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Earth matter density uncertainty in atmospheric neutrino oscillations

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The fact that muon neutrinos ν_μ oscillating into the mixture of tau neutrinos ν_τ and sterile neutrinos ν_s has been studied to explain the atmospheric ν_μ disappearance. In this scenario, the effect of Earth matter is a key in determining the fraction of ν_s. Considering that the Earth matter density has uncertainty and this uncertainty has significant effects in some neutrino oscillation cases, such as the *CP* violation in very long baseline neutrino oscillations and the day-night asymmetry for solar neutrinos, we study the effects caused by this uncertainty in the above atmospheric ν_μ oscillation scenario. We find that this uncertainty seems to have no significant effects and that the previous fitting results need not be modified.

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To explain the atmospheric muon neutrinos ν_μ disappearance, the scenario of ν_μ oscillating into ν_+ has been studied [1–3], where ν_+ is the mixture of tau neutrinos ν_τ and sterile neutrinos ν_s, and defined as $\nu_+ = \nu_\tau \cos\xi + \nu_s \sin\xi$. Since for so-called "matter effects" [4], the oscillation probabilities $P_{\nu_\mu \to \nu_\tau}$ and $P_{\nu_\mu \to \nu_s}$ are different for a muon neutrino with certain energy that travels a distance on Earth, one can expect to give a limit on ξ. The reported results from Superkamiokande have given limits on $\sin^2\xi$ [1–3].

In the calculation of $P_{\nu_\mu \to \nu_s}$, the neutron number density of Earth is a critical quantity. However, today the knowledge of Earth matter density which determines the neutron number density is only to some certain precision [5]. As to the preliminary reference Earth model (PREM) [6], the uncertainties due to the local variation have been documented [7]. Quantitatively its precision is roughly 5% averaged per spherical shell with a thickness of 100 km or so [8].

The effects of the Earth matter density uncertainty have been studied in some neutrino oscillation cases, such as the *CP* violation in very long baseline neutrino oscillations [9,10] and the day-night asymmetry for solar neutrinos [11]. One finds this uncertainty has significant effects in these cases [9–11]. Since the Earth matter is a key in determining the fraction of ν_s, this uncertainty could also have an effect on the limit of $\sin^2\xi$. In this brief report, we study the density uncertainty in Earth matter and then investigate its implications on the results of $\sin^2\xi$.

We begin our discussion with the effective Hamiltonian that governs the propagation of the neutrinos in matter. In the (2 + 2) models [12], the relevant (ν_μ, ν_+) evolution is given by the Schrödinger equation

$$i\frac{d}{dx}\begin{pmatrix} \nu_\mu \\ \nu_+ \end{pmatrix} \simeq H(x)\begin{pmatrix} \nu_\mu \\ \nu_+ \end{pmatrix} \tag{1}$$

with the effective Hamiltonian [1]

$$H(x) = \frac{\Delta m^2}{4E_\nu}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \sqrt{2}G_F\begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}\sin^2\xi N_n(x) \end{pmatrix}. \tag{2}$$

Here E_ν is the neutrino energy, Δm^2 and θ are the usual mass and mixing parameters in the $\nu_\mu \to \nu_\tau$ oscillating model, G_F is the Fermi constant, and $N_n(x) = \rho(x)N_A(1 - Y_e(x))$ is the neutron number density with $\rho(x)$ the matter density in $\mathrm{g/cm^3}$, N_A the Avogadro number and $Y_e(x)$ the electron number fraction, respectively. For increasing values of $\sin^2\xi$, we get a smooth interpolation from $\nu_\mu \to \nu_\tau$ oscillations ($\sin^2\xi = 0$) to pure $\nu_\mu \to \nu_s$ oscillations ($\sin^2\xi = 1$), passing through mixed active-sterile transitions ($0 < \sin^2\xi < 1$). Replacing $N_n(x)$ by $-N_n(x)$, we can also get the effective Hamiltonian for relevant antineutrinos.

Now we consider the uncertainty in Earth matter and its implications on the atmospheric neutrino oscillations. In some simple cases, for example, if the neutron number density $N_n(x)$ suffers from a global shift (independent of x), the induced effects on $\sin^2\xi$ and $N_n(x)$ are degenerate in the effective Hamiltonian (2), such as $\sin^2\xi \to \frac{1}{(1\% \pm 5\%)}\sin^2\xi$ when $N_n(x) \to (1\% \pm 5\%)N_n(x)$.

Generally, at a given point x on Earth, the available matter density, which determines the neutron number density, is an average value with some prescribed errors, such as the widely used PREM model [6]. We can define the average density $\hat{\rho}(x)$ as an average over all samples of density profiles $\{\rho(x)\}$

$$\hat{\rho}(x) \equiv \langle \rho(x) \rangle = \int \mathcal{D}[\rho(x)]F[\rho(x)]\rho(x), \tag{3}$$

and the error $\sigma(x)$ as a variance function

$$\sigma(x) \equiv \sqrt{\langle \rho^2(x) \rangle - \langle \rho(x) \rangle^2}, \tag{4}$$

where $F[\rho(x)]$ is the probability density of the density sample $\rho(x)$. Accordingly, the averaged probability for the α flavor neutrino oscillating into the β flavor neutrino

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should be

$$\langle P_{\nu_\alpha\to\nu_\beta}(L, E_\nu)\rangle \equiv \int \mathcal{D}[\rho(x)]F[\rho(x)]P_{\nu_\alpha\to\nu_\beta}(L, E_\nu, \rho(x), Y_e(x)), \tag{5}$$

with L the neutrino's traveling distance on Earth. Furthermore, we can write the variance as

$$\delta P_{\nu_\alpha\to\nu_\beta}(L, E_\nu) \equiv \sqrt{\int \mathcal{D}[\rho(x)]F[\rho(x)](P_{\nu_\alpha\to\nu_\beta}(L, E_\nu, \rho(x), Y_e(x)) - \langle P_{\nu_\alpha\to\nu_\beta}(L, E_\nu)\rangle)^2}. \tag{6}$$

In this brief report, we introduce a logarithmic normal distribution [13] to represent the probability density function of the Earth matter density samples

$$F[\rho(x)] = \frac{1}{\rho(x)\sqrt{2\pi s^2(x)}}\exp\left(-\frac{\ln^2[\rho(x)/\rho_0(x)]}{2s^2(x)}\right), \tag{7}$$

$$s(x) = \sqrt{\ln[1 + r^2(x)]}, \tag{8}$$

$$\rho_0(x) = \hat{\rho}(x)\exp[-\tfrac{1}{2}s^2(x)], \tag{9}$$

where $r(x) = \sigma(x)/\hat{\rho}(x)$ characterizes the precision of the Earth matter density. And then we use Monte Carlo calculations to generate the values of $F[\rho(x)]$ between 0 and 1 at a given point x along the propagating path of the neutrinos. With the chosen $\hat{\rho}(x)$ and $r(x)$, we obtain the value of $\rho(x)$ from Eq. (7) by computing the values of $s(x)$ in Eq. (8) and $\rho_0(x)$ in Eq. (9). Hence, the averaged oscillation probability (5) and the corresponding variance (6) can be calculated. Specifically, we take $Y_e(x) = 0.5$ and $r(x) = 5\%$ with $\hat{\rho}(x)$ given by the PREM in our numerical calculations.

Usually, the experiment results are reported as the event number, which can be calculated as

$$\begin{aligned} N &= \int_{E_l}^{E_u} N_0\phi_{\nu_\alpha}(E_\nu)\langle P_{\nu_\alpha\to\nu_\beta}(L, E_\nu)\rangle\sigma_\beta(E_\nu)CdE_\nu \\ &= \sum_{k=1}^{n} N_0\phi_{\nu_\alpha}(E_\nu^k)\langle P_{\nu_\alpha\to\nu_\beta}(L, E_\nu^k)\rangle\sigma_\beta(E_\nu^k)C\Delta E_\nu^k, \end{aligned} \tag{10}$$

where E_u, E_l denote the upper bound and lower limit of the detecting energy, N_0 is a normalization factor with unit conversions, ϕ_{ν_α} is the α flavor neutrino beam flux spectrum, σ_β is the charged current cross section of β flavor neutrino, C is the product of the detector's size and running time, and ΔE_ν^k is the kth energy bin size. Accordingly, we can define

$$\begin{aligned} \delta N &= \int_{E_l}^{E_u} N_0\phi_{\nu_\alpha}(E_\nu)\delta P_{\nu_\alpha\to\nu_\beta}(L, E_\nu)\sigma_\beta(E_\nu)CdE_\nu \\ &= \sum_{k=1}^{n} N_0\phi_{\nu_\alpha}(E_\nu^k)\delta P_{\nu_\alpha\to\nu_\beta}(L, E_\nu^k)\sigma_\beta(E_\nu^k)C\Delta E_\nu^k, \end{aligned} \tag{11}$$

$$r_N = \frac{\delta N}{N}, \tag{12}$$

as the variance and relative variance of the event number caused by the uncertainty in Earth matter density, respectively.

For example, in the numerical calculations we take the relevant data listed in Tables II, IV, and V of [14] and Table III of [15]. We use the mass and mixing parameters: $\Delta m^2 = 2.0 \times 10^{-3}$ eV^2, $\sin^2 2\theta = 1.0$, $\sin^2\xi = 0.19$ [3], and neutrino traveling distance $L = l - 2r\cos\Theta$, where l is the neutrino production height in atmosphere (slant

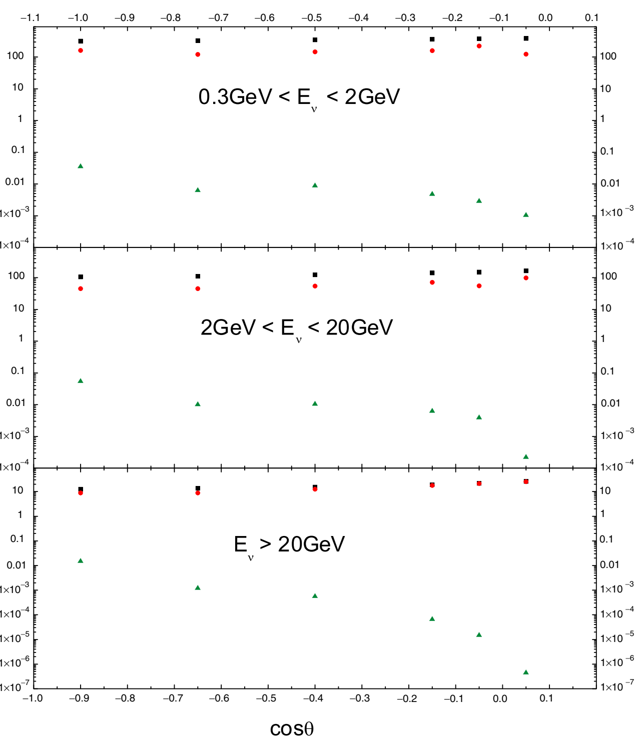

FIG. 1 (color online). Event numbers plotted as a distribution with zenith angle $\cos\Theta$ in three neutrino energy bins. The squares are Honda *et al.* [14] expectations for no oscillations. The dots are given by substituting $\langle P\rangle$ for P in Eq. (8) and up-triangles represent the variance defined in Eq. (9). These plots use the mass and mixing parameters: $\Delta m^2 = 2.0 \times 10^{-3}$ eV^2, $\sin^2 2\theta = 1.0$, $\sin^2\xi = 0.19$ [3], and the neutrino traveling distance $L = l - 2r\cos\Theta$, where l is the neutrino production height in the atmosphere (slant distance in km, listed in [15]) and $r = 6371$ km is the Earth's radius.

distance in km, listed in [15]), $r = 6371$ km is the Earth's radius and $\cos\Theta$ is the zenith angle. We plot the event number produced by the atmospheric muon neutrinos and antineutrinos in six zenith angles with three energy bins as well as their variances arising from the uncertainty of Earth matter density by using Eqs. (10) and (11). As shown in Fig. 1, we find the variance of the event number is smaller than $\sim 10^{-3}$; accordingly the relative variance is smaller than $\sim 10^{-4}$, for the longest baseline $\cos\Theta = -1.0$ with the largest matter effect, while in the other baselines, the variance and also the relative variance are much smaller. Hence we can draw a conclusion that the uncertainty of Earth matter density seems to have no significant effects in the oscillating scenario of $\nu_\mu \rightarrow \nu_\tau \cos\xi + \nu_s \sin\xi$.

In summary, considering that the Earth matter density has uncertainty and this uncertainty has significant effects in some neutrino oscillation cases, such as the *CP* violation in very long baseline neutrino oscillations and the day-night asymmetry for solar neutrinos, we study this uncertainty in the atmospheric neutrino oscillating scenario of $\nu_\mu \rightarrow \nu_\tau \cos\xi + \nu_s \sin\xi$, and analyze the effects caused by this uncertainty on the previous fitting results. We find that this uncertainty seems to have no significant effects and we need not modify the previous fitting results.

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