Is the Universe odd?

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We investigate the point-parity and mirror-parity handedness of the large angle anisotropy in the cosmic microwave background (CMB). In particular we consider whether the observed low CMB quadrupole could more generally signal odd point-parity, i.e., suppression of even multipoles. Even though this feature is "visually" present in most renditions of the WMAP dataset we find that it never supports parity preference beyond the meagre 95% confidence level. This is fortunate as point-parity handedness implies almost certainly a high level of galactic contamination. Mirror reflection parity, on the contrary, is related to the emergence of a preferred axis, defining the symmetry plane. We use this technique to make contact with recent claims for an anisotropic Universe, showing that the detected preferred axis is associated with positive (even) mirror parity. This feature may be an important clue in identifying the culprit for this unexpected signal.

The properties of physical systems when subject to parity transformations are of great interest and the concept has been extensively used in chemistry, particle physics and condensed matter systems. In this paper we examine the parity properties of the large angle cosmic microwave background (CMB) temperature as rendered by the Wilkinson Microwave Anisotropy Probe (WMAP) [1]. With the exception of the S statistic proposed in [2] this issue has strangely received almost no attention (see also the theoretical work of [3,4]). And yet there are a number of practical and theoretical considerations that make this type of analysis very topical. One should consider separately two types of parity transformations: mirror reflections (i.e., through a plane) and point reflections (relating antipodal points in the sky). They have very different implications.

Several anomalous features in the WMAP data have been reported [2,5–15], pointing toward a preferred direction in the sky, the so-called "axis of evil." The origin of this effect remains mysterious, and it could well be that it is due to foreground contamination or unsubtracted systematic errors. Unlike point reflections, mirror reflections select a preferred direction in the sky, that of the normal to the symmetry plane. Hence the search for mirror handedness entails the search for a preferred axis in the CMB fluctuations (although the converse need not be true). The first purpose of this paper is to investigate whether mirror parity could shed light upon the observed statistical anisotropy of CMB fluctuations.

Should there be a preferred parity or handedness associated with the "axis of evil" effect such a fact could be crucial in identifying the culprit. It has been suggested that the preferred axis could be the signature of a nontrivial cosmic topology [2,15–19], anisotropic expansion [20,21], or even intrinsic cosmic inhomogeneity [22]. In this context the handedness of the sky, if present, could supply selections for viable theoretical models.

A major open question in all of these studies is galactic foreground contamination. This has been ruled out in a variety of ways but remains an ongoing concern in all CMB work. Point reflection symmetry, in contrast with mirror symmetry, tests statistical homogeneity. Handedness with respect to point reflections could only be seen from a cosmic vantage point, the focal point of the symmetry. Such an observation would be cataclysmic for any theory of the Universe: even though some topological and inhomogeneous models [22] do violate translational invariance, that we might live in a privileged point remains extremely unlikely in *any* cosmology. But this leads to a very practical tool: if evidence for point reflection handedness were found this would most probably indicate foreground galactic contamination, since with respect to these the foregrounds we are in fact "at the center of the world". Worryingly, the well-documented low quadrupole (see e.g., [23–25]) could be the tip of the iceberg revealing a preference for odd point parity, and thus galactic contamination. We investigate this matter quantitatively in this paper.

We define parity with respect to reflections through the origin as $\mathbf{x}' = -\mathbf{x}$ and for reflections through a plane as $\mathbf{x}' = \mathbf{x} - 2(\mathbf{x} \cdot \mathbf{n})\mathbf{n}$ (for a mirror with normal $\mathbf{n}$). Let P be one of these parity transformations. Then from a map M one may extract a parity reversed map $\tilde{M} = PM$, and define the positive and negative parity components:

$$M^{\pm} = \frac{M \pm \tilde{M}}{2} \qquad (1)$$

One has that $M = M^+ + M^-$ and $PM^{\pm} = \pm M^{\pm}$. A measure of handedness is generally a comparison between the two components M^+ and M^-.

For this purpose we expand M^+ and M^- into spherical harmonics, defined by:

$$\frac{\Delta T}{T}(\hat{\mathbf{r}}) = \sum_\ell \delta T_\ell = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{r}}) \qquad (2)$$

and evaluate the power spectrum $\hat{C}_\ell$, defined as $(2\ell + 1)\hat{C}_\ell = \sum_m |a_{\ell m}|^2$, for each of these maps. For point reflections only even (odd) ℓ multipoles appear in M^+ (M^-) and hence only their even (odd) $\hat{C}_\ell$ components are non-zero. A sign of point-parity handedness would therefore be intermittency in the power spectrum i.e., fluctuations in power preferring alternate multipoles. Such a phenomenon would enhance one of the maps $M^\pm$ with respect to the other.

For mirror symmetries, on the other hand, only modes with even (odd) $\ell + m$ appear in M^+ (M^-). The power spectrum of $M^\pm$ is therefore

$$\hat{C}_\ell^\pm(\mathbf{n}_\ell) = \frac{1}{2\ell + 1} \sum_m p_{\ell m}^\pm |a_{\ell m}|^2 \qquad (3)$$

where $p_{\ell m}^\pm$ is 1 or 0 depending on the parity of $\ell + m$. Here $\mathbf{n}_\ell$ is the z-axis used to evaluate the expansion (2), and we stress that the decomposition into even and odd modes depends on its choice (notice that $\mathbf{n}_\ell$ may be different for different ℓ). Mirror handedness is signalled by a dominance of $\hat{C}_\ell^+$ over $\hat{C}_\ell^-$ or vice versa, and so it is another way of assessing whether some m modes are preferred over others (see [15,26,27]). As explained in [15,26,27] measures of m-preference are intrinsically measures of statistical anisotropy, since they are linked to the choice of $\mathbf{n}_\ell$. Later on in this paper we shall relate our proposal for a measure of asymmetry between $\hat{C}_\ell^+$ and $\hat{C}_\ell^-$ to other choices of measures of statistical anisotropy and their choices of $\mathbf{n}_\ell$.

We now make concrete proposals for handedness statistics, starting with point-parity. As the middle panel of Fig. 1 shows, the power spectrum $\ell(\ell + 1)\hat{C}_\ell$ displays a distinctive pattern of alternate low and high values, starting with the much publicized low quadrupole [23–25] and extending up to $\ell = 9$. Worryingly this is present in all renditions of the data. As visually striking as this feature may be, it is important to quantify its significance. We choose the following statistic to assess the significance of intermittency:

$$S_p = \sum_3^{l_{max}} \frac{\ell(\ell + 1)\hat{C}_\ell}{\ell(\ell - 1)\hat{C}_{\ell-1}} \qquad (4)$$

where the sum is over odd ℓ (i.e., considering ratios of adjoining pairs, without overlap). S_p measures point-parity preference for quasi-scale-invariant spectra, with $S_p \gg 1$ representing odd parity and $S_p \ll 1$ even parity. The fact that the best-fit spectrum is not scale-invariant induces a bias in S_p, but this is also present in Monte-Carlo simulations performed to evaluate the significance of any anomaly.
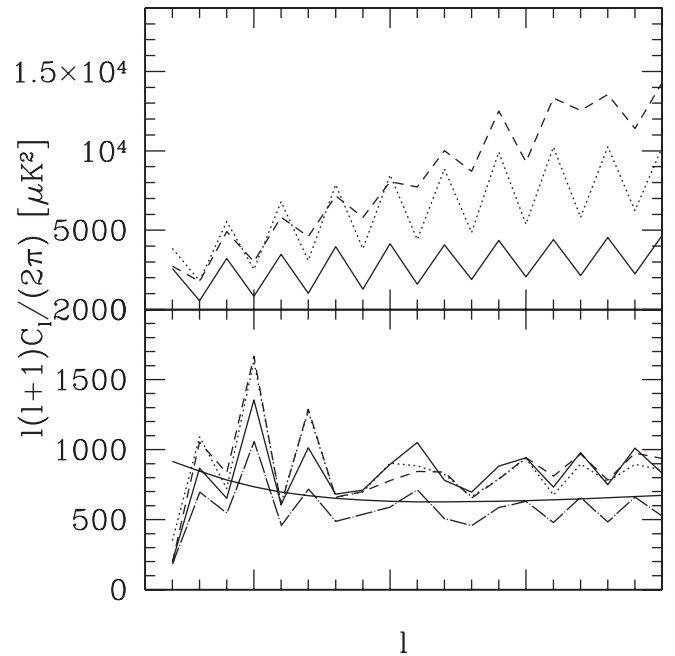

FIG. 1. Top panel: the power spectrum in the V band of templates for dust (solid), synchrotron (dotted) and free-free (dash). Bottom panel: the measured power spectrum $\hat{C}_\ell$ of WMAP, against the best-fit model (solid). We depict four renditions: ILC (dashed), TOH cleaned (solid jagged), TOH Wiener filtered (dot-dash) and LILC (dotted); see text for an explanation.

As argued above point-parity preference would signal that we live, as it were, in the center of the world, a fact most easily explained by foreground contamination. We now explain how point-parity, specifically in connection with the proposed statistic S_p, may be used as a practical tool for detecting foregrounds. In the top panel of Fig. 1 we see that galactic templates do display a clear intermittency in power, favoring even multipoles. This is promptly formalized by our statistic S_p: for $\ell_{max} = 19$ we find $S_p = 0.37, 0.51, 0.80$ for dust, synchrotron and free-free emissions, respectively, (see Fig. 2.) We have used the V band for definiteness, but other bands and choices of ℓ_{max} reveal the same strong signature of even-handedness. Apart from the free-free map, these values are anomalous well beyond the 99% confidence level.

In contrast, as already visually guessed, the CMB prefers odd point-parity, and Fig. 1 (bottom panel) reveals $S_p \gg 1$ for several ℓ ranges. If this feature were statistically significant it could be due to over-correcting for galactic foregrounds. Fortunately this is not the case, as shown by Monte-Carlo simulations. To build intuition in Table I we have displayed the average value of S_p and its r.m.s. inferred from simulations of Gaussian maps with the best-fit power spectrum, subject to the WMAP noise and beam (see also Fig. 2.) We then considered, as an example, the cleaned maps of [28] (which we denote TOHc) and evaluated S_p for this map for a variety of ℓ_{max}. By asking
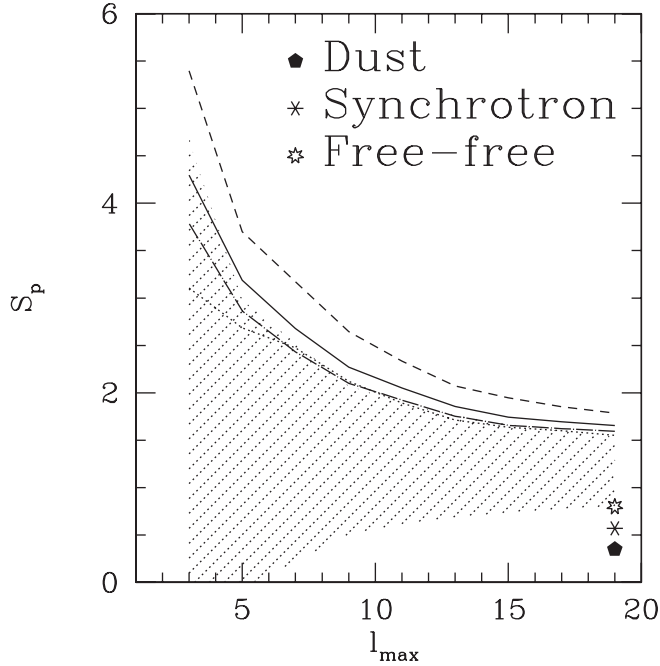
FIG. 2. The values of the S_p statistics as a function of ℓ_{max} for ILC (dashed), TOH cleaned (solid jagged), TOH Wiener filtered (dot-dash) and LILC (dotted). The shaded band represents the "1-sigma" confidence region (but notice that the actual distributions are very skewed.) The points at $\ell_{max} = 19$ refer to S_p for galactic templates as indicated in the Figure.

what percentage of simulations display a larger S_p we can evaluate the confidence level for detecting even point-parity preference. This is never above 97% and is in fact below 95% for the most visually striking features. Thus what by eye appears as a very striking feature, is actually not significant under closer scrutiny.

We have considered other renditions of the dataset: the Wiener filtered maps of [28] (TOHw); the internal linear combination maps of [29] (ILC), and the Lagrange multipliers internal linear combination maps of [9] (LILC). In the bottom panel of Fig. 1 we plot S_p for various l_{max} for all

TABLE I. The observed value of the S_p statistic in the TOHc map for different values of ℓ_{max}, against its average value and variance as obtained from simulations, and confidence levels for detecting preferred odd point parity in the CMB.

ℓ_{max}	S_p	$\bar{S}_p$	$\sigma(S_p)$	P(reject)
3	4.30	1.72	2.93	0.935
5	3.19	1.45	1.52	0.943
9	2.27	1.29	0.78	0.955
13	1.85	1.22	0.53	0.948
19	1.66	1.17	0.36	0.968
21	1.57	1.16	0.32	0.952
31	1.38	1.12	0.22	0.941
51	1.22	1.09	0.13	0.912

these maps, as well as the 1 sigma contour (which should be interpreted with care given the skewed nature of the distribution). We see that all maps have roughly consistent profiles with the exception of the ILC map, which reveals systematically larger values of S_p. This suggests that the ILC map may be more contaminated by residual artifacts than other full-sky renditions; still we never see evidence for odd point-parity beyond the 97% confidence level.

Reassured by this result we turn to mirror reflections. These depend on the choice of z-axis, and so are associated with or complementary to any statistics seeking statistical anisotropy. In [15] we advocated the use of

$$r_\ell = \max_{m\mathbf{n}} \frac{C_{\ell m}}{(2\ell + 1)\hat{C}_\ell} \tag{5}$$

where $C_{\ell 0} = |a_{\ell 0}|^2$, $C_{\ell m} = 2|a_{\ell m}|^2$ for $m > 0$ (notice that 2 modes contribute for $m \neq 0$). This statistic provides three basic quantities: the direction $\mathbf{n}_\ell$, the "shape" m_ℓ, and the ratio r_ℓ of power absorbed by multipole m_ℓ in direction $\mathbf{n}_\ell$. Essentially it seeks the direction $\mathbf{n}_\ell$ in which the highest ratio of power r_ℓ is concentrated in a single m-mode. Thus it is a statistic for both anisotropy and m-preference.

We can use this statistic to select the direction $\mathbf{n}_\ell$ in which to evaluate $\hat{C}_\ell^\pm$ and assess mirror handedness. The asymmetry between odd and even modes may then be measured by the ratio:

$$r_\ell^+ = \frac{\hat{C}_\ell^+(\mathbf{n}_\ell) - \hat{C}_\ell^-(\mathbf{n}_\ell)}{\hat{C}_\ell} \tag{6}$$

with $\hat{C}_\ell^\pm$ defined in (3), and $\mathbf{n}_\ell$ defined by (5). This complements the work of [15] in that it assesses whether or not an *existing* preferred axis is endowed with mirror-parity handedness. In [15] we found that multipoles $\ell = 2, \ldots, 5$ share a preferred axis, located roughly at $(b, l) \approx (60, -100)$ in galactic coordinates. This extended earlier claims by [2,5], who noted that $\ell = 2, 3$ are uncannily planar (i.e., $m = \pm\ell$ modes) along this axis. We pointed out [15] that the alignment of the preferred axis extends up to $\ell = 5$ but the preferred shape is not planar for $\ell = 4, 5$. The significance of preferred axes' alignment is at the 99.9% level, when the problem is reanalized from this perspective.

As Fig. 3 shows we may now add to this result the information that *all* aligned multipoles have even mirror-parity, that is $r_\ell^+ > 0$. Even though the values of χ^2 associated with these r_ℓ^+ are not anomalous it is interesting to notice that the observed r_ℓ^+ are all above the average r_ℓ^+ instead of scattering below and above it. We have evaluated the distribution of r_ℓ^+ from 5000 Monte-Carlo simulations for Gaussian maps with the best-fit power spectrum, subject to the WMAP noise and beam. This distribution is bimodal i.e., there are two peaks: one for $r_\ell^+ > 0$, another for $r_\ell^+ < 0$. We therefore represented two sets of "average and error bars" in Fig. 3, corresponding to these two peaks. Although we cannot rule out a more efficient statistic, r_ℓ^+
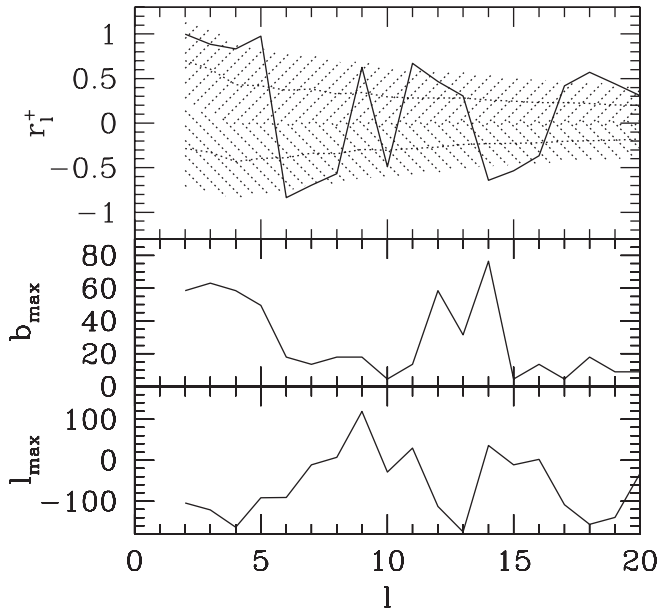
FIG. 3. The statistic r_ℓ^+ for the WMAP data and its distribution inferred from Monte-Carlo simulations. This is bimodal so we plot the peak location and width for the 2 modes (corresponding to even and odd realizations). The bottom two panels supply the preferred axis $\mathbf{n}_\ell$ in galactic coordinates (b, l).

works well (better than S_p) with galactic templates: it picks up their positive mirror-parity at a significance level of several sigma.

We can now ask what is the probability for the observed $r_\ell^+ > 0$, in the range $\ell = 2, \ldots, 5$ of aligned multipoles. We find that on its own the observed handedness is not anomalous: indeed 10% of the simulations reveal features as extreme as the one observed. However this parity feature is found *in connection* with the alignment of $\mathbf{n}_\ell$, which is indeed anomalous at the 99.9% significance level. It should therefore be regarded on the same footing as the alignment of the phases in $a_{\ell m}$ reported in [15], which is not unusual by itself (it is anomalous at the 95% confidence level) but does become interesting in that it qualifies the very anomalous alignment of the axes. Specifically, because mirror-parity and $\mathbf{n}_\ell$ alignment are statistically independent (a fact checked with Monte-Carlo simulations), the joint anomaly

is raised from 99.9% to 99.99%. This number aside, the positive mirror-parity reported in this paper is an interesting possible lead that may help identify the theoretical explanation for the anomaly.

In summary we have investigated the parity properties of the CMB temperature anisotropy, distinguishing between point and mirror-parity. Point-parity handedness would almost certainly be due to galactic foregrounds, thereby providing a contamination detection tool. We do detect even handedness in the galactic templates, but our work was motivated by the pattern of low-high values in the large angle $\hat{C}_\ell$, pointing toward odd parity. This might signal over-correcting for galactic emissions. Fortunately we do not find any evidence for odd parity in publicly available full-sky maps once we study the effect more quantitatively. Interestingly, the ILC maps [29] have the strongest odd parity signal.

Mirror parity was used as a complement to tests of statistical isotropy. If the fluctuations select a preferred axis, as has been claimed, we may ask if they also reveal mirror-parity handedness. The answer is yes: it appears that the "axis of evil" effect is endowed with even mirror parity. Thus the planarity of the quadrupole and octupole (corresponding to $\ell = 2 = |m|$ and $\ell = 3 = |m|$) is just an example for preferred even $\ell + m$ modes for $\ell = 2, \ldots, 5$. This is an interesting remark that may help in explaining the observed result, should it not be due to unmodelled residual foregrounds or systematic errors.

For example it has been suggested that a nontrivial topology induces a large wave in the sky, $\Phi(\mathbf{k}, \eta_{ls})$, with wavelength just outside the horizon. This induces a $m = 0$ mode $a_{\ell m} = A \sum \Phi(\mathbf{k}, \eta_{ls}) i^\ell j_\ell(k\Delta \eta_{ls}) Y_{\ell m}^\star(\hat{\mathbf{k}})$ which could be behind the observed axis (by destructive interference with other modes). The parity of the axis imposes strong constraints on the phase of this wave. We are currently investigating this and other [21] possibilities, in the light of the findings on mirror parity reported in this paper.

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