Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies

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We obtain the *effective* inflaton potential during slow-roll inflation by including the one-loop quantum corrections to the energy momentum tensor from scalar curvature and tensor perturbations as well as from light scalars and Dirac fermions coupled to the inflaton. During slow-roll inflation there is an unambiguous separation between super- and subhorizon contributions to the energy momentum tensor. The superhorizon part is determined by the curvature perturbations and scalar field fluctuations: both feature infrared enhancements as the inverse of a combination of slow-roll parameters which measure the departure from scale invariance in each case. Fermions and gravitons do not exhibit infrared divergences. The subhorizon part is completely specified by the *trace anomaly* of the fields with different spins and is solely determined by the space-time geometry. The one-loop corrections to the amplitude of curvature and tensor perturbations are obtained to leading order in slow roll and in the $(H/M_{\rm Pl})^2$ expansion. A complete assessment of the backreaction problem up to one loop including bosons and fermions is provided. The result validates the effective field theory description of inflation and confirms the robustness of the inflationary paradigm to quantum fluctuations. Quantum corrections to the power spectra are expressed in terms of the CMB observables: n_s, r and $dn_s/d\ln k$. Trace anomalies (especially the graviton part) dominate these quantum corrections in a definite direction: they *enhance* the scalar curvature fluctuations and *reduce* the tensor fluctuations.

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I. INTRODUCTION

Inflation is a central part of early universe cosmology passing many observational tests and becoming a predictive scenario scrutinized by current and forthcoming observations. Inflation was introduced to solve several shortcomings of the standard big bang cosmology [1–7]. It provides a mechanism for generating scalar (density) and tensor (gravitational wave) perturbations [8–13]. A distinct aspect of inflationary perturbations is that these are generated by quantum fluctuations of the scalar field(s) that drive inflation. After their wavelength becomes larger than the Hubble radius, these fluctuations are amplified and grow, becoming classical and decoupling from causal microphysical processes. Upon reentering the horizon, during the matter era, these classical perturbations seed the inhomogeneities which generate structure upon gravitational collapse [8–13]. A great diversity of inflationary models predicts fairly generic features: a Gaussian, nearly scale invariant spectrum of (mostly) adiabatic scalar and tensor primordial fluctuations, making the inflationary paradigm fairly robust. The Gaussian, adiabatic and nearly scale invariant spectrum of primordial fluctuations provide an excellent fit to the highly precise wealth of data provided by the Wilkinson microwave anisotropy probe (WMAP) [14–17]. Perhaps the most striking validation of inflation as a mechanism for generating superhorizon ("acausal") fluctuations is the anticorrelation peak in the temperature-polarization (TE) angular power spectrum at $l \sim 150$ corresponding to superhorizon scales [16,17].

The confirmation of many of the robust predictions of inflation by current high precision observations places inflationary cosmology on solid grounds. Forthcoming observations will begin to discriminate among different inflationary models, placing stringent constraints on them. There are small but important telltale discriminants among different models: non-Gaussianity, a running spectral index for scalar and tensor perturbations, an isocurvature component for scalar perturbations, the ratios for the amplitudes between tensor and scalar modes, etc. Already WMAP reports a hint of deviations from constant scaling exponents (running spectral index) and rules out the purely monomial Φ^4 potential [17].

Among the wide variety of inflationary scenarios, single field slow-roll models [18,19] provide an appealing, simple and fairly generic description of inflation. Its simplest implementation is based on a scalar field (the inflaton) whose homogeneous expectation value drives the dynamics of the scale factor, plus small quantum fluctuations. The inflaton potential is fairly flat during inflation. This flatness not only leads to a slowly varying Hubble parameter, hence ensuring a sufficient number of e folds, but also provides

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an explanation for the Gaussianity of the fluctuations as well as for the (almost) scale invariance of their power spectrum. A flat potential precludes large nonlinearities in the dynamics of the fluctuations of the scalar field. The current WMAP data seem to validate the simpler one-field slow-roll scenario [17]. Furthermore, because the potential is flat the scalar field is almost massless, and modes cross the horizon with an amplitude proportional to the Hubble parameter. This fact combined with a slowly varying Hubble parameter yields an almost scale invariant primordial power spectrum. Upon crossing the horizon the phases of the quantum fluctuations freeze out and a growing mode dominates the dynamics, i.e. the quantum fluctuations become classical (see Ref. [6] and references therein). Departures from scale invariance and Gaussianity are determined by the departures from flatness of the potential, namely, by derivatives of the potential with respect to the inflaton. These derivatives can be combined into a hierarchy of dimensionless slow-roll parameters [18] that allow an assessment of the corrections to the basic predictions of Gaussianity and scale invariance [6]. The slow-roll approximation has been recently cast as a $1/N_{e\text{ folds}}$ expansion [20], where $N_{e\text{ folds}}$ is the number of e folds before the end of inflation when modes of cosmological relevance today first crossed the Hubble radius.

The basic scenario of inflation driven by a scalar field must be interpreted as an effective field theory (EFT) [21] resulting from integrating out heavy degrees of freedom. In particular, in the effective field theory description, the classical scalar potential that determines the dynamics of the inflaton results from integrating out degrees of freedom much heavier than the scale of inflation.

Forthcoming observations have the potential of measuring the inflationary potential at least within a span in field amplitude corresponding to the 8–10 e folds during which wavelengths of cosmological relevance first cross the Hubble radius [22]. These observations will measure the full inflaton potential including all possible quantum corrections and not just the classical (tree level) potential. This possibility motivates us to assess the quantum corrections to the inflationary potential from fields lighter than the inflaton, since in the effective field theory description, the classical inflaton potential already includes contributions from heavier fields. We focus on light fields since these can exhibit infrared enhanced contributions to the effective potential as discussed in Refs. [23,24].

Our goal is to obtain the effective potential that includes the one-loop quantum corrections from fields that are light during the relevant inflationary stage.

Our program of study focuses on the understanding of quantum aspects of the basic inflationary paradigm. In previous studies we addressed the decay of inflaton fluctuations [23] and more recently [24] we focused on the quantum corrections to the equations of motion of the inflaton and the scalar fluctuations during slow-roll inflation, from integrating out not only the inflaton fluctuations but also the excitations associated with another scalar field. Since the power spectra of fields with masses $m \ll H$ are nearly scale invariant, strong infrared enhancements appear as revealed in these studies [23,24]. In addition, we find that a particular combination of slow-roll parameters which measures the departure from scale invariance of the fluctuations provides a natural infrared regularization.

The small parameter that determines the validity of inflation as an effective quantum field theory below the Planck scale is $H/M_{\rm Pl}$ where H is the Hubble parameter during inflation and therefore the scale at which inflation occurs. The slow-roll expansion is in a very well-defined sense an adiabatic approximation since the time evolution of the inflaton field is slow on the expansion scale. Thus the small dimensionless ratio $H/M_{\rm Pl}$, which is required for the validity of an EFT, is logically independent from the small dimensionless combinations of derivatives of the potential which ensure the validity of the slow-roll expansion. Present data [17] indicate a very small amplitude of tensor perturbations which is consistent with $H/M_{\rm Pl} \ll 1$.

Therefore, in this article we will invoke two independent approximations, the EFT and the slow-roll approximation. The former is defined in terms of an expansion in the ratio $H/M_{\rm Pl}$, whereas the latter corresponds to an expansion in the (small) slow-roll parameters which has recently been identified with an expansion in $1/N_{e\text{ folds}}$ [20].

It is important to highlight the main differences between slow-roll inflation and the postinflationary stage. During slow-roll inflation the dynamics of the scalar field is slow on the time scale of the expansion and consequently the change in the amplitude of the inflaton is small and quantified by the slow-roll parameters. The slow-roll approximation is indeed an adiabatic approximation. In striking contrast to this situation, during the postinflationary stage of reheating the scalar field undergoes rapid and large amplitude oscillations that cannot be studied in a perturbative expansion [25,26].

Brief summary of results.—We obtain the quantum corrections to the inflaton potential up to one loop by including the contributions from scalar and tensor perturbations of the metric as well as one light scalar and one light fermion field coupled generically to the inflaton. Therefore this study provides the most complete assessment of the general backreaction problem up to one loop that includes not only metric perturbations, but also the contributions from fluctuations of other light fields with a generic treatment of both bosonic and fermionic degrees of freedom. Motivated by an assessment of the quantum fluctuations that could be of observational interest, we focus on studying the effective inflaton potential during the cosmologically relevant stage of slow-roll inflation.

Both light bosonic fields as well as scalar density perturbations feature an infrared enhancement of their quantum corrections which is regularized by slow-roll parameters. Fermionic contributions as expected do not feature any infrared enhancement and neither does the graviton contribution to the energy momentum tensor. We find that in slow roll and for light bosonic and fermionic fields there is a clean separation between the super- and subhorizon contributions to the quantum corrections from scalar density metric and light bosonic field perturbations. For these fields the superhorizon contribution is of zero order in slow roll as a consequence of the infrared enhancement regularized by slow-roll parameters. The subhorizon contribution to the energy momentum tensor from all the fields is completely determined by the trace anomaly of minimally coupled scalars, gravitons and fermionic fields. We find the one-loop effective potential to be

$$V_{\rm eff}(\Phi_0) = V(\Phi_0)\left[1 + \frac{H_0^2}{3(4\pi)^2 M_{\rm Pl}^2}\left(\frac{\eta_v - 4\epsilon_v}{\eta_v - 3\epsilon_v} + \frac{3\eta_\sigma}{\eta_\sigma - \epsilon_v} + \mathcal{T}\right)\right] \quad (1)$$

where $V(\Phi_0)$ is the classical inflaton potential, η_v, ϵ_v, η_σ slow-roll parameters and $\mathcal{T} = \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_\Psi = -\frac{2903}{20} = -145.15$ is the total trace anomaly from the scalar metric, tensor, light scalar and fermion contributions.

The terms that feature ratios of slow-roll parameters arise from superhorizon contributions from curvature and scalar field perturbations. The last term in Eq. (1) is independent of slow-roll parameters and is completely determined by the trace anomalies of the different fields. It is the hallmark of the subhorizon contributions.

In the case when the mass of the light bosonic scalar field is much smaller than the mass of the inflaton fluctuations, we find the following result for the scalar curvature and tensor fluctuations including the one-loop quantum corrections:

$$|\Delta^{(S)}_{k,\rm eff}|^2 = |\Delta^{(S)}_k|^2\left\{1 + \frac{2}{3}\left(\frac{H_0}{4\pi M_{\rm Pl}}\right)^2 \times \left[1 + \frac{\frac{3}{8}r(n_s-1) + 2\frac{dn_s}{d\ln k}}{(n_s-1)^2} + \frac{2903}{40}\right]\right\},$$

$$|\Delta^{(T)}_{k,\rm eff}|^2 = |\Delta^{(T)}_k|^2\left\{1 - \frac{1}{3}\left(\frac{H_0}{4\pi M_{\rm Pl}}\right)^2 \times \left[-1 + \frac{1}{8}\frac{r}{n_s-1} + \frac{2903}{20}\right]\right\},$$

$$r_{\rm eff} \equiv \frac{|\Delta^{(T)}_{k,\rm eff}|^2}{|\Delta^{(S)}_{k,\rm eff}|^2} = r\left\{1 - \frac{1}{3}\left(\frac{H_0}{4\pi M_{\rm Pl}}\right)^2 \times \left[1 + \frac{\frac{3}{8}r(n_s-1) + \frac{dn_s}{d\ln k}}{(n_s-1)^2} + \frac{8709}{20}\right]\right\}. \quad (2)$$

The quantum corrections turn out to enhance the scalar curvature fluctuations and to reduce the tensor fluctuations as well as their ratio r. The quantum corrections are always small, of the order $(H_0/M_{\rm Pl})^2$, but it is interesting to see that these quantum effects are dominated by the trace anomalies and they correct both scalar and tensor fluctuations in a definite direction. Moreover, it is the tensor part of the trace anomaly which numerically yields the largest contribution.

Quantum trace (conformal) anomalies of the energy momentum tensor in gravitational fields constitute an important aspect of quantum field theory in curved backgrounds, (see for example [27] and references therein). In black hole backgrounds they are related to the Hawking radiation. It is interesting to see here that the trace anomalies appear in a relevant cosmological problem and dominate the quantum corrections to the primordial spectrum of curvature and tensor fluctuations.

In Sec. II we compute the effective potential including scalars, gravitons and fermionic fields, in Sec. III we present the quantum corrections to scalar curvature and tensor fluctuations, and in Sec. IV we present our conclusions.

II. THE EFFECTIVE POTENTIAL

In our recent calculation of the quantum corrections to the effective potential [23] different expansions appear: the expansion in the effective field theory ratio $H_0/M_{\rm Pl}$ where H_0 is the Hubble parameter during the relevant stage of inflation, and the expansion in slow-roll parameters. These expansions are logically different: the slow-roll expansion is an adiabatic expansion in the sense that the dynamics of the inflaton is slower than the universe expansion, while the (dimensionless) interaction vertices and the loop expansion are determined by the effective field theory parameter $H_0/M_{\rm Pl}$ [23].

During slow-roll inflation, the dynamics of the scale factor and the inflaton are determined by the following set of (semi) classical equations of motion:

$$H_0^2 = \frac{1}{3M_{\rm Pl}^2}\left[\frac{1}{2}(\dot{\Phi}_0)^2 + V(\Phi_0)\right], \quad (3)$$

$$\ddot{\Phi}_0 + 3H_0\dot{\Phi}_0 + V'(\Phi_0) = 0, \quad (4)$$

where $M_{\rm Pl} = 1/\sqrt{8\pi G} = 2.4 \times 10^{18}$ GeV. Slow-roll inflation is tantamount to the statement that the dynamics of the expectation value of the scalar field Φ_0 is slow on the scale of the cosmological expansion. The slow-roll approximation is indeed an adiabatic approximation in terms of a hierarchy of small dimensionless quantities related to the derivatives of the inflaton potential. Some [6,18] of these slow-roll parameters are given by[1]

[1]We follow the definitions of $\xi_V; \sigma_V$ in Ref. [17] ($\xi_V; \sigma_V$ are called $\xi_V^2; \sigma_V^3$, respectively, in [18]).

$$\epsilon_V = \frac{M_{\rm Pl}^2}{2}\left[\frac{V'(\Phi_0)}{V(\Phi_0)}\right]^2, \quad \eta_V = M_{\rm Pl}^2\frac{V''(\Phi_0)}{V(\Phi_0)},$$
$$\xi_V = M_{\rm Pl}^4\frac{V'(\Phi_0)V'''(\Phi_0)}{V^2(\Phi_0)}, \quad \sigma_V = M_{\rm Pl}^6\frac{[V'(\Phi_0)]^2V^{(IV)}(\Phi_0)}{V^3(\Phi_0)}. \tag{5}$$

The slow-roll approximation [6,18,22] corresponds to $\epsilon_V \sim \eta_V \ll 1$ with the hierarchy $\xi_V \sim \mathcal{O}(\epsilon_V^2)$; $\sigma_V \sim \mathcal{O}(\epsilon_V^3)$, namely ϵ_V and η_V are first order in slow roll, ξ_V second order in slow roll, etc. Recently a correspondence between the slow-roll expansion and an expansion in $1/N_{e\,\text{folds}}$ has been established [20] with ϵ_V, $\eta_V \sim 1/N_{e\,\text{folds}}$; $\xi_V \sim 1/N_{e\,\text{folds}}^2$; $\sigma_V \sim 1/N_{e\,\text{folds}}^3$, etc.

During slow-roll inflation the equations of motion (3) and (4) are approximated by

$$\dot\Phi_0 = -\frac{V'(\Phi_0)}{3H_0} + \text{higher order in slow roll},$$
$$H_0^2 = \frac{V(\Phi_0)}{3M_{\rm Pl}^2}\left[1 + \frac{\epsilon_V}{3} + \mathcal{O}(\epsilon_V^2, \epsilon_V\eta_V)\right]. \tag{6}$$

The scale factor is given by

$$C(\eta) = -\frac{1}{H\eta(1-\epsilon_V)}. \tag{7}$$

In the effective field theory interpretation of inflation, the classical inflaton potential $V(\Phi)$ should be understood to include the contribution from integrating out fields with masses much larger than H_0. Our goal is to obtain the one-loop quantum corrections from fields that are light during inflation. Therefore we consider that the inflaton is coupled to a light scalar field σ and to Fermi fields with a generic Yukawa-type coupling. We take the fermions to be Dirac fields but it is straightforward to generalize to Weyl or Majorana fermions. We also include the contribution to the effective potential from scalar and tensor metric perturbations, thereby considering their backreaction up to one loop.

The Lagrangian density is taken to be

$$\mathcal{L} = \sqrt{-g}\Big\{\frac{1}{2}\dot\varphi^2 - \left(\frac{\vec\nabla\varphi}{2a}\right)^2 - V(\varphi) + \frac{1}{2}\dot\sigma^2 - \left(\frac{\vec\nabla\sigma}{2a}\right)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - G(\varphi)\sigma^2 + \overline\Psi[i\gamma^\mu\mathcal{D}_\mu\Psi - m_f - Y(\varphi)]\Psi\Big\} \tag{8}$$

where $G(\Phi)$ and $Y(\Phi)$ are generic interaction terms between the inflaton and the scalar and fermionic fields. Obviously this Lagrangian can be further generalized to include a multiplet of scalar and fermionic fields and such case can be analyzed as a straightforward generalization. For simplicity we consider one bosonic and one fermionic Dirac field.

The Dirac γ^μ are the curved space-time γ matrices and the fermionic covariant derivative is given by [27–30]

$$\mathcal{D}_\mu = \partial_\mu + \tfrac{1}{8}[\gamma^c, \gamma^d]V_c^\nu(D_\mu V_{d\nu}),$$
$$D_\mu V_{d\nu} = \partial_\mu V_{d\nu} - \Gamma^\lambda_{\mu\nu}V_{d\lambda}$$

where the vierbein field is defined as $g^{\mu\nu} = V_a^\mu V_b^\nu \eta^{ab}$, η_{ab} is the Minkowski space-time metric and the curved space-time matrices γ^μ are given in terms of the Minkowski space-time ones γ^a by (Greek indices refer to curved space-time coordinates and Latin indices to the local Minkowski space-time coordinates)

$$\gamma^\mu = \gamma^a V_a^\mu, \qquad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}.$$

We will consider that the light scalar field σ has a vanishing expectation value at all times, therefore inflationary dynamics is driven by one single scalar field, the inflaton ϕ. We now separate the homogeneous expectation value of the inflaton field from its quantum fluctuations as usual by writing

$$\varphi(\vec x, t) = \Phi_0(t) + \delta\varphi(\vec x, t).$$

We will consider the contributions from the quadratic fluctuations to the energy momentum tensor. There are four distinct contributions: (i) scalar metric (density) perturbations, (ii) tensor (gravitational waves) perturbations, (iii) fluctuations of the light bosonic scalar field σ, and (iv) fluctuations of the light fermionic field Ψ.

Fluctuations in the metric are studied as usual [8,13,31–33]. Writing the metric as

$$g_{\mu\nu} = g^0_{\mu\nu} + \delta^s g_{\mu\nu} + \delta^t g_{\mu\nu}$$

where $g^0_{\mu\nu}$ is the spatially flat Friedmann-Robertson-Walker (FRW) background metric which in conformal time is given by

$$g^0_{\mu\nu} = C^2(\eta)\eta_{\mu\nu}, \qquad C(\eta) \equiv a(t(\eta))$$

and $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is the flat Minkowski space-time metric. $\delta^{s,t}g_{\mu\nu}$ correspond to the scalar and tensor perturbations, respectively, and we neglect vector perturbations. In longitudinal gauge

$$\delta^s g_{00} = C^2(\eta)2\phi, \qquad \delta^s g_{ij} = C^2(\eta)2\psi\delta_{ij},$$
$$\delta^t g_{ij} = -C^2(\eta)h_{ij} \tag{9}$$

where h_{ij} is transverse and traceless and we neglect vector modes since they are not generated in single field inflation [8,13,31–33].

Gauge invariant variables associated with the fluctuations of the scalar field and the potentials ϕ, ψ are constructed explicitly in Ref. [13] where the reader can find their expressions. Expanding up to quadratic order in the scalar fields, fermionic fields and metric perturbations the part of the Lagrangian density that is quadratic in these fields is given by

$$\mathcal{L}_Q = \mathcal{L}_s[\delta\varphi^{gi}, \phi^{gi}, \psi^{gi}] + \mathcal{L}_t[h] + \mathcal{L}_\sigma[\sigma] + \mathcal{L}_\Psi[\overline{\Psi}, \Psi],$$

where

$$\mathcal{L}_t[h] = \frac{M_{\rm Pl}^2}{8} C^2(\eta)\partial_\alpha h_i^j \partial_\beta h_j^i \eta^{\alpha\beta},$$

$$\mathcal{L}_\sigma[\sigma] = C^4(\eta)\left\{\frac{1}{2}\left(\frac{\sigma'}{C}\right)^2 - \frac{1}{2}\left(\frac{\nabla\sigma}{C}\right)^2 - \frac{1}{2}M_\sigma^2[\Phi_0]\sigma^2\right\},$$

$$\mathcal{L}_\Psi[\overline{\Psi}, \Psi] = \overline{\Psi}[i\gamma^\mu \mathcal{D}_\mu \Psi - M_\Psi[\Phi_0]]\Psi.$$

Here the prime stands for derivatives with respect to conformal time and the labels (gi) refer to gauge invariant quantities [13]. The explicit expression for $\mathcal{L}[\delta\varphi^{gi}, \phi^{gi}, \psi^{gi}]$ is given in Eq. (10.68) in Ref. [13]. The effective masses for the bosonic and fermionic fields are given by

$$M_\sigma^2[\Phi_0] = m_\sigma^2 + G(\Phi_0), \qquad M_\Psi[\Phi_0] = m_f + Y(\Phi_0). \tag{10}$$

We will focus on the study of the quantum corrections to the Friedmann equation, for the case in which both the scalar and fermionic fields are light in the sense that during slow-roll inflation,

$$M_\sigma[\Phi_0], M_\Psi[\Phi_0] \ll H_0, \tag{11}$$

at least during the cosmologically relevant stage corresponding to the 50 or so e folds before the end of inflation.

In conformal time the vierbeins V_a^μ are particularly simple

$$V_a^\mu = C(\eta)\delta_a^\mu \tag{12}$$

and the Dirac Lagrangian density simplifies to the following expression:

$$\sqrt{-g}\overline{\Psi}(i\gamma^\mu \mathcal{D}_\mu \Psi - M_\Psi[\Phi_0])\Psi = C^{3/2}\overline{\Psi}[i\not\partial - M_\Psi[\Phi_0]C(\eta)](C^{3/2}\Psi) \tag{13}$$

where $i\not\partial$ is the usual Dirac differential operator in Minkowski space-time in terms of flat space-time γ matrices.

From the quadratic Lagrangian given above the quadratic quantum fluctuations to the energy momentum tensor can be extracted.

The effective potential is identified with $\langle T_0^0\rangle$ in a spatially translational invariant state in which the expectation value of the inflaton field is Φ_0. During slow-roll inflation the expectation value Φ_0 evolves very slowly in time; the slow-roll approximation is indeed an adiabatic approximation, which justifies treating Φ_0 as a constant in order to obtain the effective potential. The time variation of Φ_0 only contributes to higher order corrections in slow roll. This is standard in any calculation of an effective potential. The energy momentum tensor is computed in the FRW inflationary background determined by the classical inflationary potential $V(\Phi_0)$, and the slow-roll parameters are also explicit functions of Φ_0. Therefore the energy momentum tensor depends implicitly on Φ_0 through the background and explicitly on the masses for the light bosonic and fermionic fields given above.

Therefore the effective potential is given by

$$V_{\rm eff}(\Phi_0) = V(\Phi_0) + \delta V(\Phi_0) \tag{14}$$

where

$$\delta V(\Phi_0) = \langle T_0^0[\Phi_0]\rangle_s + \langle T_0^0[\Phi_0]\rangle_t + \langle T_0^0[\Phi_0]\rangle_\sigma + \langle T_0^0[\Phi_0]\rangle_\Psi. \tag{15}$$

(s, t, σ, Ψ) correspond to the energy momentum tensors of the quadratic fluctuations of the scalar metric, tensor (gravitational waves), light boson field σ and light fermion field Ψ fluctuations, respectively. Since these are the expectation values of a quadratic energy momentum tensor, $\delta V(\Phi_0)$ corresponds to the one-loop correction to the effective potential.

A. Light scalar fields

We begin by analyzing the contribution to the effective potential from the light bosonic scalar field σ because this study highlights the main aspects which are relevant in the case of scalar metric (density) perturbations.

The bosonic Heisenberg field operators are expanded as follows:

$$\sigma(\vec{x}, \eta) = \frac{1}{C(\eta)\sqrt{\Omega}}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}[a_{\sigma,\vec{k}}S_\sigma(k, \eta) + a_{\sigma,\vec{k}}^\dagger S_\sigma^*(k, \eta)] \tag{16}$$

where Ω is the spatial volume.

During slow-roll inflation the effective mass of the σ field is given by Eq. (10), just as for the inflaton fluctuation. It is convenient to introduce a parameter η_σ defined to be

$$\eta_\sigma = \frac{M_\sigma^2[\Phi_0]}{3H_0^2}. \tag{17}$$

Hence, the statement that the σ field is light corresponds to the condition $\eta_\sigma \ll 1$. This dimensionless parameter plays the same role for the σ field as the parameter η_V given by Eq. (5) does for the inflaton fluctuation.

The mode functions $S_\sigma(k, \eta)$ in Eq. (16) obey the following equations up to quadratic order [24]:

$$S_\sigma''(k, \eta) + \left[k^2 + M_\sigma^2(\Phi_0)C^2(\eta) - \frac{C''(\eta)}{C(\eta)}\right]S_\sigma(k, \eta) = 0.$$

Using the slow-roll expressions Eq. (7) and in terms of η_σ, these mode equations become

$$S''_\sigma(k,\eta) + \left[k^2 - \frac{\nu_\sigma^2 - \frac{1}{4}}{\eta^2}\right] S_\sigma(k,\eta) = 0;$$

$$\nu_\sigma = \tfrac{3}{2} + \epsilon_V - \eta_\sigma + \mathcal{O}(\epsilon_V^2, \eta_\sigma^2, \eta_V^2, \epsilon_V \eta_V).$$

During slow-roll inflation Φ_0 is approximately constant, and the slow-roll expansion is an adiabatic expansion. As usual in the slow-roll approximation, the above equation for the mode functions is solved by assuming that Φ_0, hence ν_σ are constant. This is also the same type of approximation entailed in every calculation of the effective potential. Therefore during slow roll, the solution of the mode functions above is

$$S_\sigma(k,\eta) = \tfrac{1}{2}\sqrt{-\pi\eta}\, e^{i(\pi/2)[\nu_\sigma + (1/2)]} H^{(1)}_{\nu_\sigma}(-k\eta).$$

This choice of mode functions defines the Bunch-Davies (BD) vacuum, which obeys $a_{\vec{k}}|0\rangle_{\rm BD} = 0$. It is important to highlight that there is no unique choice of vacuum or initial state, a recognition that has received considerable attention in the literature, see for example [34,35] and references therein. In this study we focus on Bunch-Davies initial conditions since this has been the standard choice to study the power spectra and metric perturbations, hence we can compare our results to the standard ones in the literature, postponing for further study the assessment of different initial states.

The contribution to the effective potential from the light scalar field σ is given by

$$\langle T^0_0 \rangle_\sigma = \frac{1}{2}\left\langle \dot\sigma^2 + \left(\frac{\nabla\sigma}{C(\eta)}\right)^2 + M_\sigma^2[\Phi_0]\sigma^2 \right\rangle,$$

where the dot stands for derivative with respect to cosmic time. The expectation values are in the Bunch-Davies vacuum state and yield the following contributions:

$$\frac{1}{2}\langle(\dot\sigma)^2\rangle = \frac{H_0^4}{16\pi}\int_0^\infty \frac{dz}{z} z^2 \left|\frac{d}{dz}[z^{3/2} H^{(1)}_{\nu_\sigma}(z)]\right|^2, \tag{18}$$

$$\frac{1}{2}\left\langle\left(\frac{\nabla\sigma}{C^2(\eta)}\right)^2\right\rangle = \frac{H_0^4}{16\pi}\int_0^\infty \frac{dz}{z} z^5 |H^{(1)}_{\nu_\sigma}(z)|^2, \tag{19}$$

$$\frac{M_\sigma^2[\Phi_0]}{2}\langle\sigma^2(\vec{x},t)\rangle = \frac{3H_0^2\eta_\sigma}{2}\int_0^\infty \frac{dk}{k}\mathcal{P}_\sigma(k,t), \tag{20}$$

where $\mathcal{P}_\sigma(k,t)$ is the power spectrum of the σ field, which in terms of the spatial Fourier transform of the field $\sigma_{\vec{k}}(t)$ is given by

$$\mathcal{P}_\sigma(k,t) = \frac{k^3}{2\pi^2}\langle|\sigma^2_{\vec{k}}(t)|\rangle = \frac{H_0^2}{8\pi}(-k\eta)^3 |H^{(1)}_{\nu_\sigma}(-k\eta)|^2.$$

For a light scalar field during slow roll the power spectrum of the scalar field σ is nearly scale invariant and the index $\nu_\sigma \sim 3/2$. In the exact scale invariant case $\nu_\sigma = 3/2$,

$$z^3|H^{(1)}_{3/2}(z)|^2 = \frac{2}{\pi}[1+z^2]$$

and the integral of the power spectrum in Eq. (20) not only features logarithmic and quadratic ultraviolet divergences but also a logarithmic infrared divergence. During slow roll and for a light but massive scalar field the quantity

$$\Delta_\sigma = \tfrac{3}{2} - \nu_\sigma = \eta_\sigma - \epsilon_V + \mathcal{O}(\epsilon_V^2, \eta_\sigma^2, \epsilon_V\eta_\sigma), \ll 1$$

is a measure of the departure from scale invariance and provides a natural infrared regulator. We note that the contribution from Eq. (20) to the effective potential, which can be written as

$$\frac{3H_0^4\eta_\sigma}{16\pi}\int_0^\infty \frac{dz}{z} z^3 |H^{(1)}_{\nu_\sigma}(z)|^2,$$

is formally smaller than the contributions from Eqs. (18) and (19) by a factor $\eta_\sigma \ll 1$. However, the logarithmic infrared divergence in the exact scale invariant case leads to a single pole in the variable Δ_σ as described in detail in Refs. [23,24]. To see this feature in detail, it proves convenient to separate the infrared contribution by writing the integral above in the following form

$$\int_0^\infty \frac{dz}{z} z^3|H^{(1)}_{\nu_\sigma}(z)|^2 = \int_0^{\mu_p}\frac{dz}{z} z^3|H^{(1)}_{\nu_\sigma}(z)|^2 + \int_{\mu_p}^\infty \frac{dz}{z} z^3|H^{(1)}_{\nu_\sigma}(z)|^2.$$

In the first integral we obtain the leading order contribution in the slow-roll expansion, namely, the pole in Δ_σ, by using the small argument limit of the Hankel functions

$$z^3|H^{(1)}_{\nu_\sigma}(z)|^2 \overset{z\to 0}{=} \left[\frac{2^{\nu_\sigma}\Gamma(\nu_\sigma)}{\pi}\right]^2 z^{2\Delta_\sigma}$$

which yields

$$\int_0^{\mu_p}\frac{dz}{z} z^3|H^{(1)}_{\nu_\sigma}(z)|^2 = \frac{2}{\pi}\left[\frac{1}{2\Delta_\sigma} + \frac{\mu_p^2}{2} + \gamma - 2 + \ln(2\mu_p) + \mathcal{O}(\Delta_\sigma)\right].$$

In the second integral for small but fixed μ_p, we can safely set $\Delta_\sigma = 0$ and by introducing an upper momentum (ultraviolet) cutoff Λ_p, we finally find

$$\int_0^{\Lambda_p}\frac{dz}{z} z^3|H^{(1)}_{\nu_\sigma}(z)|^2 = \frac{1}{\pi}\left[\frac{1}{\Delta_\sigma} + \Lambda_p^2 + \ln\Lambda_p^2 + 2\gamma - 4 + \mathcal{O}(\Delta_\sigma)\right].$$

The simple pole in Δ_σ reflects the infrared enhancement arising from a nearly scale invariant power spectrum. While the terms that depend on Λ_p are of purely ultraviolet origin and correspond to the specific regularization scheme, the simple pole in Δ_σ originates in the infrared behavior and is therefore independent of the regularization scheme. A covariant regularization of the expectation value $\langle\sigma^2(\vec{x},t)\rangle$ will yield a result which features a simple pole in

Δ_σ plus terms which are ultraviolet finite and regular in the limit $\Delta_\sigma \to 0$. Such regular terms yield a contribution $\mathcal{O}(H^4\eta_\sigma)$ to Eq. (20) and are subleading in the limit of light scalar fields because they do not feature a denominator Δ_σ.

Therefore, to leading order in the slow-roll expansion and in $\eta_\sigma \ll 1$, the contribution from Eq. (20) is given by

$$\frac{M_\sigma^2[\Phi_0]}{2}\langle\sigma^2(\vec{x},t)\rangle = \frac{3H_0^4}{(4\pi)^2}\frac{\eta_\sigma}{\eta_\sigma - \epsilon_V} + \text{subleading in slow roll.}$$

In the first two contributions given by Eqs. (18) and (19) extra powers of momentum arising either from the time or spatial derivatives prevent the logarithmic infrared enhancements. These terms are infrared finite in the limit $\Delta_\sigma \to 0$ and their leading contribution during slow roll can be obtained by simply setting $\nu_\sigma = 3/2$ in these integrals, which feature quartic, quadratic and logarithmic ultraviolet divergences. A covariant renormalization of these two terms will lead to an ultraviolet and an infrared finite contribution to the energy momentum tensor of $\mathcal{O}(H_0^4)$, respectively. For the term given by Eq. (20), the infrared contribution that yields the pole in Δ_σ compensates for the $\eta_\sigma \ll 1$ in the numerator. After renormalization of the ultraviolet divergence, the ultraviolet and infrared finite contributions to this term will yield a contribution to the energy momentum tensor of order $\mathcal{O}(H_0^4\eta_\sigma)$, without the small denominator, and therefore subleading. This analysis indicates that the leading order contributions to the energy momentum tensor for light scalar fields is determined by the infrared pole $\sim 1/\Delta_\sigma$ from Eq. (20) and the fully renormalized contributions from (18) and (19), namely, to leading order in slow roll and η_σ

$$\langle T_0^0\rangle_\sigma = \frac{3H_0^4}{(4\pi)^2}\frac{\eta_\sigma}{\frac{3}{2}-\nu_\sigma} + \frac{1}{2}\left\langle \dot{\sigma}^2 + \left(\frac{\nabla\sigma}{C(\eta)}\right)^2\right\rangle_{\text{ren}}. \tag{21}$$

In the expression above we have displayed explicitly the pole at $3/2 - \nu_\sigma = \eta_\sigma - \epsilon_V$.

In calculating the second term (renormalized expectation value) to leading order in Eq. (21) we can set to zero the slow-roll parameters ϵ_V, η_V as well as the mass of the light scalar, namely $\eta_\sigma = 0$. Hence, to leading order, the second term is identified with the 00 component of the renormalized energy momentum tensor for a free massless minimally coupled scalar field in exact de Sitter space-time. Therefore we can extract this term from the known result for the renormalized energy momentum tensor for a minimally coupled free scalar boson of mass m_σ in de Sitter space-time with a Hubble constant H_0 given by [27,36,37]

$$\langle T_{\mu\nu}\rangle_{\text{ren}} = \frac{g_{\mu\nu}}{(4\pi)^2}\left\{m_\sigma^2 H_0^2\left(1-\frac{m_\sigma^2}{2H_0^2}\right)\left[-\psi\left(\frac{3}{2}+\nu\right) - \psi\left(\frac{3}{2}-\nu\right) + \ln\frac{m_\sigma^2}{H_0^2}\right] + \frac{2}{3}m_\sigma^2 H_0^2 - \frac{29}{30}H_0^4\right\},$$

$$\nu \equiv \sqrt{\frac{9}{4} - \frac{m_\sigma^2}{H_0^2}}, \tag{22}$$

where $\psi(z)$ stands for the digamma function. This expression corrects a factor of 2 in Refs. [27,38]. In Eq. (6.177) in [27] the d'Alembertian acting on $G^1(x,x')$ was neglected. However, in computing this term, the d'Alembertian must be calculated before taking the coincidence limit. Using the equation of motion yields the extra factor 2 and the expression Eq. (22). This result Eq. (22) for the renormalized energy momentum tensor was obtained by several different methods: covariant point splitting, zeta-function and Schwinger's proper time regularizations [27,38].

The simple pole at $\nu = 3/2$ manifest in Eq. (22) coincides precisely with the similar simple pole in Eq. (21) as can be gleaned by recognizing that $m_\sigma^2 = 3H^2\eta_\sigma$ as stated by Eq. (17). This pole originates in the term $m_\sigma^2\langle\sigma^2\rangle$, which features an infrared divergence in the scaling limit $\nu_\sigma = 3/2$. All the terms that contribute to the energy momentum tensor with space-time derivatives are infrared finite in this limit. Therefore, from the energy momentum tensor Eq. (22) we can extract straightforwardly the leading contribution to the renormalized expectation value in Eq. (21) in the limit $H_0 \gg m_\sigma$, and neglecting the slow-roll corrections to the scale factor. It is given by the last term in the bracket in Eq. (22). Hence, we find the leading order contribution

$$\langle T_0^0\rangle_\sigma = \frac{H_0^4}{(4\pi)^2}\left[\frac{3\eta_\sigma}{\eta_\sigma - \epsilon_V} - \frac{29}{30} + \mathcal{O}(\epsilon_V, \eta_\sigma, \eta_V)\right]. \tag{23}$$

The last term is completely determined by the trace anomaly [27,36–41] which is in turn determined by the short distance correlation function of the field and the background geometry.

Therefore, we emphasize that in the slow-roll approximation there is a clean and unambiguous separation between the contribution from superhorizon modes, which give rise to simple poles in slow-roll parameters and that of subhorizon modes whose leading contribution is determined by the trace anomaly and the short distance behavior of the field.

B. Scalar metric perturbations

The gauge invariant energy momentum tensor for quadratic scalar metric fluctuations has been obtained in Ref. [42] where the reader is referred to for details. In longitudinal gauge and in cosmic time it is given by

$$\langle T^0_0\rangle_s = M^2_{\rm Pl}\bigg[12H_0\langle\phi\dot{\phi}\rangle - 3\langle(\dot{\phi})^2\rangle + \frac{9}{C^2(\eta)}\langle(\nabla\phi)^2\rangle\bigg] + \frac{1}{2}\langle(\dot{\delta\varphi})^2\rangle + \frac{\langle(\nabla\delta\varphi)^2\rangle}{2C^2(\eta)} + \frac{V''(\Phi_0)}{2}\langle(\delta\varphi)^2\rangle + 2V'(\Phi_0)\langle\phi\delta\varphi\rangle \tag{24}$$

where the condition $\phi = \psi$ valid in scalar field inflation has been used, and the dots stand for derivatives with respect to cosmic time.

In longitudinal gauge, the equations of motion in cosmic time for the Fourier modes are [13,33]

$$\ddot{\phi}_{\vec{k}} + \bigg(H_0 - 2\frac{\ddot{\Phi}_0}{\dot{\Phi}_0}\bigg)\dot{\phi}_{\vec{k}} + \bigg[2\bigg(\dot{H}_0 - 2H_0\frac{\ddot{\Phi}_0}{\dot{\Phi}_0}\bigg) + \frac{k^2}{C^2(\eta)}\bigg]\phi_{\vec{k}} = 0,$$

$$\ddot{\delta\varphi}_{\vec{k}} + 3H\dot{\delta\varphi}_{\vec{k}} + \bigg[V''[\Phi_0] + \frac{k^2}{C^2(\eta)}\bigg]\delta\varphi_{\vec{k}} + 2V'[\Phi_0]\phi_{\vec{k}} - 4\dot{\Phi}_0\dot{\phi}_{\vec{k}} = 0, \tag{25}$$

with the constraint equation

$$\dot{\phi}_{\vec{k}} + H_0\phi_{\vec{k}} = \frac{1}{2M_{\rm Pl}}\delta\varphi_{\vec{k}}\dot{\Phi}_0. \tag{26}$$

Just as in the case of the scalar fields, we expect an infrared enhancement arising from superhorizon modes, therefore, following Ref. [42] we split the contributions to the energy momentum tensor as those from superhorizon modes, which will yield the infrared enhancement, and the subhorizon modes for which we can set all slow-roll parameters to zero. Just as discussed above for the case of the σ field, since spatiotemporal derivatives bring higher powers of the momenta, we can neglect all derivative terms for the contribution from the superhorizon modes. Therefore, the contribution from superhorizon modes which will reflect the infrared enhancement is extracted from [42]

$$\langle T^0_0\rangle_{\rm IR} \approx \tfrac{1}{2}V''[\Phi_0]\langle(\delta\varphi(\vec{x},t))^2\rangle + 2V'[\Phi_0] \times\langle\phi(\vec{x},t)\delta\varphi(\vec{x},t)\rangle. \tag{27}$$

The analysis of the solution of Eq. (25) for superhorizon wavelengths in Ref. [13] shows that in exact de Sitter space-time $\phi_{\vec{k}} \sim$ const, hence it follows that during quasi-de Sitter slow-roll inflation for superhorizon modes

$$\dot{\phi}_{\vec{k}} \sim (\text{slow roll}) \times H_0\phi_{\vec{k}}. \tag{28}$$

Therefore, for superhorizon modes, the constraint Eq. (26) yields

$$\phi_{\vec{k}} = -\frac{V'(\Phi_0)}{2V(\Phi_0)}\delta\varphi_{\vec{k}} + \text{higher orders in slow roll}. \tag{29}$$

Inserting this relation in Eq. (25) and consistently neglecting the term $\dot{\phi}_{\vec{k}}$ according to Eq. (28), we find the following equation of motion for the gauge invariant scalar field fluctuation in longitudinal gauge

$$\ddot{\delta\varphi}_{\vec{k}} + 3H_0\dot{\delta\varphi}_{\vec{k}} + \bigg[\frac{k^2}{C^2(\eta)} + 3H_0^2\eta_\delta\bigg]\delta\varphi_{\vec{k}} = 0, \tag{30}$$

where we have used the definition of the slow-roll parameters ϵ_V; η_V given in Eq. (5), and introduced

$$\eta_\delta \equiv \eta_V - 2\epsilon_V. \tag{31}$$

This is the equation of motion for a minimally coupled scalar field with mass squared $3H_0^2\eta_\delta$ and we can use the results obtained in the case of the scalar field σ above. The quantum field $\delta\varphi(\vec{x},t)$ is expanded as

$$\delta\varphi(\vec{x},\eta) = \frac{1}{C(\eta)\sqrt{\Omega}}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}[a_{\delta,\vec{k}}S_\delta(k,\eta) + a^\dagger_{\delta,\vec{k}}S^*_\delta(k,\eta)], \tag{32}$$

where the mode functions are given by

$$S_\delta(k,\eta) = \tfrac{1}{2}\sqrt{-\pi\eta}e^{i(\pi/2)[\nu_\delta+(1/2)]}H^{(1)}_{\nu_\delta}(-k\eta); \quad \nu_\delta = \tfrac{3}{2} + \epsilon_V - \eta_\delta = \tfrac{3}{2} + 3\epsilon_V - \eta_V. \tag{33}$$

In this case, the slow-roll quantity that regulates the infrared behavior is $\Delta_\delta \equiv \eta_V - 3\epsilon_V$.

Again we choose the Bunch-Davies vacuum state annihilated by the operators $a_{\delta,\vec{k}}$. Therefore, the contribution to $\langle T^0_0\rangle$ from superhorizon modes to lowest order in slow roll is given by

$$\langle T^0_0\rangle_{\rm IR} = 3H_0^2\bigg(\frac{\eta_V}{2} - 2\epsilon_V\bigg)\bigg[\int_0^\infty \frac{dk}{k}\mathcal{P}_\delta(k,\eta)\bigg]_{\rm IR} \tag{34}$$

where the power spectrum of scalar fluctuations is given by

$$\mathcal{P}_\delta(k,\eta) = \frac{k^3}{2\pi^2}\langle|\delta\varphi_{\vec{k}}(t)|^2\rangle = \frac{H_0^2}{8\pi}(-k\eta)^3|H^{(1)}_{\nu_\delta}(-k\eta)|^2 \tag{35}$$

and the subscript IR in the integral refers only to the infrared pole contribution to Δ_δ. Repeating the analysis presented in the case of the scalar field σ above, we finally find

$$\langle T^0_0\rangle_{\rm IR} = \frac{3H_0^4}{(4\pi)^2}\frac{\eta_V - 4\epsilon_V}{\eta_V - 3\epsilon_V} + \text{subleading in slow roll}. \tag{36}$$

For subhorizon modes with wave vectors $k \gg a(t)H_0$, the solutions of Eq. (25) are [13]

$$\phi_{\vec{k}}(t) \approx e^{\pm ik\eta} \Rightarrow \dot{\phi}_{\vec{k}}(t) \sim \frac{ik}{a(t)}\phi_{\vec{k}}(t). \tag{37}$$

For $k \gg a(t)H_0$ the constraint equation (26) entails that [42]

$$\phi_{\vec{k}}(t) \approx \frac{ia(t)}{2M_{\rm Pl}k}\dot{\Phi}_0\delta\varphi_{\vec{k}}. \tag{38}$$

Replacing the expressions Eqs. (37) and (38) in Eq. (24) yields that all the terms featuring the gravitational potential ϕ are suppressed with respect to those featuring the scalar field fluctuation $\delta\varphi$ by powers of $H_0a(t)/k \ll 1$ as originally observed in Ref. [42]. Therefore the contribution from subhorizon modes to $\langle T^0_{0s}\rangle$ is given by

$$\langle T^0_{0s}\rangle_{\rm UV} \simeq \frac{1}{2}\langle(\dot{\delta\varphi})^2\rangle + \frac{\langle(\nabla\delta\varphi)^2\rangle}{2a^2} \tag{39}$$

where we have also neglected the term with $V''[\Phi_0] \sim 3H_0^2\eta_V$ since $k^2/a^2 \gg H_0^2$ for subhorizon modes. Therefore, to leading order in slow roll we find the renormalized expectation value of T_{00s} is given by

$$\langle T^0_{0s}\rangle_{\rm ren} \simeq \frac{3H_0^4}{(4\pi)^2}\frac{\eta_V - 4\epsilon_V}{\eta_V - 3\epsilon_V} + \frac{1}{2}\left\langle\dot{\delta\varphi}^2 + \left(\frac{\nabla\delta\varphi}{C(\eta)}\right)^2\right\rangle_{\rm ren}. \tag{40}$$

To obtain the renormalized expectation value in Eq. (40) one can set all slow-roll parameters to zero to leading order and simply consider a massless scalar field minimally coupled in de Sitter space-time. This is precisely what we have already calculated in the case of the scalar field σ above by using the known results in the literature for the covariantly renormalized energy momentum tensor of a massive minimally coupled field [27,36–38], and we can just borrow the result from Eq. (23). We find the following final result to leading order in slow roll:

$$\langle T^0_{0s}\rangle_{\rm ren} = \frac{H_0^4}{(4\pi)^2}\left[\frac{\eta_V - 4\epsilon_V}{\eta_V - 3\epsilon_V} - \frac{29}{30} + \mathcal{O}(\epsilon_V, \eta_\sigma, \eta_V)\right]. \tag{41}$$

The last term in Eq. (41) is completely determined by the trace anomaly of a minimally coupled scalar field in de Sitter space-time [27,37–39].

C. Tensor perturbations

Tensor perturbations correspond to massless fields with two physical polarizations. The quantum fields are written as

$$h^i_j(\vec{x}, \eta) = \frac{1}{C(\eta)M_{\rm Pl}\sqrt{2\Omega}}\sum_{\lambda=\times,+}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}\epsilon^i_j(\lambda, \vec{k}) \times [a_{\lambda,\vec{k}}S_h(k, \eta) + a^\dagger_{\lambda,\vec{k}}S^*_h(k, \eta)],$$

where the operators $a_{\lambda,\vec{k}}$, $a^\dagger_{\lambda,\vec{k}}$ obey the usual canonical commutation relations, and $\epsilon^i_j(\lambda, \vec{k})$ are the two independent traceless-transverse tensors constructed from the two independent polarization vectors transverse to $\hat{\mathbf{k}}$, chosen to be real and normalized such that $\epsilon^i_j(\lambda, \vec{k})\epsilon^j_k(\lambda', \vec{k}) = \delta^i_k\delta_{\lambda,\lambda'}$.

The mode functions $S_h(k, \eta)$ obey the differential equation

$$S''_h(k, \eta) + \left[k^2 - \frac{\nu_h^2 - \frac{1}{4}}{\eta^2}\right]S_h(k, \eta) = 0; \qquad \nu_h = \tfrac{3}{2} + \epsilon_V + \mathcal{O}(\epsilon_V^2, \eta_\sigma^2, \eta_V^2, \epsilon_V\eta_V). \tag{42}$$

The solutions corresponding to the Bunch-Davies vacuum annihilated by the operators $a_{\lambda,\vec{k}}$ are

$$S_h(k, \eta) = \tfrac{1}{2}\sqrt{-\pi\eta}\,e^{i(\pi/2)[\nu_h+(1/2)]}H^{(1)}_{\nu_h}(-k\eta). \tag{43}$$

The energy momentum tensor for gravitons only depends on derivatives of the field h^i_j; therefore its expectation value in the BD vacuum does not feature infrared singularities in the limit $\epsilon_V \to 0$. The absence of infrared singularities in the limit of exact de Sitter space-time entails that we can extract the leading contribution to the effective potential from tensor perturbations by evaluating the expectation value of T_{00} in the BD vacuum in exact de Sitter space-time, namely, by setting all slow-roll parameters to zero. This will yield the leading order in the slow-roll expansion.

Because de Sitter space-time is maximally symmetric, the expectation value of the energy momentum tensor is given by [27,28]

$$\langle T_{\mu\nu}\rangle_{\rm BD} = \frac{g_{\mu\nu}}{4}\langle T^\alpha_\alpha\rangle_{\rm BD} \tag{44}$$

and T^α_α is a space-time constant, therefore the energy momentum tensor is manifestly covariantly conserved. Of course, in a quantum field theory there emerge ultraviolet divergences and the regularization procedure must be compatible with the maximal symmetry. A large body of work has been devoted to study the trace anomaly in de Sitter space-time implementing a variety of powerful covariant regularization methods that preserve the symmetry [27,37–41] yielding a renormalized value of the expectation value of the $\langle T_{\mu\nu}\rangle_{\rm BD}$ given by Eq. (44). Therefore, the full energy momentum tensor is completely determined by the trace anomaly [27,37,39].

The contribution to the trace anomaly from gravitons has been given in Refs. [27,37,39]; it is

$$\langle T^\alpha_\alpha\rangle_t = -\frac{717}{80\pi^2}H_0^4. \tag{45}$$

From this result, we conclude that

$$\langle T^0_0\rangle_t = -\frac{717}{320\pi^2}H_0^4. \tag{46}$$

This result differs by a numerical factor from that obtained in Ref. [43], presumably the difference is a result of a different regularization scheme.

D. Fermion fields

The Dirac equation in the FRW geometry is given by [see Eq. (13)]

$$[i\not\partial - M_\Psi[\Phi_0]C(\eta)](C^{3/2}\Psi(\vec{x},\eta)) = 0. \tag{47}$$

The solution $\Psi(\vec{x},\eta)$ can be expanded in spinor mode functions as

$$\Psi(\vec{x},\eta) = \frac{1}{C^{3/2}(\eta)\sqrt{\Omega}}\sum_{\vec{k},\lambda} e^{i\vec{k}\cdot\vec{x}}[b_{\vec{k},\lambda}U_\lambda(\vec{k},\eta) + d^\dagger_{-\vec{k},\lambda}V_\lambda(-\vec{k},\eta)], \tag{48}$$

where the spinor mode functions U, V obey the Dirac equations

$$\begin{aligned} &[i\gamma^0\partial_\eta - \vec{\gamma}\cdot\vec{k} - M(\eta)]U_\lambda(\vec{k},\eta) = 0,\\ &[i\gamma^0\partial_\eta + \vec{\gamma}\cdot\vec{k} - M(\eta)]V_\lambda(\vec{k},\eta) = 0, \end{aligned} \tag{49}$$

and

$$M(\eta) \equiv M_\Psi[\Phi_0]C(\eta). \tag{50}$$

Following the method of Refs. [44,45], it proves convenient to write

$$\begin{aligned} U_\lambda(\vec{k},\eta) &= [i\gamma^0\partial_\eta - \vec{\gamma}\cdot\vec{k} + M(\eta)]f_k(\eta)\mathcal{U}_\lambda,\\ V_\lambda(\vec{k},\eta) &= [i\gamma^0\partial_\eta + \vec{\gamma}\cdot\vec{k} + M(\eta)]g_k(\eta)\mathcal{V}_\lambda, \end{aligned} \tag{51}$$

with $\mathcal{U}_\lambda$; $\mathcal{V}_\lambda$ being constant spinors [44,45] obeying

$$\gamma^0\mathcal{U}_\lambda = \mathcal{U}_\lambda, \qquad \gamma^0\mathcal{V}_\lambda = -\mathcal{V}_\lambda. \tag{52}$$

The mode functions $f_k(\eta)$; $g_k(\eta)$ obey the following equations of motion:

$$\left[\frac{d^2}{d\eta^2} + k^2 + M^2(\eta) - iM'(\eta)\right]f_k(\eta) = 0, \tag{53}$$

$$\left[\frac{d^2}{d\eta^2} + k^2 + M^2(\eta) + iM'(\eta)\right]g_k(\eta) = 0. \tag{54}$$

Neglecting the derivative of Φ_0 with respect to time, namely, terms of order $\sqrt{\epsilon_V}$ and higher, the equations of motion for the mode functions are given by

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{\nu_+^2 - \frac{1}{4}}{\eta^2}\right]f_k(\eta) = 0, \tag{55}$$

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{\nu_-^2 - \frac{1}{4}}{\eta^2}\right]g_k(\eta) = 0, \qquad \nu_\pm = \frac{1}{2} \pm i\frac{M_\Psi[\Phi_0]}{H_0}. \tag{56}$$

The scalar product of the spinors $U_\lambda(\vec{k},\eta)$, $V_\lambda(\vec{k},\eta)$ yields

$$\begin{aligned} U^\dagger_\lambda(\vec{k},\eta)U_{\lambda'}(\vec{k},\eta) &= C^+(k)\delta_{\lambda,\lambda'},\\ V^\dagger_\lambda(\vec{k},\eta)V_{\lambda'}(\vec{k},\eta) &= C^-(k)\delta_{\lambda,\lambda'}, \end{aligned} \tag{57}$$

where

$$\begin{aligned} C^+(k) &= f_k^{*\prime}(\eta)f_k'(\eta) + (k^2 + M^2(\eta))f_k^*(\eta)f_k(\eta) + iM(\eta)(f_k'(\eta)f_k^*(\eta) - f_k(\eta)f_k^{*\prime}(\eta)),\\ C^-(k) &= g_k^{*\prime}(\eta)g_k'(\eta) + (k^2 + M^2(\eta))g_k^*(\eta)g_k(\eta) - iM(\eta)(g_k'(\eta)g_k^*(\eta) - g_k(\eta)g_k^{*\prime}(\eta)) \end{aligned}$$

are constants of motion by dint of the equations of motion for the mode functions $f_k(\eta)$, $g_k(\eta)$. The normalized spinor solutions of the Dirac equation are therefore given by

$$U_\lambda(\vec{k},\eta) = \frac{1}{\sqrt{C^+(k)}}[if_k'(\eta) - \vec{\gamma}\cdot\vec{k}f_k(\eta) + M(\eta)f_k(\eta)]\mathcal{U}_\lambda,$$

$$V_\lambda(\vec{k},\eta) = \frac{1}{\sqrt{C^-(k)}}[-ig_k'(\eta) + \vec{\gamma}\cdot\vec{k}g_k(\eta) + M(\eta)g_k(\eta)]\mathcal{U}_\lambda.$$

We choose the solutions of the mode equations (55) and (56) to be

$$\begin{aligned} f_k(\eta) &= \sqrt{\frac{-\pi k\eta}{2}}e^{i(\pi/2)[\nu_+ + (1/2)]}H^{(1)}_{\nu_+}(-k\eta),\\ g_k(\eta) &= \sqrt{\frac{-\pi k\eta}{2}}e^{-i(\pi/2)[\nu_- + (1/2)]}H^{(2)}_{\nu_-}(-k\eta). \end{aligned} \tag{58}$$

We also choose the Bunch-Davies vacuum state such that $b_{\vec{k},\lambda}|0\rangle_{\rm BD} = 0$; $d_{\vec{k},\lambda}|0\rangle_{\rm BD} = 0$. The choice of the mode functions Eq. (58) yields the following normalization factors:

$$C^+(k) = C^-(k) = 2k^2.$$

The energy momentum tensor for a spin $1/2$ field is given by [27]

$$T_{\mu\nu} = \frac{i}{2}[\overline{\Psi}\gamma_{(\mu}\overleftrightarrow{\mathcal{D}}_{\nu)}\Psi],$$

and its expectation value in the Bunch-Davies vacuum is equal to

$$\langle T^0_0\rangle_{\rm BD} = \frac{2}{C^4(\eta)}\int\frac{d^3k}{(2\pi)^3}\{M(\eta) - {\rm Im}[g_k'(\eta)g_k^*(\eta)]\},$$

where $M(\eta)$ and $g_k(\eta)$ are given by Eqs. (50) and (58), respectively. It is clear that this energy momentum tensor does not feature any infrared sensitivity because the index of the Bessel functions is $\nu_\pm \approx 1/2$. Of course this is expected since fermionic fields cannot feature large amplitudes due to the Pauli principle.

A lengthy computation using covariant point splitting regularization yields the following result:

$$\langle T^0_0 \rangle_\Psi = \frac{11 H_0^4}{960\pi^2}\Big\{1 + \frac{120}{11}\mathcal{M}^2(\mathcal{M}^2+1) \times \Big[-\mathrm{Re}\psi(2+i\mathcal{M}) - \frac{19}{12} - \gamma - 2\ln 2\Big]\Big\}, \quad (59)$$

$$\mathcal{M} \equiv \frac{M_\Psi[\Phi_0]}{H_0}.$$

The first term in the bracket in Eq. (59) is recognized as the trace anomaly for fermions and is the only term that survives in the massless limit [27,37–41]. For light fermion fields, $\mathcal{M} \ll 1$, and the leading contribution to the energy momentum tensor is completely determined by the trace anomaly, hence in this limit the contribution to the covariantly regularized effective potential from (Dirac) fermions is given by

$$\langle T^0_0 \rangle_\Psi = \frac{11 H_0^4}{960\pi^2}[1 + \mathcal{O}(\mathcal{M}^2)].$$

This result is valid for Dirac fermions and it must be divided by a factor 2 for Weyl or Majorana fermions.

E. Summary

In summary, we find that the effective potential at one loop is given by

$$\delta V(\Phi_0) = \frac{H_0^4}{(4\pi)^2}\Big[\frac{\eta_V - 4\epsilon_V}{\eta_V - 3\epsilon_V} + \frac{3\eta_\sigma}{\eta_\sigma - \epsilon_V} + \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_\Psi + \mathcal{O}(\epsilon_V, \eta_V, \eta_\sigma, \mathcal{M}^2)\Big],$$

where (s, t, σ, Ψ) stand for the contributions of the scalar metric, tensor fluctuations, light boson field σ and light fermion field Ψ, respectively. We have

$$\mathcal{T}_\Phi = \mathcal{T}_s = -\frac{29}{30}, \qquad \mathcal{T}_t = -\frac{717}{5}, \qquad \mathcal{T}_\Psi = \frac{11}{60}. \quad (60)$$

The terms that feature the ratios of combinations of slow-roll parameters arise from the infrared or superhorizon contribution from the scalar density perturbations and scalar fields σ respectively. The terms $\mathcal{T}_{s,t,\Psi}$ are completely determined by the trace anomalies of scalar, graviton and fermion fields, respectively. Writing $H_0^4 = V(\Phi_0)H_0^2/[3M_{\rm Pl}^2]$ we can finally write the effective potential to leading order in slow roll

$$V_{\rm eff}(\Phi_0) = V(\Phi_0)\Big[1 + \frac{H_0^2}{3(4\pi)^2 M_{\rm Pl}^2}\Big(\frac{\eta_V - 4\epsilon_V}{\eta_V - 3\epsilon_V} + \frac{3\eta_\sigma}{\eta_\sigma - \epsilon_V} - \frac{2903}{20}\Big)\Big]. \quad (61)$$

There are several remarkable aspects of this result:

(i) The infrared enhancement as a result of the near scale invariance of scalar field fluctuations, both from scalar density perturbations as well as from a light scalar field, yields corrections of zeroth order in slow roll. This is a consequence of the fact that during slow roll the particular combination $\Delta_\sigma = \eta_\sigma - \epsilon_V$ of slow-roll parameters yields a natural infrared cutoff.

(ii) The final one-loop contribution to the effective potential displays the effective field theory dimensionless parameter $H_0^2/M_{\rm Pl}^2$ confirming our previous studies [23,24],

(iii) The last term is completely determined by the trace anomaly, a purely geometric result of the short distance properties of the theory.

III. QUANTUM CORRECTIONS TO THE CURVATURE AND TENSOR FLUCTUATIONS

The quantum corrections to the effective potential lead to quantum corrections to the amplitude of scalar and tensor fluctuations.

The scalar curvature and tensor fluctuations in the slow-roll regime are given by the formulas [6]

$$|\Delta_k^{(S)}|^2 = \frac{1}{8\pi^2\epsilon_V}\Big(\frac{H}{M_{\rm Pl}}\Big)^2, \qquad |\Delta_k^{(T)}|^2 = \frac{1}{2\pi^2}\Big(\frac{H}{M_{\rm Pl}}\Big)^2, \quad (62)$$

where H stands for the Hubble parameter and ϵ_V is given by Eq. (5).

We can include the leading quantum corrections in Eq. (62) replacing in it H and ϵ_V by the corrected parameters $H_{\rm eff}$ and $\epsilon_{\rm eff}$. That is,

$$H_{\rm eff}^2 = H_0^2 + \delta H^2, \qquad \epsilon_{\rm eff} = \epsilon_V + \delta\epsilon_V \quad (63)$$

with

$$H_{\rm eff}^2 = \frac{V_{\rm eff}(\Phi_0)}{3M_{\rm Pl}^2}, \qquad \epsilon_{\rm eff} = \frac{M_{\rm Pl}^2}{2}\Big[\frac{V'_{\rm eff}(\Phi_0)}{V_{\rm eff}(\Phi_0)}\Big]^2, \quad (64)$$

and where $V_{\rm eff}(\Phi_0)$ is given by Eq. (61). We thus obtain

$$\frac{\delta H^2}{H_0^2} = \frac{1}{3}\Big(\frac{H_0}{4\pi M_{\rm Pl}}\Big)^2\Big[\frac{\eta_V - 4\epsilon_V}{\eta_V - 3\epsilon_V} + \frac{3\eta_\sigma}{\eta_\sigma - \epsilon_V} - \frac{2903}{20}\Big], \quad (65)$$

$$\frac{\delta\epsilon_V}{\epsilon_V} = \frac{2}{3}\Big(\frac{H_0}{4\pi M_{\rm Pl}}\Big)^2\Big\{\frac{\xi_V + 12\epsilon_V(2\epsilon_V - \eta_V)}{2(\eta_V - 3\epsilon_V)^2} + \frac{3\eta_\sigma}{(\eta_\sigma - \epsilon_V)^2}\Big[\eta_\sigma + \eta_V - 2\epsilon_V - \sqrt{2\epsilon_V}M_{\rm Pl}\frac{d\log M_\sigma[\Phi_0]}{d\Phi_0}\Big] - \frac{2903}{20}\Big\}.$$

Inserting Eq. (65) into Eqs. (63) and (64) yields after calculation, for the scalar perturbations,

$$|\Delta^{(S)}_{k,\mathrm{eff}}|^2 = |\Delta^{(S)}_k|^2\left[1 - \frac{\delta\epsilon_V}{\epsilon_V} + \frac{\delta H^2}{H^2}\right] = |\Delta^{(S)}_k|^2\left\{1 - \frac{1}{3}\left(\frac{H_0}{4\pi M_{\mathrm{Pl}}}\right)^2 \times\left[\frac{\xi_V + 12\epsilon_V^2 - \eta_V^2 - 5\epsilon_V\eta_V}{(\eta_V - 3\epsilon_V)^2} + \frac{3\eta_\sigma}{(\eta_\sigma - \epsilon_V)^2}\left[\eta_\sigma - 3\epsilon_V + 2\eta_V - 2\sqrt{2\epsilon_V}M_{\mathrm{Pl}}\frac{d\log M_\sigma[\Phi_0]}{d\Phi_0}\right] - \frac{2903}{20}\right]\right\}, \quad (66)$$

and for the tensor perturbations,

$$|\Delta^{(T)}_{k,\mathrm{eff}}|^2 = |\Delta^{(T)}_k|^2\left[1 + \frac{\delta H^2}{H^2}\right] = |\Delta^{(T)}_k|^2\left\{1 + \frac{1}{3}\left(\frac{H_0}{4\pi M_{\mathrm{Pl}}}\right)^2\left[\frac{\eta_V - 4\epsilon_V}{\eta_V - 3\epsilon_V} + \frac{3\eta_\sigma}{\eta_\sigma - \epsilon_V} - \frac{2903}{20}\right]\right\}, \quad (67)$$

where $M_\sigma[\Phi_0]$ and η_σ are given by Eqs. (10) and (17), respectively.

The case when the field σ is much lighter than the inflaton permits simplifications, since

$$\eta_\sigma \sim \left(\frac{m_\sigma}{m_{\mathrm{inflaton}}}\right)^2\eta_V.$$

For $m_\sigma^2 \ll m^2_{\mathrm{inflaton}}$, we can neglect terms proportional to η_σ in the expressions for $|\Delta^{(S)}_k|^2$ and $|\Delta^{(T)}_{k,\mathrm{eff}}|^2$. In this case the quantum corrections to the power spectra obtain a particularly illuminating expression when the slow-roll parameters in Eqs. (66) and (67) are written in terms of the CMB observables n_s, r and the spectral running of the scalar index using

$$\epsilon_V = \frac{r}{16}, \qquad \eta_V = \frac{1}{2}\left(n_s - 1 + \frac{3}{8}r\right),$$

$$\xi_V = \frac{r}{4}\left(n_s - 1 + \frac{3}{16}r\right) - \frac{1}{2}\frac{dn_s}{d\ln k}, \qquad \eta_V - 3\epsilon_V = \frac{1}{2}(n_s - 1). \quad (68)$$

We find from Eqs. (66) and (67),

$$|\Delta^{(S)}_{k,\mathrm{eff}}|^2 = |\Delta^{(S)}_k|^2\left\{1 + \frac{2}{3}\left(\frac{H_0}{4\pi M_{\mathrm{Pl}}}\right)^2 \times\left[1 + \frac{\frac{3}{8}r(n_s - 1) + 2\frac{dn_s}{d\ln k}}{(n_s - 1)^2} + \frac{2903}{40}\right]\right\},$$
$$|\Delta^{(T)}_{k,\mathrm{eff}}|^2 = |\Delta^{(T)}_k|^2\left\{1 - \frac{1}{3}\left(\frac{H_0}{4\pi M_{\mathrm{Pl}}}\right)^2 \times\left[-1 + \frac{1}{8}\frac{r}{n_s - 1} + \frac{2903}{20}\right]\right\}. \quad (69)$$

We see that the anomalies contribution $\frac{2903}{40} = 72.575$ and $\frac{2903}{20} = 145.15$ presumably dominate both quantum corrections. The other terms are generally expected to be smaller than these large contributions from the anomalies. These anomalous contributions are dominated in turn by the tensor part [see Eq. (60)]. Only fermions give contributions with the opposite sign. However, one needs at least 783 species of (Dirac) fermions to compensate for the tensor part.

These quantum corrections also affect the ratio r of tensor/scalar fluctuations as follows:

$$r_{\mathrm{eff}} \equiv \frac{|\Delta^{(T)}_{k,\mathrm{eff}}|^2}{|\Delta^{(S)}_{k,\mathrm{eff}}|^2} = r\left\{1 - \frac{1}{3}\left(\frac{H_0}{4\pi M_{\mathrm{Pl}}}\right)^2 \times\left[1 + \frac{\frac{3}{8}r(n_s - 1) + \frac{dn_s}{d\ln k}}{(n_s - 1)^2} + \frac{8709}{20}\right]\right\}. \quad (70)$$

We expect this quantum correction to the ratio to be negative as the anomaly contribution dominates: $\frac{8709}{20} = 435.45$.

Therefore, the quantum corrections enhance the scalar curvature fluctuations while they reduce the tensor fluctuations as well as their ratio r. The quantum corrections are small, of the order $(H_0/M_{\mathrm{Pl}})^2$, but it is interesting to see that the quantum effects are dominated by the trace anomalies and they correct both fluctuations in a definite direction.

IV. CONCLUSIONS

Motivated by the premise that forthcoming CMB observations may probe the inflationary potential, we study its quantum corrections from scalar and tensor metric perturbations as well as those from one light scalar and one light (Dirac) fermion field generically coupled to the inflaton. The reason for this study is that the measurements probe the full effective inflaton potential, namely, the classical potential plus its quantum corrections. We have focused on obtaining the quantum corrections to the effective potential during the cosmologically relevant quasi-de Sitter stage of slow-roll inflation. Both scalar metric fluctuations, as well as those from a light scalar field, feature infrared enhancements as a consequence of the nearly scale invariance of their power spectra. A combination of slow-roll parameters appropriate for each case provides a natural infrared regularization. We find that to leading order in slow roll, there is a clean and unambiguous separation between the contributions to the effective potential from superhorizon modes of the scalar metric perturbations as well as the scalar field, and those from subhorizon modes. Only the contributions to the total energy momentum tensor from curvature perturbations and the light scalar field feature an infrared enhancement, while those from gravitational waves and fermions do not feature any infrared sensitivity.

In all cases, scalar metric, tensor, light scalar and fermion fields, the contribution from subhorizon modes is determined by the trace anomaly, while the contributions from superhorizon modes, only relevant for curvature and scalar field perturbations, are infrared enhanced as the inverse of a combination of slow-roll parameters which measure the departure from scale invariance in each case.

The one-loop effective potential to leading order in slow roll is given by Eq. (61). The last term, independent of the slow-roll parameters, is completely determined by the trace anomalies of scalar, tensor and fermionic fields (therefore solely determined by the space-time geometry), while the first term reveals the hallmark infrared enhancement of superhorizon fluctuations. Using this result we have obtained the one-loop quantum corrections to the amplitude of (scalar) curvature and tensor perturbations in terms of the CMB observables n_s, r and $dn_s/d\ln k$ and the total trace anomaly $\mathcal{T}$ of the different fields.

As we anticipated in Refs. [23,24], the strength of the one-loop corrections is determined by the effective field theory parameter $(H_0/M_{\rm Pl})^2$. While this quantity is observationally of $\mathcal{O}(10^{-10})$, there is an important message in this result: the robustness of slow-roll inflation as well as the reliability of the effective field theory description. This result is consistent with those of previous studies of the bispectrum, non-Gaussianity and of isocurvature contributions [23,24,46]. The quantum corrections computed here involve two-point correlation functions and imply quantum contributions to the three (and higher) point correlation functions. Therefore, the *non-Gaussianities* will feature infrared enhancement and quantum trace anomaly contributions of the type computed here.

There is a simple interpretation of the above result: in the effective field theory approach, the "classical" inflaton potential $V(\Phi_0)$ includes contributions from integrating out the fields with scales much heavier than the scale of inflation H_0.

The contribution to the energy momentum from light fields yields the effective potential; however for fields with mass scales $\ll H_0$ the dominant scale in the problem is H_0 and on dimensional grounds the contribution to the covariantly renormalized energy momentum tensor must be $\propto H_0^4$. This argument would fail in the presence of infrared divergences, and indeed the mass terms from curvature perturbations and from the scalar field σ feature an infrared enhancement because of their nearly scale invariant power spectrum. The mass term is of first order in the slow roll, however the infrared enhancement brings about a denominator which is also of first order in slow roll yielding a ratio which is of zeroth order in slow roll. Hence, this remarkable result validates the simple power counting that yields the overall scale H_0^4 for the one-loop correction.

An important bonus of the slow-roll approximation is that the contributions from superhorizon and subhorizon modes can be unambiguously separated and the latter are completely determined by the trace anomaly, a purely geometrical result which only depends on the short distance (ultraviolet) properties. Quantum trace anomalies of the energy momentum tensor in gravitational fields constitute a nice and important chapter of quantum field theory in curved backgrounds (see [27] and references therein). Our results here show that these trace anomalies dominate the quantum corrections to a relevant cosmological problem: the primordial power spectrum of curvature and tensor fluctuations.

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