CP-violating asymmetries in B^0 decays to $K^+K^-K^0_{S(L)}$ and $K^0_SK^0_SK^0_{S(L)}$

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Decay rates and time-dependent and direct CP asymmetries in the decays $B^0 \to K^+K^-K_{S(L)}$ and $K_SK_SK_{S(L)}$ are studied. Resonant and nonresonant contributions to the three-body decays are carefully investigated. Nonresonant effects on two-body and three-body matrix elements are constrained by QCD counting rules. The predicted branching ratios are consistent with the data within the theoretical and experimental errors, though the theoretical central values are somewhat smaller than the experimental ones. Owing to the presence of *color-allowed* tree amplitudes in $B^0 \to K^+K^-K_{S(L)}$, this penguin-dominated mode may be subject to a potentially significant tree pollution and the deviation of the mixing-induced CP asymmetry from that measured in $B \to J/\psi K_S$, namely, $\Delta\sin 2\beta_{K^+K^-K_{S(L)}} \equiv \sin 2\beta_{K^+K^-K_{S(L)}} - \sin 2\beta_{J/\psi K_S}$, can be as large as $\mathcal{O}(0.10)$. In contrast, the $K_SK_SK_{S(L)}$ modes appear theoretically very clean in our picture with negligible theoretical errors in $\Delta\sin 2\beta_{K_SK_SK_{S(L)}}$. Direct CP asymmetries in $K^+K^-K_{S(L)}$ and $K_SK_SK_{S(L)}$ modes are found to be very small.

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I. INTRODUCTION

Considerable activity in search of possible new physics beyond the standard model (SM) has recently been devoted to the measurements of time-dependent CP asymmetries in neutral B meson decays into final CP eigenstates defined by

$$\frac{\Gamma(\overline{B}(t) \to f) - \Gamma(B(t) \to f)}{\Gamma(\overline{B}(t) \to f) + \Gamma(B(t) \to f)} = S_f \sin(\Delta m t) + \mathcal{A}_f \cos(\Delta m t), \quad (1.1)$$

where Δm is the mass difference of the two neutral B eigenstates, S_f monitors mixing-induced CP asymmetry, and $\mathcal{A}_f$ measures direct CP violation (in the *BABAR* notation, $C_f = -\mathcal{A}_f$). The time-dependent CP asymmetries in the $b \to sq\bar{q}$ penguin-induced two-body decays such as $B^0 \to (\phi, \omega, \pi^0, \eta', f_0)K_S$ measured by *BABAR* [1,2] and Belle [3–5] show some indications of sizable deviations from the expectation of the SM where CP asymmetry in all above-mentioned modes should be equal to $S_{J/\psi K_S} = 0.687 \pm 0.032$ [6] with a small deviation of *at most* $\mathcal{O}(0.1)$ [7,8]. Based on the framework of QCD factorization [9], the mixing-induced CP violation parameter S_f in the seven two-body modes $(\phi, \omega, \rho^0, \eta', \eta, \pi^0, f_0)K_S$ has recently been quantitatively studied in [10–12]. It is found that the sign of $\Delta S_f \equiv -\eta_f S_f - S_{J/\psi K_S}$ (η_f being the CP eigenvalue of the final state f) at short distances is positive except for the channel $\rho^0 K_S$. After including final-state rescattering effects, the central values of ΔS_f become positive for all the modes under consideration, but they tend to be rather small compared to the theoretical uncertainties involved so that it is difficult to make reliable statements on the sign at present [10].

Time-dependent CP asymmetries in the $b \to sq\bar{q}$-induced three-body decays $B^0 \to K^+K^-K_S$ and $K_SK_SK_S$ have also been measured by B factories [2,4,5,13–16] (see Table I). Three-body modes such as these were first discussed by Gershon and Hazumi [17]. While $K_SK_SK_S$ has fixed CP parity, $K^+K^-K_S$ is an admixture of CP-even and CP-odd components, rendering its CP analysis more complicated. By excluding the major CP-odd contribution from ϕK_S, the three-body $K^+K^-K_S$ final state is primarily CP even. A measurement of the CP-even fraction f_+ in the $B^0 \to K^+K^-K_S$ decay yields $f_+ = 0.89 \pm 0.08 \pm 0.06$ by *BABAR* [2] and $0.93 \pm 0.09 \pm 0.05$ by Belle [5], while the CP-odd fraction in $K^+K^-K_L$ is estimated to be $f_- = 0.92 \pm 0.07 \pm 0.06$ by *BABAR* [13]. Hence, while $\eta_f = 1$ for the $K_SK_SK_S$ mode, $\eta_f = 2f_+ - 1$ for $K^+K^-K_S$ and $\eta_f = -(2f_- - 1)$ for $K^+K^-K_L$. It is convenient to define an effective $\sin 2\beta$ via $S_f \equiv -\eta_f \sin 2\beta_{\rm eff}$. The results of $\sin 2\beta_{\rm eff}$ for $K^+K^-K_S$ obtained from the measurements of $S_{K^+K^-K_S}$ and f_+ are also shown in Table I.

In order to see if the current measurements of the deviation of $\sin 2\beta_{\rm eff}$ in KKK modes from $\sin 2\beta_{J/\psi K_S}$ signal new physics in $b \to s$ penguin-induced modes, it is of great importance to examine and estimate how much of the deviation of $\sin 2\beta_{\rm eff}$ is allowed in the SM. One of the major uncertainties in the dynamic calculations lies in the hadronic matrix elements which are nonperturbative in nature. One way to circumvent this difficulty is to impose SU(3) flavor symmetry [18,19] or isospin and U-spin symmetries [20] to constrain the relevant hadronic matrix elements. While this approach is model independent in the symmetry limit, deviations from that limit can only be computed in a model dependent fashion. In addition, it may have some weakness as discussed in [19].

TABLE I. Mixing-induced CP asymmetries $-S_f$ (top), direct CP violation $\mathcal{A}_f$ (middle), and branching ratios (in units of 10^{-6}, bottom) for $\overline{B}^0 \to K^+K^-K_S$ and $K_SK_SK_S$ decays. For effective $\sin 2\beta$ for $K^+K^-K_S$, the third error is due to the uncertainty in the fraction of CP-even contributions to the decay rate. Experimental results are taken from [2,4,5,13–16].

Final state	*BABAR*	Belle	Average
$K^+K^-K_S$ [a]	$0.42 \pm 0.17 \pm 0.03$	$0.52 \pm 0.16 \pm 0.03$	0.47 ± 0.12
$(\sin 2\beta_{\rm eff})_{K^+K^-K_S}$	$0.55 \pm 0.22 \pm 0.04 \pm 0.11$	$0.60 \pm 0.18 \pm 0.04^{+0.19}_{-0.12}$	$0.57^{+0.18}_{-0.17}$
$K^+K^-K_L$ [b]	$0.07 \pm 0.28^{+0.11}_{-0.12}$		0.07 ± 0.30
$(\sin 2\beta_{\rm eff})_{K^+K^-K_L}$	$0.09 \pm 0.33^{+0.13}_{-0.14} \pm 0.10$		0.09 ± 0.37
$K_SK_SK_S$	$0.63^{+0.28}_{-0.32} \pm 0.04$	$0.58 \pm 0.36 \pm 0.08$	0.61 ± 0.23
$K^+K^-K_S$ [a]	$-0.10 \pm 0.14 \pm 0.04$	$-0.06 \pm 0.11 \pm 0.07$	-0.08 ± 0.10
$K^+K^-K_L$ [b]	$-0.54 \pm 0.22^{+0.09}_{-0.08}$		-0.54 ± 0.24
$K_SK_SK_S$	$0.10 \pm 0.25 \pm 0.05$	$0.50 \pm 0.23 \pm 0.06$	0.31 ± 0.17
$K^+K^-K_S$	$11.9 \pm 1.0 \pm 0.8$	$14.2 \pm 1.7 \pm 2.0$	12.4 ± 1.2
$K_SK_SK_S$	$6.9^{+0.9}_{-0.8} \pm 0.6$	$4.2^{+1.6}_{-1.3} \pm 0.8$	6.2 ± 1.2 [c]

[a]with $\phi(1020)K_S$ excluded.
[b]with $\phi(1020)K_L$ excluded.
[c]with the error enlarged by a factor of $S = 1.4$.

We shall apply the factorization approach in this work as it seems to work even in the case of three-body B decays [21]. By using factorization and kaon timelike form factors extracted from the $e^+e^- \to K\overline{K}$ process, the predicted $\overline{B}^0 \to D^{(*)+}K^-K^0$ rate agrees well with the data [21]. In general, three-body B decays are more complicated than the two-body case as they receive resonant and nonresonant contributions and involve three-body matrix elements. Nonresonant charmless three-body B decays have been studied extensively [22–27] based on heavy meson chiral perturbation theory (HMChPT) [28–30]. However, the predicted decay rates are, in general, unexpectedly large. For example, the branching ratio of the nonresonant decay $B^- \to \pi^+\pi^-\pi^-$ is predicted to be of order 10^{-5} in [22,23], which is too large compared to *BABAR*'s preliminary result $(0.68 \pm 0.41) \times 10^{-6}$ [31]. The issue has to do with the applicability of HMChPT. In order to apply this approach, two of the final-state pseudoscalars have to be soft. The momentum of the soft pseudoscalar should be smaller than the chiral symmetry breaking scale $\Lambda_\chi \sim 830$ MeV. For three-body charmless B decays, the available phase space where chiral perturbation theory is applicable is only a small fraction of the whole Dalitz plot. Therefore, it is not justified to apply chiral and heavy quark symmetries to a certain kinematic region and then generalize it to the region beyond its validity. In order to have a reliable prediction for the total rate of direct three-body decays, one should try to utilize chiral symmetry to a minimum. Therefore, we will apply HMChPT only to the strong vertex and use the form factors to describe the weak vertex [32]. Moreover, we shall introduce a form factor to take care of the off-shell effect.

As shown in [10], among the aforementioned seven neutral PK_S modes, only the ωK_S and $\rho^0 K_S$ modes are expected to have a sizable deviation of the mixing-induced CP asymmetry S_f from $S_{J/\psi K_S}$. More precisely, it is found that $\Delta S_{\omega K_S} = 0.12^{+0.05}_{-0.06}$ and $\Delta S_{\rho^0 K_S} = -0.09^{+0.03}_{-0.07}$[1] in the absence of final-state interactions [10]. Although the tree contribution in these two modes is color suppressed, the large cancellation between a_4 and a_6 penguin terms renders the tree pollution relatively significant. Unlike the above-mentioned case for two-body decays, the tree contribution to the three-body decay $B^0 \to K^+K^-K_S$ is *color-allowed* and hence it has the potential for producing a large deviation from $\sin 2\beta$ measured in $B \to J/\psi K_S$. We shall see in this work that it is indeed the case. In contrast, the absence of tree pollution in $K_SK_SK_S$ renders it theoretically very clean in our picture.

The layout of the present paper is as follows. In Sec. II we apply the factorization approach to study $B^0 \to K^+K^-K_S$ and $K_SK_SK_S$ decays and discuss resonant and nonresonant contributions. Numerical results for decay rates and CP-violating parameters S_f and A_f and discussions are presented in Sec. III. Section IV contains our conclusions.

II. FORMALISM FOR CHARMLESS THREE-BODY B DECAYS

In the factorization approach, the matrix element of the $\overline{B} \to \overline{K}\,\overline{K}\,K$ decay amplitude is given by

$$\langle \overline{K}\,\overline{K}\,K|\mathcal{H}_{\rm eff}|\overline{B}\rangle = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p\langle \overline{K}\,\overline{K}\,K|T_p|\overline{B}\rangle, \qquad (2.1)$$

[1]Note that since $K^+K^-K_S$ is not a pure CP eigenstate, we define $\Delta\sin 2\beta_{\rm eff} \equiv \sin 2\beta_{\rm eff} - \sin 2\beta_{J/\psi K}$ with $\sin 2\beta_{\rm eff} = -S_f/\eta_f$. In general, the relation $\Delta S_f = \Delta\sin 2\beta_f^{\rm eff}$ holds for the final state with fixed CP parity.

where $\lambda_p \equiv V_{pb}V_{ps}^*$ and [9]

$$\begin{aligned}T_p &= a_1\delta_{pu}(\bar{u}b)_{V-A}\otimes(\bar{s}u)_{V-A} + a_2\delta_{pu}(\bar{s}b)_{V-A}\otimes(\bar{u}u)_{V-A} + a_3(\bar{s}b)_{V-A}\otimes\sum_q(\bar{q}q)_{V-A} + a_4^p\sum_q(\bar{q}b)_{V-A}\otimes(\bar{s}q)_{V-A} \\ &+ a_5(\bar{s}b)_{V-A}\otimes\sum_q(\bar{q}q)_{V+A} - 2a_6^p\sum_q(\bar{q}b)_{S-P}\otimes(\bar{s}q)_{S+P} + a_7(\bar{s}b)_{V-A}\otimes\sum_q\frac{3}{2}e_q(\bar{q}q)_{V+A} \\ &- 2a_8^p\sum_q(\bar{q}b)_{S-P}\otimes\frac{3}{2}e_q(\bar{s}q)_{S+P} + a_9(\bar{s}b)_{V-A}\otimes\sum_q\frac{3}{2}e_q(\bar{q}q)_{V-A} + a_{10}^p\sum_q(\bar{q}b)_{V-A}\otimes\frac{3}{2}e_q(\bar{s}q)_{V-A},\end{aligned} \tag{2.2}$$

with $(\bar{q}q')_{V\pm A}\equiv\bar{q}\gamma_\mu(1\pm\gamma_5)q'$, $(\bar{q}q')_{S\pm P}\equiv\bar{q}(1\pm\gamma_5)q'$ and a summation over $q=u,d,s$ being implied. The matrix element $\langle\overline{K}\,\overline{K}K|j\otimes j'|\overline{B}\rangle$ corresponds to $\langle\overline{K}K|j|\overline{B}\rangle\langle\overline{K}|j'|0\rangle$, $\langle\overline{K}|j|\overline{B}\rangle\langle\overline{K}K|j'|0\rangle$, or $\langle 0|j|\overline{B}\rangle\langle\overline{K}\,\overline{K}K|j'|0\rangle$, as appropriate, and a_i are the next-to-leading order effective Wilson coefficients. In this work, we take

$$\begin{aligned}&a_1\approx 0.99\pm 0.37i, \quad a_2\approx 0.19-0.11i, \quad a_3\approx -0.002+0.004i, \quad a_5\approx 0.0054-0.005i, \\ &a_4^u\approx -0.03-0.02i, \quad a_4^c\approx -0.04-0.008i, \quad a_6^u\approx -0.06-0.02i, \quad a_6^c\approx -0.06-0.006i, \\ &a_7\approx 0.54\times10^{-4}i, \quad a_8^u\approx(4.5-0.5i)\times10^{-4}, \quad a_8^c\approx(4.4-0.3i)\times10^{-4}, \quad a_9\approx -0.010-0.0002i, \\ &a_{10}^u\approx(-58.3+86.1i)\times10^{-5}, \quad a_{10}^c\approx(-60.3+88.8i)\times10^{-5},\end{aligned} \tag{2.3}$$

for typical a_i at the renormalization scale $\mu=m_b/2=2.1$ GeV which we are working on.

Applying Eqs. (2.1) and (2.2) and the equation of motion, we obtain the $\overline{B}^0\to K^+K^-\overline{K}^0$ decay amplitude as

$$\begin{aligned}\langle\overline{K}^0K^+K^-|T_p|\overline{B}\rangle &= \langle K^+\overline{K}^0|(\bar{u}b)_{V-A}|\overline{B}^0\rangle\langle K^-|(\bar{s}u)_{V-A}|0\rangle[a_1\delta_{pu}+a_4^p+a_{10}^p-(a_6^p+a_8^p)r_\chi] \\ &+ \langle\overline{K}^0|(\bar{s}b)_{V-A}|\overline{B}^0\rangle\langle K^+K^-|(\bar{u}u)_{V-A}|0\rangle(a_2\delta_{pu}+a_3+a_5+a_7+a_9) \\ &+ \langle\overline{K}^0|(\bar{s}b)_{V-A}|\overline{B}^0\rangle\langle K^+K^-|(\bar{d}d)_{V-A}|0\rangle\left[a_3+a_5-\frac{1}{2}(a_7+a_9)\right] \\ &+ \langle\overline{K}^0|(\bar{s}b)_{V-A}|\overline{B}^0\rangle\langle K^+K^-|(\bar{s}s)_{V-A}|0\rangle\left[a_3+a_4^p+a_5-\frac{1}{2}(a_7+a_9+a_{10}^p)\right] \\ &+ \langle\overline{K}^0|\bar{s}b|\overline{B}^0\rangle\langle K^+K^-|\bar{s}s|0\rangle(-2a_6^p+a_8^p) + \langle K^+K^-\overline{K}^0|(\bar{s}d)_{V-A}|0\rangle\langle 0|(\bar{d}b)_{V-A}|\overline{B}^0\rangle\left(a_4^p-\frac{1}{2}a_{10}^p\right) \\ &+ \langle K^+K^-\overline{K}^0|\bar{s}\gamma_5 d|0\rangle\langle 0|\bar{d}\gamma_5 b|\overline{B}^0\rangle(-2a_6^p+a_8^p),\end{aligned} \tag{2.4}$$

with $r_\chi=2m_K^2/(m_bm_s)$. In the factorization terms, the $K\overline{K}$ pair can be produced through a transition from the $\overline{B}$ meson or can be created from vacuum through V and S operators. There exist two weak annihilation contributions, where the $\overline{B}$ meson is annihilated and a final state with three kaons is created. Note that the Okubo-Zweig-Iizuka rule suppressed matrix element $\langle K^+K^-|(\bar{d}d)_{V-A}|0\rangle$ is included in the factorization amplitude since it could be enhanced through the long-distance pole contributions via the intermediate vector mesons such as ρ^0 and ω.

To evaluate the above amplitude, we need to consider the $\overline{B}\to K\overline{K}$, $0\to K\overline{K}$, and $0\to\overline{K}\,\overline{K}K$ matrix elements, the so-called two-meson transition, and two-meson and three-meson creation matrix elements in addition to the usual one-meson transition and creation ones. The two-meson transition matrix element $\langle\overline{K}^0K^+|(\bar{u}b)_{V-A}|\overline{B}^0\rangle$ has the general expression [33]

$$\begin{aligned}\langle\overline{K}^0(p_1)K^+(p_2)|(\bar{u}b)_{V-A}|\overline{B}^0\rangle &= ir(p_B-p_1-p_2)_\mu + i\omega_+(p_2+p_1)_\mu + i\omega_-(p_2-p_1)_\mu \\ &+ h\epsilon_{\mu\nu\alpha\beta}p_B^\nu(p_2+p_1)^\alpha(p_2-p_1)^\beta.\end{aligned} \tag{2.5}$$

This leads to

$$\langle K^-(p_3)|(\bar{s}u)_{V-A}|0\rangle\langle\overline{K}^0(p_1)K^+(p_2)|(\bar{u}b)_{V-A}|\overline{B}^0\rangle = -\frac{f_K}{2}[2m_3^2r+(m_B^2-s_{12}-m_3^2)\omega_+ + (s_{23}-s_{13}-m_2^2+m_1^2)\omega_-], \tag{2.6}$$

where $s_{ij}\equiv(p_i+p_j)^2$. A pole model calculation of the $\overline{B}^0\to\overline{K}^0K^+$ transition matrix element amounts to considering the strong interaction $\overline{B}^0 \to \overline{K}^0 \overline{B}_s^*$ followed by the weak transition $\overline{B}_s^* \to K^+$ and the result is [32]

$$[\langle K^-(p_3)|(\bar{s}u)_{V-A}|0\rangle\langle \overline{K}^0(p_1)K^+(p_2)|(\bar{u}b)_{V-A}|\overline{B}^0\rangle]_{\rm pole} = \frac{f_K}{f_\pi}\frac{g\sqrt{m_B m_{B_s^*}}}{s_{23}-m_{B_s^*}^2}F(s_{23}, m_{B_s^*})F_1^{B_sK}(m_3^2) \times\left[m_B + \frac{s_{23}}{m_B} - m_B\frac{m_B^2 - s_{23}}{m_3^2}\left(1-\frac{F_0^{B_sK}(m_3^2)}{F_1^{B_sK}(m_3^2)}\right)\right] \times\left[m_1^2+m_3^2-s_{13}+\frac{(s_{23}-m_2^2+m_3^2)(m_B^2-s_{23}-m_1^2)}{2m_{B_s^*}^2}\right], \tag{2.7}$$

where g is a heavy-flavor independent strong coupling which can be extracted from the recent CLEO measurement of the D^{*+} decay width, $g = 0.59 \pm 0.01 \pm 0.07$ [34], and $F_{0,1}^{B_sK}$ are the $B_s \to K$ weak transition form factors in the standard convention [35]. Since B_s^* can be far from the mass shell, it is necessary to introduce a form factor $F(s_{23}, m_{B_s^*})$ to take into account the off-shell effect of the B_s^* pole. Following [36], it is parametrized as $F(s_{23}, m_{B_s^*}) = (\Lambda^2 - m_{B_s^*}^2)/(\Lambda^2 - s_{23})$ with the cutoff parameter Λ chosen to be $\Lambda = m_{B_s^*} + \Lambda_{\rm QCD}$.

It is worth making a digression for a moment. In principle, one can apply HMChPT *twice* to evaluate the form factors r, ω_+, and ω_- [33]. However, this will lead to too large decay rates in disagreement with experiment [32]. This is because the use of HMChPT is reliable only in the kinematic region where K^+ and $\overline{K}^0$ are soft. Therefore, the available phase space where chiral perturbation theory is applicable is very limited. If the soft meson result is assumed to be applicable to the whole Dalitz plot, the decay rate will be greatly overestimated. Therefore, we employ the pole model to evaluate the aforementioned form factors. We shall apply HMChPT only *once* to the $\overline{B}^0 K^0 B_s^*$ strong vertex and introduce a form factor to take care of the momentum dependence of the strong coupling.

The resonant pole contributions to the form factors r, $\omega_\pm$, and h can be worked out from Eq. (2.7). In principle, there are also nonresonant contributions to these form factors. It turns out that the leading nonresonant contribution can be determined as follows. We notice that the same $\overline{B} \to K\overline{K}$ two-meson transition matrix element also appears in the decay $B^- \to D^0 K^0 K^-$ under factorization [21]. The data favors a 1^- configuration in the K^0K^- pair [37]. The corresponding two-meson transition matrix element is dominated by the ω_- term. Following [21] we shall include a nonresonant contribution to ω_- parametrized as

$$\omega_-^{\rm NR} = \kappa\frac{2p_B\cdot p_2}{s_{12}^2}, \tag{2.8}$$

and employ the $B^- \to D^0K^0K^-$ data and apply isospin symmetry to the $\overline{B} \to K\overline{K}$ matrix elements to determine the unknown parameter κ. The denominator in the above parametrization is inspired by the QCD counting rule which gives rise to a $1/s_{12}^2$ asymptotic behavior,[2] while the numerator $p_B \cdot p_2 = m_B E_{K^+}$ is motivated by the observation that K^+ contains an energetic u quark coming from the $b \to u$ transition.

The matrix elements involving 3-kaon creation are given by [32]

$$\langle \overline{K}^0(p_1)K^+(p_2)K^-(p_3)|(\bar{s}d)_{V-A}|0\rangle\langle 0|(\bar{d}b)_{V-A}|\overline{B}^0\rangle \approx 0,$$
$$\langle \overline{K}^0(p_1)K^+(p_2)K^-(p_3)|\bar{s}\gamma_5 d|0\rangle\langle 0|\bar{d}\gamma_5 b|\overline{B}^0\rangle = v\frac{f_B m_B^2}{f_\pi m_b}\left(1-\frac{s_{13}-m_1^2-m_3^2}{m_B^2-m_K^2}\right)F^{KKK}(m_B^2), \tag{2.9}$$

where

$$v = \frac{m_{K^+}^2}{m_u+m_s} = \frac{m_K^2 - m_\pi^2}{m_s - m_d} \tag{2.10}$$

characterizes the quark-order parameter $\langle \bar{q}q\rangle$ which spontaneously breaks the chiral symmetry. Both relations in Eq. (2.9) are originally derived in the chiral limit [32] and hence the quark masses appearing in Eq. (2.10) are referred to the scale $\sim$1 GeV. The first relation reflects helicity suppression which is expected to be even more effective for energetic kaons. For the second relation, we introduce the form factor F^{KKK} to extrapolate the chiral result to the physical region. Following [32] we shall take $F^{KKK}(q^2) = 1/[1-(q^2/\Lambda_\chi^2)]$ with $\Lambda_\chi = 0.83$ GeV being a chiral symmetry breaking scale.

We now turn to the 2-kaon creation matrix element which can be expressed in terms of timelike kaon current form factors as

$$\langle K^+(p_{K^+})K^-(p_{K^-})|\bar{q}\gamma_\mu q|0\rangle = (p_{K^+}-p_{K^-})_\mu F_q^{K^+K^-},$$
$$\langle K^0(p_{K^0})\overline{K}^0(p_{\bar{K}^0})|\bar{q}\gamma_\mu q|0\rangle = (p_{K^0}-p_{\bar{K}^0})_\mu F_q^{K^0\bar{K}^0}. \tag{2.11}$$

The weak vector form factors $F_q^{K^+K^-}$ and $F_q^{K^0\bar{K}^0}$ can be related to the kaon electromagnetic (e.m.) form factors $F_{\rm em}^{K^+K^-}$ and $F_{\rm em}^{K^0\bar{K}^0}$ for the charged and neutral kaons, re-

[2]As explained in [21], at least two hard gluon exchanges are needed: one creating the $s\bar{s}$ pair in $\overline{K}^0K^+$, the other kicking the spectator to catch up with the energetic s quark to form the K meson. This gives rise to a $1/s_{12}^2$ asymptotic behavior.

spectively. Phenomenologically, the e.m. form factors receive resonant and nonresonant contributions and can be expressed by

$$F_{\rm em}^{K^+K^-} = F_\rho + F_\omega + F_\phi + F_{\rm NR},$$
$$F_{\rm em}^{K^0\bar{K}^0} = -F_\rho + F_\omega + F_\phi + F'_{\rm NR}. \tag{2.12}$$

It follows from Eqs. (2.11) and (2.12) that

$$F_u^{K^+K^-} = F_d^{K^0\bar{K}^0} = F_\rho + 3F_\omega + \frac{1}{3}(3F_{\rm NR} - F'_{\rm NR}),$$
$$F_d^{K^+K^-} = F_u^{K^0\bar{K}^0} = -F_\rho + 3F_\omega, \tag{2.13}$$
$$F_s^{K^+K^-} = F_s^{K^0\bar{K}^0} = -3F_\phi - \frac{1}{3}(3F_{\rm NR} + 2F'_{\rm NR}),$$

where use of isospin symmetry has been made.

The resonant and nonresonant terms in Eq. (2.12) can be parametrized as

$$F_h(s_{23}) = \frac{c_h}{m_h^2 - s_{23} - im_h\Gamma_h},$$
$$F_{\rm NR}^{(\prime)}(s_{23}) = \left(\frac{x_1^{(\prime)}}{s_{23}} + \frac{x_2^{(\prime)}}{s_{23}^2}\right)\left[\ln\left(\frac{s_{23}}{\tilde{\Lambda}^2}\right)\right]^{-1}, \tag{2.14}$$

with $\tilde{\Lambda} \approx 0.3$ GeV. The expression for the nonresonant form factor is motivated by the asymptotic constraint from perturbative QCD (pQCD), namely, $F(t) \to (1/t) \times [\ln(t/\tilde{\Lambda}^2)]^{-1}$ in the large t limit [38]. The unknown parameters c_h, x_i, and x'_i are fitted from the kaon e.m. data, giving the best fit values (in units of GeV^2 for c_h) [21]:

$$c_\rho = 3c_\omega = c_\phi = 0.363, \qquad c_{\rho(1450)} = 7.98 \times 10^{-3},$$
$$c_{\rho(1700)} = 1.71 \times 10^{-3}, \qquad c_{\omega(1420)} = -7.64 \times 10^{-2},$$
$$c_{\omega(1650)} = -0.116, \qquad c_{\phi(1680)} = -2.0 \times 10^{-2}, \tag{2.15}$$

and

$$x_1 = -3.26\ {\rm GeV}^2, \qquad x_2 = 5.02\ {\rm GeV}^4,$$
$$x'_1 = 0.47\ {\rm GeV}^2, \qquad x'_2 = 0. \tag{2.16}$$

Note that the form factors $F_{\rho,\omega,\phi}$ in Eqs. (2.12) and (2.13) include the contributions from the vector mesons $\rho(770)$, $\rho(1450)$, $\rho(1700)$, $\omega(782)$, $\omega(1420)$, $\omega(1650)$, $\phi(1020)$, and $\phi(1680)$. It is interesting to note that (i) the fitted values of c_V are very close to the vector-meson dominance expression $g_{V\gamma}g_{VKK}$ for $V = \rho, \omega, \phi$ [39,40], where $g_{V\gamma}$ is the e.m. coupling of the vector meson defined by $\langle V|j_{\rm em}|0\rangle = g_{V\gamma}\epsilon_V^*$[3] and g_{VKK} is the $V \to KK$ strong coupling with $-g_{\phi K^+K^-} \simeq g_{\rho K^+K^-}/\sqrt{2} = g_{\omega K^+K^-}/\sqrt{2} \simeq$ 3.03, and (ii) the vector-meson pole contributions alone yield $F_{u,s}^{K^+K^-}(0) \approx 1, -1$ and $F_d^{K^+K^-}(0) \approx 0$ as the charged kaon does not contain the valence d quark.[4] The matrix element in the decay amplitude relevant for our purpose then has the expression

$$\langle \overline{K}^0(p_1)|(\bar{s}b)_{V-A}|\overline{B}^0\rangle\langle K^+(p_2)K^-(p_3)|(\bar{q}q)_{V-A}|0\rangle$$
$$= (s_{12} - s_{13})F_1^{BK}(s_{23})F_q^{K^+K^-}(s_{23}). \tag{2.17}$$

We also need to specify the two-body matrix element $\langle K^+K^-|\bar{s}s|0\rangle$ induced from the scalar density. It receives resonant and nonresonant contributions:

$$\langle K^+(p_2)K^-(p_3)|\bar{s}s|0\rangle \equiv f_s^{K^+K^-}(s_{23})$$
$$= \sum_i \frac{m_i\bar{f}_i g^{i\to KK}}{m_i^2 - s_{23} - im_i\Gamma_i} + f_s^{\rm NR},$$
$$f_s^{\rm NR} = \frac{v}{3}(3F_{\rm NR} + 2F'_{\rm NR}) + v\frac{\sigma}{s_{23}^2}\left[\ln\left(\frac{s_{23}}{\tilde{\Lambda}^2}\right)\right]^{-1}, \tag{2.18}$$

where the scalar decay constant $\bar{f}_i$ is defined in $\langle i|\bar{s}s|0\rangle = m_i\bar{f}_i$, $g^{i\to KK}$ is the $i \to KK$ strong coupling, and the nonresonant terms are related to those in $F_s^{K^+K^-}$ through the equation of motion.[5] The main scalar meson pole contributions are those that have dominant $s\bar{s}$ content and large coupling to $K\overline{K}$. It is found in [41] that among the f_0 mesons, only $f_0(980)$ and $f_0(1530)$ have the largest couplings with the $K\overline{K}$ pair. Note that $f_0(1530)$ is a very broad state with the width of order 1 GeV [41]. To proceed with the numerical calculations, we use $g^{f_0(980)\to KK} = 1.5$ GeV, $g^{f_0(1530)\to KK} = 3.18$ GeV, $\Gamma_{f_0(980)} = 80$ MeV, $\Gamma_{f_0(1530)} = 1.160$ GeV [41], $\bar{f}_{f_0(980)}(\mu = m_b/2) \simeq 0.39$ GeV [42], and $\bar{f}_{f_0(1530)} \simeq \bar{f}_{f_0(980)}$. The sign of the resonant terms is fixed by $f_s^{K^+K^-}(0) = v$ from a chiral perturbation theory calculation (see, for example, [43]). It should be stressed that although the nonresonant contributions to f_s^{KK} and F_s^{KK} are related through the equation of motion, the resonant ones are different and not related *a priori*. To apply the equation of motion, the form factors should be away from the resonant region. In the large s_{23} region, the nonresonant contribution dominated by the $1/s_{23}$ term is far away from the resonant one. In contrast, the $1/s_{23}^2$ term dominates in the low s_{23} region where resonant contributions cannot be ignored. The $1/s_{23}^2$ term in F_s is not necessarily conveyed to f_S through the equation of motion. Hence, the $1/s_{23}^2$

[3]The vector-meson e.m. couplings are given by $g_{\phi\gamma} = e_s m_\phi f_\phi$, $g_{\rho\gamma} = [(e_u - e_d)/\sqrt{2}]m_\rho f_\rho$, and $g_{\omega\gamma} = [(e_u + e_d)/\sqrt{2}]m_\omega f_\omega$ where e_q is the quark's charge and f_V is the vector decay constant.

[4]The sign convention is fixed by using $\langle M(q\bar{q}', p)\overline{M}(q\bar{q}', p')|\bar{q}\gamma_\mu q|0\rangle = \langle M(q\bar{q}', p)|\bar{q}\gamma_\mu q|M(q\bar{q}', -p')\rangle = (p - p')_\mu|F_q^{MM}|$ in the case of a real F_q^{MM}.

[5]The use of equations of motion also leads to

$$f_s^{K^+K^-} = -vF_s^{K^+K^-}. \tag{2.19}$$

Note that the pole contribution to $F_s^{K^+K^-}$ should be dropped in the above relation as it applies only to nonresonant contributions.

term in Eq. (2.18) is undetermined and a new parameter σ, which is expected to be of similar size as x_2, is assigned and will be determined later by fitting to the data. The corresponding matrix element is now given by

$$\langle \overline{K}^0(p_1)|\bar{s}b|\overline{B}^0\rangle\langle K^+(p_2)K^-(p_3)|\bar{s}s|0\rangle = \frac{m_B^2 - m_K^2}{m_b - m_s} F_0^{BK}(s_{23}) f_s^{K^+K^-}(s_{23}). \tag{2.20}$$

Collecting all the relevant matrix elements evaluated above, we are ready to compute the amplitude $A(\overline{B}^0 \to K_{S(L)}K^+K^-) = \pm A(\overline{B}^0 \to \overline{K}^0 K^+K^-)/\sqrt{2}$. Since under CP conjugation we have $K_S(\vec{p}_1) \to K_S(-\vec{p}_1)$, $K^+(\vec{p}_2) \to K^-(-\vec{p}_2)$, and $K^-(\vec{p}_3) \to K^+(-\vec{p}_3)$, the $\overline{B}^0 \to K_S K^+ K^-$ amplitude can be decomposed into CP-odd and CP-even components

$$\begin{aligned} A[\overline{B}^0 \to K_S(p_1)K^+(p_2)K^-(p_3)] &= A(s_{12}, s_{13}, s_{23}) \\ &= A_{CP-} + A_{CP+}, \\ A_{CP\pm} &= \frac{1}{2}[A(s_{12}, s_{13}, s_{23}) \\ &\quad \pm A(s_{13}, s_{12}, s_{23})]. \end{aligned} \tag{2.21}$$

Correspondingly, we have

$$\begin{aligned} \Gamma &= \Gamma_{CP+} + \Gamma_{CP-}, \\ \Gamma_{CP\pm} &= \frac{1}{(2\pi)^3}\frac{1}{32m_B^3}\int |A_{CP\pm}|^2 ds_{12}ds_{13} \\ &= \frac{1}{(2\pi)^3}\frac{1}{32m_B^3}\int |A_{CP\pm}|^2 ds_{12}ds_{23}. \end{aligned} \tag{2.22}$$

The vanishing cross terms due to the interference between CP-odd and CP-even components can be easily seen from the (anti)symmetric properties of the amplitude and the integration variables under the interchange of $s_{12} \leftrightarrow s_{13}$. Similar relations hold for the conjugated B^0 decay rate $\bar{\Gamma}$. The CP-even fraction f_+ is defined by

$$f_+ \equiv \frac{\Gamma_{CP+} + \overline{\Gamma}_{CP+}}{\Gamma + \overline{\Gamma}}\Big|_{\phi K_S \text{ excluded.}} \tag{2.23}$$

Note that results for the $K^+K^-K_L$ mode are identical to the $K^+K^-K_S$ ones with the CP eigenstates interchanged. For example, results for $(K^+K^-K_L)_{CP+}$ are the same as those for $(K^+K^-K_S)_{CP-}$ and hence f_+ in $K^+K^-K_S$ corresponds to f_- in $K^+K^-K_L$.

We next turn to the $\overline{B}^0 \to K_SK_SK_S, K_SK_SK_L$ decays. The decay amplitudes are given by

$$\begin{aligned} A[\overline{B}^0 \to K_S(p_1)K_S(p_2)K_{S,L}(p_3)] = \left(\frac{1}{2}\right)^{3/2}\{&\pm A[\overline{B}^0 \to K^0(p_1)\overline{K}^0(p_2)\overline{K}^0(p_3)] \pm A[\overline{B}^0 \to K^0(p_2)\overline{K}^0(p_3)\overline{K}^0(p_1)] \\ &+ A[\overline{B}^0 \to K^0(p_3)\overline{K}^0(p_1)\overline{K}^0(p_2)]\}, \end{aligned} \tag{2.24}$$

with

$$\begin{aligned} A[\overline{B}^0 \to K^0(p_1)\overline{K}^0(p_2)\overline{K}^0(p_3)] = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p\Big\{&[\langle K^0(p_1)\overline{K}^0(p_2)|(\bar{d}b)_{V-A}|\overline{B}^0\rangle\langle \overline{K}^0(p_3)|(\bar{s}d)_{V-A}|0\rangle \\ &+ \langle K^0(p_1)\overline{K}^0(p_3)|(\bar{d}b)_{V-A}|\overline{B}^0\rangle\langle \overline{K}^0(p_2)|(\bar{s}d)_{V-A}|0\rangle]\left(a_4^p + \frac{1}{2}a_{10}^p - \left(a_6^p - \frac{1}{2}a_8^p\right)r_\chi\right) \\ &+ [\langle \overline{K}^0(p_2)|\bar{s}b|\overline{B}^0\rangle\langle K^0(p_1)\overline{K}^0(p_3)|\bar{s}s|0\rangle + \langle \overline{K}^0(p_3)|\bar{s}b|\overline{B}^0\rangle\langle K^0(p_1)\overline{K}^0(p_2)|\bar{s}s|0\rangle] \\ &\times (-2a_6^p + a_8^p) + \langle K^0(p_1)\overline{K}^0(p_2)\overline{K}^0(p_3)|\bar{s}\gamma_5 d|0\rangle\langle 0|\bar{d}\gamma_5 b|\overline{B}^0\rangle(-2a_6^p + a_8^p) \\ &+ [\langle \overline{K}^0(p_2)|(\bar{s}b)_{V-A}|\overline{B}^0\rangle\langle K^0(p_1)\overline{K}^0(p_3)|(\bar{s}s)_{V-A}|0\rangle + \langle \overline{K}^0(p_3)|(\bar{s}b)_{V-A}|\overline{B}^0\rangle \\ &\times \langle K^0(p_1)\overline{K}^0(p_2)|(\bar{s}s)_{V-A}|0\rangle]\left[a_3 + a_4^p + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}^p)\right]\Big\}, \end{aligned} \tag{2.25}$$

where the last term will not contribute to the purely CP-even decay $\overline{B}^0 \to K_SK_SK_S$. Decay rates for the $K_SK_SK_S$ and $K_SK_SK_L$ modes can be obtained from Eq. (2.22) with an additional factor of 1/3! and 1/2!, respectively, for identical particles in the final state.

We now consider the CP asymmetries for $\overline{B}^0 \to K^+K^-K_{S(L)}, K_SK_SK_{S(L)}$ decays. The direct CP asymmetry and the mixing-induced CP violation are defined by

$$\mathcal{A}_{KKK} = \frac{\Gamma - \overline{\Gamma}}{\Gamma + \overline{\Gamma}} = \frac{\int |A|^2 ds_{12} ds_{23} - \int |\bar{A}|^2 ds_{12} ds_{23}}{\int |A|^2 ds_{12} ds_{23} + \int |\bar{A}|^2 ds_{12} ds_{23}},$$

$$S_{KKK,CP\pm} = \frac{2\int \mathrm{Im}(e^{-2i\beta} A_{CP\pm} \bar{A}^*_{CP\pm}) ds_{12} ds_{23}}{\int |A_{CP\pm}|^2 ds_{12} ds_{23} + \int |\bar{A}_{CP\pm}|^2 ds_{12} ds_{23}},$$

$$S_{KKK} = \frac{2\int \mathrm{Im}(e^{-2i\beta} A \bar{A}^*) ds_{12} ds_{23}}{\int |A|^2 ds_{12} ds_{23} + \int |\bar{A}|^2 ds_{12} ds_{23}} = f_+ S_{KKK,CP+} + (1 - f_+) S_{KKK,CP-}, \quad (2.26)$$

where $\bar{A}$ is the decay amplitude of $B^0 \to K^+K^-K_{S(L)}$ or $K_SK_SK_{S(L)}$. For the $K^+K^-K_S$ mode, it is understood that the contribution from ϕK_S is excluded. It is expected in the SM that $S_{KKK,CP+} \equiv \sin 2\beta_{\rm eff} \approx \sin 2\beta$, $S_{KKK,CP-} \approx -\sin 2\beta$, and hence $S_{KKK} \approx -(2f_+ - 1)\sin 2\beta$.[6]

III. NUMERICAL RESULTS AND DISCUSSIONS

To proceed with the numerical calculations, we need to specify the input parameters. For the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, we use the Wolfenstein parameters $A = 0.825$, $\lambda = 0.22622$, $\bar{\rho} = 0.207$, and $\bar{\eta} = 0.340$, corresponding to $(\sin 2\beta)_{\rm CKM} = 0.724$ [44]. For $B \to K$ form factors we shall use those derived in the covariant light-front quark model [45] with the assigned error to be 0.03, namely, $F_{0,1}^{BK}(0) = 0.35 \pm 0.03$. The parameter κ in Eq. (2.8) is determined from the $B^- \to D^0K^0K^-$ data. From the measured branching ratio $\mathcal{B}(B^- \to D^0K^0K^-) = (5.5 \pm 1.4 \pm 0.8) \times 10^{-4}$ [37], we obtain $\kappa = 3.1^{+5.1}_{-1.8}$ GeV where use of $a_1^{DKK} = 0.935$ and $a_2^{DKK}(\simeq a_2^{Dp}) = 0.4 \pm 0.2$ has been made [21]. For the quark masses and the unitarity angle γ, we shall use $m_b(m_b) = 4.2$ GeV, $m_s(m_b/2) = 80 \pm 20$ MeV, and $\gamma = (58.6 \pm 7)^\circ$ [44]. The $K_SK_SK_S$ rate sensitive to the parameter σ in Eq. (2.18) is used to determine $\sigma = (-10.4^{+5.4}_{-4.8})$ GeV^4, where the errors include the uncertainties in the $K_SK_SK_S$ decay rate, the strange quark mass, and the F_0^{BK} form factor.

Results for the decay rates and CP asymmetries in $\overline{B}^0 \to K^+K^-K_{S(L)}$, $K_SK_SK_{S(L)}$ are exhibited in Table II and Table III, respectively. The theoretical errors shown there are from the uncertainties in (i) the parameter κ which governs the nonresonant contribution to the form factor ω_- [see Eq. (2.8)], (ii) the strange quark mass m_s, the form factor F_0^{BK} and σ [see Eq. (2.18)] constrained from the $K_SK_SK_S$ rate, and (iii) the unitarity angle γ. To compute the CP-even fraction f_+ and $\sin 2\beta_{\rm eff}$ for $K^+K^-K_S$, we need to turn off the coefficient c_ϕ in Eq. (2.13). As one can see from Table II, the predicted rates for $\overline{B}^0 \to K^+K^-K_{S(L)}$ decays and the CP-even (odd) ratio $f_{+(-)}$ are in accordance with the data within errors, though the theoretical central values on rates are somewhat smaller than the experimental ones. Theoretical errors on the branching ratios are dominated by the sizable error in κ and the uncertainty in the strange quark mass as the penguin term $a_6 r_\chi$ and the parameter v are very sensitive to m_s. Note that the second error in rates (including the contribution from the uncertainty in σ) is constrained from the $K_SK_SK_S$ rate and hence is reduced significantly. For the first error, we note that the larger the value of $|\kappa|$ we have, the larger the rate on CP-odd $K^+K^-K_S$ is obtained, leading to a smaller value of $f_+(K^+K^-K_S)$. Since the central value of our $f_+(K^+K^-K_S)$ agrees well with data, κ is preferred to be around its central value.

TABLE II. Branching ratios for $\overline{B}^0 \to K^+K^-K_S, K_SK_SK_S, K_SK_SK_L$ decays and the fraction of CP-even contribution to $\overline{B}^0 \to K^+K^-K_S$, f_+ [see Eq. (2.23)]. The branching ratio of CP-odd $K^+K^-K_S$ with ϕK_S excluded is shown in parentheses. Results for $(K^+K^-K_L)_{CP\pm}$ are identical to those for $(K^+K^-K_S)_{CP\mp}$. Theoretical errors correspond to the uncertainties in (i) κ, (ii) m_s, F_0^{BK}, and σ (constrained by the $K_SK_SK_S$ rate), and (iii) γ.

Final state	$\mathcal{B}(10^{-6})_{\rm theory}$	$\mathcal{B}(10^{-6})_{\rm expt}$
$K^+K^-K_S$	$7.33^{+8.38+2.31+0.70}_{-1.08-1.59-0.10}$	12.4 ± 1.2
$(K^+K^-K_S)_{CP+}$	$5.45^{+5.29+1.48+0.05}_{-0.65-1.13-0.06}$	
$(K^+K^-K_S)_{CP-}$	$1.88^{+3.08+0.83+0.04}_{-0.43-0.46-0.04}$	
	$(0.48^{+2.98+0.54+0.03}_{-0.40-0.22-0.03})$	
$K_SK_SK_S$	input	6.2 ± 1.2
$K_SK_SK_L$	$5.74^{+6.02+2.24+0.02}_{-0.88-1.40-0.03}$	
	$f_+^{\rm theory}$	$f_+^{\rm expt}$
$K^+K^-K_S$	$0.92^{+0.06+0.04+0.00}_{-0.16-0.08-0.00}$	0.91 ± 0.07
	$f_-^{\rm theory}$	
$K^+K^-K_L$	$0.92^{+0.06+0.04+0.00}_{-0.16-0.08-0.00}$	

TABLE III. Mixing-induced and direct CP asymmetries $\sin 2\beta_{\rm eff}$ (top) and $\mathcal{A}_f$ (in %, bottom), respectively, in $B^0 \to K^+K^-K_S$ and $K_SK_SK_S$ decays. Results for $(K^+K^-K_L)_{CP\pm}$ are identical to those for $(K^+K^-K_S)_{CP\mp}$. Experimental results are taken from Table I.

Final state	$\sin 2\beta_{\rm eff}$	Expt.
$(K^+K^-K_S)_{\phi K_S \text{ excluded}}$	$0.749^{+0.080+0.024+0.004}_{-0.013-0.011-0.015}$	$0.57^{+0.18}_{-0.17}$
$(K^+K^-K_S)_{CP+}$	$0.770^{+0.113+0.040+0.002}_{-0.031-0.023-0.013}$	
$(K^+K^-K_L)_{\phi K_L \text{ excluded}}$	$0.749^{+0.080+0.024+0.004}_{-0.013-0.011-0.015}$	0.09 ± 0.34
$K_SK_SK_S$	$0.748^{+0.000+0.000+0.007}_{-0.000-0.000-0.018}$	0.65 ± 0.25
$K_SK_SK_L$	$0.748^{+0.001+0.000+0.007}_{-0.001-0.000-0.018}$	
	$\mathcal{A}_f(\%)$	Expt.
$(K^+K^-K_S)_{\phi K_S \text{ excluded}}$	$0.16^{+0.95+0.29+0.01}_{-0.11-0.32-0.02}$	-8 ± 10
$(K^+K^-K_S)_{CP+}$	$-0.09^{+0.73+0.16+0.01}_{-0.00-0.27-0.01}$	
$(K^+K^-K_L)_{\phi K_L \text{ excluded}}$	$0.16^{+0.95+0.29+0.01}_{-0.11-0.32-0.02}$	-54 ± 24
$K_SK_SK_S$	$0.74^{+0.02+0.00+0.05}_{-0.06-0.01-0.06}$	31 ± 17
$K_SK_SK_L$	$0.77^{+0.12+0.08+0.06}_{-0.28-0.11-0.07}$	

[6] Writing the CP-conjugated decay amplitude as $\bar{A} = \bar{A}_{CP+} + \bar{A}_{CP-}$, we have $\bar{A}_{CP\pm} = \pm A_{CP\pm}$ with $\lambda_p \to \lambda_p^*$. This leads to $S_{KKK,CP-} \approx -S_{KKK,CP+}$.

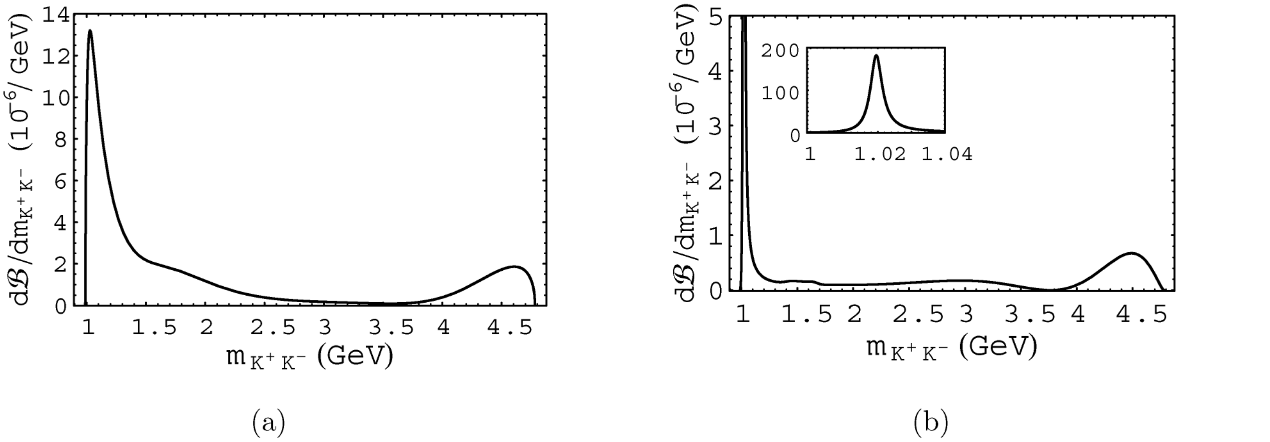

FIG. 1. The K^+K^- mass spectra for $\overline{B}^0 \to K^+K^-K_S$ decay from (a) CP-even and (b) CP-odd contributions. The insert in (b) is for the ϕ region. Results for $(K^+K^-K_L)_{CP\pm}$ are identical to those for $(K^+K^-K_S)_{CP\mp}$.

The K^+K^- mass spectra of the $\overline{B}^0 \to K^+K^-K_S$ decay from CP-even and CP-odd contributions are shown in Fig. 1. In the spectra, there are peaks at the threshold and a milder one in the large $m_{K^+K^-}$ region. For the CP-even part, the threshold enhancement arises from the $f_0(980)K_S$ and the nonresonant $f_S^{K^+K^-}$ contributions [see Eq. (2.18)], while the peak at large $m_{K^+K^-}$ comes from the nonresonant two-meson transition $\overline{B}^0 \to K^+K_S$ followed by a current-produced K^-. Since the nonresonant term [Eq. (2.8)] favors a small $m_{K^+K_s}$ region, the spectrum should peak at the large $m_{K^+K^-}$ end. For the CP-odd spectrum the bump at the large $m_{K^+K^-}$ end originates from the same two-meson transition term, while the peak on the lower end corresponds to the ϕK_s contribution, which is also shown in the insert. The full $K^+K^-K_S$ spectrum is basically the sum of the CP-even and the CP-odd parts. Note that although we include $f_0(1530)K_S$ contribution, its effect is not as prominent as one may expect from the $K^-K^+K^-$ spectrum where a large $f_X(1500)K^-$ contribution is found [46].

For the mixing-induced CP asymmetry in the $K^+K^-K_S$ mode, we compute the effective $\sin 2\beta$ in two different ways: In one way, we calculate S with ϕK_S excluded in $K^+K^-K_S$ and then apply the relation $S = -(2f_+ - 1) \times \sin 2\beta_{\rm eff}$ and the theoretical value of f_+ to obtain $\sin 2\beta_{\rm eff}$. This procedure follows closely the *BABAR* and Belle method of measuring the effective $\sin 2\beta$. In the other way, we calculate S directly for the CP-even $K^+K^-K_S$ and identify $S_{KKK,CP+}$ with $\sin 2\beta_{\rm eff}$. As for the $K_SK_SK_S$ mode, there is no such ambiguity as it is a purely CP-even state. As shown in Table III and Fig. 2, the resulting $\sin 2\beta_{\rm eff}$ is slightly different in these two different approaches.

The deviation of the mixing-induced CP asymmetry in $B^0 \to K^+K^-K_S$ and $K_SK_SK_S$ from that measured in $B \to J/\psi K_S$ (or the fitted CKM's $\sin 2\beta$ [44]), namely, $\Delta \sin 2\beta_{\rm eff} \equiv \sin 2\beta_{\rm eff} - \sin 2\beta_{J/\psi K_S({\rm CKM})}$, is calculated from Table III to be

$$\begin{aligned} \Delta \sin 2\beta_{K^+K^-K_S} &= 0.06^{+0.08}_{-0.02}(0.02^{+0.08}_{-0.02}), \\ \Delta \sin 2\beta_{K_SK_SK_S} &= 0.06^{+0.00}_{-0.00}(0.02^{+0.00}_{-0.00}). \end{aligned} \tag{3.1}$$

Note that part of the deviation comes from that between the measured $\sin 2\beta_{J/\psi K_S}$ and the fitted CKM's $\sin 2\beta$. The $K^+K^-K_S$ has a potentially sizable $\Delta \sin 2\beta$, as this penguin-dominated mode is subject to a tree pollution due to the presence of color-allowed tree contributions. For the $K_SK_SK_S$ mode, the central value and the error on $\Delta \sin 2\beta$ are small.

It is instructive to see the dependence of $\sin 2\beta_{\rm eff}$ on the K^+K^- invariant mass, $m_{K^+K^-} \equiv m_{23} = \sqrt{s_{23}}$. For the phase space integration in Eq. (2.26), for a given s_{23}, the upper and lower bounds of s_{12} are fixed. The invariant mass

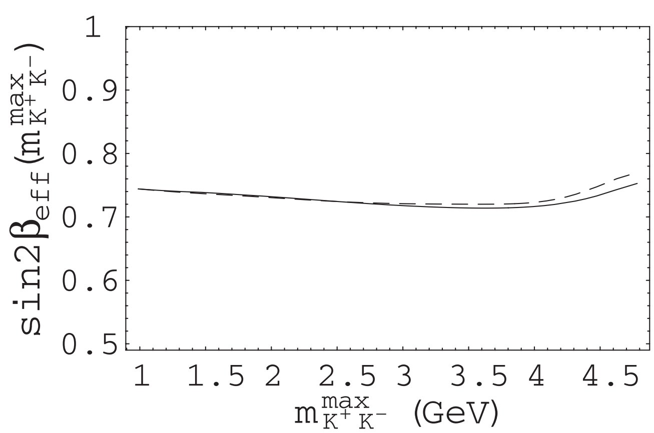

FIG. 2. Mixing-induced CP asymmetry $\sin 2\beta_{\rm eff}(m_{K^+K^-}^{\rm max})$ (see the text for the definition) versus the invariant mass $m_{K^+K^-}^{\rm max}$ for $K^+K^-K_S$ with ϕK_S excluded (solid line) and for CP-even $K^+K^-K_S$ (dashed line). When $m_{K^+K^-}^{\rm max}$ approaches the upper limit $m_B - m_{K_S}$, the whole phase space is saturated and $\sin 2\beta_{\rm eff}(m_{K^+K^-}^{\rm max})$ is reduced to the usual $\sin 2\beta_{\rm eff}$. This result also applies to the $K^+K^-K_L$ mode.

m_{23} is integrated from $m_{23}^- = m_2 + m_3$ to $m_{23}^+ = m_B - m_1$. When the variable s_{23} or m_{23} is integrated from m_{23}^- to a fixed $m_{23}^{\max}$ (of course, $m_{23}^- < m_{23}^{\max} \leq m_{23}^+$), the effective $\sin 2\beta$ thus obtained is designated as $\sin 2\beta_{\rm eff}(m_{23}^{\max})$. Figure 2 shows the plot of $\sin 2\beta_{\rm eff}(m_{K^+K^-}^{\max})$ versus $m_{K^+K^-}^{\max}$ for $K^+K^-K_S$. Since there are two different methods for the determination of $\sin 2\beta_{\rm eff}$, the results are depicted in two different curves. It is interesting that $\sin 2\beta(m_{23}^{\max})$ is slightly below $\sin 2\beta_{\rm CKM}$ at the bulk of the $m_{K^+K^-}$ region and gradually increases and becomes slightly larger than $\sin 2\beta_{\rm CKM}$ when the phase space is getting saturated. The deviation $\Delta \sin 2\beta_{K^+K^-K_S}$ arises mainly from the large $m_{K^+K^-}$ region.

Direct *CP* violation is found to be very small in both $K^+K^-K_S$ and $K_SK_SK_S$ modes. It is interesting to notice that direct *CP* asymmetry in the *CP*-even $K^+K^-K_S$ mode is only of order 10^{-3}, but it becomes 0.2×10^{-2} in $K^+K^-K_S$ with ϕK_S excluded. Since these direct *CP* asymmetries are so small, they can be used as approximate null tests of the SM.

Since direct *CP* violation in charmless *B* decays can be significantly affected by final-state rescattering [36], we have studied to what extent indications of possibly large deviations of the mixing-induced *CP* violation seen in the penguin-induced two-body decay modes from $\sin 2\beta$ determined from $B \to J/\psi K_S$ can be accounted for by final-state interactions [10]. It is natural to extend the study of final-state rescattering effects on time-dependent *CP* asymmetries to $B \to KKK_S$ decays. Final-state interactions in three-body decays are expected to be much more complicated than the two-body case. For example, the color-allowed tree decay $\overline{B}^0 \to D_s^{(*)+} D^{(*)-}$ can rescatter into a $K^+K^-K_S$ final state, where we have $D_s^{(*)+} \to K^+\bar{D}^{*0}$, $D^{(*)-} \to K_S D_s^{*-}$ followed by a $\bar{D}^{*0} D_s^* \to K^-$ fusion. These diagrams are too complicated and will not be included in this study.[7] Nevertheless, we attempt to incorporate final-state rescattering effects in a simple way by including resonance contributions to the corresponding kaon pairs in the final state [47]. We note that another attempt in this direction has recently been made by Furman *et al.* [48]. They considered rescattering of $\pi\pi$ and $K\overline{K}$ pairs in the $\pi\pi$ effective mass range from threshold to 1.1 GeV. While their predicted direct *CP* asymmetry is very small, the parameter *S* is found to be -0.64 or -0.77, depending on the set of penguin amplitudes. However, due to the limitation on phase space, the calculated branching ratios of order 1×10^{-6} for $K^+K^-K_S$ and $K_SK_SK_S$ are only small portions of the total experimental rates (see Table I) and, consequently, the predictions of *S* may be affected when the whole phase space is taken into consideration.

[7] In passing we note that these diagrams could have the effect of increasing somewhat our predictions for the rates of $3K$ final states. Although these contributions carry negligible *CP*-odd (weak) phases, they also contribute to the strong phases and hence will tend to dilute our prediction on ΔS but not necessarily on direct *CP* asymmetries.

IV. CONCLUSIONS

In the present work we have studied the decay rates and time-dependent *CP* asymmetries in the decays $B^0 \to K^+K^-K_{S(L)}$ and $K_SK_SK_{S(L)}$ within the framework of factorization. Our main results are as follows:

(1) Resonant and nonresonant contributions to the hadronic matrix elements are carefully investigated. We incorporate final-state rescattering effects in a simple way by including resonance contributions to the corresponding kaon pairs in the final state. Instead of applying heavy meson chiral perturbation theory to the matrix element for $B \to KK$, which is valid only for a small kinematic region, we consider the resonant contribution from the B_s^* pole and nonresonant contributions constrained by QCD counting rules.

(2) Using the $K_SK_SK_S$ decay rate as an input, the predicted branching ratio of $K^+K^-K_{S(L)}$ modes and the *CP*-even (odd) fraction of $B^0 \to K^+K^-K_{S(L)}$ are consistent with the data within the theoretical and experimental errors, though the theoretical central values on rates are somewhat smaller than the experimental ones.

(3) Owing to the presence of color-allowed tree contributions in $B^0 \to K^+K^-K_{S(L)}$, this penguin-dominated mode is subject to a potentially significant tree pollution and the deviation of the mixing-induced *CP* asymmetry from that measured in $B \to J/\psi K_S$, namely, $\Delta \sin 2\beta_{K^+K^-K_{S(L)}} \equiv \sin 2\beta_{K^+K^-K_{S(L)}} - \sin 2\beta_{J/\psi K_S}$, can be as large as $\mathcal{O}(0.10)$. The deviation $\Delta \sin 2\beta_{K^+K^-K_{S(L)}}$ arises mainly from the large $m_{K^+K^-}$ region.

(4) The $K_SK_SK_{S(L)}$ mode appears theoretically very clean in our picture: The uncertainties in $\Delta \sin 2\beta_{\rm eff}$ are negligible.

(5) Direct *CP* asymmetries are very small in both $K^+K^-K_{S(L)}$ and $K_SK_SK_{S(L)}$ modes.

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Note added.—After the paper was submitted for publication, *BABAR* presented a Dalitz plot study of $B^0 \to K^+K^-K_S^0$ decays [49]. The *BABAR* results constrain the tree contribution (incorporated via Eq. (2.18) in the present work) in rates and, as a result, a small $\Delta \sin 2\beta_{K^+K^-K_S}$ is preferable.

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