PHYSICAL REVIEW D **72,** 094031 (2005)

Symmetry relations in charmless $B \to PPP$ decays

Michael Gronau

Department of Physics, Technion–Israel Institute of Technology, Technion City, 32000 Haifa, Israel

Jonathan L. Rosner

Enrico Fermi Institute and Department of Physics, University of Chicago, 5640 South Ellis Avenue, Chicago, Illinois 60637, USA

(Received 28 September 2005; published 30 November 2005)

Strangeness-changing decays of B mesons to three-body final states of pions and kaons are studied, assuming that they are dominated by a $\Delta I = 0$ penguin amplitude with flavor structure $\bar{b} \to \bar{s}$. Numerous isospin relations for $B \to K\pi\pi$ and for underlying quasi-two-body decays are compared successfully with experiment, in some cases resolving ambiguities in fitting resonance parameters. The only exception is a somewhat small branching ratio noted in $B^0 \to K^{*0}\pi^0$, interpreted in terms of destructive interference between a penguin amplitude and an enhanced electroweak penguin contribution. Relations for B decays into three kaons are derived in terms of final states involving K_S or K_L, assuming that ϕK-subtracted decay amplitudes are symmetric in K and $\bar{K}$, as has been observed experimentally. Rates due to nonresonant backgrounds are studied using a simple model, which may reduce discrete ambiguities in Dalitz plot analyses.

DOI: 10.1103/PhysRevD.72.094031 PACS numbers: 13.25.Hw, 11.30.Cp, 12.15.Ji, 14.40.Nd

I. INTRODUCTION

The decays of B mesons to charmless three-body final states provide valuable information about the pattern of CP violation, as in the time-dependent studies of CP asymmetries in decays to CP eigenstates consisting of three neutral pseudoscalars [1]. Data for $B^0 \to K^+K^-K_S$ of comparable statistical weight have been presented by the *BABAR* [2–4] and Belle [5–7] collaborations. In analyzing these data it is of interest to know the CP eigenvalue of the three-body final state which depends on the K^+K^- angular momentum. In Ref. [6] isospin symmetry was utilized to relate the decays $B^+ \to K^+K^0\bar{K}^0$ (measured via $B^+ \to K^+K_SK_S$) and $B^0 \to K^+K^-K^0$ in order to conclude that the K^+K^- final state was dominated by even angular momenta.

The question of genuine three-body decays of the B meson (in contrast to quasi-two-body decays which involve resonances between two of the three bodies) arises in part because of the need to parametrize CP violation in $B \to K\bar{K}K$ and to understand nonresonant contributions arising in Dalitz plot studies. These contributions can be quite large, as measured in Dalitz plot analyses of $B^+ \to K^+K^-K^+$ [8] and $B^0 \to K^+K^-K_S$ [4]. They seem to be less significant in comparison with quasi-two-body final states in certain $B \to K\pi\pi$ decays [9,10].

In the present paper we discuss conclusions that can be drawn regarding the structure of the three-body final states for $B \to PPP$ strangeness-changing decays, where P denotes a light pseudoscalar meson. We begin by noting some relations due to isospin in the limit in which decays are dominated by a QCD-penguin amplitude with isospin-preserving flavor structure. Smaller $\Delta I = 1$ tree and electroweak penguin amplitudes will be neglected. We also analyze amplitudes for a nonresonant background using isospin symmetry and flavor SU(3). The description of B decays to a pair of charmless mesons in terms of flavor SU(3) amplitudes [11,12] has been able to correlate decay rates and CP-violating asymmetries for a wide variety of processes involving two light pseudoscalars P [13] or one pseudoscalar and one vector (V) meson [14].

We restrict our treatment for the moment in several respects. (1) We consider only $|\Delta S| = 1$ transitions and assume them to be dominated by a penguin amplitude with flavor structure $\bar{b} \to \bar{s}$. (2) We consider only final states involving pions and kaons, in order not to have to contend with octet-singlet mixing questions or possible additional flavor-singlet penguin amplitudes [15]. (3) We do not consider B_s mesons, since information on them has lagged considerably behind that on B^+ and B^0.

Earlier treatments of $B \to PPP$ decays, including model-dependent hadronic calculations of decay rates and CP asymmetries, may be found in Refs. [16–20]. Several analyses using flavor SU(3) have been performed in Refs. [21,22] in order to obtain model-independent bounds on deviations from the dominance of a single weak phase in B decays to three kaons.

Section II reviews what is known about $B \to K\pi\pi$ and $B \to K\bar{K}K$ decay rates, pointing out certain features of resonant and nonresonant contributions. Numerous isospin relations are proven for $B \to K\pi\pi$ and for corresponding quasi-two-body decays and are tested in Sec. III. Similar relations hold for $B \to K\bar{K}K$. Assuming symmetry under the interchange of K and $\bar{K}$ momenta, as observed in the data, we prove decay rate relations for processes involving K_S and K_L. Section IV compares nonresonant background amplitudes in $B \to K\pi\pi$ and $B \to K\bar{K}K$ processes using isospin symmetry and flavor SU(3) in a simple universal model. Implications for Dalitz plot analyses are noted in Sec. V, while Sec. VI summarizes, concluding with a few

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remarks about isospin-violating corrections and direct *CP* asymmetries.

II. EXPERIMENTAL STATUS

The current world averages of *CP*-averaged branching ratios from *BABAR*, Belle, and CLEO for the decays $B \to K\pi\pi$ and $B \to K\bar{K}K$ are summarized in Table I [10]. Averages involving $B^0 \to K_S\pi^+\pi^-$ and its submodes include recent Belle results [23]. Also listed are branching ratios for quasi-two-body decays for several resonances contributing to these decays. While no measurement exists so far for the branching ratio of $B^0 \to K^0\pi^0\pi^0$, a time-dependent *CP* asymmetry has been reported recently in this process [24].

The branching ratios quoted in Table I for $B^{+,0} \to K^{+,0}f_0(980)$ and $B^0 \to K_0^*(1430)^0\pi^0$ include decay branching ratios of the daughter scalar mesons into observed modes. Using $\mathcal{B}(K_0^*(1430)^0 \to K^+\pi^-) = 2/3$ we obtain

$$\mathcal{B}(B^0 \to K_0^*(1430)^0\pi^0) = 11.9 \pm 4.7, \qquad (1)$$

where branching ratios here and subsequently are quoted in units of 10^{-6}.

Dalitz plot analyses of $B^0 \to K^+K^-K^0$ [4] and $B^+ \to K^+K^-K^+$ [8] find large nonresonant contributions in these decays. In addition to the ϕK mode, where the K^+ and K^- are in a P wave, two sizable and comparable contributions have been measured: A term peaking around 1500 MeV/c^2, for which one finds in addition to a large solution also a small solution [4,8] (see Table I in Ref. [4] and Table V in Ref. [8]), and a term spreading across phase space. Both terms have a S-wave behavior in the K^+ and K^- momenta [4,8]. Contributions from higher waves were found consistent with zero. The decays $B^{+,0} \to \chi_{c0}K^{+,0}$, also having an S-wave behavior, contribute about 3% of the total branching ratios of $B^{+,0} \to K^+K^-K^{+,0}$ [4,8]. Reference [8] finds a second solution of about 8% for the fraction corresponding to $B^+ \to \chi_{c0}K^+$ (see Table V in [8]). All the above three S-wave contributions are symmetric under interchanging K^+ and K^-.

It is useful to subtract contributions for $B \to \phi K$ from the branching ratios of $B^+ \to K^+K^-K^+$ and $B^0 \to K^+K^-K^0$. Using values in Table I and [25] $\mathcal{B}(\phi \to K^+K^-) = (49.1 \pm 0.6)\%$, one finds

$$\mathcal{B}(B^+ \to K^+K^-K^+)_{\phi K-\text{subtracted}} = 25.7 \pm 1.9, \qquad (2)$$

$$\mathcal{B}(B^0 \to K^+K^-K^0)_{\phi K-\text{subtracted}} = 20.6 \pm 2.4. \qquad (3)$$

An important feature of the amplitudes corresponding to these branching ratios is their symmetry with respect to interchanging K^+ and K^- momenta, as they are superpositions of three S-wave contributions [4,8].

III. ISOSPIN RELATIONS

We assume that the dominant transition for $|\Delta S| = 1$ $B \to PPP$ decays has a flavor structure $\bar{b} \to \bar{s}$, which is isospin invariant ($\Delta I = 0$). Using isospin reflection symmetry under $u \leftrightarrow d$, we then find that each B^+ decay amplitude listed in Table I is equal (up to a possible sign) to a corresponding B^0 decay amplitude listed in the same

TABLE I. Summary of *CP*-averaged branching ratios, in units of 10^{-6}, for $B \to K\pi\pi$ and $B \to K\bar{K}K$ including quasi-two-body decays [10,23]. Pairs of processes in the same row are related by isospin reflection, except for $B^+ \to \phi K^+$ and $B^0 \to \phi K^0$ which are in different rows.

Final state in B^+ decay	Branching ratio (units of 10^{-6})	Final state in B^0 decay	Branching ratio (units of 10^{-6})
$K^+\pi^+\pi^-$	54.1 ± 3.1	$K^0\pi^+\pi^-$	44.9 ± 2.6
$K^{*0}\pi^+$	10.8 ± 0.8	$K^{*+}\pi^-$	9.8 ± 1.1
$K^+\rho^0$	$4.23^{+0.56}_{-0.57}$	$K^0\rho^0$	5.6 ± 1.1
$K^+f_0(980)$	$9.1^{+0.8}_{-1.1}$[a]	$K^0f_0(980)$	6.0 ± 0.9[a]
$K_0^*(1430)^0\pi^+$	$38.2^{+4.6}_{-4.5}$	$K_0^*(1430)^+\pi^-$	45.1 ± 6.1
$K^0\pi^+\pi^0$	<66	$K^+\pi^-\pi^0$	$35.6^{+3.4}_{-3.3}$
$K^{*+}\pi^0$	6.9 ± 2.3	$K^{*0}\pi^0$	1.7 ± 0.8
$K^0\rho^+$	<48	$K^+\rho^-$	$9.9^{+1.6}_{-1.5}$
$K_0^*(1430)^+\pi^0$	$\cdots$	$K_0^*(1430)^0\pi^0$	7.9 ± 3.1[b]
$K^+\pi^0\pi^0$	$\cdots$	$K^0\pi^0\pi^0$	$\cdots$
$K_SK_SK^+$	11.5 ± 1.3	$K^+K^-K^0$	24.7 ± 2.3
		ϕK^0	$8.3^{+1.2}_{-1.0}$
$K^+K^-K^+$	30.1 ± 1.9	$K_SK_SK_S$	6.2 ± 0.9
ϕK^+	$9.03^{+0.65}_{-0.63}$		

[a]Includes $\mathcal{B}(f_0(980) \to \pi^+\pi^-)$.
[b]Includes $\mathcal{B}(K_0^*(1430)^0 \to K^+\pi^-)$.

line. Other amplitude relations follow from our assumption that the final state is dominantly $I = 1/2$. In order to relate predictions of equal B^+ and B^0 partial widths to observed branching ratios, we use the measured ratio of B^+ and B^0 lifetimes, $\tau_+/\tau_0 = 1.076 \pm 0.008$ [10].

Relations between observed branching ratios for $B \to K\pi\pi$ and corresponding quasi-two-body decays follow directly from the above assumption. Similar amplitude relations hold for $B \to K\bar{K}K$. However, in order to rewrite these relations for decay rates involving K_S and K_L in the final state one must assume a given symmetry under interchanging K and $\bar{K}$ momenta. We will use the symmetry under $K^+ \leftrightarrow K^-$ of the amplitudes describing the ϕK-subtracted branching ratios (2) and (3).

A. $B \to K\pi\pi$

A relation which is well satisfied is

$$\mathcal{B}(B^+ \to K^+\pi^+\pi^-) = (\tau_+/\tau_0)\mathcal{B}(B^0 \to K^0\pi^+\pi^-); \tag{4}$$

$$54.1 \pm 3.1 = 48.3 \pm 2.8.$$

The discrepancy is only 1.4σ.

The above isospin relation should apply to corresponding quasi-two-body modes contributing to these decays. Thus, the following four relations hold reasonably well:

$$\mathcal{B}(B^+ \to K^{*0}\pi^+) = (\tau_+/\tau_0)\mathcal{B}(B^0 \to K^{*+}\pi^-); \tag{5}$$

$$10.8 \pm 0.8 = 10.6 \pm 1.2,$$

$$\mathcal{B}(B^+ \to K^+\rho^0) = (\tau_+/\tau_0)\mathcal{B}(B^0 \to K^0\rho^0); \tag{6}$$

$$4.23\,^{+0.56}_{-0.57} = 6.1 \pm 1.2,$$

$$\mathcal{B}(B^+ \to K^+f_0(980))\mathcal{B}(f_0 \to \pi^+\pi^-) = (\tau_+/\tau_0) \times\mathcal{B}(B^0 \to K^0f_0(980))\mathcal{B}(f_0 \to \pi^+\pi^-); \tag{7}$$

$$9.07\,^{+0.81}_{-1.06} = 6.4 \pm 0.9,$$

$$\mathcal{B}(B^+ \to K_0^*(1430)^0\pi^+) = (\tau_+/\tau_0) \times\mathcal{B}(B^0 \to K_0^*(1430)^+\pi^-); \tag{8}$$

$$38.2\,^{+4.6}_{-4.5} = 48.6 \pm 6.6.$$

The last relation disfavors a second solution, $\mathcal{B}(B^+ \to K_0^*(1430)^0\pi^+) = 8.7 \pm 2.3$ measured in [8] (see Table IV there) and is in agreement with a more recent measurement (see Table V in [26]), $\mathcal{B}(B^+ \to K_0^*(1430)^0\pi^+) = 51.6 \pm 1.7 \pm 6.8^{+1.8}_{-3.1}$.

A prediction satisfied only by an upper bound is

$$\mathcal{B}(B^+ \to K^0\pi^+\pi^0) = (\tau_+/\tau_0)\mathcal{B}(B^0 \to K^+\pi^-\pi^0); \tag{9}$$

$$<66 = 38.3^{+3.7}_{-3.6}.$$

It should not be too difficult to obtain a value for the left-hand side; a π^0 must be added to the observed final state $B^+ \to K^0\pi^+$. Corresponding predictions apply to quasi-two-body decays. The prediction

$$\mathcal{B}(B^0 \to K^{*0}\pi^0) = (\tau_0/\tau_+)\mathcal{B}(B^+ \to K^{*+}\pi^0); \tag{10}$$

$$1.7 \pm 0.8 = 6.4 \pm 2.1$$

requires more data for a statistically significant test.

Dominance of $I = 1/2$ in $K^*\pi$ final states implies

$$2\mathcal{B}(B^0 \to K^{*0}\pi^0) = \mathcal{B}(B^0 \to K^{*+}\pi^-); \tag{11}$$

$$3.4 \pm 1.6 = 9.8 \pm 1.1,$$

which is violated by 3.3σ. The smallness of the left-hand side may be due to its sensitivity to small $\Delta I = 1$ contributions, which are present in the treatment of Ref. [14]. The small branching ratio measured for $B^0 \to K^{*0}\pi^0$ is evidently the origin of the apparent discrepancies in Eqs. (10) and (11).

The prediction

$$\mathcal{B}(B^+ \to K^0\rho^+) = (\tau_+/\tau_0)\mathcal{B}(B^0 \to K^+\rho^-); \tag{12}$$

$$<48 = 10.7^{+1.7}_{-1.6}$$

is satisfied by the upper bound, while $I(K\rho) = 1/2$ implies

$$2\mathcal{B}(B^0 \to K^0\rho^0) = \mathcal{B}(B^0 \to K^+\rho^-); \tag{13}$$

$$11.3 \pm 2.2 = 9.9^{+1.6}_{-1.5}.$$

Similarly,

$$\mathcal{B}(B^+ \to K_0^*(1430)^+\pi^0) = (\tau_+/\tau_0) \times\mathcal{B}(B^0 \to K_0^*(1430)^0\pi^0) = 12.8 \pm 5.0 \tag{14}$$

awaits a measurement of the left-hand-side, while $I(K_0^*(1430)\pi) = 1/2$ implies

$$2\mathcal{B}(B^0 \to K_0^*(1430)^0\pi^0) = \mathcal{B}(B^0 \to K_0^*(1430)^+\pi^-); \tag{15}$$

$$23.7 \pm 9.3 = 45.1 \pm 6.1.$$

Finally, neither the left-hand nor right-hand side of the following prediction corresponds to a current observation:

$$\mathcal{B}(B^+ \to K^+\pi^0\pi^0) = (\tau_+/\tau_0)\mathcal{B}(B^0 \to K^0\pi^0\pi^0). \tag{16}$$

B. $B \to K\bar{K}K$

Isospin reflection symmetry implies

$$A(B^+ \to K^0\overline{K}^0K^+) = -A(B^0 \to K^+K^-K^0), \tag{17}$$

$$A(B^+ \to K^+K^-K^+) = -A(B^0 \to K^0\overline{K}^0K^0). \quad (18)$$

In order to study ϕK-subtracted amplitudes, we will use their observed symmetry under interchanging the K and $\bar{K}$ momenta mentioned at the end of Sec. II [4,8]. This permits writing relations for rates involving K_S and K_L in the final state. Note that because of Bose symmetry the amplitudes in (18), which involve two identical K mesons, are also symmetric in the two K momenta.

Using the phase convention (we neglect a tiny CP violation in K^0–$\overline{K}^0$ mixing),

$$K_S \equiv (K^0 + \overline{K}^0)/\sqrt{2}, \qquad K_L \equiv (K^0 - \overline{K}^0)/\sqrt{2}, \quad (19)$$

a symmetric state

$$|K^0\overline{K}^0\rangle_{\rm sym} \equiv [|K^0(p_1)\overline{K}^0(p_2)\rangle + |\overline{K}^0(p_1)K^0(p_2)\rangle]/\sqrt{2}, \quad (20)$$

can be expressed as

$$|K^0\overline{K}^0\rangle_{\rm sym} = (|K_S(p_1)K_S(p_2)\rangle - |K_L(p_1)K_L(p_2)\rangle)/\sqrt{2}, \quad (21)$$

while an antisymmetric state is given by

$$\begin{aligned}|K^0\overline{K}^0\rangle_{\rm anti} &\equiv [|K^0(p_1)\overline{K}^0(p_2)\rangle - |\overline{K}^0(p_1)K^0(p_2)\rangle]\sqrt{2} \\ &= [|K_L(p_1)K_S(p_2)\rangle - |K_S(p_1)K_L(p_2)\rangle]/\sqrt{2}.\end{aligned} \quad (22)$$

Similarly, a state symmetric in the three momenta, p_1, p_2, p_3, is given by

$$\begin{aligned}|K^0\overline{K}^0K^0\rangle_{\rm sym} &\equiv [|K^0K^0\overline{K}^0\rangle + |K^0\overline{K}^0K^0\rangle \\ &\quad + |\overline{K}^0K^0K^0\rangle]/\sqrt{3} \\ &= \frac{1}{2\sqrt{2}}[\sqrt{3}|K_SK_SK_S\rangle + |K_SK_SK_L\rangle_{\rm sym} \\ &\quad - |K_LK_LK_S\rangle_{\rm sym} - \sqrt{3}|K_LK_LK_L\rangle],\end{aligned} \quad (23)$$

where dependence on the three momenta has been suppressed.

Using Eq. (21), we find

$$\begin{aligned}\mathcal{B}(B^+ \to (K^0\overline{K}^0)_{\rm sym}K^+) &= \mathcal{B}(B^+ \to K_SK_SK^+) \\ &\quad + \mathcal{B}(B^+ \to K_LK_LK^+) \\ &= 2\mathcal{B}(B^+ \to K_SK_SK^+) \\ &= 23.0 \pm 2.6.\end{aligned} \quad (24)$$

Equation (17) is well satisfied for the ϕK-subtracted branching ratio

$$\begin{aligned}&\mathcal{B}(B^+ \to (K^0\overline{K}^0)_{\rm sym}K^+) = (\tau_+/\tau_0) \\ &\times \mathcal{B}(B^0 \to K^+K^-K^0)_{\phi K-\rm subtracted};\end{aligned} \quad (25)$$

$$23.0 \pm 2.6 = 22.2 \pm 2.6.$$

Finally, in order to test the prediction (18) we apply (23)

$$\begin{aligned}\mathcal{B}(B^0 \to (K^0\overline{K}^0K^0)_{\rm sym}) &= \tfrac{8}{3}\mathcal{B}(B^0 \to K_SK_SK_S) \\ &= 16.5 \pm 2.4.\end{aligned} \quad (26)$$

Equation (18) then reads

$$\begin{aligned}&\mathcal{B}(B^+ \to K^+K^-K^+)_{\phi K-\rm subtracted} = (\tau_+/\tau_0) \\ &\times \mathcal{B}(B^0 \to (K^0\overline{K}^0K^0)_{\rm sym});\end{aligned} \quad (27)$$

$$25.7 \pm 1.9 = 17.8 \pm 2.6,$$

which holds within 2.4σ. A potential discrepancy may be accounted for by small $\Delta I = 1$ amplitudes.

Equation (23) also implies predictions for branching ratios involving K_L in the final state:

$$\begin{aligned}\mathcal{B}(B^0 \to K_LK_LK_L) &= \mathcal{B}(B^0 \to K_SK_SK_S) \\ &= 3\mathcal{B}(B^0 \to K_LK_LK_S) \\ &= 3\mathcal{B}(B^0 \to K_SK_SK_L),\end{aligned} \quad (28)$$

where subtraction of ϕK contributions in the last two processes is implied. We expect $\mathcal{B}(B^0 \to K_SK_SK_L)$ to be easier to measure in comparison with $\mathcal{B}(B^0 \to K_LK_LK_S)$ and $\mathcal{B}(B^0 \to K_LK_LK_L)$.

The agreement in (25) and (27) supports the initial suggestion [6] that the K^+ and K^- in the respective processes are in dominantly symmetric even angular momentum (S-wave) states, as confirmed directly by measuring angular dependence in later experiments performing full Dalitz plot analyses [4,8]. A statistically significant discrepancy in Eq. (27), implied by reduced experimental errors, would provide evidence for nonzero contributions either from odd angular momentum states or from a $\Delta I =$ 1 amplitude.

IV. MODEL FOR A NONRESONANT BACKGROUND

The measured ϕK-subtracted rates for $B^+ \to K^+K^-K^+$ [8] and $B^0 \to K^+K^-K^0$ [4] consist each of a sum of three contributions, all symmetric in the K^+ and K^- momenta: a small $\chi_{c0}K$ term, an S-wave contribution peaking around 1500 MeV$/c^2$, and a nonresonant background amplitude also representing an S wave in K^+K^-. The latter amplitude shows no significant dependence on the K^+K^+ and K^+K^0 invariant masses in the two processes. Some dependence on the K^+K^- invariant mass is observed in $B^0 \to K^+K^-K^0$ but not in $B^+ \to K^+K^-K^+$. In a similar analysis of the nonresonant background in $B^+ \to K^+\pi^+\pi^-$ some dependence was measured on the invariant masses of $K^+\pi^-$ and of $\pi^+\pi^-$.

In the present section we will study nonresonant background amplitudes in $B \to K\pi\pi$ and $B \to K\bar{K}K$ decays, adopting a simplified assumption that these amplitudes are symmetric under interchanging the three final meson momenta. This would be the case, for instance if the nonresonant amplitudes were constant over the Dalitz plane; however, these amplitudes do not have to be constant. We start by first presenting the data and then discussing symmetry relations governing nonresonant contributions.

Table II quotes measured fractions of nonresonant background (NRB) contributions in $B \to K\pi\pi$ and $B \to K\bar{K}K$ processes. While a small NRB contribution in $B^+ \to K^+\pi^+\pi^-$, $\mathcal{B}(\mathrm{NRB}) = (2.9^{+1.1}_{-0.9}) \times 10^{-6}$, has been measured in [9] and is quoted in [10], we quote in the table a larger nonresonant fraction ($\sim 1/3$) which has been measured recently by Belle [26] (see also Ref. [8]). As we will see, a large nonresonant background in $B^+ \to K^+\pi^+\pi^-$ appears to be more consistent in our scheme with comparable large nonresonant contributions measured in $B^+ \to K^+K^-K^+$ [8] and $B^0 \to K^+K^-K^0$ [4]. Two possible solutions for the fraction of a nonresonant background were obtained in the first process, $(74.8 \pm 3.6)\%$ and $(65.1 \pm 5.1)\%$. We quote the former value, which corresponds to a fit with lower χ^2 (see Table V of Ref. [8]). These fractions and the total branching ratios given in Table I were used to calculate the nonresonant branching ratios.

A model describing nonresonant background amplitudes in $b \to s$ dominated $B \to PPP$ decays is shown in Fig. 1. The amplitudes may be categorized by whether the quark pairs $q_i\bar{q}_i (i = 1, 2)$ shown in Fig. 1 are $u\bar{u}$, $d\bar{d}$, or $s\bar{s}$. Isospin symmetry is implied by associating equal amplitudes with $u\bar{u}$ and $d\bar{d}$. This symmetry assumption may be extended to flavor SU(3) by associating the same amplitude with $s\bar{s}$. Broken SU(3) may be represented by using a smaller amplitude for an $s\bar{s}$ pair.

Table III gives the contributions to the various processes in terms of their coefficients. We use conventions for states defined in Refs. [11,12]. Quark model assignments include $B^+ = u\bar{b}$, $B^0 = d\bar{b}$, with states containing a $\bar{u}$ quark defined with a minus sign for convenience in isospin calculations. Thus, a neutral pion is $\pi^0 = (d\bar{d} - u\bar{u})\sqrt{2}$. The

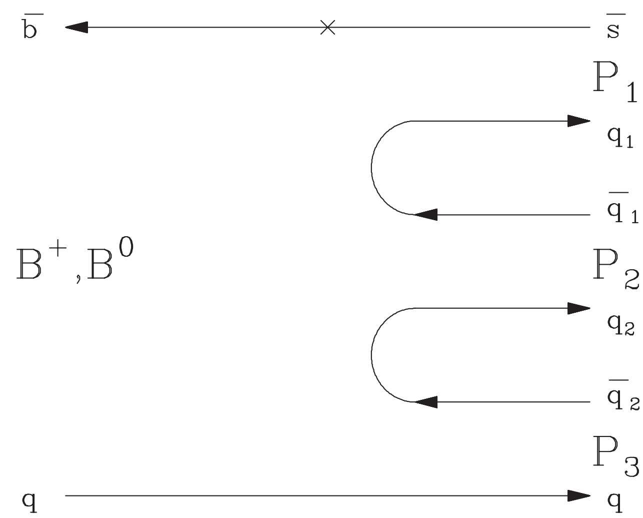

FIG. 1. Graphs describing nonresonant background in penguin-dominated $B \to P_1P_2P_3$ decays. The cross denotes a $\bar{b} \to \bar{s}$ flavor transition. Gluons or quarks associated with the penguin operator are not shown explicitly. Here q denotes u for a B^+ or d for a B^0.

entries in Table III contain factors of $2 \cdot 1/\sqrt{2} = \sqrt{2}$ for identical particles.

The coefficients in Table III imply symmetry relations between decay rates contributed by a nonresonant background in different processes. For instance, the nonresonant branching ratio in $B^+ \to K^+\pi^0\pi^0$ is predicted to be half of that measured in $B^+ \to K^+\pi^+\pi^-$. Relations applying separately to $B \to K\pi\pi$ and $B \to K\bar{K}K$ decays follow from isospin symmetry and are generally expected to hold in our model more precisely than relations between these two types of processes which assume flavor SU(3). Let us discuss some of these relations which can be tested using current measurements.

An interesting prediction follows from the two equal and opposite amplitudes present in $B^+ \to K^0\pi^+\pi^0$. When added together, the two contributions cancel. This is a key test of the S-wave nature (or any even angular momentum) of the $\pi\pi$ system for the nonresonant amplitude. An S-wave $\pi\pi$ system with charge ± 1 must be in a state of

TABLE II. Branching ratios of NRB contributions for $B \to K\pi\pi$ and $B \to K\bar{K}K$, given as fractions of total branching ratios and in units of 10^{-6}.

Decay mode	NRB fraction (%)	NRB branching ratio (units of 10^{-6})
$B^+ \to K^+\pi^-\pi^+$	$34.0 \pm 2.2^{+2.1}_{-1.8}$ [26]	18.4 ± 1.9[a]
$B^0 \to K^+\pi^-\pi^0$		<4.6 [10]
$B^+ \to K^+K^-K^+$	74.8 ± 3.6[b] [8]	22.5 ± 1.8
$B^0 \to K^+K^-K^0$	$70.7 \pm 3.8 \pm 1.7$ [4]	17.5 ± 1.9

[a]A much smaller nonresonant branching ratio, $(2.9^{+1.1}_{-0.9}) \times 10^{-6}$, is quoted in [10].
[b]A second solution, $(65.1 \pm 5.1)\%$, is obtained in [8].

TABLE III. Nonresonant background amplitudes for $B \to PPP$ decays as a function of quark pairs $q_1\bar{q}_1$ and $q_2\bar{q}_2$.

Decaying B	$q_1\bar{q}_1$	$q_2\bar{q}_2$	Final state	Coefficient of amplitude
B^+	$u\bar{u}$	$u\bar{u}$	$K^+\pi^0\pi^0$	$1/\sqrt{2}$
$= \bar{b}u$		$d\bar{d}$	$K^+\pi^-\pi^+$	-1
		$s\bar{s}$	$K^+K^-K^+$	$-\sqrt{2}$
	$d\bar{d}$	$u\bar{u}$	$K^0\pi^+\pi^0$	$-1/\sqrt{2}$
		$d\bar{d}$	$K^0\pi^0\pi^+$	$1/\sqrt{2}$
		$s\bar{s}$	$K^0\bar{K}^0K^+$	1
B^0	$u\bar{u}$	$u\bar{u}$	$K^+\pi^0\pi^-$	$1/\sqrt{2}$
$= \bar{b}d$		$d\bar{d}$	$K^+\pi^-\pi^0$	$-1/\sqrt{2}$
		$s\bar{s}$	$K^+K^-K^0$	-1
	$d\bar{d}$	$u\bar{u}$	$K^0\pi^+\pi^-$	-1
		$d\bar{d}$	$K^0\pi^0\pi^0$	$1/\sqrt{2}$
		$s\bar{s}$	$K^0\bar{K}^0K^0$	$\sqrt{2}$

$I = 2$, which cannot be reached with the penguin transition illustrated here. Thus the nonresonant contributions to $B^+ \to K^0\pi^+\pi^0$ and $B^0 \to K^+\pi^-\pi^0$ are predicted to vanish if our assumptions are valid. The current upper bound of 4.6×10^{-6} on the nonresonant branching ratio of the second process is indeed much smaller than the other nonresonant branching ratios quoted in Table II.

Table III predicts that the nonresonant decay width for $B^+ \to K^+K^-K^+$ is 2 times larger than that for $B^0 \to K^+K^-K^0$. This relation does not hold so well (we will comment on a probable reason in the next section),

$$\mathcal{B}(B^+ \to K^+K^-K^+)_{\rm NRB} = 2(\tau_+/\tau_0) \times\mathcal{B}(B^0 \to K^+K^-K^0)_{\rm NRB}; \tag{29}$$

$$22.5 \pm 1.8 = 37.7 \pm 4.1,$$

where the NRB branching ratios here and subsequently are taken from Table II. This relation tests the assumption that on the right-hand side the K^+ and K^0 in the nonresonant background are in a symmetric $I = 1$ state. In this case the two processes involve a single isospin amplitude [21], and their ratio of rates is given by the squared ratio of corresponding Clebsch-Gordan coefficients, $(\sqrt{2})^2 = 2$. The assumption of a K^+K^0 symmetric state stands in contrast to the dependence on the K^+K^- invariant mass observed in the nonresonant background for $B^0 \to K^+K^-K^0$ [4]. No such dependence was observed in the process on the left-hand side of (29).

In the SU(3) limit, one may relate the nonresonant background in the above processes and the nonresonant amplitude in $B^+ \to K^+\pi^+\pi^-$. Comparing with $B^0 \to K^+K^-K^0$, one expects

$$\mathcal{B}(B^+ \to K^+\pi^+\pi^-)_{\rm NRB} = (\tau_+/\tau_0) \times\mathcal{B}(B^0 \to K^+K^-K^0)_{\rm NRB}; \tag{30}$$

$$18.4 \pm 1.9 = 18.8 \pm 2.0,$$

which holds very well. Under the underlying SU(3) approximation for nonresonant amplitudes, one would have expected this relation to be less precise than (29). Note that in both processes appearing in (30) the measured nonresonnt background is not exactly symmetric under interchanging the three meson momenta, as assumed in our model. The agreement in (30) favors the large nonresonant background in $B^+ \to K^+\pi^+\pi^-$ given in Table II over the small value quoted in Ref. [10].

V. IMPLICATIONS FOR DALITZ PLOT ANALYSES

The isospin relations we have quoted in Sec. III are expected to be valid separately for resonant and nonresonant contributions. Thus, the amount of $K_0^*(1430)^0\pi^+$ in $B^+ \to K^+\pi^+\pi^-$, for which a two-fold ambiguity appears in the analysis of Ref. [8] (see Table IV there), should be the same as the amount of $K_0^*(1430)^+\pi^-$ measured in $B^0 \to K^0\pi^+\pi^-$ [see Eq. (8)] and should be related to the amount of $K_0^*(1430)^0\pi^0$ measured in $B^0 \to K^+\pi^-\pi^0$ [see Eq. (15)]. Similarly, the amount of nonresonant background in $B^+ \to K^+\pi^+\pi^-$, for which there seems to be some question in comparing Refs. [10,26], should be the same as in $B^0 \to K^0\pi^+\pi^-$ for which no value has been quoted yet. Also, if a nonresonant background is small in $B^0 \to K^+\pi^0\pi^-$, as shown in Table II, we would expect it to be small in $B^+ \to K^0\pi^+\pi^0$.

The isospin relation $\Gamma(B^+ \to K^+\phi) = \Gamma(B^0 \to K^0\phi)$ appears to be satisfied by present data. Thus, we expect the non-ϕ contributions in $B \to K\bar{K}K$ decays, which are related to one another by isospin reflection, also to have equal partial widths. This is confirmed by Eqs. (25) and (27).

Fits to Dalitz plots often involve discrete ambiguities in assigning amplitudes and phases to given decay channels. This is demonstrated, for instance, by the two largely different solutions for $\mathcal{B}(B^+ \to K_0^*(1430)^0\pi^+)$ measured in Ref. [8]. Isospin symmetry, which relates this process to $B^0 \to K_0^*(1430)^+\pi^-$, resolves this ambiguity. Similarly, the isospin relation $\Gamma(B^+ \to \chi_{c0}K^+) = \Gamma(B^0 \to \chi_{c0}K^0)$ is useful for eliminating a two-fold ambiguity in the measurement of the left-hand side [8]. This determines the fraction of $\chi_{c0}K^+$ in the total of all $B^+ \to K^+K^-K^+$ decays to be about 3% rather than about 8%, both solutions being permitted in [8].

One of the predictions of fully symmetric final states in nonresonant background amplitudes is that these amplitudes should be suppressed in $B^+ \to K^0\pi^+\pi^0$ and $B^0 \to K^+\pi^0\pi^-$ relative to other processes under discussion. This prediction is supported by the upper bound on $\mathcal{B}(B^0 \to K^+\pi^-\pi^0)_{\rm nonres}$ in Table II, awaiting an improvement in the upper bound.

The violation of (29) is probably related to the deviation from a fully symmetric nonresonant background amplitude measured in $B^0 \to K^+K^-K^0$ [4]. The discrepancy may be the result of the fact that nonresonant backgrounds are unstable in the fits. This is demonstrated by the large discrepancy between two values of the nonresonant background in $B^+ \to K^+\pi^+\pi^-$ measured in Refs. [9,26] using different definitions for the nonresonant background. Another ambiguity in both $B^0 \to K^+K^-K^0$ and $B^+ \to K^+K^-K^+$ is observed between a large and a very small contribution peaking around 1500 MeV$/c^2$ [4,8]. As noted, the nonresonant background in $B^+ \to K^+K^-K^+$ was found to be completely symmetric [8]. Symmetry in the two identical K^+ mesons follows from Bose symmetry.

VI. CONCLUSIONS

We have considered strangeness-changing $B \to PPP$ decays for $P = \pi, K$ under the assumption that the dominant transition is the isospin-preserving penguin amplitude with flavor structure $\bar{b} \to \bar{s}$. In this approximation pairs of B^+ and B^0 decay amplitudes to $K\pi\pi$ or $K\bar{K}K$ are related to one another under the isospin reflection $u \leftrightarrow d$, and final states have $I = 1/2$. For decays involving more than one kaon relations involving final states with K_S and K_L hold under the assumption that ϕK-subtracted amplitudes are symmetric under $K \leftrightarrow \bar{K}$, as measured in processes involving K^+ and K^-.

All the proposed isospin relations are obeyed experimentally where data exist, excluding $\mathcal{B}(B^0 \to K^{*0}\pi^0)$ which seems low relative to $\frac{1}{2}\mathcal{B}(B^0 \to K^{*+}\pi^-)$. The relations lead to predictions where data are still missing. This success led to our proposal to combine the study of Dalitz plots for isospin-related processes, which can resolve discrete ambiguities in fitting resonance parameters to given Dalitz plots.

We have presented a model for nonresonant background amplitudes in $B \to K\pi\pi$ and $B \to K\bar{K}K$ which are symmetric in the three outgoing meson momenta. Predictions characteristic to this assumption are a suppressed nonresonant background in $B^+ \to K^0\pi^+\pi^0$ and $B^0 \to K^+\pi^0\pi^-$ and simple relations between nonresonant branching ratios in several processes. This approach has the potential for resolving some ambiguities in determining nonresonant background amplitudes from fits to Dalitz plots.

We have assumed that strangeness-changing charmless decays are dominated by an isospin-preserving $\bar{b} \to \bar{s}$ amplitude. These decays involve also $\Delta I = 1$ electroweak penguin contributions, which are expected to be suppressed relative to the dominant $I = 0$ QCD-penguin amplitude [12,27], and small $\Delta I = 1$ "tree" amplitudes suppressed by λ^2 ($\lambda \approx 0.2$). The effects of these suppressed amplitudes in $B \to K\pi$ decays have been studied recently in Refs. [28,29], quoting earlier references discussing these effects. The smallness of the effect is demonstrated, for instance, by the relatively small measured deviations from $\Delta I = 0$ relations among $B \to K\pi$ decay rates. An example is the ratio of branching ratios [10,30],

$$R_n^{-1} \equiv \frac{2\mathcal{B}(B^0 \to K^0\pi^0)}{\mathcal{B}(B^0 \to K^+\pi^-)} = 1.22 \pm 0.11, \tag{31}$$

which differs only by 2σ from the $\Delta I = 0$ value of one.

Isospin breaking effects should be considered in $B \to PPP$ when data become sufficiently accurate. The first case to be studied is understanding $2\mathcal{B}(B^0 \to K^{*0}\pi^0) < \mathcal{B}(B^0 \to K^{*+}\pi^-)$. A ratio R^{*-1}, defined in analogy with R_n^{-1},

$$R_n^{*-1} \equiv \frac{2\mathcal{B}(B^0 \to K^{*0}\pi^0)}{\mathcal{B}(B^0 \to K^{*+}\pi^-)} = 0.35 \pm 0.17, \tag{32}$$

is 3.9σ below one, thus presenting a larger discrepancy from $\Delta I = 0$ than measured in R_n^{-1}. An interpretation for the small value of $\mathcal{B}(B^0 \to K^{*0}\pi^0)$ was presented in Ref. [14] in terms of destructive interference between an electroweak penguin contribution and a QCD-penguin amplitude, the ratio of which is enhanced relative to that occurring in $B \to K\pi$. An interesting and pressing question is whether such enhancement can be accounted for in the standard model of electroweak and strong interactions. While some suppression of $\mathcal{B}(B^0 \to K^{*0}\pi^0)$ can be accounted for in calculations based on QCD factorization [31], central values computed for $B \to K^*\pi$ branching ratios are consistently lower than the data by a factor 2.1 to 3.5. Small $\Delta I = 1$ contributions also may account for the 2.4σ discrepancy in the relation (27).

We have considered CP-averaged rates, disregarding in this work possible CP asymmetries. The approximate relations we have derived apply separately to B and $\bar{B}$ decays. While the $\Delta I = 0$ terms in decay amplitudes are dominated by a Cabibbo-Kobayashi-Maskawa (CKM) factor $V_{tb}^*V_{ts}$, a CKM factor $V_{ub}^*V_{us}$ smaller by λ^2 is associated with tree contributions. The two CKM factors involve different weak phases. Direct CP violation is expected if the two terms carry also different strong phases.

Potential CP asymmetries are expected from interference of decay amplitudes for $B^+ \to K^+\chi_{c0}$, where $\chi_{c0} \to \pi^+\pi^-$ and $\chi_{c0} \to K^+K^-$, with tree amplitudes in $B^+ \to K^+\pi^+\pi^-$ and $B^+ \to K^+K^+K^-$, respectively. While a large strong phase difference is induced by the χ_{c0} width [16], the asymmetries depend also on the magnitudes of the smaller tree amplitudes for which calculations are model dependent [17]. The fractions of $K^+\chi_{c0}$ in $B^+ \to K^+\pi^+\pi^-$ and $B^+ \to K^+K^+K^-$ are small, at a level of 3% [8,9] or smaller [26].

The decays $B^+ \to K^+\rho^0$, where $\rho^0 \to \pi^+\pi^-$, amount to a larger fraction of $B^+ \to K^+\pi^+\pi^-$, about 10% [8,9,26]. Tentative evidence for a CP asymmetry in these decays has been reported recently, $A_{CP}(B^+ \to K^+\rho^0) = 0.32 \pm 0.13 \pm 0.06^{+0.08}_{-0.05}$ [9], $0.30 \pm 0.11 \pm 0.03^{+0.11}_{-0.04}$ [26]. This may be compared with a prediction, $A_{CP}(B^+ \to K^+\rho^0) = 0.21 \pm 0.10$, obtained in a global SU(3) fit to all $B \to PV$ decays [14]. A CP asymmetry at a level of 10% in the processes discussed in this paper, resulting from penguin-tree interference as measured in the asymmetry for $B^0 \to K^+\pi^-$ [10], would imply a small but nonnegligible violation of the $\Delta I = 0$ relations.

ACKNOWLEDGMENTS

We thank Hai-Yang Cheng, Denis Dujmic, Guy Engelhard, Guy Raz, and Aaron Roodman for helpful discussions. This work was performed in part while M. G. visited SLAC and while J. L. R. was at the Aspen Center for Physics. The research was supported in part by the United States Department of Energy under Grant No. DE FG02 90ER40560, by the Israel Science Foundation founded by the Israel Academy of Science and Humanities, Grant No. 1052/04, and by the German-Israeli Foundation for Scientific Research and Development, Grant No. I-781-55.14/2003.

Note added in proof.—The low values of the left-hand side of (11) and the right-hand side of (32) are affected by an old CLEO null result [32]. The *BABAR* value [33], $\mathcal{B}(B^0 \to K^{*0}\pi^0) = (3.0 \pm 0.9 \pm 0.5) \times 10^{-6}$, based on a data sample more than 20 times as large, leads only to a 1.6 σ discrepancy in (11) and to a value in (32) which is only 1.8 σ below one.

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