# Flavor structure and coupling selection rule from intersecting *D*-branes

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We study flavor structure and the coupling selection rule in intersecting $D$-brane configurations on $T^2 \times T^2 \times T^2$ as well as its orbifold/orientifold compactifications. We formulate the selection rule for Yukawa couplings and its extensions to generic $n$-point couplings. We investigate the possible flavor structure, which can appear from intersecting $D$-brane configuration. Then, we show there is a certain rule among intersecting numbers for states corresponding to allowed Yukawa couplings, and also it is found that their couplings are determined by discrete Abelian symmetries. Our studies on the flavor structure and the coupling selection rule in this class of models show that there is no solution to derive realistic Yukawa matrices only from stringy 3-point couplings at the tree-level within the framework of the minimal matter content of the supersymmetric standard model.

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## I. INTRODUCTION

Understanding the origin of fermion masses and mixing angles is one of the most important issues in particle physics. Their experimental values show the hierarchical structure. Within the framework of the standard model and its extensions, fermion masses are obtained through Yukawa couplings between fermions and Higgs fields. Yukawa couplings seem naturally of O(1) in a sense. From this viewpoint, how to derive suppressed Yukawa couplings is a key point for understanding hierarchical structure among fermion masses and mixing angles.

Superstring theory is a promising candidate for unified theory including gravity. Thus, it is important to study which type of flavor structure can be derived in superstring theory. Superstring theory predicts the existence of 6D compact space in addition to our 4D space-time. Indeed, the 6D compact space is the origin of the flavor structure; that is, the flavor structure, which is derived from string models, is determined by geometrical aspects of the 6D compact space.

Several types of string models have been proposed so far. Recently, intersecting $D$-brane models have been studied extensively. In particular, many intersecting $D6$-brane models on $T^2 \times T^2 \times T^2/Z_N \times Z_M$ and $T^2 \times T^2 \times T^2/Z_N$ orientifolds as well as their $T$-dual models have been constructed. (See for an essential idea on model building Refs. [1–4].)[1] In intersecting $D$-brane models, matter fields as well as Higgs fields correspond to open string modes at intersecting points between different $D$-branes, and those are localized modes [7]. Thus, Yukawa couplings among fermions and Higgs fields depend on the distance of their intersecting points. Suppressed Yukawa couplings can be realized when intersecting points corresponding to matter fermions and Higgs fields are far from each other. That is one of the phenomenologically interesting aspects in intersecting $D$-brane models. Such behavior is qualitatively the same as in heterotic orbifold models. In general, an orbifold has fixed points on the 6D compact space, and heterotic orbifold models have twisted string modes, which are localized at orbifold fixed points. Yukawa couplings can be suppressed when matter fermions and Higgs fields correspond to different fixed points. Indeed, the same technique of conformal field theory (CFT) is used to calculate magnitudes of Yukawa couplings in both heterotic orbifold models [8–11] and intersecting $D$-brane models [12–14]. (See also [15].)

Understanding the origin of suppressed Yukawa couplings is the first important step to deriving realistic Yukawa matrices, but that is not sufficient to realize experimental values. Derivation of realistic Yukawa matrices in string models is quite nontrivial. Another important point is to study stringy selection rules for allowed Yukawa couplings. Then we would see a pattern of Yukawa matrices in a model. In heterotic orbifold models, allowed Yukawa couplings are determined by the 6D space group selection rule [8,16,17] as well as $H$-momentum conservation and gauge invariance. The space group selection rule is the selection rule for allowed Yukawa couplings due to the 6D orbifold geometry. That is quite nontrivial; that is, in some types of Yukawa couplings on orbifolds only diagonal couplings are allowed, but for certain types of Yukawa couplings on nonprime orbifolds off-diagonal couplings are also allowed. Here diagonal couplings mean the case that when we choose two states, the other state to be allowed to couple is determined uniquely. Off-diagonal couplings correspond to the case that the third states are not uniquely determined. Obviously, off-diagonal Yukawa couplings are important to realize nonvanishing mixing

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[1]See also for review , e.g. Refs. [5,6] and references therein.

 

angles in the case with the minimal and small numbers of Higgs fields. Actually, the number of possible 6D orbifolds is finite as determined by 6D crystallographic space groups, and the number of fixed points on an orbifold is also finite. Explicit results for allowed Yukawa couplings due to the space group selection rule are shown in [16,17]. Then, in principle, a systematic study is possible to classify which patterns of Yukawa matrices can be obtained within the framework of heterotic orbifold models. Indeed such analysis has been done for $Z_6$ orbifold models in [18] showing interesting results for the quark and lepton sectors.

On the other hand, patterns of Yukawa matrices have been calculated model by model explicitly within the framework of intersecting $D$-brane models so far. Although explicit models with phenomenologically interesting aspects other than Yukawa matrices have been constructed so far, it is still a challenging issue to lead nontrivial Yukawa matrices. Thus, our purpose is to study systematically the flavor structure which can be derived from intersecting $D$-brane models, and classify possible patterns of Yukawa matrices. For such purpose, we formulate a selection rule for allowed Yukawa couplings on $T^2 \times T^2 \times T^2$. In particular, we are interested in formulating the selection rule, which is useful to investigate whether off-diagonal couplings are allowed or not. Indeed, the selection rule for allowed three-point couplings has been discussed in [12]. We will formulate the coupling selection rule in a different approach, which is similar to the space group selection rule in heterotic orbifold models, showing the condition for off-diagonal couplings. Then we study which types of flavor structure can appear from intersecting $D$-brane configurations on $T^2 \times T^2 \times T^2$. Here we concentrate intersecting $D6$-brane systems (as well as $D4$-brane systems) in type IIA string theory. Since their T-duals correspond to magnetized $D$-brane systems in type IIB string theory, the following discussions would be applicable to magnetized $D$-brane systems in type IIB string theory. Furthermore, although we consider explicitly $T^2 \times T^2 \times T^2$ as 6D compact space, our discussions hold true for $T^2 \times T^2 \times T^2/Z_N \times Z_M$ and $T^2 \times T^2 \times T^2/Z_N$ orbifold backgrounds. That is because orbifold backgrounds do not realize new intersecting $D$-brane configurations, which differ from those on $T^2 \times T^2 \times T^2$, but rather constrain $D$-brane configurations on $T^2 \times T^2 \times T^2$. For orientifold backgrounds, the situation is the same except for the fact that we have to introduce mirror branes for the $D$-branes, which are not parallel to orientifold planes. Thus, our following discussions on the selection rule and possible flavor structure are applicable in $D$-brane configurations including mirror branes.

This paper is organized as follows. In Sec. II, we review briefly the space group selection rule in heterotic orbifold models. In Sec. III, we study the selection rule in intersecting $D$-brane models. In Sec. IV, we consider which types of flavor structure can appear from intersecting $D$-brane configurations. In Sec. IV A we discuss the flavor structure, that the numbers of left-handed and right-handed quarks are the same on $T^2$. In this case, we show the minimum number of Higgs fields is the same as the flavor number $N_f$, and the Higgs number is generically equal to $kN_f$ with $k \in \mathbf{N}$. The selection rule is controlled by a discrete Abelian symmetry. In Sec. IV B, we discuss generic flavor structure, that is, the asymmetric flavor structure. In that case, the coupling selection rule is also determined by a discrete Abelian symmetry. For the minimal number of Higgs field, we also show Yukawa matrices are nontrivial, but lead to a certain number of massless fermions, and diagonal entries are larger than off-diagonal entries. In Sec. IV C, we comment on how to reduce the flavor and Higgs numbers, and that would be important to obtain nontrivial fermion mass matrices. Section V is devoted to conclusion and discussion. In Appendix A, we give a simple recipe: how to obtain shift vectors, which play an important role on description of independent intersecting points and formulation of the coupling selection rule. In Appendix B, we discuss the possible number of Higgs fields when the flavor number of both left- and right-handed quarks on $T^2$ is equal to two.

## II. YUKAWA SELECTION RULE FOR HETEROTIC ORBIFOLD MODELS

Here we give a brief review on the space group selection rule for allowed Yukawa couplings in heterotic orbifold models. This will be useful to study the selection rule for allowed Yukawa couplings in intersecting $D$-brane models.

### A. Orbifold and fixed points

An orbifold is obtained by dividing a torus $T^d$ by a twist $\theta$, while $T^d$ is a division of $R^d$ by a lattice $\Lambda$, i.e. $T^d = R^d/\Lambda$, where the twist $\theta$ must be an automorphism of the lattice $\Lambda$. On such orbifold, there are the closed strings, which satisfy the following twisted boundary condition

$$X^i(\sigma = \pi) = (\theta^k X(\sigma = 0))^i + v^i, \tag{1}$$

where $v^i$ is a shift vector on the torus lattice $\Lambda$. These are refereed as the $\theta^k$ twisted string. Their zero-modes $f^i$ also satisfy the same boundary condition

$$f^i = (\theta^k f)^i + v^i \tag{2}$$

that is the fixed point on the orbifold. The fixed point $f^i$ is denoted by the corresponding space group element $(\theta^k, v^i)$. The product of two space group elements, $(\theta^{k_{(1)}}, v^i_{(1)})$ and $(\theta^{k_{(2)}}, v^i_{(2)})$, is obtained as

$$(\theta^{k_{(1)}}, v^i_{(1)})(\theta^{k_{(2)}}, v^i_{(2)}) = (\theta^{k_{(1)}}\theta^{k_{(2)}}, v^i_{(1)} + \theta^{k_{(1)}}v^i_{(2)}). \tag{3}$$

Indeed, this product of space group elements corresponds to the combination of two twisted strings with the twisted boundary conditions, $(\theta^{k_{(1)}}, v^i_{(1)})$ and $(\theta^{k_{(2)}}, v^i_{(2)})$.

As said above, the fixed point and twisted string are denoted by the corresponding space group element $(\theta^k, v^i)$. Furthermore, it is remarkable that the fixed point $f^i$ is equivalent to $f^i + \Lambda$ on the torus. That implies that the space group $(\theta^k, v^i)$ is equivalent to $(\theta^k, v^i + (1-\theta^k)\Lambda)$, that is, these belong to the same conjugacy class. Thus, independent fixed points are defined up to the conjugacy classes.

Here we give two examples. One is the 2D $Z_3$ orbifold, and the other is the 2D $Z_6$ orbifold. The 2D $Z_3$ orbifold is obtained as a division of $R^2$ by the SU(3) root lattice $\Lambda_{SU(3)}$ and its $Z_3$ automorphism $\theta$. Here we denote the two simple roots of $\Lambda_{SU(3)}$ by $e_1$ and $e_2$, and these are transformed under $\theta$ as

$$\theta e_1 = e_2, \qquad \theta e_2 = -e_1 - e_2. \tag{4}$$

The lattice $(1-\theta)\Lambda$ is spanned e.g. by $3e_1$ and $e_1 - e_2$. The three independent fixed points are denoted by

$$(\theta, ne_1), \qquad (n = 0, 1, 2). \tag{5}$$

Similarly, fixed points on the 2D $Z_6$ orbifold are obtained. The 2D $Z_6$ orbifold is obtained as a division of $R^2$ by the SU(3) lattice $\Lambda_{SU(3)}$ and the $Z_6$ twist, which transforms the SU(3) simple roots as

$$\theta e_1 = e_1 + e_2, \qquad \theta(e_1 + e_2) = e_2, \qquad \theta e_2 = -e_1. \tag{6}$$

Thus, we obtain $(1-\theta)\Lambda = \Lambda$, that is, we have only one independent fixed point under $\theta$, i.e.

$$(\theta, 0). \tag{7}$$

The $\theta^2$ twist of $Z_6$ is nothing but the $Z_3$ twist. Hence, the $\theta^2$ twist has three independent fixed points,

$$(\theta^2, ne_1), \qquad (n = 0, 1, 2). \tag{8}$$

Furthermore, the $\theta^3$ twist of $Z_6$ is the $Z_2$ twist, and the lattice $(1-\theta^3)\Lambda$ is spanned by $2e_1$ and $2e_2$. Thus, the four independent fixed points are denoted as

$$(\theta^3, n_1e_1 + n_2e_2), \qquad (n_i = 0, 1). \tag{9}$$

Similarly, we can obtain fixed points on other orbifolds, and twisted strings also correspond to those fixed points.[2] In the next subsection, we show the space group selection rule for these twisted strings.

### B. Space group selection rule

In this subsection, we give a brief review on the space group selection rule for allowed Yukawa couplings. Here we consider the condition that three twisted strings with the boundary conditions $(\theta^{k_{(a)}}, v^i_{(a)})$ for $a = 1, 2, 3$ are allowed to couple. The coupling condition due to the space group invariance is denoted as follows,

$$\prod_a (\theta^{k_{(a)}}, v^i_{(a)}) = (1, 0). \tag{10}$$

Simply, that implies the coupling is allowed when the three twisted strings combine into the untwisted closed string, which can shrink. However, here we recall that each space group element $(\theta^{k_{(a)}}, v^i_{(a)})$ is equivalent to $(\theta^{k_{(a)}}, v^i_{(a)} + (1-\theta^{k_{(a)}})\Lambda))$. This equivalence in the conjugacy class has an important meaning, as shown below in explicit models. The space group selection rule includes the point group selection rule, and the latter requires the product $\prod_a \theta^{k_{(a)}}$ to be identity.

Here we examine the space group selection rule for the two explicit models, the 2D $Z_3$ orbifold model and the 2D $Z_6$ orbifold model. First, let us consider the 2D $Z_3$ orbifold models. As shown in the previous subsection, there are three independent fixed points on the 2D $Z_3$ orbifold, $(\theta, ne_1)$ with $n = 0, 1, 2$. Let us consider the coupling of three twisted strings corresponding to fixed points, $(\theta, n_1e_1)$, $(\theta, n_2e_1)$, and $(\theta, n_3e_1)$. The space group selection rule requires

$$(\theta, n_1e_1)(\theta, n_2e_1)(\theta, n_3e_1) = (1, 0), \tag{11}$$

up to the conjugacy classes, and leads the following condition for allowed Yukawa couplings,

$$n_1 + n_2 + n_3 = 0 \pmod 3. \tag{12}$$

That implies that the couplings are allowed only two cases, (1) the case that all of three fixed points are the same, and (2) the case that all of three fixed points are different each other. Namely, these are diagonal couplings. Indeed, this coupling selection rule can be understood as the $Z_3$ symmetry.

Similarly, we examine the space group selection rule on the 2D $Z_6$ orbifold. As shown in the previous subsection, there are three twisted sectors, $\theta$-twisted, $\theta^2$-twisted, and $\theta^3$-twisted sectors. For example, let us consider the couplings among $\theta$-twisted, $\theta^2$-twisted, and $\theta^3$-twisted sectors. As shown in the previous subsection, the $\theta$-twisted, $\theta^2$-twisted, and $\theta^3$-twisted sectors have one, three, and four independent fixed points, which are denoted by $(\theta, 0)$, $(\theta^2, ne_1)$ and $(\theta^3, m_1e_1 + m_2e_2)$, respectively, where $n = 0, 1, 2$ and $m_1, m_2 = 0, 1$. Now, we examine the condition for allowed Yukawa couplings due to the space group invariance, that is,

$$(\theta, 0)(\theta^2, ne_1)(\theta^3, m_1e_1 + m_2e_2) = (1, 0), \tag{13}$$

where the space group elements are defined up to the conjugacy classes. The important point is that $(1-\theta)\Lambda = \Lambda$. As a result, the couplings among all of the twisted states are allowed, and off-diagonal couplings are allowed. This makes it clear the situation in which off-diagonal couplings are allowed, that is, when two or more independent fixed

[2] In nonprime order orbifold models, we have to take linear combinations of states corresponding directly to fixed points, in order to obtain $\theta$-eigenstates [19]. However, this is irrelevant to intersecting $D$-brane models.

points under a twisted sector belong to the same conjugacy class in different twisted sector, off-diagonal couplings among corresponding twisted states are allowed.

The extension to the selection rule for generic $n$-point couplings is straightforward, that is, $\prod_{a=1}^{n}(\theta^{k_{(a)}}, v^i_{(a)}) = (1, 0)$ for $n$ twisted strings with the boundary conditions, $(\theta^{k_{(a)}}, v^i_{(a)})$ up to $(1-\theta^{k_{(a)}})\Lambda$.

## III. COUPLING SELECTION RULE FOR INTERSECTING *D*-BRANE MODELS

Here we study the coupling selection rule for intersecting $D$-brane models. We concentrate intersecting $D6$-brane systems (as well as $D4$-brane systems) in type IIA string theory. Since their $T$-duals may correspond to magnetized $D$-branes in type IIB string theory, the following discussions would be applicable to magnetized $D$-brane systems in type IIB string theory. We consider $T^2 \times T^2 \times T^2$ as the 6D compact space, and the $D6$-branes, which wrap one-cycle on each $T^2$. Orbifold/orientifold backgrounds can be studied in the same way, as described in the introduction. Furthermore, other backgrounds like $T^4 \times T^2$ could be discussed in a similar way. The $i$th torus lattice $\Lambda^{(i)}$ is spanned by the basis $e_1^{(i)}, e_2^{(i)}$. Thus, the configuration of $D_a$-brane is described by its winding numbers for the $i$th torus,

$$(n_a^{(i)}, m_a^{(i)}), \tag{14}$$

along $e_1^{(i)}$ and $e_2^{(i)}$. Here we take g.c.d.$(n_a^{(i)}, m_a^{(i)}) = 1$ or $(n_a^{(i)}, m_a^{(i)}) = (1, 0), (0, 1)$.[3] Namely, we consider the case that the vector $\mathbf{w}_a^{(i)} = (n_a^{(i)}, m_a^{(i)})$ is the shortest vector on the lattice $\Lambda^{(i)}$ along its direction. A gauge multiplet appears on each set of $D$-branes. The gauge group U($N$) is obtained from $N$ $D$-branes.

Now, let us consider two sets of $D$-branes, $D_a$-brane and $D_b$-brane, which intersect each other. Their intersecting number on the $i$th $T^2$ is given as

$$I_{ab}^{(i)} = n_a^{(i)} m_b^{(i)} - n_b^{(i)} m_a^{(i)}, \tag{15}$$

and the total intersecting number on the 6D compact space is obtained as their product, i.e. $I_{ab} = I_{ab}^{(1)} I_{ab}^{(2)} I_{ab}^{(3)}$. At these intersecting points, there appear open strings, one of whose ends is on the $D_a$-brane and the other is on the $D_b$-brane. Such open strings can correspond to massless chiral matter fields, which have nontrivial representations under both gauge groups corresponding to $D_a$ and $D_b$ branes, that is, such massless modes have bi-fundamental representations $(N_a, \bar{N}_b)$ under $\mathrm{U}(N_a) \times \mathrm{U}(N_b)$ for positive intersecting numbers. The different signs of $I_{ab}$ correspond to charge conjugated modes, i.e., $(\bar{N}_a, N_b)$. Thus, chiral matter fields correspond to localized modes around intersecting points. Higgs fields also correspond to such modes. Here and hereafter we assume $D$-brane configurations with 4D $N = 1$ supersymmetry, because nonsupersymmetric configurations are usually unstable. Preserving supersymmetry requires certain conditions for intersecting angles, but that is irrelevant to our discussions.

Here, we study coupling selection rule among such open strings stretching different $D$-branes around intersecting points. In heterotic orbifold models, the key point for the coupling selection rule is description of boundary conditions of twisted strings (1), that is, space group elements and their conjugacy classes. Thus, let us study first how to describe boundary conditions of open strings around intersecting points. Our setup is as follows. We concentrate one 2D torus among $T^2 \times T^2 \times T^2$, e.g. the first torus, because extension to the case with $T^2 \times T^2 \times T^2$ is simple. Then we consider two sets of $D$-branes, $D_a$ and $D_b$, whose winding numbers on the first $T^2$ are obtained as the following vectors on the torus lattice $\Lambda^{(1)}$,

$$D_a{:}\mathbf{w}_a^{(1)} = (n_a^{(1)}, m_a^{(1)}), \tag{16}$$

$$D_b{:}\mathbf{w}_b^{(1)} = (n_b^{(1)}, m_b^{(1)}). \tag{17}$$

Their intersecting number on the first $T^2$ is $I_{ab}^{(1)}$. We assume the nontrivial case, i.e., $I_{ab}^{(1)} \neq 0$, and furthermore we take $I_{ab}^{(1)} > 0$, because the case with $I_{ab}^{(1)} < 0$ can be studied in the same way. One of their intersecting points is taken as the origin of the torus lattice $\Lambda^{(1)}$.

Obviously, all of the intersecting points sit along the $D_a$-brane, that is, those positions are written as $\frac{k}{I_{ab}^{(1)}} \mathbf{w}_a^{(1)}$ with $k = 0, 1, \cdots, I_{ab}^{(1)} - 1$. Note that the $D_b$-brane corresponding to $k = 0$ passes the origin of the torus lattice, but the other $D_b$-branes corresponding to $k \neq 0$ do not. On the other hand, all of the intersecting points sit along the $D_b$-brane, and those positions are written as $\frac{\ell}{I_{ab}^{(1)}} \mathbf{w}_b^{(1)}$ with $\ell = 0, 1, \cdots, I_{ab}^{(1)} - 1$. The former and the latter sets of independent intersecting points are equivalent each other on $T^2$. The equivalence on $T^2$ implies the following relation,

$$\frac{k}{I_{ab}^{(1)}} \mathbf{w}_a^{(1)} = \frac{\ell}{I_{ab}^{(1)}} \mathbf{w}_b^{(1)} + \mathbf{v}_{ab}^{(1)}, \tag{18}$$

for proper combinations of $k$ and $\ell$, where $\mathbf{v}_{ab}^{(1)}$ is a shift vector on the torus lattice. Thus, the shift vectors $\mathbf{v}_{ab}^{(1)}$ can be used to describe independent intersecting points. Note that $(\frac{k}{I_{ab}^{(1)}} + k')\mathbf{w}_a^{(1)}$ and $(\frac{\ell}{I_{ab}^{(1)}} + \ell')\mathbf{w}_b^{(1)}$ are equivalent to $\frac{k}{I_{ab}^{(1)}} \mathbf{w}_a^{(1)}$ and $\frac{\ell}{I_{ab}^{(1)}} \mathbf{w}_b^{(1)}$, respectively, for $k', \ell' \in \mathbf{Z}$. That implies that the shift vectors $\mathbf{v}_{ab}^{(1)}$ have the meaning to describe independent intersecting points up to the sublattice $\Lambda_{ab}^{(1)}$, which is spanned by $\mathbf{w}_a^{(1)}$ and $\mathbf{w}_b^{(1)}$. Namely, the $I_{ab}^{(1)}$ independent

[3] g.c.d.$(a, b)$ denotes the greatest common divisor of the integers $a$ and $b$.

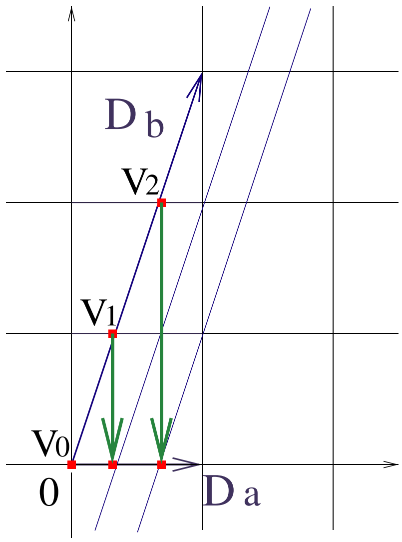


FIG. 1 (color online). A simple example of the $D$-brane configuration. The winding vectors $\mathbf{w}_a$ and $\mathbf{w}_b$ of $D_a$:(1, 0) and $D_b$:(1, 3) are shown. The points on $\mathbf{w}_a$ and $\mathbf{w}_b$ correspond to intersecting points $\frac{m}{3}\mathbf{w}_a$ and $\frac{m}{3}\mathbf{w}_b$ $(m = 0, 1, 2)$, respectively. Shift vectors $\mathbf{v}_m = (0, -m)$ are also shown.

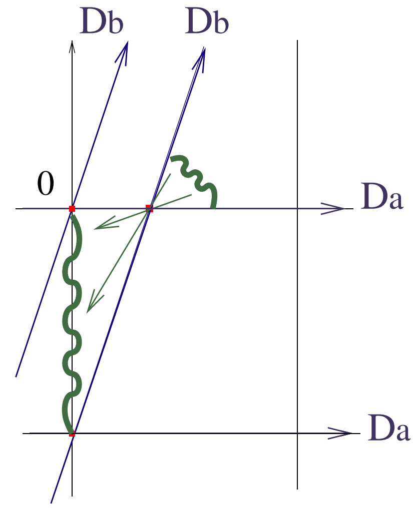


FIG. 2 (color online). Open string at the intersecting point $\frac{1}{3}\mathbf{w}_a$. We can move its endpoint on the $D_a$-brane to the origin, and the other endpoint can be moved not to the origin, but to the point $(0, -1)$.

shift vectors are coset representatives corresponding to $\Lambda^{(1)}/\Lambda^{(1)}_{ab}$. The sublattice $\Lambda^{(1)}_{ab}$ is not as dense as $\Lambda^{(1)}$,[4] and plays a role similar to $(1-\theta^k)\Lambda$ in heterotic orbifold models.

Here we give a simple example. We consider two sets of $D$-branes with winding numbers, $\mathbf{w}_a^{(1)} = (1, 0)$ and $\mathbf{w}_b^{(1)} = (1, 3)$ as shown in Fig. 1. Their intersecting number is $I^{(1)}_{ab} = 3$. Thus, the three independent intersecting points are written as $\frac{k}{3}\mathbf{w}_a^{(1)}$ with $k = 0, 1, 2$. Equivalent sets are obtained as $\frac{\ell}{3}\mathbf{w}_b^{(1)}$ with $\ell = 0, 1, 2$. On the other hand, the sublattice $\Lambda^{(1)}_{ab}$ can be spanned by (1,0) and (0,3), and that implies independent shift vectors $\mathbf{v}^{(1)}_{ab}$ up to $\Lambda^{(1)}_{ab}$ are obtained as $\mathbf{v}^{(1)}_{ab} = (0, -m)$ with $m = 0, 1, 2$. Indeed, these intersecting points and shift vectors satisfy

$$\frac{k}{3}\mathbf{w}_a^{(1)} = \frac{\ell}{3}\mathbf{w}_b^{(1)} + \mathbf{v}^{(1)}_{ab}, \tag{19}$$

up to $\Lambda^{(1)}_{ab}$, when $k = \ell = m$ as shown in Fig. 1. Therefore, independent intersecting points and the coset representatives corresponding to $\Lambda^{(1)}/\Lambda^{(1)}_{ab}$ can be described by shift vectors $\mathbf{v}^{(1)}_{ab}$ up to $\Lambda^{(1)}_{ab}$. See Appendix A, where a simple recipe how to obtain shift vectors $\mathbf{v}^{(1)}_{ab}$ for generic winding numbers is given.

The above simple example also clarifies a concrete implication of shift vectors $\mathbf{v}^{(1)}_{ab}$ for open strings. Let us consider the open string at the intersecting point $\frac{1}{3}\mathbf{w}_a^{(1)}$ between $D_a$- and $D_b$-branes, and move it such that one end sits on the origin of the torus lattice like Fig. 2. The other end cannot sit on the origin, but on the $D_b$-brane, which passes $(0, -1)$, that is nothing but the corresponding shift vector $\mathbf{v}^{(1)}_{ab}$. The same is true for the other intersecting points. Thus, we can describe open strings at different intersecting points by the following equations, which are satisfied by string endpoints, $\mathbf{x}_a$ and $\mathbf{x}_b$,

$$\mathbf{x}_a - \mathbf{x}_b = \mathbf{v}_{ab}, \tag{20}$$

when we move one of the endpoints to the same point, e.g. the origin. It is also true for the generic case, that is, independent intersecting points are represented by shift vectors $\mathbf{v}_{ab}$ up to $\Lambda_{ab}$, and endpoints of $D_a$-$D_b$ open string at the intersecting point satisfy Eq. (20).

Now we are ready to study the coupling selection rule in intersecting $D$-brane models. We consider three sets of $D$-branes, $D_a$, $D_b$, and $D_c$, with the following winding numbers,

$$D_a{:}\mathbf{w}_a^{(1)} = (n_a^{(1)}, m_a^{(1)}), \qquad D_b{:}\mathbf{w}_b^{(1)} = (n_b^{(1)}, m_b^{(1)}),$$
$$D_c{:}\mathbf{w}_c^{(1)} = (n_c^{(1)}, m_c^{(1)}). \tag{21}$$

Here we do not consider the trivial case that one of $I^{(1)}_{ab}$, $I^{(1)}_{bc}$ and $I^{(1)}_{ca}$ vanishes. Furthermore, we study the case that all of $I^{(1)}_{ab}$, $I^{(1)}_{bc}$ and $I^{(1)}_{ca}$ are positive. It is quite straightforward to extend the following discussions to other cases, e.g. the case that all of $I^{(1)}_{ab}$, $I^{(1)}_{bc}$ and $I^{(1)}_{ca}$ are negative.[5] There are

[4] Obviously we have $\mathrm{vol}(\Lambda^{(1)}_{ab}) = I^{(1)}_{ab}\mathrm{vol}(\Lambda^{(1)})$, where $\mathrm{vol}(\Lambda)$ denotes the volume of the unit cell.

[5] We will show at the end of the section that the $H$-momentum conservation requires all of $I^{(1)}_{ab}$, $I^{(1)}_{bc}$ and $I^{(1)}_{ca}$ to have the same sign.

three types of open strings, $D_a$-$D_b$, $D_b$-$D_c$, and $D_c$-$D_a$ open strings at intersecting points between $D_a$ and $D_b$ branes, $D_b$ and $D_c$ branes, and $D_c$ and $D_a$ branes, respectively. Let us study the coupling selection rule among these three open strings. Simply, these open strings can couple if the corresponding intersecting points and $D$-branes make a closed triangle. In other words, the coupling is allowed if these three open strings combine into the closed string, which can shrink without winding on the torus. Recall that the endpoints of the open string satisfy the condition (20). Thus, the above condition for allowed couplings is written as

$$\mathbf{v}^{(1)}_{ab} + \mathbf{v}^{(1)}_{bc} + \mathbf{v}^{(1)}_{ca} = 0. \tag{22}$$

Here note that shift vectors, $\mathbf{v}^{(1)}_{ab}$, $\mathbf{v}^{(1)}_{bc}$, and $\mathbf{v}^{(1)}_{ca}$, are defined up to the sublattices, $\Lambda^{(1)}_{ab}$, $\Lambda^{(1)}_{bc}$, and $\Lambda^{(1)}_{ca}$, respectively. This selection rule tells us whether off-diagonal couplings are allowed or not in intersecting $D$-brane models like heterotic orbifold models. For example, in the situation where differences between two or more independent shifts $\mathbf{v}^{(1)}_{ab}$ are on the sublattice $\Lambda^{(1)}_{bc}$, off-diagonal couplings are allowed. Thus, the difference among the sublattices, $\Lambda^{(1)}_{ab}$, $\Lambda^{(1)}_{bc}$, and $\Lambda^{(1)}_{ca}$, is important to realize off-diagonal couplings.

As an illustrating example, let us consider the $D$-brane configuration with the following winding numbers

$$D_a\colon \quad (1,0), \qquad D_b\colon \quad (-1,2), \qquad D_c\colon \quad (1,-3). \tag{23}$$

The intersecting number between $D_a$ and $D_b$-branes is obtained as $I^{(1)}_{ab} = 2$, and independent intersecting points are described by

$$\mathbf{v}^{(1)}_{ab} = (0,\ell), \qquad (\ell = 0, 1), \tag{24}$$

up to the sublattice $\Lambda^{(1)}_{ab}$, which is spanned by (1,0) and (0,2). Similarly, the intersecting number between $D_c$ and $D_a$-branes is obtained as $I^{(1)}_{ca} = 3$, and independent intersecting points are described by

$$\mathbf{v}^{(1)}_{ca} = (0,k), \qquad (k = 0, 1, 2), \tag{25}$$

up to the sublattice $\Lambda^{(1)}_{ca}$, which is spanned by (1,0) and (0,3). On the other hand, the intersecting number between $D_b$ and $D_c$ is equal to $I^{(1)}_{bc} = 1$, and the corresponding shift vector $\mathbf{v}^{(1)}_{bc}$ is obviously obtained as $\mathbf{v}^{(1)}_{bc} = 0$ up to the sublattice $\Lambda^{(1)}_{bc}$, where $\Lambda^{(1)}_{bc}$ is nothing but the torus lattice, i.e. $\Lambda^{(1)}_{bc} = \Lambda^{(1)}$. In this model, couplings among all the states are allowed. More explicitly, the following Yukawa couplings,

$$Y_{\ell k} C^{\ell}_{(ab)} C^{k}_{(ca)} C_{(bc)}, \tag{26}$$

are allowed for any $\ell$ and $k$, where $C^{\ell}_{(ab)}$ denote chiral matter fields corresponding to the $D_a$-$D_b$ open strings and the other notation of fields, $C^{k}_{(ca)}$ and $C_{(bc)}$, have a similar meaning. Here we have assumed that all of the intersecting numbers on the second and third tori are equal to one. The strength of Yukawa couplings is calculated through CFT technique in [12–14], and its dominant factor is obtained as

$$Y \sim e^{-S_{\rm cl}} \sim e^{-(\Sigma_1+\Sigma_2+\Sigma_3)}, \tag{27}$$

where $S_{\rm cl}$ denotes the action of the classical string solution $X_{\rm cl}$, which has the asymptotic behavior corresponding to local open string modes near intersecting points, and that is obtained as the triangle area $\Sigma_i$, which is swept by a string to couple on the $i$th $T^2$. Here we consider only the dominant contribution due to the minimal classical action, although other classical solutions also have subdominant contributions. That is because we are interested in the hierarchical form of Yukawa matrices. It is possible to extend the following discussions on possible forms of Yukawa matrices to full results including all of the classical solutions [12–14].

For example, in the configuration that three sets of $D$-branes, $D_a$, $D_b$, and $D_c$, intersect at the same point, "the origin" of the torus lattice, we evaluate

$$Y_{\ell k} = \begin{pmatrix} 1 & \varepsilon^4 & \varepsilon^4 \\ \varepsilon^9 & \varepsilon & \varepsilon \end{pmatrix}. \tag{28}$$

Here, the parameter $\varepsilon$ denotes the suppression factor for the Yukawa coupling corresponding to the minimum triangle. For explicit calculations of Yukawa couplings, it is simple to use a figure like Fig. 3, in particular, in the case that the number of either Higgs field or matter field is equal to one. Figure 3 shows the above $D$-brane configuration, where the single $C_{(bc)}$ field is located at the origin. Several vertical branes correspond to $D_a$-branes. Intersecting points between $D_a$ and $D_b$ are labeled by integer $\ell$ (mod 2), while intersecting points between $D_a$ and $D_c$ are labeled by integer $k$ (mod 3). Indeed, off-diagonal couplings are allowed, because all of the combinations $(\ell, k)$

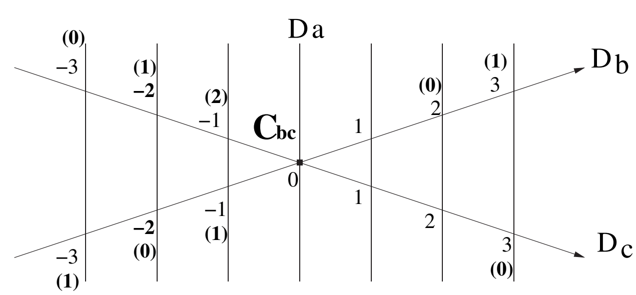


FIG. 3. $D$-brane configuration corresponding to Eqs. (23) and (28). The single $C_{(bc)}$ field is located at the origin. Intersecting points between $D_a$ and $D_b$ are labeled by integer $\ell$ (mod 2), while intersecting points between $D_a$ and $D_c$ are labeled by integer $k$ (mod 3). The parameter $\varepsilon$ in Eq. (28) denotes the suppression factor for the Yukawa coupling corresponding to the minimum triangle.

can be connected by the $D_a$-brane in Fig. 3. If $I_{ab}^{(1)}$ and $I_{ca}^{(1)}$ had g.c.d.$(I_{ab}^{(1)}, I_{ca}^{(1)}) \neq 1$, all of the combinations $(\ell, k)$ could not be connected by the $D_a$ brane in a figure like Fig. 3.

Even if the three $D$-branes do not intersect at the same point like Fig. 4, the coupling selection rule is the same, because the shift vectors do not change. However, Yukawa matrix becomes

$$Y_{\ell k} = \begin{pmatrix} \varepsilon^{d^2} & \varepsilon^{(2+d)^2} & \varepsilon^{(2-d)^2} \\ \varepsilon^{(3-d)^2} & \varepsilon^{(1-d)^2} & \varepsilon^{(1+d)^2} \end{pmatrix}, \tag{29}$$

where $d$ is a continuous parameter $(-1 \leq d \leq 1)$.

This example shows an important aspect of the coupling selection rule, that is, all of the couplings are allowed for any $D$-brane configuration with

$$\text{g.c.d.}(I_{ab}^{(1)}, I_{bc}^{(1)}) = \text{g.c.d.}(I_{ca}^{(1)}, I_{bc}^{(1)}) = \text{g.c.d.}(I_{ab}^{(1)}, I_{ca}^{(1)}) = 1. \tag{30}$$

This rule can be understood simply by drawing a figure like Fig. 3, by which we can see all of the combinations of intersecting points that are connected by the same type of $D$-branes. This can also be explained as follows. Suppose that

$$\text{g.c.d.}(n_1^{(1)}, n_2^{(1)}) = \text{g.c.d.}(n_1^{(1)}, n_3^{(1)}) = \text{g.c.d.}(n_2^{(1)}, n_3^{(1)}) = 1. \tag{31}$$

Then, the sublattice $\Lambda_{ab}^{(1)}$ is spanned by $(1, J_{ab}^{(1)})$ and $(0, I_{ab}^{(1)})$, and the sublattice $\Lambda_{ca}^{(1)}$ is spanned by $(1, J_{ca}^{(1)})$ and $(0, I_{ca}^{(1)})$, where $J_{ab}^{(1)}$ and $J_{ca}^{(1)}$ are certain integers, but irrelevant to our discussions. (See Appendix A.1.) Similarly, the sublattice $\Lambda_{bc}^{(1)}$ is spanned by $(1, J_{bc}^{(1)})$ and $(0, I_{bc}^{(1)})$. It is obvious that the combination among the sublattices $\Lambda_{ab}^{(1)}, \Lambda_{ca}^{(1)}$, and $\Lambda_{bc}^{(1)}$ corresponds to the torus lattice $\Lambda^{(1)}$, i.e. $\Lambda^{(1)} = \Lambda_{ab}^{(1)} \cup \Lambda_{ca}^{(1)} \cup \Lambda_{bc}^{(1)}$, when Eq. (30) is satisfied. This implies that all of the couplings are allowed for any $D$-brane configuration with Eq. (30). In generic winding numbers, the sublattice $\Lambda_{ab}^{(1)}$ is spanned by $(k_{ab}, J_{ab}^{(1)})$ and $(0, k'_{ab})$, and the sublattice $\Lambda_{ca}^{(1)}$ is spanned by $(k_{ca}, J_{ca}^{(1)})$ and $(0, k'_{ca})$.

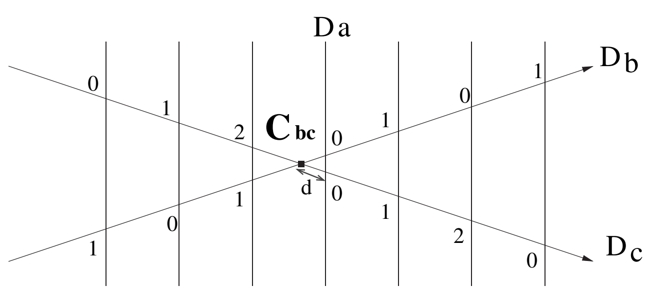


FIG. 4. The same $D$-brane configuration as Fig. 3. Three sets of $D$-branes do not intersect at the same point, and the distance between those intersecting points is parametrized by $d$.

(See Appendix A.2) Moreover, the lattice $\Lambda_{bc}^{(1)}$ is spanned by $(k_{bc}, J_{bc}^{(1)})$ and $(0, k'_{bc})$. Equation (30) implies that

$$\begin{gathered} \text{g.c.d.}(k_{ab}, k_{ca}) = \text{g.c.d.}(k_{ab}, k_{bc}) = \text{g.c.d.}(k_{ca}, k_{bc}) = 1, \\ \text{g.c.d.}(k'_{ab}, k'_{ca}) = \text{g.c.d.}(k'_{ab}, k'_{bc}) = \text{g.c.d.}(k'_{ca}, k'_{bc}) = 1, \end{gathered} \tag{32}$$

where $I_{ab}^{(1)} = k_{ab}k'_{ab}$, $I_{ca}^{(1)} = k_{ca}k'_{ca}$ and $I_{bc}^{(1)} = k_{bc}k'_{bc}$. This leads to the fact that the combination among the sublattices $\Lambda_{ab}^{(1)}, \Lambda_{ca}^{(1)}$ ,and $\Lambda_{bc}^{(1)}$ corresponds to the torus lattice $\Lambda^{(1)}$, i.e. $\Lambda^{(1)} = \Lambda_{ab}^{(1)} \cup \Lambda_{ca}^{(1)} \cup \Lambda_{bc}^{(1)}$. This implies that all of the couplings are allowed for any $D$-brane configuration with Eq. (30), and this has been shown already in [12] through a different approach. This rule will be extended into the generic case when we find generic $D$-brane configurations in Sec. IV B.

The extension to the selection rule for generic $n$-point couplings is straightforward. We consider $n$ sets of $D_i$ branes for $i = 1, 2, \cdots, n$. Such setup may include open strings at intersecting points between $D_i$ and $D_{i+1}$ branes as well as open strings between $D_n$ and $D_1$ branes. Their intersecting points are described by shift vectors $\mathbf{v}_{i,i+1}$ as well as $\mathbf{v}_{n,1}$. Then, the condition for allowed couplings is written as

$$\mathbf{v}_{1,2} + \mathbf{v}_{2,3} + \cdots + \mathbf{v}_{n-1,n} + \mathbf{v}_{n,1} = 0. \tag{33}$$

Here, recall that the shift vectors $\mathbf{v}_{i,i+1}$ are defined up to the lattice $\Lambda_{i,i+1}$, whose definition is the same as $\Lambda_{ab}$.

Finally, we comment on the coupling selection rule due to the $H$-momentum conservation. Within the bosonized formulation, the (twisted) RNS fermionic strings are written as

$$e^{iq_iH_i}, \tag{34}$$

where $H_i$ are 2D bosonized fields and $q_i$ are the so-called $H$-momenta. If $I_{ab}^{(i)} > 0$, massless space-time spinors corresponding to R modes and massless space-time scalars corresponding to NS modes have the following $H$-momenta

$$q_i = \theta_{ab}^{(i)} - \frac{1}{2} \quad \text{for R}, \tag{35}$$

$$q_i = \theta_{ab}^{(i)} - 1 \quad \text{for NS}, \tag{36}$$

respectively, where $\theta_{ab}^{(i)}\pi$ $(0 < \theta_{ab}^{(i)} < 1)$ denotes the intersecting angle on the $i$th $T^2$ between $D_a$ and $D_b$ branes. Now, let us consider the $H$-momentum conservation for Yukawa couplings among two fermions and a single scalar field, which are originated from $D_a$-$D_b$, $D_b$-$D_c$, and $D_c$-$D_a$ open strings. The $H$-momentum conservation requires that

$$(\theta_{ab}^{(i)} + \theta_{bc}^{(i)} + \theta_{ca}^{(i)})\pi = 2\pi. \tag{37}$$

This implies that the sum of the exterior angles of triangle must be equal to $2\pi$. Obviously, this is satisfied with a closed triangle. However, if one of the intersecting numbers, e.g. $I_{ab}^{(1)}$, is negative, the massless mode has the $H$-momentum opposite to Eq. (36). In this case, the $H$-momentum conservation must not be satisfied with a closed triangle configuration of $D$-branes. Moreover, if all of the intersecting numbers are negative, the $H$-momentum conservation is satisfied with a closed triangle configuration of $D$-branes. Thus, the $H$-momentum conservation is satisfied only when all of the intersecting numbers, $I_{ab}^{(i)}$, $I_{bc}^{(i)}$, and $I_{ca}^{(i)}$, have the same sign on each $i$th $T^2$. Thist includes the condition that all of the total intersecting numbers, $I_{ab}$, $I_{bc}$, and $I_{ca}$, must have the same sign. Since this means the corresponding Yukawa couplings are $(N_a, \bar{N}_b, 1) \times (1, N_b, \bar{N}_c)(\bar{N}_a, 1, N_c)$ type or its conjugate, the condition due to $H$-momentum conservation includes the condition that the Yukawa couplings must be gauge invariant.

## IV. FLAVOR STRUCTURE FROM INTERSECTING *D*-BRANE CONFIGURATIONS

In this section, we study systematically the flavor structures, which can appear from intersecting $D$-brane configurations. The flavor structure on $T^2 \times T^2 \times T^2$ is a direct product of the flavor structure on each $T^2$. Thus, we mainly investigate the flavor structures from intersecting $D$-brane configurations on each $T^2$. In Sec. IV A, we study the symmetric flavor structure, that is, the intersecting numbers for left- and right-handed quarks are the same on $T^2$. In Sec. IV B, we study the asymmetric flavor structure, that is, the intersecting numbers for left- and right-handed quarks are different from each other on $T^2$, but their total numbers on $T^2 \times T^2 \times T^2$ are the same.

### A. Symmetric flavor structure

Here we investigate the $D$-brane configurations leading to the symmetric flavor structure, that is, the intersecting numbers for left- and right-handed quarks are the same on $T^2$. Suppose that the flavor number is equal to $N_f$, e.g. on the first $T^2$ while their intersecting numbers on the other $T^2 \times T^2$ are equal to one. Then, we can easily show the number of Higgs fields, which can have allowed 3-point couplings with these quarks, is equal to $kN_f$, where $k \in \mathbf{Z}$ and $N_f \neq 2$. Let us consider three sets of $D$-branes, $D_C$, $D_L$, and $D_R$ branes with the following winding numbers,

$$\begin{gathered} D_C{:}\mathbf{w}_C = (n_C, m_C), \qquad D_L{:}\mathbf{w}_L = (n_L, m_L), \\ D_R{:}\mathbf{w}_R = (n_R, m_R). \end{gathered} \tag{38}$$

In this subsection, we omit the index (1) denoting the first $T^2$, because we discuss the $D$-brane configurations only on the first $T^2$. Open strings between $D_C$ and $D_L$ ($D_R$) correspond to left-handed (right-handed) quarks $Q_L$ ($Q_R$), and open strings between $D_L$ and $D_R$ correspond to the modes, which can have allowed 3-point couplings with left- and right-handed quarks, that is, Higgs fields $H$. Then, we consider the symmetric flavor structure

$$|I_{CL}| = |I_{RC}| = N_f. \tag{39}$$

Through a simple algebraic calculation, that implies that

$$\mathbf{w}_L \pm \mathbf{w}_R = k\mathbf{w}_C, \tag{40}$$

where $k$ is a real number. The sign on the left-hand side depends on the signs of $I_{CL}$ and $I_{RC}$.[6] Recall here that $\mathbf{w}_C$ is the shortest vector on $\Lambda$ along its direction. This implies that $k$ must be an integer. Thus, we obtain the number of Higgs fields

$$|I_{LR}| = kN_f. \tag{41}$$

Namely, the minimum number of Higgs fields is equal to $N_f$.[7] In this case, we find

$$\Lambda_{CL} = \Lambda_{RC} = \Lambda_{LR}, \tag{42}$$

and the varieties of shift vectors, $\mathbf{v}_{CL}$, $\mathbf{v}_{RC}$, and $\mathbf{v}_{LR}$ are the same. Thus, these types of $D$-brane configurations lead to only diagonal couplings for one of the $N_f$ Higgs fields. Indeed, the selection rule is determined by the discrete Abelian symmetry.

As an illustrating example, we consider the case of $N_f = 3$, e.g. with the following winding numbers,

$$\begin{aligned} D_C{:}\mathbf{w}_C &= (n_C, m_C) = (1, 0), \\ D_L{:}\mathbf{w}_L &= (n_L, m_L) = (1, 3), \\ D_R{:}\mathbf{w}_R &= (n_R, m_R) = (-2, -3). \end{aligned} \tag{43}$$

Indeed, this configuration leads to $I_{CL} = I_{RC} = I_{LR} = 3$. All of the sublattices, $\Lambda_{CL}$, $\Lambda_{RC}$, and $\Lambda_{LR}$, are the same, and spanned by (1,0) and (0,3). The intersecting points of these $D$-branes are described by the shift vectors,

$$\mathbf{v}_{CL} = (0, k_{CL}), \quad \mathbf{v}_{RC} = (0, k_{RC}), \quad \mathbf{v}_{LR} = (0, k_{LR}), \tag{44}$$

where $k_{CL}, k_{RC}, k_{LR} = 0, 1, 2 \pmod 3$. Thus, the coupling selection rule (22) leads to

$$k_{CL} + k_{RC} + k_{LR} = 0, \qquad (\mathrm{mod}\ 3). \tag{45}$$

This selection rule is the same as the one in the 2D $Z_3$ orbifold, and is described by the $Z_3$ symmetry, under which the fields $\Phi_k$ with the $Z_3$ charge $k$ transform as

$$\Phi_k \rightarrow e^{2\pi i k/3}\Phi_k, \tag{46}$$

where $k = 0, 1, 2$. Thus, only diagonal couplings are allowed for one of three Higgs fields. Explicitly, in the case that the three $D$-branes intersect at the same point, we

[6] The $H$-momentum conservation and gauge invariance require $I_{CL} = I_{RC}$, and in this case, the sign on the left-hand side must be $+$.

[7] In the case with $N_f = 2$, the minimum number of Higgs is not equal to 2, but 4. See Appendix B for such case.

obtain the Yukawa matrix

$$Y = \begin{pmatrix} H_0 & \varepsilon H_2 & \varepsilon H_1 \\ \varepsilon H_2 & H_1 & \varepsilon H_0 \\ \varepsilon H_1 & \varepsilon H_0 & H_2 \end{pmatrix}, \tag{47}$$

where $\varepsilon$ is the suppression factor. This form of Yukawa coupling is the generic form when the three sets of $D$-branes intersect at the same point, although we have shown the explicit winding numbers. When $\varepsilon$ is sufficiently suppressed, its three eigenvalues are obtained by vacuum expectation values (VEVs) of the Higgs fields, $v_i = \langle H_i \rangle$, for $i = 0, 1, 2$, and its diagonalizing matrix is obtained as[8]

$$V = \begin{pmatrix} 1 & -\frac{v_2}{v_1}\varepsilon & -\frac{v_1}{v_2}\varepsilon \\ \frac{v_2}{v_1}\varepsilon & 1 & -\frac{v_0}{v_2}\varepsilon \\ \frac{v_1}{v_2}\varepsilon & \frac{v_0}{v_2}\varepsilon & 1 \end{pmatrix}. \tag{48}$$

We apply the above Yukawa matrix to the up and down sectors of quarks with Higgs fields $H_i^{(u,d)}$ and suppression factors $\varepsilon_{u,d}$. The quark mass ratios are obtained as

$$m_u{:}m_c{:}m_t = v_0^u{:}v_1^u{:}v_2^u, \qquad m_d{:}m_s{:}m_b = v_0^d{:}v_1^d{:}v_2^d. \tag{49}$$

Moreover, the mixing angles are predicted as

$$V_{us} = \frac{m_b}{m_s}\varepsilon_d - \frac{m_t}{m_c}\varepsilon_u, \qquad V_{ub} = \frac{m_s}{m_b}\varepsilon_d - \frac{m_c}{m_t}\varepsilon_u, \qquad V_{cb} = \frac{m_d}{m_b}\varepsilon_d - \frac{m_u}{m_t}\varepsilon_u, \tag{50}$$

by the two parameters $\varepsilon_{u,d}$. This prediction does not fit the experimental values.

When the three sets of $D$-branes do not intersect at the same point, the Yukawa matrix becomes

$$Y = \begin{pmatrix} \varepsilon^{d^2} H_0 & \varepsilon^{(1-d)^2} H_2 & \varepsilon^{(1+d)^2} H_1 \\ \varepsilon^{(1+d)^2} H_2 & \varepsilon^{d^2} H_1 & \varepsilon^{(1-d)^2} H_0 \\ \varepsilon^{(1-d)^2} H_1 & \varepsilon^{(1+d)^2} H_0 & \varepsilon^{d^2} H_2 \end{pmatrix}, \tag{51}$$

where $d$ is a continuous parameter $(-1 \leq d \leq 1)$ to denote the nearest distance between three types of intersecting points. This is the generic form of the Yukawa matrix for the symmetric flavor structure with $N_f = 3$ flavor and the three Higgs fields. Application of this form to the up and down sectors of quarks does not seem to lead to fully realistic results [21].

Similarly we can discuss the $D$-brane configurations with more than three Higgs fields, i.e. the case with the $N_f k$ Higgs fields for $k > 1$. In this case, the sublattices $\Lambda_{CL}$ and $\Lambda_{RC}$ are still the same and these are spanned by $\mathbf{w}_C$ and $\mathbf{w}_L$. On the other hand, the sublattice $\Lambda_{LR}$ is spanned by $k\mathbf{w}_C$ and $\mathbf{w}_L$, and is less dense than $\Lambda_{CL}$ and $\Lambda_{RC}$. This implies that the number of coset representatives corresponding to $\Lambda/\Lambda_{LR}$ is $k$ times as large as one of $\Lambda/\Lambda_{CL}$ and $\Lambda/\Lambda_{RC}$, that is, the independent set of intersecting points for the Higgs fields is described by the joint of sets of shift vectors $\{\mathbf{v}_{CL}\} \cup \{m\mathbf{w}_C\}$ for $m = 0, 1, \cdots, (k-1)$, while the independent set of intersecting points for left- and right-handed quarks is described by the set of shift vectors $\{\mathbf{v}_{CL}\}$. However, the part $\{m\mathbf{w}_C\}$ is irrelevant to the coupling selection rule, that is, the selection rule is determined by the same $Z_{N_f}$ symmetry, and the part $\{m\mathbf{w}_C\}$ has the trivial charge under the $Z_{N_f}$ symmetry. As a result, the same couplings are allowed except we replace

$$H(\mathbf{v}_{ab}) \rightarrow H(\mathbf{v}_{ab}) + H(\mathbf{v}_{ab} + \mathbf{w}_C) + \cdots + H(\mathbf{v}_{ab} + (k-1)\mathbf{w}_C), \tag{52}$$

where $H(\mathbf{v}_{ab})$ denotes the Higgs field corresponding to the shift vector $\mathbf{v}_{ab}$ and we have omitted coefficients. As a result, only diagonal couplings are allowed for one of the Higgs fields.

Here we give an explicit model with $N_f$ flavors and $2N_f$ Higgs fields. In this case we have the following Yukawa matrix

$$Y = \begin{pmatrix} H_1 + \varepsilon^9 H_5 & \varepsilon H_4 + \varepsilon^4 H_3 & \varepsilon H_6 + \varepsilon^4 H_2 \\ \varepsilon H_4 + \varepsilon^4 H_3 & H_2 + \varepsilon^9 H_6 & \varepsilon H_5 + \varepsilon^4 H_1 \\ \varepsilon H_6 + \varepsilon^4 H_2 & \varepsilon H_5 + \varepsilon^4 H_1 & H_3 + \varepsilon^9 H_4 \end{pmatrix}, \tag{53}$$

when all of three sets of $D$-brane intersect at the same point.

Including many Higgs fields may lead to realistic Yukawa matrices for the quark and lepton sectors when we assume proper VEVs for these Higgs fields. However, this raises the question of how we can realize such proper ratios of Higgs VEVs.

### B. Asymmetric flavor structure

Here we study the asymmetric flavor structure on $T^2$, that is, the numbers of left- and right-handed quarks are different from each other, e.g. on the first $T^2$, i.e. $|I_{CL}^{(1)}| \neq |I_{RC}^{(1)}|$. The total number of left- and right-handed quarks must be the same. Thus, intersecting points, e.g. on the second $T^2$, must satisfy $|I_{CL}^{(1)} I_{CL}^{(2)}| = |I_{RC}^{(1)} I_{RC}^{(2)}|$, and the total flavor number is equal to $N_f = |I_{CL}^{(1)} I_{CL}^{(2)}| = |I_{RC}^{(1)} I_{RC}^{(2)}|$.

First, let us investigate which types of $D$-brane configurations can appear. Again, we consider three sets of $D$-branes, $D_C$, $D_L$, and $D_R$ branes with the following winding numbers on the first $T^2$,

$$D_C{:}\mathbf{w}_C^{(1)} = (n_C^{(1)}, m_C^{(1)}), \qquad D_L{:}\mathbf{w}_L^{(1)} = (n_L^{(1)}, m_L^{(1)}), \qquad D_R{:}\mathbf{w}_R^{(1)} = (n_R^{(1)}, m_R^{(1)}). \tag{54}$$

Suppose that

$$I_{CL}^{(1)} = \ell C, \qquad I_{RC}^{(1)} = rC, \tag{55}$$

[8] See e.g. [20] for similar analysis on $Z_3$ orbifold models.

where $\ell, r, C \in \mathbf{Z}$, and $\text{g.c.d.}(\ell, r) = 1$. Through a simple algebraic calculation, we can show

$$\ell \mathbf{w}_R \pm r\mathbf{w}_L = j\mathbf{w}_C, \tag{56}$$

where $j$ is an integer. Then, we can calculate

$$|I_{LR}^{(1)}| = jC. \tag{57}$$

Here the integer $j$ must satisfy

$$\text{g.c.d.}(j, \ell) = \text{g.c.d.}(j, r) = 1. \tag{58}$$

For example, if $\text{g.c.d.}(j, \ell) = C' \neq 1$, the above discussion could be applied to show that $r = C'r'$ with integer $r'$. This is inconsistent with the above condition $\text{g.c.d.}(\ell, r) = 1$. Thus, the generic $D$-brane configuration should satisfy

$$\text{g.c.d.}(I_{CL}^{(1)}, I_{RC}^{(1)}) = \text{g.c.d.}(I_{CL}^{(1)}, I_{LR}^{(1)}) = \text{g.c.d.}(I_{RC}^{(1)}, I_{LR}^{(1)}). \tag{59}$$

We have understood the generic $D$-brane configuration. Now, let us consider the selection rule for such generic case. In Sec. III, we have shown that all of the couplings are allowed in the $D$-brane configuration satisfying Eq. (30). Here we extend this into the generic case. We take $C$ as the greatest common divisor of any two intersecting numbers. Suppose that $\text{g.c.d.}(n_1^{(1)}, n_2^{(1)}) = \text{g.c.d.}(n_1^{(1)}, n_3^{(1)}) = \text{g.c.d.}(n_2^{(1)}, n_3^{(1)}) = 1$. Then, the sublattices are spanned by [9]

$$\begin{aligned} &\Lambda_{CL}^{(1)}:(1, J_{CL}^{(1)}C) \quad \text{and} \quad (0, k'_{CL}C), \\ &\Lambda_{RC}^{(1)}:(1, J_{RC}^{(1)}C) \quad \text{and} \quad (0, k'_{RC}C), \\ &\Lambda_{LR}^{(1)}:(1, J_{LR}^{(1)}C) \quad \text{and} \quad (0, k'_{LR}C), \end{aligned} \tag{60}$$

where

$$\text{g.c.d.}(k'_{CL}, k'_{RC}) = \text{g.c.d.}(k'_{CL}, k'_{LR}) = \text{g.c.d.}(k'_{RC}, k'_{LR}) = 1, \tag{61}$$

and $J_{CL}^{(1)}$, $J_{RC}^{(1)}$ and $J_{LR}^{(1)}$ are certain integers, but irrelevant to our discussions. In this case, the combination of the sublattices, $\Lambda_{CL}$, $\Lambda_{RC}$, and $\Lambda_{LR}$ corresponds to the sublattice, which is spanned by $(1, 0)$ and $(0, C)$. Therefore, the coupling selection rule is determined by the discrete $Z_C$ symmetry. Let us write this generic result on the coupling selection rule in simple words. When we label intersecting points by $j_{CL}$, $j_{RC}$, and $j_{LR}$, i.e.,

$$j_{CL} = 0, 1, \cdots, \quad I_{CL}^{(1)} - 1, j_{RC} = 0, 1, \cdots, I_{RC}^{(1)} - 1, \\ j_{LR} = 0, 1, \cdots, I_{LR}^{(1)} - 1, \tag{62}$$

the coupling selection rule is obtained as

$$j_{CL} + j_{RC} + j_{LR} = 0 \pmod{C}, \tag{63}$$

where

$$C = \text{g.c.d.}(I_{CL}^{(1)}, I_{RC}^{(1)}) = \text{g.c.d.}(I_{CL}^{(1)}, I_{LR}^{(1)}) = \text{g.c.d.}(I_{RC}^{(1)}, I_{LR}^{(1)}), \tag{64}$$

for $\text{g.c.d.}(n_1^{(1)}, n_2^{(1)}) = \text{g.c.d.}(n_1^{(1)}, n_3^{(1)}) = \text{g.c.d.}(n_2^{(1)}, n_3^{(1)}) = 1$. This selection rule is mentioned also in [12] as Ansatz.

In the generic case, the sublattices are spanned by [10]

$$\begin{aligned} &\Lambda_{CL}^{(1)}:(k_{CL}C_1, J_{CL}^{(1)}C_2) \quad \text{and} \quad (0, k'_{CL}C_2), \\ &\Lambda_{RC}^{(1)}:(k_{RC}C_1, J_{RC}^{(1)}C_2) \quad \text{and} \quad (0, k'_{RC}C_2), \\ &\Lambda_{LR}^{(1)}:(k_{LR}C_1, J_{LR}^{(1)}C_2) \quad \text{and} \quad (0, k'_{LR}C_2), \end{aligned} \tag{65}$$

where $C_1C_2 = C$ and

$$\begin{aligned} &\text{g.c.d.}(k_{CL}, k_{RC}) = \text{g.c.d.}(k_{CL}, k_{LR}) = \text{g.c.d.}(k_{RC}, k_{LR}) = 1, \\ &\text{g.c.d.}(k'_{CL}, k'_{RC}) = \text{g.c.d.}(k'_{CL}, k'_{LR}) = \text{g.c.d.}(k'_{RC}, k'_{LR}) = 1. \end{aligned} \tag{66}$$

Then, the combination of the sublattices, $\Lambda_{CL}$, $\Lambda_{RC}$, and $\Lambda_{LR}$ corresponds to the sublattice, which is spanned by $(C_1, 0)$ and $(0, C_2)$. Hence, the coupling selection rule is determined by the $Z_{C_1} \times Z_{C_2}$ symmetry. This selection rule is also written as a simple extension of Eq. (63). Indeed, we obtain the symmetry $Z_{C_1} \times Z_{C_2} = Z_{C(=C_1 \times C_2)}$ because $\text{g.c.d.}(C_1, C_2) = 1$.

We have understood the generic $D$-brane configuration and its coupling selection rule. From a phenomenological viewpoint, we are interested in $D$-brane configurations leading to small numbers of flavors, in particular, the flavor number equal to three. The $D$-brane configuration leading to the total flavor number $N_f = 3$ is realized by the intersecting numbers

$$(I_{CL}^{(1)}, I_{RC}^{(1)})(I_{CL}^{(2)}, I_{RC}^{(2)}) = (3, 1)(1, 3). \tag{67}$$

Suppose that $I_{LR}^{(1)}I_{LR}^{(2)} = 1$, that is, the number of Higgs fields is equal to one. In this case, we have the factorizable form of the Yukawa matrix

$$Y_{ij} = a_i b_j, \tag{68}$$

as already known in the literature. This is the rank-one matrix, and this makes only one of flavor massive and the others still remain massless with obviously vanishing mixing angles.

The minimal case, which can lead to nonvanishing mixing with only one Higgs field, may be the $D$-brane configuration with the intersecting numbers

$$(I_{CL}^{(1)}, I_{RC}^{(1)})(I_{CL}^{(2)}, I_{RC}^{(2)}) = (3, 2)(2, 3). \tag{69}$$

The total flavor number $N_f$ is equal to $N_f = 6$, and left- and right-handed quarks are denoted by $Q_L^{ij}$ and $Q_R^{k\ell}$, respectively, where the indices $(i, j)$ label intersecting points on the first and second tori for $i = 0, 1, 2$ and

[9]See Appendix A.1.

[10]See Appendix A.2.

$j = 0, 1$. The indices $(k\ell)$ have the same meaning. In this case with one Higgs field, the total Yukawa matrix is obtained as a direct product of the parts from the first and second tori, i.e.,

$$Y_{(i,j)(k,\ell)} = a_{ik} b_{j\ell}. \tag{70}$$

The generic form of $a_{ik}$ and $(b_{j\ell})^T$ is obtained by Eqs. (28) and (29) with different suppression factors $\varepsilon$ and $\varepsilon'$. Thus, this Yukawa matrix $Y_{(i,j)(k,\ell)}$ has the nontrivial form with the rank-four, that is, the mass ratios among massive modes and diagonalizing matrix elements are determined by geometrical aspects. However, when we apply this form to up and down sectors of quarks, it cannot realize experimental values of masses and mixing angles. The clear problem is that the flavor number is larger and two flavors remain still massless. Also, another phenomenological aspect is that off-diagonal entries are suppressed compared with diagonal entries. However, experimental values of mass ratios and mixing angles in the quark sector as well as the lepton sector satisfy $m_i/m_j \leq V_{ij}$ for $i < j$, and this implies that off-diagonal entries must not be suppressed so much in one of the up and down sectors.

The generic aspects of the asymmetric flavor structure with the intersecting numbers $(I_{CL}^{(1)}, I_{RC}^{(1)})(I_{CL}^{(2)}, I_{RC}^{(2)})$, which satisfy $I_{CL}^{(1)} = I_{RC}^{(2)}$ and $I_{RC}^{(1)} = I_{CL}^{(2)}$, are as follows. Here we consider the case that g.c.d.$(I_{CL}^{(1)}, I_{RC}^{(1)}) = 1$. The extension to the case with g.c.d.$(I_{CL}^{(1)}, I_{RC}^{(1)}) \neq 1$ is simple. We denote left- and right-handed quarks by $Q_L^{ij}$ and $Q_R^{k\ell}$, respectively, for $i, \ell = 0, 1, \cdots, I_{CL}^{(1)} - 1$ and $j, k = 0, 1, \cdots, I_{RC}^{(1)} - 1$. We suppose the minimal number of Higgs field. In this case, the full Yukawa matrix is obtained as a direct product

$$Y_{(i,j)(k,\ell)} = a_{ik} b_{j\ell}. \tag{71}$$

When all three sets of $D$-branes intersect at the same point, the factor matrix corresponding to the first $T^2$, $a_{ik}$, is derived through a simple calculation

$$a_{ik} = \varepsilon^{m^2}, \tag{72}$$

where $m$ is the integer that satisfies

$$i = m \pmod{I_{CL}^{(1)}}, \qquad k = m \pmod{I_{RC}^{(1)}}, \tag{73}$$

with the minimum $|m|$, where $m$ includes negative integers. This result may be obvious when we draw a figure like Fig. 3. The matrix $b_{j\ell}$ is also obtained in a similar way. When all three sets of $D$-branes do not intersect at the same point, the factor matrix $a_{ik}$ is obtained in the same way except by replacing $m \to m - d$, where $d$ is a continuous parameter $(-1 \leq d \leq 1)$.

The total flavor number $N_f$ is obtained as $N_f = I_{CL}^{(1)} I_{CL}^{(2)}$. The rank of the full Yukawa matrix is equal to $I_{CL}^{(1)} I_{CL}^{(1)}$ when $I_{CL}^{(1)} < I_{CL}^{(2)}$. Thus, the $I_{CL}^{(1)}(I_{RC}^{(1)} - I_{CL}^{(1)})$ modes appear massless. Off-diagonal entries are suppressed compared with diagonal entries, and not enough to realize experimental values of mixing angles.

Now let us consider the case with more than one Higgs fields. We consider the model with the intersecting numbers, $(I_{CL}^{(1)}, I_{RC}^{(1)})(I_{CL}^{(2)}, I_{RC}^{(2)}) = (3, 1)(1, 3)$. Even if the intersecting numbers for the Higgs fields are $(I_{LR}^{(1)}, I_{LR}^{(2)}) = (2, 1)$ or $(1, 2)$, the rank of mass matrix is still equal to one. For example, in the former case, the Yukawa couplings for each of the Higgs fields $H_a$ $(a = 1, 2)$ are a factorizable form like $Y_{ija} = a_{ia} b_j$, and the mass matrix $m_{ij}$ is still a factorizable form,

$$m_{ij} = (a_{i1}\langle H_1 \rangle + a_{i2}\langle H_2 \rangle) b_j, \tag{74}$$

that is, the rank-one matrix. The minimal case increasing the rank of mass matrix is $(I_{LR}^{(1)}, I_{LR}^{(2)}) = (2, 2)$, that is, the totally four Higgs fields. Then, the mass matrix becomes rank-two.

To make all of three flavors massive, we have to introduce more Higgs fields. However, recall the rule for $D$-brane configuration discussed in Sec. IV A when $I_{LR}^{(1)} = 3$ or $I_{LR}^{(2)} = 3$. If we have e.g. $I_{LR}^{(1)} = 3$ in addition to $I_{CL}^{(1)} = 3$, such $D$-brane configuration requires at least $I_{RC}^{(1)} = 3$, and the minimum case with $I_{RC}^{(1)} = 3$ corresponds to the symmetric flavor structure, which has been discussed in the previous subsection.

### C. Comments on reducing flavor and Higgs numbers

We have studied the flavor structures, which can be derived from intersecting $D$-brane configurations, including the symmetric flavor structures and asymmetric ones. For the symmetric flavor structure with $N_f = 3$, corresponding $D$-brane configurations require the number of Higgs fields to be equal to $N_f k$ with integer $k$. For the asymmetric flavor case, the $D$-brane configurations with $(I_{CL}^{(1)}, I_{RC}^{(1)})(I_{CL}^{(2)}, I_{RC}^{(2)}) = (3, 1)(1, 3)$ lead to the factorizable form of the Yukawa matrix, $Y_{ij} = a_i b_j$, that is, the rank-one matrix. This implies that even if we can realize the massless spectrum of the minimal supersymmetric standard model, we would have difficulty in deriving realistic Yukawa matrices from stringy 3-point couplings at the tree-level.[11]

For the symmetric flavor case as well as the asymmetric case, introduction of many Higgs fields may lead to realistic Yukawa matrices. However, in such case, proper values of ratios among VEVs of many Higgs fields have to be chosen. Also, the introduction of many Higgs fields may lead to a problem, because that, in general, causes flavor changing neutral currents. One solution is that we introduce extra fields $H'$, which have mass terms with

[11]Loop corrections have been studied in [22]. However, in the softly broken SUSY case, it seems difficult to obtain realistic results by loop corrections.

Higgs fields larger than the weak scale, and that only one mode remains light. If such light mode is a linear combination of original modes with proper coefficients, then realistic Yukawa matrices can be realized.

For the asymmetric flavor case, off-diagonal couplings are allowed for one Higgs field, e.g. $(I_{CL}^{(1)}, I_{RC}^{(1)})(I_{CL}^{(2)}, I_{RC}^{(2)}) = (3, 2)(2, 3)$, and the cases with more intersecting points also lead to nontrivial mixing angles. However, in such case the flavor number is larger than 3. We have to reduce the flavor number, that is, we have to introduce antigenerations and mass terms between generations and antigenerations.

Thus, it is quite important to generate mass terms among Higgs fields $H$ and extra fields $H'$, and/or generations and antigenerations of quarks. One of the stringy ways to generate mass terms is the recombination of $D$-branes. However, if those $D$-branes are stabilized to be bending, it is not clear how to treat them. Otherwise, if those are stabilized not to bend, but to be straightened, the resultant $D$-brane configurations would be classified into the $D$-brane configurations which have been studied in Secs. IV A and IV B. Anyway, the stringy way to generate mass terms is beyond our scope. Thus, here we give comments on this issue from the viewpoint of effective field theory. Within the framework of effective field theory, there may be two ways to generate mass terms; one is through symmetry breaking and the other is due to compositeness through strong dynamics.

Concerning the former scenario, suppose that we have the following type of couplings:

$$yHH'X, \qquad yQQ'X. \tag{75}$$

These couplings are originated from the $D$-brane configuration of Fig. 5 for the Higg fields, and a similar configuration for the quarks. These coupling strengths $y$ are calculated within the framework of string theory.

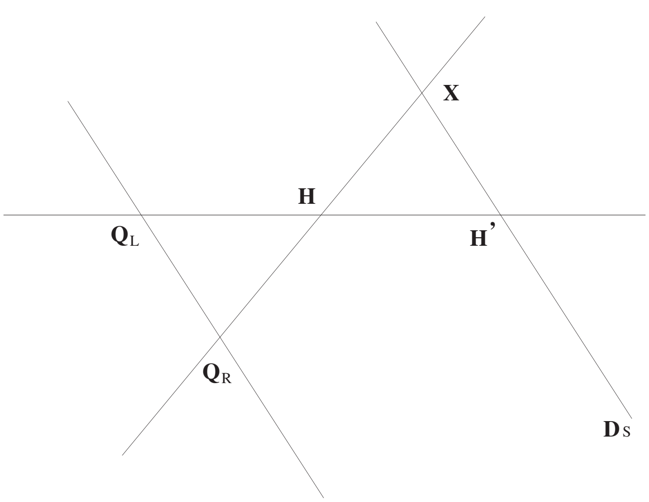


FIG. 5. $D$-brane configuration leading to mass terms between $H$ and $H'$.

Suppose that the fields $X$ develop their VEVs. Then, the above operators become effective mass terms between $H$ and $H'$, and $Q$ and $Q'$, and that can reduce the number of light flavors and light Higgs fields. Note that the VEVs of the fields $X$ break gauge symmetries, under which $X$ have nontrivial representations. Thus, the gauge group, which is obtained at the string scale, must be larger than the gauge group of the standard model.

The latter scenario to generate mass terms is similar to the former. Suppose again that we have the same type of couplings as Eq. (75). This time, we assume that the gauge coupling corresponding to the gauge group, under which both $X$ and $H'$, and $X$ and $Q'$, have nontrivial charges becomes strong. Such gauge sector corresponds to the $D_s$-brane in Fig. 5. Then, composite modes appear $(H'X)$ and $(Q'X)$. Obviously, these composite modes have effective mass terms with $H$ and $Q$. Then, the numbers of Higgs fields and flavors can be reduced.

In the composite scenario, the light modes can be composite modes, when the flavor number from composite modes is larger than the flavor number of original modes. In this case, effective Yukawa couplings can be originated from stringy $n$-point couplings for $n = 3, 4, 5, 6$ depending on which modes correspond to composite modes. Actually, such explicit models with composite modes have been studied showing an interesting form of Yukawa matrices [23,24]. It is quite important to study somehow systematically the flavor structure, which can be derived from intersecting $D$-brane configurations, considering the above scenarios to reduce the Higgs and flavor numbers through the symmetry breaking and strong dynamics. We have to classify higher dimensional operators as well as 3-point couplings. For such purpose, the discussions in the previous subsections would be useful. However, we leave it for future study.

## V. CONCLUSION

We have studied the flavor structure and the coupling selection rule within the framework of intersecting $D$-brane models on $T^2 \times T^2 \times T^2$. We have formulated the coupling selection rule in terms of shift vectors, which are coset representatives corresponding to $\Lambda/\Lambda_{ab}$. With this formulation, we can write the coupling selection rule for generic $n$-point couplings in a simple way.

We have found that generic $D$-brane configurations must satisfy the relation (59). In such a generic case, the coupling selection rule is determined by the discrete Abelian symmetry. For example, the symmetric flavor structure with $N_f = 3$ requires at least three Higgs fields, and the coupling selection rule is determined by the $Z_3$ symmetry. We may need more Higgs fields to derive realistic Yukawa matrices. However, the presence of more than one light Higgs field coupled to the same type of quarks/leptons is dangerous, because that, in general, causes unsuppressed flavor changing neutral currents.

In the asymmetric flavor structure, the case with $N_f = 3$ leads to the result that only the third family becomes massive, but the others remain massless. For the asymmetric flavor structure with more flavor, we have nontrivial Yukawa matrices, although their diagonal entries are quite larger than off-diagonal entries and some modes still remain massless.

Our results show that even if we could obtain the minimal matter content of the supersymmetric standard model at the string scale, we would face the difficultly of deriving realistic Yukawa matrices. It would be interesting to study an alternative scenario that we may have several Higgs fields and generations and antigenerations of fermions and investigate a way to generate mass terms e.g. within the effective field-theoretical way.

We have studied only 3-point couplings at the tree level. It is important to study higher dimensional operators and loop-effects. The selection rule (33) for higher dimensional operators is useful, because such operators provide effective Yukawa couplings after symmetry breaking like the Froggatt-Nielsen mechanism. Furthermore, it is also interesting to classify $D$-brane configurations with allowed $n$-point couplings for $n = 3, 4, 5, 6, \cdots$ from the viewpoint of the scenarios discussed in Sec. IV C.

In heterotic orbifold models without Wilson lines, massless spectra are degenerate on all of the fixed points, and a large number of quarks and leptons as well as Higgs fields appear. However, Wilson lines can resolve such degeneracy [16,25,26] and lead to different massless spectra between fixed points. That is useful to reduce flavor numbers. We might need such stringy way in intersecting $D$-brane models.

As 6D compact space, we consider explicitly $T^2 \times T^2 \times T^2$, but our results are the same for its orbifold compactification, $T^2 \times T^2 \times T^2/Z_N \times Z_M$ and $T^2 \times T^2 \times T^2/Z_N$. In the orientifold case, we have to introduce mirror branes for the $D$-branes, which are not parallel to orientifold planes. However, generic $D$-brane configurations including mirror branes must also satisfy the relation (59), and their allowed couplings are determined by the $Z_N$ symmetry. Our discussions can be extended to models e.g. on $T^4 \times T^2$. Moreover, it is quite important to study flavor structures in models on Calabi-Yau manifolds, but that is beyond the scope of the present paper, because how to construct intersecting $D$-brane models on Calabi-Yau manifolds is not completely clear at present.

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## APPENDIX A: SHIFT VECTORS

In this appendix, we give a simple recipe how to obtain shift vectors $\mathbf{v}_{ab}$ for generic winding numbers, e.g. for the first $T^2$. Here we omit the index (1) for the first $T^2$. We consider the two sets of $D$-branes with the following winding numbers,

$$D_a{:}\mathbf{w}_a = (n_a, m_a), \qquad D_b{:}\mathbf{w}_b = (n_b, m_b). \tag{A1}$$

### 1. Case I

First we consider the case with g.c.d.$(n_a, n_b) = 1$. In this case, we can show easily g.c.d.$(I_{ab}, n_a) =$ g.c.d.$(I_{ab}, n_b) = 1$. Thus, the set of independent intersecting points $\frac{k}{I_{ab}}\mathbf{w}_a$ $(k = 0, 1, \cdots, I_{ab} - 1)$ is equivalent to the sets of $\frac{k}{I_{ab}} n_b \mathbf{w}_a$ $(k = 0, 1, \cdots, I_{ab} - 1)$. Similarly the set of independent intersecting points $\frac{\ell}{I_{ab}}\mathbf{w}_b$ $(\ell = 0, 1, \cdots, I_{ab} - 1)$ is equivalent to the sets of $\frac{\ell}{I_{ab}} n_a \mathbf{w}_b$ $(k = 0, 1, \cdots, I_{ab} - 1)$. Obviously, we have

$$n_a \mathbf{w}_b - n_b \mathbf{w}_a = (0, I_{ab}). \tag{A2}$$

Thus, the sublattice $\Lambda_{ab}$ is spanned by $(1, J_{ab})$ and $(0, I_{ab})$, where $J_{ab}$ must be an integer. This integer $J_{ab}$ is irrelevant to description of the sets of independent shifts and coset representatives of $\Lambda/\Lambda_{ab}$. When g.c.d.$(m_a, m_b) = C_m \neq 1$, the integer $J_{ab}$ is also written as $J_{ab} = J'_{ab} C_m$ with integer $J'_{ab}$. The shift vectors describing $I_{ab}$ independent intersecting points are written as

$$\mathbf{v}_{ab} = \frac{k^{(ab)}}{I_{ab}}(n_a \mathbf{w}_b - n_b \mathbf{w}_a) = (0, k^{(ab)}), \quad (k^{(ab)} = 0, 1, \cdots, I_{ab} - 1). \tag{A3}$$

Similarly, we can obtain the shift vectors $\mathbf{v}_{ab}$ in the case with g.c.d.$(m_a, m_b) = 1$. In this case, the independent shifts are written as

$$\mathbf{v}_{ab} = \frac{k^{(ab)}}{I_{ab}}(m_b \mathbf{w}_a - m_a \mathbf{w}_b) = (k^{(ab)}, 0), \quad (k^{(ab)} = 0, 1, \cdots, I_{ab} - 1). \tag{A4}$$

When g.c.d.$(n_a, n_b) =$ g.c.d.$(m_a, m_b) = 1$, both sets of independent shift vectors are equivalent.

### 2. Case II

Here we discuss a simple recipe of how to obtain shift vectors in the case with g.c.d.$(n_a, n_b) = C_n \neq 1$ and g.c.d.$(m_a, m_b) = C_m \neq 1$, where $C_n$ and $C_m$ must satisfy g.c.d.$(C_n, C_m) = 1$. We write these winding numbers as follows,

$$n_a = n'_a C_n, \qquad n_b = n'_b C_n, \tag{A5}$$

$$m_a = m'_a C_m, \qquad m_b = m'_b C_m, \tag{A6}$$

where $n'_{a,b}, C_{n,m} \in \mathbf{Z}$. We can do the same discussion as the Appendix A.1 except we replace

$$n_{a,b} \to n'_{a,b}, \qquad m_{a,b} \to m'_{a,b}. \tag{A7}$$

For example, we obtain

$$n'_a \mathbf{w}_b - n'_b \mathbf{w}_a = \left(0, \frac{I_{ab}}{C_n}\right) = (0, C_m I'_{ab}), \tag{A8}$$

where $I'_{ab} = n'_a m'_b - n'_b m'_a$. Then the sublattice $\Lambda_{ab}$ is spanned by $(C_n, C_m J'_{ab})$ and $(0, C_m I'_{ab})$, where $J'_{ab}$ must be an integer. This integer $J'_{ab}$ is irrelevant to coset representatives of $\Lambda/\Lambda_{ab}$ like Appendix A.1. The set of independent shifts are obtained as

$$\mathbf{v}_{ab} = (k, k'), \tag{A9}$$

where $(k = 0, 1, \cdots, C_n - 1), (k' = 0, 1, \cdots, \frac{I_{ab}}{C_n} - 1)$. There are other equivalent descriptions of shift vectors.

## APPENDIX B: SYMMETRIC FLAVOR STRUCTURE WITH $N_f = 2$

In Sec. IV A, we have shown that if $|I_{CL}| = |I_{CR}| = N_f$ on $T^2$, the intersecting number $|I_{LR}|$ must satisfy $|I_{LR}| = kN_f$ with $k \in \mathbf{Z}$, that is, the minimum number of $|I_{LR}|$ is equal to $N_f$. Here, again we omit the index for the $i$th torus like Sec. IV A. However, this statement is not true for $N_f = 2$. Indeed, the case with $N_f = 2$ requires $k \geq 2$. If the following relation

$$|I_{CL}| = |I_{CR}| = |I_{LR}| = 2, \tag{B1}$$

is true, we could write

$$\mathbf{w}_C = \pm \mathbf{w}_L \pm \mathbf{w}_R, \tag{B2}$$

where the signs depend on signs of $I_{CL}, I_{CR}$, and $I_{LR}$. However, the relations (B1) and (B2) are inconsistent in the $D$-brane configurations. The relation (B2) always leads to $\mathbf{w}_C = (\text{even, even})$ except the two cases with

$$\begin{aligned} \mathbf{w}_L &= (\text{even, odd}), \qquad \mathbf{w}_R = (\text{odd, even}), \\ \mathbf{w}_L &= (\text{odd, even}), \qquad \mathbf{w}_R = (\text{even, odd}). \end{aligned} \tag{B3}$$

However, both of these cases lead to $I_{LR} = \text{odd}$. Thus, one cannot realize the $D$-brane configuration with the relation (B1).

The flavor structure with $N_f = 2$ on $T^2$ is not realistic, but this can be useful as a piece of a full flavor structure when we combine it with other flavor structures on $T^2 \times T^2$. (See Sec. IV.)


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