

**Pion-nucleon sigma term in the global color model of QCD**

Lei Chang

*Department of Physics, Peking University, Beijing 100871, China*

Yu-xin Liu\*

*Department of Physics, Peking University, Beijing 100871, China**The Key Laboratory of Heavy Ion Physics, Ministry of Education, Beijing 100871, China**Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China**CCAST(World Lab.), P. O. Box 8730, Beijing 100080, China*

Hua Guo

*Department of Technical Physics, Peking University, Beijing 100871, China**The Key Laboratory of Heavy Ion Physics, Ministry of Education, Beijing 100871, China**Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China*

(Received 18 August 2005; published 22 November 2005)

We study the pion-nucleon sigma term in vacuum and in nuclear matter in the framework of global color model of QCD. With the effective gluon propagator being taken as the  $\delta$ -function in momentum space of Munczek-Nomirovsky model, we estimate that the sigma term at chiral limit in the vacuum is  $9/2$  times the current quark mass and it decreases with the nuclear matter density. With the presently obtained in-medium pion-nucleon sigma term, we study the in-medium chiral quark condensate and obtain a reasonable variation behavior against the nuclear matter density.

DOI: [10.1103/PhysRevD.72.094023](https://doi.org/10.1103/PhysRevD.72.094023)

PACS numbers: 14.20.Dh, 11.10.Lm, 12.39.Fe, 14.40.Aq

**I. INTRODUCTION**

The pion-nucleon sigma term  $\sigma_{\pi N}$  is of fundamental importance for understanding the chiral symmetry breaking effect in nucleon [1–4], the central nuclear force [5,6] and the mass decomposition of nucleon [7,8]. Because it is related to the quark and gluon condensates in nuclear matter [9–17], the pion-nucleon sigma term is regarded to play an important role in the process of chiral symmetry restoration in nuclear matter. Recent researches show that the  $\sigma_{\pi N}$  is also important in searching for the Higgs boson [18], supersymmetric particles and dark matter [19,20]. Then the pion-nucleon sigma term has been studied in chiral perturbation theory [21–23], lattice QCD [8,24–27], various chiral models [28–34] and other models [35–37]. As for the value of the pion-nucleon sigma term, different researches show that it can be as small as  $(18 \pm 5)$  MeV [26], and as large as  $(88–90)$  MeV [38]. Generally it used to be regarded as 45 MeV [30,33], and recently suggested to be  $(50–80)$  MeV [34,37–39]. Since experiment can not measure the  $\sigma_{\pi N}$  directly and theoretical results are model dependent, the value of the sigma term has been and still is a puzzle [34]. On the other hand, when taking the sigma term as an ingredient to determine the quark and gluon condensates in nuclear matter, it was usually taken as a constant independent of the nuclear matter density [9–17]. In such a sense, the nuclear matter

density dependence of the pion-nucleon sigma term has not yet been studied.

It has been shown that the global color model (GCM) [40–45] is a quite successful effective field theory model of QCD in describing hadron properties and quark condensate [46–48] in free space (i.e., at temperature  $T = 0$ , chemical potential  $\mu = 0$ ). Meanwhile the bag constant, the radius and the mass of a nucleon in nuclear matter and the quark condensate in nuclear matter can also be evaluated in a consistent way [49–52] in the GCM. With the global color symmetry model at zero and finite chemical potential  $\mu$ , we will study the pion-nucleon sigma term in free space and in nuclear matter in this paper. As an application of the sigma term in nuclear matter, we will also discuss the density dependence of the chiral quark condensate in nuclear matter.

The paper is organized as follows. In Sec. II, we describe briefly the framework of the global color symmetry model. In Sec. III, we give our model and result of the pion-nucleon sigma term. In Sec. IV, we apply the obtained pion-nucleon sigma term to evaluate the nucleon matter density dependence of the chiral quark condensate. Finally, we summarize our work and give a brief remark in Sec. V.

**II. BRIEF DESCRIPTION OF THE GLOBAL COLOR MODEL OF QCD**

We start from the action of the global color model of QCD in Euclidean space

\*Corresponding author.

$$S_{GCM}[\bar{q}, q] = \int d^4x \{ \bar{q}(x) [\gamma \cdot \partial_x + m] q(x) \} + \frac{1}{2} \times \int d^4x d^4y [j_\mu^a(x) g_s^2 D_{\mu\nu}^{ab}(x-y) j_\nu^b(y)], \quad (1)$$

where  $m$  is current quark mass,  $j_\mu^a(x) = \bar{q}(x) \gamma_\mu \frac{\lambda^a}{2} q(x)$  denotes the color octet vector current and  $g_s^2 D_{\mu\nu}^{ab}(x-y)$  is the effective gluon propagator. Here we would like to diagonal the gluon propagator in the color matrix and choose the Landau gauge, i.e., take the effective gluon propagator as

$$g_s^2 D_{\mu\nu}^{ab}(x) = \delta_{ab} \int \frac{d^4k}{(2\pi)^4} t_{\mu\nu}(k) \mathcal{G}(k^2) e^{ik \cdot x}, \quad (2)$$

where  $t_{\mu\nu}(k) = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$  and  $\mathcal{G}(k^2)$  is the effective interaction relating to the gluon vacuum polarization introduced usually as a model input.

Introducing the auxiliary bilocal fields  $B^\theta(x, y)$  and applying the standard bosonization procedure [40], we can give the partition function at mean field approximation as

$$Z = \int \mathcal{D}B^\theta e^{-S_{eff}[B^\theta]}. \quad (3)$$

One can then identify the auxiliary field that minimizes the effective action as  $B_0^\theta$  (usually referred to as the vacuum configuration). Expansion in field fluctuations about the vacuum configuration would generate the propagating bosons (mesons etc.). Meanwhile, the vacuum configuration produces the rainbow approximation for the quark self-energy giving the rainbow Dyson-Schwinger equation

$$G^{-1}(p) = i\gamma \cdot p + m + \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} t_{\mu\nu}(p-q) \times \mathcal{G}((p-q)^2) \gamma_\mu G(q) \gamma_\nu, \quad (4)$$

where  $G(p)$  is the dressed quark propagator which can be decomposed as

$$G^{-1}(p) = i\gamma \cdot p A(p^2) + B(p^2), \quad (5)$$

with  $A(p^2)$ ,  $B(p^2)$  being scalar self-energy function and  $B(p^2) = B_0^\theta$ . Since the rainbow approximation of the Dyson-Schwinger equation determines the vacuum configuration and the quantum fluctuations about this configuration add corrections to the rainbow approximation, the GCM is a quite sophisticated and practical effective field theory model of QCD.

It is remarkable that the  $B(p^2)$  deduced within an effective interaction from the quark equation in the chiral limit ( $m \rightarrow 0$ ) has two qualitatively distinct solutions. One is the Wigner solution,  $B(p^2) \equiv 0$ , that characterizes the phase in which chiral symmetry is not broken and the dressed quarks are not confined. Another is the Nambu solution with nonzero  $B(p^2)$ , which describes the phase where chiral symmetry is spontaneously broken and prevents

quarks from involving mass pole bellow about 1–2 GeV[53]. It is also necessary to mention that the Wigner solution is always possible, but the Nambu solution is only available if the coupling is strong enough at the infrared region [53].

For studying the pion-nucleon sigma term, we should know the derivative of the constituent quark mass against the current quark mass. It is at first necessary to start from the current mass dependent quark propagator  $G(p)$ . Defining the derivative of the inverse of the quark propagator with respect to the current quark mass as

$$\Gamma(p^2) = \frac{\partial G^{-1}(p)}{\partial m}, \quad (6)$$

one can easily prove that the  $\Gamma(p^2)$  satisfies the equation

$$\Gamma(p) = 1 - \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} t_{\mu\nu}(p-q) \times \mathcal{G}((p-q)^2) \gamma_\mu G(q) \Gamma(q) G(q) \gamma_\nu. \quad (7)$$

Analyzing the Lorentz structure, one can decompose the  $\Gamma(p^2)$  function as

$$\Gamma(p^2) = i\gamma \cdot p C(p^2) + D(p^2). \quad (8)$$

This equation together with the quark equation has been used to study the nonperturbative mass-independent renormalization within the Dyson-Schwinger equation formalism [54]. Here it should be noted that not only the scalar part of the quark propagator  $B(p^2)$  depends on the current quark mass but also the vector part  $A(p^2)$ . At the same time, it is useful to simultaneously discuss the spontaneous chiral symmetry breaking and the explicit symmetry breaking.

### III. PION-NUCLEON SIGMA TERM IN THE GLOBAL COLOR MODEL OF QCD

To understand the spontaneous chiral symmetry breaking, one usually take the chiral susceptibility as a manifestation. The chiral susceptibility is defined as[55]

$$\chi(p^2) = \frac{\partial M(p^2)}{\partial m}, \quad (9)$$

where the quark mass function  $M = B/A$  can be fixed by implementing the quark equation. With the quark propagator and its derivation of the current quark mass we can give explicitly the chiral susceptibility as

$$\chi(p^2) = \frac{D(p^2)A(p^2) - B(p^2)C(p^2)}{A^2(p^2)}. \quad (10)$$

The pion-nucleon sigma term is related to the nucleon mass  $M_N$  by means of the Hellmann-Feynman theorem

$$\sigma_{\pi N} = m \frac{\partial M_N}{\partial m}. \quad (11)$$

The nucleon will be treated as three noninteracting con-

stituent quarks so as to emphasize the dominant characteristics and permit simple estimates in spirit of the chiral quark soliton model [34,40] in the isospin symmetry limit. We have then  $M_N = 3M_c$ . The quark constituent mass will be evaluated by the so-called Euclidean constituent mass which is the value of mass function matching the momentum scale and reads  $M_c = M(p^2 = M_c^2)$ . With the chiral susceptibility, the pion-nucleon sigma term can then be expressed as

$$\sigma_{\pi N} = 3m\chi(p^2 = M_c^2)_{m \rightarrow 0}. \quad (12)$$

Here we have taken the chiral limit for the chiral susceptibility on the ‘‘quark mass shell’’. We should note that quark confinement entails only that there is no ‘‘pole mass’’. The constituent quark mass  $M_c$  may then be estimated in other ways. One of the other choices could be the value of the quark dynamical mass function at  $p^2 = 0$ , that is  $\tilde{M}_c = M(p^2 = 0)$ . We will also consider this choice.

In the following practical calculation, we take the Munczek-Nomirovsky model [56] of the effective gluon propagator

$$\mathcal{G}(k^2) = 4\pi^4 \eta^2 \delta(k). \quad (13)$$

It is obvious that such a model is an infrared-dominant model that does not represent well the behavior in the large momentum region. Substituting Eq. (13) into Eqs. (4) and (7), one can have the solution with the nontrivial function  $B(p^2)$ , which is related to the dynamical quark mass, in the chiral limit

$$A = 2, \quad B = \sqrt{\eta^2 - 4p^2}, \quad (14)$$

$$C = -\frac{2}{\sqrt{(\eta^2 - 4p^2)}}, \quad D = \frac{1}{2} \frac{\eta^2 + 8p^2}{\eta^2 - 4p^2}, \quad (15)$$

for  $p^2 < \frac{\eta^2}{4}$ , and

$$A = \frac{1}{2} \left( 1 + \sqrt{1 + \frac{2\eta^2}{p^2}} \right), \quad B = 0, \quad (16)$$

$$C = 0, \quad D = \frac{p^2 + \eta^2 + p^2 \sqrt{1 + \frac{2\eta^2}{p^2}}}{p^2 - \eta^2 + p^2 \sqrt{1 + \frac{2\eta^2}{p^2}}}, \quad (17)$$

for  $p^2 \geq \frac{\eta^2}{4}$ . With Eq. (10), the chiral susceptibility in the infrared region can then be written as

$$\chi(p^2) = \frac{3}{4} \frac{\eta^2}{\eta^2 - 4p^2}. \quad (18)$$

It should be noted that there exists a singularity at  $p^2 = \frac{\eta^2}{4}$  in the chiral susceptibility in the chiral limit. Such an evident pole in the susceptibility is an artifact of the simplified nature of the Munczek-Nomirovsky model in

the chiral limit where the quark mass function becomes zero at finite momentum [55]. A realistic description would not have such a singular susceptibility. Making use of the Euclidean constituent mass concept, one has  $p^2 = M_c^2 = \frac{\eta^2}{8}$  in Eq. (18) and it gives  $\chi(p^2) = \frac{3}{2}$ . With Eq. (12), one obtains the pion-nucleon sigma term in terms of the current quark mass as

$$\sigma_{\pi N} = \frac{9}{2} m. \quad (19)$$

It is evident that, in the chiral limit approximation, the pion-nucleon sigma term is estimated to be 9/2 times the current quark mass and independent of the strength of the infrared slavery effect. If the current quark mass takes an empirical value about 10 MeV, we have apparently  $\sigma_{\pi N} = 45$  MeV. It has been shown that, if the potential representing the quark confinement can be written as an exponential form  $V_c \approx r^Z$ , the pion-nucleon sigma term can be given as  $\sigma_{\pi N} = \frac{9}{3-Z} m$  [36]. Our present result is just the case of linear confinement with  $Z = 1$ . On the other hand, as a measure of the uncertainty in such estimates of  $\sigma_{\pi N}$ , the alternative estimate [57] of the constituent mass  $\tilde{M}_c = M(p^2 = 0)$  gives  $\tilde{M}_c = \frac{\eta}{2}$ , it results in  $\chi(p^2 = 0) = \frac{3}{4}$  and  $\sigma_{\pi N} = \frac{9}{4} m$ . We consider this to overestimate the current quark mass and underestimate  $\sigma_{\pi N}$  because the expected  $p^2$  for a constituent quark in a nucleon should be nonzero.

Beyond the chiral limit, we should calculate the Eqs. (4) and (7) at finite quark mass with the Munczek-Nomirovsky model. The numerical result of the sigma term at and beyond the chiral limit is shown in Fig. 1. The Fig. 1 shows obviously that, if the current quark mass is less than 15 MeV, the effect beyond the chiral limit on the pion-nucleon sigma term can be neglected. We take then the chiral limit in the following discussions.

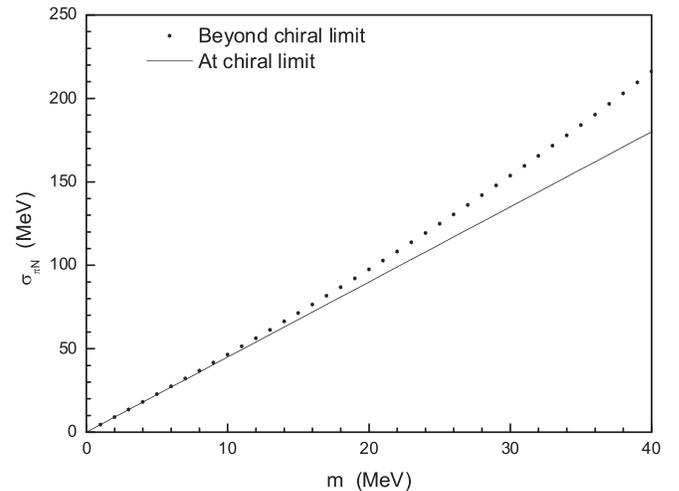


FIG. 1. The current quark mass dependence of the pion-nucleon sigma term at and beyond the chiral limit.

From the relation between the sigma term and the constituent quark mass we can infer that the sigma term is not a constant in nuclear matter since the quark mass usually changes with the density. In the GCM the quark propagator in-medium can be evaluated in a general approach. It is then appropriate to study the sigma term in-medium in the GCM.

The chemical potential is generally introduced as a Lagrangian multiplier  $e^{-\int d^4x \mu \bar{q} \gamma_4 q}$  with the partition function of GCM. With the general approach discussed above, the quark equation and its derivation against the current quark mass can be obtained. In the Munczek-Nomirovsky model (the same form as in vacuum) at chiral limit, one can easily obtain the functions  $A$ ,  $B$ ,  $C$  and  $D$ . However, it is usually difficult to numerically solve the the coupled D-S equation at finite chemical potential with an general effective gluon propagator. In order to avoid this difficulty, we have developed an approach for calculating the chemical potential dependence of the dressed quark propagator [58]. The main conclusion in the approach is that the inverse of the full dressed quark propagator at finite chemical potential can be obtained from the inverse of the dressed quark propagator at zero  $\mu$  by a replacement  $p_4 \rightarrow p_4 + i\mu$ .

Taking the replacement  $p_4 \rightarrow p_4 + i\mu$  and  $p \rightarrow \tilde{p} = (\tilde{p}, p_4 + i\mu)$ , one can obtain the functions  $A$ ,  $B$ ,  $C$  and  $D$  in Eq. (10) at finite chemical potential in the Munczek-Nomirovsky model easily. The quark chiral susceptibility in Eq. (18) can then be generalized to that at finite chemical potential  $\mu > 0$  as

$$\chi(\mu) = \frac{3}{4} \frac{\eta^2}{\eta^2 - 4\tilde{p}^2}, \quad (20)$$

where  $\tilde{p}^2 = \vec{p}^2 + (p_4 + i\mu)^2$ .

Up to now, the quark constituent mass at finite chemical potential has not yet been well defined. Analogous to that at finite temperature, one has temporal mass identified by  $M_c = M(\vec{p}^2 = 0, p_4^2 = M_c^2)$ . Another way is identifying the mass to spatial mass by  $M_c = M(\vec{p}^2 = M_c^2, p_4^2 = 0)$ . Since the temporal mass takes a complex number and so does the chiral susceptibility, we take the spatial mass. As a consequence, we have the chemical potential dependent pion-nucleon sigma term in Munczek-Nomirovsky model as

$$\sigma_{\pi N}(\mu) = \frac{9}{2} m \frac{\eta^2}{\eta^2 + 4\mu^2}. \quad (21)$$

It is apparent that, in the case of without medium (i.e.,  $\mu = 0$ ),  $\sigma_{\pi N}(0) = \frac{9}{2} m$ . Meanwhile, as the chemical potential increases, the in-medium pion-nucleon sigma term  $\sigma_{\pi N}(\mu)$  decreases monotonously. It is again remarkable that, if one takes the quark constituent mass for chemical potential  $\mu > 0$  as the value of the quark mass function with all the components of momentum being zero, one has a factor  $\frac{1}{2}$  for the right hand side of Eq. (21). This is parallel

to the situation at  $\mu = 0$  and the data can be regarded as the lower limit at a certain chemical potential.

#### IV. CHIRAL QUARK CONDENSATE IN NUCLEAR MATTER

As an application of the result of the chemical potential dependent pion-nucleon sigma term, we discuss the chiral quark condensate in hadron matter. It is well known that the in-medium chiral quark condensate can be related to the pion-nucleon sigma term with the model-independent relation [10]

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_{\text{vac}}} = 1 - \frac{1}{2|\langle \bar{q}q \rangle_{\text{vac}}|} \frac{d\varepsilon}{dm}, \quad (22)$$

where  $\rho$  is the density of the nuclear matter which can be fixed by model calculation.  $\langle \bar{q}q \rangle_\rho$  is the quark condensate in the nuclear matter with density  $\rho$  and  $\langle \bar{q}q \rangle_{\text{vac}}$  is the one in the vacuum which may depend on the current quark mass,  $\varepsilon$  is the energy density of the nuclear matter which can be approximately written as [10]

$$\varepsilon = \rho M_N + \delta\varepsilon, \quad (23)$$

where  $M_N$  is the nucleon mass, and  $\delta\varepsilon$  is the contribution to energy density from the nucleon kinetic energy and nucleon-nucleon interaction energy which is of higher order in nucleon density and is empirically small at low density. Ignoring the last term  $\delta\varepsilon$  and the quark current mass dependence of the density  $\rho$ , we have

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_{\text{vac}}} = 1 - \frac{\sigma_{\pi N}}{2m|\langle \bar{q}q \rangle_{\text{vac}}|} \rho. \quad (24)$$

Such a relation has been commonly used to study the medium density effect on the chiral quark condensate with a constant  $\sigma_{\pi N}$  (see, for example, Refs. [10,12,14]).

In the last section, we have obtained the chemical potential dependent pion-nucleon sigma term in nuclear matter. To evaluate the in-medium chiral quark condensate with Eq. (24), we should transfer the chemical potential dependence to the matter density dependence. Then it is necessary to derive the relation between the nuclear matter density and the quark chemical potential. At the mean field level, the pressure density of the matter can be written as

$$\mathcal{P}[\mu] = \text{Trln}[G^{-1}] - \frac{1}{2} \text{Tr}[\Sigma G]. \quad (25)$$

With such a pressure, one usually define the nuclear matter density as [59]

$$\rho = \frac{\partial \mathcal{P}}{\partial \mu} \quad (26)$$

As mentioned above, the inverse of the quark propagator and the quark self-energy in the Munczek-Nomirovsky model for the ‘‘Nambu-Goldstone’’ solution can be written explicitly by replacing the  $p$  with  $\tilde{p} = (\tilde{p}, p_4 + i\mu)$  in the

following forms

$$G^{-1}(p^2) = [i\gamma \cdot p A_1(p^2) + B_1(p^2)]\theta\left(\frac{\eta^2}{4} - p^2\right) + [i\gamma \cdot p A_2(p^2) + B_2(p^2)]\left[1 - \theta\left(\frac{\eta^2}{4} - p^2\right)\right], \quad (27)$$

$$\Sigma(p^2) = [i\gamma \cdot p(A_1(p^2) - 1) + B_1(p^2)]\theta\left(\frac{\eta^2}{4} - p^2\right) + [i\gamma \cdot p(A_2(p^2) - 1) + B_2(p^2)]\left[1 - \theta\left(\frac{\eta^2}{4} - p^2\right)\right], \quad (28)$$

where  $A_1(p^2)$ ,  $B_1(p^2)$  have the form of Eq. (14) and  $A_2(p^2)$ ,  $B_2(p^2)$  relate to Eq. (16), and  $\theta(\frac{\eta^2}{4} - p^2) = 1$  for  $p^2 < \frac{\eta^2}{4}$ ,  $\theta(\frac{\eta^2}{4} - p^2) = 0$  for  $p^2 \geq \frac{\eta^2}{4}$ .

It should be noted that the Eq. (26) with Eqs. (25), (27), and (28) is divergent due to the vector part of the quark propagator. To give a finite value of nuclear matter density we expand the pressure density with quark loop to second order. It is found that the second order is zero at mean field level and the relation between the nuclear matter density and the chemical potential  $\mu$  can be written as

$$\rho = \frac{2}{3\pi^2}\mu^3 + 4\frac{\partial}{\partial\mu} \int \frac{d^4q}{(2\pi)^4} [A(\tilde{q}^2) - 1]. \quad (29)$$

On the other hand, when applying the replacements  $p \rightarrow \tilde{p} = (\vec{p}, p_4 + i\mu)$  and  $q \rightarrow \tilde{q} = (\vec{q}, q_4 + i\mu)$  in Eqs. (26)–(29), one should do that not only for the argument of functions  $A_1$ ,  $B_1$ ,  $A_2$ ,  $B_2$ , but also for that in the step function  $\theta$ . It is evident that the boundary condition is arbitrary for the Heavyside step function  $\theta(\frac{\eta^2}{4} - \tilde{q}^2)$ , which cannot be defined from any criterion [60]. Then we propose a choice as

$$\text{Re}\left[\frac{\eta^2}{4} - \tilde{q}^2\right] > a\mu^2 \quad (30)$$

for the nonzero scalar function  $B$ , where  $a$  is a parameter (the choice  $a = 0$  has been taken in previous calculations with the Munczek-Nomirovsky model (see, for example, Refs. [50,52]). Because the parameter  $a$  determines the domains of different forms of function  $A$  and of the integration in Eq. (29), it influences the relation between the nuclear matter density and the chemical potential. The numerical results of  $\rho$  in terms of  $\mu$  at several parameters  $a$  are showed in Fig. 2.

Taking the relation between the nuclear matter density and the chemical potential in Eq. (29) into account, the chemical potential dependence of the pion-nucleon sigma term in nuclear matter in Eq. (21) can be rewritten as a relation between the pion-nucleon sigma term and the nuclear matter density. Basing on the result shown in

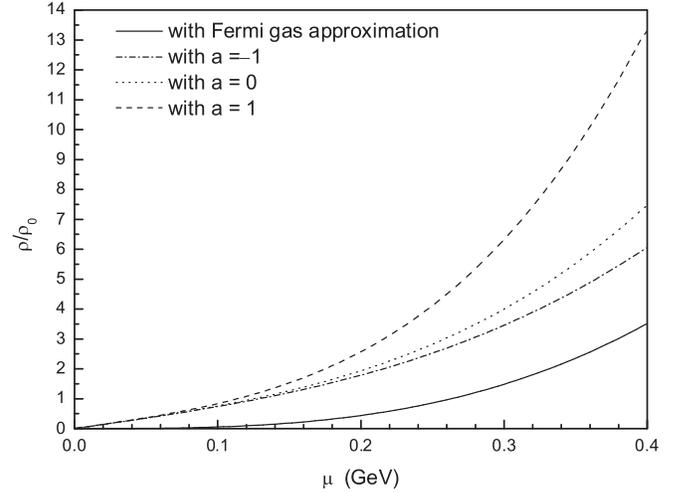


FIG. 2. Calculated relation between the density and the chemical potential of nuclear matter in several models.

Fig. 2, that the density increases monotonously with the chemical potential, one can recognize that the pion-nucleon sigma term in nuclear matter decreases monotonously with the matter density.

With the obtained relation between the pion-nucleon sigma term and the nuclear matter density  $\sigma_{\pi N}(\rho)$  being substituted into Eq. (24), we obtain the nuclear matter density dependence of the chiral quark condensate in nuclear matter. The results with current quark mass being taken as  $m = 10$  MeV and the parameter  $\eta$  being taken as  $\eta = 1.04$  GeV, with which the pion decay constant in free space (93 MeV) has been reproduced well [42], are illustrated in Fig. 3. From Fig. 3, one can recognize easily that, if the nuclear matter density dependence of the pion-nucleon sigma is neglected, the chiral quark condensate decreases linearly with the increase of the nuclear matter density and reaches zero as the density is a little larger than 4 times the normal nuclear matter density. As the nuclear matter density dependence of the sigma term is taken into account, the decreasing rate of the chiral quark condensate in nuclear matter is weakened evidently. Meanwhile, the decrease maintains monotonous, so that the ‘‘upturn’’ problem in some of the previous works does not emerge in our present work. Comparing this result with those given in Refs. [15,17], one can infer that the inclusion of the density dependence of sigma term is, in some sense, equivalent to taking the higher order corrections or the effect of pion into account appropriately. Furthermore, the degree of the reduction on the decreasing rate depends on the boundary condition we proposed. With the decrease of the boundary condition parameter  $a$  from 1 to  $-1$ , the decreasing rate gets obviously smaller. For instance, the condensate reaches zero as  $\rho \approx 5.5\rho_0$  for  $a = 1$ , however the condensate vanishes as  $\rho \approx 7.2\rho_0$  for  $a = -1$ . Recalling Eq. (30), one can know that such a phenomenon means that the decreasing rate becomes smaller as the

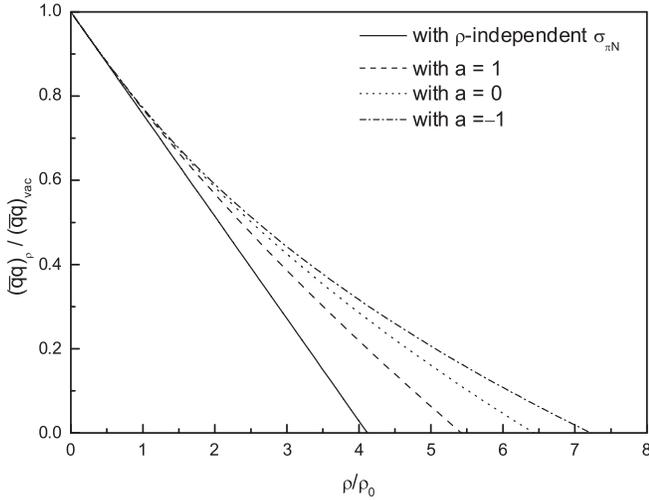


FIG. 3. The calculated density dependence of the chiral quark condensate with and without the sigma term in-medium effect being taken into account.

domain for the scalar self-energy function  $B \neq 0$  is enlarged. As *vice versa*, it indicates that the boundary condition is a manifestation of the interaction being taken into account.

## V. SUMMARY AND REMARK

In summary, we studied the pion-nucleon sigma term in vacuum and in nuclear matter in the global color model of QCD with the effective gluon propagator in Munczek-Nomirovsky model in this paper. It is estimated that the pion-nucleon sigma term at chiral limit in the vacuum is  $9/2$  times the current quark mass and the in-medium pion-nucleon sigma term decreases with the nuclear matter density. As an application of the obtained chemical potential (or hadron matter density) dependence of the pion-nucleon sigma term, we take it to study the in-medium chiral quark condensate. It shows that, with the medium effect on the sigma term being taken into account, the

linear decreasing behavior is evidently shifted to nonlinear with a much smaller descend rate. It indicates that such a variation behavior of the pion-nucleon sigma term against the medium density is reasonable and its effect on the chiral quark condensate in nuclear matter is consistent with that takes the higher order interaction into account.

Concerning our derivation of the pion-nucleon sigma term, we take the Munczek-Nomirovsky model in the framework of global color model of QCD and obtain an analytical expression in terms of the current quark mass and the chemical potential. However, since the Munczek-Nomirovsky model expresses the effective gluon propagator as a  $\delta$ -function in momentum space, it can only represent well the characteristic in infrared region but can not display the behavior in ultraviolet region. Then the study with a more realistic effective gluon propagator is necessary. Moreover, we take a nucleon simply as three non-interacting quarks to evaluate the pion-nucleon sigma term in the present work. The realistic nucleon is quark bound state with complicated interactions. To obtain the pion-nucleon sigma term more sophisticatedly needs to take into account the interaction among the quarks. The related investigations are under progress.

## ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (NSFC) under contract Nos. 10425521, 10135030, 10075002, 10275002, 10435080, the Major State Basic Research Development Program under contract No. G2000077400, the research foundation of the Ministry of Education, China (MOEC), under contract No. 305001 and the Research Fund for the Doctoral Program of Higher Education of China under grant No. 20040001010. One of the authors (Y.X.L.) thanks also the support of the Foundation for University Key Teacher by the MOEC. The authors are in debit to Dr. Craig D. Roberts for his stimulating discussions.

- 
- [1] E. Reya, Rev. Mod. Phys. **46**, 545 (1974).
  - [2] R. L. Jaffe, Phys. Rev. D **21**, 3215 (1980).
  - [3] J. Gasser, Ann. Phys. (N.Y.) **136**, 62 (1981).
  - [4] M. E. Sainio,  $\pi N$  Newsletter 16, 138 (2002).
  - [5] C. M. Maekawa, J. C. Pupin, and M. R. Robilotta, Phys. Rev. C **61**, 064002 (2000).
  - [6] M. R. Robilotta, Phys. Rev. C **63**, 044004 (2001).
  - [7] X. D. Ji, Phys. Rev. Lett. **74**, 1071 (1995).
  - [8] M. Procura, T. R. Hemmert, and W. Weise, Phys. Rev. D **69**, 034505 (2004).
  - [9] K. Tsushima, T. Maruyama, and A. Faessler, Nucl. Phys. **A535**, 497 (1991).
  - [10] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. C **45**, 1881 (1992).
  - [11] M. C. Birse, Phys. Rev. C **49**, 2212 (1994).
  - [12] H. Fujii and Y. Tsue, Phys. Lett. B **357**, 199 (1995).
  - [13] A. Delfino, J. Dey, M. Dey, and M. Malheiro, Phys. Lett. B **363**, 17 (1995).
  - [14] R. Brockmann and W. Weise, Phys. Lett. B **367**, 40 (1996).
  - [15] M. Lutz, Nucl. Phys. **A642**, c171 (1998).
  - [16] G. E. Brown and M. Rho, Phys. Rep. **363**, 85 (2002).

- [17] E. G. Drukarev, M. G. Ryskin, and V. A. Sadovnikova, *Prog. Part. Nucl. Phys.* **47**, 73 (2001); E. G. Drukarev, *Prog. Part. Nucl. Phys.* **50**, 659 (2003).
- [18] T. P. Cheng, *Phys. Rev. D* **38**, 2869 (1988).
- [19] A. Bottino, F. Donato, N. Fornengo, and S. Scopel, *Astropart. Phys.* **13**, 215 (2000).
- [20] U. Chattopadhyay, A. Corsetti, and P. Nath, *Phys. Rev. D* **66**, 035003 (2002).
- [21] V. Bernard, N. Kaiser, and U. G. Meissner, *Z. Phys. C* **60**, 111 (1993); *Phys. Lett. B* **389**, 144 (1996).
- [22] B. Borasoy and U. G. Meissner, *Phys. Lett. B* **365**, 285 (1996); *Ann. Phys. (N.Y.)* **254**, 192 (1997); B. Borasoy, *Eur. Phys. J. C* **8**, 121 (1999).
- [23] Y. Oh and W. Weise, *Eur. Phys. J. A* **4**, 363 (1999).
- [24] M. Fukugita, Y. Kuramashi, M. Okawa, and A. Ukawa, *Phys. Rev. D* **51**, 5319 (1995).
- [25] S. J. Dong, J. F. Lagae, and K. F. Liu, *Phys. Rev. D* **54**, 5496 (1996).
- [26] S. Güsken, P. Ueberholz, J. Viehoff, N. Eicker, P. Lacock, T. Lippert, K. Schilling, A. Spitz, and T. Struckmann, *Phys. Rev. D* **59**, 054504 (1999).
- [27] D. B. Leinweber, A. W. Thomas, and S. V. Wright, *Phys. Lett. B* **482**, 109 (2000); D. B. Leinweber, A. W. Thomas, and R. D. Young, *Phys. Rev. Lett.* **92**, 242002 (2004).
- [28] J.-P. Blaizot, M. Rho, and N. N. Scoccola, *Phys. Lett. B* **209**, 27 (1988).
- [29] D. I. Diakonov, V. Yu. Petrov, and M. Praszalówicz, *Nucl. Phys.* **B323**, 53 (1989).
- [30] J. Gasser, H. Leutwyler, and M. E. Sainio, *Phys. Lett. B* **253**, 252 (1991); **253**, 260 (1991).
- [31] M. C. Birse and J. A. McGovern, *Phys. Lett. B* **292**, 242 (1992).
- [32] D. B. Leinweber, A. W. Thomas, K. Tsushima, and S. V. Wright, *Phys. Rev. D* **61**, 074502 (2000).
- [33] V. E. Lyubovitskij, T. Gutsche, A. Faessler, and E. G. Drukarev, *Phys. Rev. D* **63**, 054026 (2001); T. Inoue, V. E. Lyubovitskij, T. Gutsche, and A. Faessler, *Phys. Rev. C* **69**, 035207 (2004).
- [34] P. Schweitzer, *Phys. Rev. D* **69**, 034003 (2004).
- [35] G. X. Peng, U. Lombardo, M. Loewe, and H. C. Chiang, *Phys. Lett. B* **548**, 189 (2002).
- [36] G. X. Peng, *Nucl. Phys.* **A747**, 75 (2005).
- [37] G. E. Hite, W. B. Kaufmann, and J. T. Jacob, *Phys. Rev. C* **71**, 065201 (2005).
- [38] P. Schweitzer, *Eur. Phys. J. A* **22**, 89 (2004).
- [39] M. G. Olsson and W. B. Kaufmann,  *$\pi N$  Newsletter* 16, 382 (2002).
- [40] R. T. Cahill and C. D. Roberts, *Phys. Rev. D* **32**, 2419 (1985); J. Praschifka, C. D. Roberts, and R. T. Cahill, *Phys. Rev. D* **36**, 209 (1987); R. T. Cahill, C. D. Roberts, and J. Praschifka, *Ann. Phys. (N.Y.)* **188**, 20 (1988); C. D. Roberts, R. T. Cahill, M. E. Sevier, and N. Iannella, *Phys. Rev. D* **49**, 125 (1994).
- [41] M. R. Frank, P. C. Tandy, and G. Fai, *Phys. Rev. C* **43**, 2808 (1991).
- [42] M. R. Frank and P. C. Tandy, *Phys. Rev. C* **46**, 338 (1992).
- [43] C. W. Johnson and G. Fai, *Phys. Rev. C* **56**, 3353 (1997).
- [44] P. C. Tandy, *Prog. Part. Nucl. Phys.* **39**, 117 (1997).
- [45] X. F. Lü, Y. X. Liu, H. S. Zong, and E. G. Zhao, *Phys. Rev. C* **58**, 1195 (1998).
- [46] T. Meissner, *Phys. Lett. B* **405**, 8 (1997).
- [47] L. S. Kisslinger and T. Meissner, *Phys. Rev. C* **57**, 1528 (1998).
- [48] H. S. Zong, X. F. Lü, J. Z. Gu, C. H. Chang, and E. G. Zhao, *Phys. Rev. C* **60**, 055208 (1999); H. S. Zong, J. L. Ping, H. T. Yang, X. F. Lü, and F. Wang, *Phys. Rev. D* **67**, 074004 (2003); H. S. Zong, S. Qi, W. Chen, W. M. Sun, and E. G. Zhao, *Phys. Lett. B* **576**, 289 (2003).
- [49] Y. X. Liu, D. F. Gao, and H. Guo, *Nucl. Phys.* **A695**, 353 (2001); Y. X. Liu, D. F. Gao, J. H. Zhou, and H. Guo, *Nucl. Phys.* **A725**, 127 (2003); L. Chang, Y. X. Liu, and H. Guo, *Nucl. Phys.* **A750**, 324 (2005).
- [50] P. Maris, C. D. Roberts, and S. Schmidt, *Phys. Rev. C* **57**, R2821 (1998).
- [51] A. Bender, G. I. Poulies, C. D. Roberts, S. Schmidt, and A. W. Thomas, *Phys. Lett. B* **431**, 263 (1998).
- [52] Y. X. Liu, D. F. Gao, and H. Guo, *Phys. Rev. C* **68**, 035204 (2003).
- [53] C. D. Roberts and A. G. Williams, *Prog. Part. Nucl. Phys.* **33**, 477 (1994).
- [54] K. Kusaka, H. Toki, and S. Umisedo, *Phys. Rev. D* **59**, 116010 (1999).
- [55] A. Höll, A. Krassnigg, and C. D. Roberts, *Nucl. Phys. B, Proc. Suppl.* **141**, 47 (2005).
- [56] H. J. Munczek and A. M. Nemirovsky, *Phys. Rev. D* **28**, 181 (1983).
- [57] C. D. Roberts (private communication).
- [58] H. S. Zong, L. Chang, F. Y. Hou, W. M. Sun, and Y. X. Liu, *Phys. Rev. C* **71**, 015205 (2005).
- [59] C. D. Roberts and S. M. Schmidt, *Prog. Part. Nucl. Phys.* **45**, S1 (2000).
- [60] A. Bender, W. Detmold, and A. W. Thomas, *Phys. Lett. B* **516**, 54 (2001).