

Baryons with Ginsparg-Wilson quarks in a staggered sea

Brian C. Tiburzi*

Department of Physics, Duke University, P.O. Box 90305, Durham, North Carolina 27708-0305, USA

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We determine the masses and magnetic moments of the octet baryons in chiral perturbation theory formulated for a mixed lattice action of Ginsparg-Wilson valence quarks and staggered sea quarks. Taste-symmetry breaking does not occur at next-to-leading order in the combined lattice spacing and chiral expansion. Expressions derived for masses and magnetic moments are required for addressing lattice artifacts in mixed-action simulations of these observables.

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I. INTRODUCTION

Lattice QCD has made impressive progress. As fully dynamical simulations for a wide range of observables are carried out at lighter quark masses, our confidence in lattice QCD as a predictive tool grows. There are a number of fermion discretizations used on the lattice. These are at various stages of development, and are fraught with disparate difficulties. Further confidence in lattice QCD will eventually be built through comparisons of simulations employing different lattice fermions.

Presently lattice QCD studies using dynamical staggered fermions [1] reach smaller quark masses as compared with other lattice fermions. The publicly available MILC configurations [2], moreover, have launched dynamical staggered fermions as readily accessible for lattice calculations [3–8]. The fourth-root trick is employed by these simulations to reduce the number of so-called taste degrees of freedom. In the continuum limit, there are no taste-changing interactions and the trick is kosher. On the lattice at finite a , however, the question of locality emerges and the trick remains controversial; for various recent investigations see [9–17]. Nonetheless, even with improved staggered actions the discretization effects are surprisingly large. Chiral perturbation theory has been extended to staggered actions to control the systematic errors associated with the continuum extrapolation [18–23].

Simulations that employ Ginsparg-Wilson fermions [24], on the other hand, do not suffer from the precarious theoretical situation of staggered fermions.¹ These fermions, moreover, have an exact chiral symmetry (which in practice is limited by how well the overlap operator [25–27] is approximated, or how well the domain wall fermion [28–30] is realized). Computationally, however, Ginsparg-Wilson fermions are numerically quite costly. Midground between these two lattice fermions can be found. Ginsparg-

Wilson quarks can be utilized for the valence quarks, where they are computationally less demanding. One can then calculate correlation functions in the background of the existing staggered sea of the MILC configurations. This efficacious solution has costs comparable to quenched Ginsparg-Wilson simulations, and numerical investigations in mixed-action lattice QCD have been undertaken [31–34]. Mixed-action chiral perturbation theory has recently been formulated in the meson sector [35] to analyze the lattice-spacing dependence of meson observables.

In this work, we construct the baryon chiral Lagrangian for a mixed lattice action consisting of Ginsparg-Wilson valence quarks and staggered sea quarks. We apply this Lagrangian to the computation of octet baryon masses and magnetic moments. To address the effects of finite lattice spacing, one formulates the underlying lattice theory and matches it onto a chiral effective theory [36]. To do so, we utilize a dual expansion in the quark masses and lattice spacing. We assume a hierarchy of energy scales

$$m_q \ll \Lambda_{\text{QCD}} \ll \frac{1}{a}, \quad (1)$$

and further choose the power counting scheme

$$\varepsilon^2 \sim \begin{cases} m_q/\Lambda_{\text{QCD}} \\ a^2\Lambda_{\text{QCD}}^2 \end{cases} \quad (2)$$

which is currently relevant for simulations employing improved staggered quarks [6]. The resulting quark mass and lattice-spacing dependent expressions will be useful for the analysis of numerical results from simulations of baryon observables in mixed-action lattice QCD.

Our presentation has the following organization. First we briefly review the Symanzik Lagrangian for the mixed lattice action in Sec. II. Here we recall the form of the chiral Lagrangian in the meson sector [35], and cite the relevant details for our calculation. The baryons are then included in the theory. In Sec. III, we determine the magnetic moments to $\mathcal{O}(\varepsilon)$ in the combined expansion, while in Sec. IV, we calculate the masses of the octet baryons up to $\mathcal{O}(\varepsilon^3)$. These calculations at their respective orders include the leading nonanalytic dependence on the quark masses, and are shown to be devoid of taste-symmetry

*Electronic address: bctiburzi@phy.duke.edu

¹In this work, we assume the validity of the fourth-root trick. Comparing lattice data with the behavior of observables predicted from staggered chiral perturbation theory could yield empirical evidence for or against the trick.

breaking. Taste-symmetry breaking interactions contribute at $\mathcal{O}(\varepsilon^4)$ from loops that scale generically as $\mathcal{O}(a^2 m_q \log m_q)$ and $\mathcal{O}(m_q^2 \log m_q)$. A summary ends the paper (Sec. V). For completeness we include a discussion of the finite volume corrections in the Appendix.

II. CHIRAL LAGRANGIAN

Before including baryons into the mixed-action chiral Lagrangian for Ginsparg-Wilson valence quarks and staggered sea quarks, we first give a brief review of the Symanzik Lagrangian. Next we recall the form of the chiral effective theory in the meson sector, and list results pertinent for the calculations in this paper. The relevant pieces of the baryon Lagrangian are then detailed.

A. Symanzik Lagrangian

The lattice action can be described in terms of a continuum effective field theory. This effective theory is described by the Symanzik action [37,38], which is built from continuum operators and is based on the symmetries of the underlying lattice theory. The Symanzik Lagrangian is organized in powers of the lattice spacing a , namely

$$\mathcal{L}_{\text{Sym}} = \mathcal{L} + a\mathcal{L}^{(5)} + a^2\mathcal{L}^{(6)} + a^3\mathcal{L}^{(7)} + \dots, \quad (3)$$

where $\mathcal{L}^{(n)}$ represents the contribution from dimension- n operators.² The symmetries of the mixed lattice action are respected by the Symanzik Lagrangian \mathcal{L}_{Sym} order by order in a . In the continuum limit, $a \rightarrow 0$, only the operators of \mathcal{L} survive. We consider here the case of a mixed action in partially quenched QCD (PQQCD). This type of action allows not only for the valence and sea quarks to have different masses, but to be different types of lattice fermions. An important feature of mixed-action theories concerns the general lack of symmetry between the valence and sea sectors [39]. Because different types of lattice fermions are used in each sector of the theory, the flavor symmetry of the mixed action is generally a direct product of the separate symmetries in the valence and sea sectors.

The underlying lattice action we consider is built from three flavors of Ginsparg-Wilson valence quarks and three flavors of staggered sea quarks. In the continuum limit, the Lagrangian \mathcal{L} is just the partially quenched action,³ namely

$$\mathcal{L} = \bar{Q}\not{D}Q + \bar{Q}m_q Q, \quad (4)$$

where the quark fields appear in the vector

²One should note that not all a dependence is parametrized in Eq. (3). The coefficients of operators in $\mathcal{L}^{(n)}$ depend on the gauge coupling, and hence can have a weak logarithmic dependence on a .

³We use the supersymmetric formulation of partially quenched theories that stems back to [40]. One could equivalently use the replica method [41].

$$Q = (u, d, s, j_1, j_2, j_3, j_4, l_1, l_2, l_3, l_4, r_1, r_2, r_3, r_4, \tilde{u}, \tilde{d}, \tilde{s})^T. \quad (5)$$

Notice that fermion doubling has produced four tastes for each flavor (j, l, r) of staggered quark. In a partially quenched generalization of the isospin limit, the mass matrix is given by

$$m_q = \text{diag}(m_u, m_u, m_s, m_j \xi_I, m_j \xi_I, m_r \xi_I, m_u, m_u, m_s), \quad (6)$$

with ξ_I as the 4×4 taste identity matrix. In the massless limit, the Lagrangian \mathcal{L} has a graded chiral symmetry of the form $SU(15 | 3)_L \otimes SU(15 | 3)_R$. For our discussion below, it is useful to define projection operators for the valence (V) and sea (S) sectors of the theory: $\mathcal{P}_V = \text{diag}(1, 1, 1, 0, \dots, 0, 1, 1, 1)$, and $\mathcal{P}_S = \text{diag}(0, 0, 0, 1 \xi_I, 1 \xi_I, 1 \xi_I, 0, 0, 0)$.

The Ginsparg-Wilson sector of the theory possesses an exact chiral symmetry in the limit of zero quark mass [42]. Thus there can be no operators in $\mathcal{L}^{(5)}$ involving only valence quarks because the only dimension-5 operator (after field redefinitions [43]) is a chiral symmetry breaking quark bilinear. Also there are no dimension-5 operators built from just staggered quark fields [44,45]. Finally the symmetries of the mixed action forbid bilinear operators formed from one valence and one sea quark. Thus $\mathcal{L}^{(5)} = 0$.

Next we consider the dimension-6 operators, of which there are purely gluonic operators, quark bilinears and four-quark operators. The gluonic operators can be omitted from consideration. This is because they transform as singlets under chiral transformations in the valence and sea sector, and their contribution in the effective theory will be identical to those from two-quark and four-quark operators of dimension-6 that do not break the $SU(15 | 3)_L \otimes SU(15 | 3)_R$ chiral symmetry. We decompose the dimension-6 operators into three classes as follows:

$$\mathcal{L}^{(6)} = \mathcal{L}_{\text{val}}^{(6)} + \mathcal{L}_{\text{sea}}^{(6)} + \mathcal{L}_{\text{mix}}^{(6)}. \quad (7)$$

The first term $\mathcal{L}_{\text{val}}^{(6)}$ consists of operators formed from only valence quark fields. When the valence quark masses are zero, these operators have an $SU(3 | 3)_L \otimes SU(3 | 3)_R$ chiral symmetry which is reduced to a vector symmetry when the valence masses are turned on. There is also a four-derivative valence bilinear in $\mathcal{L}_{\text{val}}^{(6)}$ that breaks the $SO(4)$ rotational invariance of Euclidean space down to the hypercubic group SW_4 . The dimension-6 terms in $\mathcal{L}_{\text{val}}^{(6)}$ for unquenched theories have been detailed long ago [46]; for a recent discussion that addresses Ginsparg-Wilson valence quarks specifically, see [47].

The second term $\mathcal{L}_{\text{sea}}^{(6)}$ consists of operators formed from only sea-quark fields. The case of one staggered flavor (corresponding to four tastes) was considered in [18,48]. More recently these results were generalized to the case of multiple flavors in [19,21]. Terms that do not break taste

symmetry have the same form as operators in $\mathcal{L}_{\text{val}}^{(6)}$. These operators have an $SU(12)_L \otimes SU(12)_R$ chiral symmetry in the massless sea-quark limit, and a vector symmetry away from zero mass. There is a single four-derivative term which reduces the rotational symmetry of taste-symmetric terms down to SW_4 . When one considers taste-symmetry breaking terms, one has four-quark operators of the form

$$[\bar{Q}(\Gamma_A \otimes \Gamma_B^*)\mathcal{P}_S Q][\bar{Q}(\Gamma_A \otimes \Gamma_B^*)\mathcal{P}_S Q] \quad (8)$$

where Γ_A denotes a Dirac matrix that acts on the spin indices of the quark fields, and Γ_B^* denotes a generator of $SU(4)$ taste that acts on the taste indices of the quark fields. We treat the sum over staggered flavors as implicit. The presence of explicit Γ_B^* matrices breaks the taste symmetry. When the collective labels A and B are unrelated, the operator respects $SO(4)$ rotational invariance. On the other hand, when the Lorentz indices contained in label A are contracted with those in B , both taste and rotational symmetries are broken and consequently the distinction between the two is blurred.

Finally operators in $\mathcal{L}_{\text{mix}}^{(6)}$ are operators that involve both valence and sea-quark fields. Because of the mixed-action symmetry, all of these terms are four-quark operators consisting of a product of a bilinear of valence quarks with a bilinear of sea quarks. These operators were constructed in [35], where it was shown that only taste-singlet sea-quark bilinears are present. The possible Dirac structures of these bilinears are constrained by the exact chiral symmetry required in the valence sector, and the axial symmetry required in the sea sector. Only vector and axial-vector bilinears are consistent with both symmetries. Ignoring color structure which has no bearing in the construction of the chiral Lagrangian, we have two terms in $\mathcal{L}_{\text{mix}}^{(6)}$

$$\begin{aligned} \mathcal{L}_{\text{mix}}^{(6)} = & C_{\text{mix}}^V (\bar{Q}\gamma_\mu \mathcal{P}_V Q)(\bar{Q}\gamma_\mu \mathcal{P}_S Q) \\ & + C_{\text{mix}}^A (\bar{Q}\gamma_\mu \gamma_5 \mathcal{P}_V Q)(\bar{Q}\gamma_\mu \gamma_5 \mathcal{P}_S Q). \end{aligned} \quad (9)$$

These terms have a $SU(3|3)_L \otimes SU(3|3)_R \otimes SU(12)_L \otimes SU(12)_R$ chiral symmetry.

Finally we consider the terms in $\mathcal{L}^{(7)}$. We do not consider operators that have a quark mass insertion because these will necessarily be higher order in our power counting. There are no operators that consist only of Ginsparg-Wilson valence quarks because we cannot build parity even, chirally symmetric operators of dimension-7. As discussed in [21], there are no dimension-7 operators built solely from staggered quark fields due to the staggered axial symmetry. Finally any operators in $\mathcal{L}^{(7)}$ that consist of both staggered quarks and Ginsparg-Wilson quarks must occur as a product of bilinears of each fermion type. Again this is because there is no symmetry relating the two sectors of the mixed-action theory. There are no such bilinears, however, because one cannot write down a dimension-7 operator consisting of a chirally symmetric

Ginsparg-Wilson bilinear and an axially symmetric staggered bilinear. Thus $\mathcal{L}^{(7)} = 0$.

B. Mesons

In this section, we review the construction of chiral perturbation theory in the meson sector of this partially quenched mixed-action theory. Partially quenched chiral perturbation theory was developed through a series of papers [49–53]. The construction of the chiral effective theory for Ginsparg-Wilson valence quarks and staggered sea quarks was carried out in [35]. We will focus only on the ingredients of this theory necessary for determining baryon properties at next-to-leading order.

As commented above, in the massless and continuum limit the Symanzik action has a graded chiral symmetry $SU(15|3)_L \otimes SU(15|3)_R$. We expect this symmetry to be spontaneously broken down to $SU(15|3)_V$ in analogy with QCD. Thus we can build an effective theory of mixed-action PQQCD written in terms of the pseudo-Goldstone modes that emerge from spontaneous chiral symmetry breaking. These modes acquire masses from the explicit chiral symmetry breaking introduced by the quark mass term in the continuum action. Additionally we shall see below that the a -dependent terms in the Symanzik action also generate masses for some of these modes. We shall generically call the pseudo-Goldstone modes mesons, and collect them in an $U(15|3)$ matrix exponential Σ

$$\Sigma = e^{2i\Phi/f} \equiv \xi^2, \quad (10)$$

that is written in terms of the meson matrix Φ . We work almost exclusively in the quark basis due to the fact that mesons composed of two sea quarks will enter our next-to-leading order calculations only as taste singlets.

At $\mathcal{O}(\varepsilon^2)$ in the combined lattice spacing and chiral expansion the effective Lagrangian of partially quenched chiral perturbation theory (PQ χ PT) that describes the mesons has the form [35]

$$\begin{aligned} \mathcal{L} = & \frac{f^2}{8} \text{str}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) - \lambda \text{str}(m_q \Sigma^\dagger + m_q^\dagger \Sigma) \\ & + \frac{1}{6} \mu_0^2 (\text{str} \Phi)^2 + a^2 \mathcal{V}. \end{aligned} \quad (11)$$

With these conventions, the pion-decay constant $f = 132$ MeV. The potential \mathcal{V} contains the effects of dimension-6 operators in the Symanzik action. It can be decomposed into three terms [35]

$$\mathcal{V} = \mathcal{U}_S + \mathcal{U}'_S + \mathcal{U}_V. \quad (12)$$

Above \mathcal{U}_S and \mathcal{U}'_S are taste-symmetry breaking potentials involving single trace and two trace operators, respectively [18,19]. These potentials involve only the mesons composed of two sea quarks and arise from $\mathcal{L}_{\text{sea}}^{(6)}$ in the Symanzik action. The potential \mathcal{U}_V contains all terms that stem from Symanzik operators involving the valence

quark fields. These operators are in $\mathcal{L}_{\text{val}}^{(6)}$ and $\mathcal{L}_{\text{mix}}^{(6)}$. As shown in [35], there is only one term in the valence potential

$$\mathcal{U}_V = -a^2 C_{\text{mix}} \text{str}(\tau_3 \Sigma \tau_3 \Sigma^\dagger), \quad (13)$$

where $\tau_3 = \mathcal{P}_S - \mathcal{P}_V$. This term only acts on mesons composed of one valence quark and one sea quark. At this order there are no contributions from operators that act on valence-valence mesons as their effects lead to a^2 renormalization of the lowest order parameters [47].

Working to tree level, we can determine the meson masses needed below for the calculation of baryon observables. When one expands the Lagrangian in Eq. (11) to leading order, one finds that mesons formed of two Ginsparg-Wilson quarks $Q\bar{Q}'$ have mass

$$m_{Q\bar{Q}'}^2 = \frac{4\lambda}{f^2} (m_Q + m_{Q'}), \quad (14)$$

which accordingly vanishes in the chiral limit. Mesons consisting of a staggered quark Q_i , i.e. of flavor Q and quark taste i , and a Ginsparg-Wilson quark \bar{Q}' have masses

$$m_{Q_i\bar{Q}'}^2 = \frac{4\lambda}{f^2} (m_Q + m_{Q'}) + \frac{16a^2 C_{\text{mix}}}{f^2}. \quad (15)$$

Notice that these masses do not depend on the quark taste. Masses of mixed mesons do not vanish in the chiral limit because the full chiral symmetry of \mathcal{L} is explicitly broken by $\mathcal{L}_{\text{mix}}^{(6)}$ down to the mixed-action chiral symmetry. The final mesons relevant to baryon observables at next-to-leading order are those with two staggered quarks in a flavor-neutral, taste-singlet combination. In the quark basis, their masses are given by

$$m_{Q\bar{Q}}^2 = \frac{8\lambda}{f^2} m_Q + \frac{64a^2}{f^2} (C_3 + C_4), \quad (16)$$

where C_3 and C_4 are parameters entering the potential \mathcal{U}_S .

Notice that in Eq. (11) the matrix Φ is not supertraceless, and we have included the singlet mass parameter μ_0 . This is a device to derive the form of the flavor-neutral propagators [53]. Partially quenched theories have a $U(1)_A$ anomaly which renders the singlet heavy. Thus the form of flavor-neutral propagators can be derived in the limit that $\mu_0 \rightarrow \infty$, and the singlet [here of $SU(15 | 3)_V$] is integrated out of the low-energy theory. For the staggered flavor-neutral propagators, the analysis has been carried out in [19], including the 1/4 factors corresponding to the fourth-root trick. There it was shown that different flavor-neutral propagators exist for the taste singlet, vector and axial-vector channels. To the order we work, the only flavor neutrals required will be composed of two Ginsparg-Wilson valence quarks. Thus as with the meson mass and decay constant in mixed-action PQ χ PT [35], only the taste-singlet flavor-neutral propagator is required. For $a, b = u, d, s$, this propagator is given by

$$\mathcal{G}_{\eta_a \eta_b} = \frac{\delta_{ab}}{k^2 + m_{aa}^2} - \frac{1}{3} \frac{(k^2 + m_{jj}^2)(k^2 + m_{rr}^2)}{(k^2 + m_{aa}^2)(k^2 + m_{bb}^2)(k^2 + m_X^2)}, \quad (17)$$

where the masses of the valence-valence mesons, m_{aa}^2 and m_{bb}^2 , are given in Eq. (14), while those of the sea-sea mesons, m_{jj}^2 and m_{rr}^2 , are given in Eq. (16). The mass m_X is defined as $m_X^2 = \frac{1}{3}(m_{jj}^2 + 2m_{rr}^2)$. The flavor-neutral propagator can be conveniently rewritten as

$$\mathcal{G}_{\eta_a \eta_b} = \delta_{ab} P_a + \mathcal{H}_{ab}(P_a, P_b, P_X), \quad (18)$$

where

$$P_a = \frac{1}{k^2 + m_{aa}^2}, \quad P_b = \frac{1}{k^2 + m_{bb}^2},$$

$$P_X = \frac{1}{k^2 + m_X^2},$$

$$\mathcal{H}_{ab}(A, B, C) = -\frac{1}{3} \left[\frac{(m_{jj}^2 - m_{aa}^2)(m_{rr}^2 - m_{aa}^2)}{(m_{aa}^2 - m_{bb}^2)(m_{aa}^2 - m_X^2)} A \right. \\ \left. + \frac{(m_{jj}^2 - m_{bb}^2)(m_{rr}^2 - m_{bb}^2)}{(m_{bb}^2 - m_{aa}^2)(m_{bb}^2 - m_X^2)} B \right. \\ \left. + \frac{(m_{jj}^2 - m_X^2)(m_{rr}^2 - m_X^2)}{(m_X^2 - m_{aa}^2)(m_X^2 - m_{bb}^2)} C \right]. \quad (19)$$

In the limit that $b \rightarrow a$, we require a separate form of the flavor-neutral propagator to handle the double pole. The functional form, however, can be related to a derivative of the single pole form, namely

$$\mathcal{H}_{aa}(A, A, C) = -\frac{1}{3} \left[\frac{\partial}{\partial m_{aa}^2} \frac{(m_{jj}^2 - m_{aa}^2)(m_{rr}^2 - m_{aa}^2)}{(m_{aa}^2 - m_X^2)} A \right. \\ \left. + \frac{(m_{jj}^2 - m_X^2)(m_{rr}^2 - m_X^2)}{(m_X^2 - m_{aa}^2)^2} C \right], \quad (20)$$

keeping in mind that $A = A(m_{aa}^2)$.

C. Baryons

Having reviewed the mixed-action Symanzik Lagrangian and the relevant pieces of meson PQ χ PT at finite a , we now extend mixed-action PQ χ PT to the baryon sector. In the continuum limit, the flavor symmetry group of the mixed-action theory is $SU(15 | 3)_V$. There are no taste-symmetry breaking interactions present when the lattice spacing is zero. Thus we can construct spin-1/2 and spin-3/2 baryon flavor tensors in the usual fashion [54–56].⁴ In general we can find the dimensionality of

⁴This is a useful way to proceed because the external states consist of three Ginsparg-Wilson quarks and have a one-to-one correspondence with the baryon interpolating fields used on the lattice. On the other hand, baryons formed from quark tastes are complicatedly related to the usual staggered quark interpolating operators used on the lattice.

these multiplets in $SU(M|N)_V$ using the Young super-tableaux [57,58]. For the spin-1/2 multiplet \mathcal{B}^{ijk} , we have a

$$\frac{1}{3}[(M+1)M(M-1) + (N+1)N(N-1)] + MN(M+N) \quad (21)$$

dimensional representation of $SU(M|N)_V$, while the spin-3/2 multiplet \mathcal{T}_μ^{ijk} furnishes a

$$\frac{1}{4}M(M+1) + \frac{1}{12}M(M+1)(2M+1) + \frac{1}{2}MN(M+N) + \frac{1}{2}(N-1)(N-2) - \delta_{N,0} \quad (22)$$

dimensional representation.

For the case at hand, $SU(15|3)_V$ PQ χ PT, we embed the spin- $\frac{1}{2}$ baryons in the **1938**-dimensional supermultiplet \mathcal{B}^{ijk} . The spin- $\frac{3}{2}$ baryons are embedded in the **1086**-dimensional supermultiplet \mathcal{T}_μ^{ijk} . For baryon PQ χ PT at next-to-leading order (and indeed at next-to-next-to-leading order as well), we shall need the states in these multiplets consisting of at most one sea quark or at most one ghost quark. To this end, we decompose the irreducible representations of $SU(15|3)_V$ into irreducible representations of $SU(3)_{\text{val}} \otimes SU(12)_{\text{sea}} \otimes SU(3)_{\text{ghost}}$ [59]. To describe these superalgebra multiplets we refer to their floors, where the floor number coincides with the number of bosonic ghost quarks contained in states. The ground floor is synonymous with zero ghost quarks. Additionally we refer to the levels of the multiplet [55] to distinguish between baryon states with differing numbers of sea quarks. Level *A* baryons do not have a sea quark, level *B* baryons have one sea quark, and so on.

The ground floor, level *A* of the multiplet \mathcal{B}^{ijk} consists of baryons that transform as a $(\mathbf{8}, \mathbf{1}, \mathbf{1})$ under $SU(3)_{\text{val}} \otimes SU(12)_{\text{sea}} \otimes SU(3)_{\text{ghost}}$. These are the octet baryons, and are embedded in the tensor \mathcal{B}^{ijk} in the standard way when all of the indices are restricted to 1–3 [60]. The first floor, level *A* of \mathcal{B}^{ijk} transforms as a $(\mathbf{6}, \mathbf{1}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{3})$. These states have been constructed explicitly in [55]. The ground floor, level *B* of the \mathcal{B}^{ijk} multiplet transforms as a $(\mathbf{6}, \mathbf{12}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{12}, \mathbf{1})$. The states constructed in [55] can be used with minimal modifications. One merely must reindex so that the range of q_{sea} is extended from 1–3 to 1–12, or merely attach a taste index to each flavor tensor.

The situation is the same with respect to the spin-3/2 baryon tensor \mathcal{T}_μ^{ijk} . Either the states have already been constructed in [55], or it is trivial to extend to the present case those which have not. The ground floor, level *A* of the spin-3/2 multiplet transforms as a $(\mathbf{10}, \mathbf{1}, \mathbf{1})$ under $SU(3)_{\text{val}} \otimes SU(12)_{\text{sea}} \otimes SU(3)_{\text{ghost}}$, and consists of the decuplet baryons which are embedded in the usual fashion. The first floor, level *A* of \mathcal{T}_μ^{ijk} transforms as a $(\mathbf{6}, \mathbf{1}, \mathbf{3})$, while the ground floor, level *B* transforms as a $(\mathbf{6}, \mathbf{12}, \mathbf{1})$.

Thus the total number of baryon states relevant for calculating loop diagrams up to next-to-next-to-leading order is 243, which is considerably smaller than the total number of baryons in the theory 3024.

To $\mathcal{O}(\varepsilon^2)$, the free Lagrangian for the \mathcal{B}^{ijk} and \mathcal{T}_μ^{ijk} fields retains the same form as in quenched and partially quenched theories [55,60] with the addition of new lattice-spacing dependent terms,

$$\begin{aligned} \mathcal{L} = & i(\overline{\mathcal{B}}\mathbf{v} \cdot \mathcal{D}\mathcal{B}) - 2\alpha_M(\overline{\mathcal{B}}\mathcal{B}\mathcal{M}_+) - 2\beta_M(\overline{\mathcal{B}}\mathcal{M}_+\mathcal{B}) \\ & - \frac{1}{2}\sigma_M(\overline{\mathcal{B}}\mathcal{B})\text{str}(\mathcal{M}_+) + a^2\mathcal{V}_B + i(\overline{\mathcal{T}}_\mu\mathbf{v} \cdot \mathcal{D}\mathcal{T}_\mu) \\ & + \Delta(\overline{\mathcal{T}}_\mu\mathcal{T}_\mu) + 2\gamma_M(\overline{\mathcal{T}}_\mu\mathcal{M}_+\mathcal{T}_\mu) \\ & - \frac{1}{2}\overline{\sigma}_M(\overline{\mathcal{T}}_\mu\mathcal{T}_\mu)\text{str}(\mathcal{M}_+) + a^2\mathcal{V}_T. \end{aligned} \quad (23)$$

The baryon potentials \mathcal{V}_B and \mathcal{V}_T arise from the operators in $\mathcal{L}^{(6)}$ of the Symanzik Lagrangian. For our next-to-leading-order calculations, we will not require the explicit form of either term. The effective contribution can be deduced from symmetry considerations alone. In the baryon Lagrangian, the mass operator is defined by

$$\mathcal{M}_+ = \frac{1}{2}(\xi^\dagger m_Q \xi^\dagger + \xi m_Q \xi). \quad (24)$$

Above, the parameter $\Delta \sim \varepsilon\Lambda_\chi$ is the mass splitting between the **1938** and **1086** in the chiral limit. The parenthesis notation used in Eq. (23) is that of [60] and is defined so that the contractions of flavor indices maintain proper transformations under chiral rotations.

The Lagrangian describing the interactions of the \mathcal{B}^{ijk} and \mathcal{T}_μ^{ijk} with the pseudo-Goldstone mesons is

$$\begin{aligned} \mathcal{L} = & 2\alpha(\overline{\mathcal{B}}S_\mu\mathcal{B}A_\mu) + 2\beta(\overline{\mathcal{B}}S_\mu A_\mu\mathcal{B}) \\ & - 2\mathcal{H}(\overline{\mathcal{T}}_\nu S_\mu A_\mu \mathcal{T}_\nu) + \sqrt{\frac{3}{2}}C[(\overline{\mathcal{T}}_\nu A_\nu \mathcal{B}) \\ & + (\overline{\mathcal{B}}A_\nu \mathcal{T}_\nu)]. \end{aligned} \quad (25)$$

The axial-vector and vector meson fields A_μ and V_μ are defined by $A_\mu = \frac{i}{2}(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi)$ and $V_\mu = \frac{1}{2}\times(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi)$. The latter appears in Eq. (23) for the covariant derivatives of \mathcal{B}_{ijk} and \mathcal{T}_{ijk} that both have the form

$$\begin{aligned} (\mathcal{D}_\mu\mathcal{B})_{ijk} = & \partial_\mu\mathcal{B}_{ijk} + (V_\mu)_{il}\mathcal{B}_{ljk} + (-)\eta_i(\eta_j+\eta_m) \\ & \times (V_\mu)_{jm}\mathcal{B}_{imk} + (-)(\eta_i+\eta_j)(\eta_k+\eta_n) \\ & \times (V_\mu)_{kn}\mathcal{B}_{ijn}. \end{aligned} \quad (26)$$

The vector S_μ is the covariant spin operator [61,62]. The interaction Lagrangian in Eq. (25) also receives lattice-spacing corrections. In calculating baryon observables, however, these lead to effects that are $\mathcal{O}(\varepsilon^2)$ higher than the next-to-leading-order results.

The parameters that appear in the mixed-action PQ χ PT Lagrangian can be related to those in χ PT by matching. One realizes that QCD is contained in the fourth root of the sea sector of the theory. Thus we can relate the parameters of χ PT by matching onto terms in the above Lagrangian restricted to one taste for each flavor of staggered sea quark. For definiteness, we restrict the indices to 4, 8, 12 corresponding to j_1 , l_1 , and r_1 . This allows the usual identifications $\alpha = \frac{2}{3}D + 2F$, $\beta = -\frac{5}{3}D + F$, and the remaining parameters α_M , β_M , σ_M , γ_M , $\bar{\sigma}_M$, C , and \mathcal{H} all to have the same numerical values as in χ PT.

III. BARYON MAGNETIC MOMENTS

In this section, we calculate the octet baryon magnetic moments in mixed-action PQ χ PT. We choose to present the calculation of magnetic moments first due to their simplicity. Recall that in the continuum limit, the baryon magnetic moments have the behavior [63–65]

$$\mu \sim \mu_0 + \alpha\sqrt{m_q} + \dots \quad (27)$$

in the chiral expansion. Terms denoted by \dots scale with a higher power of m_q , and are first encountered from one-loop graphs at next-to-next-to-leading order. At next-to-leading order then, the magnetic moments receive $\mathcal{O}(\varepsilon)$ contributions from loops.

Moving away from the continuum, we are forced to address the corrections from $\mathcal{O}(a^2)$ operators. While there are new a^2 -dependent terms in the baryon Lagrangian, there are still further terms because the vector-current operator also receives $\mathcal{O}(a^2)$ corrections in the effective theory. Potential contributions from any such terms, however, scale as $\mathcal{O}(\varepsilon^2)$ and are relevant only at next-to-next-to-leading order. We can safely ignore such contributions. This situation is similar to the calculation of magnetic moments with mixed actions of Wilson and Ginsparg-Wilson quarks [66]; see also [67]. Thus the lattice-spacing artifacts at next-to-leading order can only enter through loop effects. Of the diagrams that contribute to the electromagnetic current in PQ χ PT [68], the one-loop graphs that give a contribution to the octet magnetic moments are depicted in Fig. 1.

To calculate these diagrams and the tree-level contributions, we must extend the electric charge matrix \mathcal{Q} , which is not uniquely defined in partially quenched theories [69].



FIG. 1. Loop diagrams contributing to the octet baryon magnetic moments at $\mathcal{O}(\varepsilon)$. The photons are pictured as wiggly lines, mesons are denoted by dashed lines, and thin solid lines denote **1938** baryons, while the double lines denote **1086** baryons.

By imposing the charge matrix \mathcal{Q} to be supertraceless in $SU(15 | 3)$, no new operators involving the singlet component are introduced. There are various ways to accomplish this, however, many are not practical to implement time-wise in current lattice computations. We require that ghost charges equal their valence counterparts so that operator self-contractions from the valence quarks are completely canceled by ghosts [70]. Thus our form of the $SU(15 | 3)$ charge matrix is

$$\mathcal{Q} = \text{diag}(q_u, q_d, q_s, q_j\xi_l, q_l\xi_l, q_r\xi_l, q_u, q_d, q_s), \quad (28)$$

where to maintain supertracelessness $q_j + q_l + q_r = 0$. Taking the fourth root of the determinant in the continuum, and making the sea quarks degenerate with the valence quarks, QCD is recovered only for the specific choice of charges: $q_u = q_j = \frac{2}{3}$, and $q_d = q_s = q_l = q_r = -\frac{1}{3}$. Using unphysical charges for the valence and sea quarks provides a means to access low-energy constants, and thereby determine the physical magnetic moments, for example. Notice the choice, $q_j = q_l = q_r = 0$, is allowed by supertracelessness, and would considerably free up computation time by eliminating all operator self-contractions.

In PQ χ PT the leading contribution to the octet magnetic moments arises from two dimension-5 operators [55]⁵

$$\begin{aligned} \mathcal{L} = & \frac{ie}{2M_B} [\mu_\alpha (\bar{\mathcal{B}}[S_\mu, S_\nu] \mathcal{B} \mathcal{Q}) \\ & + \mu_\beta (\bar{\mathcal{B}}[S_\mu, S_\nu] \mathcal{Q} \mathcal{B})] F_{\mu\nu}, \end{aligned} \quad (29)$$

which can be matched onto the χ PT Lagrangian upon restricting the baryon field indices to 4, 8, 12, and taking the charges to be their physical values. In terms of the $SU(3)$ matrix B containing the octet baryons, the corresponding χ PT Lagrangian is

$$\begin{aligned} \mathcal{L} = & \frac{ie}{2M_B} [\mu_D \text{tr}(\bar{B}[S_\mu, S_\nu] \{ \mathcal{Q}, B \}) \\ & + \mu_F \text{tr}(\bar{B}[S_\mu, S_\nu] [\mathcal{Q}, B])] F_{\mu\nu}, \end{aligned} \quad (30)$$

and hence we find $\mu_\alpha = \frac{2}{3}\mu_D + 2\mu_F$, and $\mu_\beta = -\frac{5}{3}\mu_D + \mu_F$ by matching. These operators contribute to the magnetic moments at $\mathcal{O}(\varepsilon^0)$. There are additional operators that make contributions of $\mathcal{O}(\varepsilon)$ that have identical flavor structure, and differ only by the insertion of Δ/Λ_χ . These operators come with new low-energy constants and are allowed because the mass-splitting parameter is a chiral singlet. These Δ -dependent operators also function as counterterms for the one-loop divergences. We shall not keep these operators explicitly, and merely treat the leading low-energy constants as arbitrary linear functions of Δ . The ability to determine coefficients for the

⁵Here we use $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Δ -dependent operators requires the ability to vary Δ , and for this reason we treat the Δ dependence only implicitly.

Having spelled out the operators for tree-level contributions to the magnetic moments, we now comment on the computation of the loop diagrams. Because flavor-neutral mesons remain charge neutral, even with arbitrary quark charges, there are no hairpin contributions at next-to-leading order, see Fig. 1. Loop graphs containing intermediate state baryons with all valence quarks, or two valence quarks and one ghost quark are identical to those calculated for $SU(6 | 3)$ in [55] (modulo the different choice of electric charge matrix) because the ground floor, level A , and first floor, level A transform identically in each theory (both are singlets in the sea sector where the group structure is different). Finally there are the diagrams with baryons consisting of two valence quarks and one sea quark. The corresponding loop mesons are mixed mesons; they have one valence quark and one sea quark. The propagation of these modes does not break taste symmetry. This is because the Lagrangian in the mixed sector does not contain explicit taste matrices only the matrix τ_3 . Thus the mixed-meson propagator obeys [35]

$$\overline{\Phi_{Q_i Q'} \Phi_{\tilde{Q}' \tilde{Q}_j}}(k^2) = \frac{\delta_{Q' \tilde{Q}'} \delta_{Q \tilde{Q}} \delta_{ij}}{k^2 + m_{Q, Q'}^2}, \quad (31)$$

where Q_i, \tilde{Q}_j label the staggered quark flavors and tastes, and Q', \tilde{Q}' label the Ginsparg-Wilson quark flavors. The mixed-meson mass $m_{Q, Q'}^2$ is given in Eq. (15), and is independent of the quark taste i . Now because there is no taste changing in the loop, and the relevant meson and baryon masses are quark taste independent, we obtain a factor of 4 for each staggered quark flavor that can propagate in the loop. This factor is then canceled by the fourth-root trick. Explicit calculation verifies this. The baryon magnetic moments hence have the same form as in $SU(6 | 3)$ PQ χ PT [55], and depend only on one new low-energy constant C_{mix} . This low-energy constant appears in the loop meson masses that contain a sea quark and a valence quark.

Combining the tree-level and one-loop graphs, we have

$$\begin{aligned} \mu &= Q\mu_F + \alpha_D \mu_D + \frac{M_B}{4\pi f^2} \sum_{\phi} \left\{ \beta_{\phi} m_{\phi} + \beta'_{\phi} \frac{C^2}{\pi} \right. \\ &\quad \left. \times \left[\mathcal{F}(m_{\phi}, \Delta, \mu) + \frac{5}{3} \right] \right\}, \end{aligned} \quad (32)$$

where we have carried out the spin algebra in D dimensions. The nonanalytic function appearing in the above expression is defined by

$$\mathcal{F}(m, \delta, \mu) = \sqrt{\delta^2 - m^2} \log \frac{\delta - \sqrt{\delta^2 + m^2 + i\epsilon}}{\delta + \sqrt{\delta^2 + m^2 + i\epsilon}} - \delta \log \frac{m^2}{\mu^2}. \quad (33)$$

The coefficients Q , and α_D for the tree-level diagrams are

TABLE I. Tree-level coefficients for the magnetic moments of octet baryons.

	Q	α_D
p	$q_u - q_s$	$q_u + q_s$
n	$q_d - q_s$	$q_d + q_s$
Σ^+	$q_u - q_d$	$q_u + q_d$
Σ^0	0	$q_u + q_d$
Σ^-	$-q_u + q_d$	$q_u + q_d$
Ξ^0	$-q_d + q_s$	$q_d + q_s$
Ξ^-	$-q_u + q_s$	$q_u + q_s$
Λ	0	$\frac{1}{3}(q_u + q_d + 4q_s)$
$\Lambda\Sigma$	0	$\frac{1}{\sqrt{3}}(q_u - q_d)$

listed in Table I. Ordinarily Q is the baryon charge; this is no longer the case with the quark electric charge matrix \mathcal{Q} given in Eq. (28). The computed values for the β_{ϕ} , and β'_{ϕ} coefficients are listed for the octet baryons in Tables II, III, IV, V, VI, VII, VIII, and IX. The corresponding values of these coefficients for the $\Lambda\Sigma^0$ transition moment are given in Table X. In each table we have listed the values corresponding to loop mesons that have mass m_{ϕ} . In these diagrams there are valence-valence mesons, with masses given in Eq. (14), and valence-sea mesons, with masses given in Eq. (15). If a particular meson is not listed then the values for β_{ϕ} , and β'_{ϕ} are zero.

The charges q_j and q_l do not appear explicitly in these tables because, in the isospin limit, they always come in the combination $q_j + q_l$ and this is identical to $-q_r$. Again we remark that the supertracelessness of \mathcal{Q} is maintained by the computation-time simplifying choice $q_j = q_l = q_r = 0$. For this choice of charges, one is not computing the physical magnetic moments; however, the unphysical moments determined are sensitive to the physical low-energy constants, hence predictions can be made. Additionally one has complete freedom to adjust the charges q_u, q_d , and q_s as these do not contribute to the supertrace of \mathcal{Q} . One can, for example, choose these charges to isolate the low-energy constants at tree level, see Table I, or to simplify the chiral extrapolation by eliminating particular loop mesons. The form of the continuum extrapolation of baryon magnetic moments is thus highly constrained, especially since the only new low-energy constant at this order, C_{mix} , could be determined independently from the masses of mixed mesons [35].

IV. BARYON MASSES

In this section we determine the octet baryon masses in mixed-action PQ χ PT. Let us first recall the behavior of the baryon mass in the continuum, and near the chiral limit [71–74]

$$M_B \sim M_0 + \alpha m_q + \beta m_q^{3/2} + \dots, \quad (34)$$

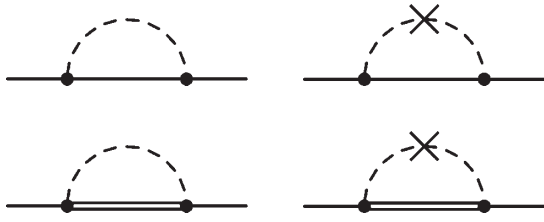


FIG. 2. Loop diagrams contributing to the octet baryon masses at $\mathcal{O}(\varepsilon^3)$. Mesons are denoted by dashed lines, flavor neutrals (hairpins) by crossed dashed lines, and thin solid lines denote **1938** baryons, while double lines denote **1086** baryons.

where we have retained the leading nonanalytic piece and the \dots denotes terms with higher powers of the quark mass. The nonanalytic term above stems from the one-loop diagrams shown in Fig. 2. A mass calculation at next-to-leading order is $\mathcal{O}(\varepsilon^3)$ in our power counting. To perform a complete $\mathcal{O}(\varepsilon^3)$ calculation, we must retain the a^2 -dependent piece of the loop meson masses and evaluate the potential \mathcal{V}_B in Eq. (23) at tree level. Remember that there are no a^3 terms in the Lagrangian.

Without writing down the explicit form of the baryon potential, we can deduce the net contribution from \mathcal{V}_B to the octet masses at tree level. Because the external states all involve Ginsparg-Wilson quarks, the action of the potential is necessarily a taste singlet at tree level. Furthermore since insertion of the mass matrix is at $\mathcal{O}(\varepsilon^4)$, there are no valence flavor matrices around. This implies that the evaluation of the baryon potential is identical for all octet states: in essence all Ginsparg-Wilson quarks are identical in the potential at this order. While there are terms in the potential that violate $SO(4)$ rotational invariance [75], the mass only picks up an indiscernible additive shift from such terms [it is the dispersion relation that is sensitive to $SO(4)$ breaking]. Therefore the effect of the potential at $\mathcal{O}(a^2)$ can be described by just one operator

$$\mathcal{V}_B \stackrel{\text{eff}}{=} -C_0(\overline{B}B). \quad (35)$$

TABLE II. The coefficients β_ϕ , and β'_ϕ for the proton.

ϕ	β_ϕ	β'_ϕ
π	$-\frac{4}{3}D^2(q_u - q_d)$	$-\frac{1}{6}(q_u - q_d)$
ju	$-\frac{4}{3}(D^2 + 3F^2)q_u - 2(D - F)^2q_d - (\frac{5}{3}D^2 - 2DF + 3F^2)q_r$	$\frac{1}{18}(2q_u + 4q_d + 3q_r)$
ru	$-\frac{2}{3}(D^2 + 3F^2)q_u - (D - F)^2q_d + (\frac{5}{3}D^2 - 2DF + 3F^2)q_r$	$\frac{1}{18}(q_u + 2q_d - 3q_r)$

TABLE III. The coefficients β_ϕ , and β'_ϕ for the neutron.

ϕ	β_ϕ	β'_ϕ
π	$\frac{4}{3}D^2(q_u - q_d)$	$\frac{1}{6}(q_u - q_d)$
ju	$-2(D - F)^2q_u - \frac{4}{3}(D^2 + 3F^2)q_d - (\frac{5}{3}D^2 - 2DF + 3F^2)q_r$	$\frac{1}{18}(4q_u + 2q_d + 3q_r)$
ru	$-(D - F)^2q_u - \frac{2}{3}(D^2 + 3F^2)q_d + (\frac{5}{3}D^2 - 2DF + 3F^2)q_r$	$\frac{1}{18}(2q_u + q_d - 3q_r)$

TABLE IV. The coefficients β_ϕ , and β'_ϕ for the Σ^+ .

ϕ	β_ϕ	β'_ϕ
K	$-\frac{4}{3}D^2(q_u - q_s)$	$-\frac{1}{6}(q_u - q_s)$
ju	$-\frac{2}{3}(D^2 + 3F^2)(2q_u + q_r)$	$\frac{1}{18}(2q_u + q_r)$
ru	$-\frac{2}{3}(D^2 + 3F^2)(q_u - q_r)$	$\frac{1}{18}(q_u - q_r)$
js	$-(D - F)^2(2q_s + q_r)$	$\frac{1}{9}(2q_s + q_r)$
rs	$-(D - F)^2(q_s - q_r)$	$\frac{1}{9}(q_s - q_r)$

TABLE V. The coefficients β_ϕ , and β'_ϕ for the Σ^0 .

ϕ	β_ϕ	β'_ϕ
K	$-\frac{2}{3}D^2(q_u + q_d - 2q_s)$	$-\frac{1}{12}(q_u + q_d - 2q_s)$
ju	$-\frac{2}{3}(D^2 + 3F^2)(q_u + q_d + q_r)$	$\frac{1}{18}(q_u + q_d + q_r)$
ru	$-\frac{1}{3}(D^2 + 3F^2)(q_u + q_d - 2q_r)$	$\frac{1}{36}(q_u + q_d - 2q_r)$
js	$-(D - F)^2(2q_s + q_r)$	$\frac{1}{9}(2q_s + q_r)$
rs	$-(D - F)^2(q_s - q_r)$	$\frac{1}{9}(q_s - q_r)$

This term and the familiar tree-level terms that are linear in the quark masses are straightforwardly evaluated. We must keep in mind that α_M , β_M , and σ_M are to be treated as arbitrary linear functions of Δ/Λ_χ . The linear terms, for example, function in part as counterterms for the one-loop diagrams.

Now we turn to the loop diagrams in Fig. 2. As with the computation of the magnetic moments, the diagrams involving baryons with three valence quarks, or two valence quarks and one ghost quark are identical to those in $SU(6 | 3)$ PQ χ PT [55], because these particular floors and levels of each multiplet are identical. Furthermore, diagrams with one sea quark flowing in the loop are evaluated in precisely the same way as before. The mixed-meson propagators are diagonal in taste and flavor, while the meson and baryon masses are taste independent. Consequently the loops come with a fourfold degeneracy when one sums over the sea-quark tastes. This degeneracy is canceled by the factor of 1/4 to implement the fourth-root trick. The only new contribution as compared to the magnetic moments is

TABLE VI. The coefficients β_ϕ , and β'_ϕ for the Σ^- .

ϕ	β_ϕ	β'_ϕ
K	$-\frac{4}{3}D^2(q_d - q_s)$	$-\frac{1}{6}(q_d - q_s)$
ju	$-\frac{2}{3}(D^2 + 3F^2)(2q_d + q_r)$	$\frac{1}{18}(2q_d + q_r)$
ru	$-\frac{2}{3}(D^2 + 3F^2)(q_d - q_r)$	$\frac{1}{18}(q_d - q_r)$
js	$-(D - F)^2(2q_s + q_r)$	$\frac{1}{9}(2q_s + q_r)$
rs	$-(D - F)^2(q_s - q_r)$	$\frac{1}{9}(q_s - q_r)$

TABLE VII. The coefficients β_ϕ , and β'_ϕ for the Ξ^0 .

ϕ	β_ϕ	β'_ϕ
K	$\frac{4}{3}D^2(q_u - q_s)$	$\frac{1}{6}(q_u - q_s)$
ju	$-(D - F)^2(2q_u + q_r)$	$\frac{1}{9}(2q_u + q_r)$
ru	$-(D - F)^2(q_u - q_r)$	$\frac{1}{9}(q_u - q_r)$
js	$-\frac{2}{3}(D^2 + 3F^2)(2q_s + q_r)$	$\frac{1}{18}(2q_s + q_r)$
rs	$-\frac{2}{3}(D^2 + 3F^2)(q_s - q_r)$	$\frac{1}{18}(q_s - q_r)$

that of flavor-neutral modes. But at this order, the flavor-neutral mesons which propagate in loops are those consisting of two Ginsparg-Wilson valence quarks. These are necessarily taste singlets; their propagators appear in Eq. (17), and involve taste-singlet sea-sea meson masses, which are a^2 dependent.

Assembling the tree-level results with the loop diagrams, we find

$$\begin{aligned}
M_B = & M_0 - 2m_u C_u - 2m_s C_s - 2\sigma_M(2m_j + m_r) - a^2 C_0 \\
& - \frac{1}{8\pi f^2} \left[\sum_\phi A_\phi m_\phi^3 + \sum_{\phi\phi'} A_{\phi\phi'} \mathcal{M}^3(m_\phi, m_{\phi'}) \right] \\
& - \frac{C^2}{8\pi^2 f^2} \left[\sum_\phi B_\phi F(m_\phi, \Delta, \mu) \right. \\
& \left. + \sum_{\phi,\phi'} B_{\phi\phi'} F(m_\phi, m_{\phi'}, \Delta, \mu) \right]. \quad (36)
\end{aligned}$$

The nonanalytic functions appearing in the expression for the octet baryon masses are defined by

$$\begin{aligned}
F(m, \delta, \mu) = & (m^2 - \delta^2) \left(\sqrt{\delta^2 - m^2} \log \frac{\delta - \sqrt{\delta^2 + m^2 + i\epsilon}}{\delta + \sqrt{\delta^2 + m^2 + i\epsilon}} \right. \\
& \left. - \delta \log \frac{m^2}{\mu^2} \right) - \frac{1}{2} \delta m^2 \log \frac{m^2}{\mu^2}, \quad (37)
\end{aligned}$$

TABLE VIII. The coefficients β_ϕ , and β'_ϕ for the Ξ^- .

ϕ	β_ϕ	β'_ϕ
K	$\frac{4}{3}D^2(q_d - q_s)$	$\frac{1}{6}(q_d - q_s)$
ju	$-(D - F)^2(2q_d + q_r)$	$\frac{1}{9}(2q_d + q_r)$
ru	$-(D - F)^2(q_d - q_r)$	$\frac{1}{9}(q_d - q_r)$
js	$-\frac{2}{3}(D^2 + 3F^2)(2q_s + q_r)$	$\frac{1}{18}(2q_s + q_r)$
rs	$-\frac{2}{3}(D^2 + 3F^2)(q_s - q_r)$	$\frac{1}{18}(q_s - q_r)$

TABLE IX. The coefficients β_ϕ , and β'_ϕ for the Λ .

ϕ	β_ϕ	β'_ϕ
K	$\frac{2}{3}D^2(q_u + q_d - 2q_s)$	$\frac{1}{12}(q_u + q_d - 2q_s)$
ju	$-\frac{2}{9}(7D^2 - 12DF + 9F^2)(q_u + q_d + q_r)$	$\frac{1}{6}(q_u + q_d + q_r)$
ru	$-\frac{1}{9}(7D^2 - 12DF + 9F^2)(q_u + q_d - 2q_r)$	$\frac{1}{12}(q_u + q_d - 2q_r)$
js	$-\frac{1}{9}(D^2 + 6DF + 9F^2)(2q_s + q_r)$	0
rs	$-\frac{1}{9}(D^2 + 6DF + 9F^2)(q_s - q_r)$	0

TABLE X. The coefficients β_ϕ , and β'_ϕ for the $\Lambda\Sigma^0$ transition.

ϕ	β_ϕ	β'_ϕ
π	$-\frac{4}{3\sqrt{3}}D^2(q_u - q_d)$	$-\frac{1}{6\sqrt{3}}(q_u - q_d)$
K	$-\frac{2}{3\sqrt{3}}D^2(q_u - q_d)$	$-\frac{1}{12\sqrt{3}}(q_u - q_d)$
ju	$\frac{4}{3\sqrt{3}}(D^2 - 3DF)(q_u - q_d)$	$-\frac{1}{6\sqrt{3}}(q_u - q_d)$
ru	$\frac{2}{3\sqrt{3}}(D^2 - 3DF)(q_u - q_d)$	$-\frac{1}{12\sqrt{3}}(q_u - q_d)$

and

$$\mathcal{M}^3(m_\phi, m_{\phi'}) = \mathcal{H}_{\phi\phi'}(m_\phi^3, m_{\phi'}^3, m_X^3), \quad (38)$$

$$F(m_\phi, m_{\phi'}, \delta, \mu)$$

$$= \mathcal{H}_{\phi\phi'}[F(m_\phi, \delta, \mu), F(m_{\phi'}, \delta, \mu), F(m_X, \delta, \mu)], \quad (39)$$

for the flavor-neutral contributions.

In Table XI, we list the coefficients C_u and C_s for the octet baryons. In Tables XII and XIII, we list the loop coefficients A_ϕ , $A_{\phi,\phi'}$, B_ϕ and $B_{\phi\phi'}$. These coefficients are grouped according to loop mesons with mass m_ϕ , and for flavor-neutral contributions are grouped according to pairs of quark-basis flavor-neutral mesons. Notice the sum on $\phi\phi'$ runs over $\eta_u\eta_u$, $\eta_u\eta_s$, and $\eta_s\eta_s$ to avoid double counting.

The lattice-spacing dependence of the octet masses too is highly constrained. There are only three free parameters: C_0 , which is the same for all octet baryons; C_{mix} , which alternately can be determined from mixed-meson masses; and the combination of parameters $C_3 + C_4$, which is already constrained from staggered meson lattice data [6].

TABLE XI. Tree-level coefficients for the octet baryon masses.

	C_u	C_d
N	$\alpha_M + \beta_M$	0
Σ	$\frac{1}{6}(5\alpha_M + 2\beta_M)$	$\frac{1}{6}(\alpha_M + 4\beta_M)$
Λ	$\frac{1}{2}(\alpha_M + 2\beta_M)$	$\frac{1}{2}\alpha_M$
Ξ	$\frac{1}{6}(\alpha_M + 4\beta_M)$	$\frac{1}{6}(5\alpha_M + 2\beta_M)$

TABLE XII. The coefficients A_ϕ and $A_{\phi\phi'}$ in PQ χ PT. Coefficients are listed for the baryon octet, and for A_ϕ are grouped into contributions from loop mesons with mass m_ϕ , while for $A_{\phi\phi'}$ are grouped into contributions from pairs of quark-basis η_q mesons.

	A_ϕ		$A_{\phi\phi'}$	
	π	K	η_s	
N	$-\frac{4}{3}(D^2 - 3DF)$	0	0	
Σ	$-\frac{2}{3}(D^2 - 3F^2)$	$-\frac{2}{3}(D^2 - 6DF + 3F^2)$	0	
Λ	$-\frac{2}{9}(D^2 - 12DF + 9F^2)$	$-\frac{2}{9}(5D^2 - 6DF - 9F^2)$	0	
Ξ	0	$-\frac{2}{3}(D^2 - 6DF + 3F^2)$	$-\frac{2}{3}(D^2 - 3F^2)$	
	ju	ru	js	rs
N	$\frac{2}{3}(5D^2 - 6DF + 9F^2)$	$\frac{1}{3}(5D^2 - 6DF + 9F^2)$	0	0
Σ	$\frac{4}{3}(D^2 + 3F^2)$	$\frac{2}{3}(D^2 + 3F^2)$	$2(D - F)^2$	$(D - F)^2$
Λ	$\frac{4}{9}(7D^2 - 12DF + 9F^2)$	$\frac{2}{9}(7D^2 - 12DF + 9F^2)$	$\frac{2}{9}(D + 3F)^2$	$\frac{1}{9}(D + 3F)^2$
Ξ	$2(D - F)^2$	$(D - F)^2$	$\frac{4}{3}(D^2 + 3F^2)$	$\frac{2}{3}(D^2 + 3F^2)$
	$\eta_u \eta_u$	$\eta_u \eta_s$	$\eta_s \eta_s$	
N	$(D - 3F)^2$	0	0	
Σ	$4F^2$	$-4(DF - F^2)$	$(D - F)^2$	
Λ	$\frac{4}{9}(2D - 3F)^2$	$-\frac{4}{9}(2D^2 + 3DF - 9F^2)$	$\frac{1}{9}(D + 3F)^2$	
Ξ	$(D - F)^2$	$-4(DF - F^2)$	$4F^2$	

TABLE XIII. The coefficients B_ϕ and $B_{\phi\phi'}$ in PQ χ PT. Coefficients are listed for the octet baryons, and for B_ϕ are grouped into contributions from loop mesons with mass m_ϕ , while for $B_{\phi\phi'}$ are grouped into contributions from pairs of quark-basis η_q mesons.

	B_ϕ							$B_{\phi\phi'}$		
	π	K	η_s	ju	ru	js	rs	$\eta_u \eta_u$	$\eta_u \eta_s$	$\eta_s \eta_s$
N	$\frac{2}{3}$	0	0	$\frac{2}{3}$	$\frac{1}{3}$	0	0	0	0	0
Σ	$\frac{1}{9}$	$\frac{5}{9}$	0	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$-\frac{4}{9}$	$\frac{2}{9}$
Λ	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{1}{3}$	0	0	0	0	0
Ξ	0	$\frac{5}{9}$	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$-\frac{4}{9}$	$\frac{2}{9}$

V. SUMMARY

Above we have included baryons into mixed-action partially quenched chiral perturbation theory for Ginsparg-Wilson valence quarks and staggered sea quarks. Working at next-to-leading order, we determined the lattice-spacing artifacts for the octet baryon magnetic moments and masses. The recipe for adding finite volume corrections is discussed in the Appendix.

To $\mathcal{O}(\varepsilon^3)$, baryon masses depend on the lattice spacing via three parameters. The first is C_0 , which is a representative coefficient of local a^2 operators in the baryon Lagrangian. The contribution from C_0 is the same for all members of the baryon octet. The second parameter is C_{mix} , which affects the masses of mesons made from a Ginsparg-Wilson quark and a staggered quark. The third parameter is a combination of low-energy constants, $C_3 + C_4$, which governs the mass of taste-singlet staggered mesons. To $\mathcal{O}(\varepsilon)$, baryon magnetic moments only depend

on the lattice spacing through loop-meson masses. Furthermore there is only one free parameter involved at next-to-leading order, C_{mix} .

A simplifying feature of the mixed-action baryon theory is that at next-to-leading order, there is no taste-symmetry violation. Beyond next-to-leading order, however, this is no longer the case. Taste-symmetry breaking interactions start at $\mathcal{O}(\varepsilon^4)$ in our power counting. They arise from two sources. Contributions that scale as $\mathcal{O}(m_q^2 \log m_q)$ can lead to taste-symmetry violation. Expanding out the sigma terms in the baryon Lagrangian (23) leads to contributions to the baryon masses that scale as $m_q^2 \log m_q$, and involve completely disconnected meson loops. In PQ χ PT these loops involve only mesons formed from two sea quarks [76–78]. For staggered sea quarks this becomes a sum over the various meson tastes (vector, axial-vector, ...) and no longer is there a cancellation of factors of 1/4 inserted from the fourth-root trick. Explicit taste-symmetry violation occurs from operators that scale as $\mathcal{O}(a^2 m_q \log m_q)$.

These arise from the loop diagrams generated by terms in \mathcal{V}_B (for which one would need to decompose C_0 into various contributing terms). Terms in \mathcal{V}_B containing taste spurions will generate taste-symmetry violation at the one-loop level. These are the only possible sources for taste-symmetry violation at next-to-next-to-leading order. The remainder of contributions to the baryon masses and other observables arise from valence-valence or valence-sea mesons in the absence of explicit taste matrices. These do not violate taste symmetry as we demonstrated above.

Nonetheless the issue of addressing lattice-spacing corrections to baryon observables calculated in mixed-action lattice QCD is very tractable. The chiral symmetry properties of Ginsparg-Wilson valence quarks effectively suppress the taste-symmetry violation from the staggered sea. While taste-symmetry violation does occur at next-to-next-to-leading order, expressions at leading order are taste symmetric and involve only a few new parameters. Mixed-action simulations in the baryon sector are a timely way of getting physical observables from lattice QCD.

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APPENDIX: FINITE VOLUME CORRECTIONS

In this Appendix, we assemble the relevant formulas for finite volume corrections. This is a trivial extension of Ref. [79], and is included for completeness. Let L denote the size of the cubic box in one spatial direction. We assume the lattice simulations are carried out in a region of parameter space where chiral physics lives inside the box, i.e. for $fL \gg 1$, and further that we are in the p regime of chiral perturbation theory, $m_\pi L \gtrsim 1$. In this case, the Poisson formula can be used to cast mode sums from loop diagrams into the infinite volume results plus finite volume modifications. We list these modifications for the octet baryon masses and magnetic moments. For finite

volume corrections in mixed-action simulations, we imagine that the Ginsparg-Wilson valence quarks will be the lightest. Thus modulo possible cancellations from additive a^2 mass shifts for the valence-sea and sea-sea mesons, the valence-valence pions should dominate the finite volume corrections.

The finite volume corrections to the magnetic moments have the form [79]

$$\delta_L \mu = -\frac{M_B}{6\pi^2 f^2} \sum_\phi [\beta_\phi \mathcal{Y}(m_\phi, 0) + \beta'_\phi C^2 \mathcal{Y}(m_\phi, \Delta)], \quad (\text{A1})$$

where

$$\mathcal{Y}(m, \Delta) = \int_0^\infty d\lambda \sum_{n=0}^\infty [3K_0(\beta_\Delta |n|L) - \beta_\Delta |n|L K_1(\beta_\Delta |n|L)], \quad (\text{A2})$$

with $\beta_\Delta^2 = m^2 + 2\Delta\lambda + \lambda^2$ and the $K_n(x)$ are modified Bessel functions. The coefficients β_ϕ and β'_ϕ are listed for the octet baryons in Tables II, III, IV, V, VI, VII, VIII, and IX. The coefficients for the $\Lambda\Sigma^0$ transition moment appear in Table X.

The finite volume corrections to the masses have the form [79]

$$\begin{aligned} \delta_L M_B = & -\frac{1}{4\pi f^2} \left[\sum_\phi A_\phi \mathcal{K}(m_\phi, 0) \right. \\ & \left. + \sum_{\phi\phi'} A_{\phi\phi'} \mathcal{K}(m_\phi, m_{\phi'}, 0) \right] - \frac{C^2}{4\pi^2 f^2} \\ & \times \left[\sum_\phi B_\phi \mathcal{K}(m_\phi, \Delta) \right. \\ & \left. + \sum_{\phi\phi'} B_{\phi\phi'} \mathcal{K}(m_\phi, m_{\phi'}, \Delta) \right], \quad (\text{A3}) \end{aligned}$$

where

$$\mathcal{K}(m, \Delta) = \int_0^\infty d\lambda \beta_\Delta^2 \sum_{n=0}^\infty \left[\frac{K_1(\beta_\Delta |n|L)}{\beta_\Delta |n|L} - K_0(\beta_\Delta |n|L) \right], \quad (\text{A4})$$

and

$$\begin{aligned} \mathcal{K}(m_\phi, m_{\phi'}, \Delta) \\ = \mathcal{H}_{\phi\phi'} [\mathcal{K}(m_\phi, \Delta), \mathcal{K}(m_{\phi'}, \Delta), \mathcal{K}(m_x, \Delta)]. \quad (\text{A5}) \end{aligned}$$

The coefficients A_ϕ and $A_{\phi\phi'}$ are listed for the octet baryons in Table XII, while B_ϕ and $B_{\phi\phi'}$ appear in Table XIII.

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