Scalar nonet quarkonia and the scalar glueball: Mixing and decays in an effective chiral approach

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We study the strong and electromagnetic decay properties of scalar mesons above 1 GeV within a chiral approach. The scalar-isoscalar states are treated as mixed states of quarkonia and glueball configurations. A fit to the experimental mass and decay rates listed by the Particle Data Group is performed to extract phenomenological constraints on the nature of the scalar resonances and to the issue of the glueball decays. A comparison to other experimental results and to other theoretical approaches in the scalar meson sector is discussed.

I. INTRODUCTION

The unique interpretation of scalar mesons constitutes an unsolved problem of hadronic QCD. Below the mass scale of 2 GeV various scalar states [1] are encountered: the isoscalar resonances $\sigma = f_0(400-1200)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, the isovectors $a_0(980)$ and $a_0(1450)$ and the isodoublets $K_0^*(800)$ and $K_0^*(1430)$. The existence of the $K_0^*(800)$ is still not well established and omitted from the summary tables of [1]. From a theoretical point of view one expects the scalar quark-antiquark ground-state nonet 0^{++}, a scalar-isoscalar glueball, which lattice QCD predicts to be the lightest gluonic meson with a mass between $1.4-1.8$ GeV [2], and possibly other exotic states (non $\bar{q}q$ states), like e.g. four-quark states or mesonic molecules [3]. Various interpretations of and assignments for the physical scalar resonances in terms of the expected theoretical states have been proposed (see, for instance, the review papers [3–5] and references therein).

In this work we follow the original assignment of Ref. [6], where in a minimal scenario the bare quarkonia states $N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) = \bar{n}n$, $S \equiv \bar{s}s$ and the bare scalar glueball G mix, resulting in the three scalar-isoscalar resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. Such a mixing scheme has been previously investigated by many authors, as, for example, in the lattice study of [7] or within the model approaches of [8–13].

The mesons $a_0(1450)$ and $K_0^*(1430)$ are considered as the $I = 1$ and $I = 1/2$ quarkonia $J^{PC} = 0^{++}$ states. In this way the low-lying scalar-quarkonia nonet is located in the energy range of 1–2 GeV, where the other p-wave nonets of tensor (2^{++}) and pseudovector mesons (1^{++}) [1] are also situated. Masses of selected scalar-quarkonia states were also estimated on the Lattice; in Ref. [14] the $I = 1$ scalar quarkonium state is predicted to have a mass of $M_{a_0} = 1.51 \pm 0.19$. This result favors the interpretation of the state $a_0(1450)$ [and not $a_0(980)$] as the isovector ground-state scalar quarkonium. However, previous lattice studies (as in [15]; see also [14,16] and references therein) find different results. Scalar resonances below 1 GeV can

possibly be interpreted as four-quark states or mesonic molecules. In [17] the hypothesis of Jaffe's four-quark states was studied on the Lattice, where the masses of four-quark states are found to be lighter than 1 GeV. Recent attempts in the context of chiral perturbation theory to describe the scalar states below 1 GeV as "dynamically generated" resonances, i.e. states which do not survive in the large N_c limit [18,19], have also been performed. In this scheme the nature of the scalars below 1 GeV can also be related to four-quark configurations. This can be viewed as a further indication that scalar-quarkonia states are located at masses above 1 GeV.

In this work, starting from an effective chiral Lagrangian [13] derived in Chiral Perturbation Theory (ChPT) [20–22], we perform a tree-level analysis of the strong and electromagnetic decays of scalar mesons settled in the energy range between 1 and 2 GeV. The scalar glueball is introduced as an extra-flavor-singlet composite field with independent couplings to pseudoscalar mesons (and to photons, although suppressed). Although a chiral approach cannot be rigorously justified at this energy scale, since loop corrections could be large, we intend to use this framework as a phenomenological tool to extract possible glueball-quarkonia mixing scenarios from the observed decays.

The scalar glueball G mixes with the scalar-quarkonia fields $N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) = \bar{n}n$ and $S \equiv \bar{s}s$ in accord with flavor blindness. We also consider in this scheme a possible direct mixing of the quarkonia fields N and S. The origin of such mixing can be driven by instantons [23]. The presence of a (even small) flavor mixing in the scalar-isoscalar sector can sensibly affect the phenomenology.

In the presented approach the glueball decay into two-pseudoscalar mesons is occurring by two mechanisms: (a) through mixing, that is the glueball G acquires a quark-onium component, which subsequently decays into two pseudoscalars; (b) direct decay of the glueball component G into two pseudoscalars without an intermediate scalar quarkonium [6,8,10]. In [6,10] this direct decay is argued to be suppressed as based on arguments of the strong coupling expansion, while in the phenomenological fit of

[8] it dominates. Here we also address the question, if the direct decay is needed to explain the decay phenomenology of the scalar-isoscalar resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$.

For consistency we also consider the strong decays of the isovector and isodoublet scalar states as well, trying to highlight the difficulties and the open issues, also comparing with previous works on this subject. Following the idea of [20] we analyze deviations from the large N_c limit in the framework of the proposed scalar-quarkonia assignment (which differs from [20]), which turn out to be relatively small in the phenomenology.

The paper is organized as follows: in Sec. II we discuss the effective Lagrangian related to the scalar-quarkonia-glueball mass spectrum and the strong and electromagnetic decays. In Sec. III we determine a phenomenological fit to the experimental data listed in [1] by first neglecting the direct decay of the glueball component. In Sec. IV we also allow for a direct glueball decay studying its influence on the results. There we use the lattice data of [24], where an approximate calculation of the glueball decays into two-pseudoscalar mesons has been performed, to constrain our analysis. Finally, in Section V we summarize our results and draw conclusions.

II. THE MODEL

A. The Lagrangian

The strong and electromagnetic decays of scalar mesons are based on an effective chiral Lagrangian $\mathcal{L}_{\text{eff}}$ as derived in chiral perturbation theory (ChPT) [20–22]. The Lagrangian involves the nonets of pseudoscalar and of scalar mesons,

$$\mathcal{P} = \frac{1}{\sqrt{2}} \sum_{i=0}^{8} P_i \lambda_i, \qquad S = \frac{1}{\sqrt{2}} \sum_{i=0}^{8} S_i \lambda_i, \qquad (1)$$

the electromagnetic field and, in addition, a new degree of freedom, the bare glueball field G, which is treated as a flavor-blind mesonic field. The lowest order effective Lagrangian $\mathcal{L}_{\text{eff}}$ in the large N_c limit including $1/N_c$ corrections reads

$$\mathcal{L}_{\text{eff}} = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger + \chi_+ \rangle + \frac{1}{2} \langle D_\mu S D^\mu S - M_S^2 S^2 \rangle$$
$$+ \frac{1}{2} (\partial_\mu G \partial^\mu G - M_G^2 G^2) + c_d^s \langle S u_\mu u^\mu \rangle$$
$$+ c_m^s \langle S \chi_+ \rangle + \frac{c_d^g}{\sqrt{3}} G \langle u_\mu u^\mu \rangle + \frac{c_m^g}{\sqrt{3}} G \langle \chi_+ \rangle$$
$$+ c_e^s \langle S F_{\mu\nu}^+ F^{+\mu\nu} \rangle + \frac{c_e^g}{\sqrt{3}} G \langle F_{\mu\nu}^+ F^{+\mu\nu} \rangle + \mathcal{L}_{\text{mix}}^P$$
$$+ \mathcal{L}_{\text{mix}}^S. \qquad (2)$$

Here the symbol $\langle ... \rangle$ denotes the trace over flavor matrices. The constants c_d^s, c_m^s, c_d^g, c_m^g, c_e^s and c_e^g define the coupling

of scalar fields and of the bare glueball to pseudoscalar mesons and photons, respectively. We use the standard notation for the basic blocks of the ChPT Lagrangian [21]: $U = u^2 = \exp(i \mathcal{P} \sqrt{2}/F)$ is the chiral field collecting pseudoscalar fields in the exponential parametrization, D_μ denotes the chiral and gauge-invariant derivative, $u_\mu = i u^\dagger D_\mu U u^\dagger$ is the chiral field, $\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$, $\chi = 2B(s + ip)$, $s = \mathcal{M} + \ldots$ and $F_{\mu\nu}^+ = u^\dagger F_{\mu\nu} Q u + u F_{\mu\nu} Q u^\dagger$, where $F_{\mu\nu}$ is the stress tensor of the electromagnetic field. The charge and the mass matrix of current quarks are denoted by $Q = e\,\text{diag}\{2/3, -1/3, -1/3\}$ and $\mathcal{M} = \text{diag}\{\hat{m}, \hat{m}, m_s\}$ (we restrict to the isospin symmetry limit with $m_u = m_d = \hat{m}$); B is the quark vacuum condensate parameter and F the pion decay constant.

The masses of the octet pseudoscalar mesons in the leading order of the chiral expansion (first term of the Lagrangian) are given by

$$M_\pi^2 = 2\hat{m}B, \qquad M_K^2 = (\hat{m} + m_s)B,$$
$$M_{\eta^8}^2 = \tfrac{2}{3}(\hat{m} + 2m_s)B. \qquad (3)$$

The contribution to the mass of η^0 in leading order is

$$M_{\eta^0}^2 = \tfrac{2}{3}(2\hat{m} + m_s)B, \qquad (4)$$

i.e. η^0 is a Goldstone boson in the large N_c and in the chiral limits.

Following [22], we encode in $\mathcal{L}_{\text{mix}}^P$ an extra-contribution to the mass of η^0 (due to the axial anomaly) and the η^0-η^8 mixing term:

$$\mathcal{L}_{\text{mix}}^P = -\tfrac{1}{2} \gamma_P (\eta^0)^2 - z_P \eta^0 \eta^8, \qquad (5)$$

(the parameters γ_P and z_P are in turn related to the parameters M_{η_1} and $\tilde{d}_m$ of [22]). The physical diagonal states η and η' are given by

$$\eta^0 = \eta' \cos\theta_P - \eta \sin\theta_P, \qquad \eta^8 = \eta' \sin\theta_P + \eta \cos\theta_P, \qquad (6)$$

where θ_P is the pseudoscalar mixing angle. We follow the standard procedure [20,22,25,26] and diagonalize the corresponding η^0-η^8 mass matrix to obtain the masses of η and η'. By using $M_\pi = 139.57$ MeV, $M_K = 493.677$ MeV (the physical charged pion and kaon masses), $M_\eta = 547.75$ MeV and $M_{\eta'} = 957.78$ MeV the mixing angle is determined as $\theta_P = -9.95°$, which corresponds to the tree-level result (see details in Ref. [25]). Correspondingly one finds $M_{\eta^0} = 948.10$ MeV and $z_P = -0.105$ GeV^2. Higher-order corrections in ChPT cause a doubling of the absolute value of the pseudoscalar mixing angle [25]); in our work we restrict to the tree-level evaluation, we therefore consistently use the corresponding tree-level result of $\theta_P = -9.95°$. In the present approach we do not include the neutral pion when considering mixing in the pseudoscalar sector, because we work in the isospin limit. This mixing is small, and can be safely neglected

when studying the decay of scalar resonances into two pseudoscalars. Similarly, for all pseudoscalar mesons we use the unified leptonic decay constant F, which is identified with the pion decay constant $F = F_\pi = 92.4$ MeV. A more accurate analysis including higher orders should use the individual couplings of the pseudoscalar mesons (for a detailed discussion see Refs. [27]).

B. Scalar-quarkonia-glueball mixing

In this subsection we discuss the glueball-quarkonia mixing. For this reason we restrict to the following part of the effective Lagrangian (2):

$$\mathcal{L}^S = \tfrac{1}{2}\langle D_\mu S D^\mu S - M_S^2 S^2 \rangle + \tfrac{1}{2}(\partial_\mu G \partial^\mu G - M_G^2 G^2) + \mathcal{L}^S_{\text{mix}} \tag{7}$$

where, besides the scalar-quarkonia nonet, also the scalar glueball field G has been introduced. In the large N_c limit all the states of the quarkonia nonet have the same nonet mass M_S and the glueball is decoupled from the quarkonia sector. Deviations from this limit are encoded in $\mathcal{L}^S_{\text{mix}}$, where the glueball-quarkonia mixing is introduced and where the degeneracy of the nonet states is lifted.

For what concerns the explicit mass term and the next-to-leading order $1/N_c$ terms in the quarkonia sector we follow [20]. Including additionally a possible breaking of the Gell-Mann-Okubo (GMO) mass relation and the glueball-quarkonia mixing under the hypothesis of flavor blindness, we have

$$\mathcal{L}^S_{\text{mix}} = e_m^S \langle S^2 \chi_+ \rangle + k_m^S S_0 \langle \chi_+ \rangle - \gamma_{S_0} \frac{M_{S_0}^2}{2} S_0^2$$
$$- \gamma_{S_8} \frac{M_{S_8}^2}{2} S_8^2 - \sqrt{3} f G S_0. \tag{8}$$

The parameter e_m^S describes the strength of flavor-symmetry breaking derived from the nonzero values of the current quark masses, the parameters k_m^S and γ_{S_0} describe the order $1/N_c$ terms. The parameter γ_{S_8} and the related term in the Lagrangian is not derived in the N_c expansion (it is in fact absent in [20]), but it describes violations of the GMO mass formula (indeed a result from higher orders in the chiral expansion). Finally, the parameter f is the glueball-quarkonia mixing strength. Note that G, being a flavor singlet, couples only to the flavor-singlet quarkonium state S_0 in the flavor-blind mixing limit. The glueball and the quarkonia sector decouple in the large N_c limit [28]: a nonzero mixing, as described by a nonvanishing parameter f, takes into account a possible deviation from the large N_c limit, as the parameters k_m^S and γ_{S_0} in the quarkonia sector.

The terms contained in $\mathcal{L}^S_{\text{mix}}$ lead to mass shifts of the nonet masses, while also introducing mixing both among quarkonia states and in the glueball-quarkonia sector. With the use of Eqs. (7) and (8) the explicit expression of the

Lagrangian $\mathcal{L}^S$ reads

$$\mathcal{L}^S = -\tfrac{1}{2}\vec{a}_0(\Box + M_{a_0}^2)\vec{a}_0 - K_0^{*\dagger}(\Box + M_{K_0^*}^2)K_0^*$$
$$- \tfrac{1}{2}G(\Box + M_G^2)G - \tfrac{1}{2}M_{S_8}^2 S_8^2 - \tfrac{1}{2}M_{S_0}^2 S_0^2 - z_S S_0 S_8$$
$$- \sqrt{3} f G S_0, \tag{9}$$

where $\vec{a}_0$ is the isovector collecting $a_0^\pm$ and a_0^0 fields; K_0^* and $K_0^{*\dagger}$ are the doublets of (K_0^{*+}, K_0^{*0}) and $(K_0^{*-}, \bar{K}_0^{*0})$ mesons, respectively, (see II A), and where the corresponding masses are given by [see Eqs. (7) and (8)]:

$$M_{a_0}^2 = M_S^2 - 4e_m^S M_\pi^2,$$
$$M_{K_0^*}^2 = M_S^2 - 4e_m^S M_K^2,$$
$$M_{S_8}^2 = M_S^2(1 + \gamma_{S_8}) - \frac{4}{3} e_m^S(4M_K^2 - M_\pi^2),$$
$$M_{S_0}^2 = M_S^2(1 + \gamma_{S_0}) - \frac{4}{3} e_m^S(2M_K^2 + M_\pi^2),$$
$$z_S = \frac{8\sqrt{2}}{3}\left(e_m^S + \frac{\sqrt{3}}{2}k_m^S\right)(M_K^2 - M_\pi^2). \tag{10}$$

By inverting we get

$$M_S^2 = M_{a_0}^2 + \frac{M_\pi^2(M_{a_0}^2 - M_{K_0^*}^2)}{(M_K^2 - M_\pi^2)},$$
$$e_m^S = \frac{M_{a_0}^2 - M_{K_0^*}^2}{4(M_K^2 - M_\pi^2)},$$
$$\gamma_{S_8} = \frac{M_{S_8}^2 - M_S^2 + \frac{4}{3} e_m^S(4M_K^2 - M_\pi^2)}{M_S^2},$$
$$\gamma_{S_0} = \frac{M_{S_0}^2 - M_S^2 + \frac{4}{3} e_m^S(2M_K^2 + M_\pi^2)}{M_S^2},$$
$$k_m^S = \frac{2}{\sqrt{3}}\left(\frac{3z_S}{8\sqrt{2}(M_K^2 - M_\pi^2)} - e_m^S\right). \tag{11}$$

We can immediately deduce M_S and e_m^S for the considered assignment (which differs from [20]) by using the experimental masses $M_{a_0} = M_{a_0(1450)} = 1.474 \pm 0.019$ GeV and $M_{K_0^*} = M_{K_0^*(1430)} = 1.416 \pm 0.006$ GeV [1]:

$$M_S = 1.479 \text{ GeV}, \qquad e_m^S = 0.199. \tag{12}$$

Note that e_m^S is positive, contrary to the other nonets [20]. The numerical values for the other constants k_m^S, γ_{S_0} and γ_{S_8} (which should be small if the large N_c limit and chiral symmetry still applies approximately for the scalar nonet) are determined by a fit to data.

Finally we discuss the mass relation in the octet sector. From the first three equations in (10) we have

$$3M_{S_8}^2 = 4M_{K_0^*}^2 - M_{a_0}^2 + 3\gamma_{S_8} M_S^2, \tag{13}$$

where the term proportional to γ_{S_8} includes deviations form the GMO limit.

Glueball-quarkonia mixing is usually set up in the basis of fields $N \equiv \sqrt{1/2}(\overline{u}u + \overline{d}d) \equiv \overline{n}n$ and $S \equiv \overline{s}s$ instead of the octet and singlet fields S_0 and S_8. The connection is given by

$$S_0 = \sqrt{2/3}N + \sqrt{1/3}S, \qquad S_8 = \sqrt{1/3}N - \sqrt{2/3}S. \tag{14}$$

Therefore, the scalar-isoscalar part of the Lagrangian (9) involving the fields N, S and G is rewritten as

$$\tilde{\mathcal{L}}^S = -\tfrac{1}{2}\Phi(\square + M_\Phi^2)\Phi \tag{15}$$

with

$$\Phi = \begin{pmatrix} N \\ G \\ S \end{pmatrix}, \qquad M_\Phi^2 = \begin{pmatrix} M_N^2 & \sqrt{2}f & \varepsilon \\ \sqrt{2}f & M_G^2 & f \\ \varepsilon & f & M_S^2 \end{pmatrix}. \tag{16}$$

The bare masses M_N, M_S and the flavor-mixing parameter ε are determined by inserting (14) into (9) with:

$$M_N^2 = \frac{2}{3}M_{S_0}^2 + \frac{1}{3}M_{S_8}^2 + \frac{2\sqrt{2}}{3}z_S,$$

$$M_S^2 = \frac{1}{3}M_{S_0}^2 + \frac{2}{3}M_{S_8}^2 - \frac{2\sqrt{2}}{3}z_S, \tag{17}$$

$$\varepsilon = \frac{\sqrt{2}}{3}(M_{S_0}^2 - M_{S_8}^2) - \frac{1}{3}z_S.$$

The inverted relations are

$$M_{S_0}^2 = \frac{2}{3}M_N^2 + \frac{1}{3}M_S^2 + \frac{2\sqrt{2}}{3}\varepsilon,$$

$$M_{S_0}^2 = \frac{1}{3}M_N^2 + \frac{2}{3}M_S^2 - \frac{2\sqrt{2}}{3}\varepsilon, \tag{18}$$

$$z_S = \sqrt{\frac{2}{3}}(M_N^2 - M_S^2) - \frac{1}{3}\varepsilon.$$

As we will show in the next section, as a result of the fit we will determine the values M_N, M_S, and ε, from which we can deduce M_{S_0}, M_{S_8} and z_S or, equivalently by (11), the parameters k_m^S, γ_{S_0} and γ_{S_8} of the Lagrangian (7).

The parameter f is the quarkonia-glueball mixing strength, analogous to the parameter z of Refs. [6–10]. The mixing strength z refers to the quantum mechanical case, where the mass matrix is linear in the bare mass terms. The connection between f and z, discussed in Refs. [9,12], leads to the approximate relation $f \simeq 2zM_G$.

The parameter ε induces a direct flavor mixing between the quarkonia states N and S. This effect is neglected in [6,8–10], where flavor mixing is considered to be of higher order. However, a substantial N-S mixing in the scalar sector is the starting point of the analysis of Refs. [29,30]. The origin of quarkonia flavor mixing is, according to [29,30], connected to instantons as in the pseudoscalar channel, but with opposite sign (see also [23]). Such a phase structure is also found in the NJL

model including the six-point t' Hooft interaction term [31,32]). The mixed physical fields are predicted to be a higher lying state of flavor structure $[N\sqrt{2} - S]/\sqrt{3}$ and a lower one with $[N + S\sqrt{2}]/\sqrt{3}$. Here we study the case $\varepsilon \neq 0$, more precisely $\varepsilon > 0$, which leads to the same phase structure as in Ref. [29,30], but the quantitative results and interpretation will differ.

For the glueball-quarkonia mixing we work in the limit of flavor blindness. The issue of flavor blindness breaking at the mixing level has been considered in [7,8,12]. This effect can be taken into account when introducing in the mass mixing matrix M_Φ^2, defined in (15), an additional parameter r as

$$M_\Phi^2 = \begin{pmatrix} M_N^2 & \sqrt{2}f \cdot r & \varepsilon \\ \sqrt{2}f \cdot r & M_G^2 & f \\ \varepsilon & f & M_S^2 \end{pmatrix}. \tag{19}$$

For $r = 1$ we regain the original expression, $r \neq 1$ takes into account a possible deviation from this limit. Determination of this parameter on the Lattice [7] results in $r = 1.20 \pm 0.07$, in the fit of [8] a value of $r = 1 \pm 0.3$ is obtained. In the microscopic quark/gluon model of [12] the value $r \sim 1.1$–1.2 is deduced. All these findings point to possible small deviations from the flavor-blind mixing configuration. Hence, in the following we will restrict to the limit $r = 1$.

The orthogonal physical states assigned as $f_1 \equiv f_0(1370)$, $f_2 \equiv f_0(1500)$ and $f_3 \equiv f_0(1710)$ resulting from Eq. (15) are obtained by diagonalization of M_Φ^2 [Eq. (16)] with the transformation matrix B as

$$BM_\Phi^2 B^T = M_f^2 = \begin{pmatrix} M_{f_1}^2 & 0 & 0 \\ 0 & M_{f_2}^2 & 0 \\ 0 & 0 & M_{f_3}^2 \end{pmatrix}, \tag{20}$$

where the eigenvalues of M_f^2 represent the masses of the physical states $f_1 \equiv f_0(1370)$, $f_2 \equiv f_0(1500)$ and $f_3 \equiv f_0(1710)$. The physical states $|i\rangle$, with $i = f_1, f_2, f_3$, are then given in terms of the bare states as

$$|i\rangle = \sum_{j=N,G,S} B_{ij}|j\rangle. \tag{21}$$

In a covariant framework, where mixing of bound states is studied on a elementary level, it is not possible to define an orthogonal mixing matrix B ($B \cdot B^T = B^T \cdot B = 1_3$) connecting the physical to the bare fields [12,31,32]. However, as shown in the model of [12] for the glueball-quarkonia system, deviations from orthogonality of the mixing matrix B are small, therefore justifying a Klein-Gordon mixing scenario as in the present approach.

C. Strong and electromagnetic decays of scalar states

The generic expression for the strong decay width of a scalar state s (both quarkonia and gluonium) into two-pseudoscalar mesons p_1 and p_2 is given by

$$\Gamma_{s \to p_1 p_2} = \frac{\lambda^{1/2}(M_s^2, M_{p_1}^2, M_{p_2}^2)}{16\pi M_s^3} \gamma_{sp_1p_2} |M_{s \to p_1 p_2}|^2, \quad (22)$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$ is the Källen triangle function. The factor $\gamma_{sp_1p_2} = 1/2$ or 1 stands for identical or different particles in the final state. In the case of a degenerate isomultiplet an average over the isospin configurations is understood.

The matrix elements $M_{s \to p_1 p_2}$ are expressed in terms of parameters c_d^s, c_m^s, c_d^g and c_m^g [see Eq. (2)]. The parameters c_d^s and c_m^s refer to the scalar-quarkonia decays, c_d^g and c_m^g to the direct glueball decays. The complete expressions for the two-pseudoscalar decay widths (matrix elements included) of the scalar resonances are given in the Appendix B.

The decay of the bare glueball states embedded in the physical f_0 states can proceed in two ways (see Sec. I). Mixing expressed by the parameter f corresponds to the conversion of a bare glueball to a quarkonia state, which in turn decays into a pseudoscalar meson pair. For $f \neq 0$ no physical f_0 state is a pure glueball, and the decays of the quarkonia components are driven by the amplitudes B_{iN} and B_{iS}, which depend on f.

The direct decay of the glueball component, without proceeding via an intermediate scalar quarkonium state, is contained in the parameters c_d^g and c_m^g. For a microscopic description of this mechanisms we refer to Refs. [6,10], where the possible transition of the glueball to two-pseudoscalar mesons is described by processes containing four internal quark/antiquark lines. The parameters c_d^g and c_m^g are attached to the gluonic amount B_{iG} of the state i, where $i = f_1, f_2, f_3$. With the normalization adopted in Eq. (2) the limit $c_d^s = c_d^g$, $c_m^s = c_m^g$ refers to a direct glueball decay strength equivalent to the decay of a flavor-singlet quarkonia state. In the large N_c limit the quarkonia decay constants c_d^s and c_m^s scale as $N_c^{1/2}$, while the glueball decay constants c_d^g and c_m^g scale as N_c [28]. Large N_c arguments therefore suggest that the direct glueball decay is suppressed with respect to quarkonia decays. When performing in the following a fit to data, we first study the large N_c limit by setting the direct glueball decay parameters c_d^g and c_m^g to zero.

The matrix element for the two-photon decay of the scalar (0^{++}) state has the manifestly gauge-invariant form

$$M_{s \to \gamma\gamma} = e^2 g_{s\gamma\gamma}(g^{\mu\nu}q_1 q_2 - q_1^\nu q_2^\mu)\epsilon_\mu(q_1)\epsilon_\nu(q_2) \quad (23)$$

where q_1 and q_2 are the photon four-momenta, $p = q_1 + q_2$ is the scalar state momentum and $g_{s\gamma\gamma}$ is the $s\gamma\gamma$ coupling constant. The decay width of the transition $s \to \gamma\gamma$ is given by

$$\Gamma_{s \to \gamma\gamma} = \frac{1}{32\pi M_s}\sum_{\text{pol}}|M_{s \to \gamma\gamma}|^2 = \frac{\pi}{4}\alpha^2 g_{s\gamma\gamma}^2 M_s^3, \quad (24)$$

where $\alpha = e^2/(4\pi) = 1/137$ is the fine structure constant.

The coupling constants $g_{s\gamma\gamma}$ for the bare states S_0, S_8, G and for the isovector state a_0^0 are directly calculated using the effective Lagrangian (2):

$$g_{S_0\gamma\gamma} = \frac{32}{3\sqrt{3}}c_e^s, \qquad g_{S_8\gamma\gamma} = \frac{8}{3}\sqrt{\frac{2}{3}}c_e^s,$$

$$g_{G\gamma\gamma} = \frac{32}{3\sqrt{3}}c_e^g, \qquad g_{a_0\gamma\gamma} = \frac{8}{3}\sqrt{2}c_e^s. \qquad (25)$$

The parameter c_e^s refers to the quarkonia components, while c_e^g contains the direct coupling of the glueball component to the electromagnetic fields. Latter coupling constant c_e^g is supposed to be suppressed, since gluons do not couple directly to the photon field. However, an intermediate state of two vector mesons for example can in the framework of vector meson dominance generate a coupling of the glueball to the two-photon final state; this coupling is supposed to be suppressed and will not be considered in the numerical analysis. Using the identities for the field transformations (14) and (21) we can derive the couplings for the bare $N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d)$ and $S \equiv \bar{s}s$ states and finally for the three physical scalar-isoscalar states $i = f_1, f_2$, and f_3:

$$g_{N\gamma\gamma} = \frac{5}{\sqrt{2}}g_{S\gamma\gamma} = \frac{40}{9}\sqrt{2}c_e^s, \qquad g_{i\gamma\gamma} = \sum_{j=N,G,S}B_{ij}g_{j\gamma\gamma}. \qquad (26)$$

III. PHENOMENOLOGICAL FIT WITHOUT DIRECT GLUEBALL DECAY

A. General considerations

In the following we determine a best fit of the parameters entering in Eqs. (2) to the experimental averages of masses and decay modes listed in Ref. [1]. We first analyze the case of a nondecaying glueball, i.e. $c_d^g = c_m^g = 0$, where the decays are dominated by the quarkonia components (as in the original work of [6]) in line with large N_c arguments. The phenomenological analysis of Ref. [10] confirmed this trend, but, as already mentioned, the fit of [8] shows a strong contribution from the direct decays of the glueball configuration.

The parameters of the model entering in the fit are the three bare masses M_N, M_G, M_S, the two mixing parameters f and ε and the two quarkonia decay parameters c_d^s and c_m^s:

$$M_N, M_G, M_S, f, \varepsilon, c_d^s, c_m^s. \qquad (27)$$

As an experimental input we use the following accepted values from [1]:

(a) The scalar-isoscalar f_0 masses with the corresponding values of

$$M_{f_1 \equiv f_0(1370)} = 1.35 \pm 0.15 \text{ GeV}, \qquad (28)$$

$$M_{f_2 \equiv f_0(1500)} = 1.507 \pm 0.005 \text{ GeV}, \qquad (29)$$

$$M_{f_3 \equiv f_0(1710)} = 1.714 \pm 0.005 \text{ GeV}. \qquad (30)$$

(b) The partial decay widths of $f_2 \equiv f_0(1500)$ with:

$$\Gamma_{f_2 \to \pi\pi} = 0.0380 \pm 0.0050 \text{ GeV}, \qquad (31)$$

$$\Gamma_{f_2 \to \overline{K}K} = 0.0094 \pm 0.0017 \text{ GeV}, \qquad (32)$$

$$\Gamma_{f_2 \to \eta\eta} = 0.0056 \pm 0.0014 \text{ GeV}. \qquad (33)$$

(c) The two accepted ratios for $f_3 \equiv f_0(1710)$:

$$\Gamma_{f_3 \to \pi\pi}/\Gamma_{f_3 \to \overline{K}K} = 0.20 \pm 0.06, \qquad (34)$$

$$\Gamma_{f_3 \to \eta\eta}/\Gamma_{f_3 \to \overline{K}K} = 0.48 \pm 0.15. \qquad (35)$$

(d) The state $f_0(1710)$ has only been observed in the decays into two-pseudoscalar mesons [1]. The decay into the final state 4π, which can be fed by higher meson resonances, is suppressed [33]. We therefore impose the additional condition that the sum of partial decay widths into two-pseudoscalar mesons $(\Gamma_{f_3})_{2P}$ saturates the total width $(\Gamma_{f_3})_{\text{tot}}$:

$$(\Gamma_{f_3})_{2P} = (\Gamma_{f_3})_{\text{tot}} = 140 \pm 10 \text{ MeV}. \qquad (36)$$

Such a constraint is necessary to obtain meaningful total decay widths: without this condition on the full width a minimum for χ^2 is obtained where $(\Gamma_{f_3})_{\text{tot}}$ is larger than 1 GeV, a clearly unacceptable solution.

(e) The two decay ratios of the $I = 1$ state $a_0(1450)$:

$$\frac{\Gamma_{a_0 \to \pi\eta'}}{\Gamma_{a_0 \to \pi\eta}} = 0.35 \pm 0.16, \qquad (37)$$

$$\frac{\Gamma_{a_0 \to KK}}{\Gamma_{a_0 \to \pi\eta}} = 0.88 \pm 0.23. \qquad (38)$$

The total width of $a_0(1450)$ into two pseudoscalars is not known, because of the uncertainty of other decay widths as for $\omega\pi\pi$.

(f) The $I = 1/2$ state $K_0^*(1450)$ decays dominantly into $K\pi$ with the corresponding width

$$\Gamma_{K_0^* \to K\pi} = 273 \pm 51 \text{ MeV}. \qquad (39)$$

The only accepted average not included in the fit is $\Gamma_{f_2 \to \eta\eta'}$ for the reason that the decay channel $\eta\eta'$ is produced at threshold. Therefore, a significant distortion due to the finite width of the state is expected.

For the $N = 12$ experimental values listed in Eqs. (28)–(39) we perform a χ^2 fit with

$$\chi^2 = \chi^2[M_N, M_G, M_S, f, \varepsilon, c_d^s, c_m^s]$$

$$= \sum_{i=1}^{N=12} \left[\frac{A_i^{\text{theory}} - A_i^{\text{exp}}}{\triangle A_i} \right]^2, \qquad (40)$$

where A_i^{exp} represents the ith experimental result, $\triangle A_i$ is its error and A_i^{theory} is the corresponding theoretical expression, depending on the parameters of (27).

For the case, where the direct glueball is suppressed (i.e. we set $c_d^g = c_m^g = 0$), two local minima for χ^2 are obtained, the consequences of which we will describe in the following. The first solution was already analyzed in [13], while the second one is a novel solution with some peculiar characteristics.

B. First solution and implications

Fit results.—From the first solution (I) we extract following fit parameters:

$$\begin{aligned} M_N &= 1.455 \text{ GeV}, & M_G &= 1.490 \text{ GeV}, \\ M_S &= 1.697 \text{ GeV}, & f &= 0.065 \text{ GeV}^2, \\ \varepsilon &= 0.211 \text{ GeV}^2, & c_d^s &= 8.48 \text{ MeV}, \\ c_m^s &= 2.59 \text{ MeV}; & \chi_{\text{tot}}^2 &= 29.01. \end{aligned} \qquad (41)$$

The corresponding fit results are reported in Table I.

Bare masses.—The bare nonstrange quarkonia field N has a mass of $M_N = 1.455$ GeV, which is, as desired, similar to the scale set by the isotriplet combination $a_0(1450)$ with a mass of $M_{a_0} = 1.474 \pm 0.019$ GeV [1]. The mass of the bare glueball $M_G = 1.490$ GeV is in agreement with the lattice results [2] and with the phenomenological analyses of [6,8,10]. The bare state S has a mass of $M_S = 1.697$ GeV, which is about ~ 200 MeV heavier than the N state, an acceptable mass difference like in the tensor meson nonet.

Mixing parameters.—For the glueball-quarkonia mixing parameter we get $f = 0.065$ GeV^2, which by the approximate relation $f \simeq 2zM_G$ [9,12] corresponds to $z \simeq 21.8$ MeV.

The results of Refs. [8,10,12] are $z = 85 \pm 10$ MeV, $z = 80$ MeV and $z \simeq 62$ MeV, respectively, i.e. of the

TABLE I. Fitted mass and decay properties of scalar mesons.

Quantity	Exp	Theory	χ_i^2
M_{f_1} (MeV)	1350 ± 150	1417	0.202
M_{f_2} (MeV)	1507 ± 5	1507	~ 0
M_{f_3} (MeV)	1714 ± 5	1714	0.003
$\Gamma_{f_2 \to \pi\pi}$ (MeV)	38.0 ± 4.6	38.52	0.011
$\Gamma_{f_2 \to \overline{K}K}$ (MeV)	9.4 ± 1.7	10.36	0.322
$\Gamma_{f_2 \to \eta\eta}$ (MeV)	5.6 ± 1.3	1.90	8.109
$\Gamma_{f_2 \to \pi\pi}/\Gamma_{f_3 \to \overline{K}K}$	0.20 ± 0.06	0.212	0.036
$\Gamma_{f_3 \to \eta\eta}/\Gamma_{f_3 \to \overline{K}K}$	0.48 ± 0.15	0.249	2.446
$\Gamma_{a_0 \to \overline{K}K}/\Gamma_{a_0 \to \pi\eta}$	0.88 ± 0.23	0.838	0.032
$\Gamma_{a_0 \to \pi\eta'}/\Gamma_{a_0 \to \pi\eta}$	0.35 ± 0.16	0.288	0.150
$\Gamma_{K_0^* \to K\pi}$ (MeV)	273 ± 51	59.10	17.590
$(\Gamma_{f_3})_{2P}$ (MeV)	140 ± 10	143.27	0.110
χ_{tot}^2	$\cdots$	$\cdots$	29.01

same order, but larger. The introduction of additional flavor mixing between the quarkonia configurations in the fit, as done here, leads to a reduction of the strength parameter f. The lattice result of Ref. [7] with 43 ± 31 MeV is in agreement with the present evaluation, but has a large uncertainty. A mixing strength of the same order is found in the lattice evaluation of [15].

The flavor-mixing parameter resulting from the fit is $\varepsilon = 0.211$ GeV^2. In the limit $f = 0$ the mixed physical states are $|f_1\rangle = 0.97|N\rangle + 0.26|S\rangle$ and $|f_3\rangle = -0.26|N\rangle + 0.97|S\rangle$ (and, of course, $|f_2\rangle = |G\rangle$). The phase structure of the mixed states is, as discussed previously, as in [23,29,30]. But here the strength of flavor mixing is smaller, resulting in mixed states, which are dominantly N or S. The influence however of (an even small) flavor mixing in strong and electromagnetic decays may be nonnegligible.

Mixing matrix.—The mixing matrix B relating the physical to the bare states in the present fit is expressed as

$$\begin{pmatrix} |f_1\rangle \equiv |f_0(1370)\rangle \\ |f_2\rangle \equiv |f_0(1500)\rangle \\ |f_3\rangle \equiv |f_0(1710)\rangle \end{pmatrix} = \begin{pmatrix} 0.86 & 0.45 & 0.24 \\ -0.45 & 0.89 & -0.06 \\ -0.24 & -0.06 & 0.97 \end{pmatrix}$$
$$\times \begin{pmatrix} |N\rangle \equiv |\bar{n}n\rangle \\ |G\rangle \equiv |gg\rangle \\ |S\rangle \equiv |\bar{s}s\rangle \end{pmatrix}. \quad (42)$$

The physical resonances are dominated by the diagonal bare components, qualitatively in line with Refs. [3,6,8,10]. Since the glueball does not contribute to the decay, the relative phase with respect to the quarkonia components is at this stage irrelevant. By inverting $f \to -f$ we would find the same results for the decays, but opposite glueball-quarkonia phases. In turn, the relative phases of the N and S components are not symmetric under $\varepsilon \to -\varepsilon$. As discussed above, in $f_0(1370)$ they are in phase, while in $f_0(1710)$ they are out of phase. The state $|f_0(1500)\rangle$ behaves like a N state with a decreased width, while the S component is small [10,34]. Thus the decay into $\overline{K}K$ is smaller than for the $\pi\pi$ channel. In the present solution (I) the N and S state components are in phase contrary to the results of [6,10]. However, the other solution (II), presented later on, shows again an opposite phase in $f_2 = f_0(1500)$, but with a large $\bar{s}s$ amount.

Large N_c constants.—From the fit parameters of (41) we determine γ_{S_0}, γ_{S_8} and k_m^S by using (11) and (18):

$$M_S = 1.479 \text{ GeV}, \qquad e_m^S = 0.199, \qquad \gamma_{S_8} = 0.225,$$
$$\gamma_{S_0} = 0.236, \qquad k_m^S = -0.818. \quad (43)$$

The values of γ_{S_0} and k_m^S are smaller than in [20]. Also, the violation of the GMO relation, encoded in γ_{S_8}, is small, indicating that higher-order corrections in the chiral expansion are possibly not too large to invalidate the present

study. In the present scenario the smallness of the glueball mixing parameter f can also be interpreted as a small violation of the large N_c limit. The present results show that large N_c and chiral symmetry can, although violated at some level, be a useful guideline to scalar meson physics.

Resonance $f_0(1370)$.—The experimental uncertainties of the $f_0(1370)$ resonance are large, no average or fit is presented in [1]. The main problem connected with this resonance is its large width (200–500 MeV) and its partial overlap with the broad low-lying $\sigma \equiv f_0(400–1200)$. However, the results from WA102 [33] indicate a large $N \equiv \bar{n}n$ component in its wave function. Results from CRYSTAL BARREL (summarized in [35] and subsequently analyzed in [36]) confirm such a trend (see also [37] for a recent review).

Predictions for the two-pseudoscalar decay modes are in acceptable agreement with the results of WA102 as shown in Table II. The measured ratio $\Gamma_{f_1 \to 4\pi}/\Gamma_{f_1 \to \pi\pi} = 34.0^{+22}_{-9}$ [33], although the errors are large, points to a dominant 4π contribution to the total width. Our prediction gives however a sizable contribution of the two-pseudoscalar decay mode and is therefore not in agreement with such a large 4π decay mode.

In the original work of [6], a quarkonium $\bar{n}n$ state has a very large two-pseudoscalar width with $\Gamma_{\bar{n}n \to \pi\pi} = 270 \pm 25$ MeV, $\Gamma_{\bar{n}n \to \overline{K}K} = 195 \pm 20$ MeV and $\Gamma_{\bar{n}n \to \eta\eta} = 95 \pm 10$ MeV, i.e. $(\Gamma_{\bar{n}n})_{2P} \sim 500$ MeV. The $f_1 \equiv f_0(1370)$ is in [6] dominantly $\bar{n}n$, therefore one expects a large value for $(\Gamma_{f_1})_{2P}$, in contrast to the presented experimental analyses listed above. A large value for $(\Gamma_{\bar{n}n})_{2P}$ is also predicted in [10]: for the mixed state $f_0(1370)$ one has $(\Gamma_{f_1})_{2P} = 115.7$ MeV comparable to the present study.

On the contrary, in the study of [8], small two-pseudoscalar partial widths are obtained by the following mechanism: the glueball decay amplitude in $f_0(1370)$ to two pseudoscalar is large and interferes destructively with the $\bar{n}n$ component [the phases quarkonia-glueball in [8] are inverted with respect to (42)]. As a result [8] the two-pseudoscalar decay width $(\Gamma_{f_1})_{2P}$ is smaller than $(\Gamma_{f_2})_{2P}$ with $(\Gamma_{f_2})_{2ps}/(\Gamma_{f_1})_{2ps} = 10.0 \pm 3.0$. At the same time $(\Gamma_{f_3})_{2P}/(\Gamma_{f_1})_{2P} = 0.7 \pm 0.2$ is obtained.

In the present fit with an inert glueball (as in the original work of [6] and as in [10], where the glueball is allowed to decay, but the quarkonia components still dominate) we find the following decay widths into two-pseudoscalar pairs:

TABLE II.   Decays of $f_1 = f_0(1370)$.

Quantity	Exp (WA102)	Theory
$\Gamma_{f_1 \to \overline{K}K}/\Gamma_{f_1 \to \pi\pi}$	0.46 ± 0.19	0.34
$\Gamma_{f_1 \to \eta\eta}/\Gamma_{f_1 \to \pi\pi}$	0.16 ± 0.07	0.06
$(\Gamma_{f_1})_{2P}$ (MeV)	"Small"	166

$$(\Gamma_{f_1})_{2P} = 166 \text{ MeV} > (\Gamma_{f_3})_{2P} = 143 \text{ MeV} > (\Gamma_{f_2})_{2P}$$
$$= 51 \text{ MeV}. \quad (44)$$

An analysis by CRYSTAL BARREL [36] also indicates sizable partial decay widths of the 4π decay channels: $\Gamma_{f_1 \to \sigma\sigma} = 120.5 \pm 65 \text{ MeV}$ and $\Gamma_{f_1 \to \rho\rho} = 62.2 \pm 28.8 \text{ MeV}$. The same analysis gives the following two-pseudoscalar partial widths [36,37]: $\Gamma_{f_1 \to \pi\pi} = 21.7 \pm 9.9 \text{ MeV}$, $\Gamma_{f_1 \to \overline{K}K} = (7.9 \pm 2.7 \text{ MeV})$ to $(21.2 \pm 7.2 \text{ MeV})$, $\Gamma_{f_1 \to \eta\eta} = 0.41 \pm 0.27 \text{ MeV}$. On the contrary, the analysis of [38] reports the ratio $\Gamma_{f_1 \to \pi\pi}/(\Gamma_{f_1})_{\text{tot}} = 0.26 \pm 0.09$, pointing to a large $\pi\pi$ (ergo to a large two-pseudoscalar) partial decay width for $f_0(1370)$. This experimental result is therefore in disagreement with the analysis of [36]. New results on $f_0(1370)$ would be crucial to disentangle the scalar puzzle and to understand if a destructive glueball/quarkonia interference as in [8] is necessary.

Resonance $f_0(1500)$.—The theoretical partial widths of the $f_0(1500)$ are in good agreement with the data (see Table I) apart from a slight underestimate of the 2η channel. We also obtain $\Gamma_{f_2 \to \eta\eta'} = 0.036 \text{ MeV}$ as compared to the experimental value of $\Gamma_{f_2 \to \eta\eta'} = 2 \pm 1 \text{ MeV}$. Taking into account the finite width of the resonance will lead to an increase of the theoretical value.

Resonance $f_0(1710)$.—For the decays of $f_0(1710)$ we summarize our results compared to the data of WA102 [33] in Table III.

The first two ratios, already included in the fit of Table I, can be reproduced. The theoretical ratio $\Gamma_{f_3 \to \eta\eta'}/\Gamma_{f_3 \to \pi\pi}$, which is not included in [1], is in complete disagreement with the WA102 result. The dominance of the $\eta\eta'$ mode over $\pi\pi$ is a solid prediction in the framework of solution I, which does not depend very much on the choice of parameters. A confirmation of the experimental result could possibly hint at a different mixing scenario or at a sizable role of direct glueball decay.

Resonance $a_0(1450)$.—The ratios of two-pseudoscalar decay modes of $a_0(1450)$, included in the fit of Table I, are well reproduced. The prediction for the two-pseudoscalar width of $(\Gamma_{a_0})_{2P} = 84.26 \text{ MeV}$ is smaller than the total width of $265 \pm 13 \text{ MeV}$. However, the experimental ratio $(\Gamma_{a_0 \to \omega\pi\pi}/\Gamma_{a_0 \to \pi\eta})$ is not known: no average or fit is listed in [1]. The experimental value from [39], which is 10.7 ± 2.3, would imply a dominant $\omega\pi\pi$ mode and in turn a

rather small two-pseudoscalar partial decay widths. This finding is in disagreement with the results of [6,40]. In [6] a value of $(\Gamma_{a_0})_{2P} = 390 \pm 110 \text{ MeV}$ is found, in the work of [40] one has $(\Gamma_{a_0})_{2P} = 420\text{–}940 \text{ MeV}$ for a_0 masses in the range of 1200–1400 MeV. For our value for $(\Gamma_{a_0})_{2P}$ we obtain the estimate: $\Gamma_{a_0 \to \omega\pi\pi}/\Gamma_{a_0 \to \pi\eta} = [(\Gamma_{a_0})_{\text{tot}} - (\Gamma_{a_0})_{2P}]/\Gamma_{a_0 \to \pi\eta} \sim 4.5$.

Resonance $K_0^*(1430)$: Our result for $\Gamma_{K_0^* \to K\pi}$ underestimates the experimental value by a factor of about 5 (see Table I). Furthermore, for the additional $K\eta$ decay channel we get $\Gamma_{K_0^* \to K\eta}/\Gamma_{K_0^* \to \pi K} = 0.026$.

In [40] a value of $\Gamma_{K_0^* \to K\pi} = 340 \text{ MeV}$ is predicted, but, as discussed above, $(\Gamma_{a_0})_{2P}$ is of the order of 1 GeV, much larger than the full width. Similarly, in [6] with $\Gamma_{K_0^* \to K\pi} = 200 \pm 20$ and $(\Gamma_{a_0})_{2P} = 390 \pm 110 \text{ MeV}$ the first result underestimates while the second overshoots the experimental value. A full analysis in the $^3P_0$ model [41] results in $\Gamma_{K^* \to \pi K} = 166 \text{ MeV}$ and $\Gamma_{N \to \pi\pi} = 271 \text{ MeV}$; unfortunately the resonance $a_0(1450)$ is not discussed in [41]. The authors of [41] also tried to adjust $\Gamma_{K_0^* \to \pi K}$ to its experimental value, and then calculate the 2π partial width of a N state, obtaining $\Gamma_{N \to \pi\pi} \sim 450 \text{ MeV}$. The last result implies a very large two-pseudoscalar and full width for a N state. A full experimental determination of all relevant decay modes involving $a_0(1450)$ and $K_0^*(1430)$ would certainly help to clarify this issue. We refer to section III D for a further discussion of this problem.

If $f_0(1370)$ is dominantly $\bar{n}n$, as in [6–10,12], there is, as discussed above, an incompatibility of the present experimental small two-pseudoscalar partial decay widths [36] and various model calculations. At the same time, a consistent understanding of the isodoublet states $K_0^*(1450)$ and the isovectors $a_0(1450)$ is still missing.

Two-photon decays.—As a further consequence we discuss the two-photon decay rates of the scalar resonances. We assume that the coupling c_e^g is suppressed with respect to c_e^s, i.e. we set the glueball-photon coupling c_e^g to zero. The ratios of radiative decay widths as a prediction of the fit are

$$\Gamma_{f_1 \to 2\gamma}:\Gamma_{f_2 \to 2\gamma}:\Gamma_{f_3 \to 2\gamma}:\Gamma_{a_0^0 \to 2\gamma} = 1:0.305:0.002:0.471,$$
$$(45)$$

which are independent of the coupling c_e^s. The result for $\Gamma_{f_2 \to 2\gamma}/\Gamma_{f_1 \to 2\gamma}$ is in qualitative agreement with the results of [8,12]. The ratio $\Gamma_{f_3 \to 2\gamma}/\Gamma_{f_1 \to 2\gamma}$, however, is considerably smaller than in the previous works. The suppression of $\Gamma_{f_3 \to 2\gamma}$ originates from the destructive interference between the N and S components, which in turn is traced to the flavor mixing with $\varepsilon > 0$ in accord with the phases of [29,30]. Another interesting prediction is the ratio $\Gamma_{a_0^0 \to 2\gamma}/\Gamma_{f_1 \to 2\gamma}$, which is relatively large.

The experimental status of the two-photon decays is still incomplete. For the $f_0(1370)$ two values are indicated

TABLE III. Decays of $f_3 = f_0(1710)$.

Quantity	Exp (WA102)	Theory
$\Gamma_{f_3 \to \overline{K}K}/\Gamma_{f_3 \to \pi\pi}$	5.0 ± 0.7	4.70
$\Gamma_{f_3 \to \eta\eta}/\Gamma_{f_3 \to \pi\pi}$	2.4 ± 0.6	1.17
$\Gamma_{f_3 \to \eta\eta'}/\Gamma_{f_3 \to \pi\pi}$	<0.18	1.59
$(\Gamma_{f_3})_{2P}$ (MeV)	"Dominant"	143.27

in PDG2000 [42]: 3.8 ± 1.5 keV and 5.4 ± 2.3 keV. However, it is not clear if the two-photon signal comes from the $f_0(1370)$ or from the high mass end of the broad $f_0(400–1200)$. The PDG currently [1] seems to favor this last possibility, but the data could also be valid for the $f_0(1370)$. We therefore interpret the two experimental values as an upper limit for the two-photon decay width of the $f_0(1370)$. Signals for two-photon decays of $f_0(1500)$ and $f_0(1710)$ have not yet been seen, the following upper limits are reported [1]:

$$\Gamma_{f_0(1500)\to 2\gamma}(\Gamma_{f_0(1500)\to\pi\pi}/\Gamma_{f_0(1500)\text{tot}}) < 0.46 \text{ keV},$$
$$\Gamma_{f_0(1710)\to 2\gamma}(\Gamma_{f_0(1710)\to K\bar{K}}/\Gamma_{f_0(1710)\text{tot}}) < 0.11 \text{ keV}. \quad (46)$$

Using the known branching ratio $\Gamma_{f_0(1500)\to 2\pi}/\Gamma_{f_0(1500)\text{tot}}$ one gets $\Gamma_{f_0(1500)\to 2\gamma} < 1.4$ keV [34]. An accepted fit for $\Gamma_{f_0(1710)\to K\bar{K}}/\Gamma_{f_0(1710)\text{tot}}$ is not reported in [1]. Using the value from [43] with $\Gamma_{f_0(1710)\to K\bar{K}}/\Gamma_{f_0(1710)\text{tot}} = 0.38^{+0.03}_{-0.13}$ we find an upper limit of the order of $\Gamma_{f_0(1710)\to 2\gamma} \sim 0.3$ keV.

For an absolute prediction of the two-photon decay widths we use $c_e^s = 0.0138$ GeV^{-1} as determined in the model approach of Ref. [12] For the nonstrange quarkonium state we get $\Gamma_{N\to 2\gamma} = 0.969$ keV, while for the isovector and mixed scalars we have:

$$\Gamma_{f_1\to 2\gamma} = 0.703 \text{ keV}, \qquad \Gamma_{f_2\to 2\gamma} = 0.235 \text{ keV},$$
$$\Gamma_{f_3\to 2\gamma} = 0.002 \text{ keV}, \qquad \Gamma_{a_0^0\to 2\gamma} = 0.362 \text{ keV}. \quad (47)$$

The results for the mixed states are below the current upper limits (the original results presented in [13] contain a slight misprint, the correct numbers are reported here. The physical considerations are not affected from this slight change).

The estimate for the 2γ decay of the bare quarkonium state $N \equiv \bar{n}n$ of 0.969 keV is smaller than the one of [34], where the following expression has been used:

$$\Gamma_{\bar{n}n\to 2\gamma}(0^{++}) = k\left(\frac{M_N(0^{++})}{M_N(2^{++})}\right)^3 \Gamma_{\bar{n}n\to 2\gamma}(2^{++}). \quad (48)$$

The coefficient k is 15/4 in a nonrelativistic calculation, but becomes smaller when considering relativistic corrections [44]. In [34] a range of values for k from 2 to 15/4 is considered. Our chiral Lagrangian approach combined with [12] points to a smaller value of k. Using our result for $\Gamma_{N\to 2\gamma}(0^{++})$ and taking the value $\Gamma_{N\to 2\gamma}(2^{++}) = 2.60 \pm 0.24$ keV [1] at $M_N(2^{++}) = 1.27$ GeV we get $k \sim 0.25$. This result is model dependent, since it relies on the parameters for the covariant description of the scalar mesons used in [12]. A fully covariant treatment may imply strong deviations from the nonrelativistic limit.

Discussion.—The largest contribution to χ^2 is due to the underestimate of the $K_0^* \to K\pi$ width: from Table I we have $\chi_{\text{tot}}^2/N = 2.42$. When excluding the data point for $\Gamma_{K_0^*\to K\pi}$ in the fit, a very similar minimum compared to (41) is found with

$$M_N = 1.442 \text{ GeV}, \qquad M_G = 1.485 \text{ GeV},$$
$$M_S = 1.695 \text{ GeV}, \qquad f = 0.080 \text{ GeV}^2,$$
$$\varepsilon = 0.225 \text{ GeV}^2, \qquad c_d^s = 8.12 \text{ MeV},$$
$$c_m^s = 3.57 \text{ MeV}; \qquad \chi_{\text{tot}}^2 = 11.19. \quad (49)$$

In this case we have $\chi_{\text{tot}}^2/N = 1.02$, corresponding to a good description of data. The discussion about the isoscalar states and $a_0(1450)$ remains unchanged.

The underestimate of $K_0^* \to K\pi$ constitutes an open problem of the scalar analysis. As already discussed, a consistent understanding of the complete scalar nonet is lacking in other approaches as well. We will further discuss this issue in Sec. III D.

Aside from this difficulty, the rest of the accepted data in [1] is well described. The quality of the current fit seemingly excludes a sizable direct decay of the scalar glueball component. Concerning results for data, which are not reported as average or fit in [1], the situation is less clear: the predicted full two-pseudoscalar width of $f_1 = f_0(1370)$ is large, when confronted with the WA102 result (see Table II and Refs. [35–37], but also the different result of [38]). The ratio $\Gamma_{f_3\to\eta\eta'}/\Gamma_{f_3\to\pi\pi}$ (Table III) is also problematic in the present solution. An accepted average, in particular, for these values, would help in clarifying these issues.

C. Second solution and implications

Fit results.—Solution II is obtained for following fit parameters:

$$M_N = 1.298 \text{ GeV}, \qquad M_G = 1.513 \text{ GeV},$$
$$M_S = 1.593 \text{ GeV}, \qquad f = 0.400 \text{ GeV}^2,$$
$$\varepsilon = 0.015 \text{ GeV}^2, \qquad c_d^s = 7.48 \text{ MeV},$$
$$c_m^s = 6.42 \text{ MeV}; \qquad \chi_{\text{tot}}^2 = 24.61. \quad (50)$$

The corresponding results are reported in Table IV.

A first look to (50) shows some peculiar differences when compared to the set of (41). In the following we discuss the implications and the differences of this second solution.

Bare masses.—The bare masses M_N and M_S are smaller than in solution I, their mass difference is still around 200 GeV [Eq. (50)]. The bare glueball mass is about ~ 1.5 GeV, as before, but now is much closer to M_S. This small mass difference leads to a strong mixing between the glueball and the bare $S \equiv \bar{s}s$ state.

Mixing parameters.—The quarkonia flavor mixing $\varepsilon = 0.015$ GeV^2 is very small in this solution and has practically no influence on the phenomenology. On the contrary $f = 0.400$ GeV^2 is much larger, leading to a strong glueball-quarkonia mixing. In this respect there is a clear difference between the two solutions. Using the approximate relation $f \simeq 2zM_G$ [9,12] we find in this case

TABLE IV. Fitted mass and decay properties of scalar mesons.

Quantity	Exp	Theory	χ_i^2
M_{f_1} (MeV)	1350 ± 150	1142	1.924
M_{f_2} (MeV)	1507 ± 5	1508	0.023
M_{f_3} (MeV)	1714 ± 5	1713	0.0254
$\Gamma_{f_2 \to \pi\pi}$ (MeV)	38.0 ± 4.6	37.31	0.019
$\Gamma_{f_2 \to \overline{K}K}$ (MeV)	9.4 ± 1.7	10.08	0.167
$\Gamma_{f_2 \to \eta\eta}$ (MeV)	5.6 ± 1.3	4.70	0.477
$\Gamma_{f_3 \to \pi\pi}/\Gamma_{f_3 \to \overline{K}K}$	0.20 ± 0.06	0.216	0.071
$\Gamma_{f_3 \to \eta\eta}/\Gamma_{f_3 \to \overline{K}K}$	0.48 ± 0.15	0.248	2.400
$\Gamma_{a_0 \to \overline{K}K}/\Gamma_{a_0 \to \pi\eta}$	0.88 ± 0.23	1.078	0.741
$\Gamma_{a_0 \to \pi\eta'}/\Gamma_{a_0 \to \pi\eta}$	0.35 ± 0.16	0.291	0.134
$\Gamma_{K_0^* \to K\pi}$ (MeV)	273 ± 51	53.75	18.480
$(\Gamma_{f_3})_{2P}$ (MeV)	140 ± 10	143.94	0.155
χ_{tot}^2	$\cdots$	$\cdots$	24.61

TABLE V. Decays of $f_1 = f_0(1370)$.

Quantity	Exp (WA102)	Theory
$\Gamma_{f_1 \to \overline{K}K}/\Gamma_{f_1 \to \pi\pi}$	0.46 ± 0.19	0.27
$\Gamma_{f_1 \to \eta\eta}/\Gamma_{f_1 \to \pi\pi}$	0.16 ± 0.07	0.02
$(\Gamma_{f1})_{2P}$ (MeV)	Small	56.79

pseudoscalar width (mostly caused by the smaller physical mass, see Table IV). In the present fit we find the following decay widths into two-pseudoscalar pairs:

$$(\Gamma_{f_3})_{2P} = 144 \text{ MeV} > (\Gamma_{f_1})_{2P} = 57 \text{ MeV} > (\Gamma_{f_2})_{2P}$$
$$= 52 \text{ MeV}. \tag{53}$$

Here the decay pattern is in better agreement with the analysis of [36,37] than the one of solution I.

Resonance $f_0(1500)$.—The theoretical partial widths of $f_0(1500)$ are in rather good agreement with the data (see Table IV). We also obtain $\Gamma_{f_2 \to \eta\eta'} = 1.5$ MeV as compared to the experimental value of $\Gamma_{f_2 \to \eta\eta'} = 2 \pm 1$ MeV without invoking further threshold corrections.

Resonance $f_0(1710)$.—For the decays of $f_0(1710)$ we summarize our results compared to the data of WA102 [33] in Table VI.

The theoretical $\eta\eta'/\pi\pi$ ratio is now smaller than in solution I. Although the prediction is still larger than the upper limit set by WA102, it does not represent such an evident mismatch.

Resonance $a_0(1450)$.—The two experimental ratios are satisfactorily described. Otherwise, the discussion of solution I is still valid here.

Resonance $K_0^*(1430)$.—The underestimate of $\Gamma_{K_0^* \to K\pi}$ is also present in the second solution (see Table IV). Furthermore, for the additional $K\eta$ decay channel we get $\Gamma_{K_0^* \to K\eta}/\Gamma_{K_0^* \to \pi K} = 0.050\,65$.

Two-photon decays.—The two-photon decay ratios resulting from solution II read:

$$\Gamma_{f_1 \to 2\gamma}{:}\Gamma_{f_2 \to 2\gamma}{:}\Gamma_{f_3 \to 2\gamma}{:}\Gamma_{a_0^0 \to 2\gamma} = 1{:}0.253{:}0.055{:}0.493. \tag{54}$$

The two γ results are in qualitative agreement with [8,12].

Following the arguments of the previous section by using the result of [12] we find:

$z \simeq 130$ MeV, which is larger than the results from other works listed in Sec. III B, but still in qualitative agreement.

Mixing matrix.—The mixing matrix B, relating the physical to the bare states, for the second solution is expressed as

$$\begin{pmatrix} |f_1\rangle \equiv |f_0(1370)\rangle \\ |f_2\rangle \equiv |f_0(1500)\rangle \\ |f_3\rangle \equiv |f_0(1710)\rangle \end{pmatrix} = \begin{pmatrix} 0.81 & 0.54 & 0.19 \\ -0.49 & 0.49 & 0.72 \\ -0.30 & -0.68 & 0.67 \end{pmatrix}$$
$$\times \begin{pmatrix} |N\rangle \equiv |\bar{n}n\rangle \\ |G\rangle \equiv |gg\rangle \\ |S\rangle \equiv |\bar{s}s\rangle \end{pmatrix}. \tag{51}$$

The large mixing parameter f causes the glueball configuration to be spread out among the f_0 states: $f_1 \equiv f_0(1370)$ is still dominantly $\bar{n}n$, $f_2 \equiv f_0(1500)$ is mostly $\bar{s}s$, but with a sizable out-of-phase $\bar{n}n$ amplitude (the opposite phase of $\bar{n}n$ and $\bar{s}s$ was first considered in [6] as a mechanism to explain the large $\pi\pi/\overline{K}K$ ratio). In $f_3 \equiv f_0(1710)$ both the gluonium and the $\bar{s}s$ components are large, where the gluonic component is slightly larger. Remarkably, although the bare level ordering is still $M_N < M_G < M_S$, the largest $\bar{s}s$ amount is contained in f_2, while the largest gluonic component is present in f_3. The mixing matrix resembles some features of the results of [7], although the bare level ordering is different.

Large N_c constants.—From the parameters in (50) we determine γ_{S_0}, γ_{S_8} and k_m^S by using (11) and (18):

$$M_S = 1.479 \text{ GeV}, \qquad e_m^S = 0.199, \qquad \gamma_{S_8} = 0.139,$$
$$\gamma_{S_0} = -0.032, \qquad k_m^S = -0.786. \tag{52}$$

The same considerations as for solution I also hold here.

Resonance $f_0(1370)$.—The results for $f_1 = f_0(1370)$ are summarized in Table V.

The main difference with respect to solution I concerning $f_1 = f_0(1370)$ is the decreased theoretical two-

TABLE VI. Decays of $f_3 = f_0(1710)$.

Quantity	Exp (WA102)	Theory
$\Gamma_{f_3 \to \overline{K}K}/\Gamma_{f_3 \to \pi\pi}$	5.0 ± 0.7	4.63
$\Gamma_{f_3 \to \eta\eta}/\Gamma_{f_3 \to \pi\pi}$	2.4 ± 0.6	1.15
$\Gamma_{f_3 \to \eta\eta'}/\Gamma_{f_3 \to \pi\pi}$	<0.18	0.36
$(\Gamma_{f_3})_{2P}$ (MeV)	Dominant	143.94

$$\Gamma_{f_1 \to 2\gamma} = 0.350 \text{ keV}, \qquad \Gamma_{f_2 \to 2\gamma} = 0.088 \text{ keV},$$
$$\Gamma_{f_3 \to 2\gamma} = 0.019 \text{ keV}, \qquad \Gamma_{a_0^0 \to 2\gamma} = 0.172 \text{ keV}. \tag{55}$$

As before, the rates are below the upper limits set by current data, but they depend on the model-dependent evaluation of [12].

Discussion.—If in this case we exclude the $K_0^* \to K\pi$ mode from the fit (which generates the by far largest contribution to χ^2), the following set of parameters is found:

$$M_N = 1.313 \text{ GeV}, \qquad M_G = 1.511 \text{ GeV},$$
$$M_S = 1.594 \text{ GeV}, \qquad f = 0.395 \text{ GeV}^2,$$
$$\varepsilon = 0.002 \text{ GeV}^2, \qquad c_d^s = 7.30 \text{ MeV}, \tag{56}$$
$$c_m^s = 6,61 \text{ MeV}; \qquad \chi_{\text{tot}}^2 = 5.97.$$

We then obtain $\chi_{\text{tot}}^2/N = 0.54$, corresponding also in this case to a good description of the remaining data. Again, there is no phenomenological evidence for a direct decay of the glueball component.

Apart from the decay $K_0^* \to K\pi$, both solutions (I and II) describe the data (28)–(38) well. There are, however, differences when comparing to the WA102 results. The second solution comes in this respect closer to these data, but the experimental results are not yet conclusive.

D. $K_0^*(1430)$

The most striking mismatch with the data is in both analyzed scenarios I and II the theoretical underestimate of the $K_0^* \to K\pi$ mode (Tables I and IV). The corresponding partial $\chi_{K_0^* \to K\pi}^2$ is by far the dominant contribution to χ_{tot}^2. This mismatch also holds when including direct glueball decays in the analysis as shown in the next section.

As a further attempt, following [41], we can also pursue another strategy: in a first fit we fix the quarkonium decay parameters c_d^s and c_m^s in order to reproduce $K_0^* \to K\pi$ [Eq. (39)] and the two ratios for a_0 given in (37) and (38). Then we obtain $c_d^s = 17.94 \text{ MeV}$ (larger by a factor 2 when compared to the previous fit in Tables I and IV). and $c_m^s = 7.35 \text{ MeV}$. With these values one has $\Gamma_{K_0^* \to \pi K} = 281 \text{ MeV}$, $\Gamma_{a_0 \to \bar{K}K}/\Gamma_{a_0 \to \pi\eta} = 0.88$ and $\Gamma_{a_0 \to \pi\eta'}/\Gamma_{a_0 \to \pi\eta} = 0.30$ and $\Gamma_{a_0 \to 2P} = 381.17 \text{ MeV}$ (already larger than the experimental result).

The corresponding decay width of the N state into two pseudoscalars is $\Gamma_{N \to 2P} \sim 900 \text{ MeV}$ (for a mass of $M_N \sim 1.4 \text{ GeV}$). We then find similar results as in [6,41], implying that the discussed trend is rather model independent. If at this stage we fix $c_d^s = 17.94 \text{ MeV}$ and $c_m^s = 7.35 \text{ MeV}$ and we make a second fit of the remaining free parameters $(M_N, M_G, M_S, f, \varepsilon, c_d^g, c_m^g)$ to the values reported in (28)–(36), we find minima with $\chi_{\text{tot}}^2 \sim 500$, which are clearly unacceptable. This is a further confirmation of the incom-

patibility of results in the scalar mesonic sector. If we attempt to reproduce the width of $K_0^*(1430)$, the other results are off. As noted above, this problem arises in other models as well.

In [1] only the $K_0^* \to K\pi$ mode is quoted in the list of the decay modes. Our suggestion is that a strong coupling to the scalar mesons below 1 GeV, $K_0^*(800)$ and σ takes place. Both states are very broad and could eventually influence the decay strengths of $K_0^*(1430)$. The theoretical description of such a phenomenon is however beyond the goals of the present tree-level study.

IV. PHENOMENOLOGY INVOLVING THE DIRECT GLUEBALL DECAY MECHANISM

A. Flavor-symmetry limit

The analysis of the previous section did not reveal a phenomenological need to include the direct two-pseudoscalar decay of the glueball component. In this section we analyze this additional mechanism when including it in the fit. We start first by including the interaction term proportional to c_d^g, while neglecting the flavor-symmetry breaking contribution c_m^g.

The ratio c_d^g/c_d^s is a measure of the direct glueball decay strength. The analogous quantity used in [8] is, because of the different normalization, related as $c_d^g/c_d^s \longleftrightarrow \sqrt{3/2} r_2$. For the different solutions in the fit of [8] the parameter r_2 varies between 1 and 5. Note that the case $r_2 = 1$ corresponds to a direct glueball decay strength into two pseudoscalars ~ 1.22 larger than the quarkonium strength. In [13] the bare masses have been kept fixed when the glueball decay parameters have been introduced. Here we release this constraint by leaving the bare masses free. In the fit we again find two different solutions, which correspond to the ones (I and II) analyzed in the previous section.

The χ^2 minimum corresponding to solution I but now with inclusion of c_d^g in the fit is

$$M_N = 1.416 \text{ GeV}, \qquad M_G = 1.493 \text{ GeV},$$
$$M_S = 1.694 \text{ GeV}, \qquad f = 0.075 \text{ GeV}^2,$$
$$\varepsilon = 0.241 \text{ GeV}^2, \qquad c_d^s = 8.73 \text{ MeV}, \tag{57}$$
$$c_m^s = 1.48 \text{ MeV}, \qquad c_d^g = -0.94 \text{ MeV};$$
$$\chi_{\text{tot}}^2 = 28.70.$$

A comparison with the previous fit (41) shows that the parameters practically did not change: with $c_d^g = -0.94 \text{ MeV}$ the direct glueball decay is suppressed, resulting in $|c_d^g/c_d^s| = 0.11 \ll 1$. Also, the total χ^2 is only slightly smaller than in (41).

The minimum corresponding to solution II reads

$$M_N = 1.401 \text{ GeV}, \qquad M_G = 1.461 \text{ GeV},$$

$$M_S = 1.609 \text{ GeV}, \qquad f = 0.376 \text{ GeV}^2,$$

$$\varepsilon = -0.082 \text{ GeV}^2, \qquad c_d^s = 7.63 \text{ MeV},$$

$$c_m^s = 7.09 \text{ MeV}, \qquad c_d^g = 1.82 \text{ MeV}; \qquad \chi_{\text{tot}}^2 = 22.87.$$

$$(58)$$

A comparison with (50) shows a clear similarity, although the bare masses are somewhat shifted. Again, direct glueball decay is suppressed with $|c_d^g/c_d^s| = 0.24$.

The inclusion of c_d^g in the fit does not lead to a drastic change of the previous fit parameters. The bare glueball decay strength is strongly suppressed with respect to the quarkonium one, thus in agreement with the analysis of the previous section and with large N_c considerations.

B. Flavor-symmetry breaking and lattice results

Inclusion of the flavor-symmetry breaking term in direct glueball decay, that is the term with c_m^g, results in nine free parameters [the set of (27) and c_d^g, c_m^g]. A direct fit of these parameters to the data generates various minima, which are not well pronounced. For example, solutions are found where the flavor-symmetry breaking term in the glueball sector c_m^g is exceedingly large, dominating the decay mechanism. Instead, to study the effect of flavor-symmetry breaking in glueball decay we resort to a first lattice study [24] to fix the decay parameters c_d^g and c_m^g.

In [24] a full two-pseudoscalar decay width of the scalar glueball with about 100 MeV is deduced. The corresponding mass of 1.7 GeV led the authors of [24] to interpret the resonance $f_0(1710)$ as mainly gluonic. However, in the cited lattice analysis it is not clear if the glueball decay mechanism occurs partially by mixing with scalar quarkonia (here parametrized by f), or by direct decay (parametrized by c_d^g and c_m^g). In [9] a scenario is studied, where the lattice results of [24] are explained by mixing only (hence c_d^g and c_m^g are set to zero). The physical state $f_0(1710)$ is mainly gluonic, but because of mixing, it acquires a large $\bar{s}s$ amount, which it turn explains the decay pattern. The corresponding mixing matrix is then similar to the results of [7] (and to some aspects of our solution II).

In [7] it is stated that the amplitudes deduced in [24] probably include significant contributions from mixing of the scalar glueball with quarkonium, although not proven. If this is the case, only the mixing mechanism contributes to glueball decay and then we are back to the previous solutions, where the constants c_d^g and c_m^g are negligible. Here we also intend to investigate the opposite case, where the decay couplings calculated in [24] arise from direct glueball decay.

The decay widths of the glueball can be expressed as [24] (see also [9]):

$$\Gamma_{G \to \pi\pi} = \frac{3\lambda^{1/2}(M_G^2, M_\pi^2, M_\pi^2)}{32\pi M_G^3}\{y_{G\pi\pi}M_\rho\}^2,$$

$$\Gamma_{G \to \overline{K}K} = \frac{\lambda^{1/2}(M_G^2, M_K^2, M_K^2)}{8\pi M_G^3}\{y_{G\overline{K}K}M_\rho\}^2, \qquad (59)$$

$$\Gamma_{G \to \eta\eta} = \frac{\lambda^{1/2}(M_i^2, M_\eta^2, M_\eta^2)}{32\pi M_G^3}\{y_{G\eta\eta}M_\rho\}^2,$$

where $M_\rho = 775$ MeV is the ρ mass. The lattice results for the decay constants $y_{G\pi\pi}$, $y_{G\overline{K}K}$ and $y_{G\eta\eta}$ are

$$y_{G\pi\pi} = 0.834^{+0.603}_{-0.579}, \qquad y_{G\overline{K}K} = 2.654^{+0.372}_{-0.402},$$

$$y_{G\eta\eta} = 3.099^{+0.364}_{-0.423}. \qquad (60)$$

Note that in the flavor-symmetry limit we would expect $y_{G\pi\pi}:y_{G\overline{K}K}:y_{G\eta\eta} = 1:1:1$. Although the errors are large, the lattice results show a sizable deviation from this limit. As already noted, it is however not clear, if and to what extent mixing with quarkonia is included in these amplitudes. Interpreting the lattice results in the context of the direct glueball decay mechanism implies a large symmetry violation parameter c_m^g.

The corresponding decays can be derived from the expressions of Appendix B by setting the glueball-quarkonia mixing to zero, that is by considering the scalar-isoscalar decay for $M_i = M_G$ and $B_{iN} = B_{iS} = 0$ and, of course, $B_{iG} = 1$. The explicit expressions for the $\pi\pi$ and $\overline{K}K$ modes of the direct glueball decay are

$$\Gamma_{G \to \pi\pi} = \frac{3\lambda^{1/2}(M_G^2, M_\pi^2, M_\pi^2)}{32\pi M_G^3}\left\{\frac{2}{\sqrt{3}F^2}([M_G^2 - 2M_\pi^2]c_d^g + 2M_\pi^2 c_m^g)\right\}^2,$$

$$\Gamma_{G \to \overline{K}K} = \frac{\lambda^{1/2}(M_G^2, M_K^2, M_K^2)}{8\pi M_G^3}\left\{\frac{2}{\sqrt{3}F^2}([M_G^2 - 2M_K^2]c_d^g + 2M_K^2 c_m^g)\right\}^2. \qquad (61)$$

Matching the expressions for the $\pi\pi$ and $\overline{K}K$ decay modes of (61) to (59) using $M_G = 1.7$ GeV (as in [24]) we obtain for the decay constants:

$$c_d^g = 1.34 \text{ MeV} \qquad \text{and} \qquad c_m^g = 24.6 \text{ MeV}. \qquad (62)$$

For a bare glueball mass of $M_G = 1.5$ GeV we find rather similar values of $c_d^g = 1.72$ MeV and $c_m^g = 25$ MeV; hence we have a rather slight dependence on M_G within a reasonable range of values. In the following we take the values evaluated at $M_G = 1.7$ GeV.

Using the values of (62) we can also determine the $\eta\eta$ decay amplitude. Compared to the lattice result of $y_{G\eta\eta}M_\rho = 2.40^{+0.28}_{-0.33}$ MeV we get the value of 2.025 GeV. The corresponding decay widths for the bare glueball are (for $M_G = 1.7$ GeV):

$$\Gamma_{G\to\pi\pi} = 7.23 \text{ MeV}, \qquad \Gamma_{G\to\bar{K}K} = 80.61 \text{ MeV}, \quad (63)$$

$$\Gamma_{G\to\eta\eta} = 18.35 \text{ MeV}, \qquad \Gamma_{G\to\eta\eta'} = 11.73 \text{ MeV}. \quad (64)$$

For a lower value of mass M_G the only significantly affected mode is $\eta\eta'$, since threshold effects become important (note that the $\eta\eta'$ mode is entirely generated by the flavor-symmetry breaking term proportional to c_m^g).

With the direct glueball decay including flavor-symmetry violation fixed by (62), we now perform a fit with the remaining free parameters M_N, M_G, M_S, f, ε, c_d^s, c_m^s. Two solutions (III and IV) are obtained, which correspond to the bare level orderings $M_N < M_G < M_S$ and $M_N < M_S < M_G$, which we analyze in the following. Although the direct glueball decay dominates in these cases, the two solutions have similarities to the ones discussed in detail in the previous section.

C. Third solution and implications

Solution III is obtained for the set of parameters:

$$
\begin{aligned}
M_N &= 1.359 \text{ GeV}, & M_G &= 1.435 \text{ GeV}, \\
M_S &= 1.686 \text{ GeV}, & f &= 0.212 \text{ GeV}^2, \\
\varepsilon &= 0.277 \text{ GeV}^2, & c_d^s &= 8.28 \text{ MeV}, \\
c_m^s &= 7.21 \text{ MeV}; & \chi_{\text{tot}}^2 &= 21.56,
\end{aligned}
\quad (65)
$$

where the fit results are listed in Table VII. The mixing matrix is similar to the one of solution I and explicitly reads

$$
\begin{pmatrix} |f_1\rangle \equiv |f_0(1370)\rangle \\ |f_2\rangle \equiv |f_0(1500)\rangle \\ |f_3\rangle \equiv |f_0(1710)\rangle \end{pmatrix} = \begin{pmatrix} 0.79 & 0.56 & 0.26 \\ -0.58 & 0.81 & 0.02 \\ -0.20 & -0.16 & 0.97 \end{pmatrix}
$$
$$
\times \begin{pmatrix} |N\rangle \equiv |\bar{n}n\rangle \\ |G\rangle \equiv |gg\rangle \\ |S\rangle \equiv |\bar{s}s\rangle \end{pmatrix}. \quad (66)
$$

TABLE VII. Fitted mass and decay properties of scalar mesons.

Quantity	Exp	Theory	χ_i^2
M_{f_1} (MeV)	1350 ± 150	1242	0.519
M_{f_2} (MeV)	1507 ± 5	1507	0.003
M_{f_3} (MeV)	1714 ± 5	1714	0.011
$\Gamma_{f_2\to\pi\pi}$ (MeV)	38.0 ± 4.6	38.50	0.010
$\Gamma_{f_2\to\bar{K}K}$ (MeV)	9.4 ± 1.7	10.38	0.332
$\Gamma_{f_2\to\eta\eta}$ (MeV)	5.6 ± 1.3	3.65	2.252
$\Gamma_{f_3\to\pi\pi}/\Gamma_{f_3\to\bar{K}K}$	0.20 ± 0.06	0.197	0.002
$\Gamma_{f_3\to\eta\eta}/\Gamma_{f_3\to\bar{K}K}$	0.48 ± 0.15	0.314	1.221
$\Gamma_{a_0\to\bar{K}K}/\Gamma_{a_0\to\pi\eta}$	0.88 ± 0.23	1.079	0.745
$\Gamma_{a_0\to\pi\eta'}/\Gamma_{a_0\to\pi\eta}$	0.35 ± 0.16	0.291	0.134
$\Gamma_{K_0^*\to K\pi}$ (MeV)	273 ± 51	71.02	16.221
$(\Gamma_{f_3})_{2P}$ (MeV)	140 ± 10	143.3	0.109
χ_{tot}^2	$\cdots$	$\cdots$	21.560

TABLE VIII. Decays of $f_1 = f_0(1370)$.

Quantity	Exp (WA102)	Theory
$\Gamma_{f_1\to\bar{K}K}/\Gamma_{f_1\to\pi\pi}$	0.46 ± 0.19	1.10
$\Gamma_{f_1\to\eta\eta}/\Gamma_{f_1\to\pi\pi}$	0.16 ± 0.07	0.17
$(\Gamma_{f_1})_{2P}$ (MeV)	Small	193.5

TABLE IX. Decays of $f_3 = f_0(1710)$.

Quantity	Exp (WA102)	Theory
$\Gamma_{f_3\to\bar{K}K}/\Gamma_{f_3\to\pi\pi}$	5.0 ± 0.7	5.08
$\Gamma_{f_3\to\eta\eta}/\Gamma_{f_3\to\pi\pi}$	2.4 ± 0.6	1.59
$\Gamma_{f_3\to\eta\eta'}/\Gamma_{f_3\to\pi\pi}$	<0.18	2.01
$(\Gamma_{f_3})_{2P}$ (MeV)	Dominant	143.3

The decay rates for $f_1 = f_0(1370)$ and $f_3 = f_0(1710)$ as compared to the WA102 data are summarized in Tables VIII and IX.

The two-photon decay ratios resulting from solution III read

$$\Gamma_{f_1\to2\gamma}:\Gamma_{f_2\to2\gamma}:\Gamma_{f_3\to2\gamma}:\Gamma_{a_0^0\to2\gamma} = 1:1.018:0.025:0.494. \quad (67)$$

The results of the fit summarized in Table VII are acceptable, apart from the already discussed underestimate of the K^* decay width. The corresponding prediction for the WA102 data on $f_0(1370)$ has problems: the predicted ratio $\Gamma_{f_1\to\bar{K}K}/\Gamma_{f_1\to\pi\pi}$ is larger than unity and the full two-pseudoscalar decay width is very large (we refer to the discussion of solution I on the issue of the latter point). For the state $f_0(1710)$ we obtain a large ratio $\Gamma_{f_3\to\eta\eta'}/\Gamma_{f_3\to\pi\pi}$, again as in solution I, in contrast to the WA102 result. For the two-photon decays we have a large ratio $\Gamma_{f_2\to2\gamma}/\Gamma_{f_1\to2\gamma}$.

D. Fourth solution and implications

Solution IV corresponds to an inverted bare level ordering and a small glueball-quarkonia mixing with

$$
\begin{aligned}
M_N &= 1.392 \text{ GeV}, & M_G &= 1.712 \text{ GeV}, \\
M_S &= 1.452 \text{ GeV}, & f &= -0.050 \text{ GeV}^2, \\
\varepsilon &= 0.232 \text{ GeV}^2, & c_d^s &= 6.66 \text{ MeV}, \\
c_m^s &= 5.84 \text{ MeV}; & \chi_{\text{tot}}^2 &= 26.330.
\end{aligned}
\quad (68)
$$

Although the fit results given in Table X are acceptable, the smallness of the glueball-quarkonia mixing is clear contrast to other phenomenological studies [8–13] and lattice result [7,24].

TABLE X.　Fitted mass and decay properties of scalar mesons.

Quantity	Exp	Theory	χ_i^2
M_{f_1} (MeV)	1350 ± 150	1330	0.017
M_{f_2} (MeV)	1507 ± 5	1507	0.002
M_{f_3} (MeV)	1714 ± 5	1714	~ 0
$\Gamma_{f_2 \to \pi\pi}$ (MeV)	38.0 ± 4.6	39.84	0.135
$\Gamma_{f_2 \to \overline{K}K}$ (MeV)	9.4 ± 1.7	10.21	0.230
$\Gamma_{f_2 \to \eta\eta}$ (MeV)	5.6 ± 1.3	4.89	0.297
$\Gamma_{f_3 \to \pi\pi}/\Gamma_{f_3 \to \overline{K}K}$	0.20 ± 0.06	0.129	1.416
$\Gamma_{f_3 \to \eta\eta}/\Gamma_{f_3 \to \overline{K}K}$	0.48 ± 0.15	0.227	2.842
$\Gamma_{a_0 \to \overline{K}K}/\Gamma_{a_0 \to \pi\eta}$	0.88 ± 0.23	1.086	0.863
$\Gamma_{a_0 \to \pi\eta'}/\Gamma_{a_0 \to \pi\eta}$	0.35 ± 0.16	0.291	0.134
$\Gamma_{K_0^* \to K\pi}$ (MeV)	273 ± 51	45.05	20.365
$(\Gamma_{f_3})_{2P}$ (MeV)	140 ± 10	142.95	0.087
χ_{tot}^2	$\cdots$	$\cdots$	26.330

TABLE XI.　Decays of $f_1 = f_0(1370)$.

Quantity	Exp (WA102)	Theory
$\Gamma_{f_1 \to \overline{K}K}/\Gamma_{f_1 \to \pi\pi}$	0.46 ± 0.19	0.73
$\Gamma_{f_1 \to \eta\eta}/\Gamma_{f_1 \to \pi\pi}$	0.16 ± 0.07	0.13
$(\Gamma_{f1})_{2P}$ (MeV)	Small	99.47

TABLE XII.　Decays of $f_3 = f_0(1710)$.

Quantity	Exp (WA102)	Theory
$\Gamma_{f_3 \to \overline{K}K}/\Gamma_{f_3 \to \pi\pi}$	5.0 ± 0.7	7.78
$\Gamma_{f_3 \to \eta\eta}/\Gamma_{f_3 \to \pi\pi}$	2.4 ± 0.6	1.76
$\Gamma_{f_3 \to \eta\eta'}/\Gamma_{f_3 \to \pi\pi}$	<0.18	1.00
$(\Gamma_{f_3})_{2P}$ (MeV)	Dominant	142.95

The mixing matrix reads

$$
\begin{pmatrix} |f_1\rangle \equiv |f_0(1370)\rangle \\ |f_2\rangle \equiv |f_0(1500)\rangle \\ |f_3\rangle \equiv |f_0(1710)\rangle \end{pmatrix} = \begin{pmatrix} 0.82 & -0.07 & 0.57 \\ -0.57 & \sim 0 & 0.82 \\ -0.06 & 0.99 & 0.04 \end{pmatrix}
$$
$$
\times \begin{pmatrix} |N\rangle \equiv |\bar{n}n\rangle \\ |G\rangle \equiv |gg\rangle \\ |S\rangle \equiv |\bar{s}s\rangle \end{pmatrix}. \qquad (69)
$$

In this solution the state $f_0(1710)$ is very close to a pure gluonic configuration, which is traced to the small mixing parameter f. The states $f_0(1370)$ and $f_0(1500)$ are in turn dominated by the quarkonia components, but with strong mixing between $\bar{n}n$ and $\bar{s}s$. The decay rates for $f_1 = f_0(1370)$ and $f_3 = f_0(1710)$ as compared to the WA102 data are listed in Tables XI and XII.

Finally, the two-photon decay ratios resulting from solution IV read:

$$
\Gamma_{f_1 \to 2\gamma} : \Gamma_{f_2 \to 2\gamma} : \Gamma_{f_3 \to 2\gamma} : \Gamma_{a_0^0 \to 2\gamma} = 1 : 0.273 : 0.008 : 0.382.
$$
$$
(70)
$$

E. Discussion of solutions III and IV

The study of the previous subsections shows that a direct glueball decay with large violation of flavor symmetry is feasible. This feature is based on an interpretation of first lattice results, which corresponds to the limiting case of a direct glueball decay as contained in the parameters c_d^g and c_m^g. This interpretation is in contrast to the phenomenological study of [9] and to the comments given in [24]. Furthermore, such a large value for the flavor-symmetry breaking parameter c_m^g is not in agreement with large N_c arguments. In any case, considering that the interpretation of lattice results is not unique, from a phenomenological point of view it is interesting to analyze the situation, where a strong, flavor-symmetry violating glueball decay is present.

A sizable direct glueball decay is, as already discussed, the result of the phenomenology given in [8]. However, in [8] the direct glueball decay pattern is flavor blind, which would correspond to a large c_d^g but to a suppressed value for c_m^g. In this sense the solutions III and IV differ from the analysis of [8].

Care should also be taken when considering the two-photon decay in this scheme: if the two-pseudoscalar amplitudes are sizable, the same can also be expected for the transitions into two vector mesons (although not studied here). Invoking vector meson dominance sizable corrections to the two-photon final state are expected. The use of the limit $c_e^g = 0$ in the present case is therefore questionable.

V. CONCLUSIONS

In this paper we analyzed the two-pseudoscalar and the two-photon decays of the scalar states between 1–2 GeV in the framework of a chiral Lagrangian, where the glueball has been included as a flavor-blind composite mesonic field with independent couplings to pseudoscalar fields.

In a first step we have set the glueball-pseudoscalar couplings to zero and performed a fit to the accepted averages of PDG2004 [1]. We find two possible solutions (I and II), which, apart from the underestimate of the $K_0^* \to K\pi$ mode, show good agreement with the data. The solutions I and II differ in the bare isoscalar masses, in the mixing matrix (in scheme I) the state $f_0(1500)$ has the largest gluonic amount, while in scheme II the state $f_0(1710)$ has the main gluonic component), in some predictions concerning other decay modes, which have been compared to the experimental results of [33]. From a phenomenological point of view, there is no striking hint for a direct glueball-pseudoscalar coupling.

We then enlarged our analysis by including the direct glueball decay parameter (c_d^g) in the flavor-symmetry limit in the fit. A small value for this parameter is obtained, which in turn confirms the suppression of the direct glueball decay in agreement with large N_c arguments and with [6,10], but contrary to the study of [8]. In a last step we also

included the second glueball decay parameter (c_m^g) in the fit, which involves flavor-symmetry breaking in the direct glueball decay. The minima in the fit are less pronounced, therefore we utilized the lattice results of [24] to determine the direct glueball decay parameters c_d^g and c_m^g. The lattice data are interpreted such as they are matched by the direct glueball decay mechanism, although this procedure might be a limiting case. The resulting fits also generate a good description of the data, where either the $f_0(1500)$ or the $f_0(1710)$ contain a dominant glueball component. These solutions however should at present taken with some care and require further input either from an enlarged, reliable database or lattice constraints. For this reason our preferred solutions are I and II presented in section III.

Although the presence of a strong direct glueball decay cannot be verified directly, the presence of a sizable glueball-quarkonia mixing is essential to be in accord with the data. The magnitude is different in the two proposed solutions (smaller in I, largest in II), but is in line with other models and in magnitude qualitatively consistent with lattice results [7,15].

The starting point of the Lagrangian has been outlined in [20]. In the present work we added the glueball degree of freedom, both for mixing and decays. As a result of the fit, we also considered deviations from the large N_c limit. We find that large N_c arguments are still useful as a guideline in the scalar sector as well. Also a small deviation from the GMO octet mass relation is found.

A direct isoscalar $\overline{n}n$-$\overline{s}s$ quarkonia mixing has been introduced in the theoretical analysis. Instanton solutions of the QCD vacuum are believed to generate strong flavor mixing in both the isoscalar and the scalar sectors. The presence of such mixing is established in the pseudoscalar nonet, while it is still an open question in the scalar-isoscalar mesonic sector. Our two proposed solutions differ in this point: while in the first one this flavor mixing sensibly affects the results, in the second solution it turns out to be negligible.

The problem of the $K_0^* \to K\pi$ mode has been discussed; although no final statement to this puzzle can be said, we compared predictions of various approach and attempted to highlight the difficulty in the scalar sector.

Although many experimental results can be reproduced, and the presence of a scalar glueball and its mixing with scalar quarkonia explains many features of the scalar meson spectroscopy, further work, both theoretically and experimentally, is needed to rule out some mixing scenarios in favor of others.

APPENDIX A: ASPECTS OF THE CHIRAL LAGRANGIAN

We use the following phase conventions for the pseudoscalar $P = \{\pi^\pm, \pi^0, K^\pm, K^0, \overline{K^0}, \eta^0, \eta^8\}$ and scalar $S = \{a_0^\pm, a_0^0, K_0^{*\pm}, K_0^{*0}, \overline{K_0^{*0}}, S^0, S^8\}$ meson fields (neglecting the mixing of the third and the eighth component):

$$\pi^\pm = \frac{1}{\sqrt{2}}(P_1 \mp iP_2), \qquad \pi^0 = P_3,$$

$$K^\pm = \frac{1}{\sqrt{2}}(P_4 \mp iP_5), \qquad K^0 = \frac{1}{\sqrt{2}}(P_6 - iP_7), \quad \text{(A1)}$$

$$\overline{K^0} = \frac{1}{\sqrt{2}}(P_6 + iP_7), \qquad \eta^0 = P_0, \qquad \eta^8 = P_8,$$

$$a_0^\pm = \frac{1}{\sqrt{2}}(S_1 \mp iS_2), \qquad a_0^0 = S_3,$$

$$K_0^{*\pm} = \frac{1}{\sqrt{2}}(S_4 \mp iS_5), \qquad K_0^{*0} = \frac{1}{\sqrt{2}}(S_6 - iS_7),$$

$$\overline{K_0^{*0}} = \frac{1}{\sqrt{2}}(S_6 + iS_7).$$

APPENDIX B: TWO-BODY $s \to p_1 p_2$ AND $s \to \gamma\gamma$ TRANSITIONS (MATRIX ELEMENTS AND DECAY WIDTHS)

1. Scalar-isoscalar strong decays

The strong decay widths of the scalar states are derived at tree-level from the following term of the Lagrangian (2):

$$\mathcal{L}_{\text{decay}}^{\text{strong}} = c_d^s \langle Su_\mu u^\mu \rangle + c_m^s \langle S\chi_+ \rangle + \frac{c_d^g}{\sqrt{3}} G\langle u_\mu u^\mu \rangle$$
$$+ \frac{c_m^g}{\sqrt{3}} G\langle \chi_+ \rangle. \tag{B1}$$

The decay expression for the scalar-isoscalar states $|i\rangle$ with $i = f_1, f_2, f_3$ into $\pi\pi$, $\overline{K}K$, $\eta\eta$ and $\eta\eta'$ are given by the following expressions:

$$\Gamma_{i\to\pi\pi} = \Gamma_{i\to\pi^+\pi^-} + \Gamma_{i\to\pi^0\pi^0} = \frac{3}{2}\Gamma_{i\to\pi^+\pi^-}$$
$$= \frac{3\lambda^{1/2}(M_i^2, M_\pi^2, M_\pi^2)}{32\pi M_i^3}|M_{i\pi^+\pi^-}|^2, \tag{B2}$$

$$\Gamma_{i\to K\overline{K}} = \Gamma_{i\to K^+K^-} + \Gamma_{i\to K^0\overline{K}^0} = 2\Gamma_{i\to K^+K^-}$$
$$= \frac{\lambda^{1/2}(M_i^2, M_K^2, M_K^2)}{8\pi M_i^3}|M_{iK^+K^-}|^2, \tag{B3}$$

$$\Gamma_{i \to \eta\eta} = \frac{\lambda^{1/2}(M_i^2, M_\eta^2, M_\eta^2)}{32\pi M_i^3} |M_{i\eta\eta}|^2, \qquad \text{(B4)}$$

$$\Gamma_{i \to \eta\eta'} = \frac{\lambda^{1/2}(M_i^2, M_\eta^2, M_{\eta'}^2)}{16\pi M_i^3} |M_{i\eta\eta'}|^2, \qquad \text{(B5)}$$

where $\lambda(x, y, z)$ is the Källen triangle function:

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz. \qquad \text{(B6)}$$

The matrix elements $M_{i\pi^+\pi^-}$, $M_{iK^+K^-}$, $M_{i\eta\eta}$ and $M_{i\eta\eta'}$ are given by

$$M_{i \to \pi^+\pi^-} = -\frac{2B_{iN}}{F^2\sqrt{2}}\{[M_i^2 - 2M_\pi^2]c_d^s + 2M_\pi^2 c_m^s\}$$
$$- \frac{2B_{iG}}{F^2\sqrt{3}}\{[M_i^2 - 2M_\pi^2]c_d^g + 2M_\pi^2 c_m^g\}, \qquad \text{(B7)}$$

$$M_{i \to K^+K^-} = -\frac{B_{iN} + \sqrt{2}B_{iS}}{F^2\sqrt{2}}\{[M_i^2 - 2M_K^2]c_d^s + 2M_K^2 c_m^s\}$$
$$- \frac{2B_{iG}}{F^2\sqrt{3}}\{[M_i^2 - 2M_K^2]c_d^g + 2M_K^2 c_m^g\}, \qquad \text{(B8)}$$

$$M_{i \to \eta\eta} = -\frac{2c_d^s}{F^2\sqrt{2}}[M_i^2 - 2M_\eta^2]\{B_{iN}\sin^2\delta_P + B_{iS}\cos^2\delta_P\sqrt{2}\}$$
$$- \frac{4c_m^s}{F^2\sqrt{2}}\{M_\pi^2 B_{iN}\sin^2\delta_P$$
$$+ [2M_K^2 - M_\pi^2]B_{iS}\cos^2\delta_P\sqrt{2}\}$$
$$- \frac{2B_{iG}}{F^2\sqrt{3}}\{c_d^g[M_i^2 - 2M_\eta^2] + 2c_m^g(M_\pi^2$$
$$+ 2[M_K^2 - M_\pi^2]\cos^2\delta_P)\}, \qquad \text{(B9)}$$

$$M_{i \to \eta\eta'} = \frac{\sin 2\delta_P}{F^2\sqrt{2}}\{c_d^s[M_i^2 - M_\eta^2 - M_{\eta'}^2][B_{iN} - B_{iS}\sqrt{2}]$$
$$+ 2c_m^s[M_\pi^2 B_{iN} + (M_\pi^2 - 2M_K^2)B_{iS}\sqrt{2}]$$
$$- 4\sqrt{2/3}\,c_m^g B_{iG}[M_K^2 - M_\pi^2]\}, \qquad \text{(B10)}$$

where $\delta_P = \theta_P - \theta_P^I$ and θ_P^I is the ideal mixing angle with $\sin\theta_P^I = 1/\sqrt{3}$; the quantities B_{ij} are the elements of mixing matrix relating physical states $i = f_1, f_2, f_3$ and bare states $j = N, G, S$ [see definitions in Eqs. (14) and (21)].

2. Isovector and isodoublet strong decays

The decay rates for $a_0(1450)$ into $K\overline{K}$, $\pi\eta$ and $\pi\eta'$ are

$$\Gamma_{a_0 \to K\overline{K}} = \frac{1}{3}[\Gamma_{a_0^+ \to K^+\overline{K}^0} + \Gamma_{a_0^- \to K^-K^0} + \Gamma_{a_0^0 \to K^+K^-}$$
$$+ \Gamma_{a_0^0 \to K^0\overline{K}^0}]$$
$$= \Gamma_{a_0^+ \to K^+\overline{K}^0} = \frac{\lambda^{1/2}(M_{a_0}^2, M_K^2, M_K^2)}{16\pi M_{a_0}^3} |M_{a_0^+ K^+\overline{K}^0}|^2, \qquad \text{(B11)}$$

$$\Gamma_{a_0 \to \pi\eta} = \frac{1}{3}[\Gamma_{a_0^+ \to \pi^+\eta} + \Gamma_{a_0^- \to \pi^-\eta} + \Gamma_{a_0^0 \to \pi^0\eta}]$$
$$= \Gamma_{a_0^+ \to \pi^+\eta} = \frac{\lambda^{1/2}(M_{a_0}^2, M_\pi^2, M_\eta^2)}{16\pi M_{a_0}^3} |M_{a_0^+ \pi^+\eta}|^2, \qquad \text{(B12)}$$

$$\Gamma_{a_0 \to \pi\eta'} = \frac{1}{3}[\Gamma_{a_0^+ \to \pi^+\eta'} + \Gamma_{a_0^- \to \pi^-\eta'} + \Gamma_{a_0^0 \to \pi^0\eta'}]$$
$$= \Gamma_{a_0^+ \to \pi^+\eta'} = \frac{\lambda^{1/2}(M_{a_0}^2, M_\pi^2, M_{\eta'}^2)}{16\pi M_{a_0}^3} |M_{a_0^+ \pi^+\eta'}|^2. \qquad \text{(B13)}$$

The matrix elements $M_{a_0^+ K^+\bar{K}^0}$, $M_{a_0^+ \pi^+\eta}$ and $M_{a_0^+ \pi^+\eta'}$ are given by

$$M_{a_0^+ K^+\bar{K}^0} = -\frac{1}{F^2}([M_{a_0}^2 - 2M_K^2]c_d^s + 2M_K^2 c_m^s), \qquad \text{(B14)}$$

$$M_{a_0^+ \pi^+\eta} = \frac{\sin\delta_P\sqrt{2}}{F^2}([M_{a_0}^2 - M_\pi^2 - M_\eta^2]c_d^s + 2M_\pi^2 c_m^s), \qquad \text{(B15)}$$

$$M_{a_0^+ \pi^+\eta'} = -\frac{\cos\delta_P\sqrt{2}}{F^2}([M_{a_0}^2 - M_\pi^2 - M_{\eta'}^2]c_d^s + 2M_\pi^2 c_m^s). \qquad \text{(B16)}$$

The decay rates for $K_0^*(1430)$ into $K\pi$ and $K\eta$ are (considering the isodoublet $\{K_0^{*+}, K_0^{*0}\}$):

$$\Gamma_{K_0^* \to K\pi} = \frac{1}{2}[\Gamma_{K_0^{*+} \to K^0\pi^+} + \Gamma_{K_0^{*+} \to K^+\pi^0} + \Gamma_{K_0^{*0} \to K^0\pi^0}$$
$$+ \Gamma_{K_0^{*0} \to K^+\pi^-}]$$
$$= \frac{3}{2}\Gamma_{K_0^{*+} \to K^0\pi^+}$$
$$= \frac{3\lambda^{1/2}(M_{K_0^*}^2, M_K^2, M_\pi^2)}{32\pi M_{K_0^*}^3} |M_{K_0^{*+} K^0\pi^+}|^2, \qquad \text{(B17)}$$

$$\Gamma_{K_0^* \to K\eta} = \frac{1}{2}[\Gamma_{K_0^{*+} \to K^+\eta} + \Gamma_{K_0^{*0} \to K^0\eta}]$$

$$= \Gamma_{K_0^{*+} \to K^+\eta} = \frac{\lambda^{1/2}(M_{K_0^*}^2, M_K^2, M_\eta^2)}{16\pi M_{K_0^*}^3}|M_{K_0^{*+}K^+\eta}|^2,$$

(B18)

The matrix elements $M_{K_0^{*+}K^0\pi^+}$ and $M_{K_0^{*+}K^+\eta}$ are given by

$$M_{K_0^{*+}K^0\pi^+} = -\frac{1}{F^2}([M_{K_0^*}^2 - M_\pi^2 - M_K^2]c_d^s$$

$$+ [M_\pi^2 + M_K^2]c_m^s),$$

(B19)

$$M_{K_0^{*+}K^+\eta} = \frac{\cos\delta_P}{F^2}([M_{K_0^*}^2 - M_K^2 - M_\eta^2]c_d^s$$

$$+ [3M_K^2 - M_\pi^2]c_m^s)$$

$$+ \frac{\sin\delta_P}{F^2\sqrt{2}}([M_{K_0^*}^2 - M_K^2 - M_\eta^2]c_d^s$$

$$+ [M_K^2 + M_\pi^2]c_m^s).$$

(B20)

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