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3-3-1 models with unique lepton generations

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We study previously unconsidered 3-3-1 models which are characterized by each lepton generation having a different representation under the gauge group. Flavor-changing neutral currents in the lepton sector occur in these models. To satisfy constraints on $\mu \rightarrow 3e$ decays, the Z' must be heavier than 2 to 40 TeV, depending on the model and assignments of the leptons. These models can result in very unusual Higgs decay modes. In most cases the $\mu\tau$ decay state is large (in one case, it is the dominant mode), and in one case, the $\Phi \rightarrow s\bar{s}$ rate dominates.

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I. INTRODUCTION

An interesting extension of the standard model is based on the gauge group $SU(3)_c \times SU(3)_L \times U(1)$ (3-3-1). In the original, minimal version of the model [1,2], the leptons are put into antitriplets of $SU(3)_L$, two generations of quarks are put into triplets and the third generation of quarks is put into an antitriplet. With this structure, the anomalies will all cancel if and only if the number of generations is a multiple of three. The model has an automatic Peccei-Quinn symmetry [3,4], and the fact that one quark family has different quantum numbers than the other two may explain the heavy top quark mass [5]. An unusual feature of this model is that $\sin^2\theta_W$ must be less than $1/4$. Since it is an increasing function of q^2, the scale of $SU(3)_L$ breaking must be relatively low, and cannot arbitrarily be moved up to a high scale.

This minimal model contains doubly charged gauge fields (bileptons) as well as isosinglet quarks with exotic charges. The phenomenology of these models is very rich and has been the subject of extensive study [6]. A completely different class of models was proposed in Refs. [7,8], in which the embedding of the charge operator into $SU(3)_L$ is different. In these models, there are no exotic charges for the quarks, and the gauge bosons are all either neutral or singly charged. In all of these models, one still treats the lepton generations identically, and treats one quark generation differently than the other two. A comprehensive review of the gauge, fermion and scalar sectors of all of these models can be found in Refs. [9,10].

In Ref. [9], a detailed analysis of the anomalies in 3-3-1 models showed that there are two anomaly-free sets of fermion representations in which the lepton generations are *all* treated differently. The phenomenology of these models has never been studied in the literature. With leptons in different representations, one might expect lepton-flavor-changing-neutral processes.

In this paper, we discuss the phenomenology of these two models. In Sec. II, the various 3-3-1 models are presented, as well as the possible representations for fermions in these models. A set of anomaly-free models will be found, and it will be noted that two of them have very different representations for the lepton families. In Sec. III, we will consider the scalar sector of these "unique lepton generation" models, and in Sec. IV will present the mass matrices for the leptons, look at the possible variations that can occur, and find the Yukawa couplings to the scalars. The phenomenology of lepton-number violating μ and τ decays will be discussed in Sec. V, and for Higgs decays in Sec. VI. Our most interesting result will be that many of these models have fairly large branching ratios for the Higgs boson decaying into a muon and a tau, and in one model it may be the dominant decay. In Sec. VII, we will examine lepton-number violation due to gauge boson exchange, and the resulting bounds on the gauge boson masses. Finally, in Sec. VIII we present our conclusions.

II. MODELS

As discussed in Ref. [9], if one assumes that the isospin $SU(2)_L$ of the standard model is entirely embedded in $SU(3)_L$, then all models can be characterized by the charge operator

$$Q = T_{3L} + \frac{2}{\sqrt{3}} b T_{8L} + X I_3 \tag{1}$$

where I_3 is the unit matrix and $T_{iL} = \lambda_{iL}/2$, where the λ_{iL} are the Gell-Mann matrices. X is fixed by anomaly cancellation and the coefficient can be absorbed in the hypercharge definition. Different models are characterized by different values of b.

In the original Frampton, Pisano, and Pleitez [1,2] model, $b = 3/2$, leading to doubly charged gauge bosons and fermions with exotic charges. The fermion representations, with the $SU(3) \times U(1)$ quantum numbers, are

$$L_i = \begin{pmatrix} e_i \\ \nu_i \\ e_i^c \end{pmatrix} : (3^*, 0) \tag{2}$$

for the leptons ($i = 1, 2, 3$) and

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$$Q_{1,2} = \begin{pmatrix} u \\ d \\ D \end{pmatrix}, \begin{pmatrix} c \\ s \\ S \end{pmatrix} : (3, -1/3), \qquad (3)$$

$$Q_3 = \begin{pmatrix} b \\ t \\ T \end{pmatrix} : (3^*, 2/3) \qquad (4)$$

with all of the quark conjugate fields being isosinglets. D, S, T are quarks with charges given by $-4/3$, $-4/3$, $5/3$.

A simple variant of this model [11] changes the lepton structure by replacing the e^c with a heavy lepton E^+ and adding e^c and E^- as singlets.

If one wishes to avoid exotic electric charges, one must choose $b = 1/2$. In that case, the fermion structure is very different. Following [9], we can find six sets of fermions, which contain the antiparticles of all charged particles. The first four are leptons and the last two are quarks. Noting e_i, d_i, u_i as standard model fermions, and E_i, D_i, U_i as exotic fermions, the four sets of leptons are

$$L_1 = \begin{pmatrix} \nu_i \\ e_i^- \\ E_i^- \end{pmatrix}; e_i^+; E_i^+ \qquad (5)$$

with $SU(3) \times U(1)$ quantum numbers $(3, -2/3)$, $(1, 1)$, $(1, 1)$,

$$L_2 = \begin{pmatrix} e_i^- \\ \nu_i \\ N_i^0 \end{pmatrix}; e_i^+ \qquad (6)$$

with $SU(3) \times U(1)$ quantum numbers $(3^*, -1/3)$, $(1, 1)$, and N_i^0 is a heavy neutrino,

$$L_3 = \begin{pmatrix} e_i^- \\ \nu_i \\ N_1^0 \end{pmatrix}; \begin{pmatrix} E_i^- \\ N_2^0 \\ N_3^0 \end{pmatrix}; \begin{pmatrix} N_4^0 \\ E_i^+ \\ e_i^+ \end{pmatrix} \qquad (7)$$

with $SU(3) \times U(1)$ quantum numbers $(3^*, -1/3)$, $(3^*, -1/3)$, $(3^*, 2/3)$, and there are four heavy neutrino states (some may be conjugates of another), and

$$L_4 = \begin{pmatrix} \nu_i \\ e_i^- \\ E_{1i}^- \end{pmatrix}; \begin{pmatrix} E_{2i}^+ \\ N_1^0 \\ N_2^0 \end{pmatrix}; \begin{pmatrix} N_3^0 \\ E_{2i}^- \\ E_{3i}^- \end{pmatrix}; e_i^+; E_{1i}^+; E_{3i}^+ \qquad (8)$$

with $SU(3) \times U(1)$ quantum numbers $(3, -2/3)$, $(3, 1/3)$, $(3, -2/3)$, $(1, 1)$, $(1, 1)$, $(1, 1)$.

The two sets of quarks are

$$Q_1 = \begin{pmatrix} d_i \\ u_i \\ U_i \end{pmatrix}; d_i^c; u_i^c; U_i^c \qquad (9)$$

with $SU(3) \times U(1)$ quantum numbers $(3^*, 1/3)$, $(1, 1/3)$, $(1, -2/3)$, $(1, -2/3)$, and

$$Q_2 = \begin{pmatrix} u_i \\ d_i \\ D_i \end{pmatrix}; u_i^c; d_i^c; D_i^c \qquad (10)$$

with $SU(3) \times U(1)$ quantum numbers $(3, 0)$, $(1, -2/3)$, $(1, 1/3)$, $(1, 1/3)$.

The anomalies for these six sets are [9] found in Table I. With this table, anomaly-free models (without exotic charges) can be constructed. As noted in Ref. [9], there are two one-family and eight three-family models that are anomaly free. Of the eight three-family models, four treat the lepton generations identically, two treat two of the lepton generations identically and in two, the lepton generations are all different. It is the latter two that will be the subject of this study.

Note that one can easily see from Table I that there are only two one-family models. The first consists of $Q_2 + L_3$. This structure is perhaps most familiar to grand unified model builders, since the 27 fields are contained in the 27-dimensional fundamental representation of E_6. In addition to analyses of E_6 models, an analysis of this model, in the context of 3-3-1 models, can be found in Refs. [9,12].

The second one-family structure is $Q_1 + L_4$. This model is related to $SU(6) \times U(1)$ unified models, and is analyzed in Ref. [13]. Note that both of these one-family models are simply triplicated to become three-family models.

There are two other three-family models in which all of the leptons are treated the same way (but now the quark generations are treated differently). These were the first models analyzed once it was recognized that 3-3-1 models without exotic charges (i.e. with $b = 1/2$) could be constructed. The first is $3L_2 + Q_1 + 2Q_2$. As in the original 3-3-1 models, one generation of quarks is treated differently than the other two, and thus three families are needed to cancel anomalies. These were analyzed in Ref. [8]. The second such model is $3L_1 + 2Q_1 + Q_2$, which also requires three families for anomaly cancellation. This model has been analyzed in Ref. [14].

Two models involve simple replication of the two one-family models, but take two copies of the first one-family model and one copy of the second, or vice versa, i.e. $2(Q_2 + L_3) + (Q_1 + L_4)$ and $2(Q_1 + L_4) + (Q_2 + L_3)$. Since the lepton generations are not all different, we will not consider these models further, although they have not, to our knowledge, been studied.

The two models of interest treat all of the lepton generations differently. They are model A: $L_1 + L_2 + L_3 + Q_1 + 2Q_2$ and model B: $L_1 + L_2 + L_4 + 2Q_1 + Q_2$. Note

TABLE I. Anomalies for the fermion families.

Anomalies	L_1	L_2	L_3	L_4	Q_1	Q_2
$[SU(3)_c]^2 U(1)_X$	0	0	0	0	0	0
$[SU(3)_L]^2 U(1)_X$	$-2/3$	$-1/3$	0	-1	1	0
$[grav]^2 U(1)_X$	0	0	0	0	0	0
$[U(1)_X]^3$	10/9	8/9	6/9	12/9	$-12/9$	$-6/9$

that each model has two "simple" lepton families (L_1 and L_2 above), and one more complicated family. We now analyze the phenomenology of these two models. Note that one cannot determine which (e, μ, τ) lepton belongs to which representation, and so we will consider all six possible permutations for each model.

III. THE SCALAR SECTOR

The scalar sector of 3-3-1 models has been extensively studied [11,15]. Here, one can see a substantial advantage to $b = 1/2$ models. In the original $b = 3/2$ models, the minimal Higgs sector consists of three $SU(3)_L$ triplets plus an $SU(3)_L$ sextet. In the $b = 1/2$ models, three triplets are sufficient. One triplet breaks the $SU(3)_L \times U(1)$ gauge symmetry down to the standard model, and the other two are necessary to break the $SU(2)_L$ symmetry and to give the fermions mass. A very comprehensive analysis of the scalar sector in all previously considered models can be found in Ref. [15].

Although the models we are considering are $b = 1/2$ models, it is not *a priori* obvious that three triplets will suffice to give the leptons mass, since the different families have very different structure. Our model A has five charged leptons (the e, μ, τ and two exotic leptons), and model B has seven charged leptons (with four exotic leptons). Fortunately, as will be seen in Sec. IV, three triplets will suffice to give the charged leptons mass. We will not consider neutrino masses in this study since the number of fields and the various options (which exotic neutrinos correspond to which right-handed neutrinos, for example) will rule out any substantial predictive power.

The first stage of breaking from $SU(3)_L \times U(1)$ to $SU(2) \times U(1)$ is carried out by a triplet Higgs, Φ_A, which is a $(3, 1/3)$ under the $SU(3)_L \times U(1)$ group, and its vacuum expectation value (vev) is given by

$$\langle \Phi_A \rangle = \begin{pmatrix} 0 \\ 0 \\ V \end{pmatrix}. \tag{11}$$

Note that the second component of the triplet is neutral, and could also get a vev, but that can be removed by a gauge transformation. Five of the gauge bosons acquire masses of $O(V)$, while the remaining four are massless at this stage. One can easily see that this vev will give masses of $O(V)$ to the U and D exotic quarks, and in previously considered models, to the E exotic leptons as well. These masses are phenomenologically constrained to be substantially larger than the electroweak scale.

The second stage of symmetry breaking requires two Higgs triplets, Φ_1 and Φ_2 with quantum numbers $(3, -2/3)$ and $(3, 1/3)$ respectively. If one only wished to break the gauge symmetry, then one triplet would suffice. However, giving mass to the fermions requires a second doublet. This is not too surprising, since the quark masses in the standard model necessitate a Higgs doublet H and $i\tau_2 H^*$ to give masses to the down and up quarks, respectively. In $SU(2)$ $\bar{2} = 2$, but this does not apply in $SU(3)$. Thus the low-energy theory is a two-doublet model. The vevs of these doublets are

$$\langle \Phi_1 \rangle = \begin{pmatrix} v_1/\sqrt{2} \\ 0 \\ 0 \end{pmatrix}; \qquad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2/\sqrt{2} \\ 0 \end{pmatrix} \tag{12}$$

where $v_1^2 + v_2^2 = (246 \text{ GeV})^2$. Note that the third component of Φ_2 could acquire a nonzero vev, but this will not involve $SU(2)$ breaking and will be irrelevant.

IV. YUKAWA COUPLINGS

With the fermion representations discussed in Sec. II and the scalar representations discussed in Sec. III, we can now write down the Yukawa couplings and mass matrices for the charged leptons. Let us first write down the fermion representations more explicitly.

For model A, the fields, followed by their $SU(3)_L \times U(1)$ quantum numbers, are (with the subscript L understood)

$$\psi_i = \begin{pmatrix} \nu_i \\ e_i \\ E_i \end{pmatrix}, (3, -2/3); \quad e_i^c, (1, 1); \quad E_i^c, (1, 1), \tag{13}$$

$$\psi_j = \begin{pmatrix} e_j \\ \nu_j \\ N_j^0 \end{pmatrix}, (\bar{3}, -1/3); \quad e_j^c, (1, 1), \tag{14}$$

$$\psi_k = \begin{pmatrix} e_k \\ \nu_k \\ N_{1k}^0 \end{pmatrix}, (\bar{3}, -1/3); \qquad \psi_k' = \begin{pmatrix} E_k \\ N_{2k}^0 \\ N_{3k}^0 \end{pmatrix}, (\bar{3}, -1/3);$$
$$\psi_k'' = \begin{pmatrix} N_{4k}^0 \\ E_k^c \\ e_k^c \end{pmatrix}, (\bar{3}, 2/3) \tag{15}$$

where the N^0 could be a conjugate of either the ν or another N^0, and the generation labels i, j and k are all distinct. Note that the model contains five charged leptons: the standard three plus two exotic leptons.

For model B, the fields are

$$\psi_i = \begin{pmatrix} \nu_i \\ e_i \\ E_i \end{pmatrix}, (3, -2/3); \quad e_i^c, (1, 1); \quad E_i^c, (1, 1), \tag{16}$$

$$\psi_j = \begin{pmatrix} e_j \\ \nu_j \\ N_j^0 \end{pmatrix}, (\bar{3}, -1/3); \quad e_j^c, (1, 1), \tag{17}$$

$$\psi_k = \begin{pmatrix} \nu_k \\ e_k \\ E_{1k} \end{pmatrix}, (3, -2/3); \qquad \psi'_k = \begin{pmatrix} E^c_{2k} \\ N^0_{1k} \\ N^0_{2k} \end{pmatrix}, (3, 1/3);$$
$$\psi''_k = \begin{pmatrix} N^0_{3k} \\ E_{2k} \\ E_{3k} \end{pmatrix}, (3, -2/3); \quad e^+_i; \quad E^c_{1k}; \quad E^c_{3k} \tag{18}$$

where the last three fields are singlets. Note that this model has seven charged leptons: the standard three plus four exotics.

From these representations, and the scalar fields (with their vevs) in Sec. III, we can write down the mass matrices for the charged leptons. The mass matrix for model A is 5×5 and for model B is 7×7. From these matrices, the Yukawa couplings to each scalar field can be trivially obtained by replacing the vev with the field. The Yukawa couplings and full mass matrices are given in the Appendix. If one takes the limit in which $v_1 = v_2 = 0$, then each of these matrices has three zero eigenvalues, indicating that the exotic leptons all get masses of $O(V)$. Since V must be large, we can take the limit as $V \to \infty$, and find the effective mass matrices for the three standard model leptons. Note that we do not know, *a priori*, which of the leptons is in the first, second, or third rows, so each model will have six permutations.

For model A, we find that the mass matrix is of the form

$$M_A = \frac{1}{\sqrt{2}} \begin{pmatrix} h_1 v_2 & h_2 v_2 & 0 \\ h_3 v_1 & h_4 v_1 & h_5 v_2 \\ h_6 v_1 & h_7 v_1 & h_8 v_2 \end{pmatrix} \tag{19}$$

where the h_i are constants. The Yukawa coupling matrices are then

$$\begin{pmatrix} 0 & 0 & 0 \\ h_3 & h_4 & 0 \\ h_6 & h_7 & 0 \end{pmatrix} \Phi_1 + \begin{pmatrix} h_1 & h_2 & 0 \\ 0 & 0 & h_5 \\ 0 & 0 & h_8 \end{pmatrix} \Phi_2. \tag{20}$$

For model B, the mass matrix is of the form

$$M_B = \frac{1}{\sqrt{2}} \begin{pmatrix} h'_1 v_2 & h'_2 v_2 & h'_3 v_2 \\ h'_4 v_1 & h'_5 v_1 & h'_6 v_1 \\ h'_7 v_2 & h'_8 v_2 & h'_9 v_2 \end{pmatrix} \tag{21}$$

and the Yukawa coupling matrices are

$$\begin{pmatrix} 0 & 0 & 0 \\ h'_4 & h'_5 & h'_6 \\ 0 & 0 & 0 \end{pmatrix} \Phi_1 + \begin{pmatrix} h'_1 & h'_2 & h'_3 \\ 0 & 0 & 0 \\ h'_7 & h'_8 & h'_9 \end{pmatrix} \Phi_2. \tag{22}$$

These Yukawa coupling matrices are certainly unusual. Note that diagonalizing the mass matrices will *not* diagonalize the Yukawa coupling matrices, and thus one will have lepton-flavor-changing neutral currents (FCNC) in the Higgs sector. This is just the Glashow-Weinberg theorem [16]. To determine the size of the lepton-flavor violation, one simply must diagonalize the mass matrix and read off the Yukawa coupling matrices in the diagonalized basis.

Unfortunately, such a procedure will not be useful. The matrices have far too many free parameters. Worse, in general fine-tuning will be needed. We define "fine-tuning" as a situation in which several terms add together to give a term that is much smaller than any individual term. In general, fine-tuning will be needed to give the electron a small mass,[1] and it is unclear how this fine-tuning will affect the Yukawa coupling matrices.

In order to avoid fine-tuning, and to give the matrices a nontrivial structure, we will assume that the matrices will have a Fritzsch structure [17]. The original Fritzsch matrix was of the form

$$\begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix} \tag{23}$$

where $C \sim m_\tau$, $B \sim \sqrt{m_\mu m_\tau}$ and $A \sim \sqrt{m_e m_\mu}$. This matrix has the correct eigenvalues, is parameter free and does not have fine-tuning. It was shown in Ref. [18] that a wide variety of matrices, such as those with nonzero values in the 1,1 and 2,2 elements, will (if one requires that there be no fine-tuning) yield the same flavor-changing-neutral structure as the Fritzsch structure. We expect that the general case will give the same qualitative results.

Since the matrices we are considering are not symmetric, we will write the desired mass matrix as

$$\begin{pmatrix} 0 & a\sqrt{m_e m_\mu} & 0 \\ b\sqrt{m_e m_\mu} & 0 & c\sqrt{m_\mu m_\tau} \\ 0 & d\sqrt{m_\mu m_\tau} & e m_\tau \end{pmatrix} \tag{24}$$

where a, b, c, d and e are all of order 1. In general, with multiple scalars, the individual Yukawa couplings would be of this form, with $\sum a = \sum b = \sum c = \sum d = \sum e = 1$.

So, for a given model, and a given choice of permutations of i, j and k, one compares this matrix with the mass matrices M_A and M_B, and reads off the values of a, b, c, d and e. Then the mass matrices are diagonalized, and the Yukawa coupling matrices in the diagonal basis are determined. It turns out that the procedure is only consistent for model A if j is the second generation, and thus we have a total of 4 Yukawa coupling matrices for model A (two choices of Φ_1 or Φ_2, and the choice between $i = 1$, $k = 3$ or $i = 3$, $k = 1$), and 12 Yukawa coupling matrices for model B (two choices of Φ and six permutations of *i, j, k*). However, the results are simplified in model B by the fact that if we permute the first and third indices, the Yukawa coupling matrices are identical, so there are only six different matrices.

[1]There are trivial exceptions. For example, in M_A, if h_1 is very small, and all off-diagonal terms vanish, then there is no fine-tuning (and no flavor-changing neutral currents).

TABLE II. Yukawa coupling matrices to Φ_1 and Φ_2 for model A. All entries are to be divided by $\sqrt{v_1^2 + v_2^2}/\sqrt{2} = 175$ GeV. The specific models are discussed in the text.

Scalar	A1	A2
Φ_1	$\begin{pmatrix} 0 & -\sqrt{m_e m_\mu} & -m_\mu\sqrt{\frac{m_e}{m_\tau}} \\ 0 & -m_\mu & -m_\mu\sqrt{\frac{m_\mu}{m_\tau}} \\ \sqrt{m_e m_\tau} & \sqrt{m_\mu m_\tau} & m_\mu \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \sqrt{m_e m_\tau} \\ -\sqrt{m_e m_\mu} & -m_\mu & \sqrt{m_\mu m_\tau} \\ -m_\mu\sqrt{\frac{m_e}{m_\tau}} & -m_\mu\sqrt{\frac{m_\mu}{m_\tau}} & m_\mu \end{pmatrix}$
Φ_2	$\begin{pmatrix} m_e & \sqrt{m_e m_\mu} & m_\mu\sqrt{\frac{m_e}{m_\tau}} \\ 0 & 0 & 0 \\ -\sqrt{m_e m_\tau} & -\sqrt{m_\mu m_\tau} & m_\tau + m_\mu \end{pmatrix}$	$\begin{pmatrix} m_e & 0 & -\sqrt{m_e m_\tau} \\ \sqrt{m_e m_\mu} & 0 & -\sqrt{m_\mu m_\tau} \\ m_\mu\sqrt{\frac{m_e}{m_\tau}} & 0 & m_\tau + m_\mu \end{pmatrix}$

TABLE III. Yukawa coupling matrices to Φ_1 and Φ_2 for model B. All entries are to be divided by $\sqrt{v_1^2 + v_2^2}/\sqrt{2} = 175$ GeV. The specific models are discussed in the text.

Scalar	B1	B2	B3
Φ_1	$\begin{pmatrix} 0 & -\sqrt{m_e m_\mu} & \sqrt{m_e m_\tau} \\ 0 & -m_\mu & \sqrt{m_\mu m_\tau} \\ 0 & -m_\mu\sqrt{\frac{m_\mu}{m_\tau}} & m_\mu \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -\sqrt{m_e m_\tau} \\ 0 & 0 & -\sqrt{m_\mu m_\tau} \\ 0 & 0 & m_\tau + m_\mu \end{pmatrix}$	$\begin{pmatrix} m_e & \sqrt{m_e m_\mu} & m_\mu\sqrt{\frac{m_e}{m_\tau}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
Φ_2	$\begin{pmatrix} m_e & \sqrt{m_e m_\mu} & -\sqrt{m_e m_\tau} \\ 0 & 0 & -\sqrt{m_\mu m_\tau} \\ 0 & 0 & m_\tau + m_\mu \end{pmatrix}$	$\begin{pmatrix} m_e & 0 & \sqrt{m_e m_\tau} \\ 0 & -m_\mu & \sqrt{m_\mu m_\tau} \\ 0 & -m_\mu\sqrt{\frac{m_\mu}{m_\tau}} & m_\mu \end{pmatrix}$	$\begin{pmatrix} 0 & \sqrt{m_e m_\mu} & -m_\mu\sqrt{\frac{m_e}{m_\tau}} \\ 0 & -m_\mu & -m_\mu\sqrt{\frac{m_\mu}{m_\tau}} \\ 0 & -m_\mu\sqrt{\frac{m_\mu}{m_\tau}} & m_\tau + 2m_\mu \end{pmatrix}$

The Yukawa couplings are given in Table II for model A and in Table III for model B. We label models A1 and A2 as corresponding to $(i, j, k) = (e, \mu, \tau)$ or (τ, μ, e), respectively, and we label models B1, B2 and B3 as corresponding to (e, μ, τ), (e, τ, μ) or (μ, e, τ), respectively.

Note that we have tacitly assumed that the two Higgs triplets in the low-energy sector do not mix. This is for simplicity. One can easily find the couplings of one of the physical Higgs bosons by including an appropriate (and unknown) mixing angle. In our discussion of the phenomenology, this angle will play an important role, and it must be kept in mind.

Note how unusual some of these Yukawa coupling matrices are. For example, in model B3's coupling to Φ_1, the Yukawa couplings to $\tau - \tau$, $\mu - \tau$ and $\mu - \mu$ all vanish, leading to an effectively leptophobic Higgs boson. We now turn to the lepton-flavor-changing phenomenology of these models.

V. LEPTONIC FLAVOR-CHANGING DECAYS

In all of these models, there are Higgs-mediated lepton FCNC arising from the off-diagonal terms in the Yukawa coupling matrices. This will lead to μ and τ decays which violate the lepton number. The leptonic decays of the τ^- are into $e^-e^-e^+$, $\mu^-\mu^-\mu^+$, $e^-e^-\mu^+$, $\mu^-\mu^-e^+$, $e^-\mu^-\mu^+$, $e^-\mu^-\mu^+$ and the μ decay is into $e^-e^-e^+$.

The decay rate calculations are straightforward [19,20]. Given the experimental upper bound on the decay rate for each of these processes, one can find a lower bound on the mass of the exchanged Higgs boson. The rate is inversely proportional to the Higgs mass to the fourth power. Examining all of the Yukawa coupling matrices in the previous section, we find that this lower bound is always less than 4.9 GeV. Since the experimental lower bound is more than an order of magnitude higher, these bounds are not competitive.

One can still have one-loop radiative decays. Again, the bounds from τ decays ($\tau \to e\gamma, \tau \to \mu\gamma$) do not give strong bounds. The strongest is from $\tau \to \mu\gamma$ in models A1, A2, B1, B2 in which the first three involve coupling to Φ_2 and the last to Φ_1. However, even this lower bound is only 50 GeV, and is marginally competitive with current experimental bounds.

A much stronger bound comes from $\mu \to e\gamma$. Here a τ can be in the loop. The formula for the decay rate [21] is

$$\Gamma_{\mu\to e\gamma} = h_{\mu\tau}^2 h_{e\tau}^2 \frac{\alpha m_\tau^2 m_\mu^3}{128\pi^4}\left[\frac{\ln(m_h/m_\tau)}{m_h^2}\right]^2 \qquad (25)$$

where the h_{ij} are the Yukawa couplings, and m_h is the scalar mass. This result does not change if the relevant scalar is a pseudoscalar.

Plugging in, one finds a lower bound of 230 GeV on the exchanged scalar mass for models A1, A2, B1 and B2, regardless of which scalar is used. However, for several reasons this bound is quite uncertain. First, we have a Fritzsch ansatz, and without that assumption the Yukawa couplings are only order of magnitude. Second, we have ignored mixing angles, which could also lower the Yukawa couplings substantially. Third, these models can have heavy leptons in the loop, and cancellations are possible. Thus, the numerical bound should be taken with a grain of salt, but it is clear that $\mu \rightarrow e\gamma$ may be quite close to detection in these models.

Note that model B3 was not included in the above paragraph. In the coupling to Φ_1, there is no bound coming from muon decay; in the coupling to Φ_2, there is a bound of 7.3 GeV on the Higgs mass. So the model is unconstrained by muon decay, and the Higgs bosons in this model could be very light.

We now turn to lepton-number violation in Higgs decays.

VI. LEPTON-NUMBER VIOLATING HIGGS DECAYS

We have a two-Higgs model in the low-energy sector. Here, mixing between the Higgs scalars (which will generically occur and depend on parameters of the scalar potential) can have a major effect on the branching ratios of Higgs bosons. For the moment, we will ignore these effects, but they are important and will be discussed shortly.

In the conventional two-Higgs model, one Higgs doublet couples to the $Q = 2/3$ quarks, and the other to the $Q = -1/3$ quarks and the charged leptons. The latter's primary decay into fermions is thus to $b\bar{b}$, with the $\tau^+\tau^-$ decay being a factor of $3m_b^2/m_\tau^2 \sim 25$ smaller. Of course, the primary decay mode could be WW, WW^*, ZZ or ZZ^*, depending on the mass of the Higgs. Here we will only look at the primary fermionic decays, which are relevant if the Higgs mass is not too much larger than its current lower bound (if it is larger, the fermionic decay branching ratios might be small, but certainly detectable at the CERN LHC). The primary fermionic decay mode of the Higgs that couples to $Q = 2/3$ fields would be into $t\bar{t}$ if kinematically accessible, and $c\bar{c}$ if not. It will not couple to the charged leptons. If the mixing angle is not too small, then the latter field's primary fermionic decay is also into $b\bar{b}$.

In both models under consideration, one of the quark generations has a different structure than the other two. The unique generation is generally assumed to be the third generation, an assumption we concur with. If it is not, there will be flavor-changing effects in the kaon sector which will be phenomenologically problematic.

Then, again ignoring mixing, the scalar that couples to $b\bar{b}$ will *not* couple to the charged leptons. The field coupling to the charged leptons will couple to the strange quark and to the top quark. If its mass is below 360 GeV, then its primary fermionic decay is into the charged leptons and the strange quark.[2] In this case, we can calculate the fermionic branching ratios for the five models under consideration, and show these in Table IV.

TABLE IV. The fermionic branching fraction into various final states for the Higgs that does not couple to the b quarks in the various models. We have explicitly assumed no mixing between the Higgs scalars, and that top quark decays are not kinematically accessible. The decay into gauge bosons will dominate if they are kinematically accessible.

Model	$\mu\mu$	$\mu\tau$	$\tau\tau$	$s\bar{s}$
A1	0	0.05	0.94	0.01
A2	0	0.06	0.93	0.01
B1	0.04	0.72	0.04	0.20
B2	0	0.06	0.93	0.01
B3	0	0	0	1.00

The results in Table IV are interesting. In models A1, A2 and B2, we see that the inversion of the bottom-top quark doublet takes the field that would "normally" decay into $b\bar{b}$, and (since the top quark is too heavy) makes its primary decay mode $\tau^+\tau^-$. This would be a very dramatic signature. In model B3, in which the Higgs is leptophobic (and in which, as shown in the last section, radiative muon decay does not bound the Higgs mass), there are no leptonic decays, and the primary decay mode would be into $s\bar{s}$. The most unusual model is B1, in which the *primary* decay mode is into $\mu\tau$. This monochromatic muon would give a very dramatic signature.

All of these signatures are quite dramatic. How realistic is this scenario? Abandoning the use of the Fritzsch ansatz will have effects of $O(1)$ on these results, but will not change the general results. However, the assumption of no mixing between the doublets will have a substantial effect on the scalars (the pseudoscalar will not, in general, have this mixing, and thus the results of the above paragraph will apply). For the scalars, mixing means that the branching ratio into $b\bar{b}$ is not negligible. For models A1, A2 and B2, the fermionic branching ratio into $b\bar{b}$ relative to $\tau^+\tau^-$ is approximately $25\sin^2\theta$, and thus the individual branching ratios must be reduced accordingly. For model B3, the fermionic branching ratio into $b\bar{b}$ is approximately $1000\sin^2\theta$, and thus the primary decay mode will almost certainly be into $b\bar{b}$, unless the angle is extremely small. For B1, the fermionic branching ratio into $b\bar{b}$ is approximately $400\sin^2\theta$, and thus it is likely that $b\bar{b}$ decays will dominate, although the remarkable $\mu\tau$ decay mode will still be substantial. Note that the signature for $\mu\tau$ decays is very clean, and branching ratios of 10^{-4} can be detected. As a result, in all of these models except B3, the Higgs decay into $\mu\tau$ is detectable.

VII. BOUNDS ON THE GAUGE BOSON SECTOR

The electroweak Lagrangian (with the kinetic terms dropped) may be written in the form

[2]Actually, if it is between 270 and 360 GeV, then the three body decay through a virtual top into tbW will dominate.

$$\mathcal{L} = \sum_i \overline{\psi}_i \left(\frac{g}{2} \lambda_\alpha A_\alpha^\mu + g' X B^\mu \right) \psi_i$$
$$= \sum_i \overline{\psi}_i \begin{pmatrix} D_1^\mu & \frac{g}{\sqrt{2}} W^{+\mu} & \frac{g}{\sqrt{2}} K^{+\mu} \\ \frac{g}{\sqrt{2}} W^{-\mu} & D_2^\mu & \frac{g}{\sqrt{2}} K^{0\mu} \\ \frac{g}{\sqrt{2}} K^{-\mu} & \frac{g}{\sqrt{2}} \overline{K}^{0\mu} & D_3^\mu \end{pmatrix} \psi_i$$

where

$$D_1^\mu = g\left(\frac{A_3^\mu}{2} + \frac{A_8^\mu}{2\sqrt{3}} \right) + g' X B^\mu,$$
$$D_2^\mu = g\left(-\frac{A_3^\mu}{2} + \frac{A_8^\mu}{2\sqrt{3}} \right) + g' X B^\mu, \tag{26}$$
$$D_3^\mu = -g \frac{A_8^\mu}{\sqrt{3}} + g' X B^\mu$$

and the sum is over all ψ in the model. With the relationship $\sin^2\theta_W = 3g'^2/(3g^2 + 4g'^2)$ defining the electroweak mixing angle, we find that the diagonal terms reduce to combinations of the expected neutral gauge bosons A^μ and Z^μ, plus a new boson, the Z'^μ. The photon and Z have the same couplings and Feynman rules as the standard model, and therefore display no unusual characteristics. However, the Z' has vector and axial couplings which depend on the particular lepton generation, Eqs. (5)–(8), leading to FCNC.

In terms of the $SU(3)_L \otimes U(1)_X$ gauge bosons, we find that the low-energy fields are given by [15,22,23]

$$A_\mu = S_W A_\mu^3 + C_W\left(\frac{T_W}{\sqrt{3}} A_\mu^8 + \sqrt{1 - \frac{T_W^2}{3}} B_\mu \right),$$

$$Z_\mu = C_W A_\mu^3 - S_W\left(\frac{T_W}{\sqrt{3}} A_\mu^8 + \sqrt{1 - \frac{T_W^2}{3}} B_\mu \right),$$
$$Z'_\mu = -\sqrt{1 - \frac{T_W^2}{3}} A_\mu^8 + \frac{T_W}{\sqrt{3}} B_\mu,$$

where $S_W = \sin\theta_W$, $C_W = \cos\theta_W$, and $T_W = \tan\theta_W$. These fields have the eigenvalues

$$M_{A_\mu}^2 = 0; \qquad M_{Z_\mu}^2 \simeq \frac{g^2}{2}\left[\frac{3g^2 + 4g'^2}{3g^2 + g'^2} \right](v_1^2 + v_2^2);$$
$$M_{Z'_\mu}^2 \simeq \frac{2[3g^2 + g'^2]}{9} V^2. \tag{27}$$

The Z' has a vertex factor of the form $-i\frac{1}{2}\gamma_\mu(C_V - C_A\gamma_5)$ where the $C_{V,A}$ are family dependent, and given in Table V.

A recent analysis of precision electroweak (EW) bounds in 3-3-1 models without exotic electric charges [22] gave a lower bound of 1400 GeV on the mass of the Z'. Since the $SU(3)_L \otimes U(1)_X$ representations are different for each lepton family, one expects Z'-mediated FCNC. As discussed

TABLE V. The C_V and C_A for the various lepton families. A common factor of $e/6\cos\theta_W$ has been factored out of each. Note that C_A is the same for $L_{1,3,4}$.

Family	C_V	C_A
L_1,L_4	$\frac{10S_W}{\sqrt{3-4S_W^2}} + \frac{\sqrt{3-4S_W^2}}{S_W}$	$\frac{\sqrt{3-4S_W^2}}{S_W} - \frac{2S_W}{\sqrt{3-4S_W^2}}$
L_2	$\frac{8S_W}{\sqrt{3-4S_W^2}} - \frac{\sqrt{3-4S_W^2}}{S_W}$	$-\frac{\sqrt{3-4S_W^2}}{S_W} - \frac{4S_W}{\sqrt{3-4S_W^2}}$
L_3	$\frac{6S_W}{\sqrt{3-4S_W^2}} - 3\frac{\sqrt{3-4S_W^2}}{S_W}$	$\frac{\sqrt{3-4S_W^2}}{S_W} - \frac{2S_W}{\sqrt{3-4S_W^2}}$

in the last section, the mixing matrix between the $SU(3)_L$ eigenstates and the mass eigenstates will have too many free parameters. To estimate the size of the Z' FCNC, we therefore again use the Fritsch ansatz. Failure to use this ansatz results in too many parameters. This results in a mixing matrix with no free parameters but the lepton masses. To determine the FCNC couplings of the Z', one picks the model and diagonalizes. Using the C_V and C_A in Table V, one reads off the couplings for each particle. These couplings will be a linear combination of the family couplings. Since the C_V differ for each family, there will be FCNC.

The most stringent bound on $M_{Z'}$ is found from $\mu \to 3e$ decays. The formula for this decay rate is

$$\Gamma = \frac{\pi m_\mu^5}{108}\left(\frac{e}{24\pi C_W M_{Z'}} \right)^4 [3(C_{Ve\mu}^2 + C_{Ae\mu}^2)(C_{Vee}^2 + C_{Aee}^2) + 4C_{Ve\mu}C_{Ae\mu}C_{Vee}C_{Aee}]. \tag{28}$$

Given that we do not know which family corresponds to which lepton, we try all possibilities. This provides bounds that range from 2 TeV in model B2, to between 20 and 40 TeV in the other models. A similar calculation using $\tau \to 3\mu$ or $\mu \to e\gamma$ provides much weaker lower bounds. Thus precision EW bounds will not be relevant in these models.

A bound of 20 to 40 TeV is discouraging since the Z' will be beyond the reach of the LHC, and because fine-tuning will be needed to explain a new hierarchy problem. Nonetheless, model B2 does not need substantial fine-tuning, and the Higgs decays in any of the models will provide distinct signatures.

VIII. CONCLUSIONS

We have studied a pair of 3-3-1 models that have not previously been examined. The defining characteristic of these models is that each lepton generation has a unique structure. This leads to FCNC decays mediated by the light Higgs and Z' boson. Z' mediated $\mu \to 3e$ provides a lower bound of 2 TeV for M_Z' in model B2, and between 20 and 40 TeV in the others. These models will all have interesting Higgs decay signatures. In particular, $\Phi \to \mu\tau$ could show up clearly at the LHC.

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APPENDIX

Here are the full mass matrices for the charged leptons in the 3-3-1 models studied in this paper. For model A we have

$$\begin{pmatrix} h_1 v_2 & h_2 v_2 & 0 & h_3 v_2 & 0 \\ h_7 v_1 & h_8 v_1 & -g_1 v_2 & h_9 v_1 & g_2 V \\ h_{10} v_1 & h_{11} v_1 & -g_3 v_2 & h_{12} v_1 & g_4 V \\ h_4 V & h_5 V & 0 & h_6 V & 0 \\ h_{13} v_1 & h_{14} v_1 & -g_5 v_2 & h_{15} v_1 & g_6 V \end{pmatrix} \tag{A1}$$

where the ordering is e_i, e_j, e_k, E_i, E_k.

The relevant terms in the Lagrangian are

$$\begin{aligned}\mathcal{L}_{Y,AA} = {}& (h_4 \overline{\psi}_{iL} e_{iR} + h_5 \overline{\psi}_{iL} e_{jR} + h_6 \overline{\psi}_{iL} E_{iR})\Phi_A \\ & + \epsilon_{\alpha\beta\gamma}(g_2 \overline{\psi}^{\alpha}_{jL}(\psi''^{c}_{kL})^{\beta} + g_4 \overline{\psi}^{\alpha}_{kL}(\psi''^{c}_{kL})^{\beta} \\ & + g_6 \overline{\psi'}^{\alpha}_{kL}(\psi''^{c}_{kL})^{\beta})\Phi^{\gamma}_A,\end{aligned} \tag{A2}$$

$$\begin{aligned}\mathcal{L}_{Y,A1} = {}& (h_7 \overline{\psi}_{jL} e_{iR} + h_8 \overline{\psi}_{jL} e_{jR} + h_9 \overline{\psi}_{jL} E_{iR} + h_{10} \overline{\psi}_{kL} e_{iR} \\ & + h_{11} \overline{\psi}_{kL} e_{jR} + h_{12} \overline{\psi}_{kL} E_{iR} + h_{13} \overline{\psi}'_{kL} e_{iR} \\ & + h_{14} \overline{\psi}'_{kL} e_{jR} + h_{15} \overline{\psi}'_{kL} E_{iR})\Phi^{*}_1,\end{aligned} \tag{A3}$$

$$\begin{aligned}\mathcal{L}_{Y,A2} = {}& (h_1 \overline{\psi}_{iL} e_{iR} + h_2 \overline{\psi}_{iL} e_{jR} + h_3 \overline{\psi}_{iL} E_{iR})\Phi_2 \\ & + \epsilon_{\alpha\beta\gamma}(g_1 \overline{\psi}^{\alpha}_{jL}(\psi''^{c}_{kL})^{\beta} + g_3 \overline{\psi}^{\alpha}_{kL}(\psi''^{c}_{kL})^{\beta} \\ & + g_5 \overline{\psi}'^{\alpha}_{kL}(\psi''^{c}_{kL})^{\beta})\Phi^{\gamma}_2.\end{aligned} \tag{A4}$$

Similarly, the mass matrix for model B is

$$\begin{pmatrix} h_1 v_2 & h_2 v_2 & h_3 v_2 & h_4 v_2 & h_5 v_2 & g_4 V & h_6 v_2 \\ h_{13} v_1 & h_{14} v_1 & h_{15} v_1 & h_{16} v_1 & h_{17} v_1 & 0 & h_{18} v_1 \\ h_{19} v_2 & h_{20} v_2 & h_{21} v_2 & h_{22} v_2 & h_{23} v_2 & g_5 V & h_{24} v_2 \\ h_7 V & h_8 V & h_9 V & h_{10} V & h_{11} V & -g_1 v_2 & h_{12} V \\ h_{25} V & h_{26} V & h_{27} V & h_{28} V & h_{29} V & -g_2 v_2 & h_{30} V \\ h_{31} v_2 & h_{32} v_2 & h_{33} v_2 & h_{34} v_2 & h_{35} v_2 & g_6 V & h_{36} v_2 \\ h_{37} V & h_{38} V & h_{39} V & h_{40} V & h_{41} V & -g_3 v_2 & h_{42} V \end{pmatrix} \tag{A5}$$

with the ordering e_i, e_j, e_k, E_i, E_{1k}, E_{2k}, E_{3k}.

The relevant terms in the Lagrangian are

$$\begin{aligned}\mathcal{L}_{Y,BA} = {}& (h_7 \overline{\psi}_{iL} e_{iR} + h_8 \overline{\psi}_{iL} e_{jR} + h_9 \overline{\psi}_{iL} e_{kR} + h_{10} \overline{\psi}_{iL} E_{iR} + h_{11} \overline{\psi}_{iL} E_{1kR} + h_{12} \overline{\psi}_{iL} E_{3kR} + h_{25} \overline{\psi}_{kL} e_{iR} + h_{26} \overline{\psi}_{kL} e_{jR} \\ & + h_{27} \overline{\psi}_{kL} e_{kR} + h_{28} \overline{\psi}_{kL} E_{iR} + h_{29} \overline{\psi}_{kL} E_{1kR} + h_{30} \overline{\psi}_{kL} E_{3kR} + h_{37} \overline{\psi}''_{kL} e_{iR} + h_{38} \overline{\psi}''_{kL} e_{jR} + h_{39} \overline{\psi}''_{kL} e_{kR} + h_{40} \overline{\psi}''_{kL} E_{iR} \\ & + h_{41} \overline{\psi}''_{kL} E_{1kR} + h_{42} \overline{\psi}''_{kL} E_{3kR})\Phi_A + \epsilon_{\alpha\beta\gamma}(g_4 \overline{\psi}'^{\alpha}_{kL}(\psi^{c}_{iL})^{\beta} + g_5 \overline{\psi}'^{\alpha}_{kL}(\psi^{c}_{kL})^{\beta} + g_6 \overline{\psi}'^{\alpha}_{kL}(\psi''^{c}_{kL})^{\beta})(\Phi^{*}_A)^{\gamma},\end{aligned} \tag{A6}$$

$$\mathcal{L}_{Y,B1} = (h_{13} \overline{\psi}_{jL} e_{iR} + h_{14} \overline{\psi}_{jL} e_{jR} + h_{15} \overline{\psi}_{jL} e_{kR} + h_{16} \overline{\psi}_{jL} E_{iR} + h_{17} \overline{\psi}_{jL} E_{1kR} + h_{18} \overline{\psi}_{jL} E_{3kR})\Phi^{*}_1, \tag{A7}$$

$$\begin{aligned}\mathcal{L}_{Y,B2} = {}& (h_1 \overline{\psi}_{iL} e_{iR} + h_2 \overline{\psi}_{iL} e_{jR} + h_3 \overline{\psi}_{iL} e_{kR} + h_4 \overline{\psi}_{iL} E_{iR} + h_5 \overline{\psi}_{iL} E_{1kR} + h_6 \overline{\psi}_{iL} E_{3kR} + h_{19} \overline{\psi}_{kL} e_{iR} + h_{20} \overline{\psi}_{kL} e_{jR} \\ & + h_{21} \overline{\psi}_{kL} e_{kR} + h_{22} \overline{\psi}_{kL} E_{iR} + h_{23} \overline{\psi}_{kL} E_{1kR} + h_{24} \overline{\psi}_{kL} E_{3kR} + h_{31} \overline{\psi}''_{kL} e_{iR} + h_{32} \overline{\psi}''_{kL} e_{jR} + h_{33} \overline{\psi}''_{kL} e_{kR} + h_{34} \overline{\psi}''_{kL} E_{iR} \\ & + h_{35} \overline{\psi}''_{kL} E_{1kR} + h_{36} \overline{\psi}''_{kL} E_{3kR})\Phi_2 + \epsilon_{\alpha\beta\gamma}(g_1 \overline{\psi}'^{\alpha}_{kL}(\psi^{c}_{iL})^{\beta} + g_2 \overline{\psi}'^{\alpha}_{kL}(\psi^{c}_{kL})^{\beta} + g_3 \overline{\psi}'^{\alpha}_{kL}(\psi''^{c}_{kL})^{\beta})(\Phi^{*}_2)^{\gamma}.\end{aligned} \tag{A8}$$

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