Erratum: Exclusive double-charmonium production from e^+e^- annihilation into a virtual photon [Phys. Rev. D 67, 054007 (2003)]

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In Ref. [1], we calculated the cross sections for exclusive double-charmonium production from e^+e^- annihilation into a virtual photon. We made an error in the calculation by omitting a relative minus sign between diagrams that differ by the interchange of identical fermions in the final state. That error was also made in Refs. [2,3]. In Ref. [1], the error was the omission of a relative minus sign between the sum of the four Feynman diagrams in Fig. 1 and the sum of the two QED Feynman diagrams in Fig. 2. We also made a separate error in the calculation of the cross section for $\eta_c + h_c$. We show corrected text in italics and corrected numbers in boldface type, except in display equations.

The expression in Eq. (17) for the coefficient A in the amplitude for $J/\psi + \eta_c$ should be corrected as follows:

$$A = \frac{128\pi\alpha_s}{s^2}(\langle O_1\rangle_{J/\psi}\langle O_1\rangle_{\eta_c})^{1/2}\left(\frac{N_c^2-1}{2N_c^2} + \frac{e_c^2\alpha}{N_c\alpha_s} + \frac{1}{r^2}\frac{e_c^2\alpha}{\alpha_s}\right). \tag{17}$$

The sign in front of the $1/r^2$ term has been reversed. In the expression for the coefficient Y in Eq. (19b), the overall sign should also be reversed:

$$Y = -\frac{\alpha}{\alpha_s}\left(1 + \frac{\alpha}{3\alpha_s}\right)^{-1}. \tag{19b}$$

In the subsequent sentence, the numerical value of Y should have the opposite sign: $Y = \mathbf{-0.0344}$.

In Sec. II, the descriptions for the changes in various cross sections due to the QED contribution should be corrected as follows:

(i) Section II C (S wave + S wave) above Eq. (20): If we set $\sqrt{s} = 10.6$ GeV and $m_c = 1.4$ GeV, the electromagnetic correction *increases* the cross section by **29**%.

(ii) Section II D (S wave + P wave) above Eq. (23): If we set $\sqrt{s} = 10.6$ GeV and $m_c = 1.4$ GeV, the QED corrections change the cross sections by **+5.0**%, **−5.5**%, and **+11**% for $J = 0$, 1, and 2, respectively.

(iii) Section II F (S wave + D wave) below Eq. (26): If we set $\sqrt{s} = 10.6$ GeV and $m_c = 1.4$ GeV, the QED correction *increases* the cross section by about **15**%.

(iv) Section II F (S wave + D wave) below Eq. (27): The QED contribution increases the cross section by **41**%.

There are several corrections to the text in Sec. V:

(i) The relativistic corrections increase the central values of the cross sections by about **2** for $J/\psi + \eta_c$, by about **5** for $J/\psi + \eta_c(2S)$, by about **4** for $\psi(2S) + \eta_c$, and by about **8** for $\psi(2S) + \eta_c(2S)$.

(ii) The largest individual correction factor for $J/\psi + \eta_c$ is $(\mathbf{1.28})^2$ coming from the expansion of the amplitude. The corresponding factors for $J/\psi + \eta_c(2S)$, $\psi(2S) + \eta_c$, and $\psi(2S) + \eta_c(2S)$ are $(\mathbf{1.80})^2$, $(\mathbf{1.64})^2$, and $(\mathbf{2.16})^2$, respectively.

TABLE II. Cross sections in fb for e^+e^- annihilation into double-charmonium states $H_1 + H_2$ without relativistic corrections. The errors are only those from variations in the next-to-leading order pole mass $m_c = 1.4 \pm 0.2$ GeV.

$H_2\backslash H_1$	J/ψ	$\psi(2S)$	$h_c(1P)$	$\psi_1(1D)$	$\psi_2(1D)$
η_c	**3.78** ± **1.26**	**1.57** ± **0.52**	**0.308** ± **0.017**	**0.106** ± **0.025**	1.04 ± 0.23
$\eta_c(2S)$	**1.57** ± **0.52**	**0.65** ± **0.22**	**0.128** ± **0.007**	**0.044** ± **0.010**	0.43 ± 0.09
$\chi_{c0}(1P)$	**2.40** ± **1.02**	**1.00** ± **0.42**	0.053 ± 0.019		
$\chi_{c1}(1P)$	**0.38** ± **0.12**	**0.16** ± **0.05**	0.258 ± 0.064		
$\chi_{c2}(1P)$	**0.69** ± 0.13	**0.29** ± **0.06**	0.017 ± 0.002		
$\eta_{c2}(1D)$	**0.35** ± 0.05	**0.14** ± 0.02			

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TABLE III. Cross sections in fb for e^+e^- annihilation into S-wave double-charmonium states $H_1 + H_2$ including relativistic corrections. The errors are only those from variations in the next-to-leading order pole mass $m_c = 1.4 \pm 0.2$ GeV.

$H_2 \setminus H_1$	J/ψ	$\psi(2S)$
η_c	$\mathbf{7.4^{+10.9}_{-4.1}}$	$\mathbf{6.1^{+9.5}_{-3.4}}$
$\eta_c(2S)$	$\mathbf{7.6^{+11.8}_{-4.1}}$	$\mathbf{5.3^{+9.1}_{-2.9}}$

(iii) The lower bound provided by Eq. (46) is about an order of magnitude larger than the central value **3.8** fb of the calculated cross section for $J/\psi + \eta_c$ in Table II.
(iv) The proportion of $J/\psi + \eta_c$, $J/\psi + \eta_c(2S)$, and $J/\psi + \chi_{c0}(1P)$ events, 1.00:0.63 ± 0.25:0.58 ± 0.24, is *consistent* with the proportions 1.00:**0.41**:**0.63** of the cross sections in Table II.
(v) The cross sections for $J/\psi + \chi_{cJ}(1P)$ for $J = 1$ and 2 are predicted to be smaller than for $J = 0$ by factors of about **0.16** and **0.29**, respectively.
(vi) The prediction in Table II for the cross section for $J/\psi + \eta_{c2}(1D)$ is smaller than that for $J/\psi + \eta_c$ by about a factor **0.09**.
(vii) The prediction in Table II for the cross section for $\psi_2(1D) + \eta_c$ is smaller than that for $J/\psi + \eta_c$ only by about a factor **0.27**.

In Ref. [1], we also made an error in the calculation of the cross section for $\eta_c + h_c$. The result that was obtained for the ratio R in Eq. (23) was zero. The correct result is

$$R[\eta_c + h_c] = \frac{\pi^2\alpha_s^2}{144} X^2 H(r) r^4 (1 - r^2)^{1/2} \frac{\langle O_1\rangle_{\eta_c}\langle O_1\rangle_{h_c}}{m_c^8}, \tag{23}$$

where the function $H(r)$ is

$$H(r) = 2r^2(r^2 - 2)^2 + (3r^4 - 6r^2 + 4)^2.$$

The dependence on α appears only in the overall factor X^2 because the QED contribution comes from diagrams with the same topology as the QCD diagrams in Fig. 1 of Ref. [1]. The QED contribution from the photon-fragmentation diagrams in Fig. 2 of Ref. [1] vanishes because the + parity of h_c does not allow the direct coupling to a single virtual photon. The pure QCD contribution to the result in Eq. (23) is obtained by substituting $X = 4/9$. This result was first calculated correctly in Ref. [4]. The angular distribution for $\eta_c + h_c$ is

$$\frac{dR}{dx}[\eta_c + h_c(\lambda)] = \frac{\pi^2\alpha_s^2}{144} X^2 r^4 (1 - r^2)^{1/2} \frac{\langle O_1\rangle_{\eta_c}\langle O_1\rangle_{h_c}}{m_c^8} H(\lambda, x),$$

where

$$H(0, x) = \tfrac{3}{4}(1 - x^2)(3r^4 - 6r^2 + 4)^2, \qquad H(\pm 1, x) = \tfrac{3}{8}(1 + x^2) r^2 (r^2 - 2)^2.$$

The error in the calculations produced errors in the numerical predictions for some of the cross sections given in Tables II and III of Ref. [1]. Those tables are reproduced here, with the corrected values given in boldface.

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