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Erratum: Mass zeros in the one-loop effective actions of QED in 1 + 1 and 3 + 1 dimensions [Phys. Rev. D 62, 125007 (2000)]

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There are a number of changes in Sec. IV due to the fact that Abelian (anti) self-dual fields are harmonic functions and so are not square integrable on noncompact Euclidean space-times. Taking this into account, the conclusions of Sec. IV remain unchanged. The amended Sec. IV may be accessed at hep-th/0010008. The changes are itemized below.

Above Eq. (1.4) and also six lines below it replace $A_\mu \in L^n(\mathbb{R}^2)$, $n > 2$ with $A_\mu \in \cap_{n>2} L^n(\mathbb{R}^2)$.

In the paragraph above Sec. II, replace the first two sentences with "For Euclidean QED$_4$ we will present evidence in Sec. IV that $\ln\det_{\mathrm{QED}_4}$ vanishes for at least one value of m^2 for a class of smooth, square-integrable background gauge fields $F_{\mu\nu}$ provided $A_\mu \in \cap_{n>4} L^n(R^4)$."

Equation (4.3) should read

$$\frac{1}{4\pi^2}\int d^4x \mathbf{E}\cdot\mathbf{B} = -\frac{1}{16\pi^2}\int d^4x {}^*F_{\mu\nu}F^{\mu\nu} = \lim_{m^2\to 0} m^2 \mathrm{Tr}[(H_+ + m^2)^{-1} - (H_- + m^2)^{-1}].$$

Delete the last two sentences in the text under Eq. (4.4). Begin the paragraph that follows with "Now let us specialize to the case of smooth square-integrable background gauge fields for which all zero modes have either positive or negative chirality."

In Eqs. (4.5), (4.6), (4.8), and (4.10) replace D^2 with H_- and the factor $2m^2$ in Eq. (4.5) with m^2.

The three lines following Eq. (4.6) are replaced with "consistent with definition (2.1). From Eq. (4.3), Eq. (4.5) reduces to."

Replace Eq. (4.7) with

$$\lim_{m^2\to 0} m^2 \frac{\partial}{\partial m^2} \ln\det_{\mathrm{QED}_4} = -\frac{1}{32\pi^2}\int d^{4x}\left({}^*F_{\mu\nu}F^{\mu\nu} + \frac{2}{3}F^2_{\mu\nu}\right) + \lim_{m^2\to 0} m^2 \mathrm{Tr}[(H_- + m^2)^{-1} - (P^2 + m^2)^{-1}].$$

Replace Eq. (4.9) with

$$\ln\det_{\mathrm{QED}_4} = -\frac{1}{32\pi^2}\int d^4x\left({}^*F_{\mu\nu}F^{\mu\nu} + \frac{2}{3}F^2_{\mu\nu}\right)\ln m^2 + R(m^2).$$

Under Eq. (4.10), remove the text in the remaining paragraph beginning with "In four dimensions..." and replace it with "This and the tendency for infrared divergences to be less severe in higher dimensions lead us to conjecture that Eq. (4.8) is true. In the case when the fermionic zero modes all have negative chirality the roles of H_+ and H_- are interchanged. Thus, in either case, if

$$\left|\int d^4x {}^*F_{\mu\nu}F^{\mu\nu}\right| > \frac{2}{3}\int d^4x F^2_{\mu\nu},$$

then Eq. (4.9) indicates that $\ln\det_{\mathrm{QED}_4}$ becomes negative as $m^2 \to 0$, which is a reflection of paramagnetism [22]."

Above Eq. (4.11) replace $A_\mu \in L^n(\mathbb{R}^4)$, $n > 4$ with $A_\mu \in \cap_{n>4} L^n(\mathbb{R}^4)$.

In Eq. (4.12) the factor $1/2880m^4$ should be $1/2880\pi^2 m^4$.

Four lines above Eq. (4.15) delete "for (anti-) self-duals will be $O(\int d^4x B^6/m^8)$" and replace with "will be $O(\int d^4x F^6_{\mu\nu}/m^8)$."

In the sentence above Eq. (4.15) delete "and (anti-) self-dual fields."

Replace Eq. (4.15) with

$$\ln\det_{\mathrm{QED}_4} = \frac{1}{240\pi^2 m^2}\int d^4x(\partial_\alpha F_{\mu\nu})^2 + R_2 + R_4 + R_5.$$

Replace the remaining paragraph under Eq. (4.15) with "Here R_2 is the remainder from the second-order term which is of order $\int d^4x(\partial_\alpha\partial_\beta F_{\mu\nu})^2/m^4$; R_4 is the remainder from the fourth-order term and is of order $\int d^4x F^4_{\mu\nu}/m^4$, and R_5, the remainder from $\ln\det_5$, is of order $\int d^4x F^6_{\mu\nu}/m^8$. Therefore, provided $A_\mu \cap_{n>4} L^n(\mathbb{R}^4)$ and the integrals of $(\partial_\alpha F_{\mu\nu})^2$, $(\partial_\alpha\partial_\beta F_{\mu\nu})^2$, $F^4_{\mu\nu}$, and $F^6_{\mu\nu}$ are finite, $\ln\det_{\mathrm{QED}_4}$ becomes positive before dropping off to zero. This establishes that $\ln\det_{\mathrm{QED}_4}$ has at least one zero for $m^2 > 0$ for the class of fields considered if Eq. (4.8) is true."

