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Remark about a non-BPS Dp-brane at the tachyon vacuum moving in a curved background

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This paper is devoted to the study of the dynamics of a non-BPS Dp-brane at the tachyon vacuum that moves in the curved background.

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I. INTRODUCTION

One of the most interesting problems in string theory is the study of the time dependent process. Even if this problem is far from being solved in the full generality one can find many examples where we can obtain some interesting results. The most celebrated problem is the time dependent tachyon condensation in the open string theory.[1] Another example of the time dependent process is the study of the motion of the probe D-brane in given supergravity background. It turns out that the dynamics of such a probe has a lot of common with the time dependent tachyon condensation [2].[2] In our previous works [4,8,13,14] we have studied the dynamics of a non-BPS Dp-brane in the Dk-brane and in NS5-brane background in the effective field theory description. We have shown that generally, when we take the time dependent tachyon into account, it is very difficult to obtain an exact time dependence of the tachyon and radion mode. On the other hand we argued in [4], where we studied the properties of the world volume theory of BPS D-branes and non-BPS Dp-branes in the near horizon limit of N Dk-branes or NS5-branes, that the problem simplifies considerably in the case when the tachyon reaches its homogeneous vacuum value $T_{\min}$ that is defined as $V(T_{\min}) = 0$, $\partial_i T_{\min} = 0$ where $V(T)$ is a tachyon potential. Since the analysis in [4] was performed in the near horizon region of given background configuration of D-branes one can ask the question how this description changes when we do not restrict to this particular situation. This paper is then devoted to the study of the situation when the non-BPS Dp-brane at the tachyon vacuum moves in general spatial dependent background.

An analysis of the properties of the Dirac-Born-Infeld (DBI) non-BPS tachyon effective action at the tachyon vacuum was previously performed in [21–26]. However this analysis was mainly focused on the problem of the space-time filling non-BPS Dp-brane. Our goal on the other hand is to study the dynamics of the non-BPS Dp-brane where the world volume tachyon reaches its minimum and when this Dp-brane is embedded in a general background.

As it is believed the final state of the D-brane decay comprises the dust of massive closed strings known as a tachyon matter [27–29]. Another interesting aspect of the low energy theory is found in the sector with net electric flux that carries fundamental string charges. Generally, when the D-brane decays, the classical solution of the system is characterized as a two component fluid system: One is pressureless electric flux lines, known as string fluid, while the other is a tachyon matter [30–34]. As we claimed above the string fluid and tachyon matter must have a natural interpretation via closed string states. In fact, it was shown that string fluid reproduces the classical behavior of fundamental string remarkably well. Dynamics of such a configuration has been shown to be exactly that of Nambu-Goto string [21,22]. Natural construction from this however, hampered by the degeneracy of the string fluid.

More recently the macroscopic interpretation for the combined system of string fluid and tachyon matter was proposed in [25,35]. The basic idea was to consider a macroscopic number of long fundamental strings lined up along one particular direction and turn on oscillators along each of these strings. The proposed map is to identify energy of electric flux lines as coming from the winding mode part of the fundamental strings, while attributing the tachyon matter energy to oscillator part.

While the analysis performed in [25,35] is very interesting and certainly deserves generalization to the Dp-brane moving in general background (we hope to return to this problem in the future) the goal of this paper is more modest. As is clear from the analysis given in [25,35] the crucial point in mapping the string fluid and the tachyon matter to the fundamental strings degrees of freedom is an existence of the nonzero electric flux. On the other hand we know that the tachyon condensation also occurs when the electric flux is zero and the resulting configuration should correspond to the gas of massive closed strings [29]. Because of the remarkable success of the tachyon effective action in the description of the open string tachyon condensation one could hope that the classical effective field theory analysis should be able to capture some aspects of the closed strings a non-BPS Dp-brane decays into. We

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[1]For recent review and extensive list of references, see [1].

[2]Similar problems have been discussed in [3–20].

will see that this is indeed the case. More precisely, in Section II we will solve the equation of motion for the non-BPS Dp-brane at the tachyon vacuum moving in the Dk-brane background and we will argue that the solution is the same as the collective motion of the gas of massless particles. Then in Section III we will demonstrate the equivalence between the homogeneous tachyon condensation and the gas of massless particles for space-time, where the metric components are functions of coordinates transverse to Dp-brane, following [22]. As we will argue in the conclusion this result is in perfect agreement with the open-closed string conjecture presented in [32,36]. In order to find the solution corresponding to the macroscopic fundamental string we will consider the solution with nonzero electric flux aligned along one spatial direction on the world volume of the Dp-brane. We will show in Section IV that this solution can be interpreted as a gas of the macroscopic strings stretched along this direction that move in given supergravity background. Then the dynamics of a non-BPS Dp-brane with nonzero electric flux that moves in Dk-brane background will be studied in Section V. In the conclusion (VI) we outline our result and suggest possible extension of this work.

II. HAMILTONIAN FORMULATION OF THE NON-BPS Dp-BRANE

As we claimed in the introduction the main goal of this paper is to study the tachyon effective action at the tachyon vacuum. Even if the Lagrangian for a non-BPS Dp-brane in its tachyon vacuum vanishes [37–41], the dynamics of this configuration is still nontrivial [21–26] as follows from the fact that the Hamiltonian for a non-BPS Dp-brane at the tachyon vacuum is nonzero.

More precisely, let us introduce the Hamiltonian for a non-BPS Dp-brane that is moving in 9 + 1-dimensional background with the metric

$$ds^2 = -N^2 dt^2 + g_{ab}(dx^a + L^a dt)(dx^b + L^b dt),$$
$$a, b = 1, \ldots, 9 \qquad (1)$$

and with the spatial dependent dilaton.[3] Let us now consider the non-BPS action in the form

$$S = -\int d^{p+1}\xi e^{-\Phi} V(T)\sqrt{-\det \mathbf{A}}, \qquad (2)$$

where

$$\mathbf{A}_{\mu\nu} = G_{MN}\partial_\mu X^M \partial_\nu X^N + F_{\mu\nu} + W(T)\partial_\mu T\partial_\nu T, \qquad (3)$$

where $M, N = 0, 1, \ldots, 9$ and where $V(T)$, $W(T)$ are functions of T that vanish for $T_{\min} = \pm\infty$. Let us fix the gauge by $\xi^\mu = x^\mu$, $\mu = 0, 1, \ldots, p$. In what follows we will also use the notation $\mathbf{x} = (x^1, \ldots, x^p)$. With the metric (1) the components of the matrix $\mathbf{A}$ take the form

$$\begin{aligned}
\mathbf{A}_{00} &= -N^2 + g_{ij}L^i L^j + g_{IJ}\partial_0 X^I \partial_0 X^J + W(\partial_0 T)^2,\\
\mathbf{A}_{0i} &\equiv E_i^+ = g_{ij}L^j + g_{IJ}\partial_0 X^I \partial_i X^J + F_{0i} + W\partial_0 T\partial_i T,\\
\mathbf{A}_{i0} &\equiv -E_i^- = g_{i0} + g_{ij}L^j + g_{IJ}\partial_i X^I \partial_0 X^J - F_{0i}\\
&\quad + W\partial_i T\partial_0 T,\\
\mathbf{A}_{ij} &= g_{ij} + g_{IJ}\partial_i X^I \partial_j X^J + F_{ij} + W\partial_i T\partial_j T,
\end{aligned} \qquad (4)$$

where $i, j = 1, \ldots, p$ and $I, J = p + 1, \ldots, 9$. Then we can write

$$\det\mathbf{A} = \mathbf{A}_{00}\det\mathbf{A}_{ij} + E_i^+ D_{ij} E_j^-, \qquad D_{ij} = (-1)^{i+j}\triangle_{ji}, \qquad (5)$$

where $\triangle_{ji}$ is the determinant of the matrix with jth row and ith column omitted. From (2) we obtain the canonical momenta as

$$\begin{aligned}
\pi^i &= \frac{\delta\mathcal{L}}{\delta\partial_0 A_i} = \frac{Ve^{-\Phi}}{\sqrt{-\det\mathbf{A}}}\frac{E_j^+ D_{ji} + D_{ij}E_j^-}{2},\\
\pi_T &= \frac{\delta\mathcal{L}}{\delta\partial_0 T} = \frac{e^{-\Phi}VW}{\sqrt{-\det\mathbf{A}}}\Big(\dot{T}\det\mathbf{A}_{ij}\\
&\quad - \frac{E_j^+ D_{ji}\partial_i T - \partial_i T D_{ij} E_j^-}{2}\Big),\\
p_I &= \frac{\delta\mathcal{L}}{\delta\partial_0 X^I} = \frac{e^{-\Phi}V}{\sqrt{-\det\mathbf{A}}}\Big(g_{IJ}\partial_0 X^J\det\mathbf{A}_{ij}\\
&\quad - \frac{E_j^+ D_{ji} g_{IJ}\partial_i X^J + g_{JI}\partial_i X^J D_{ij}E_j^-}{2}\Big).
\end{aligned} \qquad (6)$$

Note also that π^i satisfies the Gauss law constraint $\partial_i\pi^i = 0$. The Hamiltonian density is then obtained following Legendre transformation

$$\mathcal{H}(\mathbf{x}) = \pi^i E_i + \pi_T\dot{T} + p_I\dot{X}^I - \mathcal{L}. \qquad (7)$$

After some length and tedious algebra we obtain the Hamiltonian density as a function of canonical variables in the form

$$\begin{aligned}
\mathcal{H} &= N\sqrt{\mathcal{K}} - \pi^i F_{ij}L^j - p_K L^K + (\pi_T\partial_i T + p_K\partial_i X^K)L^i,\\
\mathcal{K} &= \pi^i g_{ij}\pi^j + W^{-1}\pi_T^2 + p_I g^{IJ}p_J + b_i g^{ij} b_j\\
&\quad + (\pi^i\partial_i T)^2 + (\pi^i\partial_i X^K)g_{KL}(\pi^j\partial_j X^L)\\
&\quad + e^{-2\Phi}V^2\det\mathbf{A}_{ij},\\
b_i &= F_{ik}\pi^k + \pi_T\partial_i T + \partial_i X^K p_K.
\end{aligned} \qquad (8)$$

The form of the Hamiltonian density (8) considerably simplifies in a situation when the tachyon reaches its global minimum ($V(T_{\min}) = W(T_{\min}) = 0$) and also when its spatial derivatives are equal to zero: $\partial_i T = 0$. This state is interpreted as a final state of the unstable Dp-brane decay that does not contain any propagating open string degrees

[3] In this paper we will consider the case when the metric and dilaton are functions of the coordinates transverse to Dp-brane world volume. This restriction is relevant for the study of the probe non-BPS Dp-brane in the Dk-brane background.

of freedom. On the other hand we see that even in this case there is still nontrivial dynamics as follows from the form of the Hamiltonian density (8).

To see this more clearly we begin with an explicit example of an unstable Dp-brane in its tachyon vacuum that moves in the background of N coincident Dk-branes. The metric, the dilaton (Φ), and the R-R field (C) for a system of N coincident Dk-branes is given by

$$g_{\alpha\beta} = H_k^{-1/2}\eta_{\alpha\beta}, \qquad g_{mn} = H_k^{1/2}\delta_{mn},$$
$$(\alpha, \beta = 0, 1, \ldots, k, m, n = k+1, \ldots, 9),$$
$$e^{2\Phi} = H_k^{(3-k)/2}, \qquad C_{0\ldots k} = H_k^{-1}, \tag{9}$$
$$H_k = 1 + \frac{\lambda}{r^{7-k}}, \qquad \lambda = N g_s l_s^{7-k},$$

where H_k is a harmonic function of N Dk-branes satisfying the Green function equation in the transverse space. We will consider a non-BPS Dp-brane with $p < k$ that is inserted in the background (9) with its spatial section stretched in directions $(x^1, \ldots, x^p)$. For zero electric flux and for tachyon equal to T_m the Hamiltonian density (8) takes the form

$$\mathcal{H} = N\sqrt{p_I g^{IJ} p_J + \partial_i X^K p_K g^{ij}\partial_j X^L p_L} = N\sqrt{\mathcal{K}(\mathbf{x})}. \tag{10}$$

Using (10) the canonical equations of motion take the form

$$\partial_0 X^K(\mathbf{x}) = \frac{\delta H}{\delta p_K(\mathbf{x})} = N\frac{g^{KL}p_L + \partial_i X^K g^{ij}\partial_j X^L p_L}{\sqrt{\mathcal{K}(\mathbf{x})}} \tag{11}$$

and

$$\partial_0 p_K(\mathbf{x}) = -\frac{\delta H}{\delta X^K(\mathbf{x})} = -\frac{\delta N}{\delta X^K(\mathbf{x})}\sqrt{\mathcal{K}} - \frac{1}{2\sqrt{\mathcal{K}}}\left(\frac{\delta g^{IJ}}{\delta X^K}p_I p_J + \partial_i X^I P_I \frac{\delta g^{ij}}{\delta X^K}\partial_j X^J P_J\right) + \partial_i\left[\frac{N P_K g^{ij}\partial_j X^L p_L}{\sqrt{\mathcal{K}}}\right], \tag{12}$$

where $N = \sqrt{-g_{00}}$, g_{ij}, g_{IJ}, and Φ are given in (9).

To further simplify the problem we restrict ourselves to the case of homogeneous modes on the world volume of non-BPS Dp-brane. Then the equations of motions (11) take the form

$$\partial_0 X^m = \frac{p_m}{H_k^{3/4}\sqrt{\mathcal{K}}}, \qquad \partial_0 Y^u = \frac{H_k^{1/4}p_u}{\sqrt{\mathcal{K}}}, \tag{13}$$

where $Y^u, u, v = p+1, \ldots, k$ are world volume modes that characterize the transverse position of Dp-brane that is parallel with the world volume of Dk-branes and $X^m, m = k+1, \ldots, 9$ are world volume modes that parametrize transverse positions both to the Dk-branes and to Dp-brane. Thanks to the manifest rotation invariance in transverse R^{9-k} space we will restrict ourselves to the motion in the (x^8, x^9) plane where we introduce the cylindrical coordinates

$$X^8 = R\cos\theta, \qquad X^9 = R\sin\theta. \tag{14}$$

Note also since the Hamiltonian does not explicitly depend on Y^u and θ the corresponding conjugate momenta p_u, p_θ are conserved. As a next step we use the fact that the energy density

$$\mathcal{E} = \sqrt{-g_{00}}\sqrt{\mathcal{K}} \tag{15}$$

is conserved and replace $\mathcal{K}$ with $\mathcal{E}$ and also express p_R as a function of R and conserved quantities $\mathcal{E}$, p_u, p_θ

$$p_R = \pm\sqrt{H_k}\sqrt{\mathcal{E}^2 - p_u^2 - \frac{p_\theta^2}{R^2 H_k}}. \tag{16}$$

Then the equation of motion (13) can be written as

$$\partial_0 Y^u = \frac{p_u}{\mathcal{E}}, \qquad \partial_0\theta = \frac{p_\theta}{R^2\sqrt{H_k}\mathcal{E}},$$
$$\partial_0 R = \pm\frac{\sqrt{\mathcal{E}^2 - p_u^2 - \frac{p_\theta^2}{R^2 H_k}}}{\sqrt{H_k}\mathcal{E}}. \tag{17}$$

In order to study the general properties of the radial motion of the probe non-BPS Dp-brane we will present the similar analysis as was performed in [42]. First of all, note that the Hamiltonian density for the background (9) takes the form

$$\mathcal{H} = \sqrt{-g_{00}}\sqrt{p_u g^{uv} p_v + p_r g^{rr} p_r + p_\theta g^{\theta\theta} p_\theta} = \sqrt{p_u^2 + \frac{p_R^2}{H_k} + \frac{p_\theta^2}{R^2 H_k}} \tag{18}$$

that implies that $\mathcal{H}$ is an increasing function of p_R so that the allowed range of R for the classical motion can be found by plotting the effective potential $V_{\text{eff}}(R)$ that is defined as

$$V_{\text{eff}}(R) = \mathcal{H}(p_R = 0) = \sqrt{p_u^2 + \frac{p_\theta^2}{R^2 H_k}} \tag{19}$$

against R and finding those R for which $\mathcal{E} \geq V_{\text{eff}}(R)$. The properties of V_{eff} depend on H_k that is a monotonically decreasing function of R with the limit $H_k \to \frac{\lambda}{R^{7-k}}$ for $R \to 0$ and with $H_k \to 1$ for $R \to \infty$. For $p_\theta \neq 0$ we obtain following asymptotic behavior of the potential (19) for $R \to 0$

(a) **k=6**
In this case we obtain

$$V_{\text{eff}} \to \frac{|p_\theta|}{\sqrt{\lambda}\sqrt{R}} \tag{20}$$

and hence for nonzero p_θ the potential diverges at the origin.

(b) **k=5**
Now the potential in the limit $R \to 0$ approaches to

$$V_{\rm eff} = \sqrt{p_u^2 + \frac{p_\theta^2}{\lambda}}. \qquad (21)$$

(c) **k < 5**
In this case the effective potential takes the form

$$V_{\rm eff} \approx \sqrt{p_u^2 + \frac{p_\theta^2 R^{5-k}}{\lambda}} \qquad (22)$$

that again implies that potential approaches the constant $\sqrt{p_u^2}$ in the limit $R \to 0$.

On the other hand for $R \to \infty$ we get

$$V_{\rm eff} \to \sqrt{p_\mu^2}. \qquad (23)$$

More precisely, looking at the form of the potential for $k = 6$, 5 it is easy to see that these potentials are decreasing functions of R. On the other hand for $k < 5$ it can be shown that $V_{\rm eff}$ has extremum at

$$R_{\rm max} = \left(\frac{\lambda(5-k)}{2}\right)^{1/(7-k)}. \qquad (24)$$

Collecting these results we obtain following pictures for the dynamics of the non-BPS Dp-brane in its tachyon vacuum moving in Dk-brane background. In the first case we consider the non-BPS Dp-brane that moves towards the stack of N Dk-branes from the asymptotic infinity $R = \infty$ at $t = -\infty$. It reaches its turning point at

$$1 - \frac{p_u^2}{\mathcal{E}^2} - \frac{p_\theta^2}{\mathcal{E}^2 R_T^2 H_k} = 0 \Rightarrow R_T^2 + \frac{\lambda}{R_T^{5-k}} = \frac{p_\theta^2}{\mathcal{E}^2(1-\frac{p_u^2}{\mathcal{E}^2})}, \qquad (25)$$

and then it moves outwards. On the other hand from the existence of local maxima (24) for $k < 5$ it is clear that the Dp-brane can be in bounded region near the stack of N Dk-branes. To see this more precisely let us solve the third equation in (17) in the limit $\frac{\lambda}{R^{7-k}} \gg 1$. In this case we obtain the following equation

$$\frac{dR}{\sqrt{(1-\frac{p_u^2}{\mathcal{E}^2})R^{7-k} - \frac{p_\theta^2}{\mathcal{E}^2\lambda}R^{2(6-k)}}} = \pm\frac{dt}{\sqrt{\lambda}} \qquad (26)$$

that has the solution

$$R^{5-k} = \frac{\lambda(\mathcal{E}^2 - p_u^2)}{p_\theta^2}\frac{1}{1 + (\mp\frac{\mathcal{E}^2-p_u^2}{2\mathcal{E}p_\theta}t + \sqrt{R_0^{k-5}-1})^2}. \qquad (27)$$

We see that now Dp-brane leaves the world volume of Dk-branes at $t = -\infty$ and moves outwards until its turning point at $\dot{R} = 0$ and then it moves towards the stuck of Dk-branes that it again reaches at $t = \infty$. The precise analysis of the dynamics of the Dp-brane in the region $\frac{\lambda}{R^{7-k}} \gg 1$ was performed in [4] where more details can be found.

As is clear from (21) the effective potential takes a very simple form when $p_\theta = 0$. In this case the differential equation for R is equal to

$$\dot{R} = \pm\frac{\sqrt{\mathcal{E}^2 - p_u^2}}{\sqrt{H_k}\mathcal{E}} \qquad (28)$$

that can be explicitly solved in terms of hypergeometric functions. However in order to gain better physical meaning of this physical situation it is useful to consider the case when $p_u = 0$. Then the Eq. (28) can be rewritten in more suggestive form

$$-H_k^{-1/2}dt^2 + H_k^{1/2}dR^2 = 0 \qquad (29)$$

that is an equation of the radial geodesics in Dk-brane background.

In summary, we have found that a non-BPS Dp-brane where the tachyon reaches its vacuum value moves in the background of N Dk-branes as a gas of massless particles that are confined to the world volume of the original Dp-brane. In the next section we will present more detailed arguments that support validity of this correspondence.

III. NON-BPS Dp-BRANE AT THE TACHYON VACUUM AS A GAS OF MASSLESS PARTICLES

Let us consider the curved background with the metric

$$ds^2 = -N^2dt^2 + g_{ab}(dx^a + L^a dt)(dx^b + L^b dt), \quad a, b = 1, \ldots, 9, \qquad (30)$$

where we presume that N, L^a, g_{ab}, and the dilaton Φ are functions of the coordinates transverse to the Dp-brane. As we know from the previous section the dynamics of the non-BPS Dp-brane at the tachyon vacuum is governed by the Hamiltonian

$$\begin{aligned} H &= \int d\mathbf{x}\mathcal{H}, \\ \mathcal{H} &= N\sqrt{\mathcal{K}(\mathbf{x})} + p_K\partial_i X^K L^i - p_K L^K, \\ \mathcal{K} &= p_I g^{IJ} p_J + \partial_i X^K p_K g^{ij}\partial_j X^L p_L. \end{aligned} \qquad (31)$$

It is now straightforward to determine the canonical equations of motions

$$\partial_0 X^K(\mathbf{x}) = N\frac{g^{KL}p_L + \partial_i X^K g^{ij}\partial_j X^L p_L}{\sqrt{\mathcal{K}(\mathbf{x})}} + \partial_i X^K L^i - L^K \qquad (32)$$

and

$$\begin{aligned}\partial_0 p_K(\mathbf{x}) = &-\frac{\delta N}{\delta X^K(\mathbf{x})}\sqrt{\mathcal{K}} - \frac{1}{2\sqrt{\mathcal{K}}}\Big(\frac{\delta g^{IJ}}{\delta X^K} p_I p_J \\ &+ \partial_i X^K p_K \frac{\delta g^{ij}}{\delta X^K}\partial_j X^L p_L\Big) \\ &+ \partial_i\Big[\frac{N p_K g^{ij}\partial_j X^L p_L}{\sqrt{\mathcal{K}}}\Big] + \partial_i[p_K L^i] \\ &- p_L \partial_i X^L \frac{\delta L^i}{\delta X^K} + p_L \frac{\delta L^L}{\delta X^K}. \end{aligned} \tag{33}$$

As we argued in the previous section the Dp-brane at the tachyon vacuum with zero electric flux has similar properties as a homogeneous gas of the massless particles embedded in the background of N Dk-branes. Now we would like to show that this correspondence holds in more general situations. To see this we will closely follow a very nice analysis performed in [22].

We begin with an action for the massive particle in general space-time

$$S = -m\int d\tau\sqrt{-g_{MN}\dot{Z}^M\dot{Z}^N} = -m\int d\tau\sqrt{\mathbf{A}}, \tag{34}$$

where $\dot{Z} \equiv \frac{dZ}{d\tau}$ and where Z^M are embedding coordinates for the massive particle. As a next step we fix the gauge in the form $\tau = Z^0$ so that the action (34) takes the form

$$\begin{aligned}S &= -m\int d\tau\sqrt{N^2 - g_{st}L^sL^t - 2g_{st}L^t\dot{Z}^s - g_{st}\dot{Z}^s\dot{Z}^t} \\ &= -m\int d\tau\sqrt{\mathbf{A}},\ s,t = 1,\ldots,9.\end{aligned} \tag{35}$$

Then the conjugate momenta are

$$P_s = \frac{\delta S}{\delta \dot{Z}^s} = \frac{m(g_{st}\dot{Z}^t + g_{st}L^t)}{\sqrt{\mathbf{A}}} \tag{36}$$

and consequently the Hamiltonian takes the form

$$H = P_s\dot{Z}^s - L = N\sqrt{P_s g^{st} P_t + m^2} - P_s L^s. \tag{37}$$

Using the Hamiltonian formalism we can take the limit $m \to 0$ and we obtain the Hamiltonian for a massless particle moving in general background

$$H = N\sqrt{P_r g^{rs} P_s} - P_s L^s. \tag{38}$$

Then the canonical equations of motion for the massless particle take the form

$$\begin{aligned}\dot{Z}^s &= \frac{\delta H}{\delta P_s} = N\frac{g^{st}P_t}{\sqrt{P_r g^{rs} P_s}} - L^s, \\ \dot{P}_s &= -\frac{\delta H}{\delta Z^s} \\ &= -\frac{\delta N}{\delta Z^s}\sqrt{P_r g^{rt} P_t} - \frac{N}{2\sqrt{P_r g^{rt} P_t}}\frac{\delta g^{rt}}{\delta Z^s}P_r P_t + P_r\frac{\delta L^r}{\delta Z^s}.\end{aligned} \tag{39}$$

Following [22] we will now presume that there exists the solution of the equation of motion (39) given as $Z^s(\tau)$, $P_s(\tau)$. Consider then the following field configuration on the Dp-brane:

$$p_I(x^0,\ldots,x^p) = P_I(\tau)f(x^0,\ldots,x^p)|_{\tau=x^0}, \tag{40}$$

where f is an arbitrary function of the variables $(x^i - Z^i(\tau))$ for $i = 1,\ldots,p$. Then it is clear that

$$(\partial_0 f + \partial_i f \partial_\tau Z^i)|_{\tau=x^0} = 0. \tag{41}$$

We also demand that X^I obey

$$(\partial_i X^I P_I + P_i)|_{\tau=x^0} = 0 \tag{42}$$

but are otherwise unspecified. Inserting the ansatz (40) into (31) we obtain that the Hamiltonian density takes the form

$$\mathcal{H}(x^0,\ldots,x^p) = (N(X)\sqrt{P_s g^{st}(X) P_t} - P_s L^s)f(x^0,\ldots,x^p). \tag{43}$$

We see that the expression in the bracket has the form of the Hamiltonian for the massless particle where the metric components still depend on X^I that are arbitrary functions of t, $\mathbf{x}$. It turns out however that in order to obey the equation of motion for general space-time we should perform the identification

$$X^K(x^0,\ldots,x^p) = Z^K(\tau). \tag{44}$$

Then the equation of motion (33) can be written as

$$\begin{aligned}&\Big(\partial_\tau P_K + \frac{\delta N}{\delta Z^K}\sqrt{P_r g^{rt} P_t} + \frac{N}{2\sqrt{P_r g^{rt} P_t}} \\ &\times\Big(\frac{\delta g^{IJ}}{\delta Z^K}P_I P_J + p_i\frac{\delta g^{ij}}{\delta Z^K}p_j\Big) - P_L\frac{\delta L^L}{\delta Z^K}\Big)f \\ &- P_K\partial_i f\Big(\partial_\tau Z^i - \frac{N g^{ij}P_j}{\sqrt{P_r g^{rt} P_t}} + L^i\Big)\partial_i f = 0.\end{aligned} \tag{45}$$

We see that this equation is obeyed since the expressions in the brackets are equal to zero thanks to the fact that Z^s, P_s obey the equations of motion (39). On the other hand from (42) and (44) we get that $P_i = 0$ and hence the configuration on a non-BPS Dp-brane in the tachyon vacuum corresponds to the motion of massless particles that have nonzero transverse momenta only. Then the Eq. (33) takes the form

$$\partial_0 X^K(\mathbf{x}) = \partial_\tau Z^K(\tau) = N\frac{g^{KL}P_K}{\sqrt{P_s g^{st} P_t}} - L^K \tag{46}$$

that is clearly obeyed since Z^K obeys (39).

The final question, and the most difficult one, is regarded to the form of the function $f(x^0,\ldots,x^p)$. We have seen that its form is not determined from the Dp-brane equations of motion. The most natural choice is

$$f(x^0, \ldots, x^p) = \prod_{i=1}^{p} \delta(x^i - Z^i(x^0)). \quad (47)$$

As follows from (43) the energy is localized along the line $x^i = Z^i(x^0)$ for $i = 1, \ldots, p$. Using also the identification (44) we see that in the full 9 + 1-dimensional space-time this solution describes the world line $x^s = Z^s(\tau)$ for $s = 1, \ldots, 9$. In other words, the Dp-brane world volume theory contains a solution whose dynamics are exactly that of the massless particle in (9 + 1) dimensions.

As in the case of Nambu-Gotto (NG) string solution given in [22] the freedom of replacing the δ function by an arbitrary function of $x^i - Z^i(\tau)$ is slightly unusual. Very nice and detailed discussion considering this issue was given in [35]. According to this paper the solution with arbitrary function f should be regarded as a system of high density of massless particles, or more precisely as a system of high density of pointlike solutions of the closed string equations of motion.

IV. MOTION OF NON-BPS Dp-BRANE WITH NONZERO ELECTRIC FLUX

As we have seen in the previous section the case when the non-BPS Dp-brane in the tachyon vacuum moves in the general background with zero electric flux can be interpreted as a motion of the gas of massless particles. In order to find the solution of the D-brane equations of motion having the interpretation as a fundamental macroscopic string we should rather consider the case when we switch on the electric flux as well. In fact, let us again consider the Hamiltonian for a non-BPS Dp-brane at the tachyon vacuum that moves in curved background

$$\mathcal{H} = N\sqrt{\pi_i g^{ij}\pi_j + p_I g^{IJ} p_J + b_i g^{ij} b_j + (\pi^i \partial_i X^K) g_{KL} (\pi^j \partial_j X^L)} + p_K \partial_i X^K L^i - p_K L^K, \quad (48)$$

where

$$b_i = F_{ij}\pi^j + \partial_i X^K p_K. \quad (49)$$

Note that π^i also obeys the Gauss law constraint

$$\partial_i \pi^i = 0. \quad (50)$$

Now canonical equations of motion take the form

$$\partial_0 A_i(\mathbf{x}) = E_i(\mathbf{x}) = \frac{\delta H}{\delta \pi^i(\mathbf{x})} = \frac{N}{\sqrt{\mathcal{K}}}(g_{ij}\pi^j - F_{ik}g^{kj}b_j + \partial_i X^K g_{KL}(\pi^j \partial_j X^L)), \quad (51)$$

$$\partial_0 \pi^i(\mathbf{x}) = -\frac{\delta H}{\delta A_i(\mathbf{x})} = -\partial_j \left[\frac{N}{\sqrt{\mathcal{K}}}(\pi^j g^{ik} b_k - \pi^i g^{jk} b_k)\right], \quad (52)$$

$$\partial_0 X^I(\mathbf{x}) = \frac{\delta H}{\delta p_I(\mathbf{x})} = \frac{N}{\sqrt{\mathcal{K}}}(g^{IK} p_K + \partial_i X g^{ij} b_j) + \partial_i X^K L^i - L^K, \quad (53)$$

$$\begin{aligned}\partial_0 p_I(\mathbf{x}) = -\frac{\delta H}{\delta X^I(\mathbf{x})} &= \partial_i \left[\frac{N}{\sqrt{\mathcal{K}}}(\pi^i g_{IK}\partial_j X^K \pi^j + p_I g^{ij} b_j)\right] + \frac{\delta N}{\delta X^I}\sqrt{\mathcal{K}} \\ &- \frac{\sqrt{N}}{2\sqrt{\mathcal{K}}}\left(\pi^i \frac{\delta g_{ij}}{\delta X^I}\pi^j - p_K \frac{\delta g^{KL}}{\delta X^I} p_L - b_i \frac{\delta g^{ij}}{\delta X^I} b_j - (\pi^i \partial_i X^K)\frac{\delta g_{KL}}{\delta X^I}(\pi^j \partial_j X^L)\right) + \partial_i [p_K L^i] \\ &- p_L \partial_i X^L \frac{\delta L^i}{\delta X^K} + p_L \frac{\delta L^L}{\delta X^K}. \end{aligned} \quad (54)$$

Following [22] we will now try to find the solution of the equation of motion given above that can be interpreted as the fundamental string solution. To begin with let us consider the Nambu-Goto action for fundamental string

$$S = -\int d\tau d\sigma \sqrt{-\det G_{\alpha\beta}}, \qquad G_{\alpha\beta} = G_{MN}\partial_\alpha Z^M \partial_\beta Z^N, \quad (55)$$

where $\alpha, \beta = \sigma, \tau$. We fix the gauge so that $Z^0 = \tau$, $Z^1 = \sigma$ so that

$$G_{\alpha\beta} = g_{\alpha\beta} + g_{st}\partial_\alpha Z^s \partial_\beta Z^t, \quad (56)$$

where $s, t = 2, \ldots, 9$. Then the Hamiltonian takes the form

$$H_{NG} = \int d\sigma \mathcal{H}_{NG}(\sigma), \quad (57)$$

where the Hamiltonian density $\mathcal{H}_{NG}$ is equal to

$$\mathcal{H}_{NG} = N\sqrt{\mathcal{K}_{NG}} - P_s L^s + P_s \partial_\alpha X^s L^\alpha,$$
$$\mathcal{K}_{NG} = g_{\sigma\sigma} + P_s g^{st} P_t + \partial_\sigma Z^s P_s g^{\sigma\sigma} \partial_\sigma Z^t P_t + \partial_\sigma Z^s \partial_\sigma Z^t g_{st}. \quad (58)$$

Now the equation of motion of the fundamental string takes the form

$$\partial_\tau Z^s = \frac{\delta H_{NG}}{\delta P_s} = \frac{N(g^{st}P_t + \partial_\sigma Z^s g^{\sigma\sigma}\partial_\sigma Z^t P_t)}{\sqrt{\mathcal{K}_{NG}}} - L_s + \partial_\sigma X^s L^\sigma,$$
$$\partial_\tau P_s = -\frac{\delta H}{\delta Z^s} = -\frac{\delta N}{\delta Z^s}\sqrt{\mathcal{K}_{NG}} - \frac{N}{2\sqrt{\mathcal{K}_{NG}}}\left(\frac{\delta g_{\sigma\sigma}}{\delta Z^s} + P_r \frac{g^{rt}}{\delta Z^s} P_t + \partial_\sigma Z^r P_r \frac{g^{\sigma\sigma}}{\delta Z^s}\partial_\sigma Z^t P_t + \frac{\delta g^{rt}}{\delta Z^s}\partial_\sigma Z^r \partial_\sigma Z^t\right)$$
$$+ \partial_\sigma\left[\frac{N(P_s g^{\sigma\sigma}\partial_\sigma Z^r P_r + g_{st}\partial_\sigma Z^t)}{\sqrt{\mathcal{K}_{NG}}}\right] + P_t \frac{\delta L^t}{\delta X^s} - P_t \partial_\sigma X^t \frac{\delta L^\sigma}{\delta X^s} + \partial_\sigma[P_s L^\sigma]. \quad (59)$$

For future use we also define

$$P = -\sum_{s=2}^{9} P_s \partial_\sigma Z^s, \qquad Z^1(\tau, \sigma) = \sigma. \quad (60)$$

Let $Z^s(\tau, \sigma)$, $P_s(\tau, \sigma)$, $s = 2, \ldots, 9$ be the solutions of the equation of motion (59). As was shown in [22] it is natural to consider the following field configuration on Dp-brane

$$\pi_i(x^0, \ldots, x^p) = \partial_\sigma Z^i(\tau, \sigma) f(x^0, \ldots, x^p)|_{(\tau,\sigma)=(x^0,x^1)},$$
$$p_I(x^0, \ldots, x^p) = P_I(\tau, \sigma) f(x^0, \ldots, x^p)|_{(\tau,\sigma)=(x^0,x^1)}, \quad (61)$$

where $i = 1, \ldots, p$. Following [22] we presume that $f(x^0, \ldots, x^p)$ is an arbitrary function of variables $(x^m - Z^m(x^0, x^1))$ for $m = 2, \ldots, p$ and hence satisfies:

$$\partial_\sigma Z^i \partial_i f|_{(\tau,\sigma)=(x^0,x^1)} = 0,$$
$$(\partial_0 f + \partial_i f \partial_\tau Z^i)|_{(\tau,\sigma)=(x^0,x^1)} = 0. \quad (62)$$

We also presume that the fields $X^I(x^0, \ldots, x^p)$ and $F_{ij}(x^0, \ldots, x^p)$ are subject to the following set of conditions:

$$(\partial_\sigma Z^j \partial_j X^I - \partial_\sigma Z^I)|_{(\tau,\sigma)=(x^0,x^1)} = 0,$$
$$(F_{ij}\partial_\sigma Z^j + \partial_i X^I P_I + P_i)|_{(\tau,\sigma)=(x^0,x^1)} = 0. \quad (63)$$

With this notation we can easily find that

$$\pi^i \partial_i X^I(x^0, \ldots, x^p) = \partial_\sigma Z^I(\tau = x^0, \sigma = x^1) f(x^0, \ldots, x^p),$$
$$b_i(x^0, \ldots, x^p) = -P_i(\tau = x^0, \sigma = x^1) f(x^0, \ldots, x^p),$$
$$\sqrt{\mathcal{K}}(x^0, \ldots, x^p) = \sqrt{\mathcal{K}_{NG}}(\tau = x^0, \sigma = x^1, X^I) \times f(x^0, \ldots, x^p). \quad (64)$$

We see that due to the nontrivial dependence of the metric on transverse coordinates X^I the expression $\sqrt{\mathcal{K}_{NG}}$ still depends on X^I. As in the case of the particlelike solution studied in the previous section it is clear that in the curved space-time we should demand that the coordinates X^I are related to Z^I as:

$$X^I(x^0, x^1, x^m = Z^m(x^0, x^1)) = Z^I(x^0, x^1). \quad (65)$$

This condition implies that

$$\mathcal{H}(x^0, \ldots, x^p) = \mathcal{H}_{NG}(\tau = x^0, \sigma = x^1) f(x^0, \ldots, x^p). \quad (66)$$

Then we can show exactly as in [22] that the ansatz (61) together with (65) obeys the equations of motion (51)–(54) as well as the Gauss law constraint (50). The interpretation of this solution is the same as in the flat space [35]. First, the spatial choice of the function f

$$f(x^0, \ldots, x^p) = \prod_{m=2}^{p} \delta(x^m - Z^m(x^0, x^1)) \quad (67)$$

gives the solution that corresponds to the stretched string in the x^1 direction that moves the background (30). On the other hand the solutions with the general form of the functions f should be interpreted as the configurations of the high density of fundamental strings moving in (30) and that are confined to the world volume of the original Dp-brane.

V. NON-BPS Dp-BRANE AT THE TACHYONIC VACUUM WITH NONZERO FLUX IN Dk-BRANE BACKGROUND

Let us again return to the spatial case of the motion of the non-BPS Dp-brane in its tachyon vacuum in the Dk-brane background (9). As in Section II we demand that all world volume fields are homogeneous $\partial_i X^I = 0$ and the electric flux has a nonzero component in the x^1 direction only:

$$A_1 = f(t), \qquad F_{ij} = 0. \quad (68)$$

For homogeneous fields and for the gauge fields given in (68) we get that $b_i = 0$ and consequently $\mathcal{K}$ in (48) is equal to

$$\mathcal{K} = (\pi_1)^2 H_k^{1/2} + H_k^{-1/2} p_m p_m + H_k^{1/2} p_u p_u, \quad (69)$$

where $p_m, m = k + 1, \ldots, 9$ are momenta conjugate to coordinates X^m transverse to Dk-brane and to Dp-brane while $p_u, u = p + 1, \ldots, k$ are momenta conjugate to coordinates Y^u transverse to Dp-brane but parallel to the Dk-brane. With this ansatz the equation of motions (51)–(54) take the form

$$\partial_0 A_1(\mathbf{x}) = E_1(\mathbf{x}) = \frac{\pi^1}{H_k^{3/4}\sqrt{\mathcal{K}}}, \tag{70}$$

$$\partial_0 \pi^i(\mathbf{x}) = 0, \tag{71}$$

$$\partial_0 X^m(\mathbf{x}) = \frac{p_m}{H_k^{3/4}\sqrt{\mathcal{K}}}, \tag{72}$$

$$\partial_0 Y^u(\mathbf{x}) = \frac{p_u H_k^{1/4}}{\sqrt{\mathcal{K}}}. \tag{73}$$

It is clear that the solution of (71) consistent with the presumption that all fields are homogeneous is the constant electric flux $\pi_1 = \Pi$. Note however that E_1 is time dependent as follows from (70) since generally metric components depend on the coordinates $X^m(t)$. To find the trajectory of a non-BPS Dp-brane we express $\sqrt{\mathcal{K}}$ using the conserved energy density as

$$\mathcal{E} = \sqrt{-g_{00}}\sqrt{\mathcal{K}} \Rightarrow \sqrt{\mathcal{K}} = \frac{\mathcal{E}}{\sqrt{-g_{00}}} \tag{74}$$

so that

$$\partial_0 X^m = \frac{p_m}{H_k \mathcal{E}}, \qquad \partial_0 Y^u = \frac{p_u}{\mathcal{E}}, \qquad E_1 = \frac{\Pi}{H_k \mathcal{E}}. \tag{75}$$

Since the Hamiltonian density does not depend on Y^u we get that $p_u = \text{const}$. Using manifest rotation invariance of the transverse R^{9-k} space we restrict ourselves to the motion in (x^8, x^9) plane where we also introduce the R and θ coordinates defined as

$$X^8 = R\cos\theta, \qquad X^9 = R\sin\theta. \tag{76}$$

Using the fact that p_θ is conserved we express p_R from $\mathcal{E}$ as

$$p_R = \pm\sqrt{H_k}\sqrt{\mathcal{E}^2 - \Pi^2 - p_u^2 - \frac{p_\theta^2}{R^2 H_k}} \tag{77}$$

so that we get

$$\dot{R} = \frac{p_R}{\sqrt{H_k}\mathcal{E}} = \pm\frac{\sqrt{\mathcal{E}^2 - \Pi^2 - p_u^2 - \frac{p_\theta^2}{R^2 H_k}}}{\sqrt{H_k}\mathcal{E}}. \tag{78}$$

Since the Eq. (78) has the same form as the Eq. (17) (if we identify $\mathcal{E}^2 - \Pi^2$ in (78) with $\mathcal{E}^2$ in (17)), then the analysis of the Eq. (17) performed in Section II holds for (78) as well. Then we can interpret the solution with nonzero electric flux Π as a solution describing the motion of the homogeneous gas of the macroscopic strings stretched in the x^1 direction that are confined to the world volume of a non-BPS Dp-brane and that move in Dk-brane background.

VI. CONCLUSION

We have studied the dynamics of the non-BPS Dp-brane at the tachyon vacuum and when this Dp-brane moves in the background where metric and dilaton are functions of the coordinates transverse to Dp-brane. We have shown that in the case when there is not any electric flux present on the world volume of this Dp-brane, its dynamics are equivalent to the dynamics of the homogeneous gas of massless particles that are confined on the world volume of the unstable Dp-brane. At this place we should ask the question how this result is related to the analysis performed in [29] where it was shown that the end product of the tachyon condensation should be the gas of massive closed strings. A relevant problem has been discussed in [36]. According to this paper there exists the spread of the energy density from the plane of the brane due to the internal oscillation of the final state of the closed strings. In the classical limit we have delta function localized D-brane and hence according to a previous remark the state of closed strings without oscillator excitations. This however also implies that the classical description of such closed strings is given by masslesslike solution of the equation of motion when the modes on the world volume of fundamental string are not a function of σ.[4] In other words, the classical result given in this paper can be considered as a manifestation of the *Open-Closed Duality Conjecture* proposed in [32].

In order to find macroscopic fundamental string solutions we had to, as in the flat space-time, consider the nonzero electric flux on the world volume of the non-BPS Dp-brane. Then we have shown that the dynamics of the unstable D-brane at the tachyon vacuum with the nonzero electric flux corresponds to the dynamics of the gas of stretched fundamental strings.

In concussion, we would like to stress that the results presented in this paper give the modest contribution to the study of the tachyon condensation. On the other hand we hope that they could be helpful for better understanding of the general properties of the tachyon condensation in curved space-time.

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[4]For recent review of some aspects of classical string solutions, see [43].

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