin 2\beta$ measurement in $B_d^0(t) \to \phi K_s$ was predicted to show a small deviation from the SM value [if Eq. (16) is assumed] but it was found that $\sin 2\beta$ measured in $B_d^0(t) \to \eta' K_s$ could have significant deviation from the SM prediction.

(iii) The decays with the quark transition $b \to s\bar{u}u$ and $b \to s\bar{c}c$ were found to be unaffected.

(iv) We found a new source of weak phase from our model in $B_s - \bar{B}_s$ mixing while $B_d - \bar{B}_d$ mixing was not affected.

We studied the implication of the 2-3 symmetry extended to the up sector and found that we could generate the right sign and size of the CKM matrix element V_{cb} and V_{ts}. We then studied FCNC effects in the lepton sector. The lepton flavor violating decays $\tau \to \mu \bar{l}(\bar{q}) l(q)$ were found to be below the present experimental limits by a factor of 2×10^{-2}–10^{-1}. Finally, we found that the breaking of 2-3 symmetry in the lepton sector could lead to deviations of the atmospheric neutrino mixing angle from the maximal value by $\sim$2 degrees.

ACKNOWLEDGMENTS

This work is financially supported by NSERC of Canada. We thank David London, R. N. Mohapatra and Sandip Pakvasa for useful discussion.

[1] C. S. Lam, Phys. Lett. B **507**, 214 (2001); T. Kitabayashi and M. Yasue, Phys. Rev. D **67**, 015006 (2003); W. Grimus and L. Lavoura, Phys. Lett. B **572**, 189 (2003); J. Phys. G **30**, 73 (2004); Y. Koide, Phys. Rev. D **69**, 093001 (2004).

[2] R. N. Mohapatra, in *Proceedings of the 32nd SLAC Summer Institute on Particle Physics, Menlo Park, CA, 2004*, eConf C040802, L011 (2004) (http://www-conf.slac.stanford.edu/ssi/2004); J. High Energy Phys. 10 (2004) 027; W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura, H. Sawanaka, and M. Tanimoto, Nucl. Phys. **B713**, 151 (2005); A. Ghosal, Mod. Phys. Lett. A **19**, 2579 (2004).

[3] B. Aubert *et al.* (*BABAR* Collaboration), hep-ex/0507087; Phys. Rev. D **71**, 111102 (2005); **71**, 091102 (2005); Phys. Rev. Lett. **94**, 191802 (2005); **94**, 161803 (2005).

[4] K. Abe *et al.* (Belle Collaboration), hep-ex/0507037; K. F. Chen *et al.* (Belle Collaboration), Phys. Rev. D **72**, 012004 (2005); K. Abe *et al.* (Belle Collaboration), hep-ex/0409049.

[5] Heavy Flavor Average Group, http://www.slac.stanford.edu/xorg/hfag/rare.

[6] Experiment: B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **89**, 281802 (2002); hep-ex/0408062; hep-ex/0408080; hep-ex/0408081; A. Bornheim *et al.* (CLEO Collaboration), Phys. Rev. D **68**, 052002 (2003); Y. Chao *et al.* (Belle Collaboration), Phys. Rev. D **69**, 111102 (2004); Theory: A. J. Buras, R. Fleischer, S.

Recksiegel, and F. Schwab, Eur. Phys. J. C **32**, 45 (2003); Phys. Rev. Lett. **92**, 101804 (2004); Nucl. Phys. **B697**, 133 (2004); Acta Phys. Pol. B **36**, 2015 (2005); V. Barger, C. W. Chiang, P. Langacker, and H. S. Lee, Phys. Lett. B **598**, 218 (2004); S. Mishima and T. Yoshikawa, Phys. Rev. D **70**, 094024 (2004); Y. L. Wu and Y. F. Zhou, Phys. Rev. D **71**, 021701 (2005); H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D **71**, 014030 (2005); Y. Y. Charng and H.-n. Li, Phys. Rev. D **71**, 014036 (2005); X. G. He and B. H. J. McKellar, hep-ph/0410098; C. S. Kim, S. Oh, and C. Yu, Phys. Rev. D **72**, 074005 (2005).

[7] S. Baek, P. Hamel, D. London, A. Datta, and D. A. Suprun, Phys. Rev. D **71**, 057502 (2005).

[8] T. E. Browder, A. Datta, X. G. He, and S. Pakvasa, Phys. Rev. D **57**, 6829 (1998).

[9] For a study of triple products in the SM and with new physics, see A. Datta and D. London, Int. J. Mod. Phys. A **19**, 2505 (2004).

[10] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **93**, 231804 (2004). Note that the earlier Belle measurements of the same quantities do not show any signs of a nonzero triple-product asymmetry; see K.-F. Chen *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 201801 (2003).

[11] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **91**, 171802 (2003); K.-F. Chen *et al.* (Belle Collaboration), Phys. Rev. Lett. **94**, 221804 (2005).

[12] Particle Data Group, in Ref. [20]; B. Aubert *et al.* (*BABAR* Collaboration), hep-ex/0408093; K. Senyo *et al.* (Belle Collaboration), hep-ex/0505067.

[13] S. Baek, A. Datta, P. Hamel, O. F. Hernandez, and D. London, Phys. Rev. D **72**, 094008 (2005).

[14] See, for example, T. Moroi, J. High Energy Phys. 03 (2000) 019; N. Akama, Y. Kiyo, S. Komine, and T. Moroi, Phys. Rev. D **64**, 095012 (2001); D. Chang, A. Masiero, and H. Murayama, Phys. Rev. D **67**, 075013 (2003), and references therein.

[15] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Nucl. Phys. **B591**, 313 (2000); **B606**, 245 (2001).

[16] A. Datta, M. Imbeault, D. London, V. Page, N. Sinha, and R. Sinha, Phys. Rev. D **71**, 096002 (2005); A. Datta and D. London, Phys. Lett. B **595**, 453 (2004).

[17] Alakabha Datta (unpublished).

[18] D. Atwood, L. Reina, and A. Soni, Phys. Rev. D **55**, 3156 (1997).

[19] A. K. Das and C. Kao, Phys. Lett. B **372**, 106 (1996).

[20] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004), http://pdg.lbl.gov/pdg.html.

[21] A. Datta, Phys. Rev. D **66**, 071702 (2002).

[22] A. Datta, X. G. He, and S. Pakvasa, Phys. Lett. B **419**, 369 (1998).

[23] H. J. Lipkin, Phys. Lett. B **254**, 247 (1991); **544**, 145 (2002); **283**, 421 (1992); A. Datta, H. J. Lipkin, and P. J. O'Donnell, Phys. Lett. B **544**, 145 (2002); H. J. Lipkin, hep-ph/0507225.

[24] R. Fleischer, Eur. Phys. J. C **10**, 299 (1999); A. Datta and D. London, J. High Energy Phys. 04 (2004) 072; Phys. Lett. B **584**, 81 (2004); J. Albert, A. Datta, and D. London, Phys. Lett. B **605**, 335 (2005).

[25] D. Black, T. Han, H. J. He, and M. Sher, Phys. Rev. D **66**, 053002 (2002).