# Estimates for pion-photon transition distributions

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In the scaling regime, amplitudes for backward virtual Compton scattering and hadron annihilation into two photons depend on structure functions that describe the hadron-photon transition at the partonic level. We construct simple analytical models for the vector and axial-vector pion-photon transition distribution amplitudes based on double distributions. Via a sum rule, these models are tuned to reproduce the pion-photon transition form factors. To obtain reasonable estimates of the distributions at a low scale, we modify the models to saturate the positivity bounds using empirically parametrized parton distributions. We also determine a model-independent contribution to the axial-vector transition distribution using chiral perturbation theory.

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## I. INTRODUCTION

The detailed study of off-diagonal exclusive processes in perturbative QCD [1–5] has led to a wide variety of theoretical and experimental work. QCD factorization theorems for such processes enable the physical matrix elements to be written as convolutions of perturbatively calculable hard scattering amplitudes with what are now commonly referred to as generalized parton distributions. Typically these distributions arise from hadronic matrix elements that are off-diagonal in momentum space and contain quark or gluon operators separated by a lightlike distance. Parton distributions, which arise in inclusive deep-inelastic scattering, involve diagonal matrix elements of lightlike separated operators. Elastic form factors, on the other hand, are nondiagonal matrix elements of local operators. The physics encompassed by generalized distributions is thus that of both inclusive and exclusive processes at the partonic level. There are a number of insightful reviews on the subject from a variety of perspectives, see [6–11].

In the case of ordinary parton distributions, which we shall generically denote $q(x)$, a wealth of phenomenology has resulted from simple analytic parametrizations. For example, one might assume at some low input scale $\mu$ the form

$$q(x) = Ax^a(1-x)^b,$$

and constrain the parameters $A$, $a$, and $b$ by particle number sum rules and fits to experimental data at higher scales. Typically there is considerable information buried in these few parameters, since one also generally assumes the vanishing of sea quark distributions at the input scale, and these distributions are then radiatively generated by perturbative evolution. Nonetheless reliable phenomenology has emerged from such *Ansätze*, and global analysis of structure function data has allowed for better tuned input parametrizations.

For generalized parton distributions, one has functions which we shall generically write as $F(x, \xi, t)$. In addition to the longitudinal momentum fraction $x$, these functions depend on the four-momentum transfer squared, $t$, and the longitudinal momentum transfer, $\xi$. Lorentz covariance imposes rather stringent constraints on the $x$-moments of $F(x, \xi, t)$, namely, the $n$th moment must be a polynomial in $\xi$ of at most $n$th degree. The coefficients of these polynomials are in general form factors that depend on $t$. For example, the zeroth moment is some form factor $F(t)$. On the other hand, when the four-momentum transfer goes to zero, we have the reduction relation $F(x, 0, 0) = q(x)$, which relates the forward limit of the generalized distribution to the ordinary parton distribution. One might be inclined to choose an input parametrization at the scale $\mu$ of the form

$$F(x, \xi, t) = q(x)F(t),$$

and use existing parton phenomenology and form factor data to fix the input distribution. Without $\xi$ dependence, the polynomiality property of the $x$-moments is trivial. Further the reduction relation in the forward limit is guaranteed by the form factor's normalization. Some problems with the resulting phenomenology are clear: one relies completely on perturbative evolution for the distribution to acquire $\xi$ dependence, whereas one knows that this dependence arises at any scale from the interference between light-cone wave functions of differing longitudinal momenta [12,13]. Furthermore, there is no transparent way to improve the input parametrizations consistent with the constraints, i.e. once the parton distribution and form factor are known, the input is fixed.

For generalized parton distribution phenomenology, one needs a different way to construct and constrain input parametrizations beyond the factorized form above. In an alternate approach, one formulates the generalized distributions in terms of underlying double distributions [14,15].

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The generalized parton distributions are then obtained as projections of these Lorentz invariant double distributions, and this feature maintains the polynomiality properties of generalized parton distribution moments. Consequently the double distribution formulation is attractive from the perspective of generating input parametrizations. Calculations of model double distributions have been pursued [16–20]. There is a drawback to using models based on double distributions: the positivity bounds [12,15,21–24] become obscure [25].

In this work we use the formalism of double distributions to provide estimates of pion-photon transition distribution amplitudes [26]. Unlike generalized parton distributions, the off-diagonality of these transition distributions is in particle state, not just momentum. But dissimilar to other nondiagonal distributions considered previously [27–29], the $t$-channel exchange involved in transition distribution amplitudes carries away nearly all the quantum numbers of the initial state. These distributions enter, e.g., in the pion annihilation process, $\pi^+\pi^- \to \gamma^*\gamma$, and backward virtual Compton scattering, $\gamma^*\pi^+ \to \pi^+\gamma$, in the kinematic region where the real photon is nearly collinear to the initial $\pi^+$. This region is analogous to the scaling limit of virtual Compton scattering, and QCD factorization applies. The amplitudes for the annihilation process and backward Compton scattering factorize into a convolution of a hard scattering kernel with a soft matrix element, the latter describes the hadron to photon transition [26]. Analogous proton to pion transition distribution amplitudes appear in the scaling limit of $\overline{p}N \to \gamma^*\pi$ [30].

This paper has the following organization. First in Sec. II we define the pion-photon transition distributions of [26] in terms of double distributions. Next in Sec. III, we present a simple quark model and use it to calculate transition double distributions. While the model can reproduce the pion-photon transition form factors, the partonic content must be modified to give reasonable phenomenology for processes at large momentum transfer. In Sec. IV, we investigate the positivity bounds for pion-photon transition distributions. Saturating these bounds by including empirically parametrized parton distributions in a factorization approximation results in phenomenologically reasonable input distributions satisfying known constraints. A conclusion ends the paper (Sec. V).

## II. DEFINITIONS

To begin, we build up the vector and axial-vector pion-photon transition distributions by considering matrix elements of towers of twist-two operators. This approach leads us to define the transition distribution amplitudes in terms of double distributions. Consequently Lorentz covariance will be manifest, and this formulation will ultimately enable us to obtain model estimates that satisfy known constraints.

### A. Vector operators

The pion-to-photon matrix elements of vector twist-two operators can be decomposed in a fully Lorentz covariant fashion in terms of various twist-two form factors $W_{nk}(t)$, namely

$$\langle\gamma(P')|\overline{\psi}(0)\tau^-\gamma^{\{\mu}i\overleftrightarrow{D}^{\mu_1}\cdots i\overleftrightarrow{D}^{\mu_n\}}\psi(0)|\pi^+(P)\rangle = -\frac{ie}{f\pi}\epsilon^*_\nu\overline{P}_\rho\Delta_\sigma\varepsilon^{\nu\rho\sigma\{\mu}\sum_{k=0}^{n}\frac{n!}{k!(n-k)!} \times W_{nk}(t)\overline{P}^{\mu_1}\cdots\overline{P}^{\mu_{n-k}}\left(-\frac{\Delta}{2}\right)^{\mu_{n-k+1}}\cdots\left(-\frac{\Delta}{2}\right)^{\mu_n\}}, \quad (1)$$

where the action of $\{\cdots\}$ on Lorentz indices produces the symmetric, traceless part of the tensor, $f_\pi = 132$ MeV is the pion decay constant, $\overline{P}$ is defined to be the average momentum between the initial and final states, $\overline{P}^\mu = \frac{1}{2}\times(P'+P)^\mu$, and $\Delta$ is the momentum transfer, $\Delta^\mu = (P'-P)^\mu$, with $t = \Delta^2$. The field $\psi$ is an isodoublet of the up and down quark fields, and $\tau^\pm$ are the usual isospin raising and lowering operators. $T$-invariance restricts the form factors to be real, but places no further restriction on the index $k$ in the sums.

Ordinarily ambiguity is present in such decompositions of twist-two matrix elements [31–34]. This situation stems from the generality of writing down form factors of currents that are not conserved, and leads to the freedom to define various distributions with the same physical content. Here, however, there is only one possible grouping of Lorentz structures in the above decomposition due to parity. Nonetheless the twist-two currents are not conserved, and there is no ambiguity in the above decomposition into form factors $W_{nk}(t)$.[1] We now define a double distribution $W(\beta, \alpha; t)$ that is the generating function of the twist-two form factors

$$W_{nk}(t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\,\beta^{n-k}\alpha^k W(\beta, \alpha; t). \quad (2)$$

There is no $\alpha$-symmetry of this double distribution because the values of $k$ are unrestricted in Eq. (1).

With the operator product expansion, we can relate the moments in Eq. (1) to matrix elements of bilocal operators. By construction, the double distributions appear in the decomposition of the lightlike separated quark bilinear operator

$$\langle\gamma(P')|\overline{\psi}(-z/2)\tau^-\not{z}\psi(z/2)|\pi^+(P)\rangle = \frac{ie}{f_\pi}z_\mu\epsilon^*_\nu\overline{P}_\rho\Delta_\sigma\varepsilon^{\mu\nu\rho\sigma}\int_{-1}^{1}d\beta\int_{-1+|\beta|}^{1-|\beta|}d\alpha\, e^{-i\beta\overline{P}\cdot z+i\alpha\Delta\cdot z/2} \times W(\beta, \alpha; t), \quad (3)$$

where $z^\mu$ is a lightlike vector.

[1]Electromagnetic gauge invariance is maintained for each twist-two operator as can be verified by replacing $\epsilon^*_\nu$ with $P'_\nu$.

The light-cone correlation function is obtained by Fourier transforming with respect to the light-cone separation $z^-$

$$\mathcal{M}_V(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\overline{P}^+ z^-} \langle \gamma(P')|\overline{\psi}(-z^-/2)\tau^-\gamma^+ \times \psi(z^-/2)|\pi^+(P)\rangle. \tag{4}$$

Above, the variable $\xi$ is the skewness parameter defined by $\xi = -\Delta^+/(2\overline{P}^+)$. Unlike the case of generalized parton distributions, we cannot assume without loss of generality that $\xi > 0$ because time reversal does not relate the negative $\xi$ distributions to those at positive $\xi$. The correlation function in Eq. (4) can be written in terms of the vector pion-photon transition distribution amplitude $V(x,\xi,t)$ [26]

$$\mathcal{M}_V(x,\xi,t) = \frac{ie}{2\overline{P}^+ f_\pi} \epsilon^*_\nu \overline{P}_\rho \Delta_\sigma \varepsilon^{+\nu\rho\sigma} V(x,\xi,t). \tag{5}$$

Unlike the double distribution, the transition amplitude is a quantity that enters directly into convolutions that describe physical processes. Inserting the decomposition equation (3) into the correlator in Eq. (5), we can express the transition amplitude as a projection of the double distribution

$$V(x,\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(x-\beta-\xi\alpha) W(\beta,\alpha;t), \tag{6}$$

from which we interpret the $\xi$-dependence of $V(x,\xi,t)$ as arising from different slices of the Lorentz invariant double distribution.

The transition distribution satisfies an important sum rule constraint. As with the case of generalized parton distributions, integrating $\mathcal{M}_V(x,\xi,t)$ over $x$ produces the local matrix element. Thus we can relate the vector transition distribution to the vector form factor $F_V$ accessible in the weak decay of the pion. Furthermore matrix elements of the flavor diagonal vector operators define transition distributions $V^u$ and $V^d$, and these are related to the $\pi^0 - \gamma$ electromagnetic transition form factor, via [26]

$$\int_{-1}^{1} dx[Q_u V^u(x,\xi,t) - Q_d V^d(x,\xi,t)] = \sqrt{2} f_\pi F_{\pi\gamma}(t). \tag{7}$$

In our model, isospin algebra will enable us to utilize this sum rule constraint.

### B. Axial-vector operators

The analysis of matrix elements of the axial-vector twist-two operators proceeds quite similarly. There are, however, additional contributions from so-called internal bremsstrahlung processes that must be separated out from the matrix elements. These contributions can be determined from chiral perturbation theory in a model-independent way, while the structure dependent terms lead to the axial transition distribution which is modeled in subsequent sections.

To discuss the axial transition distribution, it is useful to recall the form of the transition matrix element of the local axial current. For the pion transition to a real photon we have, see, e.g., [35]

$$\begin{aligned}\langle \gamma(P')|\overline{\psi}(0)\tau^-\gamma^\mu\gamma_5\psi(0)|\pi^+(P)\rangle &= e\epsilon^*_\nu\Big[ f_\pi\Big(g^{\mu\nu} - 2\frac{\Delta^\nu\Delta^\mu}{t-M_\pi^2}\Big) \\ &\quad + F_A(t)(P'^\mu\Delta^\nu - g^{\mu\nu}P'\cdot\Delta)\Big].\end{aligned} \tag{8}$$

The terms in the above decomposition proportional to the pion decay constant arise at tree-level in chiral perturbation theory and are depicted in Fig. 1. The last term, usually called the structure dependent piece, involves the axial transition form factor $F_A(t)$. Electromagnetic gauge invariance is only present for the structure dependent term. For each process, the remaining bremsstrahlung diagrams are required for gauge invariance. The leading contributions in chiral perturbation theory stem from the axial-vector operator $A^\mu$

$$A^\mu = \frac{f^2}{4}\mathrm{Tr}(\Sigma^\dagger\tau^- i\partial^\mu\Sigma - \Sigma\tau^- i\partial^\mu\Sigma^\dagger) \tag{9}$$

and the electromagnetically gauged axial-vector operator. Here $f \approx f_\pi$ at leading order, and $\Sigma$ is an exponential matrix of the pion fields

$$\Sigma = \exp\sqrt{2}i\boldsymbol{\pi}\cdot\boldsymbol{\tau}/f. \tag{10}$$

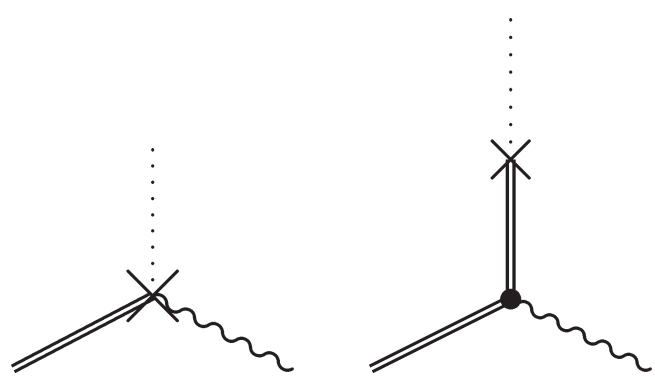

FIG. 1. Tree-level contributions to axial-vector matrix elements in chiral perturbation theory. The pion corresponds to the double line, the photon to the wiggly line, while the axial current is represented by the dotted line. In the local limit, the cross represents the axial current and the depicted contributions are proportional to $f_\pi$. These are separated out from the structure dependent axial form factor. When the cross represents the axial-vector twist-two operators, the same separation can be made, but it involves terms proportional to $f_\pi$ times moments of the pion light-cone distribution amplitude.

There are analogous contributions to the axial-vector twist-two matrix elements and we shall separate them out in order to define a structure dependent axial transition distribution. To this end, let us determine the relevant tree-level contribution in chiral perturbation theory.

Following [36,37], we map the axial-vector twist-two operators onto those in chiral perturbation theory. At leading order[2] we find

$$\mathcal{O}_5^{\mu\mu_1\cdots\mu_n} = a_5^{(n)}\frac{f^2}{4}\mathrm{Tr}(\Sigma^\dagger\tau^- i\partial^{\{\mu}i\partial^{\mu_1}\cdots i\partial^{\mu_n\}}\Sigma - \Sigma\tau^- i\partial^{\{\mu}i\partial^{\mu_1}\cdots i\partial^{\mu_n\}}\Sigma^\dagger), \tag{11}$$

where the $a_5^{(n)}$ are a set of low-energy constants which in general are not constrained by chiral symmetry. They are moments of the pion light-cone distribution amplitude $\phi_\pi(x)$ [39,40], specifically

$$a_5^{(n)} = \int_0^1 dx\left(x-\frac{1}{2}\right)^n\phi_\pi(x). \tag{12}$$

We demonstrate this as follows [38]. Using the operators in Eq. (11), we determine

$$\langle 0|\overline{\psi}(0)\tau^-\gamma^{\{\mu}\gamma_5 i\overleftrightarrow{D}^{\mu_1}\cdots i\overleftrightarrow{D}^{\mu_n\}}\psi(0)|\pi^+(P)\rangle = if_\pi a_5^{(n)}P^{\{\mu}P^{\mu_1}\cdots P^{\mu_n\}}. \tag{13}$$

Contracting with the lightlike vectors $z_\mu z_{\mu_1}\cdots z_{\mu_n}$, and utilizing the operator product expansion, we arrive at

$$\langle 0|\overline{\psi}(-z/2)\tau^-\not{z}\gamma_5\psi(z/2)|\pi^+(P)\rangle = if_\pi P\cdot z\int_0^1 d\beta e^{-i[\beta-(1/2)]P\cdot z}\phi_\pi(\beta). \tag{14}$$

And thus the familiar definition [41] of the pion distribution amplitude emerges via a Fourier transformation, namely

$$\int\frac{dz^-}{2\pi}e^{i[x-(1/2)]z^-P^+}\langle 0|\overline{\psi}(-z^-/2)\tau^-\not{z}\gamma_5\psi(z^-/2)|\pi^+(P)\rangle = if_\pi\phi_\pi(x). \tag{15}$$

Integration over $x$ produces the matrix element of the axial current which defines the pion decay constant. Thus we require the normalization

$$\int_0^1 dx\phi_\pi(x) = 1, \tag{16}$$

which tells us further that $a_5^{(0)} = 1$, see Eq. (12).

Armed with the leading-order operators $\mathcal{O}_5^{\mu\mu_1\cdots\mu_n}$ in Eq. (11), we can calculate the tree-level chiral contributions to the axial twist-two matrix elements shown in Fig. 1. The first diagram, for example, can be determined by electromagnetically gauging the operators in Eq. (11), and then calculating the axial pion-to-photon transition matrix element. One finds

$$\langle\gamma(P')|\mathcal{O}_5^{\mu\mu_1\cdots\mu_n}|\pi^+(P)\rangle = ef(n+1)a_5^{(n)}\epsilon^{*\{\mu}P^{\mu_1}\cdots P^{\mu_n\}}. \tag{17}$$

The factor of $(n+1)$ tells us that ultimately this set of diagrams will be related to the derivative of the pion distribution amplitude. The leading-twist contribution from such terms, however, is a gauge artifact because $z\cdot\epsilon^* = 0$ in light-cone gauge. Thus in the decomposition of axial twist-two transition matrix elements, we should ignore structures proportional to $\epsilon^{*\mu}$. The contribution at leading twist from the second diagram is not a gauge artifact, and can be calculated using the pion electromagnetic current and the operators $\mathcal{O}_5^{\mu\mu_1\cdots\mu_n}$ in Eq. (11). This calculation enables us to write down the decomposition of axial twist-two matrix elements that is the analogue of Eq. (8):

$$\begin{aligned}\langle\gamma(P')|\overline{\psi}(0)\tau^-\gamma^{\{\mu}\gamma_5 i\overleftrightarrow{D}^{\mu_1}\cdots i\overleftrightarrow{D}^{\mu_n\}}\psi(0)|\pi^+(P)\rangle &= e\epsilon_\nu^*\Bigg[-2f_\pi a_5^{(n)}\frac{\Delta^\nu}{t-M_\pi^2}\Delta^{\{\mu}(-\Delta)^{\mu_1}\cdots(-\Delta)^{\mu_n\}}\\ &+\frac{1}{f_\pi}\Delta^\nu P'^{\{\mu}\sum_{k=0}^{n}\frac{n!}{k!(n-k)!}\\ &\times B_{nk}(t)P'^{\mu_1}\cdots P'^{\mu_{n-k}}\left(-\frac{\Delta}{2}\right)^{\mu_{n-k+1}}\cdots\left(-\frac{\Delta}{2}\right)^{\mu_n\}}\Bigg],\end{aligned} \tag{18}$$

where we have omitted any terms involving the structure $g^{\mu\nu}$, and used $\epsilon\cdot P' = 0$ for the real photon. Here the $B_{nk}(t)$ are structure dependent form factors of the axial twist-two operators. As with the vector operators, we define a double distribution $B(\beta,\alpha;t)$ to be the generating function for these form factors

$$B_{nk}(t) = \int_{-1}^1 d\beta\int_{-1+|\beta|}^{1-|\beta|}d\alpha\beta^{n-k}(\alpha+\beta)^k B(\beta,\alpha;t). \tag{19}$$

Using this definition and the relation of the $a_5^{(n)}$ to the pion distribution amplitude, we find that matrix elements of the bilocally separated quark operator have the form

$$\begin{aligned}\langle\gamma(P')|\overline{\psi}(-z/2)\tau^-\not{z}\gamma_5\psi(z/2)|\pi^+(P)\rangle &= -2ef_\pi\frac{\Delta\cdot z\Delta\cdot\epsilon^*}{t-M_\pi^2}\int_0^1 d\beta e^{i[\beta-(1/2)]\Delta\cdot z}\phi_\pi(\beta)\\ &+\frac{e}{f_\pi}P'\cdot z\Delta\cdot\epsilon^*\int_{-1}^1 d\beta\\ &\times\int_{-1+|\beta|}^{1-|\beta|}d\alpha e^{-i\beta\overline{P}\cdot z+i\alpha\Delta\cdot z/2}B(\beta,\alpha;t).\end{aligned} \tag{20}$$

[2]At higher orders in chiral perturbation theory, one has contributions with two extra derivatives contracted, or equivalently, by the equations of motion, insertions of the quark mass matrix. These as well as the one-loop contributions can be systematically calculated. For meson distribution amplitudes, the one-loop corrections have been determined in [38]. We restrict our attention, however, only to those terms which survive in the chiral limit.

The light-cone correlation function is obtained by Fourier transforming with respect to the light-cone separation $z^-$

$$\mathcal{M}_A(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle \gamma(P')|\bar{\psi}(-z^-/2)\tau^-\gamma^+\gamma_5 \times \psi(z^-/2)|\pi^+(P)\rangle, \tag{21}$$

and can be written in terms of the pion distribution amplitude and the axial pion-photon transition distribution $A(x,\xi,t)$ as

$$\mathcal{M}_A(x,\xi,t) = e f_\pi \mathrm{sign}(\xi)\theta(|\xi| - |x|)\frac{\Delta\cdot\epsilon^*}{t - M_\pi^2}\phi_\pi\left(\frac{x+\xi}{2\xi}\right) + \frac{e}{2f_\pi}\Delta\cdot\epsilon^*(1-\xi)A(x,\xi,t), \tag{22}$$

where

$$A(x,\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(x - \beta - \xi\alpha) B(\beta,\alpha;t). \tag{23}$$

The pion pole term derived above is analogous to the one encountered in the nucleon's $\tilde{E}(x,\xi,t)$ generalized parton distribution [42–44]. In our case, instead of the pion-nucleon coupling, $g_{\pi N}(0)$, we have the pion electromagnetic form factor, $F(0) = Q_{\pi^+} = 1$.

Integration of the amplitude $\mathcal{M}_A(x,\xi,t)$ over the variable $x$ produces the plus component of the local axial current operator. The normalization of the pion distribution amplitude guarantees that the pion pole term in Eq. (8) is correctly reproduced. Additionally we arrive at the sum rule constraint that relates the transition distribution of Eq. (22) to the axial transition form factor in Eq. (8)

$$\int_{-1}^{1} dx A(x,\xi,t) = f_\pi F_A(t). \tag{24}$$

## III. QUARK MODEL FOR THE PION-PHOTON TRANSITION

In this section, we use a quark model to calculate pion-photon transition double distributions. Our main goal is to obtain realistic functional dependence of these distributions on the momentum transfer within a covariant framework. Moreover these models will serve as the core for constructing phenomenological estimates of the transition distribution amplitudes by incorporating realistic $x$-dependence.

The model consists of a triplet of pions $\boldsymbol{\pi}$ and a doublet of constituent quarks $q$. Their Lagrangian has the form

$$\mathcal{L} = \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \frac{1}{2}M_\pi^2\boldsymbol{\pi}\cdot\boldsymbol{\pi} + \bar{q}(i\not{\partial} - m)q - ig\boldsymbol{\pi}\cdot\bar{q}\gamma_5\boldsymbol{\tau}q, \tag{25}$$

where we keep the quarks degenerate and omit color indices from the quarks, as these will produce only trivial color factors below. To consider electromagnetic interactions, we must add the photon's coupling to each of the particles in the above Lagrangian. We note that, unlike the chirally invariant operators considered in the previous section, the Lagrangian above does not respect chiral symmetry.[3] In this quark model, the quark-pion coupling $g$ is given by $g = m/f_\pi$. This model was used long ago to calculate pion-photon transition form factors [46–49]. Additionally a variety of partonic applications of this quark model have been pursued, see, e.g., [50,51].

To apply the quark model to pion-photon transition distributions we must come up with a prescription to handle the QCD matrix elements in Eqs. (1) and (18). To make any progress in calculating the double distributions, we must use the parton model simplification for the QCD gauge covariant derivative: $D^\mu \to \partial^\mu$. Further we replace the elementary quark fields $\psi$ with constituent quarks $q$. Working in the impulse approximation, we have the quark model contributions depicted in Fig. 2, and we address the determination of the vector and axial-vector distributions separately.

### A. Vector distributions

For the matrix elements of vector twist-two operators, we only have contributions from the first two diagrams. Explicitly the first depicted diagram has the form

$$-iegN_cQ_d\epsilon_\alpha^* \int \frac{d^4k}{(2\pi)^4} \frac{[(\not{k}+m)\gamma_5(\not{k}-\not{P}'+m)\gamma^\alpha(\not{k}+\not{\Delta}+m)\Gamma^{\mu\mu_1\ldots\mu_n}]}{[k^2-m^2+i\varepsilon][(k+\Delta)^2-m^2+i\varepsilon][(k-P)^2-m^2+i\varepsilon]}, \tag{26}$$

[3]Our choice of a chirally nonsymmetric Lagrangian for the constituent quarks is based on practical considerations. If one uses chiral quarks [45], the pion couples derivatively and the asymptotic falloff of the pion-photon transition form factor is contrary to experiment and perturbative QCD. Moreover, the pion-photon transition amplitudes are complicated by divergences in the chiral quark model. For these reasons we have chosen Eq. (25) as our model.

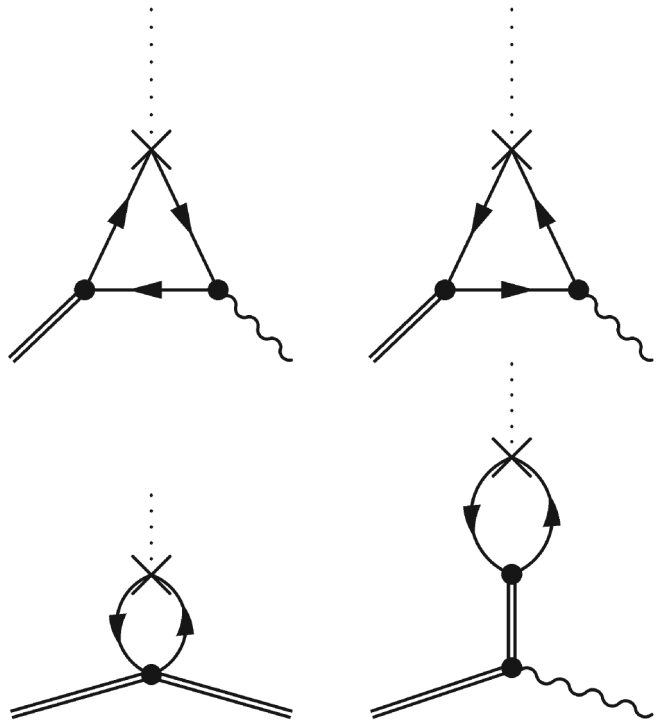

FIG. 2. Impulse approximation to the twist-two matrix elements. Here the twist-two operators with momentum insertion are denoted by a cross. The initial-state pion has momentum $P$, while the final-state photon has momentum $P'$. Only the first two diagrams contribute to the vector twist-two matrix elements. The last two diagrams depend on the moments of the model's (divergent) distribution amplitude. These plus the spectator reduced contributions from the first two diagrams must be isolated to deduce the remaining, structure dependent axial transition distribution.

where $N_c$ is the number of colors. Above we have already performed the trace over isospin and color indices. The symmetric, traceless tensor $\Gamma^{\mu\mu_1\cdots\mu_n}$ in Eq. (26) is the momentum-space transcription of the parton model operator $\mathcal{O}^{\mu\mu_1\cdots\mu_n}$ and is given by

$$\Gamma^{\mu\mu_1\cdots\mu_n} = \gamma^{\{\mu}(k+\Delta/2)^{\mu_1}\cdots(k+\Delta/2)^{\mu_n\}}. \tag{27}$$

Deriving the double distribution can be done directly by casting the above contribution into the form of Eq. (1), and then using the definition in Eq. (2). Taking the contribution from both the first and second diagrams, we find the vector transition double distribution has the form

$$W(\beta,\alpha;t) = \frac{m^2}{4\pi^2}\left\{m^2 + \frac{1}{2}\beta(\alpha+\beta-1)M_\pi^2 - [1-\alpha^2-\beta(2-\beta)]\frac{t}{4}\right\}^{-1}. \tag{28}$$

Notice in the chiral limit $M_\pi \to 0$, the invariant masses of the initial and final states are the same and consequently $\alpha$-symmetry of the double distribution emerges. Using isospin algebra, we can calculate the distributions for flavor diagonal twist-two operators. We find

$$W^u(\beta,\alpha;t) = W^d(\beta,\alpha;t) = 2W(\beta,\alpha;t), \tag{29}$$

where $W(\beta,\alpha;t)$ is the flavor nondiagonal distribution given in Eq. (28). Using these relations we can make contact with the sum rule in Eq. (7), namely

$$F_{\pi\gamma}(t) = \frac{\sqrt{2}}{f_\pi}\int_0^1 d\beta \int_{-1+\beta}^{1-\beta} d\alpha W(\beta,\alpha;t). \tag{30}$$

In the limit $M_\pi \to 0$, we find that the quark model reproduces the normalization of the form factor $F_{\pi\gamma}(0) = (2\sqrt{2}\pi^2 f_\pi)^{-1}$ required by the axial anomaly in the chiral limit [52–54]. Notice the variable $\beta$ is strictly positive because there are no explicit sea quark contributions in this model.

At this stage, we shall fit the constituent quark mass parameter $m$ by requiring that the model pion-photon transition form factor comes close to the experimentally parametrized form. In [55], a simple dipole form is used to describe the experimental data

$$F_{\pi\gamma}(t) = \frac{F_{\pi\gamma}(0)}{1-t/\Lambda^2}, \tag{31}$$

where the dipole mass is $\Lambda = 776$ MeV. We will tune the quark mass at this stage to reproduce the form factor. Eventually, however, we will augment the model distribution with empirical parton distributions. This will necessitate that the model parameters be readjusted. Using the pion mass $M_\pi = 0.14$ GeV, we find that the form factor at low momentum transfer is well reproduced for $m = 0.18$ GeV. The limiting and asymptotic behaviors cannot simultaneously be well described with the model, however, our concern is to find viable models only at low momentum transfer and thus we have fit accordingly. In Fig. 3, we

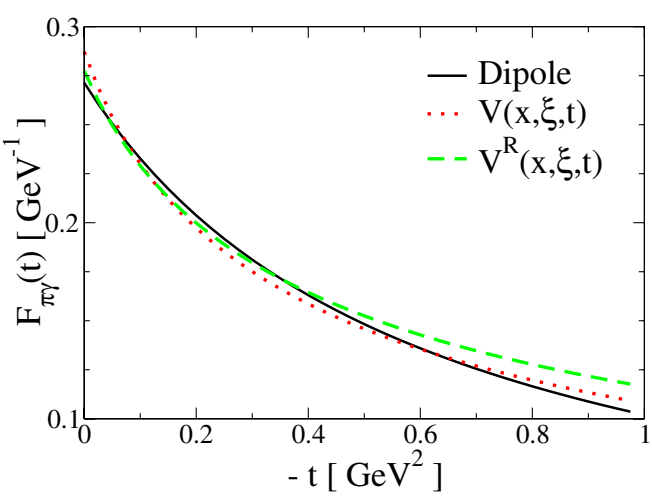


FIG. 3 (color online). Comparison of form factor fits. We compare fits of model pion-photon transition form factors to the empirically parametrized form in Eq. (31). Fits for the basic model $V(x,\xi,t)$ derived from Eq. (28) and the more realistic model $V^R(x,\xi,t)$ in Eq. (45) are shown.

show our fit to the experimentally parametrized transition form factor.

### B. Axial distributions

In considering the matrix elements of the axial-vector twist-two operators, all of the diagrams in Fig. 2 contribute. The first two diagrams give contributions quite similar to that in Eq. (26) with the insertion of an additional $\gamma_5$ from the twist-two operator. We must be careful to remove contributions that are not structure dependent, in the sense of the decomposition in Eq. (8). Contributions from the first two diagrams where the spectators propagator can be reduced (by canceling against factors in the numerator) are not structure dependent [48]. This procedure is complicated by the fact that moments of the pion light-cone distribution amplitude in this model are divergent. We implicitly use a regularization scheme to render the distribution amplitude finite, and remind the reader of this by showing the third diagram in the figure. The exact form of the scheme used is irrelevant; subsequent regularized contributions from the fourth diagram, and those reduced contributions from the first two diagrams are completely discarded because they are not structure dependent. In essence, we do not care what the model says about the pion light-cone distribution amplitude, for we have already determined the model-independent form for this contribution using chiral perturbation theory, cf. Eq. (22).

These technicalities aside, the structure dependent axial transition distribution amplitude can be determined from finite parts of the first two diagrams. We find

$$B(\beta, \alpha; t) = \frac{3m^2(\alpha + \beta)}{4\pi^2}\left\{m^2 + \frac{1}{2}\beta(\alpha + \beta - 1)M_\pi^2 - [1 - \alpha^2 - \beta(2 - \beta)]\frac{t}{4}\right\}^{-1}, \tag{32}$$

which is very similar to the vector transition double distribution. The differences result from the differing spin structure factors present in the interference of the valence light-cone wave functions of the pion and photon, and an overall numerical factor that depends on how the charge factors from the first two diagrams combine. For the parameter value $m = 0.18$ GeV, we find $F_A(0)/F_V(0) = 0.98$ and $F_A(0) = 0.026$. These are consistent with earlier findings for this model [48], but are more than a factor of 2 too large compared with experiment [56].

At this point, we do not plot the vector or axial-vector transition distribution amplitudes derived in this model. The main goal of this section has been to determine the $t$-dependence of the vector and axial form factors in a covariant manner. We will obtain realistic estimates of transition distributions based on these models and further considerations below.

## IV. POSITIVITY AND INPUT PARAMETRIZATIONS

The vector and axial-vector double distributions determined above are presumably at some very low hadronic scale intrinsic to the model. It is not clear how to determine this scale from the model transition distributions alone. One could in turn calculate the pion parton distributions in the model, and then use the evolution equations to evolve empirical parametrizations down to a scale where the first few moments of the model pion parton distribution agree and identify this scale with the model. This procedure is not unique; many models can reproduce the empirical quark distributions at higher scales.[4] Also the use of perturbative evolution is questionable at best at low scales. While the evolution kernels for generalized parton distributions are known at next-to-leading order [57–61], we will not use the evolution approach for our simple model. Perturbative evolution cannot generate the nonperturbative small-$x$ physics which our model lacks, and we believe the small-$x$ physics is crucial for relating to data in the scaling regime. The relevant leading-twist amplitudes are convolutions of transition distribution amplitudes weighted by a hard scattering kernel that emphasizes a region where either the initial- or final-state wave function is evaluated near the end point. In fact, the imaginary part of the amplitude is directly proportional to an overlap of light-cone wave functions, where either the initial or final state has $x = 0$.

In order to be useful for phenomenological estimates, a more pragmatic solution amounts to augmenting the model with realistic parametrizations of the pion and photon quark distributions at a low scale. We shall do this by implanting the realistic distribution via factorization of the $\beta$-dependence of the double distributions [16]. This choice, while unrealistic for the double distributions, may indeed be less problematic for the transition distribution amplitudes since there is ample allowance for interplay of $x$, $\xi$, and $t$ dependence. From the perspective of model building, we are merely attempting to satisfy the known constraints and generate an input distribution at a moderately low scale. One can then evolve upward to the scale relevant to make contact with data and eventually learn how to improve the input parametrization.

To implant realistic partonic behavior and estimate the transition distribution amplitudes, we shall use a factorization *Ansatz* motivated by the positivity constraints. Just like generalized parton distributions, the transition distribution amplitudes satisfy constraints arising from the norm on

[4]There is no guarantee, moreover, that the scale of the implicit quark distribution of the photon is not mismatched with the determined scale of the pion's quark distribution. One should thus calculate the model's quark distributions for both the pion and photon and determine their mutual scale by a simultaneous fit.

Hilbert space: the positivity bounds [12,15,21–24]. Using the method described in [21], we can derive the basic bounds satisfied by both the vector and axial-vector amplitudes. For the positively charged pion, we find

$$\theta(x-\xi)|\mathcal{M}_{V,A}(x,\xi,t)| \leq \sqrt{d^{\gamma}\left(\frac{x-\xi}{1-\xi}\right)u^{\pi^+}\left(\frac{x+\xi}{1+\xi}\right)}. \tag{33}$$

Here we have used $f^H(x)$ to denote the parton distribution of the flavor $f$ in the state $H$. The vector and axial-vector distributions derived in the quark model above self-consistently satisfy the positivity bounds. That is, the constraints in Eq. (33) are met using the quark distributions of the pion and photon determined within the model.[5] For useful phenomenology, however, these model parton distributions must be modified, and hopefully in such a way that the bound in Eq. (33) remains satisfied. As an inspired guess, we choose to saturate the bounds in the forward limit by making a factorization *Ansatz* of the form

$$W^R(\beta,\alpha;t) = N_V \frac{W(\beta,\alpha;t)}{\int_{-1+\beta}^{1-\beta} d\alpha W(\beta,\alpha;0)}\sqrt{d^{\gamma}(\beta)u^{\pi^+}(\beta)}, \tag{34}$$

for the vector distribution and

$$B^R(\beta,\alpha;t) = N_A \frac{B(\beta,\alpha;t)}{\int_{-1+\beta}^{1-\beta} d\alpha B(\beta,\alpha;0)}\sqrt{d^{\gamma}(\beta)u^{\pi^+}(\beta)}, \tag{35}$$

for the axial-vector distribution. Here $N_V$ and $N_A$ are constants to be determined below. When $\{t,\xi\}\to 0$, integration over $\beta$ and $\alpha$ of the *Ansätze* yields

$$\begin{aligned} V^R(x,0,0) &= N_V\sqrt{d^{\gamma}(x)u^{\pi^+}(x)}, \\ A^R(x,0,0) &= N_A\sqrt{d^{\gamma}(x)u^{\pi^+}(x)}, \end{aligned} \tag{36}$$

where $V^R(x,\xi,t)$ and $A^R(x,0,0)$ are the realistic vector and axial-vector transition distribution amplitudes formed from the factorized double distributions. From Eq. (36), one sees our attempt to input realistic light-cone wave functions for the pion and photon consistent with positivity. The resulting form is reasonable provided there is little interference arising from the transverse momentum components of the wave functions. We are not advocating that this is the case. In fact because the positivity bounds are stable under evolution and the square-root *Ansatz* is certainly not true at a high scale, the form is not expected to hold at a low scale. We are, however, only estimating the distributions to the best of our ability given reasonable $x$-dependence of parton distributions. Improving these estimates sensibly is difficult and the subject of an ongoing investigation. In fact for our model, the $\alpha$ integration in Eq. (35) introduces a singularity proportional to $1/\beta$ in the double distribution. This suggests that interference between the various Fock components in the axial distribution cannot be neglected. In our model for the axial distribution, saturation of the positivity bounds in the forward limit is not possible. To tame the singularity, we shall add a factor of $\beta$ to the numerator of Eq. (35) as the simplest way to introduce the requisite interference:

$$B^R(\beta,\alpha;t) = N_A \frac{\beta B(\beta,\alpha;t)}{\int_{-1+\beta}^{1-\beta} d\alpha B(\beta,\alpha;0)}\sqrt{d^{\gamma}(\beta)u^{\pi^+}(\beta)}. \tag{37}$$

In the literature, there exists a number of analytic parametrizations of quark distributions that have been constrained to fit data. Our preference is to choose those models which are at a reasonably low scale, so that evolving up to the relevant scale for comparison with experiment allows some of the crudeness in the factorization *Ansätze* to be evolved away. For the pion, in Ref. [62] simple analytic parametrizations of the valence and sea quark distributions are presented. As the sea quark distributions are unconstrained experimentally, they have been related, using a constituent quark model, to the radiatively generated parton distributions in the proton, see [63,64]. For our model estimates, we need the up distribution in the positively charged pion,

$$u^{\pi^+}(x) = \frac{1}{2}v^{\pi^+}(x) + \overline{q}^{\pi^+}(x). \tag{38}$$

This distribution is given in terms of the valence and sea distributions defined in [62] that are given by

$$v^{\pi^+}(x,\mu^2 = 0.40\ \mathrm{GeV}^2) = 1.391x^{-0.447}(1-x)^{0.426}, \tag{39}$$

$$\overline{q}^{\pi^+}(x,\mu^2 = 0.40\ \mathrm{GeV}^2) = 0.417x^{-0.793}(1-2.466\sqrt{x} + 3.855x)(1-x)^{4.454}. \tag{40}$$

The input scale $\mu^2 = 0.40\ \mathrm{GeV}^2$ is obtained from using next-to-leading order QCD evolution.

For the real photon, we use the analytic parametrization presented in [65]. The photon's quark distribution consists of a perturbative pointlike contribution and a nonperturbative hadronic contribution. The former is chosen to vanish at the input scale $\mu^2 = 0.40\ \mathrm{GeV}^2$. The hadronic piece is determined using a vector meson dominance model [66], and by relating the quark distributions in vector mesons to

[5] This can be easily understood due to the simplicity of the quark model used. When one confronts the light-cone correlation functions $\mathcal{M}_{V,A}$ directly instead of via the double distributions, one finds that the contributions for $x > \xi$ stem from the light-cone time ordering where the spectator quark is on shell. The remaining two propagators then reduce to the model light-cone wave functions for the initial and final states. A simple application of the Cauchy-Schwarz inequality gives the bound in Eq. (33), because the quark distributions are transverse momentum integrals of the squares of light-cone wave functions.

those in the pion. For the down quark distribution of the photon

$$d^{\gamma}(x) = \frac{1}{4\pi} G_d^2 d^{\pi^0}(x), \tag{41}$$

where $G_d^2 \approx 0.25$, and $d^{\pi^0} = \frac{1}{2}(d^{\pi^+} + d^{\pi^-})$. In terms of the parametrized forms for the pion parton distribution used above, $d^{\pi^0} = \frac{1}{4}[v^{\pi^+} + 4\overline{q}^{\pi^+}]$.

The terms motivated by the positivity bound in the region $\xi < x < 1$ all concern the quark distributions of the pion and photon and hence contribute to the transition distribution amplitudes in the kinematic region $-\xi < x < 1$. Assuming the distributions vanish outside this range is inconsistent with the partonic input. To arrive at the antiquark contributions in the kinematic region $-1 < x < \xi$, we make a similar set of *Ansätze* by considering the positivity bound in the region $-1 < x < -\xi$. In this region, we have

$$\theta(\xi - x)|\mathcal{M}_{V,A}(x, \xi, t)| \leq \sqrt{\overline{d}^{\gamma}\left(\frac{x-\xi}{1-\xi}\right)\overline{u}^{\pi^+}\left(\frac{x+\xi}{1+\xi}\right)}. \tag{42}$$

In terms of the parametrizations used above, $\overline{u}^{\pi^+}(x) = \overline{q}^{\pi^+}(x)$ and $\overline{d}^{\gamma}(x) = d^{\gamma}(x)$. For lack of more information, we are forced to assume the antiquark contributions have the same $t$-dependence as the quark contributions, and thus take the antiquark distributions to be of the form

$$\overline{W}^R(\beta, \alpha; t) = N_V \frac{W(\beta, \alpha; t)}{\int_{-1+\beta}^{1-\beta} d\alpha W(\beta, \alpha; 0)} \sqrt{\overline{d}^{\gamma}(\beta)\overline{u}^{\pi^+}(\beta)}, \tag{43}$$

for the vector distribution and

$$\overline{B}^R(\beta, \alpha; t) = N_A \frac{\beta B(\beta, \alpha; t)}{\int_{-1+\beta}^{1-\beta} d\alpha B(\beta, \alpha; 0)} \sqrt{\overline{d}^{\gamma}(\beta)\overline{u}^{\pi^+}(\beta)}, \tag{44}$$

for the axial-vector distribution. A factor of $\beta$ has been inserted because the axial distribution becomes singular if one saturates the positivity bound in the forward limit. We have defined these functions for positive $\beta$ but have used a barred notation to remind the reader that they are antiquark contributions. The realistic transition distribution amplitudes are formed from the barred and unbarred double distributions via

$$V^R(x, \xi, t) = \int_0^1 d\beta \int_{-1+\beta}^{1-\beta} [\delta(x - \beta - \xi\alpha) W^R(\beta, \alpha; t) - \delta(x + \beta + \xi\alpha)\overline{W}^R(\beta, \alpha; t)], \tag{45}$$

$$A^R(x, \xi, t) = \int_0^1 d\beta \int_{-1+\beta}^{1-\beta} [\delta(x - \beta - \xi\alpha) B^R(\beta, \alpha; t) - \delta(x + \beta + \xi\alpha)\overline{B}^R(\beta, \alpha; t)], \tag{46}$$

where the minus sign arises from the opposite ordering of the antiquark creation and annihilation operators, see [12].

Using these parametrized parton distribution functions in the *Ansätze* equations (34), (37), (43), and (44), we can determine the constants $N_V$ and $N_A$ by using the experimental numbers for $F_V(0)$ derived from conservation of vector current, and $F_A(0)$. Using $F_V(0) = 0.0259$, and $F_A(0) = 0.0115$ [56], we find $N_V = 0.33$ and $N_A = 0.48$. There is no reason to suppose that the constituent quark mass parameter $m$ remains the same when augmenting the double distribution with parametrized quark distributions. Just as we did for the basic model, we can determine $m$ for the realistic model by requiring a reasonable fit to the small momentum transfer behavior of the pion-photon transition form factor $F_{\pi\gamma}(t)$. In Fig. 3, we have shown the form factor determined from $V^R(x, \xi, t)$ for the value $m = 0.20$ GeV, as well as the earlier fit, and the empirical dipole form. We are able to match the low momentum behavior of the transition form factor. Having determined the model parameters for the vector and axial-vector distributions in Eqs. (45) and (46), we can now plot the estimates of transition distribution amplitudes. The vector distribution $V^R(x, \xi, t)$ is plotted as a function of $x$ at fixed $t = -0.10$ GeV$^2$ for a few values of $\xi$ in Fig. 4. The same is done in Fig. 5 for the function $A^R(x, \xi, t)$. One should note that, in the symmetric kinematics, we have the relation

$$t = -\frac{\Delta_\perp^2 - 2\xi(1-\xi)M_\pi^2}{1-\xi^2}. \tag{47}$$

Thus for fixed $t$, we must have

$$-\frac{1}{1 - 2M_\pi^2/t} \leq \xi \leq 1, \tag{48}$$

and unlike the case of deeply virtual Compton scattering, the maximal skewness is not kinematically restricted. In

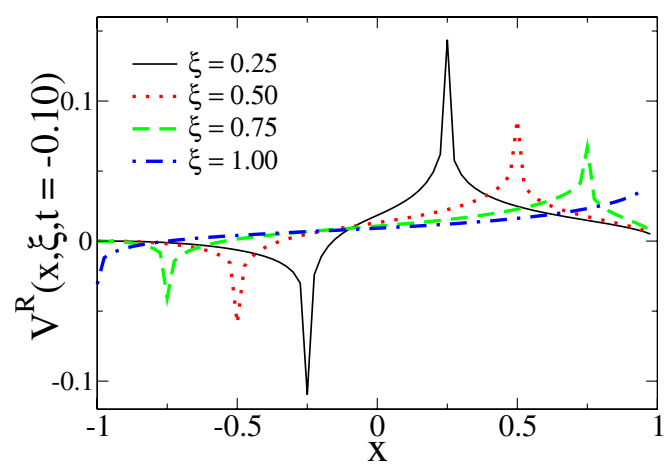


FIG. 4 (color online). Vector pion-photon transition distribution amplitude. The function $V^R(x, \xi, t)$ is plotted at fixed $t = -0.10$ GeV$^2$ for a few values of $\xi$. Notice the distribution is only qualitatively $x$ antisymmetric.

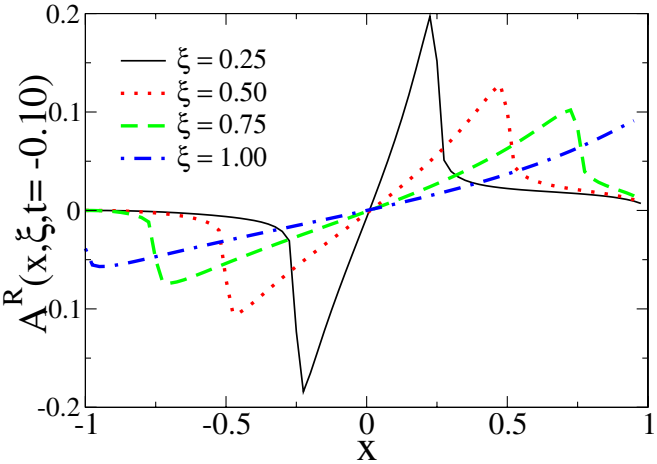


FIG. 5 (color online). Axial-vector pion-photon transition distribution amplitude. The function $A^R(x, \xi, t)$ is plotted at fixed $t = -0.10\ \mathrm{GeV}^2$ for a few values of $\xi$. The distribution is only qualitatively $x$ antisymmetric.

the figures, the minimal skewness at $t = -0.10\ \mathrm{GeV}^2$ is $\xi_{\mathrm{min}} = -0.72$. We have not plotted the distributions for negative values of $\xi$. In our model the differences between distributions at positive and negative values of $\xi$ are minimal. The approximate $\xi$-symmetry that emerges is due to the factorization *Ansätze* which evaluate the pion and photon parton distributions symmetrically.

## V. CONCLUSION

Above we have written the vector and axial-vector pion-photon transition distribution amplitudes in terms of transition double distributions. These double distributions were defined from the matrix elements of the relevant twist-two operators. For the axial-vector transition distribution, we determined the form of a model-independent contribution depending on the pion light-cone distribution amplitude from chiral perturbation theory.

Using a quark model, we calculated the transition double distributions, and demonstrated that the falloff of the pion-photon electromagnetic transition form factor could be well reproduced by tuning the constituent quark mass parameter. Motivated by the positivity bounds, we then took a factorized *Ansatz* for the double distributions in order to establish the renormalization scale of the model, and to incorporate parametrized parton distributions with realistic $x$-dependence. Model parameters were then retuned to produce the pion-photon electromagnetic transition form factor and the pion axial form factor. We then plotted the resultant input distributions which should lead to reasonable phenomenological estimates for pion-photon transitions in the scaling regime. It would be interesting to compare our estimates with other calculations. There are a number of existing realistic studies of pion structure that could be extended to transition distributions amplitudes, e.g., [67–71].

Improving the factorization *Ansatz* employed above may ultimately be necessary to best describe the $x$, $\xi$, and $t$ dependence of the data. There is ample room for improvement, e.g., we assumed the same $t$-dependence and absolute normalization for quark and antiquark contributions. More realistic models, data from lattice QCD, and experimental data can help to motivate better input parametrizations. Furthermore the only difference between vector and axial-vector distributions in our model arises from the spin structure of the valence light-cone wave functions, whereas realistically the difference stems from the interference from all Fock components. One needs to address whether our approximation is good enough for the input distributions at a low scale. We already found suggestions of sizable interference between Fock components in the axial distribution. Finally approximate $\xi$-symmetry of the distributions emerges due to the simplicity of our *Ansätze*. Better input parametrizations must address how to treat the initial and final states differently in a covariant fashion.

The study of off-diagonal exclusive reactions leads to phenomenology involving generalized parton distributions. Unlike ordinary parton distributions, input parametrizations cannot simply be written down at will, as constraints on the distributions are nontrivial. Using double distributions, we have formulated a basic input parametrization for pion-photon transition distribution amplitudes. Contact with future data, lattice QCD data, or other models will help one to tune this type of input parametrization. This procedure in turn gives one a better sense of how to construct reasonable input parametrizations of generalized parton distributions in general.

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