PHYSICAL REVIEW D 72, 097502 (2005)

# $\eta_c$-glueball mixing and resonance X(1835)

Nikolai Kochelev[1,2,*] and Dong-Pil Min[1,†]

[1]School of Physics and Center for Theoretical Physics, Seoul National University, Seoul 151-747, Korea
[2]BLTP, JINR, Dubna, Moscow region, 141980 Russia
(Received 6 October 2005; published 8 November 2005)

The mixing of $\eta_c$ and the lowest mass pseudoscalar glueball is estimated within the framework of the instanton liquid model. It is demonstrated that the mixing is large and may explain the difference between the observed mass of the glueball candidate X(1835) and the theoretical prediction of the QCD sum rule analysis.

DOI: 10.1103/PhysRevD.72.097502 PACS numbers: 12.39.Mk, 12.38.Lg, 12.40.Yx

Recently in [1] we presented our arguments to consider X(1835) resonance observed by the BES Collaboration [2] in the reactions $J/\Psi \to \gamma p\bar{p}$ and $J/\Psi \to \gamma\eta'\pi^+\pi^-$ as the lowest mass pseudoscalar glueball [3]. Our interpretation is based on the appearance of the parity doublet structure for high mass hadronic excitations, which can be explained naturally within the instanton model for the QCD vacuum. Thus, we have considered the doublet $[\mathrm{X}(1835), f_0(1710)]$ as the parity doublet of the lowest mass glueballs [6]. Furthermore, the contribution of X(1835) to the flavor singlet axial vector coupling of the proton and its influence to the proton spin problem with the large observed coupling of X(1835) to the $p\bar{p}$ channel were given there.

However, we left unexplained one doubt in interpreting the X(1835) as the lowest pseudoscalar glueball. That is the magnitude of its mass, which is lower than the predicted values of the quenched lattice approach, 2.1–2.5 GeV [10], and the QCD sum rules, $2.05 \pm 0.19$ GeV [11], $2.2 \pm 0.2$ GeV [12].

In this report we provide our conjecture that the $\eta_c$-glueball mixing can be a key factor of adjusting the mass of the lowest pseudoscalar glueball to its experimental value.

The $\eta_c$ has the same quantum numbers as the X(1835), $(I = 0, J^{PC} = 0^{-+})$ and the mass 2.98 GeV which is quite close to the lattice and QCD sum rule estimations of the pseudoscalar glueball mass. So the mixing of the $\eta_c$ with the glueball can be large due to the possible $c\bar{c}$ annihilation to two gluons. We will estimate this mixing by using the instanton model for QCD vacuum [13,14]. The effective interaction responsible for the mixing follows from the gluon-gluon effective interaction induced by instantons [13–15]:

$$\mathcal{L}_{\mathrm{eff}} = \int dU d\rho n(\rho) e^{-(2\pi^2/g_s)\rho^2 U_{ab}\bar{\eta}_{b\alpha\beta}G^a_{\alpha\beta}(x)} + (I \to \bar{I}), \tag{1}$$

where $n(\rho)$ is the effective instanton density, $\rho$ is the instanton size, $g_s$ is a strong coupling constant, $\bar{\eta}_{b\alpha\beta}$ is the 't Hooft symbol, and $U$ is the orientation matrix of the instanton in $SU(3)_c$ color space. The last term in Eq. (1) represents the contribution coming from anti-instantons. We are going to calculate the contribution of the diagram illustrated in Fig. 1 to the nondiagonal matrix element

$$\frac{\Delta M^2_{\eta_c G_P}}{2} = -\langle\eta_c|\mathcal{L}_{\mathrm{eff}}|G_P\rangle. \tag{2}$$

In the vacuum dominance approximation, which is very suitable in the estimation of instanton contributions to various hadron decays (see [16,17]), the contribution corresponding to the diagram in Fig. 1 to the matrix element, Eq. (2), can be written in the following form:

$$\frac{\Delta M^2_{\eta_c G_P}}{2} = \int d\rho n(\rho)\left(\frac{\pi^3\rho^4}{8\alpha_s(\rho)}\right)^2\langle\eta_c|G^a_{\alpha\beta}\tilde{G}^a_{\alpha\beta}|0\rangle \times\langle 0|G^b_{\mu\nu}\tilde{G}^b_{\mu\nu}|G_P\rangle. \tag{3}$$

In the framework of the instanton model of QCD vacuum the matrix element $\langle\eta_c|G^a_{\alpha\beta}\tilde{G}^a_{\alpha\beta}|0\rangle$ has been calculated in [17] following the approach of [18]

$$\langle\eta_c|G^a_{\alpha\beta}\tilde{G}^a_{\alpha\beta}|0\rangle = \frac{64\alpha_s(2m_c)m_c^{3/2}|\Psi(0)|}{\pi^3\rho^4\sqrt{6}} I_{\eta_c}(\rho), \tag{4}$$

$$I_{\eta_c}(\rho) \simeq \frac{\pi^2 A_0\rho^4\log(1 + 1/(m_c\rho))}{1 + B_0(m_c\rho)^4\log(1 + 1/(m_c\rho))}, \tag{5}$$

where $A_0 = 0.213$, $B_0 = 0.124$. In Eq. (4) the $\Psi(0)$ is the ${}^1S_0$ wave function of charmonium at the origin. Our main result is

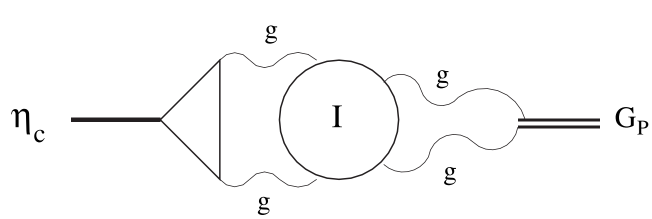


FIG. 1. The $\eta_c$-pseudoscalar glueball $G_P$ mixing induced by the instanton. The symbol $I$ denotes the instanton.

*Electronic address: kochelev@theor.jinr.ru
†Electronic address: dpmin@phya.snu.ac.kr

 

$$\frac{\Delta M^2_{\eta_c G_P}}{2} \simeq \int d\rho n(\rho) \frac{m_c^{3/2}\pi^3\rho^4}{\sqrt{6}\alpha_s^2(\rho)} |\Psi(0)| I_{\eta_c}(\rho) \times \langle 0|\alpha_s G^b_{\mu\nu}\tilde{G}^b_{\mu\nu}|G_P\rangle. \tag{6}$$

According to the instanton liquid model by Shuryak [19], the instanton density is given by

$$n(\rho) = n_0\delta(\rho - \rho_c), \tag{7}$$

where $n_0 \approx 0.5$ fm$^{-4}$ and $\rho_c \approx 1/3$ fm. We adopt our parameter values to fit the properties of charmonium, $m_c = 1.25$ GeV, $|\Psi(0)| = 0.19$ GeV$^{3/2}$ as in [17], and of the strong coupling constant at the average instanton size $\alpha_s(\rho_c) = 0.52$ as in [14]. The coupling of X(1835) to gluons was obtained in our previous paper [1]

$$f_{G_P} = \langle 0|\alpha_s G^b_{\mu\nu}\tilde{G}^b_{\mu\nu}|G_P\rangle \simeq 2.95 \text{ GeV}^3. \tag{8}$$

This value is consistent with the result of the recent QCD sum rule analysis $f_{G_P} = 2.9 \pm 1.4$ GeV$^3$ [12]. Our estimate of the mixing is

$$\Delta M^2_{\eta_c G_P} \simeq 1.54 \text{ GeV}^2. \tag{9}$$

Now we are in the position to evaluate the effect of mixing on the mass of the pseudoscalar glueball by using the following decomposition of physical charmonium and glueball states:

$$\begin{aligned} |\eta_c\rangle &= |G_P^0\rangle \sin\theta + |\eta_c^0\rangle \cos\theta, \\ |G_P\rangle &= |G_P^0\rangle \cos\theta - |\eta_c^0\rangle \sin\theta, \end{aligned} \tag{10}$$

where $|\eta_c^0\rangle$ and $|G_P^0\rangle$ are bare states. Let us assume that bare masses of glueball and $\eta_c$ are following

$$M^0_{G_P} = 2 \text{ GeV}, \qquad M^0_{\eta_c} = 2.9 \text{ GeV}. \tag{11}$$

These values lie inside the range of the QCD sum rules expectation [11,12,20,21] if one admits about 10% accuracy in the predictions of this approach due to uncertainties in the values of various gluon condensates, the mass of the charm quark and $\alpha_s$, high dimension operator contributions, etc. [22]. As the result of the mixing Eq. (9), the physical masses and mixing angle are

$$M_{G_P} \simeq 1.87 \text{ GeV}, \qquad M_{\eta_c} \simeq 2.98 \text{ GeV}, \qquad \theta \simeq 17°. \tag{12}$$

Therefore, the mixing leads to the increasing of the $\eta_c$ mass to its experimental value and decreasing of the pseudoscalar glueball mass towards the mass of glueball candidate X(1835). The value of the mixing angle, Eq. (12), is rather large and should be taken into account in the calculation of different properties of $\eta_c$ and the pseudoscalar glueball with decay modes. In this connection we may point out that the mixing might be present behind the observed large decay rates of $\eta_c$ to $p\bar{p}$ and $\eta'\pi\pi$ final states [23] due to the large coupling of the glueball to these channels.

In principle, the fine-tuning of the parameters allows us to bring the mass of the glueball to the observed one. However, we think that such a procedure is beyond the accuracy of our approach based on the vacuum dominance approximation and definite instanton model for QCD vacuum. Furthermore, before the tuning process, some additional effects such as the glueball mixing with $\eta_c(2S)$, $\eta'$, and others, which are beyond the scope of the present paper, should be taken into account.

In summary, we have shown that the instanton induced mixing of the charmonium and the pseudoscalar glueball is large and may explain the difference between the experimental mass of the glueball candidate X(1835) and the prediction of QCD sum rules. This observation provides the additional argument in favor of our suggestion in [1] to treat the X(1835) as the lowest mass pseudoscalar glueball.

We are grateful to A. E. Dorokhov, S. B. Gerasimov, Xiao-Gang He, R. N. Faustov, H. Forkel, S. Narison, A. A. Pivovarov, San Fu Tuan, and Shi-Lin Zhu for useful discussions. This work was supported by the Brain Pool program of the Korea Research Foundation through KOFST, Grant No. 042T-1-1, and in part by grants from the Russian Foundation for Basic Research, No. RFBR-03-02-17291 and No. RFBR-04-02-16445 (N. K.). N. K. is very grateful to the School of Physics, SNU, for their warm hospitality during this work.

[1] Nikolai Kochelev and Dong-Pil Min, hep-ph/0508288.
[2] J. Z. Bai *et al.* (BES Collaboration), Phys. Rev. Lett. **91**, 022001 (2003); M. Ablikim *et al.* (BES Collaboration), hep-ex/0508025.
[3] The possibility of a strong X(1835) coupling to gluons was pointed out by Rosner in [4]. The values of this coupling and X(1835) decay modes were discussed recently in [5] as well.
[4] J. L. Rosner, hep-ph/0508155; Phys. Rev. D **68**, 014004 (2003).
[5] Xiao-Gang He, Xue-Qian Li, Xiang Liu, and J. P. Ma, hep-ph/0509140.
[6] There are various points of view on the structure of the scalar meson $f_0(1710)$. Some authors treat it as a scalar glueball [7] or a standard quark-antiquark state [8], and others suggest their strong mixture [9]. In our consideration of mixing the pseudoscalar glueball with the $\eta_c$ meson the structure of the scalar $f_0(1710)$ is not crucial.
[7] M. Chanowitz, hep-ph/0506125.
[8] A. V. Anisovich *et al.*, Phys. Rev. D **62**, 051502(R) (2000);

V. V. Anisovich and A. V. Sarantsev, Eur. Phys. J. A **16**, 229 (2003).
[9] F. E. Close and A. Kirk, Phys. Lett. B **483**, 345 (2000).
[10] C. J. Morningstar and M. Peardon, Phys. Rev. D **56**, 4043 (1997); A. Hart and M. Teper, Phys. Rev. D **65**, 034502 (2002); Weon-Jong Lee and D. Weingarten, Phys. Rev. D **61**, 014015 (2000); C. Michael, hep-lat/0302001.
[11] S. Narison, Nucl. Phys. **B509**, 312 (1998).
[12] H. Forkel, Phys. Rev. D **71**, 054008 (2005).
[13] T. Schäfer and E. V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).
[14] D. Diakonov, Prog. Part. Nucl. Phys. **51**, 173 (2003).
[15] A. I. Vainshtein, V. I. Zakharov, V. A. Novikov, and M. A. Shifman, Usp. Fiz. Nauk **136**, 553 (1982) [Sov. Phys. Usp. **24**, 195 (1982)]; *ITEP Lectures on Particle Physics and Field Theory* (World Scientific, Singapore, 1999), Vol. 1, pp. 201–299.
[16] N. I. Kochelev and V. Vento, Phys. Rev. Lett. **87**, 111601 (2001).
[17] V. Zetocha and T. Schafer, Phys. Rev. D **67**, 114003 (2003).
[18] M. Anselmino and S. Forte, Phys. Lett. B **323**, 71 (1994).
[19] E. V. Shuryak, Nucl. Phys. **B203**,93 (1982); **B203**, 116 (1982); **B203**, 140 (1982).
[20] N. Brambilla *et al.*, hep-ph/0412158.
[21] K. Zyablyuk, J. High Energy Phys. 01 (2003) 081.
[22] Unfortunately, the available lattice results for the lowest mass pseudoscalar glueball have been performed in the quenched approximation [10]. It is rather hard to estimate the accuracy of such an approximation in the pseudoscalar channel where the light quark exchange between instantons plays an essential role (see the discussion in [1]).
[23] S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).