Decays $\overline{B} \to D^{**}\pi$ and the Isgur-Wise functions $\tau_{1/2}(w)$, $\tau_{3/2}(w)$

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We perform a phenomenological analysis of the decays $\overline{B} \to D^{**}\pi$, where D^{**} is a P-wave excited meson with total angular momentum $j = \frac{1}{2}$ or $\frac{3}{2}$ for the light cloud, recently measured by the Belle Collaboration in the modes $\overline{B}^0 \to D^{**+}\pi^-$ (Class I) and $B^- \to D^{**0}\pi^-$ (Class III). Making the reasonable assumption of naive factorization, that we test in $B \to D(D^*)\pi$ decays, Class I decays allow to extract the Isgur-Wise form factors $\tau_{1/2}(w)$, $\tau_{3/2}(w)$ at $w \cong w_{\max}$ ($q^2 \cong 0$). We obtain $\tau_{1/2}(w_{\max}) < 0.20$, $\tau_{3/2}(w_{\max}) = 0.31 \pm 0.12$. We discuss the question of the w dependence of these IW functions. We find agreement with the Bakamjian-Thomas quark model of form factors and, extrapolating at $w = 1$, with Bjorken and Uraltsev sum rules. We discuss also Class III decays, where the D^{**0} ($j = \frac{1}{2}$) emission diagram contributes. We extract the corresponding $f_{D_{1/2}}$ decay constant, that is in agreement with theoretical estimates at finite mass. Finally, we must warn that $1/m_Q$ corrections could be large and upset the results of the present stage of this analysis. On the other hand, we confront present data on the semileptonic rate of B mesons to excited states with theoretical expectations.

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I. INTRODUCTION.

Our main purpose in this paper is to extract information on the lowest ($n = 0$) heavy quark Isgur-Wise functions $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$, that correspond to the transitions $\frac{1}{2} \to \frac{1}{2}^+$ or $\frac{3}{2}^+$, quantities that are of importance in heavy quark physics, for instance in small velocity sum rules (SR). We will first make a naive estimation of these quantities from $\overline{B}^0 \to D^{**+}\pi^-$ decays assuming factorization and the heavy quark limit. We will moreover discuss the more involved decays $B^- \to D^{**0}\pi^-$ in which there is D^{**0} emission, in particular, the sign of the interference between the π^- and D^{**0} emission diagrams.

There are four P-wave D^{**} mesons corresponding to the coupling of the total quark spin $S = 0, 1$ and the orbital momentum $\ell = 1$. In the language of the heavy quark limit, where the total angular momentum j of the light quark is a good quantum number ($j = \frac{1}{2}$ or $\frac{3}{2}$), these states can be denoted by D_J^j where J is the total angular momentum of the state. There are then four possibilities $(j, J) = (\frac{1}{2}, 0), (\frac{1}{2}, 1), (\frac{3}{2}, 1)$, or $(\frac{3}{2}, 2)$.

According to the states of charge, the $\overline{B} \to D^{**}\pi$ decays are of three classes, following the classification of B. Stech and collaborators [1,2]:

(i) $\overline{B}^0 \to D^{**+}\pi^-$ (Class I) where only the π emission diagram contributes;

(ii) $\overline{B}^0 \to D^{**0}\pi^0$ (Class II) where only the D^{**0} diagram contributes;

(iii) $B^- \to D^{**0}\pi^-$ (Class III), where both diagrams of π emission and D^{**0} emission contribute.

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It is worthy to recall that isospin symmetry relates the amplitudes of the three classes in this particular case, namely [3],

$$A(B^- \to D^{**0}\pi^-) = A(\overline{B}^0 \to D^{**+}\pi^-) - \sqrt{2}A(\overline{B}^0 \to D^{**0}\pi^0). \quad (1)$$

The Belle Collaboration has obtained in the past very interesting results on Class III decays $B^- \to D^{**0}\pi^-$ [4]. Four states were indeed observed, two narrow states corresponding to the $j = \frac{3}{2}$ states, that decay into $D(D^*)$ in the D-wave, and two very wide states that decay into $D(D^*)$ in the S-wave.

The interesting news is that recently, at the Beijing ICHEP 04 Conference, the Belle Collaboration has presented results on Class I decays $\overline{B}^0 \to D^{**+}\pi^-$ [5]. Interestingly, the narrow states have been observed, while only limits on the decays into wide states have been obtained, indicating a much smaller BR. The Belle data have recently drawn the attention of the theory [6].

An enormous theoretical effort has been dedicated in the last five years to the understanding of nonleptonic two-body B decays in the cases of the *emission* of a light meson like π or ρ in the so-called QCD factorization approach [7], in the perturbative QCD factorization approach [8] or within the soft collinear effective theory [9]. These methods have been applied to two-body decays into ground state mesons. In the present paper we are dealing with decays into excited D^{**} mesons, with both π or D^{**} emission diagrams. For Class I decays we have only the π emission diagram, and in this case we could in principle use the QCD methods of these papers. However, in Class III decays there is the diagram of D^{**} emission, a meson composed of heavy-light quarks, for which there are no rigorous results. Moreover, we are dealing with the first measurements of these decays that hopefully will be refined in the future. For these reasons, as a preliminary study, we will stick to the naive factorization approach [1,2] in order to investigate if there is a sensible description of the decays $\overline{B} \to D^{**}\pi$ within this simple phenomenological approach.

Class I decays and the π emission diagram of Class III are related to the $\overline{B} \to D^{**}\pi$ form factors that, in the heavy quark limit, reduce to two Isgur-Wise (IW) functions $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$ [10]. These form factors are of a significant theoretical importance, since they are related, at zero recoil $w = 1$, to the slope of the elastic IW function through Bjorken [10,11] and Uraltsev SR [12].

In a recent paper we have tried to use the Belle data on Class III decays to extract $\tau_{1/2}(w_{\max})$ and $\tau_{3/2}(w_{\max})$, where $w(q^2 \cong 0) \cong w_{\max}$ [13]. This calculation relied on a strong hypothesis, namely, that the diagram of D^{**0} emission should be small. We got some results on $\tau_{1/2}(w_{\max})$, $\tau_{3/2}(w_{\max})$ that were extrapolated to $\tau_{1/2}(1)$, $\tau_{3/2}(1)$ assuming the w-dependence of the form factors given by the Bakamjian-Thomas (BT) class of relativistic quark models that yield covariant form factors exhibiting heavy quark symmetry [14]. We obtained $\tau_{1/2}(1) \sim \tau_{3/2}(1)$, at odds with the expectations of Uraltsev SR.

However, to neglect the D^{**0} diagram is a rough approximation that could be unfounded [15]. It is well known that in some cases the color-suppressed diagrams like the D^{**0} emission one are often not as suppressed as one could expect on naive grounds. Therefore, our determination of $\tau_{1/2}(w_{\max})$, $\tau_{3/2}(w_{\max})$ has to be reconsidered using only Class I decays, now measured, and where only the π emission diagram contributes. Hence the interest of the new results on Class I decays is one on which we will first concentrate. Below we will come back to the interpretation of the results on Class III decays, taking into account D^{**0} emission. In what follows there is some unavoidable overlap with our Appendix B of Ref. [13] from which some points are worthy to be recalled.

The paper is organized as follows. In Sec. II we extract $\tau_{1/2}(w_{\max})$, $\tau_{3/2}(w_{\max})$ from Class I decays $\overline{B}^0 \to D^{**+}\pi^-$ using factorization, and discuss the question of the w-dependence and the extrapolation of $\tau_{1/2}(w)$, $\tau_{3/2}(w)$ at $w = 1$, the comparison with Bjorken and Uraltsev SR and with BT quark models. In Sec. III we combine the results of Sec. II with the measured rates of Class III decays $B^- \to D^{**0}\pi^-$ in order to extract the needed value for the decay constant $f_{D_{1/2}}$, nonvanishing in the heavy quark limit, and we compare with theoretical predictions. We treat with special care the question of the interference between the π^- and D^{**0} emission diagrams. In Sec. IV we make predictions for Class II decays $\overline{B}^0 \to D^{**0}\pi^0$. In Sec. V we discuss the implications for the semileptonic decays $\overline{B}^0 \to D^{**+}\ell^-\overline{\nu}$ and compare with the existing data, and in Sec. VI we conclude. In Appendix A we reproduce the Belle data for Class I and Class III decays to make clear in the text how we extract the rates $\overline{B} \to D^{**}\pi$ of the different modes and how we treat the experimental errors. In Appendix B we use the factorization model to compare with the data on $\overline{B} \to D(D^*)\pi$, where all modes have been measured, and we extract the effective coefficients a_1 and a_2 that enter in the factorization model. In Appendix C we discuss the corrections to factorization and finally Appendix D is devoted to the question of the $1/m_Q$ corrections to the heavy quark limit.

II. EXTRACTION OF $\tau_{1/2}(w_{\max})$, $\tau_{3/2}(w_{\max})$ FROM CLASS I DECAYS

Let us now consider the Class I decays measured by the Belle Collaboration, where for the wide states the masses of the results of Class III decays are assumed (Table V of Appendix A).

Assuming that these states decay essentially into two-body modes, i.e. $B(D_2^{3/2} \to (D + D^*)\pi)$, $B(D_1^{3/2} \to D^*\pi)$, $B(D_0^{1/2} \to D\pi)$, $B(D_1^{1/2} \to D^*\pi)$, the following branching ratios are given by a Clebsch-Gordan coefficient

$$B(D_1^{3/2+} \to D^{*0}\pi^+) = B(D_0^{1/2+} \to D^0\pi^+) = B(D_1^{1/2+} \to D^{*0}\pi^+) = \frac{2}{3}. \quad (2)$$

To estimate $B(D_2^{3/2+} \to D^0\pi^+)$ and $B(D_2^{3/2+} \to D^{*0}\pi^+)$, we use the spin counting of the nonrelativistic quark model. *In the limit of heavy quark symmetry*, i.e. assuming the pairs (D, D^*), $(D_2^{3/2}, D_1^{3/2})$, and $(D_1^{1/2}, D_0^{1/2})$ to be degenerate, simple angular momentum calculations give

$$\Gamma(D_2^{3/2}) = \Gamma(D_1^{3/2}) \qquad \Gamma(D_0^{1/2}) = \Gamma(D_1^{1/2}) \quad (3)$$

$$\Gamma(D_2^{3/2} \to D^*\pi) = \frac{3}{2}\Gamma(D_2^{3/2} \to D\pi). \quad (4)$$

This last relation gives the needed spin counting coefficient.

It is easy to obtain this factor by realizing that to have the D- wave one needs (1 denoting the quark emitting a pion) the operator (taking Oz along the pion momentum) to emit a pion in the D-wave $(\sigma_1^z k_\pi)\exp(iz_1 k_\pi) \to ik_\pi^2 \sigma_1^z z_1$. We have then, for the nonvanishing amplitudes

$$\begin{aligned} M(D_2^{3/2} \to D\pi) &= \langle 10, 10|20\rangle\langle 00|\sigma_1^z|10\rangle\langle 00|\mathcal{Y}_1^z|10\rangle \\ M(D_2^{3/2(\pm 1)} \to D^{*(\pm 1)}\pi) &= \langle 10, 1 \pm 1|2 \pm 1\rangle \\ &\quad \times \langle 1 \pm 1|\sigma_1^z|1 \pm 1\rangle\langle 00|\mathcal{Y}_1^z|10\rangle \end{aligned} \quad (5)$$

that gives

$$M(D_2^{3/2} \to D\pi) = \sqrt{\frac{2}{3}}\langle 00|\mathcal{Y}_1^z|10\rangle$$
$$M(D_2^{3/2(\pm1)} \to D^{*(\pm1)}\pi) = \pm\frac{1}{\sqrt{2}}\langle 00|\mathcal{Y}_1^z|10\rangle \tag{6}$$

and hence (4).

We now take into account the actual masses. Since both $D_2^{3/2} \to D\pi$ and $D_2^{3/2} \to D^*\pi$ proceed through the D-wave, we will have, in an obvious notation, in the isospin symmetry limit,

$$\frac{\Gamma(D_2^{3/2+} \to D^0\pi^+)}{\Gamma(D_2^{3/2+} \to D^{*0}\pi^+)} = \frac{\Gamma(D_2^{3/20} \to D^+\pi^-)}{\Gamma(D_2^{3/20} \to D^{*+}\pi^-)} = \frac{2}{3}\frac{p^5}{p^{*5}} \cong 2.5. \tag{7}$$

This estimation is in agreement with the present world averages [16]

$$\frac{\Gamma(D_2^{3/2+} \to D^0\pi^+)}{\Gamma(D_2^{3/2+} \to D^{*0}\pi^+)} = 1.9 \pm 1.1 \pm 0.3$$
$$\frac{\Gamma(D_2^{3/20} \to D^+\pi^-)}{\Gamma(D_2^{3/20} \to D^{*+}\pi^-)} = 2.3 \pm 0.6. \tag{8}$$

Therefore, we obtain the branching ratios

$$B(D_2^{3/2+} \to D^0\pi^+) \cong 0.48$$
$$B(D_2^{3/2+} \to D^{*0}\pi^+) \cong 0.19. \tag{9}$$

From these BR, adding the errors in quadrature, we find roughly

$$B(\overline{B}^0 \to D_2^{3/2+}\pi^-) = (6.4 \pm 0.8) \times 10^{-4} \quad (\text{from } D_2^{3/2+} \to D^0\pi^+)$$
$$B(\overline{B}^0 \to D_2^{3/2+}\pi^-) = (12.9 \pm 3.3) \times 10^{-4} \quad (\text{from } D_2^{3/2+} \to D^{*0}\pi^+) \tag{10}$$

We realize that the $B(\overline{B}^0 \to D_2^{3/2+}\pi^-)$ differs if one obtains it from $D_2^{3/2+} \to D^0\pi^+$ or from $D_2^{3/2+} \to D^{*0}\pi^+$, and the values are consistent only within 2σ. Using (2) for the other modes, and taking into account the large uncertainty from both results (10), one finds

$$B(\overline{B}^0 \to D_2^{3/2+}\pi^-) = (10.9 \pm 5.3) \times 10^{-4}$$
$$B(\overline{B}^0 \to D_1^{3/2+}\pi^-) = (5.5 \pm 1.4) \times 10^{-4}$$
$$B(\overline{B}^0 \to D_0^{1/2+}\pi^-) < 1.8 \times 10^{-4}$$
$$B(\overline{B}^0 \to D_1^{1/2+}\pi^-) < 1.0 \times 10^{-4}. \tag{11}$$

Assuming factorization of π^- emission, as it is reasonable within the QCD factorization scheme in the heavy quark limit [7], and assuming that the states 1^+ are unmixed, we find for the decay rates

$$\Gamma = \frac{G_F^2}{16\pi}|V_{cb}V_{ud}^*|^2\frac{p}{m_B^2}|M(B \to D^{**}\pi)|^2 \tag{12}$$

where

$$|M(\overline{B}^0 \to D_2^{3/2+}\pi^-)|^2 = 2m_Dm_B(m_B + m_D)^2(w_0^2 - 1)^2a_1^2f_\pi^2|\tau_{3/2}(w_0)|^2$$
$$|M(\overline{B}^0 \to D_1^{3/2+}\pi^-)|^2 = 2m_Dm_B(m_B - m_D)^2(w_0 + 1)^2(w_0^2 - 1)a_1^2f_\pi^2|\tau_{3/2}(w_0)|^2$$
$$|M(\overline{B}^0 \to D_1^{1/2+}\pi^-)|^2 = 4m_Dm_B(m_B - m_D)^2(w_0^2 - 1)a_1^2f_\pi^2|\tau_{1/2}(w_0)|^2$$
$$|M(\overline{B}^0 \to D_0^{1/2+}\pi^-)|^2 = 4m_Dm_B(m_B + m_D)^2(w_0 - 1)^2a_1^2f_\pi^2|\tau_{1/2}(w_0)|^2 \tag{13}$$

with

$$w_0 \cong \frac{m_B^2 + m_D^2}{2m_Bm_D} \tag{14}$$

the subindex 0 denoting the value of w for $q^2 = m_\pi^2 \cong 0$ and m_D the mass of the corresponding D_J^j state. The short distance QCD factor a_1 is close to 1 (Appendix B).

It is interesting to notice that the rates (12) and (13) are given, assuming the D^{**} for a given $j = \frac{1}{2}$ or $\frac{3}{2}$ to be degenerate, by the expressions

$$\Gamma(\overline{B}^0 \to D_2^{3/2+}\pi^-) = \Gamma(\overline{B}^0 \to D_1^{3/2+}\pi^-) = \frac{G_F^2}{16\pi}|V_{cb}V_{ud}^*|^2m_B^3a_1^2f_\pi^2\frac{(1-r)^5(1+r)^7}{16r^3}\left|\tau_{3/2}\left(\frac{1+r^2}{2r}\right)\right|^2 \tag{15}$$

$$\Gamma(\overline{B}^0 \to D_1^{1/2+}\pi^-) = \Gamma(\overline{B}^0 \to D_0^{1/2+}\pi^-) = \frac{G_F^2}{16\pi}|V_{cb}V_{ud}^*|^2m_B^3a_1^2f_\pi^2\frac{(1-r)^5(1+r)^3}{2r}\left|\tau_{1/2}\left(\frac{1+r^2}{2r}\right)\right|^2 \tag{16}$$

where $r = \frac{m_D^{3/2}}{m_B}$ and $r = \frac{m_D^{1/2}}{m_B}$ respectively in the first and second relations. The equalities $\Gamma(\overline{B} \to D_2^{3/2}\pi) = \Gamma(\overline{B} \to D_1^{3/2}\pi)$, $\Gamma(\overline{B} \to D_1^{1/2}\pi) = \Gamma(\overline{B} \to D_0^{1/2}\pi)$ follow from heavy quark symmetry since the B meson is spinless, there is a single helicity amplitude for each decay, and there is the emission of a longitudinally polarized D^{**}.

Using the central values for the masses, but taking into account the errors in (11) and $|V_{cb}| = 0.040 \pm 0.002$ (Appendix B), we find from the different modes,

$$\begin{aligned} &\overline{B} \to D_2^{3/2}\pi \qquad |\tau_{3/2}(1.31)| = 0.32 \pm 0.10 \\ &\overline{B} \to D_1^{3/2}\pi \qquad |\tau_{3/2}(1.32)| = 0.23 \pm 0.04 \\ &\overline{B} \to D_0^{1/2}\pi \qquad |\tau_{1/2}(1.37)| < 0.20 \\ &\overline{B} \to D_1^{1/2}\pi \qquad |\tau_{1/2}(1.32)| < 0.16. \end{aligned} \tag{17}$$

Within 1σ there is consistency between the different determinations of $|\tau_{3/2}(w_0)|$ and $|\tau_{1/2}(w_0)|$, but errors increase considering both determinations. Since besides the statistical errors there are systematic errors (from experiment and theory), we consider it safer to take the union of the domains (17) rather than their intersection. We conclude safely that we will have the numbers

$$|\tau_{3/2}(1.31)| = 0.31 \pm 0.12 \qquad |\tau_{1/2}(1.37)| < 0.20. \tag{18}$$

A. Extrapolation at $w = 1$ and comparison with Bjorken and Uraltsev sum rules

Our results have been obtained at $w_{\max}$. It is of interest to know the values of $\tau_{1/2}(w)$, $\tau_{3/2}(w)$ at $w = 1$. There are a number of values for the slopes of these IW functions in the literature.

We should keep in mind that we have two rather loose constraints on $\tau_{1/2}(w)$, $\tau_{3/2}(w)$, namely, the values at $w_{\max}$ (18) and the qualitative idea that the $n = 0$ IW functions should give a main contribution to Bjorken and Uraltsev sum rules.

Let us consider, *as an illustration*, the parametrization obtained within BT quark models (last reference [14])

$$\begin{aligned} \tau_{1/2}(w) &= \tau_{1/2}(1)\left(\frac{2}{w+1}\right)^{2\sigma_{1/2}^2} \qquad \sigma_{1/2}^2 = 0.83 \\ \tau_{3/2}(w) &= \tau_{3/2}(1)\left(\frac{2}{w+1}\right)^{2\sigma_{3/2}^2} \qquad \sigma_{3/2}^2 = 1.5 \end{aligned} \tag{19}$$

we obtain at zero recoil

$$|\tau_{3/2}(1)| = 0.46 \pm 0.18 \qquad |\tau_{1/2}(1)| < 0.26 \tag{20}$$

to be compared with the values in the BT model

$$|\tau_{3/2}(1)|^{\rm BT} = 0.54 \qquad |\tau_{1/2}(1)|^{\rm BT} = 0.22. \tag{21}$$

We find agreement for $|\tau_{3/2}(1)|$ within errors, and $|\tau_{1/2}(1)|$ could be consistent with the BT model.

Bjorken [10,11] and Uraltsev [12] sum rules write, respectively,

$$\begin{aligned} \rho^2 &= \frac{1}{4} + \sum_n |\tau_{1/2}^{(n)}(1)|^2 + 2\sum_n |\tau_{3/2}^{(n)}|^2 \\ \sum_n |\tau_{3/2}^{(n)}(1)|^2 &- \sum_n |\tau_{1/2}^{(n)}(1)|^2 = \frac{1}{4}. \end{aligned} \tag{22}$$

It is understood that these SR are truncated at some n that corresponds to a scale $\Delta \sim 1$ GeV and it is then natural to assume that the ground state dominates. Keeping thus the $n = 0$ states, with which we are dealing here, we get contributions to Bjorken and Uraltsev SR that lie in the following ranges:

$$\begin{aligned} &0.40 < \frac{1}{4} + |\tau_{1/2}(1)|^2 + 2|\tau_{3/2}(1)|^2 < 1.13 \\ &0.01 < |\tau_{3/2}(1)|^2 - |\tau_{1/2}(1)|^2 < 0.41. \end{aligned} \tag{23}$$

Therefore, the $n = 0$ states could give an important contribution to the SR and, considering this piece as dominant, low values for ρ^2 are not excluded nor the value $\frac{1}{4}$ for the right-hand side of Uraltsev SR.

There are other theoretical estimates of the IW functions $\tau_{1/2}(w)$, $\tau_{3/2}(w)$, mainly within the QCD sum rules approach [17–20]. The pioneering calculations of $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$ [18] show indeed that at large w the slope of $\tau_{1/2}(w)$ is much smaller than the one of $\tau_{3/2}(w)$, as in the BT model, while the values at $w = 1$ were found to be roughly equal for both IW functions, $\tau_{1/2}(1) = \tau_{3/2}(1) \cong 0.24$. On the other hand, next-to-leading calculations of the function $\tau_{1/2}(w)$ have later been performed [19], giving $\tau_{1/2}(1) = 0.35 \pm 0.10$ and a slope of the order $\sigma_{1/2}^2 = 0.5$, in our notation (19). In view of the importance of the corrections, it would be interesting to have the corresponding calculation for $\tau_{3/2}$. This latter value for $\tau_{1/2}(1)$ is larger than the value obtained in the present paper. On the other hand, in Ref. [20] there is a calculation of both $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$ ($\zeta(w) = 2\tau_{1/2}(w)$, $\tau(w) = \sqrt{3}\tau_{3/2}(w)$ [21]) and gives $\tau_{3/2}(1) = 0.43 \pm 0.08$, $\tau_{1/2}(1) = 0.13 \pm 0.04$ and the slopes $\sigma_{3/2}^2 = 0.90 \pm 0.05$, $\sigma_{1/2}^2 = 0.50 \pm 0.05$. These results imply a sizeable contribution to Uraltsev SR, are in agreement with the determinations of the present paper, and are qualitatively consistent with the BT model results (21) and with (19), $\tau_{3/2}(w)$ being steeper than $\tau_{1/2}(w)$. One should notice that a different interpolating field for $\tau_{1/2}$ is used by [20] from the one in [18,19], and that radiative corrections are absent. Recently, a lattice determination has obtained the values $\tau_{1/2}(1) = 0.38(5)$ and $\tau_{3/2}(1) = 0.53(8)$, with unknown systematic errors [22]. These values imply a sizeable contribution to Uraltsev SR. Compared with the BT determination (21), $\tau_{3/2}(1)$ is in fair agreement, while $\tau_{1/2}(1)$ is larger, and in agreement with the QCDSR determination [19]. *A fortiori*, this latter value is much larger than the QCDSR result [20],

TABLE I. The values at zero recoil $\tau_j(1)$ and slopes σ_j^2 ($j = \frac{1}{2}, \frac{3}{2}$) in the different theoretical approaches, compared with the phenomenological determination at $w_{\max}$ of the present paper, extrapolated at $w = 1$ with the slopes of the BT quark model (19).

Theoretical method	$\tau_{1/2}(1)$	$\sigma_{1/2}^2$	$\tau_{3/2}(1)$	$\sigma_{3/2}^2$
QCDSR (NLO) [19]	0.35 ± 0.10	0.5	$\cdots$	$\cdots$
QCDSR [20]	0.13 ± 0.04	0.50 ± 0.05	0.43 ± 0.08	0.90 ± 0.05
BT Quark Model [14]	0.22	0.83	0.54	1.5
Lattice [22]	0.38(5)	$\cdots$	0.53(8)	$\cdots$
Present paper	<0.26	0.83 (input)	0.46 ± 0.18	1.5 (input)

and also than the present phenomenological limit (18) obtained in the present paper, with the extrapolation from $w_{\max}$ assumed here. We summarize the situation in Table I.

III. INTERFERENCE WITH D^{**0} EMISSION IN CLASS III DECAYS

Let us now consider Class III decays measured by the Belle Collaboration, which we summarize in Table VI of Appendix A.

We have here the same BR as for the modes of Class I, namely,

$$\begin{aligned} B(D_1^{3/20} \to D^{*+}\pi^-) &= B(D_0^{1/20} \to D^+\pi^-) \\ &= B(D_1^{1/20} \to D^{*+}\pi^-) = \frac{2}{3} \\ B(D_2^{3/20} \to D^+\pi^-) &\cong 0.48 \\ B(D_2^{3/20} \to D^{*+}\pi^-) &\cong 0.19. \end{aligned} \tag{24}$$

We find, adding the errors in quadrature,

$$\begin{aligned} B(B^- \to D_2^{3/20}\pi^-) &= (7.1 \pm 1.6) \times 10^{-4} \\ &\quad (\text{from } D_2^{3/20} \to D^+\pi^-) \\ B(B^- \to D_2^{3/20}\pi^-) &= (9.5 \pm 2.5) \times 10^{-4} \\ &\quad (\text{from } D_2^{3/20} \to D^{*+}\pi^-). \end{aligned} \tag{25}$$

We realize that the values obtained for $B(B^- \to D_2^{3/20}\pi^-)$ from $D_2^{3/20} \to D^+\pi^-$ or $D_2^{3/20} \to D^{*+}\pi^-$ agree within 1σ. Using (24) for the other modes, and taking into account the uncertainty from both results (25) one finds

$$\begin{aligned} B(B^- \to D_2^{3/20}\pi^-) &= (8.7 \pm 3.2) \times 10^{-4} \\ B(B^- \to D_1^{3/20}\pi^-) &= (10.2 \pm 2.3) \times 10^{-4} \\ B(B^- \to D_0^{1/20}\pi^-) &= (9.1 \pm 2.9) \times 10^{-4} \\ B(B^- \to D_1^{1/20}\pi^-) &= (7.5 \pm 1.7) \times 10^{-4}. \end{aligned} \tag{26}$$

Comparing these BR with the corresponding Class I (11) we see that there is a large difference for the $j = \frac{1}{2}$ states, while there is consistency for the $j = \frac{3}{2}$ states. This is interesting and seems to indicate that the D^{**0} emission diagram could be very important [15]. This is likely because the decay constants of $j = \frac{1}{2}$ states do not vanish, while those of $j = \frac{3}{2}$ states vanish in the heavy quark limit [3,23,24]

$$f_{1/2} \neq 0 \qquad f_{3/2} = 0. \tag{27}$$

As demonstrated in [23], the equality $f_{3/2} = 0$ follows intuitively from the fact that the multiplet $j = \frac{3}{2}$ contains two states with $J = 1, 2$ and there is no current coupling the vacuum to $J = 2$. On the other hand, $f_{1/2} \neq 0$ follows because the $D^{**}(J = 0)$ is a system of widely unequal masses, and vector current conservation does not hold. We assume, following [15], that the decays $B^- \to D_0^{1/20}\pi^-$ and $B^- \to D_1^{1/20}\pi^-$ have a sizeable contribution from $D_J^{1/20}$ ($J = 0, 1$) emission via, respectively, the vector and axial current.

We now consider both diagrams for Class III decays and we will take care of the delicate question of the relative sign between the π emission and the D^{**0} emission diagrams.

The decays $B \to D_0^{1/2}\pi$ and $B \to D_1^{1/2}\pi$ are respectively S-wave parity conserving and P-wave parity violating. Let us define in a homogeneous way the needed matrix elements ($q = p - p'$) [23]. For π emission we need the current matrix elements

$$\begin{aligned} \langle \pi(q)|A_\mu|0\rangle &= f_\pi q_\mu \\ \langle D_0^{1/20}(p')|A_\mu|B^-(p)\rangle &= \sqrt{m_D m_B}\, 2(v' - v)_\mu \tau_{1/2}(w) \\ \langle D_1^{1/20}(p', \varepsilon)|V_\mu|B^-(p)\rangle &= \sqrt{m_D m_B}\, 2[(w-1)\varepsilon_\mu^* \\ &\quad - (\varepsilon^* \cdot v)v'_\mu]\tau_{1/2}(w) \end{aligned} \tag{28}$$

while for D^{**} emission [23,25]

$$\begin{aligned} \langle D_0^{1/20}(p')|V_\mu|0\rangle &= f_{D_{1/2}} p'_\mu \\ \langle D_1^{1/20}(p', \varepsilon)|A_\mu|0\rangle &= -f_{D_{1/2}} m_{D_{1/2}} \varepsilon_\mu^* \\ \langle \pi^-(q)|V_\mu|B^-(p)\rangle &= \left(p_\mu + q_\mu - \frac{m_B^2 - m_\pi^2}{p'^2} p'_\mu\right) f_+^{\pi B}(p'^2) \\ &\quad + \frac{m_B^2 - m_\pi^2}{p'^2} p'_\mu f_0^{\pi B}(p'^2). \end{aligned} \tag{29}$$

The minus sign for the definition of the $D_1^{1/2}$ decay constant comes from the Clebsch-Gordan convention of Isgur and Wise (coupling the orbital angular momentum $\ell = 1$ with the light quark spin $s_q = \frac{1}{2}$ to give $j = \frac{1}{2}$) that yields the definitions (28). From (29), as predicted by heavy quark symmetry, one obtains

$$\langle D_0^{1/20}(p')|A^3|B^-(p)\rangle = -\langle D_1^{1/20}(p',\varepsilon)|V^0|B^-(p)\rangle = \sqrt{m_D m_B}2v'^3\tau_{1/2}(w) \qquad (30)$$

and corresponds to the convention

$$S_3^c|D_1^{1/2+}\rangle = -|D_0^{1/2+}\rangle. \qquad (31)$$

Likewise, one must have

$$\langle D_0^{1/20}(p')|V^3|0\rangle = -\langle D_1^{1/20}(p',\varepsilon)|A^0|0\rangle = f_{D_{1/2}}p'^3. \qquad (32)$$

This is the convention that we have used in [23,25] (there is a misprint in formula (14) of [23], corrected in [25]).

We find for the rates (only one helicity amplitude contributes to the $B^- \to D_1^{1/20}\pi^-$ transition):

$$\Gamma(B^- \to D_0^{1/20}\pi^-) \cong \frac{G_F^2}{16\pi}|V_{cb}V_{ud}^*|^2\frac{p}{m_B^2}[a_1\sqrt{m_D m_B}2(m_B + m_D)(w_0 - 1)f_\pi\tau_{1/2}(w_0) + a_2 m_B^2 f_{D_{1/2}}f_0^{\pi B}(m_D^2)]^2 \qquad (33)$$

$$\Gamma(B^- \to D_1^{1/20}\pi^-) \cong \frac{G_F^2}{4\pi}|V_{cb}V_{ud}^*|^2\frac{p}{m_B^2}\frac{p^2}{m_D^2}[a_1\sqrt{m_D m_B}(m_B - m_D)f_\pi\tau_{1/2}(w_0) + a_2 m_B m_D f_{D_{1/2}}f_+^{\pi B}(m_D^2)]^2. \qquad (34)$$

We will use in these expressions the color-allowed and color-suppressed factors, respectively, of the order $a_1 \cong 1$, $a_2 \cong 0.3$ (B8). The powers of p indicate that the decays $\overline{B} \to D_0^{1/2}\pi$ and $\overline{B} \to D_1^{1/2}\pi$ occur, respectively, in the S and P waves.

The relative sign between both terms in (33) and (34) is crucial. Let us give an argument that shows that the interference is constructive. Instead of considering the π, let us consider the pseudoscalar D meson, composed of heavy-light quarks. Our assumption is that the form factors and decay constants between ground state mesons do not change sign when going from heavy mesons made of heavy-light quarks to light mesons made of equal mass quarks. This is a very sensible continuity hypothesis that is satisfied in the quark model, since there are no nodes in these ground state wave functions, and the extrapolation in reduced mass is smooth. On the other hand, this smooth continuity in mass is commonly used in lattice calculations, and it is also observed, considering, for example, the decay constants f_{D_s} or f_K and varying the c or s quark masses.

In [23,25], we did demonstrate (using duality in $B^0 - \overline{B}^0$ mixing and also within the operator product expansion) the following sum rules in the heavy quark limit of QCD for heavy-light form factors and decay constants, *valid for all values of w*:

$$\sum_n f^{(n)}\xi^{(n)}(w) = 2\sum_n f_{1/2}^{(n)}\tau_{1/2}^{(n)}(w) = f^{(0)} \qquad (35)$$

where n denotes a radial quantum number, $f^{(0)} = f$ is the ground state decay constant and $\xi^{(0)}(w) = \xi(w)$ the elastic Isgur-Wise function. The decay constants $f^{(n)}$ and $f_{1/2}^{(n)}$ scale like $\frac{1}{\sqrt{m_Q}}$. In particular, we have demonstrated that the rigorous SR (35) are satisfied within relativistic BT quark models and in the nonrelativistic quark model. Within BT quark models, we have shown that a main contribution to the SR (35) comes from the $n = 0$ states, that has the same sign as the whole sum [25] and the same is true in the nonrelativistic quark model,

$$\mathrm{Sign}[f\xi(w)] = \mathrm{Sign}[f_{1/2}\tau_{1/2}(w)], \qquad (36)$$

where we have used the notations $\tau_{1/2}^{(0)}(w) = \tau_{1/2}(w)$, $f_{1/2}^{(0)} = f_{1/2}$. Multiplying the equalities (36) by $\xi(w)\tau_{1/2}(w)$, we have $\mathrm{Sign}\{f\tau_{1/2}(w)[\xi(w)]^2\} = \mathrm{Sign}\{f_{1/2}\xi(w)[\tau_{1/2}(w)]^2\}$ and therefore

$$\mathrm{Sign}[f\tau_{1/2}(w)] = \mathrm{Sign}[f_{1/2}\xi(w)]. \qquad (37)$$

Heavy quark scaling implies for $\overline{B} \to D$ form factors:

$$\frac{\sqrt{4m_D m_B}}{m_D + m_B}f_+^{DB}(q^2) = \frac{\sqrt{4m_D m_B}}{m_D + m_B}\frac{f_0^{DB}(q^2)}{1 - \frac{q^2}{(m_D+m_B)2}} = \xi(w) \qquad (38)$$

and therefore $\mathrm{Sign}[f_+^{DB}(q^2)] = \mathrm{Sign}[f_0^{DB}(q^2)] = \mathrm{Sign}[\xi(w)]$. Our continuum assumption linking heavy-light mesons to light mesons implies then, within a definite phase convention for unequal masses:

$$\mathrm{Sign}[f_+^{\pi B}(q^2)] = \mathrm{Sign}[f_0^{\pi B}(q^2)] = \mathrm{Sign}[\xi(w)]$$
$$\mathrm{Sign}[f_D] = \mathrm{Sign}[f_\pi]. \qquad (39)$$

From (37) and (39) we get

$$\mathrm{Sign}[f_\pi\tau_{1/2}(w)] = \mathrm{Sign}[f_{1/2}f_+^{\pi B}(q^2)] = \mathrm{Sign}[f_{1/2}f_0^{\pi B}(q^2)]. \qquad (40)$$

Therefore, a relative constructive sign between the two contributions in (33) and (34) follows from (40).

We need some input on the form factors $f_0^{\pi B}(q^2)$ and $f_+^{\pi B}(q^2)$. We could use the simple theoretically motivated pole-dipole parametrization for $f_0^{\pi B}(q^2)$, $f_+^{\pi B}(q^2)$ of the large energy effective theory (LEET) [26]:

$$f_0^{\pi B}(q^2) = \left(1 - \frac{q^2}{m_B^2}\right) f_+^{\pi B}(q^2) \cong \frac{0.3}{1 - \frac{q^2}{m_B^2}}. \tag{41}$$

However, there is an empirical parametrization, inspired by (41), that fits the lattice data on these form factors, proposed by Becirevic and Kaidalov [27],

$$\begin{aligned} f_+^{\pi B}(q^2) &= \frac{c_B(1-\alpha_B)}{(1-\frac{q^2}{m_{B^*}^2})(1-\alpha_B\frac{q^2}{m_{B^*}^2})} \\ f_0^{\pi B}(q^2) &= \frac{c_B(1-\alpha_B)}{(1-\frac{q^2}{\beta_B m_{B(0^+)}^2})}. \end{aligned} \tag{42}$$

A fit to the lattice data [28] yields two sets of values for these parameters. We choose one of them, the other one yielding very comparable results

$$\begin{aligned} c_B = 0.51(8)(1) \qquad \alpha_B = 0.45(17)^{+0.06}_{-0.13} \\ \beta_B = 1.20(13)^{+0.15}_{-0.00} \end{aligned} \tag{43}$$

that corresponds to

$$f_+^{\pi B}(0) = f_0^{\pi B}(0) = 0.28(6)(5). \tag{44}$$

Concerning the QCD coefficient a_2, in Appendix B we have made an analysis of the well-measured decays $\overline{B} \to D(D^*)\pi$ in all its charged modes. Since the perturbative estimation of $a_2 \cong 0.2$ [29] appears to give too small Class II branching ratios, we have to consider, following [2,3], nonperturbative contributions to a_1 and a_2 (see the discussion in Appendices B and C), that yield the values

$$a_1 \cong 1 \qquad a_2 \cong 0.3. \tag{45}$$

Moreover, we adopt, like in Appendix B, the parametrization for the form factors $f_+^{\pi B}(q^2)$, $f_0^{\pi B}(q^2)$ given by (42)–(44).

Once we know the sign of the interference between the two terms in (33) and (34), we proceed as follows. We extract the decay constant $f_{D_{1/2}}$ from these formulas comparing to the Class I ones

$$\Gamma(B^- \to D_0^{1/20}\pi^-) = \frac{G_F^2}{16\pi}|V_{cb}V_{ud}^*|^2 \frac{p}{m_B^2}[a_1\sqrt{m_D m_B}2(m_B + m_D)(w_0 - 1)f_\pi \tau_{1/2}(w_0)]^2 \tag{46}$$

$$\Gamma(B^- \to D_1^{1/20}\pi^-) = \frac{G_F^2}{4\pi}|V_{cb}V_{ud}^*|^2 \frac{p}{m_B^2}\frac{p^2}{m_D^2}[a_1\sqrt{m_D m_B}(m_B - m_D)f_\pi \tau_{1/2}(w_0)]^2. \tag{47}$$

Adding the theoretical errors in quadrature and using the QCD coefficients (45), we find

$$\begin{aligned} f_{D_{1/2}} &= (206 \pm 120)\ \text{MeV} \quad (\text{from } \overline{B} \to D_0^{1/2}\pi) \\ f_{D_{1/2}} &= (196 \pm 93)\ \text{MeV} \quad (\text{from } \overline{B} \to D_1^{1/2}\pi). \end{aligned} \tag{48}$$

Both determinations are roughly consistent. In view of the large systematic uncertainties, we proceed as in (18), taking the union of both domains rather than the intersection. We thus keep the safe range

$$f_{D_{1/2}} = (206 \pm 120)\ \text{MeV}. \tag{49}$$

A. Comparison with theoretical estimates of $f_{D_{1/2}}$.

The value (49) is in reasonable agreement with the calculation of QCDSR [17] that gives, for decay constants of D mesons with 0^- and 0^+ quantum numbers, including $1/m_Q$ and α_s corrections, the following numbers

$$f_D = (195 \pm 20)\ \text{MeV} \qquad f_{D(0^+)} = (170 \pm 20)\ \text{MeV}. \tag{50}$$

There are also calculations within QCDSR in the heavy quark limit, without including α_s corrections [18], that give a larger value for $f_{D(0^+)}$, consistent within 1σ with (49)

$$\begin{aligned} \sqrt{m_D} f_D &= (0.21 \pm 0.03)\ \text{GeV}^{3/2} \\ \sqrt{m_{D(0^+)}} f_{D(0^+)} &= (0.46 \pm 0.06)\ \text{GeV}^{3/2}. \end{aligned} \tag{51}$$

Another estimation using QCDSR in the heavy quark limit [30] gives a larger value,

$$\sqrt{m_{D(0^+)}} f_{D(0^+)} = (0.570 \pm 0.08)\ \text{GeV}^{3/2} \tag{52}$$

and correcting for the $B\pi$ continuum [30] one gets the results

$$\sqrt{m_D} f_D = 0.35 \text{ GeV}^{3/2}$$
$$\sqrt{m_{D(0^+)}} f_{D(0^+)} = (0.36 \pm 0.10) \text{ GeV}^{3/2} \qquad (53)$$

that are consistent with (49).

Within the Bakamjian-Thomas class of relativistic quark models [14], the decay constants of heavy-light mesons in the heavy quark limit have been computed [25]. One finds heavy quark scaling $\sqrt{m_D} f_D =$ const. for the $D(D^*)$ states and for the doublet of $j^P = \frac{1}{2}^+$ states. Within the specific spectroscopy model of Godfrey and Isgur [31], one finds the following values for the lowest $n = 0$ states:

$$\sqrt{m_D} f_D = (0.670 \pm 0.020) \text{ GeV}^{3/2}$$
$$\sqrt{m_{D_{1/2}}} f_{D_{1/2}} = (0.640 \pm 0.020) \text{ GeV}^{3/2}. \qquad (54)$$

These values for $\frac{1}{2}^-$ and $\frac{1}{2}^+$ doublets are close. From the masses $m_{D(D^*)}$ and $m_{D_{1/2}}$ one finds, in the heavy quark limit

$$f_D \cong f_{D^*} \cong (474 \pm 14) \text{ MeV} \qquad (55)$$

$$f_{D_{1/2}} \cong (417 \pm 13) \text{ MeV}. \qquad (56)$$

These values for f_D, f_{D^*} are *much larger* than the estimations given by lattice QCD [32,33], even adding a 10% error due to quenching (used in Appendix B):

$$f_D = (216 \pm 36) \text{ MeV} \qquad f_{D^*} = (258 \pm 52) \text{ MeV}. \qquad (57)$$

For $f_{D(0^+)}$ one finds, in lattice QCD, keeping only the statistical error [34],

$$f_{D(0^+)} = (122 \pm 43) \text{ MeV}. \qquad (58)$$

This latter value is consistent with the value obtained in the quark model of Veseli and Dunietz [24]:

$$f_{D(0^+)} = (139 \pm 30) \text{ MeV}. \qquad (59)$$

These theoretical estimations of the $f_{D_{1/2}}$ or $f_{D(0^+)}$ decay constants, that become equal in the heavy quark limit, are not homogeneous in their methods. To make the panorama somewhat clearer, we summarize the results in Tables II and III. In Table II we give the results of the different methods *at finite mass*, together with the phenomenological determination of the present paper. In Table III we give the results of the methods in the heavy quark limit, dividing the invariant $\sqrt{m_Q} f_{1/2}$ by $\sqrt{m_{D(0^+)}}$ with $m_{D(0^+)} =$ 2290 MeV of the Belle experiment (Appendix A). Although, of course, this choice is somewhat arbitrary, one can thus qualitatively compare it with the finite mass results.

The table shows a very scattered set of results, but there is the general trend that the decay constant is much larger in the methods that use the heavy quark limit. The largest value is obtained by the BT models. The subleading correction is negative. Also, we observe that the phenomenological determination of the present paper, that has a large error, agrees within errors with the methods including finite mass corrections.

IV. PREDICTIONS FOR CLASS II DECAYS

Let us finish our discussion giving predictions for the rates of the color-suppressed decays, using the range (49)

TABLE II. Theoretical predictions for the decay constant $f_{D(0^+)}$ in the different methods, at *finite mass*, compared with the phenomenological determination of the present paper.

Theoretical method	$f_{D(0^+)}$
QCD Sum Rules (finite mass) [17]	(170 ± 20) MeV
Lattice QCD (finite mass) [34]	(122 ± 43) MeV
Veseli-Dunietz quark model (finite mass) [24]	(139 ± 30) MeV
Present phenomenological determination from $B \to D^{**}\pi$	(206 ± 120) MeV

TABLE III. Theoretical predictions for $\sqrt{m_Q} f_{1/2}$ in the heavy quark limit, divided by $\sqrt{m_{D(0^+)}}$, with $m_{D(0^+)} = 2290$ MeV of the Belle experiment.

Theoretical method	$\frac{\sqrt{m_Q} f_{1/2}}{\sqrt{m_{D(0^+)}}}$
QCD Sum Rules [18]	(304 ± 40) MeV
QCD Sum Rules [30]	(377 ± 53) MeV
QCD Sum Rules with correction for $B\pi$ continuum [30]	(238 ± 66) MeV
Bakamjian-Thomas quark model [25]	(417 ± 13) MeV

for the decay constant $f_{D_{1/2}}$. From the rates

$$\Gamma(\overline{B}^0 \to D_0^{1/20}\pi^0) = \frac{G_F^2}{16\pi}|V_{cb}V_{ud}^*|^2 \frac{p}{m_B^2}\frac{1}{2}[a_2 m_B^2 f_{D_{1/2}} f_0^{\pi B}(m_D^2)]^2$$
$$\Gamma(\overline{B}^0 \to D_1^{1/20}\pi^0) = \frac{G_F^2}{4\pi}|V_{cb}V_{ud}^*|^2 \frac{p}{m_B^2}\frac{p^2}{m_D^2}\frac{1}{2}[a_2 m_B m_D f_{D_{1/2}} f_+^{\pi B}(m_D^2)]^2 \tag{60}$$

we obtain the branching ratios

$$BR(\overline{B}^0 \to D_0^{1/20}\pi^0) = (2.8 \pm 2.0) \times 10^{-4}$$
$$BR(\overline{B}^0 \to D_1^{1/20}\pi^0) = (2.2 \pm 1.5) \times 10^{-4}. \tag{61}$$

The central values are large enough that they could, in principle, be measured. These rates are independent of the IW function $\tau_{1/2}(w)$, while they depend on the nonvanishing decay constant $f_{D_{1/2}}$.

Heavy quark symmetry plus factorization predicts

$$BR(\overline{B}^0 \to D_2^{3/20}\pi^0) = BR(\overline{B}^0 \to D_1^{3/20}\pi^0) = 0 \tag{62}$$

because of the vanishing of the $f_{3/2}$ decay constants (27). However, it is worth noticing that, because of the large experimental errors and theoretical uncertainties (spin counting, etc.), from the BR (26) and using the isospin relation (1) we can only have a rather loose upper bound

$$BR(\overline{B}^0 \to D_2^{3/20}\pi^0) < 4 \times 10^{-4}$$
$$BR(\overline{B}^0 \to D_1^{3/20}\pi^0) < 3 \times 10^{-4}. \tag{63}$$

V. THE RATE TO EXCITED STATES IN SEMILEPTONIC *B* DECAYS.

The values of the functions $\tau_{1/2}(w)$, $\tau_{3/2}(w)$ at $w = 1$ and their w-dependence give predictions for the semileptonic (SL) decay $\overline{B} \to D^{**}\ell\nu$ branching ratios in the heavy quark limit. The differential decay rates for $\overline{B} \to D_J^j$ ($j = \frac{1}{2}, \frac{3}{2}$)$\ell\nu$ write [25]

$$\frac{d\Gamma(B \to D_2^{3/2}\ell\nu)}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2 2r^3(w+1)(w^2-1)^{3/2}[(w+1)(1-r)^2 + 3w(1+r^2-2rw)]|\tau_{3/2}(w)|^2$$
$$\frac{d\Gamma(B \to D_1^{3/2}\ell\nu)}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2 2r^3(w+1)(w^2-1)^{3/2}[(w-1)(1+r)^2 + w(1+r^2-2rw)]|\tau_{3/2}(w)|^2$$
$$\frac{d\Gamma(B \to D_1^{1/2}\ell\nu)}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2 4r^3(w-1)(w^2-1)^{1/2}[(w-1)(1+r)^2 + 4w(1+r^2-2rw)]|\tau_{1/2}(w)|^2$$
$$\frac{d\Gamma(B \to D_0^{1/2}\ell\nu)}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2 4r^3(w^2-1)^{3/2}(1-r)^2|\tau_{1/2}(w)|^2 \tag{64}$$

where $r = \frac{m_D}{m_B}$ and D denotes the corresponding D_J^j meson.

For completeness, we write down the corresponding formulas for the ground state:

$$\frac{d\Gamma(B \to D\ell\nu)}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2 r^3(w^2-1)^{3/2}(1+r)^2|\xi(w)|^2$$
$$\frac{d\Gamma(B \to D^*\ell\nu)}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2 r^3(1+w)(w^2-1)^{1/2}[(w+1)(1-r)^2 + 4w(1+r^2-2rw)]|\xi(w)|^2. \tag{65}$$

The situation is given in Table IV, a slight modification of the predictions of Ref. [25].

To make predictions for the SL rates we need an input on the IW functions $\tau_{1/2}(w)$, $\tau_{3/2}(w)$. First, we must take into account that it is reasonable to expect that the $n = 0$ IW functions give a sizeable contribution to Bjorken and Uraltsev SR. Making this assumption, Uraltsev SR is very constraining on the difference $|\tau_{3/2}(1)|^2 - |\tau_{1/2}(1)|^2$, that should be not far away from $\frac{1}{4}$. This is the case for the BT model values (21), and, more importantly, consistent with the values that we have found from nonleptonic decays (20) using the Belle data. As an example, we will then adopt the values $|\tau_{3/2}(1)|$ and $|\tau_{1/2}(1)|$ given by the BT model (21) and allow nevertheless a $\pm 50\%$ departure for the values of the slopes. The lower values of the slopes would be in agreement with Ref. [20]. This gives the range of model predictions for SL rates in the table.

Although the $B \to D_J^{3/2}\ell\nu$ rates are in reasonable agreement, for the $B \to D_J^{1/2}\ell\nu$ rates there is a problem. We find $BR[\overline{B} \to (D_0^{1/2} + D_1^{1/2})\ell\nu] = (1.2 \pm 0.4) \times 10^{-3}$. The

TABLE IV. Comparison between rates for $\overline{B} \to D_J^j \ell \nu$ decays and the model described in the text. The data are from (a) ALEPH [35,36], (b) DELPHI [36–38], and (c) CLEO [39] experiments. For the elastic IW function we adopt the values (B5) and (B6) of Appendix B. Wide stands for unidentified $(D + D^*)\pi$ events forming a wide bump. In the text we discuss a new DELPHI analysis [40] and very recent data from the Belle Collaboration [41]

Semileptonic mode	Experiment	Model
$\overline{B} \to D\ell\nu$	$(2.14 \pm 0.20) \times 10^{-2}$	$(1.95 \pm 0.45) \times 10^{-2}$
$\overline{B} \to D^*\ell\nu$	$(5.44 \pm 0.23) \times 10^{-2}$	$(5.90 \pm 1.10) \times 10^{-2}$
$\overline{B} \to D_2^{3/2}\ell\nu$	(a) $(2.4 \pm 1.1) \times 10^{-3}$ (b) $(4.4 \pm 2.4) \times 10^{-3}$ (c) $(3.0 \pm 3.4) \times 10^{-3}$	$(6.3^{+3.0}_{-2.0}) \times 10^{-3}$
$\overline{B} \to D_1^{3/2}\ell\nu$	(a) $(7.0 \pm 1.6) \times 10^{-3}$ (b) $(6.7 \pm 2.1) \times 10^{-3}$ (c) $(5.6 \pm 1.6) \times 10^{-3}$	$(4.0^{+1.2}_{-1.4}) \times 10^{-3}$
$\overline{B} \to D_1^{1/2}\ell\nu$	(b) $(2.3 \pm 0.7) \times 10^{-2}$ (wide $D^{**} \to (D + D^*)\pi$)	$(6 \pm 2) \times 10^{-4}$
$\overline{B} \to D_0^{1/2}\ell\nu$		$(6 \pm 2) \times 10^{-4}$

DELPHI experiment gives, once the $BR[\overline{B} \to (D_2^{3/2} + D_1^{3/2})\ell\nu]$ is subtracted, a large branching ratio of B decays into "wide" D^{**} mesons decaying into $(D + D^*)\pi$. We will call this branching ratio $BR_{\rm wide}^{\rm DELPHI}[\overline{B} \to (D + D^*)\pi\ell\nu] = (2.3 \pm 0.7) \times 10^{-2}$, which we report in the last line of Table IV. On the one hand, this BR is 1 order of magnitude larger than our prediction for $BR[\overline{B} \to (D_0^{1/2} + D_1^{1/2})\ell\nu]$. On the other hand, keeping only to the DELPHI experiment, one obtains the sum $BR[\overline{B} \to (D + D^*)\ell\nu] + BR[\overline{B} \to (D_2^{3/2} + D_1^{3/2})\ell\nu] + BR_{\rm wide}[\overline{B} \to (D+ D^*)\pi\ell\nu] = (11 \pm 1.6) \times 10^{-2}$, that are already saturated with errors, however large, total semileptonic width $BR[\overline{B} \to \ell\nu + \text{anything}] = (10.73 \pm 0.28) \times 10^{-2}$. There are therefore two problems. On the one hand, other wide states besides $D_0^{1/2} + D_1^{1/2}$ have to contribute to $BR_{\rm wide}[\overline{B} \to (D + D^*)\pi\ell\nu]$. These could be radial excitations or higher orbital excitations, that are in principle allowed due to the large phase space available. The experimental width is so large that it includes high masses. Moreover, the multiplicity of higher excitations grows with the mass. On the other hand, it is curious that considering only the modes $\overline{B} \to (D + D^*)\ell\nu$, $\overline{B} \to (D + D^*)\pi\ell\nu$ the total semileptonic width is already saturated, and one could wonder why there is no place for decays into multipion modes $\overline{B} \to (D + D^*) + n\pi$ $(n > 1)$ and why they are not observed. Presumably, due to phase space, these could not come from modes of the type $D\rho$ but could come from various $D^{**}\pi$.

A recent new analysis by DELPHI [40] confirms and makes more precise this situation. The total branching ratio into D^{**} (narrow and broad) is measured to be

$$BR(\overline{B}^0 \to D^{**}\ell\overline{\nu}) = (2.7 \pm 0.7 \pm 0.2)\% \qquad (66)$$

with the decay final states dominated by the $D(D^*)\pi$ channels. The dominant contributing channel is a broad state decaying into $D^*\pi$, i.e. a state D^{**} or $J^P = 1^+$ with a mass $M = 2445 \pm 34 \pm 10$ MeV and a width $\Gamma = 234 \pm 74 \pm 25$ MeV. On the other hand, broad $D\pi$ states favor a production with a maximum close to threshold. Moreover, DELPHI bounds the branching ratios into $D\pi\pi$ and $D^*\pi\pi$ final states.

Very interesting recent data have been published by Belle at the last Lepton-Photon Conference [41], that gives the following branching ratios

$$\begin{aligned} BR(B^- \to D^+\pi^-\ell^-\overline{\nu}) &= (0.54 \pm 0.07 \pm 0.07 \pm 0.06) \times 10^{-2} \\ BR(B^- \to D^{*+}\pi^-\ell^-\overline{\nu}) &= (0.67 \pm 0.11 \pm 0.09 \pm 0.03) \times 10^{-2} \\ BR(\overline{B}^0 \to D^0\pi^+\ell^-\overline{\nu}) &= (0.33 \pm 0.06 \pm 0.06 \pm 0.03) \times 10^{-2} \\ BR(\overline{B}^0 \to D^{*0}\pi^+\ell^-\overline{\nu}) &= (0.65 \pm 0.12 \pm 0.08 \pm 0.05) \times 10^{-2}. \end{aligned} \qquad (67)$$

These values are to be compared with the theoretical expectations of Table IV. We find, considering only the central values of the model and taking into account the relevant branching fractions of the different D_J^j computed in Sec. II,

$$\begin{aligned} BR(B^- \to D^+\pi^-\ell^-\overline{\nu}) &= BR(\overline{B}^0 \to D^0\pi^+\ell^-\overline{\nu}) = 0.34 \times 10^{-2} \\ BR(B^- \to D^{*+}\pi^-\ell^-\overline{\nu}) &= BR(\overline{B}^0 \to D^{*0}\pi^+\ell^-\overline{\nu}) = 0.43 \times 10^{-2} \end{aligned} \qquad (68)$$

In view of the uncertainties, there is a fair agreement between our predictions and the Belle data.

In conclusion, there seems to be a potential problem concerning the DELPHI semileptonic data on $B \to (D_1^{1/2} + D_0^{1/2})\ell\nu$ decays that should be addressed in future experiments. If $BR_{\rm wide}[B \to (D + D^*)\pi\ell\nu]$ had to be attributed to $D_1^{1/2} + D_0^{1/2}$, then $\tau_{1/2}(1)$ would be much larger than $\tau_{3/2}(1)$, in contradiction with the expectations of Uraltsev SR. This is at odds with the Belle nonleptonic data studied in the present paper. The study of the D_J^j wide states is not an easy experimental task. A recent Tevatron D0 experiment sees clearly the narrow states $j = \frac{3}{2}$ in SL B decays, but has not given a measurement of the wide ones, $j = \frac{1}{2}$ [42]. On the other hand, we find agreement with the very recent Belle data on $\overline{B} \to D(D^*)\pi\ell\nu$.

VI. CONCLUSION

In conclusion, we have shown within a simple factorization model, tested in the well-measured $B \to D(D^*)\pi$ decays, that one can extract information on the Isgur-Wise functions at zero recoil $\tau_{1/2}(1)$, $\tau_{3/2}(1)$ from nonleptonic data on $\overline{B}^0 \to D^{**+}\pi^-$ (Class I). Combining with $B^- \to D^{**0}\pi^-$ (Class III), one can obtain the nonvanishing decay constant $f_{D_{1/2}}$ of $D^{**}(j = \frac{1}{2})$. Special care has been taken in the determination of the interference sign between the π and D^{**} emission diagrams. The ranges obtained for $\tau_{1/2}(1)$, $\tau_{3/2}(1)$ are consistent for both types of modes, with the expectations of Bjorken and Uraltsev sum rules, and with the predictions of the Bakamjian-Thomas quark model of form factors. Moreover, the range of values found for the decay constant $f_{D_{1/2}}$ agrees with most theoretical expectations. We predict sizeable rates of the Class II decays $\overline{B}^0 \to D^{**0}\pi^0$ that could be measured in the near future. On the contrary, for $D^{**}(j = \frac{3}{2})$, Class II decays should be suppressed due to the vanishing of the decay constant $f_{3/2}$ in the heavy quark limit.

We must warn that $1/m_Q$ corrections could be large and could upset the results of the present stage of this analysis, as discussed in Appendix D. Also, we point out a problem with present DELPHI data of semileptonic decays for the total rate to excited states. Very recent Belle data on $\overline{B} \to D^{(*)}\pi\ell\nu$ seem in good agreement with the theoretical expectations.

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Note added in proof—When this paper was completed, we took notice of the related works of H.-Y. Cheng and C.-W. Hwang [46].

APPENDIX A: BELLE DATA ON $\overline{B} \to D^{**}\pi$ DECAYS

For the sake of clarity on how we extract the branching ratios of the different $\overline{B} \to D^{**}\pi$ decay modes, and how we handle the errors in the text, we reproduce here the Belle data on Class I [5] and Class III decays [4].

APPENDIX B: TESTING THE FACTORIZATION MODEL IN $\overline{B} \to D(D^*)\pi$

In this appendix, in order to check qualitatively our simple factorization model applied to $\overline{B} \to D^{**}\pi$ decay, we use it to describe the well-measured decays into the

TABLE V. Data of the Belle Collaboration for the masses, widths, and branching ratios to D^{**} states D_J^j for Class I decays $\overline{B}^0 \to D^{**+}\pi^-$ [5].

$M_2^{3/2}$	$(2459.5 \pm 2.3 \pm 0.7^{+4.9}_{-0.5})$ MeV
$\Gamma_2^{3/2}$	$(48.9 \pm 5.4 \pm 4.2 \pm 1.0)$ MeV
$B(\overline{B}^0 \to D_2^{3/2\,+}\pi^-)B(D_2^{3/2\,+} \to D^0\pi^+)$	$(3.08 \pm 0.33 \pm 0.09^{+0.15}_{-0.02}) \times 10^{-4}$
$B(\overline{B}^0 \to D_2^{3/2\,+}\pi^-)B(D_2^{3/2\,+} \to D^{*0}\pi^+)$	$(2.45 \pm 0.42^{+0.35+0.39}_{-0.45-0.17}) \times 10^{-4}$
$M_1^{3/2}$	$(2428.2 \pm 2.9 \pm 1.6 \pm 0.6)$ MeV
$\Gamma_1^{3/2}$	$(34.9 \pm 6.6^{+4.1}_{-0.9} \pm 4.1)$ MeV
$B(\overline{B}^0 \to D_1^{3/2\,+}\pi^-)B(D_2^{3/2\,+} \to D^{*0}\pi^+)$	$(3.68 \pm 0.60^{+0.71+0.65}_{-0.40-0.30}) \times 10^{-4}$
$M_0^{1/2}$	$(2290 \pm 22 \pm 20)$ MeV
$\Gamma_0^{1/2}$	$(276 \pm 21 \pm 18 \pm 60)$ MeV
$B(\overline{B}^0 \to D_0^{1/2\,+}\pi^-)B(D_0^{1/2\,+} \to D^0\pi^+)$	$<1.2 \times 10^{-4}$
$M_1^{1/2}$	$(2428 \pm 2.9 \pm 1.6 \pm 0.6)$ MeV
$\Gamma_1^{1/2}$	$(380 \pm 100 \pm 100)$ MeV
$B(\overline{B}^0 \to D_1^{1/2\,+}\pi^-)B(D_1^{1/2\,+} \to D^{*0}\pi^+)$	$<0.7 \times 10^{-4}$

TABLE VI. Data of the Belle Collaboration for the masses, widths, and branching ratios to D^{**} states D_J^j for Class III decays $B^- \to D^{**0}\pi^-$ [4].

$M_2^{3/2}$	$(2461.6 \pm 2.1 \pm 0.5 \pm 3.3)$ MeV
$\Gamma_2^{3/2}$	$(45.6 \pm 4.4 \pm 6.5 \pm 1.6)$ MeV
$B(B^- \to D_2^{3/2\,0}\pi^-)B(D_2^{3/2\,0} \to D^+\pi^-)$	$(3.4 \pm 0.3 \pm 0.6 \pm 0.4) \times 10^{-4}$
$B(B^- \to D_2^{3/2\,0}\pi^-)B(D_2^{3/2\,0} \to D^{*+}\pi^-)$	$(1.8 \pm 0.3 \pm 0.3 \pm 0.2) \times 10^{-4}$
$M_1^{3/2}$	$(2421.4 \pm 1.5 \pm 0.4 \pm 0.8)$ MeV
$\Gamma_1^{3/2}$	$(23.7 \pm 2.7 \pm 0.2 \pm 4.0)$ MeV
$B(B^- \to D_1^{3/2\,0}\pi^-)B(D_1^{3/2\,0} \to D^{*+}\pi^-)$	$(6.8 \pm 0.7 \pm 1.3 \pm 0.3) \times 10^{-4}$
$M_0^{1/2}$	$(2308 \pm 17 \pm 15 \pm 28)$ MeV
$\Gamma_0^{1/2}$	$(276 \pm 21 \pm 18 \pm 60)$ MeV
$B(B^- \to D_0^{1/2\,0}\pi^-)B(D_0^{1/2\,0} \to D^+\pi^-)$	$(6.1 \pm 0.6 \pm 0.9 \pm 1.6) \times 10^{-4}$
$M_1^{1/2}$	$(2427.0 \pm 26 \pm 20 \pm 15)$ MeV
$\Gamma_1^{1/2}$	$(384^{+107}_{-75} \pm 24 \pm 70)$ MeV
$B(B^- \to D_1^{1/2\,0}\pi^-)B(D_1^{1/2\,0} \to D^{*+}\pi^-)$	$(5.0 \pm 0.4 \pm 1.0 \pm 0.4) \times 10^{-4}$

ground state $\overline{B} \to D(D^*)\pi$. The data of the Particle Data Group [16] is given in Table VI, together with our predictions that follow from the following simple formulas.

From the definitions

$$\begin{gathered}\langle\pi^-(q)|A_\mu|0\rangle = f_\pi q_\mu \qquad \langle D^0(p')|A_\mu|B^-(p)\rangle = \sqrt{m_D m_B}\xi(w)(v+v')_\mu \\ \langle D^{*0}(p',\varepsilon)|V_\mu|B^-(p)\rangle = \sqrt{m_D m_B}\xi(w)[(w+1)\varepsilon^*_\mu - (\varepsilon^*\cdot v)v'_\mu] \\ \langle D^0(p')|A_\mu|0\rangle = f_D p'_\mu \qquad \langle D^{*0}(p',\varepsilon)|V_\mu|0\rangle = f_D m_{D^*}\varepsilon^*_\mu \\ \langle\pi^-(q)|V_\mu|B^-(p)\rangle = \left(p_\mu + q_\mu - \frac{m_B^2-m_\pi^2}{p'^2}p'_\mu\right)f_+^{\pi B}(p'^2) + \frac{m_B^2-m_\pi^2}{p'^2}p'_\mu f_0^{\pi B}(p'^2)\end{gathered} \tag{B1}$$

we obtain the matrix elements, satisfying the isospin relation (1),

$$\begin{gathered}A(B^- \to D^0\pi^-) = a_1\sqrt{m_D m_B}(m_B - m_D)(w_0+1)f_\pi\xi(w_0) + a_2 m_B^2 f_D f_0^{\pi B}(m_D^2) \\ A(\overline{B}^0 \to D^+\pi^-) = a_1\sqrt{m_D m_B}(m_B - m_D)(w_0+1)f_\pi\xi(w_0) \qquad A(\overline{B}^0 \to D^0\pi^0) = -a_2\frac{1}{\sqrt{2}}m_B^2 f_D f_0^{\pi B}(m_D^2)\end{gathered} \tag{B2}$$

$$\begin{gathered}A(B^- \to D^{*0}\pi^-) = \frac{p}{m_{D^*}}[a_1\sqrt{m_{D^*}m_B}(m_B + m_{D^*})f_\pi\xi(w_0) + a_2 2m_{D^*}m_B f_{D^*}f_+^{\pi B}(m_{D^*}^2)] \\ A(\overline{B}^0 \to D^{*+}\pi^-) = a_1\frac{p}{m_{D^*}}\sqrt{m_{D^*}m_B}(m_B + m_{D^*})f_\pi\xi(w_0) \qquad A(\overline{B}^0 \to D^{*0}\pi^0) = -a_2\frac{1}{\sqrt{2}}\frac{p}{m_{D^*}}2m_{D^*}m_B f_{D^*}f_+^{\pi B}(m_{D^*}^2).\end{gathered} \tag{B3}$$

For the decay constants f_D, f_{D^*} we use the values of lattice calculations within the quenched approximation $f_D = 216(11)(5)$ MeV [32], $f_{D^*} = 258(14)(6)$ MeV [33], where the first error is statistical and the second is systematic. Assuming a 10% uncertainty due to the quenching approximation and adding all the errors in quadrature, we adopt the values

$$\begin{gathered}f_D = (0.216 \pm 0.036)\ \text{GeV} \\ f_{D^*} = (0.258 \pm 0.052)\ \text{GeV}.\end{gathered} \tag{B4}$$

For the form factors $f_0^{\pi B}(q^2)$, $f_+^{\pi B}(q^2)$ we use (42)–(44), and for the Isgur-Wise function, we use the parametrization given by the BT model (last reference of [14]),

$$\xi(w) \cong \left(\frac{2}{w+1}\right)^{2\rho^2} \tag{B5}$$

that we have used in [43] to fit Belle data on $B \to D^*\ell\nu$ [44], that gives $\mathcal{F}^*(1)|V_{cb}| = 0.036 \pm 0.002$ and $\rho^2 = 1.15 \pm 0.18$. In conclusion, from $\mathcal{F}^*(1) \cong 0.91$, we adopt the ranges

$$|V_{cb}| = 0.040 \pm 0.002 \qquad \rho^2 = 1.15 \pm 0.18. \tag{B6}$$

We add the theoretical errors in quadrature, that gives, in amplitude, a 10% error for Class I decays and a 30% error

TABLE VII. Data on the branching ratios of $B \to D(D^*)\pi$ from PDG 2004 [16], and the predictions of the factorization model with the perturbative values $a_1 \cong 1$, $a_2 \cong 0.2$. The errors in the predictions come from the uncertainty on the decay constants of $D(D^*)$ mesons, the value of $|V_{cb}|$, the Isgur-Wise function, and the form factors $f_+^{\pi B}(q2)$, $f_0^{\pi B}(q2)$ (42)–(44). The theoretical errors are added in quadrature.

Modes $B \to D(D^*)\pi$	Experiment	Factorization $a_1 \cong 1$, $a_2 \cong 0.2$
$BR(\overline{B}^0 \to D^+\pi^-)$	$(2.76 \pm 0.25) \times 10^{-3}$	$(3.4 \pm 0.7) \times 10^{-3}$
$BR(\overline{B}^0 \to D^0\pi^0)$	$(2.7 \pm 0.8) \times 10^{-4}$	$(0.6 \pm 0.4) \times 10^{-4}$
$BR(B^- \to D^0\pi^-)$	$(4.98 \pm 0.29) \times 10^{-3}$	$(5.2 \pm 1.0) \times 10^{-3}$
$BR(\overline{B}^0 \to D^{*+}\pi^-)$	$(2.76 \pm 0.21) \times 10^{-3}$	$(3.3 \pm 0.7) \times 10^{-3}$
$BR(\overline{B}^0 \to D^{*0}\pi^0)$	$(2.7 \pm 0.5) \times 10^{-4}$	$(0.8 \pm 0.5) \times 10^{-4}$
$BR(B^- \to D^{*0}\pi^-)$	$(4.6 \pm 0.4) \times 10^{-3}$	$(5.3 \pm 1.0) \times 10^{-3}$

for Class II decays. Adopting the values

$$a_1 \cong 1, \qquad a_2 \cong 0.2 \tag{B7}$$

obtained by the perturbative calculations [29], and considering the uncertainties given in (B4) and (B6), we obtain the predictions of Table VII.

A first remark on the experimental data of Table VII is that the isospin relation (1) is roughly satisfied by real amplitudes, i.e. without the need of final-state interactions (FSI) phases, like in the naive factorization model. However, it is clear from this table that Class II decays are underestimated using the perturbative value $a_2 \cong 0.2$. There is no theoretical reason, unlike the case of Class I decays, in which there is emission of the light meson π [7], to have approximate factorization in the case of $D(D^*)$ emission. There are nonperturbative corrections to factorization, as pointed out in Appendix C, that suggest an effective value for a_2. In Table VIII we give the results for

$$a_1 \cong 1, \qquad a_2 \cong 0.3 \tag{B8}$$

that agree with the data within errors.

Now Class II decays are in better agreement and the overall picture seems reasonable. In the estimation of the $\overline{B} \to D^{**}\pi$ decays in Sec. III we adopt the values (B8) for a_1 and a_2.

APPENDIX C: CORRECTIONS TO FACTORIZATION IN $\overline{B} \to D^{**}\pi$ DECAYS

In the simple-minded factorization approach one adopts the perturbative Wilson coefficients $a_1 \cong 1$, $a_2 \cong 0.2$. Moreover, in this approach, the amplitudes of Class II decays to $j = \frac{3}{2}$ states vanish in the heavy quark limit $A(\overline{B}^0 \to D_{3/2}^{**0}\pi^0) = 0$ due to the vanishing of the decay constant $f_{3/2}$.

Let us here discuss how the analysis would be modified by taking into account nonperturbative corrections to factorization, following Neubert [3]. In Ref. [3] only the decays $\overline{B} \to D_{3/2}^{**}\pi$ are discussed. We extend the formalism to $\overline{B} \to D_{1/2}^{**}\pi$.

In the case of the $j = \frac{1}{2}$ states D^{**} emission is allowed. Therefore, we write the amplitudes of Class I and Class II decays $\overline{B} \to D_{1/2}^{**}\pi$ ($J = 0, 1$) in the form

$$\begin{aligned} A(\overline{B}^0 \to D_J^{1/2+}\pi^-) &= a_1^{\mathrm{eff},1/2} A_{\mathrm{fact}}(\overline{B}^0 \to D_J^{1/2+}\pi^-) \\ A(\overline{B}^0 \to D_J^{1/20}\pi^0) &= a_2^{\mathrm{eff},1/2} A_{\mathrm{fact}}(\overline{B}^0 \to D_J^{1/2+}\pi^-) \end{aligned} \tag{C1}$$

with

$$\begin{aligned} a_1^{\mathrm{eff},1/2} &= \left[c_1(\mu) + \frac{c_2(\mu)}{N_c}\right][1 + \varepsilon_1^{1/2}(\mu)] + c_2(\mu)\varepsilon_8^{1/2}(\mu) \\ a_2^{\mathrm{eff},1/2} &= \left[c_2(\mu) + \frac{c_1(\mu)}{N_c}\right][1 + \varepsilon_1^{1/2}(\mu)] + c_1(\mu)\varepsilon_8^{1/2}(\mu) \end{aligned} \tag{C2}$$

where the hadronic parameters $\varepsilon_1^{1/2}$, $\varepsilon_8^{1/2}$ describing the nonfactorizable contributions are given by the matrix elements

TABLE VIII. Same as in Table VII with the effective values $a_1 \cong 1$, $a_2 \cong 0.3$.

Modes $B \to D(D^*)\pi$	Experiment	Factorization $a_1 \cong 1$, $a_2 \cong 0.3$
$BR(\overline{B}^0 \to D^+\pi^-)$	$(2.76 \pm 0.25) \times 10^{-3}$	$(3.4 \pm 0.7) \times 10^{-3}$
$BR(\overline{B}^0 \to D^0\pi^0)$	$(2.7 \pm 0.8) \times 10^{-4}$	$(1.3 \pm 0.9) \times 10^{-4}$
$BR(B^- \to D^0\pi^-)$	$(4.98 \pm 0.29) \times 10^{-3}$	$(6.0 \pm 1.2) \times 10^{-3}$
$BR(\overline{B}^0 \to D^{*+}\pi^-)$	$(2.76 \pm 0.21) \times 10^{-3}$	$(3.3 \pm 0.7) \times 10^{-3}$
$BR(\overline{B}^0 \to D^{*0}\pi^0)$	$(2.7 \pm 0.5) \times 10^{-4}$	$(1.7 \pm 1.1) \times 10^{-4}$
$BR(B^- \to D^{*0}\pi^-)$	$(4.6 \pm 0.4) \times 10^{-3}$	$(6.3 \pm 1.3) \times 10^{-3}$

$$\langle D_J^{1/2+}\pi^-|(\bar{d}u)(\bar{c}b)|\bar{B}^0\rangle = [1+\varepsilon_1^{1/2}(\mu)] \times A_{\rm fact}(\bar{B}^0 \to D_J^{1/2+}\pi^-)$$
$$\langle D_J^{1/2+}\pi^-|(\bar{d}t^a u)(\bar{c}t^a b)|\bar{B}^0\rangle = \frac{1}{2}\varepsilon_8^{1/2}(\mu) \times A_{\rm fact}(\bar{B}^0 \to D_J^{1/2+}\pi^-). \quad \text{(C3)}$$

We make explicit the upper script $j=\frac{1}{2}$, since the situation is quite different for the decays into $j=\frac{3}{2}$ states. Following [3], we consider the large-N_c counting rules

$$c_1 = 1 + O(1/N_c^2) \qquad c_2 = O(1/N_c)$$
$$\varepsilon_1 = O(1/N_c^2) \qquad \varepsilon_8 = O(1/N_c). \quad \text{(C4)}$$

Keeping the terms up to order $1/N_c$ included, one finds

$$a_1^{\rm eff,1/2}(\mu) \cong a_1^{\rm pert}(\mu) \cong 1$$
$$a_2^{\rm eff,1/2}(\mu) \cong a_2^{\rm pert}(\mu) + \varepsilon_8^{1/2}(\mu) \cong 0.2 + \varepsilon_8^{1/2}(\mu). \quad \text{(C5)}$$

The departures relative to the naive approximation presented above are given by the nonperturbative coefficient $\varepsilon_8^{1/2}(\mu)$. These quantities do not affect Class I decays, but only Class II and Class III [cf. the isospin relation (1)].

In the analysis of Appendix B on the well-measured $B \to D(D^*)\pi$ decays, we have found $a_1^{\rm eff} \cong 1$ and $a_2^{\rm eff} \cong 0.3$. Although there is no firm theoretical argument, we have adopted in the text the same values, i.e.,

$$a_1^{\rm eff,1/2} \cong 1 \qquad a_2^{\rm eff,1/2} \cong 0.3. \quad \text{(C6)}$$

Going now to the case of $j=\frac{3}{2}$ states, we define

$$A(\bar{B}^0 \to D_J^{3/2+}\pi^-) = a_1^{\rm eff,3/2} A_{\rm fact}(\bar{B}^0 \to D_J^{3/2+}\pi^-)$$
$$-\sqrt{2}(\bar{B}^0 \to D_J^{3/20}\pi^0) = \varepsilon_8^{3/2} A_{\rm fact}(\bar{B}^0 \to D_J^{3/2+}\pi^-) \quad \text{(C7)}$$

where, due to (C4),

$$a_1^{\rm eff,3/2} = \left[c_1(\mu) + \frac{c_2(\mu)}{N_c}\right][1+\varepsilon_1^{3/2}(\mu)] + c_2(\mu)\varepsilon_8^{3/2}(\mu) \cong a_1^{\rm pert}(\mu) \cong 1. \quad \text{(C8)}$$

The hadronic coefficients $\varepsilon_1^{3/2}(\mu)$, $\varepsilon_8^{3/2}(\mu)$ are defined as in (C3). We have kept the notation $\varepsilon_8^{3/2}$ in (C7) because, switching off nonperturbative corrections, this coefficient does not have as a limit a nonvanishing perturbative coefficient, unlike $a_1^{\rm eff,3/2}$. Since the amplitude $A(\bar{B}^0 \to D_J^{3/20}\pi^0)$ vanishes in the heavy quark limit, because $f_{3/2}=0$, the amplitude chosen for the normalization is the one that is allowed, the π^- emission one.

Taking into account a nonvanishing coefficient $\varepsilon_8^{3/2}$ in (C7),

$$\Gamma(\bar{B}^0 \to D_2^{3/20}\pi^0) = \Gamma(\bar{B}^0 \to D_1^{3/20}\pi^0) = \frac{G_F^2}{16\pi}|V_{cb}V_{ud}^*|^2 \frac{1}{2}[\varepsilon_8^{3/2}]^2 m_B^3 f_\pi^2 \frac{(1-r)^5(1+r)^7}{16r^3}\left|\tau_{3/2}\left(\frac{1+r^2}{2r}\right)\right|^2 \quad \text{(C9)}$$

where $r=\frac{m_D^{3/2}}{m_B} \cong 0.46$. From the upper limits (63) and the central value (18) for $\tau_{3/2}(\frac{1+r^2}{2r})$ we can infer an upper limit for $|\varepsilon_8^{3/2}|$, namely,

$$|\varepsilon_8^{3/2}| < 0.90. \quad \text{(C10)}$$

As expected, since the upper bounds (63) are rather loose, we obtain a large upper bound on $|\varepsilon_8^{3/2}|$.

APPENDIX D: REMARKS ON $1/m_Q$ CORRECTIONS

Using the formalism of [21] and assuming factorization, one can in principle compute the analytical expressions of the $1/m_Q$ ($Q=b$ or c) corrections to the rates $B \to D_J^j\pi$. Let us consider Class I decays, $\bar{B}^0 \to D_J^{j+}\pi^-$. Many subleading form factors contribute and, although a theoretical effort has been made in their estimation for the $j=\frac{3}{2}$ states within the QCD sum rules approach [45], we do not have presently at our disposal an estimation of all the subleading form factors defined in [21]. Therefore, we are not able at present to make an estimation of these corrections. However, a *formal* expansion can be done for these decays to pions, and subleading quantities can be estimated in some approximation, as we explain now.

Let us consider the most important contributions at $w = w_{\max} = w_0$, that correspond to $q^2 \cong 0$, the value for pion decays:

$$w \cong w_0 = \frac{m_B^2 + m_D^2}{2m_B m_D} = \frac{1+r^2}{2r} \cong 1.3 \quad \text{(D1)}$$

where m_D is the mass of the corresponding D_J^j meson and $r = m_D/m_B$. Therefore, one can express all the mass factors in terms of w_0 and a common overall scale.

Then, for π decays, using (D1), there are two small parameters that characterize the corrections to the rates, namely,

$$w_0 - 1 \cong 0.3 \quad \text{and} \quad \frac{\Lambda_{\rm QCD}}{2m_Q} \quad (Q=c,b). \quad \text{(D2)}$$

It is therefore convenient to classify the subleading corrections to the rates as being of successive orders

$$(w_0-1)^s\left(\frac{\Lambda_{\rm QCD}}{2m_Q}\right)^t. \quad \text{(D3)}$$

We decide to retain only the subleading orders contributing to the rate ($t = 1$) with the dominant order in $(w_0 - 1)$, namely, $s = -1/2$.

Using the formulas of Ref. [21], neglecting higher orders of the type (D3), and keeping the dominant order $(w_0 - 1)^{-1/2}(\frac{\Lambda_{QCD}}{2m_Q})$, we find

$$\Gamma(\overline{B}^0 \to D_J^{j+}\pi^-) \cong \Gamma_0(\overline{B}^0 \to D_J^{j+}\pi^-)(1 + \delta_J^j) \quad \text{(D4)}$$

with

$$\delta_2^{3/2} = 0 \qquad \delta_1^{3/2} = \frac{2\sqrt{2}\Delta E_{3/2}}{\sqrt{w_0 - 1}}\frac{1}{2m_c}$$
$$\delta_1^{1/2} = \frac{\sqrt{2}\Delta E_{1/2}}{\sqrt{w_0 - 1}}\left(\frac{3}{2m_b} - \frac{1}{2m_c}\right) \quad \text{(D5)}$$
$$\delta_0^{1/2} = \frac{3\sqrt{2}\Delta E_{1/2}}{\sqrt{w_0 - 1}}\left(\frac{1}{2m_b} + \frac{1}{2m_c}\right)$$

and Γ_0 denotes the leading rate. Numerically, some of these terms of order $1/m_Q$ are not small, as they are, respectively, of the order

$$\delta_2^{3/2} = 0 \qquad \delta_1^{3/2} \cong 0.7 \qquad \delta_1^{1/2} \cong 0 \qquad \delta_0^{1/2} \cong 1.3 \quad \text{(D6)}$$

for $\Delta E_{3/2} \cong \Delta E_{1/2} \cong 0.4$ GeV, $m_c \cong 1.5$ GeV, $m_b \cong 4.8$ GeV, and $w_0 - 1 \cong 0.3$.

For $j = \frac{3}{2}$ states, the trend is not in the right direction to explain the different central values (11). This gives an idea of the type of uncertainties induced by the $1/m_Q$ corrections that are large. One can guess that the extraction of $|\tau_{1/2}(w_0)|^2$, $|\tau_{3/2}(w_0)|^2$ made in Sec. II is uncertain by about a factor 2 due to these corrections. Therefore, one could assume a reasonable additional 40% uncertainty on the values (18) given for $|\tau_{1/2}(w_0)|$, $|\tau_{3/2}(w_0)|$.

Although these estimations give large corrections, this impression could be wrong, since many subleading form factors contribute [21] and we do not know the magnitude or sign of the neglected terms. Moreover, we have taken only the leading order in the expansion (D3). The aim of this exercise has been to emphasize that the corrections in $1/m_Q$ are possibly large. If this was actually the case, this would upset the results of the present stage of this analysis.

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