Weak annihilation topologies and final state interactions in $D \to PP$ decays

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(Received 10 September 2005; published 14 November 2005)

We study two-body $D \to PP$ decays, assuming that each decay process go through the *bare* amplitude followed by elastic SU(3) rescattering, where the *bare* amplitude consists of (i) the color-allowed and color-suppressed factorization amplitudes and (ii) the short-distance weak annihilation amplitudes. We have performed the χ^2 fit on 14 branching ratios of $D \to PP$ decays in the formalism of the above mentioned model. The final state interactions can be well accounted for by the short-distance annihilation topologies and SU(3) rescatterings. The two SU(3) rescattering phase differences are $\delta \equiv \delta_{27} - \delta_8 \simeq -46°$ and $\sigma \equiv \delta_{27} - \delta_1 \simeq -21°$, where δ_{27}, δ_8, and δ_1 are the rescattering phases of final states corresponding to the representations **27**, **8**, and **1**, respectively. We find that the $D^0 \to K^0\overline{K}^0$ decay occurs mainly due to the nonzero short-distance weak annihilation effects, originating from SU(3) symmetry-breaking corrections to the distribution amplitudes of the final-state kaons, but receives tiny effects from other modes via SU(3) rescattering. Our results are in remarkable accordance with the current data.

DOI: 10.1103/PhysRevD.72.096001 PACS numbers: 11.30.Hv, 12.39.St, 13.25.Ft

I. INTRODUCTION

It is known that the naive factorization approximation fails to describe the color-suppressed D decays. The results can be improved if the Fierz-transformed terms characterized by $1/N_c$ are discarded [1]. The short-distance (SD) weak annihilation effects, which may mimic some nonresonant final state interactions, have recently been emphasized in two-body B decays [2–5]. In D decays, the SD weak annihilation contributions involving gluon emission from the final-state quarks, which arise from the $(V - A) \otimes (V - A)$ four-quark operators, vanish. Nevertheless, if the gluon is emitted from the initial quarks, the SD weak annihilation effects are not zero (see the results shown in Sec. II B) and may give sizable corrections to the amplitudes. Such effects were first noticed by Li and Yeh [2,3] and recently discussed in B decays [4,5]. One therefore expects that the SD weak annihilation may play an important role in D decays because the energy released to the final-state particles is not as large as that in B decays. Unfortunately, the SD weak annihilation topologies, in general, are not calculable in the QCD factorization approach.[1]

The color-suppressed $\overline{B}^0 \to D^{(*)0}(\pi^0, \eta, \omega)$, $\overline{B}^0 \to D^0(\eta', \overline{K}^0)$, and $B^- \to D_s K^-$ decay modes have recently been observed by the Belle, CLEO, and *BABAR* collaborations [8–12]. These branching ratios (BRs) are much larger than the expectation in the factorization-based analysis [13]. Using the isospin amplitude analysis of $\overline{B}^0 \to D^0\pi^-$, $\overline{B}^0 \to D^+\pi^-$ and $\overline{B}^0 \to D^0\pi^0$, one can obtain that the rescattering phase difference of isospin amplitudes $A_{3/2}$ and $A_{1/2}$ is about 30° [14]. It may indicate that long-distance (LD) final state interactions (FSIs) are not negligible even in B meson decays [14]. Analogously, larger FSIs could be expected in D meson decays since the energy released in D decays is much less than that in B decays as mentioned above. For illustrating this point, we perform the isospin decomposition for $D^0 \to K^-\pi^+, \overline{K}^0\pi^0$ and $D^+ \to \overline{K}^0\pi^+$ decay amplitudes:

$$\begin{aligned} A(D^0 \to K^-\pi^+) &= \sqrt{\frac{1}{3}}A_{3/2} + \sqrt{\frac{2}{3}}A_{1/2}, \\ A(D^0 \to \overline{K}^0\pi^0) &= \sqrt{\frac{2}{3}}A_{3/2} - \sqrt{\frac{1}{3}}A_{1/2}, \\ A(D^+ \to \overline{K}^0\pi^+) &= \sqrt{3}A_{3/2}, \end{aligned} \tag{1.1}$$

where the isospin amplitudes with isospin 3/2 and 1/2 are denoted as $A_{3/2}$ and $A_{1/2}$, respectively. The relative rescattering phase between $A_{3/2}$ and $A_{1/2}$, denoted as ϕ, satisfies the following relation,

[1] One may introduce the transverse momenta of quarks ($\mathbf{k}_\perp$) to regulate the endpoint divergence, where $\mathbf{k}_\perp$ is naturally constrained by the infrared cutoff $\sim 1/R$ with R the meson's radius (some other discussions can be found in Ref. [4]). However, the result may suffer from the gauge problem and is part of higher twist contribution. It is interesting to note that in deeply inelastic scattering (DIS) processes, by introducing a generalized special propagator [6] for massive quarks [7], the separation of the hard part (T) from the soft part (parton distributions) is manifestly gauge invariant for different orders in $1/Q$ (twist). An important feature of using the special propagator technique is that the $\mathbf{k}_\perp$ contributions should be moved into T, such that, after combining with the gluon field A^α in T, a covariant derivative of color gauge invariance can be achieved and classified as a high-twist contribution.

$$\cos\phi = \frac{|A(D^0 \to K^-\pi^+)|^2 - 2|A(D^0 \to \overline{K}^0\pi^0)|^2 + \frac{1}{3}|A(D^+ \to \overline{K}^0\pi^+)|^2}{2\sqrt{2}|A_{1/2}||A_{3/2}|}. \tag{1.2}$$

Substituting the data for BRs of $D^0 \to K^-\pi^+, \overline{K}^0\pi^0$ and $D^+ \to \overline{K}^0\pi^+$ modes, which are $(3.8 \pm 0.09)\%$, $(2.30 \pm 0.22)\%$ and $(2.82 \pm 0.19)\%$ [15], respectively, into Eq. (1.2), one can obtain the rescattering phase $\phi \approx 94°$, much larger than that in the charmful two-body B decays. The above result indicates that FSIs should be significant in D meson decays.

In this article, we will assume that the FSIs in $D \to PP$ are described by SD weak annihilation topologies and elastic (LD) SU(3) rescatterings. Analogously, the elastic final-state rescattering picture has been extended from SU(2)-type to SU(3)-type in B decays [14,16]. We presume that each $D \to PP$ decay process go through the "*bare*" amplitude followed by elastic SU(3) rescattering, where the *bare* amplitude describing the SD-dominant contributions consists of (i) the usual factorization amplitudes of color-allowance and color-suppression, which can be calculated using the factorization approach, and (ii) the SD weak annihilation topologies (W-exchange or W-annihilation) which present the endpoint singularities are regulated by introducing the complex phenomenological parameter X_A [4] in the QCD factorization approach (see the detailed description in Sec. II B).

Interestingly, the SD weak annihilation amplitudes are dominated by the topologies of gluon emission arising from the *initial-state quarks* of the weak vertex, while the total amplitudes vanish in order of α_s if the gluon is emitted from the *final-state quarks*. On the other hand, the elastic SU(3) rescatterings are mainly generated by gluon exchange between the final-state mesons. Therefore, it could be expected that the possible double counting is negligible between the two possible sources for FSIs. We will give a detailed discussion for possible rescattering sources in Sec. V.

We consider the SU(3) breaking effects in the *bare* amplitude level, but, for simplicity, do not distinguish the breaking influence on the two SU(3) rescattering phases, defined as $\delta \equiv \delta_{27} - \delta_8$ and $\sigma \equiv \delta_{27} - \delta_1$. In other words, in description of decay amplitudes, masses vary according to SU(3) breaking, and meson productions differ in strength as reflected in the decay constants and form factors.

The rest of this article is organized as follows. In Sec. II, neglecting elastic SU(3) FSIs, we first sketch the factorization amplitudes as well as the SD weak annihilation contributions in two-body D decays. Section III is devoted to the formulation of SU(3) rescatterings. We give the numerical analysis in Sec. IV. The discussions and summary are presented in Sec. V. The detailed results for the factorization amplitudes, SD weak annihilation amplitudes and tensor approach for the SU(3) final-state decomposition are collected in Appendices A, B, and C, respectively.

II. THE *BARE* AMPLITUDES

Here we present factorization and SD weak annihilation amplitudes for $D \to PP$ decays. The relevant effective Hamiltonian for the charmed meson decays is

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q,q'=d,s} V_{uq}V^*_{cq'}(c_1O_1 + c_2O_2) + \text{h.c.}, \tag{2.1}$$

where G_F is the weak coupling constant, and the current-current operators read

$$O_1 = (\overline{u}q)_{V-A}(\overline{q}'c)_{V-A}, \qquad O_2 = (\overline{u}c)_{V-A}(\overline{q}'q)_{V-A}, \tag{2.2}$$

with $(\overline{u}q)_{V-A} \equiv \overline{u}\gamma^\mu(1-\gamma^5)q$. V_{uq} and $V^*_{cq'}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements given by

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^2(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{2.3}$$

in the Wolfenstein parametrization.

The two ingredients of the "*bare*" amplitude for describing the decay processes are (i) the factorization amplitudes, which are made of the color-allowed external W-emission tree amplitude ($\mathcal{T}$) and/or the color-suppressed internal W-emission amplitude (C), and (ii) the weak annihilation amplitudes which consist of W-exchange and/or W-annihilation topologies.

A. Factorization amplitudes

Taking $D^0 \to K^-\pi^+, \overline{K}^0\pi^0$ as examples, the factorization amplitudes can be written as the following general forms:

$$\mathcal{T}^{K^-\pi^+} = \frac{G_F}{\sqrt{2}} V_{ud}V^*_{cs}a_1 if_\pi(m_D^2 - m_K^2)F_0^{DK}(m_\pi^2), \tag{2.4}$$

$$C^{\overline{K}^0\pi^0} = \frac{G_F}{\sqrt{2}} V_{ud}V^*_{cs}a_2 if_K(m_D^2 - m_\pi^2)F_0^{D\pi}(m_K^2), \tag{2.5}$$

where the superscripts denote the decay modes. Here, the nonfactorizable effects, including the radiative corrections to the weak vertex and the spectator interactions, are absorbed into the parameters $a_{1,2}$ which amount to replace N_c (equals the number of color) by $N_c^{\rm eff}$ such that

$$a_{1,2} = c_{2,1} + c_{1,2}\frac{1}{N_c^{\rm eff}}. \tag{2.6}$$

We have summarized the factorization decay amplitudes in Appendix A, where the physical η' and η states are related to the SU(3) octet state η_8 and singlet state η_0 by

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix}\begin{pmatrix} |\eta_8\rangle \\ |\eta_0\rangle \end{pmatrix}, \tag{2.7}$$

with the mixing angle $\vartheta = -15.4°$ [17], and

$$\begin{aligned} |\eta_0\rangle &= \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d + \bar{s}s\rangle, \\ |\eta_8\rangle &= \frac{1}{\sqrt{6}}|\bar{u}u + \bar{d}d - 2\bar{s}s\rangle. \end{aligned} \tag{2.8}$$

Introducing the decay constants f_8 and f_0 by

$$\langle 0|A_\mu^0|\eta_0\rangle = if_0 p_\mu, \qquad \langle 0|A_\mu^8|\eta_8\rangle = if_8 p_\mu, \tag{2.9}$$

we have

$$\begin{aligned} f_{\eta'}^u &= \frac{f_8}{\sqrt{6}}\sin\vartheta + \frac{f_0}{\sqrt{3}}\cos\vartheta, \\ f_{\eta'}^s &= -2\frac{f_8}{\sqrt{6}}\sin\vartheta + \frac{f_0}{\sqrt{3}}\cos\vartheta, \end{aligned} \tag{2.10}$$

and

$$\begin{aligned} f_{\eta}^u &= \frac{f_8}{\sqrt{6}}\cos\vartheta - \frac{f_0}{\sqrt{3}}\sin\vartheta, \\ f_{\eta}^s &= -2\frac{f_8}{\sqrt{6}}\cos\vartheta - \frac{f_0}{\sqrt{3}}\sin\vartheta, \end{aligned} \tag{2.11}$$

where

$$\begin{aligned} \langle 0|\bar{u}\gamma_\mu\gamma_5 u|\eta^{(\prime)}(p)\rangle &= if_{\eta^{(\prime)}}^u p_\mu, \\ \langle 0|\bar{s}\gamma_\mu\gamma_5 s|\eta^{(\prime)}(p)\rangle &= if_{\eta^{(\prime)}}^s p_\mu. \end{aligned} \tag{2.12}$$

The form factors for $B \to \eta^{(\prime)}$ transitions are assumed to be

$$\begin{aligned} F_0^{D\eta} &= F_0^{D\pi}\left(\frac{\cos\vartheta}{\sqrt{6}} - \frac{\sin\vartheta}{\sqrt{3}}\right), \\ F_0^{D\eta'} &= F_0^{D\pi}\left(\frac{\sin\vartheta}{\sqrt{6}} + \frac{\cos\vartheta}{\sqrt{3}}\right). \end{aligned} \tag{2.13}$$

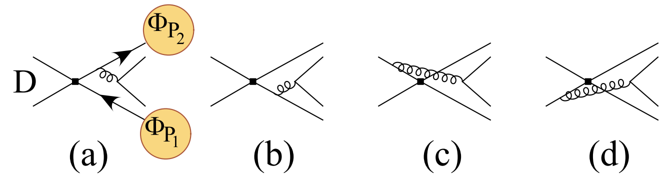

FIG. 1 (color online). Annihilation corrections to $D \to P_1P_2$, where (a) and (b) correspond to A_1^f, while (c) and (d) give rise to A_1^i.

B. SD weak annihilation amplitudes

The SD weak annihilation contributions [4,5] to $D \to P_1P_2$, graphically shown in Fig. 1, are represented as

$$\frac{G_F}{\sqrt{2}}\sum_{q,q'=d,s} V_{uq}V_{cq'}^*\langle P_1P_2|\mathcal{T}_\mathcal{B}|D\rangle \equiv A_{\mathcal{T}_\mathcal{B}}(P_1P_2). \tag{2.14}$$

In general, $\langle P_1P_2|\mathcal{T}_\mathcal{B}|D\rangle$ consists of $icf_Df_{P_1}f_{P_2}b_{1,2}$, where c contains factors of $\pm 1, \pm 1/\sqrt{2}$, $1/\sqrt{6}$, or $-2/\sqrt{6}$, arising from the flavor structures of final-state mesons, and

$$b_{1,2} = \frac{C_F}{N_c^2}c_{1,2}A_1^i(P_2P_1), \tag{2.15}$$

with the convention adopted here that P_2 (P_1) contains a quark (antiquark) arising from the weak vertex with longitudinal momentum fraction x ($\bar{y}$). Here the basic building blocks for annihilation amplitudes originating from operators $(\bar{q}_1c)_{V-A}(\bar{q}_2q_3)_{V-A}$ are denoted as $A_1^{i,f}$, where the superscript $i(f)$ indicates gluon emission from the initial-(final-) state quarks in the weak vertex, given by

$$\begin{aligned} A_1^i(P_2P_1) = {}& \pi\alpha_s\int_0^1 dxdy\Big\{\Phi_{P_2}(x)\Phi_{P_1}(y)\Big[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2y}\Big] \\ & + r_\chi^{P_1}r_\chi^{P_2}\Phi_{P_2}^p(x)\Phi_{P_1}^p(y)\frac{2}{\bar{x}y}\Big\}, \\ A_1^f(P_2P_1) = {}& 0. \end{aligned} \tag{2.16}$$

with $r_\chi^{P_i}$ being defined as

$$r_\chi^{P_i}(\mu) = \frac{2m_{P_i}^2}{m_c(\mu)(m_{q_1}(\mu) + m_{q_2}(\mu))}, \tag{2.17}$$

and m_{q_1,q_2} the current quark masses of the meson constituents in the $\overline{\rm MS}$ scheme. The relevant two-parton light-cone distribution amplitudes (LCDAs), up to twist-3, of a light pseudoscalar meson P are defined as [18]

$$\langle P(p)|\bar{q}_2(z_2)\gamma_\mu\gamma_5 q_1(z_1)|0\rangle = -if_P p_\mu \int_0^1 dx e^{i(xp\cdot z_2+\bar{x}p\cdot z_1)}\Phi_P(x),$$
$$\langle P(p)|\bar{q}_2(z_2)i\gamma_5 q_1(z_1)|0\rangle = f_P\mu_P \int_0^1 dx e^{i(xp\cdot z_2+\bar{x}p\cdot z_1)}\Phi_P^p(x), \tag{2.18}$$
$$\langle P(p)|\bar{q}_2(z_2)\sigma_{\mu\nu}\gamma_5 q_1(z_1)|0\rangle = if_P\mu_P(p_\mu z_\nu - p_\nu z_\mu)\int_0^1 dx e^{i(xp\cdot z_2+\bar{x}p\cdot z_1)}\frac{\Phi_P^\sigma(x)}{6},$$

where $z = z_2 - z_1$, $\mu_P = m_P^2/(m_{q_1} + m_{q_2})$, f_P is the decay constant, and x(or $\bar{x} = 1 - x$) is the collinear momentum fraction carried by the quark q_2 (or antiquark $\bar{q}_1$). Here and below we do not explicitly show the gauge factors

$$\mathrm{P}\exp\left[ig_s\int_0^1 dt(z_1 - z_2)_\mu A^\mu(tz_1 + (1-t)z_2)\right] \tag{2.19}$$

in between the quark fields. The leading-twist LCDA $\Phi_P(x)$ is of twist-2, while $\Phi_P^p(x)$ and $\Phi_P^\sigma(x)$ are of twist-3. LCDAs appearing in the calculation of weak annihilation contributions are in the form of

$$\langle P(p)|\bar{q}_{2,\beta}(z_2)q_{1,\alpha}(z_1)|0\rangle = \frac{if_P}{4}\int_0^1 dx e^{i(xp\cdot z_2+\bar{x}p\cdot z_1)}\left\{\not{p}\gamma_5\Phi_P(x) - \mu_P\gamma_5\left(\Phi_P^p(x) - \sigma_{\mu\nu}p^\mu z^\nu\frac{\Phi_P^\sigma(x)}{6}\right)\right\}_{\alpha\beta}. \tag{2.20}$$

Neglecting three-particle contributions, the twist-3 distribution amplitudes in the asymptotic limit are related to each other by equations of motion, so that

$$\Phi_P^p(x) = 1, \qquad \frac{\Phi_P^{\sigma\prime}(x)}{6} = (\bar{x} - x)\Phi_P^p, \qquad \frac{\Phi_P^\sigma(x)}{6} = (x\bar{x})\Phi_P^p. \tag{2.21}$$

Using the above simplification, one can get the corresponding projector of Eq. (2.20) in the momentum space [4,5,19]

$$M_{\alpha\beta}^P = \frac{if_P}{4}\left(\not{p}\gamma_5\Phi_P(x) - \mu_P\gamma_5\frac{\not{k}_2\not{k}_1}{k_2\cdot k_1}\Phi_P^p(x)\right)_{\alpha\beta}, \tag{2.22}$$

and further obtain the basic building blocks for annihilation amplitudes given in Eq. (2.16), where the momenta of the quark q_1 and antiquark $\bar{q}_2$ in a meson are parametrized as

$$k_1^\mu = xEn_-^\mu + k_\perp^\mu + \frac{k_\perp^2}{4xE}n_+^\mu, \qquad k_2^\mu = \bar{x}En_-^\mu - k_\perp^\mu + \frac{k_\perp^2}{4\bar{x}E}n_+^\mu, \tag{2.23}$$

respectively. For simplicity, we have introduced two lightlike vectors $n_-^\mu \equiv (1,0,0,-1)$, $n_+^\mu \equiv (1,0,0,1)$. If neglecting the meson mass squared, we have $p^\mu = En_-^\mu$ where E is the energy of the meson. We refer the reader to Refs. [4,5] for the detailed technique of calculating weak annihilation contributions.

The LCDAs normalized at the scale μ can be expanded in Gegenbauer polynomials of forms

$$\Phi_P(x,\mu) = 6x(1-x)\left[1 + \sum_{n=1}^\infty a_n^P(\mu)C_n^{(3/2)}(2x-1)\right], \tag{2.24}$$

$$\Phi_P^p(x,\mu) = 1 + \sum_{n=1}^\infty a_n^{P,p}(\mu)C_n^{(1/2)}(2x-1), \tag{2.25}$$

$$\Phi_P^\sigma(x,\mu) = 6x(1-x)\left[1 + \sum_{n=1}^\infty a_n^{P,\sigma}(\mu)C_n^{(3/2)}(2x-1)\right]. \tag{2.26}$$

In the numerical analysis, we truncate the expansion of Φ_P at $n = 1$ and just take the asymptotic approximation for Φ_P^p and Φ_P^σ. Note that a_1^P is nonzero only for the kaon. For the kaon containing an $\bar{s}$ quark, we have the replacement $x \leftrightarrow \bar{x}$ in Eq. (2.24). The annihilation corrections to $D^0 \to K^0\overline{K}^0$, as an example, thus read

$$\begin{aligned}A_{\mathcal{T}_{\mathcal{B}}}(K^0\overline{K}^0) &= i\frac{G_F}{\sqrt{2}}f_D f_K^2\frac{C_F}{N_c^2}\pi\alpha_s c_1\left\{V_{us}V_{cs}^*\int_0^1 dxdy\left[\Phi_{\overline{K}^0}(x)\Phi_{K^0}(y)\left(\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y}\right) + (r_\chi^K)^2\Phi_{\overline{K}^0}^p(x)\Phi_{K^0}^p(y)\frac{2}{\bar{x}y}\right]\right.\\ &\quad\left. + V_{ud}V_{cd}^*\int_0^1 dxdy\left[\Phi_{K^0}(x)\Phi_{\overline{K}^0}(y)\left(\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y}\right) + (r_\chi^K)^2\Phi_{K^0}^p(x)\Phi_{\overline{K}^0}^p(y)\frac{2}{\bar{x}y}\right]\right\}\\ &= i\frac{G_F}{\sqrt{2}}f_D f_K^2\frac{C_F}{N_c^2}\pi\alpha_s c_1 V_{us}V_{cs}^* 36 a_1^K(4X_A + 33 - 4\pi^2),\end{aligned} \tag{2.27}$$

where use of $V_{ud}V_{cd}^* = -V_{us}V_{cs}^*$ has been made, and $\int_0^1 dz/z \to X_A$ has been used to parametrize the logarithmically divergent integrals [4,5], which can be regulated by including the transverse momentum of the quark in the endpoint region of integrals, but however may suffer from some theoretical problems (see discussions in the introduction). It is interesting to note that $A_{\mathcal{T}_{\mathcal{B}}}(K^0\overline{K}^0)$ is proportional to a_1^K. As will be seen in Sec. IV, the magnitude of a_1^K has a large impact on the $D^0 \to K^0\overline{K}^0$ branching ratio. Two remarks are in order. First, the simplified form of the projector in Eq. (2.22) cannot be justified if considering higher Gegenbauer moment corrections to Φ_P^p and Φ_P^σ. We have checked that the amplitude corrections due to $a_1^{K,p}$ and $a_1^{K,\sigma}$ are numerically negligible if the magnitudes of $a_1^{K,p}$ and $a_1^{K,\sigma}$ are not too large. Second, we do not consider a_2^P, since distinguishing a_2^π, a_2^K, and $a_2^{\eta_8}$ is not numerically significant in the present study [4], and, moreover, partial effects due to a_2^P can be absorbed in X_A. The detailed expressions for SD weak annihilation amplitudes are collected in Appendix B.

III. SU(3) RESCATTERINGS

From the isospin amplitude analysis of $D^0 \to K^-\pi^+, \overline{K}^0\pi^0$ and $D^+ \to \overline{K}^0\pi^+$, as discussed in Sec. I, we know that the LD FSIs effects may be significant in D meson decays. Considering elastic SU(3) rescatterings in D decays, for instance, $D^0 \to K^-\pi^+$, $D^0 \to \overline{K}^0\pi^0$, and $D^0 \to \overline{K}^0\eta_8$ can be generated via

$$D^0 \to K^-\pi^+ \begin{array}{l} \to K^-\pi^+ \\ \to \overline{K}^0\pi^0 \\ \to \overline{K}^0\eta_8 \end{array}, \qquad D^0 \to \overline{K}^0\pi^0 \begin{array}{l} \to K^-\pi^+ \\ \to \overline{K}^0\pi^0 \\ \to \overline{K}^0\eta_8 \end{array}, \qquad D^0 \to \overline{K}^0\eta_8 \begin{array}{l} \to K^-\pi^+ \\ \to \overline{K}^0\pi^0 \\ \to \overline{K}^0\eta_8 \end{array}.$$

Taking into account elastic SU(3) FSIs, the decay amplitudes $\mathbf{A}_i^{\mathrm{FSI}}$ are given by [20–22]

$$\mathbf{A}_i^{\mathrm{FSI}} = \sum_l \mathbf{S}_{il}^{1/2}\mathbf{A}_l^{\mathrm{bare}} = (\mathbf{U}^{\mathrm{T}}\mathbf{S}_{\mathrm{diag}}^{1/2}\mathbf{U})_{il}\mathbf{A}_l^{\mathrm{bare}}, \tag{3.1}$$

where $\mathbf{S}$ is strong interaction scattering matrix, and $\mathbf{A}_l^{\mathrm{bare}}(= \mathbf{A}_l^{\mathrm{fac}} + \mathbf{A}_l^{\mathcal{T}_{\mathcal{B}}})$ are approximated in terms of the factorization and SD weak annihilation amplitudes. Note that $\mathbf{S}$ is unitary. The SU(3) final-state rescatterings for $D \to P_1P_2$ are described by the product $\mathbf{8}\otimes\mathbf{8}$. Since the P_1P_2 states obey the Bose symmetry, only the symmetric states given by the representation $\mathbf{36}(= \mathbf{27}\oplus\mathbf{8}\oplus\mathbf{1})$ in $\mathbf{8}\otimes\mathbf{8}(= \mathbf{36}\oplus\mathbf{28})$ decomposition are relevant, whereas states given by the representation $\mathbf{28}(= \mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{8})$ vanish.

In the present study, we will use δ_{27}, δ_8, and δ_1 to stand for the respective rescattering phases of **27**, **8**, and **1** states. The detailed derivation for $\mathbf{U}$ matrices and the corresponding SU(3) eigen-amplitudes is exhibited in Appendix C. Thus the $\mathbf{S}^{1/2}$ matrices and decay amplitudes can be recast into the following 5 subsets (see also Refs. [16,22]):

(i) subset 1 ($K^-\pi^+ - \overline{K}^0\pi^0 - \overline{K}^0\eta_8$ rescatterings),

$$\mathbf{S}_{(K\pi)^0}^{1/2}e^{-i\delta_{27}} = \begin{pmatrix} \frac{2+3e^{-i\delta}}{5} & \frac{3(1-e^{-i\delta})}{5\sqrt{2}} & \frac{\sqrt{3}(1-e^{-i\delta})}{5\sqrt{2}} \\ \frac{3(1-e^{-i\delta})}{5\sqrt{2}} & \frac{7+3e^{-i\delta}}{10} & \frac{\sqrt{3}(-1+e^{-i\delta})}{10} \\ \frac{\sqrt{3}(1-e^{-i\delta})}{5\sqrt{2}} & \frac{\sqrt{3}(-1+e^{-i\delta})}{10} & \frac{9+e^{-i\delta}}{10} \end{pmatrix}, \tag{3.2}$$

$$\mathbf{A}_{(K\pi)^0}^{\mathrm{bare}} = \begin{pmatrix} A_{K^-\pi^+}^{\mathrm{bare}} \\ A_{\overline{K}^0\pi^0}^{\mathrm{bare}} \\ A_{\overline{K}^0\eta_8}^{\mathrm{bare}} \end{pmatrix}, \tag{3.3}$$

(ii) subset 2 ($K^+\pi^- - K^0\pi^0 - K^0\eta_8$ rescatterings),

$$\mathbf{S}_{(K\pi)^0}^{1/2} = \mathbf{S}_{(K\pi)^0}^{1/2}, \tag{3.4}$$

$$\mathbf{A}_{(K\pi)^0}^{\mathrm{bare}} = \begin{pmatrix} A_{K^+\pi^-}^{\mathrm{bare}} \\ A_{K^0\pi^0}^{\mathrm{bare}} \\ A_{K^0\eta_8}^{\mathrm{bare}} \end{pmatrix}, \tag{3.5}$$

(iii) subset 3 ($K^0\pi^+ - K^+\pi^0 - K^+\eta_8$ rescatterings),

$$\mathbf{S}_{(K\pi)^+}^{1/2}e^{-i\delta_{27}} = \begin{pmatrix} \frac{2+3e^{-i\delta}}{5} & -\frac{3(1-e^{-i\delta})}{5\sqrt{2}} & -\frac{\sqrt{3}(1-e^{-i\delta})}{5\sqrt{2}} \\ -\frac{3(1-e^{-i\delta})}{5\sqrt{2}} & \frac{7+3e^{-i\delta}}{10} & -\frac{\sqrt{3}(-1+e^{-i\delta})}{10} \\ -\frac{\sqrt{3}(1-e^{-i\delta})}{5\sqrt{2}} & -\frac{\sqrt{3}(-1+e^{-i\delta})}{10} & \frac{9+e^{-i\delta}}{10} \end{pmatrix}, \tag{3.6}$$

$$\mathbf{A}_{(K\pi)^+}^{\mathrm{bare}} = \begin{pmatrix} A_{K^0\pi^+}^{\mathrm{bare}} \\ A_{K^+\pi^0}^{\mathrm{bare}} \\ A_{K^+\eta_8}^{\mathrm{bare}} \end{pmatrix}, \tag{3.7}$$

(iv) subset 4 ($\pi^+\pi^0 - \pi^+\eta_8 - K^+\overline{K}^0$ rescatterings),

$$\mathbf{S}^{1/2}_{(\pi\pi)^+} e^{-i\delta_{27}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{3+2e^{-i\delta}}{5} & -\frac{\sqrt{6}(1-e^{-i\delta})}{5} \\ 0 & -\frac{\sqrt{6}(1-e^{-i\delta})}{5} & \frac{2+3e^{-i\delta}}{5} \end{pmatrix}, \tag{3.8}$$

$$\mathbf{A}^{\,\mathrm{bare}}_{(\pi\pi)^+} = \begin{pmatrix} A^{\mathrm{bare}}_{\pi^+\pi^0} \\ A^{\mathrm{bare}}_{\pi^+\eta_8} \\ A^{\mathrm{bare}}_{K^+\overline{K}^0} \end{pmatrix}, \tag{3.9}$$

(v) subset 5 ($\pi^+\pi^- - \pi^0\pi^0 - \eta_8\eta_8 - K^+K^- - K^0\overline{K}^0 - \pi^0\eta_8$ rescatterings),

$$\mathbf{S}^{1/2}_{(\pi\pi)^0} e^{-i\delta_{27}} \begin{pmatrix} \frac{5e^{-i\sigma}+8e^{-i\delta}+7}{20} & \frac{5e^{-i\sigma}+8e^{-i\delta}-13}{20\sqrt{2}} & \frac{5e^{-i\sigma}-8e^{-i\delta}+3}{20\sqrt{2}} & \frac{5e^{-i\sigma}-4e^{-i\delta}-1}{20} & \frac{5e^{-i\sigma}-4e^{-i\delta}-1}{20} & 0 \\ \frac{5e^{-i\sigma}+8e^{-i\delta}-13}{20\sqrt{2}} & \frac{5e^{-i\sigma}+8e^{-i\delta}+27}{40} & \frac{5e^{-i\sigma}-8e^{-i\delta}+3}{20\sqrt{2}} & \frac{5e^{-i\sigma}-4e^{-i\delta}-1}{20\sqrt{2}} & \frac{5e^{-i\sigma}-4e^{-i\delta}-1}{20\sqrt{2}} & 0 \\ \frac{5e^{-i\sigma}-8e^{-i\delta}+3}{20\sqrt{2}} & \frac{5e^{-i\sigma}-8e^{-i\delta}+3}{20\sqrt{2}} & \frac{5e^{-i\sigma}+8e^{-i\delta}+27}{40} & \frac{5e^{-i\sigma}+4e^{-i\delta}-9}{20\sqrt{2}} & \frac{5e^{-i\sigma}+4e^{-i\delta}-9}{20\sqrt{2}} & 0 \\ \frac{5e^{-i\sigma}-4e^{-i\delta}-1}{20} & \frac{5e^{-i\sigma}-4e^{-i\delta}-1}{20\sqrt{2}} & \frac{5e^{-i\sigma}+4e^{-i\delta}-9}{20\sqrt{2}} & \frac{5e^{-i\sigma}+8e^{-i\delta}+7}{20} & \frac{5e^{-i\sigma}-4e^{-i\delta}-1}{20} & \frac{4\sqrt{3}(e^{-i\delta}-1)}{20} \\ \frac{5e^{-i\sigma}-4e^{-i\delta}-1}{20} & \frac{5e^{-i\sigma}-4e^{-i\delta}-1}{20\sqrt{2}} & \frac{5e^{-i\sigma}+4e^{-i\delta}-9}{20\sqrt{2}} & \frac{5e^{-i\sigma}-4e^{-i\delta}-1}{20} & \frac{5e^{-i\sigma}+8e^{-i\delta}+7}{20} & \frac{4\sqrt{3}(1-e^{-i\delta})}{20} \\ 0 & 0 & 0 & \frac{4\sqrt{3}(e^{-i\delta}-1)}{20} & \frac{4\sqrt{3}(1-e^{-i\delta})}{20} & \frac{4(2e^{-i\delta}+3)}{20} \end{pmatrix}, \tag{3.10}$$

$$\mathbf{A}^{\,\mathrm{bare}}_{(\pi\pi)^0} = \begin{pmatrix} A^{\mathrm{bare}}_{\pi^+\pi^-} \\ A^{\mathrm{bare}}_{\pi^0\pi^0} \\ A^{\mathrm{bare}}_{\eta_8\eta_8} \\ A^{\mathrm{bare}}_{K^+K^-} \\ A^{\mathrm{bare}}_{K^0\overline{K}^0} \\ A^{\mathrm{bare}}_{\pi^0\eta_8} \end{pmatrix}, \tag{3.11}$$

where $\delta \equiv \delta_{27} - \delta_8$, $\sigma \equiv \delta_{27} - \delta_1$, and we have included the identical particle factor $1/\sqrt{2}$ in the amplitudes $\mathcal{A}^{\mathrm{bare}}_{\pi^0\pi^0}$ and $\mathcal{A}^{\mathrm{bare}}_{\eta_8\eta_8}$. Here $\mathbf{S}^{1/2}$ matrices have been factored out an overall phase $e^{i\delta_{27}}$ since only phase differences affect physical results. Note that we do not list $D^+ \to \overline{K}^0\pi^+$, which does not belong to any above subset, i.e., does not rescatter with other PP modes. Note also that in the subset 4, $D^+ \to \pi^+\pi^0$ does not rescatter with other modes, too.

IV. RESULTS

In this section, we will first introduce the relevant parameters in the fit and then give the numerical results together with a brief discussion. The 2-body D meson decay rates are given by

$$\Gamma(D \to P_1P_2) = \frac{|\vec{p}_c|}{8\pi m_D^2}|A^{\mathrm{FSI}}|^2, \tag{4.1}$$

where $\vec{p}_c$ is the center-of-mass momentum of decay particles. In the numerical analysis, we perform the best multimode χ^2 fit for measured branching ratios, defined as

$$\chi^2 = \sum_i \left(\frac{y_i - x_i}{\Delta x_i}\right)^2, \tag{4.2}$$

where y_i and $x_i \pm \Delta x_i$ denote the theoretical results and measurements, respectively. On the theoretical side, input parameters relevant for our numerical analysis are listed in Table I [15,23–25]. As listed in Table II, we take the current data [15] for the 14 $K\pi$, $\pi\pi$, KK, $K\eta^{(\prime)}$, and $\pi\eta^{(\prime)}$ BRs as inputs. The modes involving η or η' are related to η_8 and η_0 via the mixing angle ϑ. The SU(3) FSI picture is not suitable to be extended to the U(3) scenario since $\mathrm{U}_A(1)$ symmetry is broken by anomaly, i.e., η' is not a Goldstone boson. The weak annihilation effect for SU(3) channels is parametrized in terms of X_A, while that for decay modes involving η_0 is distinguished to be X'_A. However we do not distinguish $1/N_c^{\mathrm{eff}}$ because it is numerically small, as seen in our analysis. The scale for the factorization amplitudes is taken to be $\mu = m_c$, i.e.,

TABLE I. Summary of input parameters [15,23–25] on the theoretical side of the fit.

Running quark masses [GeV] and the strong coupling constant				
$m_c(m_c)$	m_s(1GeV)	m_u(1GeV)	m_d(1GeV)	α_s(1GeV)
1.35	0.12	0.004	0.009	0.517

The Wolfenstein parameter and D-meson lifetimes [$10^{-15}s$]		
λ	$\tau(D^+)$	$\tau(D^0)$
0.2196	1040 ± 7	410.3 ± 1.5

Pseudoscalar-meson decay constants [MeV]				
f_π	f_K	f_{η_8}	f_{η_0}	f_D
131	160	168	157	220 ± 20

The form factor (at $q^2 = 0$) and $\eta - \eta'$ mixing angle	
$F_0^{DK}(0)$	ϑ
0.76 ± 0.03	$-14.5°$

The Wilson coefficients for D decays			
$c_1(m_c)$	$c_2(m_c)$	c_1(1GeV)	c_2(1GeV)
1.216	-0.422	1.275	-0.510

TABLE II. The branching ratios in units of 10^{-3}: data ($\mathcal{B}_{\rm Exp}$) [15] vs fitted results ($\mathcal{B}_{\rm FSI}$). The individual χ_i^2 values of decay modes corresponding to the best fit are listed. For comparison, taking the best fit parameters and $F_0^{DK}(0) = 0.76$ into account, we then give (i) $\mathcal{B}_{\rm Fact}$ by means of setting $\delta = \sigma = 0$ and neglecting the weak annihilation corrections, (ii) $\mathcal{B}_{\rm NoAnn}$ by means of neglecting only the SD weak annihilation corrections, and (iii) $\mathcal{B}_{\rm NoFSI}$ by means of setting $\delta = \sigma = 0$. Note that $D^0 \to \eta'\eta'$ is kinematically forbidden. The errors in $\mathcal{B}_{\rm FSI}$ and χ_i^2 are due to the variation of $F_0^{DK}(0)$.

Decay modes	$\mathcal{B}_{\rm Exp}$	$\mathcal{B}_{\rm Fact}$	$\mathcal{B}_{\rm NoAnn}$	$\mathcal{B}_{\rm NoFSI}$	$\mathcal{B}_{\rm FSI}$	χ_i^2
$D^0 \to K^-\pi^+$	38.0 ± 0.9	62.20	56.20	56.69	$38.02^{+0.05}_{-0.09}$	$0.00^{+0.04}_{-0.00}$
$D^0 \to \overline{K}^0\pi^0$	23.0 ± 2.2	10.39	14.05	16.69	$23.83^{+0.01}_{-0.19}$	$0.14^{+0.01}_{-0.06}$
$D^0 \to \overline{K}^0\eta$	7.7 ± 1.1	2.75	3.83	2.67	$7.92^{+0.17}_{-0.05}$	$0.04^{+0.09}_{-0.02}$
$D^0 \to \overline{K}^0\eta'$	18.8 ± 2.8	2.40	3.14	16.11	$19.82^{+0.17}_{-0.05}$	$0.13^{+0.05}_{-0.01}$
$D^+ \to K^0\pi^+$	$\cdots$	0.14	0.21	0.09	$0.27^{+0.01}_{-0.02}$	$\cdots$
$D^+ \to K^+\pi^0$	$\cdots$	0.46	0.39	0.59	0.39 ± 0.05	$\cdots$
$D^+ \to K^+\eta$	$\cdots$	0.12	0.10	0.08	0.07 ± 0.00	$\cdots$
$D^+ \to K^+\eta'$	$\cdots$	0.11	0.12	0.19	0.20 ± 0.01	$\cdots$
$D^+ \to \overline{K}^0\pi^+$	28.2 ± 1.9	28.15	28.15	28.15	$28.15^{+0.05}_{-0.03}$	0.00 ± 0.00
$D^0 \to K^+\pi^-$	0.138 ± 0.011	0.36	0.31	0.24	0.15 ± 0.00	$0.68^{+0.28}_{-0.16}$
$D^0 \to K^0\pi^0$	$\cdots$	0.03	0.06	0.03	0.08 ± 0.00	$\cdots$
$D^0 \to K^0\eta$	$\cdots$	0.007	0.02	0.006	0.03 ± 0.01	$\cdots$
$D^0 \to K^0\eta'$	$\cdots$	0.006	0.009	0.06	0.07 ± 0.00	$\cdots$
$D^+ \to \pi^+\pi^0$	2.6 ± 0.7	2.27	2.27	2.27	$2.27^{+0.03}_{-0.04}$	$0.22^{+0.05}_{-0.04}$
$D^+ \to K^+\overline{K}^0$	5.9 ± 0.6	11.67	10.47	6.77	$5.73^{+0.29}_{-0.23}$	$0.08^{+0.36}_{-0.04}$
$D^+ \to \pi^+\eta$	3.0 ± 0.6	0.77	2.61	0.45	2.61 ± 0.01	$0.42^{+0.02}_{-0.01}$
$D^+ \to \pi^+\eta'$	5.1 ± 1.0	3.47	3.02	4.15	$3.31^{+0.14}_{-0.16}$	$3.19^{+0.58}_{-0.47}$
$D^0 \to \pi^+\pi^-$	1.38 ± 0.05	4.73	4.14	2.76	1.37 ± 0.01	$0.02^{+0.01}_{-0.00}$
$D^0 \to \pi^0\pi^0$	0.84 ± 0.22	0.36	0.71	0.22	$0.73^{+0.03}_{-0.04}$	$0.26^{+0.18}_{-0.11}$
$D^0 \to K^+K^-$	3.89 ± 0.14	4.58	3.92	5.43	$3.85^{+0.01}_{-0.00}$	$0.08^{+0.00}_{-0.04}$
$D^0 \to K^0\overline{K}^0$	0.71 ± 0.19	0	0.00	0.65	$0.68^{+0.04}_{-0.06}$	$0.03^{+0.23}_{-0.03}$
$D^0 \to \pi^0\eta$	$\cdots$	0.09	0.35	0.25	0.68 ± 0.01	$\cdots$
$D^0 \to \pi^0\eta'$	$\cdots$	0.09	0.15	0.01	$0.05^{+0.03}_{-0.02}$	$\cdots$
$D^0 \to \eta\eta$	$\cdots$	0.10	0.44	0.33	$1.17^{+0.00}_{-0.02}$	$\cdots$
$D^0 \to \eta\eta'$	$\cdots$	0.13	0.25	1.29	1.95 ± 0.05	$\cdots$
$D^0 \to \eta'\eta'$	$\cdots$	0	0	0	0	$\cdots$

$1/N_c^{\rm eff} = 1/N_c^{\rm eff}(m_c)$, while the scale for SD weak annihilation amplitudes is 1 GeV. We use the world average value of $F_0^{DK}(0) = 0.76 \pm 0.03$ [23]. For the q^2 dependence of form factors, we adopt the pole dominance assumption:

$$F_0(q^2) = \frac{F_0(0)}{1 - q^2/m_*^2}, \tag{4.3}$$

with taking m_* as the mass of the lowest-lying scalar charmed meson in the corresponding channel. The above form is consistent with the recent QCD sum rule study for $B \to$ light meson transitions [26]. We assume $m_* = 2.3$ GeV [27] (or 2.2 GeV) for F_0^{DK} (or $F_0^{D\pi}$). The results for fitted parameters, which are (i) two FSI phases, δ and σ, (ii) the form factor $F_0^{D\pi}$, (iii) SD weak annihilation parameter X_A and X_A', and (iv) $1/N_c^{\rm eff}$, are cataloged in Table III. Output observables are given in Table II. The errors of outputs correspond to the variation of $F_0^{DK}(0)$, while the errors due to uncertainties of D lifetimes are negligible.

The nonfactorizable effects are lumped into the effective number of color $N_c^{\rm eff}$, of which the deviation from N_c measures such effects. $1/N_c^{\rm eff}$ could be complex. However, it is assumed to be real due to its small value: $1/N_c^{\rm eff} < -1/15(\simeq -0.067)$ in the fit, consistent with the very earlier large-N_c approach for describing hadronic D

TABLE III. The $\chi^2_{\rm min}$/d.o.f. and fitted parameters, where we obtain a twofold solution for X_A' which is relevant only for decay modes involving η_0. The errors are due to the variation of $F_0^{DK}(0)$.

	Best fit results
$\chi^2_{\rm min}$/d.o.f.	$(5.3^{+1.3}_{-0.5})/5$
δ	$(-46 \pm 2)°$
σ	$(-21 \pm 1)°$
$N_c^{\rm eff}$	-21^{+6}_{-18}
$F_0^{D\pi}(0)$	0.83 ± 0.02
a_1^K	$-0.15^{+0.00}_{-0.01}$
$\lvert X_A\rvert$	3.84 ± 0.06
$\arg(X_A)$	$(-138 \pm 3)°$
$\lvert X_A'\rvert$	$2.45^{+0.07}_{-0.46}$ [or 2.18 ± 0.19]
$\arg(X_A')$	$(-138 \pm 3)°$ [or $(130 \pm 3)°$]

decays [1]. It is interesting to note that we obtain the weak annihilation parameter $|X_A| = 3.84 \pm 0.06$ ($|X'_A| = 2.45^{+0.07}_{-0.46}$ or 2.18 ± 0.19) with a large phase $(-138 \pm 3)°$ [$(-138 \pm 3)°$ or $(130 \pm 3)°$], compared with the similar parameter $|X_A| \sim 4.5$ given in B decays [5,28,29]. Note that we obtain a twofold solution for X'_A. As seen in Table II, the weak annihilation topologies have a large impact on branching ratios. This analysis gives moderate rescattering phases $\delta \simeq -46°$ and $\sigma \simeq -21°$.

One can see from Table II that the $D \to PP$ data can be nicely fitted by the present picture.

A. $D^+ \to \pi^+\pi^0$ vs $D^+ \to \overline{K}^0\pi^+$

Consider the ratio

$$R_1 = 2\left|\frac{V_{cs}}{V_{cd}}\right|^2 \frac{\Gamma(D^+ \to \pi^+\pi^0)}{\Gamma(D^+ \to \overline{K}^0\pi^+)}. \qquad (4.4)$$

The data show $R_1 = 3.46 \pm 1.17$, whereas $R_1 = 1$ in the SU(3) limit. It is interesting to note that both $D^+ \to \pi^+\pi^0$ and $D^+ \to \overline{K}^0\pi^+$ amplitudes are identical during SU(3) rescattering because they do not rescatter with other decay modes. Moreover, these two amplitudes have no SD weak annihilation corrections. To take into account the BRs and their ratio

$$R_1 = \left|\frac{(a_1 + a_2)f_\pi(m_D^2 - m_\pi^2)F_0^{D\pi}(m_\pi^2)}{a_1 f_\pi(m_D^2 - m_K^2)F_0^{DK}(m_\pi^2) + a_2 f_K(m_D^2 - m_\pi^2)F_0^{D\pi}(m_K^2)}\right|^2, \qquad (4.5)$$

a small $1/N_c^{\rm eff}$ and $F_0^{D\pi}(0) \gtrsim F_0^{DK}(0)$ are preferred (see also the discussion in footnote 3).

B. $D^0 \to \pi^+\pi^-$ vs $D^0 \to K^+K^-$

The experiments have measured the ratio

$$R_2 = \frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} = 2.82 \pm 0.01, \qquad (4.6)$$

which is a long-standing puzzle because the conventional factorization approach yields $R_2 = 1$ in the SU(3) limit (see discussions in Ref. [30]). We found that the SD weak annihilation contributions together with FSIs interfere destructively to the $D^0 \to \pi^+\pi^-$ amplitude, but constructively to the $D^0 \to K^+K^-$ amplitude, such that the ratio can be accounted for.

C. $D^0 \to K^0\overline{K}^0$

In the limit of SU(3) symmetry, the $D^0 \to K^0\overline{K}^0$ amplitude vanishes. It was explained in Ref. [31] that the nonsmall branching ratio of this mode may be owing to long-distance FSIs. Nevertheless, here we conclude that $D \to \overline{K}^0K^0$ occurs mainly due to nonzero SD weak annihilation effects originating from SU(3) symmetry-breaking corrections to the distribution amplitudes of the kaons.[2] Moreover, we find $a_1^K = -0.15^{+0.00}_{-0.01}$ in the best fit, which is consistent with the result given in Ref. [33] but in contrast with that in Ref. [34] where the value is positive.[3] Note that it has been argued in Refs. [34,35] that the result given in Ref. [33] is less reliable.

D. D decays involving η or η'

It should be stressed that the SD weak annihilation and SU(3) rescattering effects enter the amplitudes in different ways. For instance, $\mathcal{B}(D^0 \to \overline{K}^0\eta)$ and $\mathcal{B}(D^+ \to \pi^+\eta)$ are mainly enhanced by SU(3) rescattering, whereas $\mathcal{B}(D^0 \to \overline{K}^0\eta')$ receives contributions mainly from the SD weak annihilation. This mechanism can be further tested experimentally from the relative values of the $D^0 \to \pi^0\eta$, $\pi^0\eta'$, $\eta\eta$, and $\eta\eta'$ branching ratios.

V. DISCUSSIONS AND SUMMARY

We have built up a simple model that the $D \to PP$ decay processes go through "*bare*" amplitudes followed by elastic SU(3) rescatterings, where the *bare* amplitude consists of (i) the usual factorization amplitudes of color-allowance and color-suppression, discussed in Sec. II A, and (ii) the SD weak annihilation amplitudes (W-exchange and/or W-annihilation) presented in Sec. II B. A similar model estimate was proposed by Chernyak and Zhitnisky [36], who considered the *bare amplitude* followed by SU(2) rescattering for $D \to K\pi$.

In terms of quark-graph amplitudes in the diagrammatic approach [37–42], the topologies relevant to $D \to PP$ decays are the tree topology "T," the color-suppressed tree topology "C", and weak annihilation topologies (W-exchange and/or W-annihilation), shown in the first row of Fig. 2. It has been stressed in Ref. [41] that in the diagrammatic approach even though the SD weak annihilation contributions are neglected, it is still possible for that the weak annihilation topologies receive sizable contributions from the LD final-state rescatterings of the (color-suppressed) tree amplitude T (C), as sketched in the second and third rows of Fig. 2 (some estimates for LD effects see Refs. [30,43–45]). Moreover, it should be stressed that the SD weak annihilation amplitudes $A_{\mathcal{T}_\mathcal{B}}$ have sizable magnitudes comparable to the factorization amplitudes $\mathcal{A}_{\rm fac}$; due to the structure of $(V-A) \otimes (V-A)$ operators in the

[2] In spirit, our conclusion agrees with the result in Ref. [32], where the authors used the chiral perturbation theory to calculate the weak annihilation effects and found that the result is proportional to m_s.

[3] There also exists a solution of positive $a_1^K \simeq 0.19$, $\delta \simeq -39°$, $\sigma \simeq -12°$, $N_c^{\rm eff} \simeq -14$, $X_A = 2.7e^{i92°}$, $X'_A = 1.7e^{-i104°}$ [or $X'_A = 3.5e^{i152°}$] and $F_0^{D\pi}(0)/F_0^{DK}(0) \simeq 1.05$ with a larger $\chi_{\rm min} \simeq 11.4$.

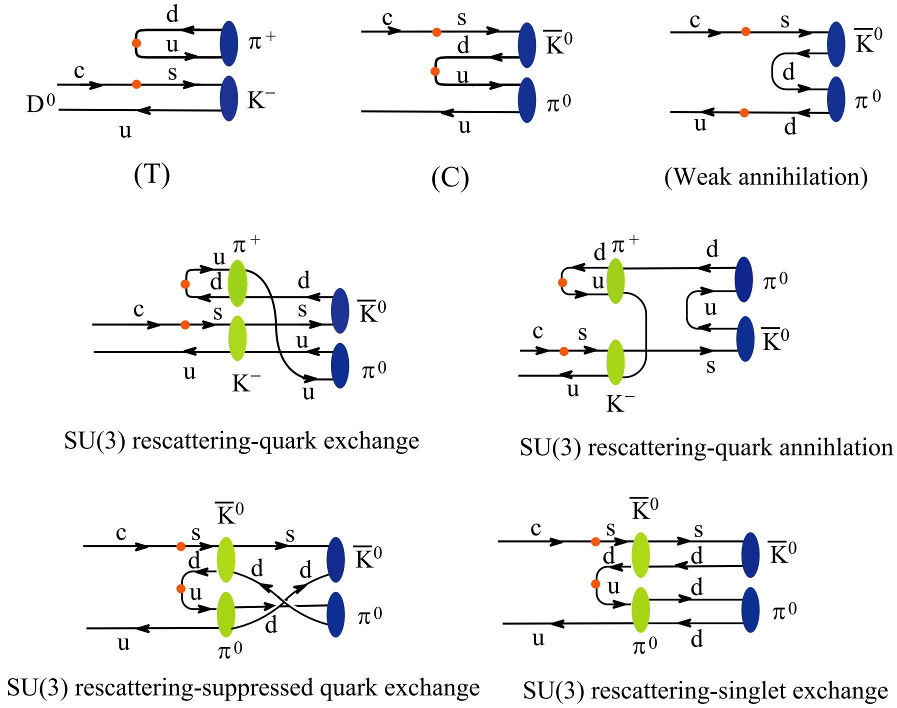

FIG. 2 (color online). Topologies relevant to $D \to K\pi$. The second and third rows correspond to the long-distance SU(3) rescattering contributions to $D^0 \to \overline{K}^0\pi^0$ originating from the tree amplitude, where the quark exchange and singlet exchange contribute to C, the suppressed quark exchange to T, and the quark annihilation to the weak annihilation. The dots denote the quark fields contained in $(V - A) \otimes (V - A)$ four-quark operators.

weak Hamiltonian relevant to the D decays, as given in (2.16) the SD annihilation contributions are dominated by the topologies of gluon emission arising from the initial-state quarks of the weak vertex, whereas the contributions vanish in order of α_s if the gluon is emitted from the final-state quarks, i.e., the amplitudes drawn in Figs. 1(a) and 1(b) cancel each other.

We expect that the possible double counting is reasonably negligible between the LD rescatterings and SD weak annihilation amplitudes due to the following three reasons: (i) The LD rescatterings mainly contain *gluon exchanges between the two final-state mesons*, as depicted in Fig. 2, while the gluon emission originating from the *initial-state quarks* of the weak vertex gives rise to the nonzero SD weak annihilation amplitudes, as shown in Figs. 1(c) and 1(d). (ii) The LD FSIs are dominated by rescatterings of the (color-suppressed) tree amplitudes which are quite different from the mechanism of the SD weak annihilation amplitudes. (iii) The LD rescattering and SD weak annihilation contribute to amplitudes in different ways; for instance, as seen explicitly in Table II, the LS rescattering (SD weak annihilation) interfere constructively (destructively) in the $D^+ \to K^0\pi^+$ and $D^0 \to \pi^0\pi^0$ amplitudes. Finally, it should be noted that, in B decays, one may worry the double counting problem since the nonzero weak annihilation is due to the gluon attached to the *final-state quarks* in the $2(S - P) \otimes (S + P)$ weak vertex.

The strong phase can be generated from the radiative corrections to the weak vertex and the spectator interactions. Such effects were lumped into $N_c^{\rm eff}$ as we calculated the factorization amplitudes. However, since the magnitude of $1/N_c^{\rm eff}$ is very small obtained in our analysis, it is thus reasonable to neglect the resulting strong phase; choosing a real number of $N_c^{\rm eff}$, we have a very nice fit since $\chi^2_{\rm min}/{\rm d.o.f.} = (5.3^{+1.3}_{-0.5})/5$ for negative a_1^K (or $\simeq 11.4/5$ for positive a_1^K). In other words, the LD rescattering effects should be approximately absent from "$N_c^{\rm eff}$."

Our remaining results are briefly summarized as follows.

(i) The two modest rescattering phase differences are $\delta \equiv \delta_{27} - \delta_8 \simeq -46°$ and $\sigma \equiv \delta_{27} - \delta_1 \simeq -21°$, where the σ phase enters only in the $\pi^+\pi^-$-$\pi^0\pi^0$-K^+K^--$K^0\overline{K}^0$-$\pi^0\eta_8$-$\eta_8\eta_8$ rescattering subset.

(ii) We obtain the weak annihilation parameter $|X_A| = 3.84 \pm 0.06$ [$|X_A'| = 2.45^{+0.07}_{-0.46}$ or 2.18 ± 0.19] with a large phase $(-138 \pm 3)°$ [$(-138 \pm 3)°$ or $(130 \pm 3)°$], where a twofold solution exists for X_A'.

(iii) The $D^0 \to K^0\overline{K}^0$ decay occurs mainly due to the short-distance weak annihilation effects, arising from SU(3) symmetry-breaking corrections to the distribution amplitudes of the final-state kaons, but receives negligible contributions from other modes via SU(3) rescattering.

(iv) Our results are in good agreement with the experimental measurements. The predictions for the branching ratios of some unmeasured modes can be used to test our model in the near future.

ACKNOWLEDGMENTS

We are grateful to Chuan-Hung Chen, Hai-Yang Cheng, Dao-Neng Gao, and Chien-Wen Hwang for useful discussions. This work was supported in part by the National Science Council of R. O. C. under Grant Nos. NSC93-2112-M-033-004 and NSC94-2112-M-033-001.

APPENDIX A: FACTORIZATION AMPLITUDES

$$\begin{aligned}
\mathcal{A}_{\rm fac}(D^0 \to K^-\pi^+) &= i\frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^* a_1 f_\pi (m_D^2 - m_K^2) F_0^{DK}(m_\pi^2),\\
\mathcal{A}_{\rm fac}(D^0 \to \overline{K}^0\pi^0) &= i\frac{G_F}{2} V_{ud}V_{cs}^* a_2 f_K (m_D^2 - m_\pi^2) F_0^{D\pi}(m_K^2),\\
\mathcal{A}_{\rm fac}(D^0 \to \overline{K}^0\eta_8) &= i\frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^* a_2 f_K [\cos\vartheta (m_D^2 - m_\eta^2) F_0^{D\eta}(m_K^2) + \sin\vartheta (m_D^2 - m_{\eta'}^2) F_0^{D\eta'}(m_K^2)],\\
\mathcal{A}_{\rm fac}(D^0 \to \overline{K}^0\eta_0) &= i\frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^* a_2 f_K [-\sin\vartheta (m_D^2 - m_\eta^2) F_0^{D\eta}(m_K^2) + \cos\vartheta (m_D^2 - m_{\eta'}^2) F_0^{D\eta'}(m_K^2)],
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{\rm fac}(D^+ \to K^0\pi^+) &= i\frac{G_F}{\sqrt{2}} V_{us}V_{cd}^* a_2 f_K (m_D^2 - m_\pi^2) F_0^{D\pi}(m_K^2),\\
\mathcal{A}_{\rm fac}(D^+ \to K^+\pi^0) &= -i\frac{G_F}{2} V_{us}V_{cd}^* a_1 f_K (m_D^2 - m_\pi^2) F_0^{D\pi}(m_K^2),\\
\mathcal{A}_{\rm fac}(D^+ \to K^+\eta_8) &= i\frac{G_F}{\sqrt{2}} V_{us}V_{cd}^* a_1 f_K [\cos\vartheta (m_D^2 - m_\eta^2) F_0^{D\eta}(m_K^2) + \sin\vartheta (m_D^2 - m_{\eta'}^2) F_0^{D\eta'}(m_K^2)],\\
\mathcal{A}_{\rm fac}(D^+ \to K^+\eta_0) &= i\frac{G_F}{\sqrt{2}} V_{us}V_{cd}^* a_1 f_K [-\sin\vartheta (m_D^2 - m_\eta^2) F_0^{D\eta}(m_K^2) + \cos\vartheta (m_D^2 - m_{\eta'}^2) F_0^{D\eta'}(m_K^2)],
\end{aligned}$$

$$\mathcal{A}_{\rm fac}(D^+ \to \overline{K}^0\pi^+) = i\frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^* [a_1 f_\pi (m_D^2 - m_K^2) F_0^{DK}(m_\pi^2) + a_2 i f_K (m_D^2 - m_\pi^2) F_0^{D\pi}(m_K^2)],$$

$$\begin{aligned}
\mathcal{A}_{\rm fac}(D^0 \to K^+\pi^-) &= i\frac{G_F}{\sqrt{2}} V_{us}V_{cd}^* a_1 f_K (m_D^2 - m_\pi^2) F_0^{D\pi}(m_K^2),\\
\mathcal{A}_{\rm fac}(D^0 \to K^0\pi^0) &= i\frac{G_F}{2} V_{us}V_{cd}^* a_2 f_K (m_D^2 - m_\pi^2) F_0^{D\pi}(m_{K^0}^2),\\
\mathcal{A}_{\rm fac}(D^0 \to K^0\eta_8) &= i\frac{G_F}{\sqrt{2}} V_{us}V_{cd}^* a_2 f_K [\cos\vartheta (m_D^2 - m_\eta^2) F_0^{D\eta}(m_K^2) + \sin\vartheta (m_D^2 - m_{\eta'}^2) F_0^{D\eta'}(m_K^2)],\\
\mathcal{A}_{\rm fac}(D^0 \to K^0\eta_0) &= i\frac{G_F}{\sqrt{2}} V_{us}V_{cd}^* a_2 f_K [-\sin\vartheta (m_D^2 - m_\eta^2) F_0^{D\eta}(m_K^2) + \cos\vartheta (m_D^2 - m_{\eta'}^2) F_0^{D\eta'}(m_K^2)],
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{\rm fac}(D^+\to\pi^+\pi^0) &= -i\frac{G_F}{2}V_{ud}V_{cd}^*(a_1+a_2)f_\pi(m_D^2-m_\pi^2)F_0^{D\pi}(m_\pi^2),\\
\mathcal{A}_{\rm fac}(D^+\to\pi^+\eta_8) &= i\frac{G_F}{\sqrt{2}}V_{ud}V_{cd}^*\{a_1f_\pi[\cos\vartheta(m_D^2-m_\eta^2)F_0^{D\eta}(m_\pi^2)+\sin\vartheta(m_D^2-m_{\eta'}^2)F_0^{D\eta'}(m_\pi^2)]\\
&\quad+a_2[f_\eta^u\cos\vartheta(m_D^2-m_\pi^2)F_0^{D\pi}(m_\eta^2)+f_{\eta'}^u\sin\vartheta(m_D^2-m_\pi^2)F_0^{D\pi}(m_{\eta'}^2)]\}\\
&\quad+i\frac{G_F}{\sqrt{2}}V_{us}V_{cs}^*a_2[f_\eta^s\cos\vartheta(m_D^2-m_\pi^2)F_0^{D\pi}(m_\eta^2)+f_{\eta'}^s\sin\vartheta(m_D^2-m_\pi^2)F_0^{D\pi}(m_{\eta'}^2)],\\
\mathcal{A}_{\rm fac}(D^+\to K^+\overline{K}^0) &= i\frac{G_F}{\sqrt{2}}V_{us}V_{cs}^*a_1f_K(m_D^2-m_K^2)F_0^{DK}(m_K^2),\\
\mathcal{A}_{\rm fac}(D^+\to\pi^+\eta_0) &= i\frac{G_F}{\sqrt{2}}V_{ud}V_{cd}^*\{a_1f_\pi[-\sin\vartheta(m_D^2-m_\eta^2)F_0^{D\eta}(m_\pi^2)+\cos\vartheta(m_D^2-m_{\eta'}^2)F_0^{D\eta'}(m_\pi^2)]\\
&\quad+a_2[-f_\eta^u\sin\vartheta(m_D^2-m_\pi^2)F_0^{D\pi}(m_\eta^2)+f_{\eta'}^u\cos\vartheta(m_D^2-m_\pi^2)F_0^{D\pi}(m_{\eta'}^2)\}\\
&\quad+i\frac{G_F}{\sqrt{2}}V_{us}V_{cs}^*a_2[-f_\eta^s\sin\vartheta(m_D^2-m_\pi^2)F_0^{D\pi}(m_\eta^2)+f_{\eta'}^s\cos\vartheta(m_D^2-m_\pi^2)F_0^{D\pi}(m_{\eta'}^2)],
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{\rm fac}(D^0\to\pi^+\pi^-) &= i\frac{G_F}{\sqrt{2}}V_{ud}V_{cd}^*a_1f_\pi(m_D^2-m_\pi^2)F_0^{D\pi}(m_\pi^2),\\
\mathcal{A}_{\rm fac}(D^0\to\pi^0\pi^0) &= -i\frac{G_F}{2}V_{ud}V_{cd}^*a_2f_\pi(m_D^2-m_\pi^2)F_0^{D\pi}(m_\pi^2),\\
\mathcal{A}_{\rm fac}(D^0\to\eta_8\eta_8) &= iG_FV_{ud}V_{cd}^*a_2\{f_\eta^u[\cos^2\vartheta(m_D^2-m_\eta^2)F^{D\eta}(m_\eta^2)+\sin\theta\cos\vartheta(m_D^2-m_{\eta'}^2)F_0^{D\eta'}(m_\eta^2)]\\
&\quad+f_{\eta'}^u[\sin^2\vartheta(m_D^2-m_{\eta'}^2)F_0^{D\eta'}(m_{\eta'}^2)+\sin\vartheta\cos\vartheta(m_D^2-m_\eta^2)F_0^{D\eta}(m_{\eta'}^2)]\},\\
\mathcal{A}_{\rm fac}(D^0\to K^+K^-) &= i\frac{G_F}{\sqrt{2}}V_{us}V_{cs}^*a_1f_K(m_D^2-m_K^2)F_0^{DK}(m_K^2),\\
\mathcal{A}_{\rm fac}(D^0\to K^0\overline{K}^0) &= 0,\\
\mathcal{A}_{\rm fac}(D^0\to\pi^0\eta_8) &= i\frac{G_F}{2}V_{ud}V_{cd}^*a_2\{-f_\pi[\cos\vartheta(m_D^2-m_\eta^2)F_0^{D\eta}(m_\pi^2)+\sin\vartheta(m_D^2-m_{\eta'}^2)F_0^{D\eta'}(m_\pi^2)]\\
&\quad+[f_\eta^u\cos\vartheta(m_D^2-m_\pi^2)F_0^{D\pi}(m_\eta^2)+f_{\eta'}^u\sin\vartheta(m_D^2-m_\pi^2)F_0^{D\pi}(m_{\eta'}^2)]\},\\
\mathcal{A}_{\rm fac}(D^0\to\pi^0\eta_0) &= i\frac{G_F}{2}V_{ud}V_{cd}^*a_2\{-f_\pi[-\sin\vartheta(m_D^2-m_\eta^2)F_0^{D\eta}(m_\pi^2)+\cos\vartheta(m_D^2-m_{\eta'}^2)F_0^{D\eta'}(m_\pi^2)]\\
&\quad+[-f_\eta^u\sin\vartheta(m_D^2-m_\pi^2)F_0^{D\pi}(m_\eta^2)+f_{\eta'}^u\cos\vartheta(m_D^2-m_\pi^2)F_0^{D\pi}(m_{\eta'}^2)]\},\\
\mathcal{A}_{\rm fac}(D^0\to\eta_8\eta_0) &= i\sqrt{2}G_FV_{ud}V_{cd}^*a_2\{f_\eta^u[-\sin2\vartheta(m_D^2-m_\eta^2)F^{D\eta}(m_\eta^2)+\cos2\theta(m_D^2-m_{\eta'}^2)F_0^{D\eta'}(m_\eta^2)]\\
&\quad+f_{\eta'}^u[-\sin2\vartheta(m_D^2-m_{\eta'}^2)F_0^{D\eta'}(m_{\eta'}^2)+\cos2\vartheta(m_D^2-m_\eta^2)F_0^{D\eta}(m_{\eta'}^2)]\},\\
\mathcal{A}_{\rm fac}(D^0\to\eta_0\eta_0) &= iG_FV_{ud}V_{cd}^*a_2\{f_\eta^u[\sin^2\vartheta(m_D^2-m_\eta^2)F^{D\eta}(m_\eta^2)-\sin\theta\cos\vartheta(m_D^2-m_{\eta'}^2)F_0^{D\eta'}(m_\eta^2)]\\
&\quad+f_{\eta'}^u[\cos^2\vartheta(m_D^2-m_{\eta'}^2)F_0^{D\eta'}(m_{\eta'}^2)-\sin\vartheta\cos\vartheta(m_D^2-m_\eta^2)F_0^{D\eta}(m_{\eta'}^2)]\}. \qquad \text{(A1)}
\end{aligned}$$

APPENDIX B: WEAK ANNIHILATION AMPLITUDES

Here the basic building blocks for annihilation amplitudes corresponding to Fig. 1 are denoted as $A_1^{i(f)}(P_2P_1)$, where the superscript $i(f)$ indicates gluon emission from the initial (final) state quarks, and P_2 (P_1) contains a quark (antiquark) arising from the weak vertex with longitudinal momentum fraction x and $\bar{y}$, respectively, so that the building blocks read

$$A_1^i(P_2P_1) = \pi\alpha_s\int_0^1 dxdy\bigg[\Phi_{P_2}(x)\Phi_{P_1}(y)\bigg(\frac{1}{y(1-x\bar{y})}+\frac{1}{\bar{x}^2y}\bigg)+r_\chi^{P_2}r_\chi^{P_1}\Phi_{P_2}^p(x)\Phi_{P_1}^p(y)\frac{2}{\bar{x}y}\bigg], \qquad \text{(B1)}$$

$$A_1^f(P_2P_1) = 0. \tag{B2}$$

A_1^i can be further expressed in terms of X_A as follows:

$$A_1^i = \begin{cases} \pi\alpha_s\Big[18\Big(X_A - 4 + \frac{\pi^2}{3}\Big) + 2r_\chi^K r_\chi^{P_1} X_A^2 \\ \quad + 54a_1^K\Big(X_A + \frac{4}{3} - \frac{\pi^2}{3}\Big)\Big]; & \text{if } P_2 = K^-, \overline{K}^0, P_1 = \pi, \eta_{8,0}, \\ \pi\alpha_s\Big[18\Big(X_A - 4 + \frac{\pi^2}{3}\Big) + 2r_\chi^K r_\chi^{P_2} X_A^2 + 18a_1^K(X_A + 29 - 3\pi^2)\Big]; & \text{if } P_2 = \pi, \eta_{8,0}, P_1 = K^+, K^0, \\ \pi\alpha_s\Big[18\Big(X_A - 4 + \frac{\pi^2}{3}\Big) + 2r_\chi^K r_\chi^{P_1} X_A^2 - 54a_1^K\Big(X_A + \frac{4}{3} - \frac{\pi^2}{3}\Big)\Big]; & \text{if } P_2 = K^+, K^0, P_1 = \pi, \eta_{8,0}, \\ \pi\alpha_s\Big[18\Big(X_A - 4 + \frac{\pi^2}{3}\Big) + 2r_\chi^{P_2} r_\chi^{P_1} X_A^2]; & \text{if } P_2 = \pi, \eta_{8,0}, \\ \pi\alpha_s\Big[18\Big(X_A - 4 + \frac{\pi^2}{3}\Big) + 2r_\chi^K r_\chi^{P_2} X_A^2 - 18a_1^K(X_A + 29 - 3\pi^2)\Big]; & \text{if } P_2 = \pi, \eta_{8,0}, P_1 = K^-, \overline{K}^0, \\ \pi\alpha_s\Big[18\Big(X_A - 4 + \frac{\pi^2}{3}\Big) + 2(r_\chi^K)^2 X_A^2 - 18a_1^K(4X_A + 33 - 4\pi^2) \\ \quad + 54(a_1^K)^2(X_A - 71 + 7\pi^2)\Big]; & \text{if } P_2 = K^{+,0}, P_1 = \overline{K}^0, \\ \pi\alpha_s\Big[18\Big(X_A - 4 + \frac{\pi^2}{3}\Big) + 2(r_\chi^K)^2 X_A^2 + 18a_1^K(4X_A + 33 - 4\pi^2) \\ \quad + 54(a_1^K)^2(X_A - 71 + 7\pi^2)\Big]; & \text{if } P_2 = K^-, (\overline{K}^0), P_1 = K^+(K^0), \end{cases}; \tag{B3}$$

where $X_A \to X_A'$ for processes containing η_0. The complete weak annihilation amplitudes are given by $A_{\mathcal{T}_\mathcal{B}}(K^0\pi^+) = c_2V_{ud}V_{cs}^*[18(X_A - 4 + \frac{\pi^2}{3}) + 2r_\chi^K r_\chi^\pi X_A^2 + 18a_1^K(X_A + 29 - 3\pi^2)]$, $A_{\mathcal{T}_\mathcal{B}}(K^+\pi^0) = \frac{1}{\sqrt{2}}A_{\mathcal{T}_\mathcal{B}}(K^0\pi^+)$, $A_{\mathcal{T}_\mathcal{B}}(K^+\eta_8) = -\frac{1}{\sqrt{6}}c_2V_{ud}V_{cs}^*[18(X_A - 4 + \frac{\pi^2}{3}) + 2r_\chi^K r_\chi^{\eta_8} X_A^2 - 18a_1^K(7X_A + 37 - 5\pi^2)]$,

$$\begin{aligned}
A_{\mathcal{T}_\mathcal{B}}(K^-\pi^+) &= i\frac{G_F}{\sqrt{2}}f_Df_\pi f_K\frac{C_F}{N_c^2}c_1V_{ud}V_{cs}^*A_1^i(K^-\pi^+), \\
A_{\mathcal{T}_\mathcal{B}}(\overline{K}^0\pi^0) &= -i\frac{G_F}{2}f_Df_\pi f_K\frac{C_F}{N_c^2}c_1V_{ud}V_{cs}^*A_1^i(\overline{K}^0\pi^0), \\
A_{\mathcal{T}_\mathcal{B}}(\overline{K}^0\eta_8) &= i\frac{G_F}{2\sqrt{3}}f_Df_\pi f_{\eta_8}\frac{C_F}{N_c^2}c_1V_{ud}V_{cs}^*\{-2A_1^i(\eta_8\overline{K}^0) + A_1^i(\overline{K}^0\eta_8)\}, \\
A_{\mathcal{T}_\mathcal{B}}(\overline{K}^0\eta_0) &= i\frac{G_F}{\sqrt{6}}f_Df_\pi f_{\eta_0}\frac{C_F}{N_c^2}c_1V_{ud}V_{cs}^*\{A_1^i(\eta_0\overline{K}^0) + A_1^i(\overline{K}^0\eta_0)\}, \\
A_{\mathcal{T}_\mathcal{B}}(K^0\pi^+) &= i\frac{G_F}{\sqrt{2}}f_Df_\pi f_K\frac{C_F}{N_c^2}c_2V_{us}V_{cd}^*A_1^i(\pi^+K^0), \\
A_{\mathcal{T}_\mathcal{B}}(K^+\pi^0) &= i\frac{G_F}{2}f_Df_\pi f_K\frac{C_F}{N_c^2}c_2V_{us}V_{cd}^*A_1^i(\pi^0K^+), \\
A_{\mathcal{T}_\mathcal{B}}(K^+\eta_8) &= i\frac{G_F}{2\sqrt{3}}f_Df_\pi f_{\eta_8}\frac{C_F}{N_c^2}c_2V_{us}V_{cd}^*\{A_1^i(\eta_8K^+) - 2A_1^i(K^+\eta_8)\}, \\
A_{\mathcal{T}_\mathcal{B}}(K^+\eta_0) &= i\frac{G_F}{\sqrt{6}}f_Df_\pi f_{\eta_0}\frac{C_F}{N_c^2}c_2V_{us}V_{cd}^*\{A_1^i(\eta_0K^+) + A_1^i(K^+\eta_0)\}, \\
A_{\mathcal{T}_\mathcal{B}}(\overline{K}^0\pi^+) &= 0,
\end{aligned}$$

$$A_{\mathcal{T}_{\mathcal{B}}}(K^0\pi^+) = c_2 V_{ud} V_{cs}^* \bigg[18\bigg(X_A - 4 + \frac{\pi^2}{3}\bigg) + 2r_\chi^K r_\chi^\pi X_A^2 + 18a_1^K(X_A + 29 - 3\pi^2) \bigg],$$

$$A_{\mathcal{T}_{\mathcal{B}}}(K^+\pi^0) = \frac{1}{\sqrt{2}} A_{\mathcal{T}_{\mathcal{B}}}(K^0\pi^+),$$

$$A_{\mathcal{T}_{\mathcal{B}}}(K^+\eta_8) = -\frac{1}{\sqrt{6}} c_2 V_{ud} V_{cs}^* \bigg[18\bigg(X_A - 4 + \frac{\pi^2}{3}\bigg) + 2r_\chi^K r_\chi^{\eta_8} X_A^2 - 18a_1^K(7X_A + 37 - 5\pi^2) \bigg],$$

$$A_{\mathcal{T}_{\mathcal{B}}}(K^+\pi^-) = i\frac{G_F}{\sqrt{2}} f_D f_\pi f_K \frac{C_F}{N_c^2} c_1 V_{us} V_{cd}^* A_1^i(\pi^- K^+),$$

$$A_{\mathcal{T}_{\mathcal{B}}}(K^0\pi^0) = -i\frac{G_F}{2} f_D f_\pi f_K \frac{C_F}{N_c^2} c_1 V_{us} V_{cd}^* A_1^i(\pi^0 K^0),$$

$$A_{\mathcal{T}_{\mathcal{B}}}(K^0\eta_8) = i\frac{G_F}{2\sqrt{3}} f_D f_\pi f_{\eta_8} \frac{C_F}{N_c^2} c_1 V_{us} V_{cd}^* \{A_1^i(\eta_8 K^0) - 2A_1^i(K^0\eta_8)\},$$

$$A_{\mathcal{T}_{\mathcal{B}}}(K^0\eta_0) = i\frac{G_F}{\sqrt{6}} f_D f_\pi f_{\eta_0} \frac{C_F}{N_c^2} c_1 V_{us} V_{cd}^* \{A_1^i(\eta_0 K^0) + A_1^i(K^0\eta_0)\},$$

$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^+\pi^0) = i\frac{G_F}{2} f_D f_\pi^2 \frac{C_F}{N_c^2} c_2 V_{ud} V_{cd}^* \{A_1^i(\pi^0\pi^+) - A_1^i(\pi^+\pi^0)\} = 0,$$

$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^+\eta_8) = i\frac{G_F}{2\sqrt{3}} f_D f_\pi f_{\eta_8} \frac{C_F}{N_c^2} c_2 V_{ud} V_{cd}^* \{A_1^i(\pi^+\eta_8) + A_1^i(\eta_8\pi^+)\},$$

$$A_{\mathcal{T}_{\mathcal{B}}}(K^+\overline{K}^0) = i\frac{G_F}{\sqrt{2}} f_D f_K^2 \frac{C_F}{N_c^2} c_2 V_{ud} V_{cd}^* A_1^i(K^+\overline{K}^0),$$

$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^+\eta_0) = i\frac{G_F}{\sqrt{6}} f_D f_\pi f_{\eta_0} \frac{C_F}{N_c^2} c_2 V_{ud} V_{cd}^* \{A_1^i(\pi^+\eta_0) + A_1^i(\eta_0\pi^+)\},$$

$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^+\pi^-) = i\frac{G_F}{\sqrt{2}} f_D f_\pi^2 \frac{C_F}{N_c^2} c_1 V_{ud} V_{cd}^* A_1^i(\pi^-\pi^+),$$

$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^0\pi^0) = i\frac{G_F}{2} f_D f_\pi^2 \frac{C_F}{N_c^2} c_1 V_{ud} V_{cd}^* A_1^i(\pi^0\pi^0),$$

$$A_{\mathcal{T}_{\mathcal{B}}}(\eta_8\eta_8) = i\frac{G_F}{6} f_D f_{\eta_8}^2 \frac{C_F}{N_c^2} c_1 (V_{ud} V_{cd}^* + 4V_{us} V_{cs}^*) A_1^i(\eta_8\eta_8),$$

$$A_{\mathcal{T}_{\mathcal{B}}}(K^+K^-) = i\frac{G_F}{\sqrt{2}} f_D f_K^2 \frac{C_F}{N_c^2} c_1 V_{us} V_{cs}^* A_1^i(K^- K^+),$$

$$A_{\mathcal{T}_{\mathcal{B}}}(K^0\overline{K}^0) = i\frac{G_F}{\sqrt{2}} f_D f_K^2 \frac{C_F}{N_c^2} c_1 \{V_{us} V_{cs}^* A_1^i(\overline{K}^0 K^0) + V_{ud} V_{cd}^* A_1^i(K^0\overline{K}^0)\},$$

$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^0\eta_8) = -i\frac{G_F}{2\sqrt{6}} f_D f_\pi f_{\eta_8} \frac{C_F}{N_c^2} c_1 V_{ud} V_{cd}^* \{A_1^i(\pi^0\eta_8) + A_1^i(\eta_8\pi^0)\},$$

$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^0\eta_0) = -i\frac{G_F}{2\sqrt{3}} f_D f_\pi f_{\eta_0} \frac{C_F}{N_c^2} c_1 V_{ud} V_{cd}^* \{A_1^i(\pi^0\eta_0) + A_1^i(\eta_0\pi^0)\},$$

$$A_{\mathcal{T}_{\mathcal{B}}}(\eta_8\eta_0) = i\frac{G_F}{6} f_D f_{\eta_8} f_{\eta_0} \frac{C_F}{N_c^2} c_1 (V_{ud} V_{cd}^* - 2V_{us} V_{cs}^*) \{A_1^i(\eta_0\eta_8) + A_1^i(\eta_8\eta_0)\},$$

$$A_{\mathcal{T}_{\mathcal{B}}}(\eta_0\eta_0) = i\frac{G_F}{3} f_D f_0^2 \frac{C_F}{N_c^2} c_1 (V_{ud} V_{cd}^* + V_{us} V_{cs}^*) A_1^i(\eta_0\eta_0).$$

The above weak annihilation amplitudes can be further expressed in terms of $X_A^{(\prime)}$ as follows (in units of $i\frac{G_F}{\sqrt{2}} f_D f_{P_1} f_{P_2} \frac{C_F}{N_c^2} \pi\alpha_s$):

$$A_{\mathcal{T}_{\mathcal{B}}}(K^-\pi^+) = c_1 V_{ud} V_{cs}^* \left[18\left(X_A - 4 + \frac{\pi^2}{3}\right) + 2 r_\chi^K r_\chi^\pi X_A^2 + 54 a_1^K \left(X_A + \frac{4}{3} - \frac{\pi^2}{3}\right)\right],$$
$$A_{\mathcal{T}_{\mathcal{B}}}(\overline{K}^0\pi^0) = \frac{1}{\sqrt{2}} A_{\mathcal{T}_{\mathcal{B}}}(K^-\pi^+),$$
$$A_{\mathcal{T}_{\mathcal{B}}}(\overline{K}^0\eta_8) = -\frac{1}{\sqrt{6}} c_1 V_{ud} V_{cs}^* \left[18\left(X_A - 4 + \frac{\pi^2}{3}\right) + 2 r_\chi^K r_\chi^{\eta_8} X_A^2 - 18 a_1^K (5X_A + 62 - 7\pi^2)\right],$$
$$A_{\mathcal{T}_{\mathcal{B}}}(\overline{K}^0\eta_0) = \frac{2}{\sqrt{3}} c_1 V_{ud} V_{cs}^* \left[18\left(X_A' - 4 + \frac{\pi^2}{3}\right) + 2 r_\chi^K r_\chi^{\eta_0} X_A'^2 + 9 a_1^K (2X_A' - 25 + 2\pi^2)\right],$$
$$A_{\mathcal{T}_{\mathcal{B}}}(K^0\pi^+) = c_2 V_{ud} V_{cs}^* \left[18\left(X_A - 4 + \frac{\pi^2}{3}\right) + 2 r_\chi^K r_\chi^\pi X_A^2 + 18 a_1^K (X_A + 29 - 3\pi^2)\right],$$
$$A_{\mathcal{T}_{\mathcal{B}}}(K^+\pi^0) = \frac{1}{\sqrt{2}} A_{\mathcal{T}_{\mathcal{B}}}(K^0\pi^+),$$
$$A_{\mathcal{T}_{\mathcal{B}}}(K^+\eta_8) = -\frac{1}{\sqrt{6}} c_2 V_{ud} V_{cs}^* \left[18\left(X_A - 4 + \frac{\pi^2}{3}\right) + 2 r_\chi^K r_\chi^{\eta_8} X_A^2 - 18 a_1^K (7X_A + 37 - 5\pi^2)\right],$$
$$A_{\mathcal{T}_{\mathcal{B}}}(K^+\eta_0) = \frac{2}{\sqrt{3}} c_2 V_{ud} V_{cs}^* \left[18\left(X_A' - 4 + \frac{\pi^2}{3}\right) + 2 r_\chi^K r_\chi^{\eta_0} X_A'^2 - 9 a_1^K (2X_A' - 25 + 2\pi^2)\right],$$

$$A_{\mathcal{T}_{\mathcal{B}}}(\overline{K}^0\pi^+) = 0,$$

$$A_{\mathcal{T}_{\mathcal{B}}}(K^+\pi^-) = \frac{V_{us} V_{cd}^* c_1}{V_{ud} V_{cs}^* c_2} A_{\mathcal{T}_{\mathcal{B}}}(K^0\pi^+), \qquad A_{\mathcal{T}_{\mathcal{B}}}(K^0\pi^0) = -\frac{1}{\sqrt{2}} \frac{V_{us} V_{cd}^* c_1}{V_{ud} V_{cs}^* c_2} A_{\mathcal{T}_{\mathcal{B}}}(K^0\pi^+),$$
$$A_{\mathcal{T}_{\mathcal{B}}}(K^0\eta_8) = \frac{V_{us} V_{cd}^* c_1}{V_{ud} V_{cs}^* c_2} A_{\mathcal{T}_{\mathcal{B}}}(K^+\eta_8), \qquad A_{\mathcal{T}_{\mathcal{B}}}(K^0\eta_0) = \frac{V_{us} V_{cd}^* c_1}{V_{ud} V_{cs}^* c_2} A_{\mathcal{T}_{\mathcal{B}}}(K^+\eta_0),$$

$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^+\pi^0) = 0,$$
$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^+\eta_8) = \sqrt{\frac{2}{3}} c_2 V_{ud} V_{cd}^* \left[18\left(X_A - 4 + \frac{\pi^2}{3}\right) + 2 r_\chi^\pi r_\chi^{\eta_8} X_A^2\right],$$
$$A_{\mathcal{T}_{\mathcal{B}}}(K^+\overline{K}^0) = c_2 V_{ud} V_{cd}^* \left[18\left(X_A - 4 + \frac{\pi^2}{3}\right) + 2 (r_\chi^K)^2 X_A^2 - 18 a_1^K (4X_A + 33 - 4\pi^2) + 54 (a_1^K)^2 (X_A - 71 + 7\pi^2)\right],$$
$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^+\eta_0) = \frac{2}{\sqrt{3}} c_2 V_{ud} V_{cd}^* \left[18\left(X_A' - 4 + \frac{\pi^2}{3}\right) + 2 r_\chi^\pi r_\chi^{\eta_0} X_A'^2\right],$$
$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^+\pi^-) = c_1 V_{ud} V_{cd}^* \left[18\left(X_A - 4 + \frac{\pi^2}{3}\right) + 2 (r_\chi^\pi)^2 X_A^2\right],$$
$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^0\pi^0) = \frac{1}{\sqrt{2}} A_{\mathcal{T}_{\mathcal{B}}}(\pi^+\pi^-), \qquad A_{\mathcal{T}_{\mathcal{B}}}(\eta_8\eta_8) = -\frac{1}{\sqrt{2}} A_{\mathcal{T}_{\mathcal{B}}}(\pi^+\pi^-),$$
$$A_{\mathcal{T}_{\mathcal{B}}}(K^+K^-) = c_1 V_{us} V_{cs}^* \left[18\left(X_A - 4 + \frac{\pi^2}{3}\right) + 2 (r_\chi^K)^2 X_A^2 + 18 a_1^K (4X_A + 33 - 4\pi^2) + 54 (a_1^K)^2 (X_A - 71 + 7\pi^2)\right],$$
$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^0\eta_8) = -\frac{1}{\sqrt{3}} c_1 V_{ud} V_{cd}^* \left[18\left(X_A - 4 + \frac{\pi^2}{3}\right) + 2 (r_\chi^\pi)^2 X_A^2\right],$$
$$A_{\mathcal{T}_{\mathcal{B}}}(\pi^0\eta_0) = -\sqrt{\frac{2}{3}} c_1 V_{ud} V_{cd}^* \left[18\left(X_A' - 4 + \frac{\pi^2}{3}\right) + 2 r_\chi^\pi r_\chi^{\eta_0} X_A'^2\right],$$
$$A_{\mathcal{T}_{\mathcal{B}}}(\eta_8\eta_0) = -\frac{\sqrt{2}}{3} c_1 V_{ud} V_{cd}^* \left[18\left(X_A' - 4 + \frac{\pi^2}{3}\right) + 2 r_\chi^{\eta_8} r_\chi^{\eta_0} X_A'^2\right], \qquad A_{\mathcal{T}_{\mathcal{B}}}(\eta_0\eta_0) = 0, \tag{B4}$$

where X_A is treated as a universal parameter for SU(3) channels, while for decay modes involving η_0, it is distinguished to be X'_A.

APPENDIX C: SU(3) FINAL STATE INTERACTIONS—$8 \otimes 8$ DECOMPOSITION

To describe elastic SU(3) final state interactions among $D \to P_1P_2$ decays, we adopt the notations:

$$\mathbf{q} = q^i = \begin{pmatrix} q^1 \\ q^2 \\ q^3 \end{pmatrix} \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix} \tag{C1}$$

and

$$\bar{\mathbf{q}} = q_j = (\,q_1 \quad q_2 \quad q_3\,) \equiv (\,\bar{u} \quad \bar{d} \quad \bar{s}\,). \tag{C2}$$

The octet final-state pseudoscalar mesons P_1 and P_2, which are viewed as composites of quarks in the quark model, can be represented by the matrix

$$\Pi = \mathbf{q} \otimes \bar{\mathbf{q}} - \frac{1}{3}\mathbf{1}\,\mathrm{Tr}(\mathbf{q} \otimes \bar{\mathbf{q}}) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix}, \tag{C3}$$

where Π^i_j is the **8** representation, while $\Pi^i_i = 0$. The SU(3) final-state rescatterings for $D \to P_1P_2$ are described by the product $\mathbf{8} \otimes \mathbf{8}$. Since the P_1P_2 states obey the Bose symmetry, only the symmetric states given by the representation $\mathbf{36}(= \mathbf{27} \oplus \mathbf{8} \oplus \mathbf{1})$ in $\mathbf{8} \otimes \mathbf{8}(= \mathbf{36} \oplus \mathbf{28})$ decomposition are relevant, whereas states given by the representation $\mathbf{28}(= \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{8})$ vanish. The weak decay amplitudes $\mathbf{A}_i^{\mathrm{FSI}}$ for $D \to P_1P_2$ with FSIs are given by

$$\mathbf{A}_i^{\mathrm{FSI}} = \sum_l \mathbf{S}_{il}^{1/2}\mathbf{A}_l^{\mathrm{bare}} = (\mathbf{U}^{\mathrm{T}}\mathbf{S}_{\mathrm{diag}}^{1/2}\mathbf{U})_{il}\mathbf{A}_l^{\mathrm{bare}}, \tag{C4}$$

where $\mathbf{A}_l^{\mathrm{bare}} = \mathbf{A}_l^{\mathrm{fac}} + \mathbf{A}_l^{\mathcal{T}\mathcal{B}}$ are defined in Eqs. (3.3), (3.5), (3.7), (3.9), and (3.11). In orthonormal bases of SU(3), the $\mathbf{S}_{\mathrm{diag}}^{1/2}$ matrix, describing the SU(3) FSIs, can be recast into the following form

$$\mathbf{S}_{\mathrm{diag}}^{1/2} = e^{i\delta_{27}}\sum_{a=1}^{27}|T(27);a\rangle\langle T(27);a| + e^{i\delta_8}\sum_{b=1}^{8}|T(8);b\rangle\langle T(8);b| + e^{i\delta_1}|T(1)\rangle\langle T(1)|, \tag{C5}$$

where $|T(27);a\rangle$, $|T(8);b\rangle$, and $|T(1)\rangle$ are orthonormal SU(3) bases in the irreducible representation **36**. Using the tensor approach [46,47], the **36** states are described by $\Pi^{\{i}_{\{k}\Pi^{j\}}_{l\}}$ with $\{i, j\}$ being symmetric in indices i, j, and can be decoupled into three types of irreducible tensors: (i) **1**, an irreducible tensor of rank (0, 0), equals to $\Pi^i_k\Pi^k_i \equiv T^{ik}_{ki}$. (ii) **8**, an irreducible tensor of rank (1, 1), is equivalent to $T^{mj}_{km} - (1/3)\delta^j_k T^{ml}_{lm} = \Pi^m_k\Pi^j_m - (1/3)\delta^j_k\Pi^m_i\Pi^i_m \equiv U^j_k$. (iii) **27**, an irreducible tensor of rank (2, 2), is given by $T^{ij}_{kl} + T^{ji}_{kl} - (1/5)(\delta^i_k T^{mj}_{lm} + \delta^j_k T^{mi}_{lm} + \delta^i_l T^{mj}_{km} + \delta^j_l T^{mi}_{km}) + (1/20)(\delta^i_k\delta^j_l + \delta^j_k\delta^i_l)T^{mn}_{nm} \equiv V^{ij}_{kl}$. We summarized the orthonormal states in the representation **36** together with their quantum numbers S and I as follows.

(i) In the representation **1**, the normalized state $|T(1)\rangle$ is

$$(S = 0, I = 0) : \frac{1}{\sqrt{8}}(\sqrt{2}|\pi^+\pi^-\rangle + |\pi^0\pi^0\rangle + |\eta_8\eta_8\rangle + \sqrt{2}|K^+K^-\rangle + \sqrt{2}|\bar{K}^0K^0\rangle). \tag{C6}$$

(ii) In the representation **8**, the normalized states $|T(8);b\rangle$ are

$$\left(S = 1, I = \frac{1}{2}\right): \quad \sqrt{\frac{1}{10}}(\sqrt{6}(|K^0\pi^+\rangle + \sqrt{3}|K^+\pi^0\rangle - |K^+\eta_8\rangle), \quad \sqrt{\frac{1}{10}}(\sqrt{6}(|K^+\pi^-\rangle - \sqrt{3}|K^0\pi^0\rangle - |K^0\eta_8\rangle); \tag{C7}$$

$$\left(S = -1, I = \frac{1}{2}\right): \quad \sqrt{\frac{1}{10}}(\sqrt{6}(|K^-\pi^+\rangle - \sqrt{3}|\bar{K}^0\pi^0\rangle - |\bar{K}^0\eta_8\rangle), \quad \sqrt{\frac{1}{10}}(-\sqrt{6}(|\bar{K}^0\pi^-\rangle - \sqrt{3}|K^-\pi^0\rangle + |K^-\eta_8\rangle); \tag{C8}$$

$$(S=0,I=1):\quad \sqrt{\frac{2}{5}}|\pi^+\eta_8\rangle+\sqrt{\frac{3}{5}}|\overline{K}^0K^+\rangle,$$
$$\sqrt{\frac{1}{10}}(\sqrt{3}|K^+K^-\rangle-\sqrt{3}|\overline{K}^0K^0\rangle+2|\pi^0\eta_8\rangle),$$
$$\sqrt{\frac{2}{5}}|\pi^-\eta_8\rangle+\sqrt{\frac{3}{5}}|K^0K^-\rangle; \tag{C9}$$

$$(S=0,I=0):\sqrt{\frac{1}{10}}(-2|\pi^+\pi^-\rangle-\sqrt{2}|\pi^0\pi^0\rangle+\sqrt{2}|\eta_8\eta_8\rangle+|K^+K^-\rangle+|\overline{K}^0K^0\rangle). \tag{C10}$$

(iii) In the representation **27**, the normalized states $|T(27);a\rangle$ are

$$(S=2,I=1):\frac{1}{\sqrt{2}}|K^+K^+\rangle,\quad |K^+K^0\rangle,\quad \frac{1}{\sqrt{2}}|K^0K^0\rangle; \tag{C11}$$

$$(S=-2,I=1):\frac{1}{\sqrt{2}}|\overline{K}^0\overline{K}^0\rangle,\quad |\overline{K}^0K^-\rangle,\quad \frac{1}{\sqrt{2}}|K^-K^-\rangle; \tag{C12}$$

$$\left(S=1,I=\frac{3}{2}\right):\quad |K^+\pi^+\rangle,\quad \frac{1}{\sqrt{3}}(|K^0\pi^+\rangle-\sqrt{2}|K^+\pi^0\rangle),$$
$$\frac{1}{\sqrt{3}}(|K^+\pi^-\rangle+\sqrt{2}|K^0\pi^0\rangle),\quad |K^0\pi^-\rangle; \tag{C13}$$

$$\left(S=-1,I=\frac{3}{2}\right):\quad |\overline{K}^0\pi^+\rangle,\quad \frac{1}{\sqrt{3}}(|K^-\pi^+\rangle+\sqrt{2}|\overline{K}^0\pi^0\rangle),$$
$$\frac{1}{\sqrt{3}}(|\overline{K}^0\pi^-\rangle-\sqrt{2}|K^-\pi^0\rangle),\quad |K^-\pi^-\rangle; \tag{C14}$$

$$\left(S=1,I=\frac{1}{2}\right):\quad \frac{1}{\sqrt{30}}(\sqrt{2}|K^0\pi^+\rangle+|K^+\pi^0\rangle+3\sqrt{3}|K^+\eta_8\rangle),$$
$$\frac{1}{\sqrt{30}}(\sqrt{2}|K^+\pi^-\rangle-|K^0\pi^0\rangle+3\sqrt{3}|K^0\eta_8\rangle); \tag{C15}$$

$$\left(S=-1,I=\frac{1}{2}\right):\quad \frac{1}{\sqrt{30}}(\sqrt{2}|K^-\pi^+\rangle-|\overline{K}^0\pi^0\rangle+3\sqrt{3}|\overline{K}^0\eta_8\rangle),$$
$$\frac{1}{\sqrt{30}}(\sqrt{2}|\overline{K}^0\pi^-\rangle+|K^-\pi^0\rangle+3\sqrt{3}|K^-\eta_8\rangle); \tag{C16}$$

$$(S=0,I=2):\frac{1}{\sqrt{2}}|\pi^+\pi^+\rangle,\quad |\pi^0\pi^+\rangle,\quad \frac{1}{\sqrt{3}}(|\pi^-\pi^+\rangle-\sqrt{2}|\pi^0\pi^0\rangle,\quad |\pi^0\pi^-\rangle,\quad \frac{1}{\sqrt{2}}|\pi^-\pi^-\rangle; \tag{C17}$$

$$(S=0,I=1):\quad \sqrt{\frac{2}{5}}|\overline{K}^0K^+\rangle-\sqrt{\frac{3}{5}}|\pi^+\eta_8\rangle,\quad \sqrt{\frac{1}{5}}(|K^+K^-\rangle-|\overline{K}^0K^0\rangle-\sqrt{3}|\pi^0\eta_8\rangle),$$
$$\sqrt{\frac{2}{5}}|K^0K^-\rangle-\sqrt{\frac{3}{5}}|\pi^-\eta_8\rangle; \tag{C18}$$

$$(S=0,I=0):\frac{1}{4\sqrt{15}}(2|\pi^+\pi^-\rangle+\sqrt{2}|\pi^0\pi^0\rangle+9\sqrt{2}|\eta_8\eta_8\rangle-6|K^+K^-\rangle-6|\overline{K}^0K^0\rangle). \tag{C19}$$

Using the above results, one can immediately obtain the relevant **U** matrices and the corresponding SU(3) eigen-amplitudes in D decays:

$$\mathbf{A}^{\mathrm{SU}(3)}_{(\overline{K\pi})^0} = \begin{pmatrix} |\mathbf{27}, S=-1, I=3/2, I_z=+1/2\rangle \\ |\mathbf{27}, S=-1, I=1/2, I_z=+1/2\rangle \\ |\mathbf{8}, S=-1, I=1/2, I_z=+1/2\rangle \end{pmatrix} = \mathbf{U}_{(\overline{K\pi})^0}\mathbf{A}^{\mathrm{bare}}_{(\overline{K\pi})^0} = \begin{pmatrix} \sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ \sqrt{\frac{1}{15}} & -\sqrt{\frac{1}{30}} & \frac{3}{\sqrt{10}} \\ \sqrt{\frac{3}{5}} & -\sqrt{\frac{3}{10}} & -\sqrt{\frac{1}{10}} \end{pmatrix}\mathbf{A}^{\mathrm{bare}}_{(\overline{K\pi})^0}, \tag{C20}$$

$$\mathbf{A}^{\mathrm{SU}(3)}_{(K\pi)^0} = \begin{pmatrix} |\mathbf{27}, S=1, I=3/2, I_z=-1/2\rangle \\ |\mathbf{27}, S=1, I=1/2, I_z=-1/2\rangle \\ |\mathbf{8}, S=1, I=1/2, I_z=-1/2\rangle \end{pmatrix} = \mathbf{U}_{(K\pi)^0}\mathbf{A}^{\mathrm{bare}}_{(K\pi)^0} = \begin{pmatrix} \sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ \sqrt{\frac{1}{15}} & -\sqrt{\frac{1}{30}} & \frac{3}{\sqrt{10}} \\ \sqrt{\frac{3}{5}} & -\sqrt{\frac{3}{10}} & -\sqrt{\frac{1}{10}} \end{pmatrix}\mathbf{A}^{\mathrm{bare}}_{(K\pi)^0}, \tag{C21}$$

$$\mathbf{A}^{\mathrm{SU}(3)}_{(K\pi)^+} = \begin{pmatrix} |\mathbf{27}, S=1, I=3/2, I_z=1/2\rangle \\ |\mathbf{27}, S=1, I=1/2, I_z=1/2\rangle \\ |\mathbf{8}, S=1, I=1/2, I_z=1/2\rangle \end{pmatrix} = \mathbf{U}_{(K\pi)^+}\mathbf{A}^{\mathrm{bare}}_{(K\pi)^+} = \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} & 0 \\ \sqrt{\frac{1}{15}} & \sqrt{\frac{1}{30}} & \frac{3}{\sqrt{10}} \\ \sqrt{\frac{3}{5}} & \sqrt{\frac{3}{10}} & -\sqrt{\frac{1}{10}} \end{pmatrix}\mathbf{A}^{\mathrm{bare}}_{(K\pi)^+}, \tag{C22}$$

$$\mathbf{A}^{\mathrm{SU}(3)}_{(\pi\pi)^+} = \begin{pmatrix} |\mathbf{27}, S=0, I=2, I_z=1\rangle \\ |\mathbf{27}, S=0, I=1, I_z=1\rangle \\ |\mathbf{8}, S=0, I=1, I_z=1\rangle \end{pmatrix} = \mathbf{U}_{(\pi\pi)^+}\mathbf{A}^{\mathrm{bare}}_{(\pi\pi)^+} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\sqrt{\frac{3}{5}} & \sqrt{\frac{2}{5}} \\ 0 & \sqrt{\frac{2}{5}} & \sqrt{\frac{3}{5}} \end{pmatrix}\mathbf{A}^{\mathrm{bare}}_{(\pi\pi)^+}, \tag{C23}$$

$$\mathbf{A}^{\mathrm{SU}(3)}_{(\pi\pi)^0} = \begin{pmatrix} |\mathbf{27}, S=0, I=2, I_z=0\rangle \\ |\mathbf{27}, S=0, I=0, I_z=0\rangle \\ |\mathbf{8}, S=0, I=0, I_z=0\rangle \\ |\mathbf{1}, S=0, I=0, I_z=0\rangle \\ |\mathbf{27}, S=0, I=1, I_z=0\rangle \\ |\mathbf{8}, S=0, I=1, I_z=0\rangle \end{pmatrix} = \mathbf{U}_{(\pi\pi)^0}\mathbf{A}^{\mathrm{bare}}_{(\pi\pi)^0} = \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} & 0 & 0 & 0 & 0 \\ \frac{1}{2\sqrt{15}} & \frac{1}{2\sqrt{30}} & \frac{3\sqrt{3}}{2\sqrt{10}} & -\frac{\sqrt{3}}{2\sqrt{5}} & -\frac{\sqrt{3}}{2\sqrt{5}} & 0 \\ -\sqrt{\frac{2}{5}} & -\sqrt{\frac{1}{5}} & \sqrt{\frac{1}{5}} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 \\ \frac{1}{2} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \sqrt{\frac{1}{5}} & -\sqrt{\frac{1}{5}} & -\sqrt{\frac{3}{5}} \\ 0 & 0 & 0 & \sqrt{\frac{3}{10}} & -\sqrt{\frac{3}{10}} & \sqrt{\frac{2}{5}} \end{pmatrix}\mathbf{A}^{\mathrm{bare}}_{(\pi\pi)^0}, \tag{C24}$$

where $\mathbf{A}^{\mathrm{bare}}_{(\overline{K\pi})^0}$, $\mathbf{A}^{\mathrm{bare}}_{(K\pi)^0}$, $\mathbf{A}^{\mathrm{bare}}_{(K\pi)^+}$, $\mathbf{A}^{\mathrm{bare}}_{(\pi\pi)^+}$, and $\mathbf{A}^{\mathrm{bare}}_{(\pi\pi)^0}$ have been defined in Eqs. (3.3), (3.5), (3.7), (3.9), and (3.11), respectively.

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