

Reheating the Universe after string theory inflationLev Kofman^{1,*} and Piljin Yi^{2,†}¹*Canadian Institute for Theoretical Astrophysics, 60 St George Str, Toronto, Ontario, M5S 3H8, Canada*²*School of Physics, Korea Institute for Advanced Study, 207-43, Cheongryangri-Dong, Dongdaemun-Gu, Seoul 130-722, Korea*

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In string theory realizations of inflation, the endpoint of inflation is often brane-antibrane annihilation. We consider the processes of reheating of the standard model universe after brane inflation. We identify the channels of inflaton energy decay, cascading from tachyon annihilation through massive closed string loops, KK modes, and brane displacement moduli to the lighter standard model particles. Cosmological data constrains scenarios by putting stringent limits on the fraction of reheating energy deposited in gravitons and nonstandard sector massive relics. We estimate the energy deposited into various light degrees of freedom in the open and closed string sectors, the timing of reheating, and the reheating temperature. Production of gravitons is significantly suppressed in warped inflation. However, we predict a residual gravitational radiation background at the level $\Omega_{\text{GW}} \sim 10^{-8}$ of the present cosmological energy density. We also extend our analysis to multiple throat scenarios. A viable reheating would be possible in a single throat or in a certain subclass of multiple throat scenarios of the KKLMMT type inflation model, but overproduction of massive Kaluza-Klein (KK) modes poses a serious problem. The problem is quite severe if some inner manifold comes with approximate isometries (angular KK modes) or if there exists a throat of modest length other than the standard model throat, possibly associated with some hidden sector (low-lying KK modes).

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I. GENERALITIES: REHEATING THE UNIVERSE

The transfer of inflaton energy into radiation energy in the process of (p)reheating after inflation is a vital part of any model of early universe inflation. According to the inflationary scenario, the universe at early times expands quasiexponentially in a vacuum-like state without entropy or particles. During this stage of inflation, all energy is contained in a classical slowly moving inflaton field ϕ . Eventually the inflaton field decays and transfers all of its energy to relativistic particles to start the thermal history of the hot Friedmann universe.

The quantum field theory (QFT) of (p)reheating, i.e. the theory of particle creation from the inflaton field in an expanding universe, describes a process where the quantum effects of particle creation are not small, but instead produce a spectacular process where all the particles of the universe are created from the classical inflaton. The theory of particle creation and their subsequent thermalization after inflation has a record of theoretical developments within QFT. The four-dimensional QFT Lagrangian $\mathcal{L}(\phi, \chi, \psi, A_i, h_{ik}, \dots)$ contains the inflaton part with the potential $V(\phi)$ and other fields which give subdominant contributions to gravity. In chaotic inflationary models, soon after the end of inflation the almost homogeneous inflaton field $\phi(t)$ coherently oscillates with a very large amplitude of the order of the Planck mass around the minimum of its potential. Because of the interactions of other fields with the inflaton in \mathcal{L} , the inflaton field decays and transfers all of its energy to relativistic particles. If the

creation of particles is sufficiently slow (for instance, if the inflaton is coupled only gravitationally to the matter fields), the decay products simultaneously interact with each other and come to a state of thermal equilibrium at the reheating temperature T_r . This gradual reheating can be treated with the perturbative theory of particle creation and thermalization [1]. However, for wide range of couplings the particle production from a coherently oscillating inflaton occurs in the nonperturbative regime of parametric excitation [2]. This picture, with variation in its details, is extended to other inflationary models. For instance, in hybrid inflation (including D -term inflation) inflaton decay proceeds via a tachyonic instability of the inhomogeneous modes which accompany symmetry breaking [3]. One consistent feature of preheating—nonperturbative copious particle production immediately after inflation—is that the process occurs far away from thermal equilibrium.

Since hybrid inflation is the closest field theory prototype of string theory brane inflation, it will be especially instructive to refer to the theory of tachyonic preheating. Hybrid inflation involves multiple scalar fields Φ in the inflaton sector. It can be realized in brane inflation [4] where the interbrane distance is the slow rolling inflaton, while the subsequent dynamics of the branes are described by tachyonic instability in the Higgs direction. Tachyonic instability is very efficient so that the backreaction of rapidly generated fluctuations does not allow homogeneous background oscillations to occur because all the energy of the oscillating field is transferred to the energy of long-wavelength scalar field fluctuations within a single oscillation. However, this does not preclude the subsequent decay of the Higgs and inflaton inhomogeneities into other particles, and thus very fast reheating. Particles are gen-

*Electronic address: kofman@cita.utoronto.ca†Electronic address: piljin@kias.re.kr

erated in out-of-equilibrium states with very high occupation numbers, well outside of the perturbative regime.

Recent developments in string theory inflation, while in the early stages, have to address the issue of the endpoint of inflation, specifically (p)reheating immediately after inflation. As in the QFT theory of reheating, string theory reheating should be compatible with the thermal history of the universe. We can use this criterion to constrain the models. Yet, we are especially interested in the specific string theory effects of reheating which may be, in principle, observationally testable. From a theoretical point of view, reheating after string theory inflation deals with the theory of particle creation in string theory, which is an exciting topic by itself.

In this paper we study the transfer of energy into radiation from string theory inflation based on brane-antibrane annihilation. Brane annihilation is a typical endpoint of brane inflation. We assume a “warped” realization of brane inflation, constructed at the top of the ground of the KKLT stabilized vacuum [5]. The models of string theory inflation with warping branch into different possibilities. We mostly study reheating in the brane-antibrane warped inflation of [6], but the methods shall be relevant for the models like [7,8]. Reheating in other versions of string inflation, like $D3 - D7$ inflation of [9], potentially can be described by QFT reheating [10], while the racetrack inflation of [11] relies on field theory entirely.

The warped geometry of string theory inflation has some common features with the warped Randall-Sundrum five-dimensional braneworld. Based on this analogy, reheating in warped string theory inflation was modeled on the braneworld formalism in the recent paper [12].

Successful reheating means the complete conversion of inflaton energy into thermal radiation without any dangerous relics, in order to provide a thermal history of the universe compatible with big bang nucleosynthesis (BBN), baryo/leptogenesis, and other observations. Dangerous relics can be massless or massive, and they are each a danger in their own way. Too many massless relics like radiation of gravitational waves is excluded by BBN, while too many massive relics overclose the universe. Therefore, we have to monitor undesirable relics in the string theory inflationary scenario. There are significant differences between string theory reheating and QFT reheating in this respect. Indeed, in string theory we expect excitation of all modes interacting gravitationally, gravitational waves, moduli fields, and KK modes, and we need their complete decay or extra tuning to go through the needle eye of observational constraints on potential non-standard model (SM) particles.

We study reheating after brane-antibrane annihilation in warped inflation, more specifically, in the well-known ten-dimensional model of Klebanov-Strassler (KS) throat geometry. We begin with a single throat case to identify systematically the channels of energy transfer from infla-

tion to radiation. There are several processes of energy cascading, from $D - \bar{D}$ pair annihilation and closed string loop decay to excitation of KK modes, and from them to the excitation of open string modes of the SM branes. Then we extend our investigation to the multiple throat case, associated with different energy scales. We shall estimate the decay rate of each channel, its energy scales, the reheating temperature, and the abundance of dangerous relics. Therefore, we have to combine the string theory picture of all relevant excitations, their QFT description when it is adequate, and elements of the theory of early universe thermodynamics. This is a challenge for string theorists and cosmologists.

The plan of the paper is as follows. In Sec. II we lay out the background model, specifically KS geometry with extra ingredients needed for a KKLT stabilized string theory solution. We discuss the effect of warping on the local string modes, which will be major players in the reheating process. In Sec. III we study qualitatively the cascade of energy from $D - \bar{D}$ pairs to radiation. Section IV wraps up the thermalization process in the single throat model. In Sec. V we consider the multiple throat case.

II. WARPED COMPACTIFICATION WITH HIERARCHY

In finding a realistic model of universe in string theory, one of the more severe constraints comes from the so-called moduli problems in cosmology. If there are light scalar fields around, especially those associated with moduli fields of compactification, inflation process could easily lead to accumulation of energy in these light modes which then interferes with either exit from inflation or with low energy physics at a later stage of cosmological evolution.

A simple way out of these moduli problems, which are being investigated in string theory, involves turning on antisymmetric tensor fields along compactified internal dimensions. In a generic situation with all possible fluxes turned on and all possible nonperturbative corrections included, it is believed that the only massless degrees of freedom surviving the flux compactification would be that of 4-dimensional gravity or its supersymmetry completion in the case of supersymmetric vacua.

A common feature of these flux compactification is the warping. In this paper, we will be employing the well-understood case of IIB compactification, where the warping can be summarized as the ten-dimensional metric of the form,

$$G = H^{-1/2}(y)g_{\mu\nu}(x)dx^\mu dx^\nu + H^{1/2}(y)\mathcal{G}_{IJ}dy^I dy^J, \quad (2.1)$$

where \mathcal{G} is a Calabi-Yau metric on 6-dimensional compact manifold, and the warp factor H depends only on the internal Calabi-Yau direction. Note that as far as the internal manifold goes, this way of writing the metric is a mere convention. Unless the physical process concerned depends crucially on underlying supersymmetric structure,

we may as well rewrite the metric as

$$G = H^{-1/2}(y)g_{\mu\nu}dx^\mu dx^\nu + h_{IJ}dy^I dy^J \quad (2.2)$$

since $h \equiv H^{1/2}\mathcal{G}$ is the relevant physical metric of the compact direction. This form of the metric makes it very clear that the most important consequence of the warp factor, i.e., a generation of hierarchy via exponential redshift is really coming from the $H^{-1/2}$ in front of the first piece. A precise form of this warp factor is found by solving a second order equation for H with various source term, such as contribution from the NS-NS and RR fluxes, D -branes, and orientifold planes.

A. Klebanov-Strassler throats

An example of the warped geometry is the well-understood KS solution [13][14][15]. KS geometry is one of the building blocks of the Kachru-Kalosh-Linde-Trivedi (KKLT) stabilized solution, which also includes warped instanton branes and $\bar{D}3$ at the tip of the throat. Using this as a background (asymptotic of late time cosmological evolution) inflation can be realized by inclusion of additional elements: mobile $D3$ -brane in the throat attracted to another $\bar{D}3$ -brane around the bottom. This $D3 - \bar{D}3$ interacting pair provides an inflation in 4d description, the endpoint of which is their annihilation. We also need other $D3$ -branes around the bottom for SM phenomenology. Geometry of the model is shown in Fig. 1.

The Klebanov-Strassler throat starts from a conifold part of the Calabi-Yau metric \mathcal{G} . The local form of the metric is conical,

$$\mathcal{G}_{IJ}dy^I dy^J = dr^2 + r^2 ds_{T^{1,1}}^2, \quad (2.3)$$

where the five-dimensional manifold $T^{1,1}$ is a S^1 fibred over a product manifold $S^2 \times S^2$. The metric is clearly singular at origin $r = 0$ because the size of $T^{1,1}$ vanishes there.

Actual geometry involves a deformation that blows up one S^3 , as a fibration of S^1 over one S^2 , to keep it finite size at origin while allowing the other S^2 collapse to zero size.

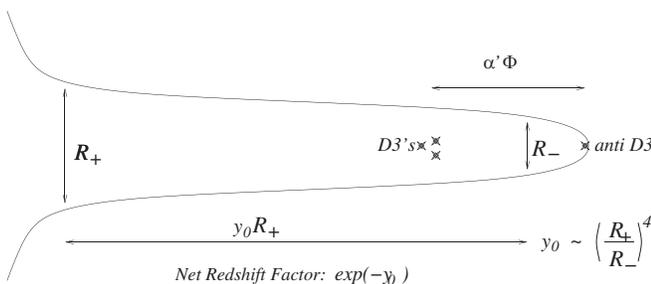


FIG. 1. Radial geometry of a Klebanov-Strassler throat. For most of this paper, we consider KKLMNT-like inflation scenario where unstable D -brane system of $D3$ -branes and anti- $D3$ -branes near the bottom of the throat drives inflation, possibly with some leftover $D3$'s.

Thus, at the bottom $r = 0$, the geometry is roughly that of

$$S^3 \times R^3. \quad (2.4)$$

Corresponding to S^3 is a 3-cycle on which RR 3-form flux $F_{(3)}$ is supported, while R^3 is part of its dual 3-cycle on which NS-NS 3-form flux $H_{(3)}$ is supported. Denoting these two 3-cycles as A and B , we have the quantization condition

$$\frac{1}{2\pi\alpha'} \int_A F_{(3)} = 2\pi M, \quad \frac{1}{2\pi\alpha'} \int_B H_{(3)} = 2\pi K \quad (2.5)$$

with integers M and K .

Note that the B cycle should extend outside of this local conifold geometry and the quantization condition on $H_{(3)}$ refers to this entire 3-cycle. However, assuming that most of $H_{(3)}$ flux resides within this conical region, the warp factor has been solved explicitly. Away from the conifold point $r = 0$, we have the following approximate solution [16],

$$H = \frac{1}{r^4} (R_+^4 + R_-^4 \log(r/R_+)^4) \quad (2.6)$$

with

$$R_+^4 \equiv \frac{27\pi}{4} \alpha'^2 g_s M K, \quad R_-^4 \equiv \frac{3}{8\pi} \frac{27\pi}{4} \alpha'^2 g_s^2 M^2. \quad (2.7)$$

The constant part of H is to be determined by gluing this local geometry to the rest of the compact Calabi-Yau metric near $r = R_+$. But for simplicity we have set it to zero.

The two radii, R_{\pm} , are both tunable, but with string coupling g_s small or with K sufficiently large, R_- can be considerably smaller than R_+ . This is the regime of interest for us since $R_+ > R_-$ typically generate a large hierarchy, be it between string scale and inflation scale or between string scale and electroweak scale.

The local geometry in terms of the physical metric is then

$$h \simeq (R_+^4 + R_-^4 \log(r/R_+)^4)^{1/2} \left(\frac{dr^2}{r^2} + ds_{T^{1,1}}^2 \right) = (R_+^4 - 4R_-^4 y)^{1/2} (dy^2 + ds_{T^{1,1}}^2). \quad (2.8)$$

Up to the change of overall size due to the logarithm, this metric describes a line parameterized by $y = -\log(r/R_+)$, times $T^{1,1}$. Together with smooth completion near $r = 0$ this is called the Klebanov-Strassler throat. The radius of $T^{1,1}$ varies from R_+ near the top of the throat ($r \sim R_+$) to R_- near the bottom of the throat. At the bottom of the throat radius of S^3 is R_- , while radius of S^2 is shrinking to zero.

This approximate metric based on the singular conifold must be replaced by the one based on deformed conifold as we reach the size of $T^{1,1}$ to be R_- . We will denote the value

of r there as $r_0 = R_+ e^{-y_0}$, which takes the value

$$1 \gg e^{-y_0} \equiv \frac{r_0}{R_+} = e^{-(R_+^4/4R_+^4)+1} = e^{-(2\pi K/3g_s M)+1}. \quad (2.9)$$

For $r < r_0$ the value of H does not change much, and we then have a redshift factor between the top and the bottom of the throat

$$1 \gg H^{-1/4}(r_0) \simeq \frac{r_0}{R_-} = \frac{R_+}{R_-} e^{-(R_+^4/4R_-^4)+1} = \frac{R_+}{R_-} e^{-y_0}. \quad (2.10)$$

This small factor is responsible for the generation of the hierarchy between the top and bottom of the Klebanov-Strassler throat.

B. Hierarchy and the decay cascade of local string modes

We identified the cascade of energy transfer from $D - \bar{D}$ annihilation to SM particles as shown in Fig. 2. The first part of the process is the creation of closed strings loops. Then they decay into lighter KK modes. KK modes interact with SM branes to excite SM particles. Residual scalar excitations of a SM brane decay further into SM fermions. Each step of this cascade will be described in Sec. III. Here we give a qualitative description of excitation modes (particles) in the warp geometry, which will be crucial for quantitative estimations of Sec. III.

The hierarchy is generated because the total metric has the form

$$G = H^{-1/2} g + h. \quad (2.11)$$

Cascading Energy from Inflaton to Radiation

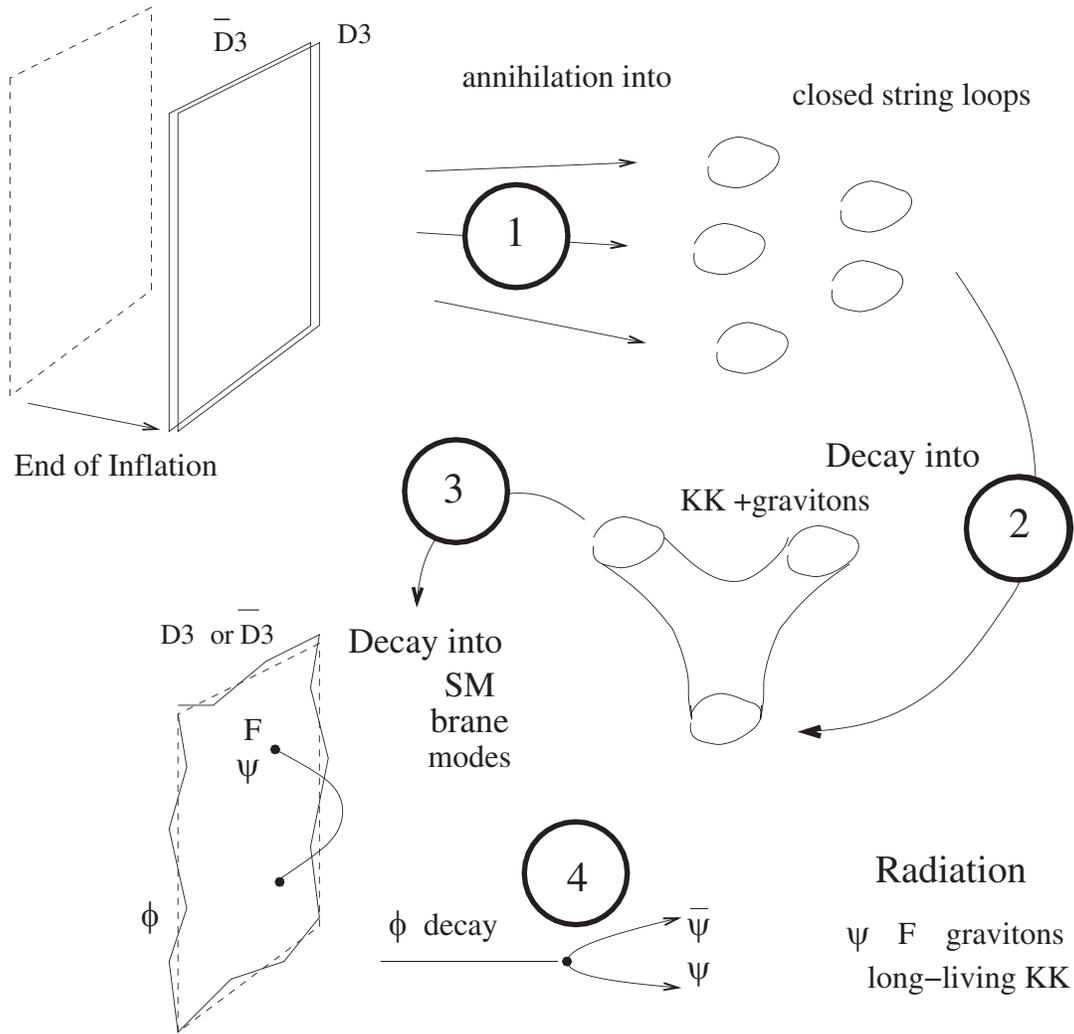


FIG. 2. Identifying the channels of D -brane decay

This structure implies that the conserved energy E in the noncompact spacetime depends on the position of the quanta in the compact manifold as

$$E^2 \sim H^{-1/2}(y). \quad (2.12)$$

This can be seen from the on shell condition for particle of mass m ,

$$H^{1/2}g^{\mu\nu}p_\mu p_\nu + h^{IJ}p_I p_J = -m^2, \quad (2.13)$$

where p_μ would be the conserved momenta in the spacetime for a Minkowskii metric g .

For degrees of freedom arising from closed strings, the right-hand side would be represented by the oscillator contribution, and we have

$$-g^{\mu\nu}p_\mu p_\nu = H^{-1/2}(y)\left(h^{IJ}p_I p_J + \frac{N}{\alpha'}\right) \quad (2.14)$$

with an integer quantized oscillator number N , and the internal Kaluza-Klein momenta p_I . This crude formula is valid only when we can regard the geometry h and the warp factor H to be both sufficiently slowly varying so that we can regard internal geometry h as nearly flat, and H as nearly constant. Furthermore in this rough scaling argument we also ignore change in string quantization due to fluxes.

In fact, it is doubtful whether these assumptions are justifiable for most of the closed string modes we are familiar with in old Calabi-Yau compactification without flux. In the absence of flux, the Calabi-Yau compactification gives two types of closed string excitations. One class is the oscillator modes which we usually ignore for low energy purpose since the mass thereof are all fundamental string scale $\frac{1}{\sqrt{\alpha'}}$ at least. The other class is Kaluza-Klein modes which arise from Fourier analysis of 10D supergravity modes on the compact internal manifold. For large volume, these latter modes are most relevant. These KK modes are expressed in terms of eigenmodes of various kinds on internal manifold, which take nontrivial wave function throughout the underlying Calabi-Yau manifold, and can in no way be deemed to be localized in some part of the internal manifold except for those with extremely large mass.

When the flux compactification involves a warped throat of the kind we discussed with large redshift factor, however, a new class of closed string modes emerges. While the conifold region is a very small piece of the Calabi-Yau manifold, \mathcal{G} , its associated throat geometry is not necessarily small since the physical metric is $h = H^{1/2}\mathcal{G}$ with an exponentially large H . Thanks to this, there are both oscillator modes and KK modes which are localized deep down the KS throat. Mass scales of such local modes are

$$m_{KK} \sim H^{-1/4}(r_0)\frac{1}{R_-}, \quad m_{\text{oscillator}} \sim H^{-1/4}(r_0)\frac{1}{\sqrt{\alpha'}}, \quad (2.15)$$

respectively. Provided that R_- is somewhat larger than the string scale $\sqrt{\alpha'}$, it makes sense to consider such localized and thus redshifted modes.

For these modes at least, the above scaling of the on shell condition is applicable. While curvature of h , gradient of H , and existence of fluxes will change the form of the on shell condition, none of them will have as dramatic of an effect as the factor $H^{-1/2}$ in front of the right-hand side. In terms of the linear coordinate $y = -\log(r/R_+)$, we have

$$H^{-1/4} \simeq e^{-y} \quad (2.16)$$

up to a prefactor, and this exponential dependence dominates any kinematics of the closed string modes.

For any massive modes of mass m^2 in the unrescaled unit, located deep down the throat, we have the on shell condition,

$$-p_\mu p^\mu \sim e^{2y_0-2y} m^2 \quad (2.17)$$

with the right-hand side increasing exponentially as we move up the throat, away from the conifold toward the bulk of Calabi-Yau manifold. This shows roughly how the mass quantization differs between local modes and the rest. For modes deep down the KS throat, the mass gap scales either as $\sim e^{-y_0}/R_-$ or as $\sim e^{-y_0}/\sqrt{\alpha'}$, while everywhere else the mass gap scales as $1/R$, with R being the linear size of the Calabi-Yau manifold, and $1/\sqrt{\alpha'}$.¹

Therefore, for energy distributed among local modes to escape the throat, one must assemble the energy into a few quanta with exponentially large kinetic energy. The required kinetic energy must be larger than its rest mass by a factor of $e^{y_0} \gg 1$. Otherwise, the quanta would be simply reflected by a Liouville-like wall. The strength of these Liouville-like walls is dependent on the mass of the particle: the heavier the local mode, the stiffer is the wall. Thus, in the presence of this mass-dependent potential barrier, the localized heavy mode will have tendency to decay near the point y_c

$$e^{y_0-y_c} \sim \frac{E}{m} \quad (2.18)$$

to modes with smaller mass scale.

Any heavy localized mode present deep inside the KS throat would eventually decay to lighter modes within the same throat. For R_-^2 substantially larger than α' , the lightest modes will be KK modes localized at the bottom of the throats, and thus in the intermediate time scale, energy will be deposited in these modes. While they are light, they also are massive with characteristic mass scale e^{-y_0}/R_- , and

¹Here we are assuming that all moduli are fixed by the flux compactification. For some moduli, notably those to be fixed by nonperturbative mechanism, the associated mass scale could be considerably lower. However, this separation of two scales associated two types would-be moduli are in principle independent of this hierarchy we address, and will be taken to be insignificant.

thus are also confined within the throat. The associated potential wall is less stiff, and the energy can be stored in a somewhat larger volume at the bottom.

On a quantitative level, what was said above can be seen in terms of eigenmodes which obey the oscillator-like equation [17,18]. Here instead of writing the full KK mode equation, we rely on a simple massless scalar field eigenmode equation

$$\left[\frac{H(y)}{\sqrt{h}} \partial_I h^{IJ} \frac{\sqrt{h}}{H(y)} \partial_J + H^{1/2}(y) \nabla_\mu \nabla^\mu \right] \Phi = 0. \quad (2.19)$$

Factorizing the eigenmode into outer/inner space parts, and replacing four-dimensional Lorentz invariant operator $\nabla_\mu \nabla^\mu$ by its eigenvalue m_{KK}^2 , we get equation for the KK eigenmode $\Phi_{m_{\text{KK}}}$. This m_{KK}^2 is KK mass as seen by four-dimensional observer, $m_{\text{KK}}^2 = \omega^2 - \vec{k}^2$. Taking the approximate form

$$H \simeq e^{4y}, \quad h = R^2(dy^2 + ds_{T_{1,1}}^2), \quad (2.20)$$

where R is a slowly varying function ranging from R_+ to R_- which we take to be constant effectively. In other words, we can approximate geometry with that of AdS warped geometry of Randall-Sundrum [19] type, times the internal compact manifold $T_{1,1}$ of a definite size. Singular boundaries of the Randall-Sundrum geometry are naturally smoothed out by having the additional internal dimensions. Attachment to Calabi-Yau manifold and the cigarlike capping of the bottom, respectively, replace UV and IR branes.

Denoting the quantized and dimensionless angular momentum on $T_{1,1}$ by L^2 , this equation simplifies to

$$\left[e^{4y} \partial_y e^{-4y} \partial_y + m_{\text{KK}}^2 R^2 e^{2y} - L^2 \right] \Phi_{m_{\text{KK}};L} = 0, \quad (2.21)$$

where L^2 is a contribution from angular momentum. Its spectrum depends on the isometry of $T_{1,1}$. For instance, contribution from the S^q sphere will give $L^2 = l(l+q-1)$, where l are integer numbers.

As usual with such warped geometry, the equation can be transformed to a Bessel equation by taking

$$z \equiv e^y, \quad \Phi_{m_{\text{KK}};L} = z^2 \Psi_{m_{\text{KK}};L} \quad (2.22)$$

which gives

$$\left[\partial_z^2 + \frac{1}{z} \partial_z + m_{\text{KK}}^2 R^2 - \frac{4+L^2}{z^2} \right] \Psi_{m_{\text{KK}};L} = 0. \quad (2.23)$$

Thus the eigenmodes to this simplified KK equation are given by linear combinations of Bessel functions

$$J_{\pm\nu}(m_{\text{KK}} R e^y) \quad (2.24)$$

with $\nu^2 = 4 + L^2$ (for $L = 0$ we shall take combination of functions J_2 and Y_2). This shows that, with the length of the interval in z coordinate of order $\simeq e^{y_0}$, the mass eigenvalues are quantized in units of

$$\Delta m_{\text{KK}} \sim \frac{e^{-y_0}}{R} \quad (2.25)$$

as promised. The mass spectrum is roughly

$$m_{\text{KK}} \sim n \frac{e^{-y_0}}{R} \quad (2.26)$$

with integers n , and for large n , the wave function $\Phi_{m_{\text{KK}};L}(y)$ is oscillating near the bottom of the throat, while far away from the bottom (small y) it has the asymptotics $\Phi_{m_{\text{KK}};L}(y) \sim e^{(2 \pm \sqrt{4+L^2})y}$.

In the long run, the decay will further proceed until energy is mostly retained by massless or nearly massless modes of the string theory in question. Assuming complete stabilization of Calabi-Yau moduli by flux, the only such modes are 4-dimensional gravity and possibly light open string modes associated with stable D -branes which may exist the bottom of the throat. The question is then how the final distribution of energy will look between the bulk gravitational sector and the localized open string sector of stable D -branes.

One important point in pursuing this question is that the only light degrees of freedom is the 4-dimensional gravity, but this couples to the modes localized at the bottom of the KS throat very weakly. The redshift factor $H^{-1/4}$ reduces the effective scale of energy-momentum by an exponential factor and pushes down the inflation scale and subsequent reheating scale as well. This is essentially the physics of the Randall-Sundrum scenario, realized in string theory setting, and can be understood from the fact that at the 4-dimensional graviton has the wave function profile $H^{-1/2}(y)$ in the internal direction. Any localized mode at the bottom of the KS throat will have very small wave function overlap with the 4D graviton and thus cannot generate much gravitational energy.

On the other hand, open strings on D -branes will couple to KK modes without this exponential suppression but there still is a volume suppression if they are, for example, on $D3$ -branes transverse to the compact Calabi-Yau. In order to see how much energy is deposited to what species of particles, we must pay more close attention to interaction at the bottom of the KS throat, which will be discussed in a later section.

C. Subtleties

The above Eq. (2.23) is usually obtained via vast simplification of actual KK mode equations. In particular, the capping of the throat near $y = y_0$ is not faithfully reflected, which leaves the determination of precise spectrum difficult. On the other hand, the robust part of the above estimate is that the lowest energy is of order $\sim e^{-y_0}/R_-$ and also that subsequent gap between adjacent eigenmodes is also $\sim e^{-y_0}/R_-$.

An important subtlety to bear in mind is how far one should trust this equal-spaced spectrum. This linear analy-

sis suggests that a throat of length $R_+ y_0$ has an exponentially large number of states of order e^{+y_0} due to its very small mass gap. If we take some state of midrange value of the mass m' such that $1/R_+ \gg m' \gg e^{-y_0}/R_-$, the number of states it can decay to is of order $(m'R_-)e^{y_0}$ and this will induce a very large width to the eigenmode thus obtained. For large enough m' , it is therefore reasonable to expect that this linear analysis is misleading.

This problem is also manifested in the shape of the eigenmode. While it takes a simple innocuous form in the conformal coordinate z , its behavior in the physical coordinate y is much more drastic for high-lying modes. With $mR_- e^{-y_0} \gg 1$, the Bessel functions oscillate wildly near $y = y_0$, and its derivative could be larger than the local string scale. So once we have m_{KK} larger than the local string scale, it is unclear whether supergravity approximation can be trusted.

Thankfully, however, analysis of the present work is not affected by this ambiguity. We will consider closed string oscillator modes with perhaps up to 100 oscillators, which eventually will decay to low-lying KK modes. As we will see later, energy deposited to KK modes will quickly thermalize among themselves, and since the energy scale of initial state right after inflation is of order $\sim 1/4\pi^3 g_s \alpha'^2 < 1/\alpha'^2$, relevant KK modes are those below string scales. We never rely on very high scale KK modes whose precise nature would need more careful analysis. For bulk estimate of reheating processes, even details of low-lying KK modes do not enter other than their numbers.

One important exception to this is how the lowest energy eigenvalues depend on the angular momentum L^2 . Later we will consider L^2 as an approximately conserved quantum number in the full Calabi-Yau compactification; this precise spectrum at low end could be important for lifetime estimate of long-lived KK mode carrying such quantum numbers. We leave it to future study.

III. DECAY OF D -BRANES AT THE END OF A BRANE INFLATION

One attractive class of stringy inflation models involve the unstable D -brane system [20] whose elevated vacuum energy drives the inflation [21]. While introduction of an unstable brane system is a novel element, this difference does not seem to generate much new flavor in terms of studying 4-dimensional low energy effective theory during inflation. As is typical with inflation, maintaining a sufficient number of e-folding and at the same time having a graceful exit is not a small problem, and these are just translated to more geometrical constraints on the underlying string theory.

When it comes to the reheating process after the end of the inflationary era, however, the behavior of brane inflation could be very different from ordinary field theory models. Sometimes the so-called tachyon effective action

is invoked, but one cannot take this tachyon effective action too literally. While this low energy approach has been immensely successful, mathematical results one finds with it must be reinterpreted with care. For instance, the so-called tachyon matter is known to survive the decay process and takes up all initial energy in the unstable brane system, and behaves like a perfect fluid of very massive noninteracting particles. However, the system in question started with an open string description which should not be valid by the time D -brane has decayed. This so-called tachyon matter turns out to be a coarse-grained view on the underlying physical state, namely, a certain distribution of highly excited closed string states.

In the conventional inflationary models, the reheating question centers on how effective an inflaton decay can excite other degrees of freedom. Here the situation is reversed. The initial reheating process is such that 100% of the energy density responsible for the inflationary phase is converted to heavy degrees of freedom that have nothing to do with what drove the inflation. The right question to ask here is how this huge amount of energy density is eventually distributed among different light degrees of freedom. Since the initial stage of decay produces a lot of heavy closed strings, there is an inherent danger of closed strings dominating the process. Assuming some kind of braneworlds scenario for standard model sector, this would be a disaster for brane inflation to be viable, or alternatively might work as a very efficient and simple tool for eliminating many stringy inflation scenarios.

In this section, we will describe a very efficient and viable reheating process for the case of single-throat scenario. In later sections we will discuss under what circumstances the multithroat scenario can offer a viable reheating process.

A. $D - \bar{D}$ annihilation and closed strings production

The endpoint of inflation is $D3 - \bar{D}3$ pair annihilation, the step 1 at Fig. 2. A complete description of a non-Bogomol'nyi-Prasad-Sommerfield monopoles (BPS) system in string theory is a complicated problem [22]. The hallmark of the D -brane decay, as opposed to decay of inflaton in field theoretical models of inflation, is the fast and complete conversion of unstable D -brane energy into massive closed string modes [23–25]. This is what we find from tachyon dynamics in low energy [26–29] but this is also supported by stringy computation [30–32] using the decaying boundary state [33–36].

The pair of isolated $D3 - \bar{D}3$ branes annihilates into excitations of close strings loops with the average energy E , in the bosonic string theory [31] E is

$$\frac{E}{V_3} \simeq \sum_N \int d^{(d-3)}k_{\perp} D(N) e^{-2\pi\omega_{n,k}}, \quad (3.1)$$

where $\omega_{n,k} = \sqrt{k_{\perp}^2 + 4N}$, V_3 is the three-dimensional vol-

ume of the branes, \vec{k}_\perp is the momentum of the closed strings transverse to the branes, $D(N)$ is the number of closed strings oscillator states, for large N

$$D(N) \simeq N^{-q} e^{4\pi\sqrt{N}}, \quad (3.2)$$

in bosonic theory $q = 27/4$, $d = 25$; in superstring theory $d = 9$ and q is not known.

It is interesting to compare this result with the inflaton decay in the QFT tachyonic preheating, which is characterized by the high occupation numbers. In brane-antibrane annihilation, the product states are characterized by two numbers, oscillator state n and momentum k . The analog of the occupation number would be $e^{-2\pi\omega_{N,k}}$, which is less than unity. However, QFT has no analog of the oscillator number. If we sum over n for a given \vec{k} , in principle we can get the number exceeding unity.

With a view toward reheating from brane inflation, we will not need much of the details of the decay. Let us summarize the main characteristics of closed strings from decay of unstable D -branes:

Initially, massive oscillator modes are produced. The probability of a particular closed string mode with energy $\omega_{N,k}$ to be produced scales as $e^{-2\pi\omega_{N,k}}$, but this exponential suppression is exactly canceled by the Hagedorn growth of density of state $D(N)$ at larger energy. The upshot is that for each oscillator level N , roughly the same amount of energy is deposited.

Since the boundary state is formulated at $g_s \rightarrow 0$ limit, one must introduce cutoff to emulate backreaction of the boundary state to production of closed strings. For unstable $D0$, this natural cutoff is m_s/g_s with the local string scale m_s , and with this cutoff the energy in the produced heavy closed string account for all energy in the unstable D -branes. The highest oscillator state which is expected to be excited is estimated from $\omega_{\max} \sim 1/g_s$. For the string coupling $g_s \sim 0.1$ we get $N_{\max} \sim 100$.

The probability distribution $e^{-2\pi\omega_{N,k}}$ also implies a narrow velocity dispersion. The usual string on shell condition $\omega \sim \sqrt{p^2 + m_s^2 N}$ implies that the transverse velocity of the typical closed string mode is at most of order $\sim \sqrt{m_s}/\sqrt{m} \sim \sqrt{g_s}$. Thus, kinetic energy of the produced closed strings tends to be smaller than its rest mass. The string loops are nonrelativistic, and only very slowly can move around their birthplace in the volume V_{d-3} .

If the unstable D -brane system decays well inside the KS throat with a redshift factor $H^{-1/4}(r_0)$ and a large radius R_- , all of above should remain true qualitatively. The main difference is in the string scale m_s . Since we are discussing energy in terms of the 4-dimensional metric g , the string scale m_s that appears above should be related to the fundamental string tension by

$$m_s^2 = \frac{H^{-1/4}(r_0)}{2\pi\alpha'} \sim e^{-2y_0} \frac{1}{\alpha'}. \quad (3.3)$$

Thus, the decay of a D -brane at the bottom of a KS throat produces strings of mass m and energy E of order $e^{-y_0}/(g_s\sqrt{\alpha'})$ at most.

From the four-dimensional perspective, the energy density of the closed loops is the same as the energy density of the $D3 - \bar{D}3$ pair. We have to take into account redshift of the brane tension in the warped geometry by the factor e^{-4y_0}

$$\epsilon = 2T_3 e^{-4y_0} = \frac{e^{-4y_0}}{4\pi^3 g_s \alpha'^2}, \quad (3.4)$$

which is the scale of the warped inflation.

The timing of $D3 - \bar{D}3$ annihilation we estimate as

$$\Delta t_1 \sim e^{y_0} \sqrt{\alpha'}. \quad (3.5)$$

Strictly speaking, the string computation was done for the decay process involving the 1-point function of closed strings, and for this reason can be relied on only for unstable $D0$ or unstable D -branes wrapped on a small torus. The same computation for more generic cases is inconclusive because the 1-point emission is not the dominant decay channel. In fact, energy output from such 1-point emission can be computed and can be shown to be well short of the expected energy output. Mathematically this happens because the produced string mode cannot carry momentum longitudinal to the D -brane. However, this is an artifact of 1-point emission processes.

For higher dimensional cases, the dominant process should involve simultaneous production of two or more closed strings, which has more phase space volume along the longitudinal direction. Although such a multipoint amplitude would be suppressed by powers of string couplings, the gain from phase space volume can easily overcome this. For instance, consider an unstable $D1$ which is infinitely extended. Two point decay would be suppressed by one more power of g_s , but the available phase volume grows linearly with the energy of the created pair of particles. Introducing a natural cutoff $1/g_s$ in string scale, the phase volume will be of order $1/g_s$. This cancels the coupling suppression from the string diagram easily; we expect this process to dominate the decay process with the quantitatively same characteristics of produced closed strings.

Consideration of the low energy approach using the tachyon effective action has been developed independent of the string theory computation and tells us pretty much the same story. The so-called tachyon matter [33,34] and string fluid [26,27,29,37], which emerges from this low energy approach, have been studied in depth and compared to the closed string side [25,38], for unstable D -branes of arbitrary dimensions. The result shows that all energy is converted into heavy closed string excitations, possibly together with long fundamental strings.

B. Closed strings decay to local KK modes

Now that we identified the initial state right after the end of brane inflation, we must consider the subsequent decay of the massive closed string thus produced. This corresponds to step 2 at the Fig. 2.

The local closed string modes will then decay to whichever are lighter degrees of freedom around. There are two types of light modes at the bottom of throats. One are KK modes, which would be lighter than oscillator modes, as long as R_- is not too near the string scale. Another are light open strings modes associated with extra D -branes. One easy way to achieve this is to have some extra $D3$ -branes surviving at the end of annihilation. While decay amplitude to a pair of open string modes would carry one less factors of $g_s^{1/2}$, it is also suppressed by volume effect $\sim(\alpha'/R_-)^{3/2}$. Even with $R_-/\sqrt{\alpha'}$ slightly larger than 1, the latter effect will easily compensate for the former, and will favor decay to KK modes first. In this work, we assume that R_- is large enough to justify field theory and gravity analysis employed here. Even if it is necessary to extrapolate to smaller value R_- , our guess is that basic qualitative estimates we offer in this paper would remain valid. After all, what really matters at the end of the day is that energy settles in some open string sector that would contain the standard model, and non-SM dangerous relics.

In the flat background massive closed string with oscillation number n decays into two closed strings with oscillation numbers $N - N'$ and N' , $N' < N$, with the coupling strength g_s . Subsequently, $N - N'$ and N' states further cascade into states with the smaller oscillator numbers. The final states will be KK modes, a fraction of which is in massless gravitational radiation in the bulk. Gravi-tensor projection of the bulk gravitons to four dimensions describe usual gravitational waves in four dimensions. Therefore, in the inflationary scenarios which have $D - \bar{D}$ annihilation at the end, we shall address the problem of overproduction of gravitational radiation. One of the possibilities to overcoming the problem is to arrange annihilation not between a $D - \bar{D}$ pair, but a system of \bar{D} and several D branes, to dilute the amount of gravitons.

However, decay of closed strings in the warped geometry is significantly different from that in the flat background. Although the energy of the unstable D -brane system is all deposited to the highly excited strings, none of these modes can overcome the potential barrier toward the top of the KS throat, since there is e^{y_0} factor difference between strings scales between the top and the bottom.

The only exception to this would be the part of energy radiated into massless modes. In the current setup the only massless graviton is the 4-dimensional variety, whose wave function comes with the same exponential suppression at the bottom of the throat. Another way to say this is that the effective tension of the unstable D -brane is redshifted by a power of e^{-y_0} relative to the Planck scale which weakens its coupling to the 4-dimensional gravity. It also can be

seen in the following way. The wave function of a massless four-dimensional graviton (zero KK mode), after rescaling with respect to the warp factor, is homogeneous relative to inner dimensions. Therefore, decay of the closed strings into gravitons is suppressed by the four-dimensional Planck mass, while decay into massive KK modes is suppressed by the redshifted string scale. Therefore, we have a very specific effect of the warped geometry of the exponentially suppressed production of gravitational waves from $D - \bar{D}$ annihilation. To the first approximation, no energy is deposited to the 4-dimensional graviton directly. More specifically, the energy fraction deposit in gravitational waves radiation is e^{-2y_0} of the total energy of radiation. This figure depends on the energy scale of inflation. Recall that the amount of energy density in gravitational waves at the moment of BBN shall be no more than several percent of the total radiation energy density.

Apart from the 4-dimensional gravitons, the coupling to which is universally small deep inside the throat, the next light degrees of freedom are local KK modes. To see this clearly and also for a later purpose of computing branching ratio, let us expand the 10-dimensional Einstein action according to the compactification. Starting with 10-dimensional action,

$$\frac{1}{2g_s^2\kappa_{10}^2} \int dX^{9+1} \sqrt{-G} R_G \quad (3.6)$$

with $\kappa_{10}^2 = (2\pi)^7 \alpha'^4/2$, taken as before $G = H^{-1/2}g + h$ but allow small fluctuations in g and h . The former gives the 4-dimensional gravitons while the latter is prototype for KK modes from the compact manifold. Expanding the 10 dimensional action and keeping terms quadratic to the small fluctuations, the first term

$$\frac{1}{16\pi G_N} \int dx^{3+1} \sqrt{-g} R_g \quad (3.7)$$

is the Einstein action in 4 dimensions with the effective Newton's constant G_N such that

$$\frac{1}{16\pi G_N} = \frac{\tilde{V}_6}{2g_s^2\kappa_{10}^2}, \quad \tilde{V}_6 \equiv \int dy^5 \sqrt{h} H^{-1/2}. \quad (3.8)$$

Note that the KS throat region contribution to $\sim\tilde{V}_6$ is of order R_+^6 , so we have a bound $V_6 > R_+^6$. Large R_{\pm} will necessarily make the volume of the compact manifold somewhat larger than the fundamental string scale and weaken gravity further, regardless of the exponential redshift factor.

In our convention, the gravitational redshift due to the warp factor is manifest on the matter effective action. If we expand $h = h_{\text{KS}} + \delta h$, the remaining terms have the following general structure,

$$\frac{1}{2g_s^2\kappa_{10}^2} \int dx^{3+1} \sqrt{-g} \left(\int dy^6 \sqrt{-h} H^{-1/2} \delta h \nabla_g^2 \delta h + \int dy^6 \sqrt{h} H^{-1} \delta h \nabla_{h_{KS}}^2 \delta h \right), \quad (3.9)$$

which are the kinetic term and the mass term, respectively, of internal KK modes. Further expanding δh in terms of eigenmodes localized at the bottom of the throat,

$$\delta h = \sum \phi_{\text{KK}}^{(n)}(x) \delta h_{(n)}(y) + \dots \quad (3.10)$$

where $\delta h_{(n)}$ denote local KK modes, all supported at the bottom $r \simeq r_0$.

As a matter of convenience we will normalize $\delta h_{(n)}$ to produce canonical kinetic term for $\phi_{\text{KK}}^{(n)}$, so that these second class of terms produce,

$$\frac{1}{2} \int dx^{3+1} \sqrt{-g} \sum_n \left((\nabla \phi_{\text{KK}}^{(n)})^2 - (m_{\text{KK}}^{(n)})^2 (\phi_{\text{KK}}^{(n)})^2 \right). \quad (3.11)$$

The KK masses $m_{\text{KK}}^{(n)}$ of the localized modes are of order

$$m_{\text{KK}}^{(n)} \sim e^{-y_{(n)}} \frac{1}{R_-}, \quad (3.12)$$

where we introduced $y_{(n)}$ whereby $\delta h_{(n)}$ occupy region at the bottom of the throat $y > y_{(n)}$.

As we saw above, the initial product of D -brane decay is numerous heavy closed strings of mass $\sim e^{-y_0}/g_s \sqrt{\alpha'}$ or smaller. With not too small R_- , local KK modes are much lighter and more numerous than these local oscillator modes, and will couple to the closed string modes in the usual 3-point diagram to these oscillator modes with the coupling being essentially g_s . The KK modes thus produced can have energy or mass up to $\sim e^{-y_0}/g_s \sqrt{\alpha'}$, which implies that the energy is distributed in KK quanta and excites the throat up to $y_0 - y \sim \log(R_-/g_s \sqrt{\alpha'})$.

In contrast, the 4-dimensional graviton, which is the only light mode that is not localized at the bottom of the throat, has much higher mass scale of its own,

$$M_P \simeq \frac{1}{g_s} \frac{\sqrt{V_6}}{\kappa_{10}} \sim \frac{1}{g_s} \frac{R_+^3}{(\alpha')^{3/2}} \frac{\varepsilon}{\sqrt{\alpha'}} \sim \left(\frac{27\pi MK}{4g_s^{1/3}} \right)^{3/4} \frac{\varepsilon}{\sqrt{\alpha'}}, \quad (3.13)$$

which is exponentially larger by a factor of e^{y_0} than that of other scattering processes among local string modes and local KK modes. Here the numerical constant ε which came from the definition of κ_{10} is

$$\varepsilon \equiv \left(\frac{2}{(2\pi)^7} \right)^{1/2}. \quad (3.14)$$

Up to this stage, the primary decay channel of the closed string produced from the D -brane decay will be into local KK modes.

Finally, we estimate the timing of closed strings cascading into light KK modes. The rate of decay of the individual close string is about $\Gamma \sim g_s^2 e^{-y_0}/\sqrt{\alpha'}$. The longest process is the closed string with the oscillator number N_{max} goes through N_{max} decays, so the upper bound on the net decay time is $\Delta t_2 \sim N_{\text{max}}/\Gamma$. Since $N_{\text{max}} \sim 1/g_s^2$, we estimate

$$\Delta t_2 \sim g_s^{-4} e^{y_0} \sqrt{\alpha'}. \quad (3.15)$$

This timing is longer than the brane annihilation Δt_1 . KK modes are ultrarelativistic. Indeed, closed strings have energy $m_{\text{closed}} \sim \frac{e^{-y_0}}{g_s \sqrt{\alpha'}}$, while KK modes have the mass $m_{\text{KK}} \sim \frac{\sqrt{\alpha'}}{R_-} \frac{e^{-y_0}}{\sqrt{\alpha'}}$, so that $\frac{m_{\text{closed}}}{m_{\text{KK}}} \sim \frac{R_-}{\sqrt{\alpha'}} \frac{1}{g_s}$.

Next we proceed with the decay of KK mode into open strings modes, which can be associated with the SM particles.

C. KK modes decay to open string modes

Let us note that upon exciting KK modes, the corresponding metric perturbation along the compact direction has the size

$$\delta h(y) \sim \frac{g_s \kappa_{10} H^{1/4}(r_0)}{R_-^3} \phi_{\text{KK}}^{(n)} \quad (3.16)$$

for each KK eigenmode with the canonically normalized 4-dimensional massive fields $\phi_{\text{KK}}^{(n)}$. This is what couples to open string degrees of freedom directly as we will see shortly. With this, let us consider how KK modes couple and decay to open string modes on a D -brane transverse to the Calabi-Yau direction.

One may be alarmed to see the exponentially large factor $H^{1/4}$ in this expression, but all this does is rescale the dimensionful parameters in terms of the local redshifted scale, since

$$\frac{\kappa_{10} H^{1/4}(r_0)}{R_-^3} \sim \frac{\alpha'^2 H^{1/4}(r_0)}{R_-^3} \simeq \frac{(e^{-y_0}/R_-)^3}{(e^{-y_0}/\sqrt{\alpha'})^4}. \quad (3.17)$$

The mass scales of relevant ϕ_{KK} 's are anywhere between e^{-y_0}/R_- and $e^{-y_0}/\sqrt{\alpha'}$, so as long as R_- is not too small in string unit, the fluctuation is small and treatable as a perturbation.

With this in mind, let us consider a probe $D3$ brane located at the bottom of the tip. The Born-Infeld action can be written as,

$$-\frac{1}{8\pi^3 g_s \alpha'^2} \int dx^{3+1} H^{-1}(r) \times \sqrt{-\text{Det}(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} + H^{1/2}(r) h_{IJ} \partial_\mu Y^I \partial_\nu Y^J)} \quad (3.18)$$

where $r = r(Y)$ and $Y^I(x)$ represent transverse fluctuation of $D3$ brane along the compact directions. The primary interaction between δh and D -brane appears from the

leading expansion,

$$\frac{1}{8\pi^3 g_s \alpha'^2} \int dx^{3+1} \sqrt{-g} (H^{-1/2}(Y) \times \left(\frac{1}{2} h_{IJ} \partial_\mu Y^I \partial_\nu Y^J g^{\mu\nu} - H^{-1}(Y) \right)). \quad (3.19)$$

Here we have chosen the longitudinal coordinate system such that

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta g_{\mu\nu}, \quad (3.20)$$

where δg represents the 4-dimensional graviton. For the moment we set $\delta g = 0$ so that $g = \eta$.

In addition, there is another potential term from the minimal coupling to the background R-R 4-form potential C_4 . For this let us recall that KS geometry comes with 3 types of fluxes: NS-NS 3-form flux $H_3 = dB_2$, R-R 3-form flux F_3 , and 5-form flux, which is related to the previous two by

$$F_5 = dC_4 + B_2 \wedge F_3 \quad (3.21)$$

and is constrained to be self-dual. A minimal C_4 for this self-duality constraint to hold can be determined from B_2 and F_3 . Relying on the explicit solution of Ref. [16], we find

$$C_4 = c H^{-1} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3. \quad (3.22)$$

$D3$ branes have the usual minimal coupling to C_4 . The proportionality constant c is such that this contribution from R-R coupling cancels the tension term from the Born-Infeld piece exactly for a probe $D3$ brane and double it for probe anti- $D3$ brane.² Thus, the potential term H^{-1} would be either canceled or doubled in actual dynamics.

Expanded up to quadratic order in Y , and canonically normalizing Y fields to \hat{Y} , we have the following general form of action

$$\frac{1}{2} \int dx^{3+1} \sqrt{-g} \left((\partial \hat{Y})^2 - \mu^2 \hat{Y}^2 + \frac{(e^{-y_0}/R_-)^3}{(e^{-y_0}/\sqrt{\alpha'})^4} \phi_{\text{KK}} (\partial \hat{Y})^2 \right), \quad (3.23)$$

where we ignored a multiplicative constant of order one in front of the last, cubic interaction term. The mass term is $\mu_{\bar{D}3} \sim e^{-y_0}/R_-$ for anti- $D3$ -brane, but for $D3$ -brane, $\mu_{D3} = 0$ within the present framework.

With a single throat containing both inflation and standard model, there should be a further hierarchy generating mechanism. One such possibility is to have supersymmetry unbroken to a much lower energy scale. For this, we must then consider leftover $D3$'s rather than anti- $D3$'s, for otherwise we would end up breaking supersymmetry at the inflation scale. The story of SM-brane phenomenology is

out of the scope of this paper; we just assume small but nonzero μ which is larger than TeV scale.

In fact, it is not even necessary to assume that the transverse scalar fields are the main decay channel for the KK modes into SM brane world. *The above three-point coupling is generic enough to work on any low-lying degrees of freedom on the SM brane world.* For instance, world volume fermions would also have such a 3-point coupling, and can absorb energy from KK modes directly. Since we are only interested in a rough estimate of the reheating process, we will work with the scalar Y and its specific form of coupling while keeping in mind that the result applies to all standard model sector. The interaction between ϕ_{KK} and Y has the general form

$$\frac{1}{\Lambda} \phi_{\text{KK}} (\partial \hat{Y})^2 \quad (3.24)$$

with

$$\frac{1}{\Lambda} = e^{y_0} \frac{\alpha'^2}{R_-^3} = \left(\frac{71 g_s^2 M^2}{32} \right)^{3/4} e^{y_0} \sqrt{\alpha'}. \quad (3.25)$$

This type of interaction is typical for interaction of the radion in the braneworlds scenarios [39]. The rate of decay of KK modes into Y is

$$\Gamma_{\text{KK}} = \frac{m_{\text{KK}}^3}{32\pi\Lambda^2} \sqrt{1 - \frac{4\mu^2}{m_{\text{KK}}^2}}. \quad (3.26)$$

For the anti- $D3$ brane we have a problem of decaying into Y particles, since $\mu_{\bar{D}3}$ and m_{KK} are comparable.

On the other hand the mass $\mu_{\bar{D}3}$ can be arranged to be smaller than m_{KK} , and therefore $D3$ -branes as the SM brane is preferable. We proceed assuming the timing of decay of KK modes is $\Delta t_3 = 1/\Gamma_{\text{KK}}$

$$\Delta t_3 = 32\pi \left(\frac{R_-}{\sqrt{\alpha'}} \right)^9 e^{y_0} \sqrt{\alpha'} = 32\pi \left(\frac{71 g_s^2 M^2}{32} \right)^{9/4} e^{y_0} \sqrt{\alpha'}. \quad (3.27)$$

If we adopt R_- to be larger than $\sqrt{\alpha'}$, then so far this is the longest process in the chain of energy transfer from inflaton field. As we encounter many times throughout the paper, once again our conclusion depends on a specific range of parameters. Our choice here is based on justification of computation in low curvature KS geometry. As we mentioned above, KK modes can also decay directly into SM fermions, with the rate of decay similar to (3.26).

Before proceeding with the decay open string modes into SM particle, we shall discuss separately a special case of KK modes, which can be long living and most dangerous for the whole scenario.

IV. THERMALIZATION AND DANGEROUS KK RELICS

In the previous section we consider decay of massive (but light) KK modes into excitations of the SM brane, and

²This is the same cancellation/doubling that is usually employed in deriving a slow role potential for a $D3$ -anti- $D3$ pair.

it was found that in the model whose radius at the tip of the KS throat is significantly larger than $\sqrt{\alpha'}$ (as expected in the supergravity description of KS geometry), decay time of KK modes (3.27) is significant.

KK modes are self-interacting. Therefore, let us check if KK modes are thermalized or not before they decay into open string modes. In this section we address thermalization of KK modes. Then we consider residual decay of open string modes into SM particles and thermalization of SM particles. Most importantly, we identify the problem of long-living KK modes, which is a serious problem for the string inflation scenarios.

A. Thermalization of KK modes

To check whether or not KK modes are thermalized, we have to compare the time of relaxation of KK modes towards their thermal equilibrium τ and time of their decay Δt_3 .

Relaxation time towards thermal equilibrium is

$$\tau \sim \frac{1}{n\sigma v}, \quad (4.1)$$

where n is the 3d number density of KK modes, σ is cross-section of their rescattering and v is their typical velocity. We have to estimate each factor in (4.1).

In principle, each factor in (4.1) depends on the expansion of the universe. We, however, argue that it can be ignored. Indeed, let us estimate the value of the Hubble parameter H immediately after brane-antibrane annihilation. From the Einstein equation

$$3H^2 \simeq \frac{1}{M_P^2} \frac{e^{-4y_0}}{4\pi^3 g_s \alpha'^2}, \quad (4.2)$$

where we use Eq. (3.4) for the energy density of the universe at the end of inflation. From here

$$H \sim \frac{e^{-2y_0}}{\sqrt{12\pi^3 g_s}} \frac{1/\sqrt{\alpha'}}{M_P} \frac{1}{\sqrt{\alpha'}}. \quad (4.3)$$

The inverse Hubble parameter, which is a typical time of expansion, is suppressed by the small factor e^{-2y_0} , while all time intervals of interaction including τ , as we will see below, are suppressed by factor e^{-y_0} only. Therefore here expansion can be ignored.

Number density can be estimated by the ratio of the total energy density ϵ and energy per KK mode. ϵ can be taken to be the energy density after the brane annihilation (3.4), and energy per KK mode to be comparable with the energy of closed string loops $e^{-y_0}/\sqrt{\alpha'}$, from which they originated. Thus,

$$n \sim \frac{e^{-3y_0}}{4\pi^3 g_s \alpha'^{3/2}}. \quad (4.4)$$

KK modes have masses $m_{\text{KK}} \sim \frac{e^{-y_0}}{R}$ while they are created

from closed strings of the mass $\frac{e^{-y_0}}{\sqrt{\alpha'}}$, so that KK modes are relativistic $v = 1$.

Interaction of KK modes can be derived in the following way. Again, take 10d action (3.6) with the ansatz $G = H^{-1/2}g + h$ as before in Sec. III B. The four-dimensional action, extended up to nonlinear terms with respect to h , contains the term $\frac{1}{M_P^2} \int d^{3+1}x \sqrt{-g}(h\nabla h)^2$. After decomposition $h = h_{\text{KS}} + \delta h$ it contains, in principle, three-legs and four-legs interactions, which are comparable. We take four-leg interaction $(\delta h \nabla \delta h)^2$. The cross-section of this interaction σ is the ratio of squares of coupling and energy, the coupling is $\frac{g_s^2 e^{2y_0} k^2}{\alpha'^2}$, thus coupling is rather strong $\sim g_s^2$. This is because KK modes are very energetic, $k \sim \frac{e^{-y_0}}{\sqrt{\alpha'}}$, so that momentum factors compensate the suppression by the local string mass. The estimate of relaxation time is

$$\tau \sim \frac{4\pi^3}{g_s} e_0^y \sqrt{\alpha'}. \quad (4.5)$$

The relaxation time of KK modes rescattering is much shorter than the time of decay of unstable KK modes Δt_3 . Therefore, we can treat all KK modes as particles in thermal equilibrium. We encounter a rather unusual situation in cosmology when *non-SM particles are set in the thermal equilibrium first, even before SM particles are produced!*

Therefore, the issue of reheat temperature is split into two issues: what is the reheat temperature of KK particles, and what is the reheat temperature of SM particles?

To calculate reheat temperature of KK particles T_{KK} , we have to convert the energy-density KK particles into thermal energy of KK plasma. Energy density of KK particles

$$\epsilon \simeq c \frac{M_P^2}{t^2}, \quad (4.6)$$

where numerical coefficient c depends on the equation of state, for the radiation equation of state of KK particles $c = \frac{3}{4}$. The end of inflation fixes the initial value of $t = t_0$ in (4.6). We have to compare (4.6) energy density (4.6) at the moment $t = t_0 + \tau$ with thermal energy of KK particles. Suppose KK particles are thermalized being relativistic. Then their energy is

$$\epsilon = \frac{\pi^2}{30} g_{\text{KK}} T_{\text{KK}}^4, \quad (4.7)$$

where g_{KK} is a number of degrees of freedom of KK particles.

A subtle point is that $t_0 \gg \tau$. Indeed, the end of inflation t_0 is defined by equalizing (4.6) with the energy-density of the original $D3 - \bar{D}3$ pair (3.4)

$$\epsilon = c \frac{M_P^2}{t_0^2} = \frac{e^{-4y_0}}{4\pi^3 g_s \alpha'^2}, \quad (4.8)$$

or similarly from $t_0 \sim 1/H$ with H from (4.3). We find

$$t_0 \sim \sqrt{4\pi^3 c g_s} e^{2y_0} \frac{M_P}{1/\sqrt{\alpha'}} \sqrt{\alpha'} \quad (4.9)$$

thus $t_0 \gg \tau$.

Therefore, comparing $c \frac{M_P^2}{t_0^2}$ with (4.7), we have

$$T_{\text{KK}} \sim \frac{e^{-y_0}}{g_s^{1/4} g_{\text{KK}}^{1/4}} \frac{1}{\sqrt{\alpha'}}. \quad (4.10)$$

Thus, reheat temperature of KK modes up to a factor $g_{\text{KK}}^{-1/4}$ is comparable with the inflaton energy scale.

It is instructive to compare T_{KK} with the mass of KK particles $m_{\text{KK}} \sim \frac{e^{-y_0}}{R_-}$. Unless g_{KK} is very large, the reheat temperature of KK particles is larger than their mass, so they are thermalized as relativistic particles. A new interesting element here is g_{KK} factor. As we have seen in Sec. III B, KK modes have energies up to $e^{-y_0}/g_s \sqrt{\alpha'}$. The number of degrees of KK modes g_{KK} of the mass m_{KK} is then proportional to $R_-/g_s \sqrt{\alpha'}$. However, formula (4.7) is valid only for relativistic KK modes, i.e. for modes with masses less than T_{KK} . For nonrelativistic KK modes we have to use formula $\epsilon_{\text{KK}} = m_{\text{MM}}, n_{\text{KK}}$ with $n_{\text{KK}} = \left(\frac{m_{\text{KK}} T}{2\pi}\right)^{3/2} e^{-m_{\text{KK}}/T}$.

B. Disappearance of open string relic

Among the product of decaying KK modes into the open string sector of the SM brane, there was the scalar Y , describing displacement of the SM brane. Specific field content of such scalars are dependent upon how SM is realized, but nevertheless we must worry about such particles as undesirable moduli fields in the cosmological sense. We need a mechanism to get rid of Y , even if such a field exists in the SM brane world. Fortunately, the gauge theory associated with the brane contains fermions ψ in the adjoint representation. Decay of ϕ_{KK} can go also into fermions; we are interested here in the troublemakers Y . The scalars Y interact with the fermions with the three-linear coupling $\sqrt{g_s} \bar{\psi} Y \psi$. Therefore, Y shall completely decay into lighter fermions and the rate of decay is

$$\Gamma(Y \rightarrow \bar{\psi}\psi) = \frac{g_s \mu_{D3}}{8\pi}. \quad (4.11)$$

Remember that the mass μ_{D3} here is introduced on the phenomenological ground beyond the brane inflation scenario. For μ_{D3} exceeding the TeV scale, cosmological constraints are satisfied with a large margin. Thus decay of the scalars Y by itself is not an essential process of the reheating, as long as SM phenomenology from string theory is successful.

C. Reheating of SM particles

Let us calculate the reheat temperature of the SM universe in the scenario. The longest process in the chain of energy transfer is the decay of KK modes into open string

modes given by time Δt_3 (3.27). Now we can compute the reheat temperature of the SM sector after the warped brane inflation.

For this we have to convert the energy density stored in the thermal radiation of KK modes

$$\epsilon_{\text{KK}} = \frac{3M_P^2}{4(t_0 + \Delta t_3)^2} \quad (4.12)$$

into thermal energy of SM plasma

$$\epsilon_{\text{SM}} = \frac{\pi^2}{30} g_* T^4. \quad (4.13)$$

Again, as in Sec. IVA, we shall compare t_0 from (4.9) with the decay time of KK modes Δt_3 from (3.27). The answer is model-dependent. In principle, R_- can be as small as the string scale, see e.g. discussion in [40]. In this case Δt_3 can be smaller than t_0 and estimation of SM particles temperature is similar to that of KK modes of Sec. IVA, just with replacement of g_{KK} . However, in the toy model of KS geometry in supergravity limit with $R_- \geq 10\sqrt{\alpha'}$, for inflation at energy scale 10^{14} GeV with $e^{-y_0} \sim 10^{-4}$, we have Δt_3 greater than t_0 by a large margin.

Energy transfer is completed when the Hubble parameter $H(t) \sim 1/t$ drops below the rate Γ_{KK} , i.e. when t in (4.13) is equal to Δt_3 . In (4.13) g_* is the number of SM degrees of freedom, $g_* \sim 100$. We have

$$T_r \approx 0.1 \sqrt{M_P \Gamma_{\text{KK}}} \sim 0.01 e^{-y_0/2} \left(\frac{\sqrt{\alpha'}}{R_-}\right)^{9/2} \sqrt{\frac{1/\sqrt{\alpha'}}{M_P}} M_P. \quad (4.14)$$

This estimation is not sensitive to the equation of state for KK modes; if they would have the matter equation of state, it changes only numerical coefficients in (4.12). Reheat temperature (4.14) is suppressed by two small factors, the redshift factor $e^{-y_0/2}$, and by the ratio of scales $\frac{\sqrt{\alpha'}}{R_-}$. Reheat temperature of the SM sector is lower than the reheat temperature (4.10) of KK particles

Suppose the energy scale of brane inflation at 10^{14} GeV, $e^{-y_0} \sim 10^{-4}$, and geometry is such that $R_- \sim 10\sqrt{\alpha'}$. We get $T_r \sim 10^7$ GeV. Now suppose the brane inflation is the low energy TeV scale inflation with $e^{-y_0} \sim 10^{-15}$, and $R_- \sim 10\sqrt{\alpha'}$. Then $T_r \sim 100$ GeV, which is cosmologically acceptable. If we choose however, $R_- \leq 100\sqrt{\alpha'}$, reheat temperature drops below the MeV scale, the temperature of BBN, which is unacceptable. Reheat temperature in the scenario is generally lower than that in QFT inflation. Therefore, we conclude that *the space of warped brane inflation is constrained from the lowest value of reheat temperature.*

D. Problem of long-living KK modes

Above we silently assumed that KK modes all decay into open string modes through three-legs interaction (3.24),

which make the decay complete. This, however, is not true for a special class of KK modes for which three-legs interactions like (3.24) are forbidden. Now we are coming to the critical problem of the whole scenario we identified so far, the problem of long-living KK modes. They are massive so that after universe expansion dilutes energy density of radiation they become dominant. Unless extreme fine-tuning, these particles will have unacceptably large energy density Ω_{KK} .

In the particular model of warped inflation with KS geometry, there are exact isometries S^3 and S^2 of six dimensional interior manifold X^6 at the bottom of the throat. Suppose more generally we have S^q isometry inside X^6 , and θ_A are angular coordinates of S^q . KK modes will be described by the harmonic expansion of rank two tensor with respect to the eigenmodes $\delta h^{(L)}(\theta_A)$

$$\nabla_A \nabla^A h^{(L)} = L^2 h^{(L)}, \quad (4.15)$$

(we drop δ from δh for transparency), with the Laplace operator on S^q in the right-hand side of the equation. The eigenmodes $\delta h^{(L)}$ are characterized by the set of conserved quantum numbers, associated with the conservation of angular momentum and its projections. For example, for the eigenmode of spin s and S^3 with isometry $\text{SO}(4) \simeq \text{SU}(2) \times \text{SU}(2)$, we have [41] $L^2 = (l+1)^2 - (s+1)$.

The amplitudes of interactions of KK modes include three-legs decay interactions with the factor $\int d\Omega_q h^{(L)}$, which vanishes. The only nonvanished amplitudes are the annihilation and inverse annihilation amplitudes $\int d\Omega_q h^{(L)} h^{(L)}$, which conserve the angular momentum.

In the KS geometry, around the bottom of the throat we have isometry of S^3 sphere with the radius R_- . As long as we place the SM brane at S^3 , the whole S^3 isometry is distorted by the presence of $D3$ brane, and some quantum numbers of the original $\text{SO}(3)$ isometry are not precisely conserved. However, residual $\text{SO}(2)$ isometry is intact. Another two directions have isometry of S^2 but its radius shrinks. This would blow up the corresponding KK mass, so that corresponding KK modes will stay away from the bottom.

Rescattering of KK modes with angular momentum brings them into the thermal equilibrium with the rest of KK sector. However, in an expanding universe the number density of KK modes with angular momentum freezes out. Indeed, massive KK particles sooner or later becomes nonrelativistic. Number density of nonrelativistic particles $n_{\text{KK}} = \left(\frac{m_{\text{KK}} T}{2\pi}\right)^{3/2} e^{-m_{\text{KK}}/T}$ is decreasing exponentially as temperature T diluted with expansion, rate of annihilation and inverse annihilation is exponentially decreasing, and when it becomes comparable with the expansion rate of the universe, the abundance of long-living KK modes freezes out.

The problem of KK modes with angular momentum in KS geometry has similarity with the problem of heavy KK modes in supergravity noted in [42].

Suppose the angular KK modes are stable. Then the figure of the primary interest would be the present day abundance Ω_{KK} of the stable KK modes. To estimate Ω_{KK} , we have to use the standard theory of freeze-out species in expanding universe, see e.g. [43]. In principle, particles may freeze-out being relativistic or nonrelativistic, depending on the details of the model. The simplest estimation of Ω_{KK} is for relativistic freeze-out. In this case we have

$$\Omega_{\text{KK}} \simeq 0.16 \frac{g_{st}}{g_{\text{KK}}} \frac{m_{\text{KK}}}{eV}, \quad (4.16)$$

where g_{st} and g_{KK} are the numbers of degrees of freedom of freeze-out stable KK modes and all KK modes correspondingly. The value of Ω_{KK} is of order 10^{22} for the 10^{14} GeV scale inflationary throat.

However, the isometries one finds in the KS throat are only approximate in the context of Calabi-Yau compactification, and therefore there is no absolutely stable KK angular modes. While a KS throat that extends infinitely to the UV region does have exact $\text{SU}(2) \times \text{SU}(2)$ isometry, the actual internal manifold involves cutting-off the UV region by a finite Calabi-Yau manifold. This has the effect of distorting the small r part of the KS throat and destroying the isometries. This then propagates toward the IR end of the throats in such a way that, even at the bottom of the throat, the angular momentum quantum numbers are not strictly conserved. In dual field theory language, the attachment to a compact Calabi-Yau induces certain global symmetry breaking perturbation. Reference [44] estimated the leading supersymmetry preserving operators to be of dimension 7.

This then translates to the typical width of the lowest angular momentum mode to be

$$m(m/M_P)^6 \quad (4.17)$$

or equivalently to lifetime of such long-living relic

$$\left(\frac{M_P}{m}\right)^7 \frac{1}{M_P}. \quad (4.18)$$

With an inflation scale m lower than 10^{12} GeV this could easily cause a problem. Since KK modes live longer than 100 sec, their energy release destroys BBN. If they live longer, then there is too much dark matter content.

Furthermore, this problem is potentially much more severe if the standard model is realized in another longer throat with such approximate isometries. As will be described in next section, energy transfer from the inflation throat to a standard model throat occurs via quantum processes of tunneling and oscillation, the other throat with standard model is likely to be of much lower energy scale. The energy transfer then will involve highly excited KK modes in the standard model throat and thus will produce relatively large amounts of the such angular KK modes. With a longer throat, the approximate isometry is

more protected, and the lifetime of such angular KK modes would be significantly longer.

We should warn the readers that, most likely, the existence of such approximate isometries is not generic. It is true that all known Sasaki-Einstein manifolds which could play the role of $T_{1,1}$ of KS throats are equipped with some angular isometries, if smaller than that of $T_{1,1}$. But this is probably the result of limited technology on our part in finding explicit examples. In dual field theory language, such global symmetries are not required supersymmetry in any intrinsic way. It is easy to envision that generic throats in type IIB flux compactification involve no such approximate isometries at all. Nevertheless, our present consideration should exclude a certain subset of such inflationary scenarios involving relatively symmetry throats. In particular, the face value model based on the KS throat has the problem of long-living angular KK modes.

V. ISSUES WITH MULTITHROAT SCENARIOS

So far we studied in some detail how remnant of D -brane annihilation cascade down to the standard model sector, assuming that a standard model is realized at the bottom of the inflation throat as a brane world. Apart from potential problems with long-lived relics associated with approximate isometries at the bottom of the throat, realistic reheating seems possible, although its detail differ from conventional reheating process. For one thing, the reheating process involves two distinct phases, where localized string modes and KK modes are first created and thermalized and then later this energy is transferred to open string sector and is thermalized at lower temperature.

The SM throat requires the choice of warping factor $e^{-y_0} \sim 10^{-15}$. On the other hand, the KKLMMT model suggests an inflationary throat to have $e^{-y_0} \sim 10^{-4}$ to have a right amplitude of cosmological fluctuations. This is one of the motivations to consider the multiple throat scenarios. This constraint can be relaxed if we admit another source of cosmological inhomogeneities related to the modulated cosmological fluctuations [45]. Notice also that the scale of inflation is tightly related to the SUSY-breaking scale, and the TeV scale gravitino requires low-scale inflation [46].

This single-throat scenario is certainly the simplest possibility, and some variant of it might work for the real world provided that supersymmetry generates further hierarchy from the inflation scale down to the TeV scale. On the other hand, with a single-throat scenario like this it is a little bit unclear how supersymmetry would be broken at the right scale and how to generate the small and positive cosmological constant which is observed in today's universe. For model building purposes, it gives us more room to consider flux compactification scenarios with more than one such throats. Here we comment on new issues in reheating such as a multithroat brane world.

One immediate fact is that there is never a thermal equilibrium between any such pair of throats, due to the

large redshift factors. Classically the localized degrees of freedom in any of the throats are effectively confined to the bottom of the throat, and can interact with those in another throat only via highly suppressed mixing operators. Wave functions of such localized particles of mass m may have exponentially small tails toward the top of the throat $(m/M_P)^d$ for some positive number d , so the interaction amplitude between two different throats should carry suppression $(m/M_P)^{2d}$. In turn, the associated probability goes like $(m/M_P)^{4d}$. Most crudely, the time scale of such interaction is then

$$\Delta t \sim \frac{1}{m} \left(\frac{M_P}{m} \right)^{4d}. \quad (5.1)$$

A hint on what the number d might be can be found in a work by Dimopoulos *et al.* [47,48], which estimated the tunneling rate from one Randall-Sundrum universe to another glued at the UV brane. The most optimistic estimate one finds there is $d = 1$.

We expect an even lower tunneling rate for angular modes. This extra suppression for angular modes can also be seen from the behavior of the wave function outside the throats as well. In between throats, that is, in the ‘‘big’’ CY volume, we can put the warping to be almost constant, $H(y) \sim H_0$, and the wave equation reads as

$$[\partial_y^2 + m_{\text{KK}}^2 R^2 H_0^{1/2} - L^2] \Phi_{m_{\text{KK}};L} = 0. \quad (5.2)$$

Thus, in between throats, the solution of the KK wave equation (2.19) is trivial, for $L = 0$ modes it is simply a slow sinusoidal form (in fact more or less a small constant since $m_{\text{KK}}R$ is very small), while for angular KK modes it is exponential in y , $\Phi_{m_{\text{KK}};L} \sim e^{\pm Ly}$. As the wave function traverses the classically forbidden region, it will be more strongly suppressed because of this behavior. For the actual decay exponent with smooth Calabi-Yau (rather than the singular boundary condition), we must do a more careful analysis of the whole wave function.

While the rate of decay of angular KK modes is seemingly thus more suppressed, this will probably lead to an overestimate of the lifetime of actual angular modes. One problem is that there is the possibility that higher angular modes will decay a pair of lower ones classically without having to tunnel. To settle this, one must study the lowest lying KK spectrum and L^2 dependence thereof. The other problem is that since the angular isometries are approximate, the dominant decay channel might be to change the angular momentum to a lower value within the same throat and then tunnel to another throat. For this, one must take into account perturbation of type that lead to (4.18). Thus, one should not take this estimation of the net decay rate too seriously for larger values of L^2 .

Let us return to the formula (5.1). With $d = 1$, this time scale is clearly much larger than $1/H \sim M_P/m^2$ at the exit of inflation, and thus the two throats are not in thermal equilibrium. While the $1/H$ increases with later evolution,

so does Δt at much faster rate. Any exchange of energy between two such throats is possible only via possible decay of heavy particles in one throat into lighter ones in the other.

This warns against one easy mistake in dealing with multithroat cases. In the early universe one often invokes the equipartition principle and assumes that energy is deposited in each and every degree of freedom in equal amount. A variant of this may be employed to argue that if throat 1 comes with many more degrees of freedom than throat 2, then more energy is deposited to throat 1. However, we are also familiar with the fact that some subsystem can be frozen away from thermal equilibrium and will evolve on its own if its interaction rate with the rest is much smaller than H . The above estimate of interaction between distinct throats tells us that such naive counting based on intuition from thermal equilibrium physics cannot be trusted.

One can make this a little more precise using simple quantum mechanics. Consider throat 0 which drove inflation and contains massive KK modes of scale m after branes annihilation, and a longer throat 1 with mass scale $\mu \ll m$. The dominant decay channel of heavy particle of mass m in throat 0 is found by realizing that, since the two throats are not completely disconnected, KK modes of throat 0 will mix and oscillate quantum mechanically with KK modes of throat 1 with similar mass, say $m' \sim m \gg \mu$. With a generic form of the mixing mass matrix,

$$H = \begin{pmatrix} m & \epsilon \\ \epsilon & m' \end{pmatrix} \quad (5.3)$$

we expect $\epsilon \sim m(m/M_p)^{2d} \ll m$. The oscillation of the initial state in throat 0 will induce amplitude of KK modes in throat 1 which is roughly

$$\langle 1 | e^{-iHt} | 0 \rangle \sim e^{-imt} \frac{\epsilon}{\Delta m} \sin(\Delta mt) \quad (5.4)$$

with $\Delta m \simeq m_1 - m_2$. Part of the quantum state oscillating into the throat 1 will then decay since there are many lighter states in throat 1. Naively one may think that having many decay channels in throat 1 is very helpful in transferring energy into throat 1 from throat 0, since there is a lot of phase volume in throat 1.

However, to maintain consistency, the above mass matrix must be modified to reflect the decay width of the state in throat 1.

$$\tilde{H} = \begin{pmatrix} m & \epsilon \\ \epsilon & m' - i\Gamma \end{pmatrix} \quad (5.5)$$

With N number of states available as decay products in throat 1, the width should scale like $\Gamma \sim \lambda m' N$ with a small but finite coupling constant λ . With the new \tilde{H} that has the decay process built in, we can estimate how fast the initial state in throat 0 loses its amplitude,

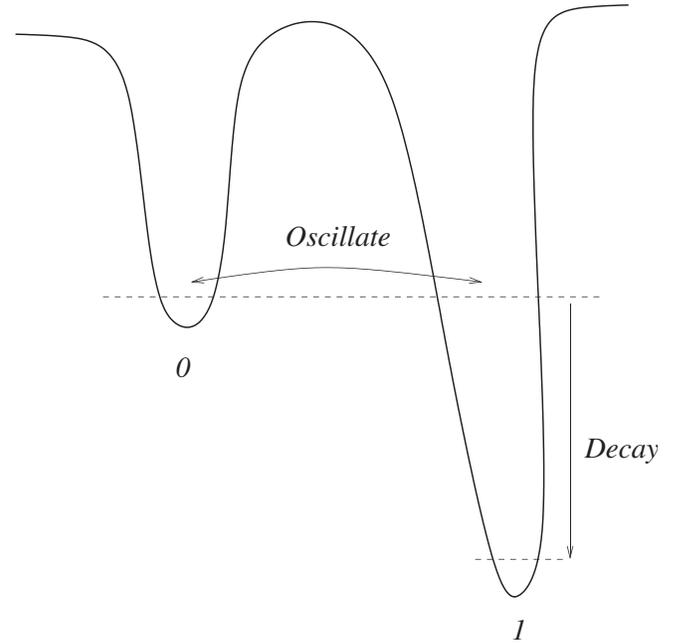


FIG. 3. KK modes in the inflation throat can decay to another throat via quantum oscillation. While the oscillation amplitude depends on mass eigenvalue distribution in the 2nd throat, it will also be suppressed further if the state in throat 1 has a large width.

$$e^{-i\tilde{H}t} | 0 \rangle \sim e^{-\tilde{\Gamma}t} e^{-imt} \left((1 - O(\epsilon^2)) | 0 \rangle + \frac{\epsilon}{\Gamma - i\Delta m} \times \sin \Delta mt | 1 \rangle \right), \quad (5.6)$$

where the effective width

$$\tilde{\Gamma} \equiv \frac{\epsilon^2}{\Delta m^2 + \Gamma^2} \Gamma \quad (5.7)$$

of the state $| 0 \rangle$ due to the oscillation is actually suppressed as the width Γ of state $| 1 \rangle$ increases. See Fig. 3

While somewhat counterintuitive, this can be understood from the basics of the quantum oscillation. The two states mix with each other well if they are almost degenerate while the mixing is suppressed by the mass difference. What this simple computation tells us is that the suppression depends on difference in the complex mass. Another way to see this is that the decay of state $| 0 \rangle$ occurs at second order in the perturbation theory. It must first oscillate into state $| 1 \rangle$ and experience the decay width Γ and then come back to $| 0 \rangle$. The standard 2nd order perturbation due to the mixing $H_{01} = \epsilon$ gives

$$\text{Im} \frac{\langle 0 | \tilde{H}_{01} | 1 \rangle \langle 1 | \tilde{H}_{10} | 0 \rangle}{\tilde{H}_{11} - \tilde{H}_{00}} \quad (5.8)$$

as the width, which is exactly $\tilde{\Gamma}$.

Finally, we must take into account that there are also roughly N number of states that can be used for such

oscillation, so the net width of state $|0\rangle$ is more like

$$N\tilde{\Gamma}. \quad (5.9)$$

Assuming large N , so that Δm is smaller than Γ , we find

$$N\tilde{\Gamma} \sim \frac{\epsilon^2}{\Gamma} N \sim \frac{\epsilon^2}{\lambda m} \quad (5.10)$$

which shows that, to leading approximation, the effect of having a large number of state in the low energy throat completely washes out.³ Any throat that is significantly longer than the inflation throat would receive energy as dictated by the mixing of typical KK wave functions, irrespective of how many decay channels each comes with. In particular, having a large number of open string decay channels in throat 1 will make matters worse, since these will contribute to Γ but not to the mixing.

Such a universal nature of decay into different throats will make the reheating process quite delicate. For instance, suppose the compact manifold involves three different throats. We will label them as 0 the inflation throat, 1 the SUSY-breaking throat, and 2 the standard model throat, with the mass scales $m \gg \mu_1 \gg \mu_2$, possibly with large number of additional open string degrees of freedom in throat 2. The above consideration tells us that energy deposits into throat 1 and throat 2 are largely determined by each mixing mass matrix element ϵ 's and initial energy level m , and independent of $\mu_{1,2}$. This would result in a significant deposit of energy in throat 1 as well as in throat 2, provided that the process is fast enough. In terms of (5.1), $d = 1$ with conventional inflation scale would suffice.

With μ_1 at some intermediate scale, furthermore, whatever energy deposited there will behave as massive particles as the universe expands and cools down. Compared to the radiations in throat 2 (mostly photons and other light standard model degrees of freedom), the density of such massive particles dilute rather slowly. Even a small amount of deposit in throat 2 will quickly overtake and dominate energy density of the universe rendering reasonable BBN impossible. While energy deposited in throat 1 will eventually decay to throat 2, its decay width would be dictated by $(\mu_1/M_P)^{4d}$, which is much smaller than $(m/M_P)^{4d}$. Relics in throat 1 would be very difficult to remove. We believe that this would cause a very serious problem whenever we build models based on KKLMMT-like inflation with warp factors playing crucial roles in hierarchy, determination of cosmological constant, and low scale of supersymmetry breaking. See Fig. 4

³This estimate would not apply to a relatively stable state of lower mass $\sim \mu$ in throat 1 at its bottom. However, the corresponding ϵ would be far more suppressed at $\sim (m\mu/M_P^2)^d$, since the state $|1\rangle$ in that case would be living at the bottom of throat 1.

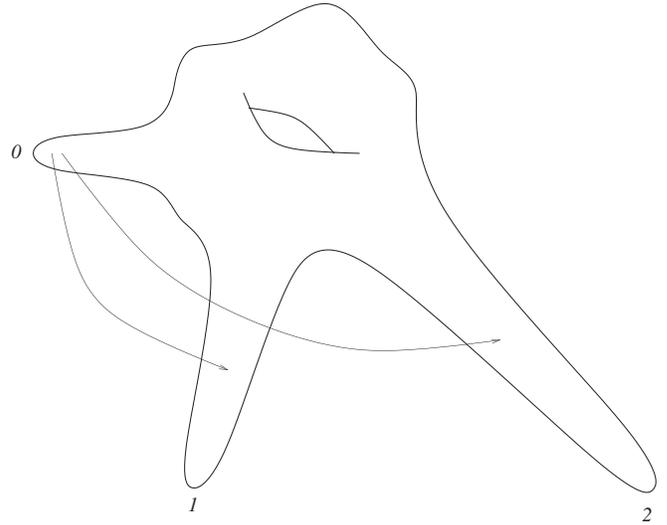


FIG. 4. KK modes in the inflation throat deposit energy to lower energy throats. The branching ratio will be largely determined by how throats are distributed in the internal manifold with respect to the inflation throat and less sensitive to the field content of each throat. Energy deposited in throat 1 will later decay to throat 2, but at much more suppressed rate because its mass scale is far lower than that of the inflation throat.

Notice that this problem with low-lying KK modes is independent from the angular KK modes. While the problem with angular KK modes is there only if the base manifold of the throat happens to be quite symmetric, this problem is much more generic, provided that there are some intermediate-length throats containing a hidden sector.

VI. SUMMARY

It is fair to say that string theory inflation is at a crossroads. There are several potential directions for building realistic inflationary models. The endpoint of all inflationary models is the reheating of the universe. Successful reheating means almost a complete conversion of inflaton energy into thermalized radiation without any dangerous relics, in order to provide a thermal history of the universe compatible with big bang nucleosynthesis (BBN), baryo/leptogenesis, and other observations. All of this provides us with constraints on string theory inflationary scenarios.

In this paper we investigated reheating after brane-antibrane annihilation. The starting point was the KKLMMT model of warped brane inflation with a background KS geometry. Control of the calculations is possible in the regime of low curvature KS geometry with all scales (like the radius at the bottom of the throat) larger than the string scale. Apart from the justification of computation, we do not have much reason to believe these constraints. In fact, for large R_+/R_- to generate hierarchy, one may prefer R_- nearer to the string scale [40]. KS geometry also admits isometries of the inner manifold.

Another important parameter is the warping factor, which defines the energy scale of inflation. The single-throat scenario with low energy inflation just above the SM scale is the simplest possibility if there are alternative mechanisms to generate cosmological fluctuations (like modulated fluctuations). Otherwise we can have a multiple-throats scenario with inflation and the SM sector located in different energy scale throats. Thus, the models bifurcate into single-throat or multiple-throat scenarios, with different ranges of parameters to stay in or out of low curvature geometry.

We followed the way the energy of annihilated branes transfers into SM radiation. The first step after annihilation is the excitation of closed string loops, located near the bottom of the throat, where the local string mass scale is reduced by the warp factor e^{-y_0} . They quickly decay into 10d gravitons (and particles in their supermultiplets). From a four-dimensional perspective, 10d gravitons are manifested as usual 4d gravitational radiation and massive KK modes.

The generation of 4d gravitational radiation is a universal prediction of string theory brane inflation. In principle, it is a source of concern because BBN excludes more than a few percent of radiation in forms other than photons and 3 light neutrino species. Massless radiation relics today are photons with the fraction of the total energy density $\Omega_\gamma \simeq 5 \times 10^{-5}$ and very light neutrinos with $\Omega_\nu \simeq 1.6 \times 10^{-2}$. Successful BBN potentially allows an extra few percent of energy density in light species, for instance background relic gravitons. Hence the energy density of relic gravitons cannot exceed the limit $\Omega_{\text{GW}} \leq 5 \times 10^{-6}$. For instance, in brane inflation models without warping and just a pair of $D3 - \bar{D}3$ branes, overproduction of gravitons is a problem, which can be cured by annihilation of $\bar{D}3$ with a stack of N $D3$ -branes to dilute the abundance of gravitons by $1/N$. In warped geometry, the situation with gravitons is very different. The decay rate of closed loops into gravitons (i.e. homogeneous KK modes) is suppressed by M_p , while decay into KK modes is suppressed by the local string mass $e^{-y_0}/\sqrt{\alpha'}$ at the bottom of the throat, which is much larger than M_p . As a result, the fraction of energy deposited into gravitons is reduced by the factor e^{-2y_0} . For inflation at 10^{14} GeV this is $e^{-2y_0} \sim 10^{-8}$. We therefore conclude that the warped brane inflation scenario predicts a residual background of relic gravitational radiation at the level $\Omega_{\text{GW}} \sim e^{-2y_0}$. The wavelengths are located around the energy scale of closed loop excitations after inflation, redshifted by the expansion of the universe, and up to some numerical factors close to that of the relic photons, around mm scale.

The bulk of the energy after the decay of closed string loops is in KK modes. They interact and reach thermal equilibrium with a temperature comparable with the infla-

tion energy scale. A much more serious concern comes from the angular KK modes. Indeed, the 6d compact manifold in the KS throat geometry has isometry, near the bottom $S^3 \times S^2$ isometry. Conservation of angular momentum associated with these isometries forbids complete decay of angular KK modes, but allows only their annihilation. As a result, a residual amount of massive angular KK modes freezes out in an expanding universe, posing a problem for the scenario.

In the context of CY compactification isometries are only approximate, which allows the angular KK modes to decay. However, the width of their decay is suppressed by the factor $(m/M_p)^7$. While this gives a fast enough decay time for angular KK modes in 10^{14} GeV inflation, it is too long a time for angular KK modes in the SM throat.

In fact, there are two independent KK mode problems. One has to do with the approximately conserved angular momentum which we just discussed. This is there for both the single and multiple throat cases as long as a long throat has an approximate isometry. The other problem comes from a low-lying KK mode created in some intermediate throat (low-scale inflation throat or SUSY-breaking hidden-sector throat) in multithroat scenarios. Their decay is via tunneling only, and is thus suppressed. This suppression has little to do with any isometry and is thus more problematic, at least for generic multithroat cases.

It is an interesting exercise to calculate the cosmological abundance of residual KK modes, which we considered only briefly for the single-throat case in Sec. IV. Let us mention the cosmological relevance of the issue. The energy density of massive relics Ω_{KK} dilutes slower than that of radiation, therefore Ω_{KK} sooner or later begins to dominate the expansion of the universe. Special conditions are needed to tune Ω_{KK} to be the cold dark matter component, to constitute today $\Omega_{\text{CDM}} \simeq 0.27$. If KK particles are abundant in the very early universe it becomes matter dominated too early to allow successful BBN. This is a typical situation with SUSY moduli fields and gravitinos. A similar problem is inherited by simple versions of rolling tachyon inflation [49]. On the other hand, it is fair to keep a possibility that Ω_{KK} is a candidate for the cold dark matter, if KK particles are long-lived and are created in exactly the right amount [50].

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