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# Gravitational collapse in asymptotically anti–de Sitter or de Sitter backgrounds

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We study here the gravitational collapse of a matter cloud with a nonvanishing tangential pressure in the presence of a nonzero cosmological term $\Lambda$. It is investigated how $\Lambda$ modifies the dynamics of the collapsing cloud and whether it affects the cosmic censorship. Conditions for bounce and singularity formation are derived. It is seen that when the tangential pressure vanishes, the bounce and singularity conditions reduce to the dust case studied earlier. The collapsing interior is matched to an exterior which is asymptotically de Sitter or anti–de Sitter, depending on the sign of the cosmological constant. The junction conditions for matching the cloud to the exterior are specified. The effect of $\Lambda$ on apparent horizons is studied in some detail and the nature of central singularity is analyzed. The visibility of singularity and implications for the cosmic censorship conjecture are discussed. It is shown that for a nonvanishing cosmological constant, both black hole and naked singularities do form as collapse end states in spacetimes which are asymptotically de Sitter or anti–de Sitter.

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## I. INTRODUCTION

The gravitational collapse of a matter cloud which is pressureless dust and its dynamical evolution, as governed by the Einstein's equations, were first studied in detail by Oppenheimer and Snyder [1]. In recent years there have been extensive studies on gravitational collapse to examine the final fate of a collapsing cloud in order to investigate the end state of such a collapse in terms of the formation of a black hole or naked singularity. These studies throw important light on the nature of cosmic censorship and possible mathematical formulations for the same (for some recent reviews, see e.g. [2]). Such studies have already helped to rule out several possible versions of cosmic censorship, where a precise and well-defined formulation itself has been a major unresolved problem so far. While understanding the nature of dynamical gravitational collapse within the framework of Einstein's gravity is a problem with considerable astrophysical significance, the understanding of cosmic censorship, if it is valid in some form, is another major motivation for such collapse studies.

Most of these investigations so far have, however, assumed a vanishing cosmological term ($\Lambda$). The cosmological constant is sometimes thought of as a constant term in the Lagrangian density of general relativity, and it is also theorized that $\Lambda$ may be related to the energy density of vacuum (see [3] and references therein). Recent astronomical observations of high redshift type Ia supernovae [4] strongly indicate that the universe may be undergoing an accelerated expansion and it is believed that this may be due to a nonvanishing *positive* cosmological constant. On the other hand, the proposed anti–de Sitter/conformal field theory (AdS/CFT) conjecture [5] in string theory has generated interest in the possibility of spacetimes with a *negative* cosmological constant. The conjecture relates string theory in a spacetime where the noncompact part is asymptotically AdS, to a conformal field theory in a space isomorphic to the boundary of AdS. Whereas the obvious effect of a positive cosmological constant is to slow down gravitational collapse, that of the negative cosmological constant is to supplement the gravitational forces. Dust models with $\Lambda$ are known in the literature [6] and there have been some studies on dust collapse with $\Lambda$ in recent years (see e.g. [7]). Collapse models with a cosmological term have also been studied earlier by some authors in other contexts [8]. These dust model studies with $\Lambda$ indicate that the cosmological term has a significant influence on dynamical collapse. Also, the study of horizons in spacetimes with $\Lambda$ have shown the cosmological term to have nontrivial effects [9].

In the above context, it is pertinent to study the dynamical collapse of matter clouds when $\Lambda \neq 0$. Our purpose here is to examine a class of collapsing models which incorporate pressure, and which are asymptotically either de Sitter or anti–de Sitter geometry, depending on the sign of the cosmological term. We discuss here a sufficiently general fluid model which allows pressure to be nonzero, and which also allows the cosmological term to be nonvanishing. Specifically, we study models with a nonvanishing tangential pressure $p_\theta$ [10], together with $\Lambda \neq 0$. Collapse models with a tangential pressure have been studied extensively, but not with a nonzero $\Lambda$ [11]. Allowing the collapse to develop from regular initial conditions, we investigate the bounce and singularity formation conditions, the junction conditions at the boundary of the cloud so as to match it to a suitable exterior, and we also consider how the apparent horizons are affected by the presence of $\Lambda$. The possibility that the presence of $\Lambda$ could

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restore the cosmic censorship conjecture (for reviews on CCC see [12] and references therein) is discussed to investigate how the final end state of gravitational collapse is affected by $\Lambda$. The present model generalizes the Lemaitre-Tolman-Bondi (LTB) dust [13] collapse studies with $\Lambda$, by introducing nonzero pressures in the collapsing cloud. It turns out that both black holes and naked singularities do form as collapse end states in the presence of a nonzero $\Lambda$, in spacetimes which are asymptotically de Sitter or anti–de Sitter.

The relevant form of Einstein equations, conditions to ensure that the collapse develops from a regular initial data, and the necessary energy conditions are introduced in Sec. II, together with the details of the tangential pressure model. In Sec. III, we derive the evolution of the collapsing matter shells, and explicitly give the conditions when a singularity is formed and when the bounce of a particular shell occurs during the collapse evolution. The reduction to $\Lambda \neq 0$ dust collapse case, when the tangential pressures are put to zero, is also demonstrated. For the collapsing solution to be physically plausible, it must satisfy certain junction conditions at the boundary hypersurface where the interior collapsing cloud joins with a suitable exterior spacetime. In Sec. IV we study the matching of the collapsing interior to the exterior Schwarzschild–de Sitter or anti–de Sitter spacetime, and in Sec. V we discuss briefly the effect of $\Lambda$ on the apparent horizons of the fluid model. In Sec. VI the nature of the singularity in the tangential pressure fluid and dust models is considered when $\Lambda \neq 0$, in terms of its being hidden within a black hole, or whether it would be visible to outside observers. The final Sec. VII gives some conclusions.

## II. EINSTEIN EQUATIONS, REGULARITY, AND ENERGY CONDITIONS

The general spherically symmetric metric in the coordinates $(t, r, \theta, \phi)$ is given as

$$ds^2 = -e^{2\nu(t,r)}dt^2 + e^{2\psi(t,r)}dr^2 + R^2(t, r)d\Omega^2, \quad (1)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the line element on a two-sphere.

We take the matter field to be of type I, which is a broad class including most of the physically reasonable matter fields such as dust, perfect fluids, massless scalar fields, and others. These matter fields are characterized by the energy-momentum tensor which admits one timelike and three spacelike eigenvectors [14]. We choose our coordinates $(t, r, \theta, \phi)$ to be along these eigenvectors, which makes the coordinate system to be comoving, that is, the coordinate system moves with the matter. A general spherically symmetric metric in the comoving coordinates $(t, r, \theta, \phi)$ must admit three arbitrary functions of $t$ and $r$, which are given by $\nu(t, r)$, $\psi(t, r)$, and $R(t, r)$ in the above metric. Comoving coordinates are used in the present study to simplify the analysis (for a different approach see for example [15]), and because the gravitational collapse studies so far have widely used it. In the comoving coordinates the energy-momentum tensor for the matter field under consideration becomes diagonal, regularity and energy conditions may be formulated in a relatively simple way, and the study of apparent horizons and the nature of singularity is simplified since collapse happens in finite proper time.

In the comoving frame the energy-momentum tensor for any matter field which is type I is given in a diagonal form,

$$T^t_t = -\rho; \qquad T^r_r = p_r; \qquad T^\theta_\theta = T^\phi_\phi = p_\theta. \quad (2)$$

The quantities $\rho$, $p_r$, and $p\theta$ are the matter density, radial pressure, and the tangential pressure, respectively. In the present study, we give a perfect fluid interpretation to the cosmological constant [3]. Also, it is assumed that the net energy-momentum tensor given by

$$T^{(\text{net})}_{\mu\gamma} = T^{(\text{matter})}_{\mu\gamma} + T^{(\Lambda)}_{\mu\gamma} \quad (3)$$

satisfies the weak energy condition [14]. It means that the net energy density as measured by any local observer is non-negative. Then, for any timelike vector $V^\mu$ we have

$$T^{(\text{net})}_{\mu\gamma} V^\mu V^\gamma \geq 0. \quad (4)$$

This amounts to

$$\rho + \Lambda \geq 0; \qquad \rho + p_r \geq 0; \qquad \rho + p\theta \geq 0. \quad (5)$$

It is important to note that, if, instead of considering the cosmological term as vacuum energy density, we had considered it to be a part of the spacetime geometry in Einstein's equations, then we would have imposed the energy condition only on the matter field. However, as we shall see below, both these interpretations of cosmological constants are equivalent, since the dynamical equations and hence the evolution of the system remain unchanged.

The evolution of the matter cloud is determined by the Einstein equations, and for the metric (1) these are given by (in units of $8\pi G = c = 1$)

$$\rho + \Lambda = \frac{F'}{R^2 R'}, \qquad p_r - \Lambda = -\frac{\dot{F}}{R^2 \dot{R}}, \quad (6)$$

$$\nu'(\rho + p_r) = 2(p_\theta - p_r)\frac{R'}{R} - p_r', \quad (7)$$

$$-2\dot{R}' + R'\frac{\dot{G}}{G} + \dot{R}\frac{H'}{H} = 0, \quad (8)$$

$$G - H = 1 - \frac{F}{R}, \quad (9)$$

where ($\dot{}$) and ($'$) represent partial derivatives with respect to $t$ and $r$ respectively and

$$G(r, t) = e^{-2\psi}(R')^2, \qquad H(r, t) = e^{-2\nu}\dot{R}^2. \quad (10)$$

Here $F(r, t)$ is an arbitrary function, and in spherically symmetric spacetimes, it has the interpretation of the mass function of the collapsing cloud, in the sense that this represents the total gravitational mass within a shell of comoving radius $r$ [16]. The boundary of the collapsing cloud is labeled by the comoving coordinate $r_\pi$. In order to preserve regularity at the initial epoch we require $F(t_i, 0) = 0$.

It can be seen from Eq. (6) that the density of the matter blows up when $R = 0$ or $R' = 0$. The case $R' = 0$ corresponds to shell-crossing singularities. The shell-cross singularities are generally considered to be weak and possibly removable singularities. Hence we shall consider here only the shell-focusing singularities (taking $R' > 0$), where the physical radius of all the matter shells goes to a zero value ($R = 0$). Let us use the scaling independence of the coordinate $r$ to write

$$R(t, r) = rv(t, r), \tag{11}$$

where $v$ is the scale factor. We have

$$v(t_i, r) = 1; \qquad v(t_s(r), r) = 0; \qquad \dot{v} < 0, \tag{12}$$

where $t_i$ and $t_s$ stand for the initial and the singular epochs, respectively. This means we scale the radial coordinate $r$ in such a way that at the initial epoch $R = r$, and at the singularity, $R = 0$. The condition $\dot{v} < 0$ signifies that we are dealing with gravitational collapse. From the point of view of initial data, at the initial epoch $t = t_i$, we have five functions of coordinate $r$ given by $\nu_0(r)$, $\psi_0(r)$, $\rho_0(r)$, $p_{r_0}(r)$, and $p_{\theta 0}(r)$. Note that initial data are not all mutually independent. To preserve regularity and smoothness of initial data we must make some assumptions about the initial pressures at the regular center $r = 0$. Let the gradients of pressures vanish at the center, that is, $p'_{r_0}(0) = p'_{\theta_0}(0) = 0$. The difference between radial and tangential pressures at the center should also vanish, i.e. $p_{r_0}(0) - p_{\theta_0}(0) = 0$. It is seen that we have a total of five field equations with seven unknowns, $\rho$, $p_r$, $p_\theta$, $\psi$, $\nu$, $R$, and $F$, giving us the freedom to choose two free functions. The selection of these functions, subject to the given initial data and weak energy condition, determines the matter distribution and metric of the spacetime and thus leads to a particular dynamical evolution of the initial data.

Spherically symmetric collapse models, where the radial pressure is taken to be vanishing but the tangential pressure could be nonzero, have been studied in some detail over the past few years [11]. The main motivation in the present consideration is to study bounce and singularity formation conditions for the case when we have a nonvanishing $\Lambda$ term present, when pressures are also introduced and allowed to be nonzero within a collapsing cloud. One would also like to understand how $\Lambda$ affects the junction conditions at the boundary where the cloud is matched to an exterior spacetime, the horizon formation, and the nature of central singularity, when nonzero pressures are present within the cloud.

Taking $p_r = 0$ in Eq. (6) gives $F(t, r)$ and $\rho$ in the following forms:

$$F(t, r) = r^3 \mathcal{M}(r) + \frac{\Lambda}{3} R^3, \tag{13}$$

$$\rho = \frac{3\mathcal{M} + r[\mathcal{M}_{,r}]}{v^2(v + rv')}. \tag{14}$$

Here $\mathcal{M}(r)$ is an arbitrary function of $r$ subject to the energy conditions. There remains the freedom to choose one function, since there are six equations with seven unknowns. In order to work within the framework of a specific class of models, we take $\nu(t, r)$ in the specific form,

$$\nu(t, r) = c(t) + \nu_0(R). \tag{15}$$

The conditions imposed here, namely, that of vanishing radial pressures, and Eq. (15) specifying a form of $\nu$ may be considered to be strong assumptions. However, this enables us to make our study in sufficient generality with sufficiently rich structure as we shall see below, with nonzero pressures introduced into the collapse model. A physical mechanism by which we can have nonvanishing tangential pressures is illustrated by the Einstein cluster [17]. This is a spherically symmetric cluster of counterrotating particles which has nonzero tangential stresses within the collapsing cloud.

Also, one can rederive the dust collapse models with a nonvanishing cosmological term, by putting $\nu_0(R) = 0$ in Eq. (15) and redefining the comoving time coordinate. It is thus clear that the class of models considered generalizes the dust collapse models with a nonvanishing cosmological constant. In general, as $v \to 0$, $\rho \to \infty$. Thus the density blows up at the singularity $R = 0$ which will be a curvature singularity as expected. Using Eq. (15) in Eq. (8), we have

$$G(t, r) = b(r) e^{2\nu_0(R)}. \tag{16}$$

Here $b(r)$ is an arbitrary, $C^2$ function of $r$. In correspondence with the dust models, we can write

$$b(r) = 1 + r^2 b_0(r), \tag{17}$$

where $r^2 b_0(r)$ is the energy distribution function for the collapsing shells. Using the given initial data and Eq. (7) one can obtain the function $\nu_0(R)$. Substituting Eq. (15) in Eq. (7) we get the intrinsic equation of state,

$$p_\theta = \frac{R}{2} \nu_{,R} \rho. \tag{18}$$

Finally, using Eqs. (13), (15), and (16) in Eq. (9), we have

$$\sqrt{R}\dot{R} = -a(t)e^{\nu_0(R)}\sqrt{(1+r^2b_0)Re^{2\nu_0} - R + r^3\mathcal{M} + \frac{\Lambda}{3}R^3}. \tag{19}$$

Here $a(t)$ is a function of time and by a suitable scaling of the time coordinate, we can always make $a(t) = 1$. We deal here with the collapse models, and so the negative sign is due to the fact that $\dot{R} < 0$, which represents the collapse condition.

## III. SINGULARITY FORMATION AND REBOUNCE

One of our main purposes here is to examine how the introduction of a nonvanishing cosmological term modifies the collapse dynamics. For example, in the case of dust collapse, once the collapse initiates from an initial epoch, there cannot be any reversal or a bounce, and the gravity forces the cloud to collapse necessarily to a singularity. However, this need not be the case when the cosmological term is nonvanishing, and we have to reexamine the collapse dynamics in order to find how the collapse evolves. This we do here for the particular class of tangential collapse models as specified above, which also generalizes the dust collapse case.

The evolution of a particular shell may be deduced from Eq. (19). Rewriting Eq. (19) in terms of the scale factor we have

$$\dot{v}^2 = \frac{e^{2\nu_0}[vj(r,v) + \mathcal{M} + \frac{\Lambda}{3}v^3]}{v} = V(r,v), \tag{20}$$

where

$$j(r,v) = \frac{b(r)e^{2\nu_0(rv)} - 1}{r^2}. \tag{21}$$

The right-hand side of Eq. (20) may be thought of as an effective potential $[V(r,v)]$ for a shell. The allowed regions of motion correspond to $V(r,v) \geq 0$, as $\dot{v}^2$ is non-negative, and the dynamics of the shell may be studied by finding the turning points. If we start from an initially collapsing state ($\dot{v} < 0$), we will have a rebounce if we get $\dot{v} = 0$, before the shell has become singular. This can happen when $V(r,v) = 0$. Hence, to study the various evolutions for a particular shell we must analyze the roots of the equation $V(r,v) = 0$ keeping the value $r$ to be fixed. It will be seen that the cosmological constant appearing in the effective potential does play an important role in the evolution of a shell.

To clarify these ideas, let us consider a smooth initial data, where the initial density, pressure, and energy distributions are expressed as only even powers of $r$. Such a consideration, that the initial data be smooth, is often justified on physical grounds. So we take

$$\rho(t_i,r) = \rho_{00} + \rho_2r^2 + \rho_4r^4 + \cdots, \tag{22}$$

$$p_\theta(t_i,r) = p_{\theta_2}r^2 + p_{\theta_4}r^4 + \cdots, \tag{23}$$

$$b_0(r) = b_{00} + b_{02}r^2 + \cdots. \tag{24}$$

With the above form of smooth initial data to evolve in time using the Einstein equations, we can explicitly integrate Eq. (7) at the initial epoch to get

$$\nu_0(R) = p_{\theta_2}R^2 + \frac{(p_{\theta_4} - \rho_2p_{\theta_2})}{2}R^4 + \cdots. \tag{25}$$

We have neglected here higher order terms in the expansion, since at present we want to concentrate on the evolution of shells near $r = 0$. The conditions when the treatment is applicable to the whole cloud will be discussed later. Equation (20) near the center of the cloud ($r \ll r_\pi$) may be written as

$$\dot{v}^2 = \frac{(1+2p_{\theta_2}r^2v^2)[(\Lambda + 6p_{\theta_2}b(r))v^3 + 3b_0(r)v + 3\mathcal{M}]}{3v}. \tag{26}$$

The first factor in $V(r,v)$ is initially positive, because it is the $|g_{00}|$ term. As the collapse evolves, the scale factor ($v$) reduces from 1 at the initial epoch to 0 at the time of singularity. Hence it is clear that the first factor can never become zero, and hence does not contribute to a bounce of the shell. The main features of the evolution of the cloud basically derive from the second factor in $V(r,v)$.

The second factor in the effective potential expression is a cubic equation which in general has three roots. Only positive real roots correspond to physical cases. Since the coefficient in the second power is zero, we may conclude that if all three roots are real then at least one of them has to be positive and at least one negative. We observe that $V(r,0) = \mathcal{M} > 0$. Hence, any region between $R = rv = 0$ and the first positive zero of $V(r,v)$ always becomes singular during collapse. The region between the unique positive roots is forbidden since in those regions $\dot{v}^2 < 0$. For a particular shell to bounce it must therefore lie, during the initial epoch ($v = 1$), in a region to the right of the second positive root. We will now analyze the various cases for $\Lambda \neq 0$ in detail and derive the necessary conditions.

(1) If

$$b_0(r) \geq 0, \qquad (\Lambda + 6p_{\theta_2}b(r)) > 0, \tag{27}$$

then from Descartes's rule of signs (see for example [18]) we see that there are no positive roots. Thus a singularity always forms from initial collapse.

(2) If

$$b_0(r) \geq 0, \qquad (\Lambda + 6p_{\theta_2}b(r)) < 0, \tag{28}$$

we infer from the sign rule that there is exactly one positive root ($\alpha(r)$). The other two roots are negative or complex conjugates. The allowed space of dynamics is $[0, \alpha]$. Thus $\alpha \geq 1$ would always ensure a singularity. However $\alpha < 1$ implies an unphysical situation initially, where $\dot{v}^2 < 0$.

(3) If

$$b_0(r) \leq 0, \qquad (\Lambda + 6p_{\theta_2}b(r)) < 0, \tag{29}$$

there is exactly one positive root ($\beta(r)$). Again all shells in the allowed dynamical space $[0, \beta]$ become singular starting from initial collapse.

(4) If

$$b_0(r) < 0, \qquad (\Lambda + 6p_{\theta_2}b(r)) > 0, \tag{30}$$

there are three possibilities:

(4.1) If

$$\mathcal{M}^2 > -4\frac{b_0^3(r)}{9(\Lambda + 6p_{\theta_2}b(r))}, \tag{31}$$

then there are no positive roots and a singularity is always the final outcome of the collapse for shells under consideration.

(4.2) If

$$\mathcal{M}^2 < -4\frac{b_0^3(r)}{9(\Lambda + 6p_{\theta_2}b(r))}, \tag{32}$$

then there are two positive roots [$\gamma_1(r)$ and $\gamma_2(r)$ respectively]. The space of allowed dynamics is $[0, \gamma_1]$ and $[\gamma_2, \infty)$. The region $(\gamma_1, \gamma_2)$ is forbidden. Shells in the $[0, \gamma_1]$ region initially, always become singular. But shells initially belonging to the region $[\gamma_2, \infty)$ will undergo a bounce and subsequent expansion starting from initial collapse. This bounce occurs when their geometric radius approaches $R_{\rm bounce} = r\gamma_2$. Using the definitions

$$\varrho = \sqrt{\frac{-4b_0(r)}{\Lambda + 6p_{\theta_2}b(r)}}, \tag{33}$$

$$\vartheta = \frac{1}{3}\cos^{-1}\left[-\sqrt{\frac{-9\mathcal{M}(r)^2(\Lambda + 6p_{\theta_2}b(r))}{4b_0^3(r)}}\right], \tag{34}$$

the condition for a particular shell to become singular or undergo a bounce may be explicitly written in terms of the initial data and $\Lambda$ as

$$1 < \varrho\cos\vartheta, \qquad \text{singularity}, \tag{35}$$

$$1 > \varrho\cos\left(\vartheta + \frac{4\pi}{3}\right), \qquad \text{bounce}. \tag{36}$$

Here $\varrho\cos\vartheta$ is $\gamma_1(r)$ and $\varrho\cos(\vartheta + \frac{4\pi}{3})$ is $\gamma_2(r)$, which are the two roots of the potential function. Note that contrary to the dust models with $\Lambda$, there may be a bounce for both positive and negative values of the cosmological constant.

(4.3) If

$$\mathcal{M}^2 = -4\frac{b_0^3(r)}{9(\Lambda + 6p_{\theta_2}b(r))}, \tag{37}$$

the positive roots are equal. There is no forbidden region, and there will be a bounce if $1 > \frac{\varrho}{2}$. Figure 1 illustrates a particular choice of initial data and $\Lambda$ which causes a bounce in the central ($r = 0$) shell.

To analyze the evolution of shells far from the center, in general one has to resort to numerical methods, and it is difficult to analytically give a simple expression for the singularity or bounce conditions. Nevertheless, the analysis as given here becomes valid for the entire cloud all the way till the boundary when the geometrical radius of the cloud boundary at the initial epoch [$R(t_i, r_\pi) = r_\pi$] is itself small relative to the initial data coefficients (i.e. $\rho_n r_\pi^n$, $p_{\theta_n} r_\pi^n \ll 1$). Also, if we choose the initial data such that the higher coefficients in the power series expansion are zero (i.e. $p_{\theta_{n\geq 4}}$, $\rho_{k\geq 4} = 0$) and $\rho_2 r_\pi^2$, $p_{\theta_2} r_\pi^2 \ll 1$, then the analysis is again applicable to the whole cloud. Thus the results derived can be considered quite general in these circumstances and applicable to the cloud as a whole.

In this context, if all shells in the collapsing cloud satisfy Eqs. (30), (32), and (36), the complete cloud undergoes bounce starting from the initial collapse. To avoid shell crossings the sufficient condition would be

$$\forall\ r \epsilon [0, r_\pi), \qquad \gamma_2(r + \delta) \geq \gamma_2(r), \tag{38}$$

where $\delta$ is an infinitesimal increment in the comoving radius. It is seen that in all the cases discussed, it is not $\Lambda$ alone, but $\Lambda + 6p_{\theta_2}b(r)$ along with $b_0(r)$ that determines the evolution of the shell. This is in contrast to the dust models with a nonzero $\Lambda$, where solely the cosmological constant decided the evolution of a shell for a given energy function. It is also interesting to note that unlike the dust collapse models with $\Lambda$, there could be a bounce in the fluid model with vanishing radial pressures for both positive and negative values of the cosmological constant. This is due to the contribution from the tangential pressure.

It can be seen now that one can rederive the known bounce conditions in the $\Lambda \neq 0$ dust collapse case

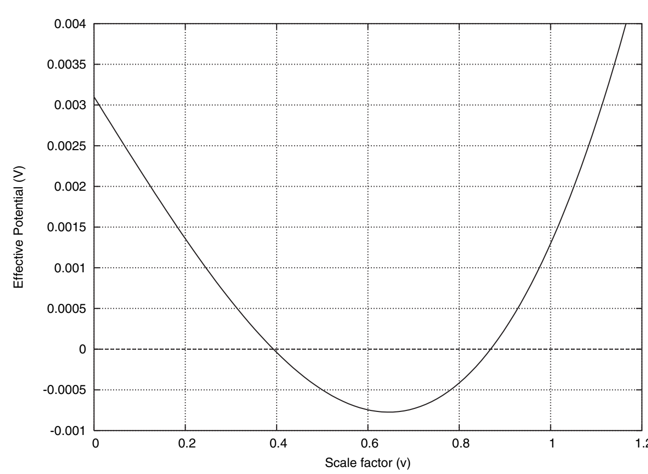


FIG. 1. The effective potential profile [$V(r, v)$] for the $r = 0$ shell with $(\Lambda + 6p_{\theta_2}) = 7.1978 \times 10^{-3}$, $b_{00} = -3 \times 10^{-3}$, and $\mathcal{M} = 1.033 \times 10^{-3}$ in appropriate units.

(Deshingkar *et al.* [7]) as a special case of the consideration here, when $p_\theta = 0$. In that case,

$$\dot{v}^2 = \frac{\Lambda v^3 + 3b_0(r)v + 3\mathcal{M}}{3v} = V(r, v). \tag{39}$$

The study now becomes valid for all shells, without any approximation. For example, following similar steps as for the fluid model, one obtains for $\Lambda > 0$,

(1) If

$$b_0(r) > 0, \tag{40}$$

the singularity always forms from initial collapse.

(2) If

$$b_0(r) = 0, \tag{41}$$

then again it is found that all shells become singular from collapse.

(3) If

$$b_0(r) < 0, \tag{42}$$

there are two scenarios possible. For

$$\mathcal{M}^2 > -4\frac{b_0^3}{9\Lambda}, \tag{43}$$

all shells become singular. For

$$\mathcal{M}^2 < -4\frac{b_0^3}{9\Lambda}, \tag{44}$$

there are two positive roots ($\psi_1$ and $\psi_2$ respectively) for $V(r, v)$. Shells belonging to $[0, \psi_1]$ always become singular while those belonging to $[\psi_2, \infty)$ undergo bounce starting from initial collapse.

## IV. SPACETIME MATCHING

As we pointed out above, there is a strong physical motivation to study and investigate the gravitational collapse phenomenon in a background which is asymptotically either a de Sitter or anti–de Sitter metric. For this purpose, the collapsing cloud has to be matched at the boundary to a suitable exterior spacetime which has the desired properties.

In the present case, we shall show below that the exterior vacuum spacetime of the collapsing region may be described by the Schwarzschild–de Sitter (SdS) or Schwarzschild–anti–de Sitter (SAdS) metric, depending on whether the cosmological constant is taken to be positive or negative. The collapsing interior cloud which has a nonzero tangential pressure is then to be smoothly matched to an exterior spacetime in order to generate the full spacetime. The necessary and sufficient conditions to achieve a smooth matching are given by the Israel-Darmois junction conditions ([19–21]), which we shall use below.

Let the interior of the collapsing cloud be described by the metric,

$$S^-: ds_-^2 = -e^{2\nu(t,r)}dt^2 + e^{2\psi(t,r)}dr^2 + R^2(t, r)d\Omega^2. \tag{45}$$

The exterior vacuum solution can be given as the Schwarzschild–de Sitter/anti–de Sitter spacetime (as decided by the sign chosen for the cosmological term) as given by

$$S^+: ds_+^2 = -\mathcal{D}(\mathcal{R})dT^2 + \mathcal{D}^{-1}(\mathcal{R})d\mathcal{R}^2 + \mathcal{R}^2 d\Omega^2, \tag{46}$$

where $\mathcal{D}(\mathcal{R})$ is given by

$$\mathcal{D}(\mathcal{R}) = 1 - \frac{2M}{\mathcal{R}} \mp \frac{|\Lambda|\mathcal{R}^2}{3}. \tag{47}$$

The negative sign precedes $|\Lambda|$ for a SdS exterior and the positive for the SAdS exterior. Note that as $\mathcal{R} \to \infty$, the $\mp|\Lambda|\mathcal{R}^2/3$ term dominates over the $\frac{2M}{\mathcal{R}}$ term, and the spacetime approaches asymptotically the de Sitter metric $(-|\Lambda|\mathcal{R}^2/3)$, or the anti–de Sitter metric $(+|\Lambda|\mathcal{R}^2/3)$, as the case may be.

Let $\Pi$ denote the boundary hypersurface. The equations of the boundary hypersurface considered as an embedding in the interior or exterior spacetimes are

$$\Pi^-: r - r_\pi = 0, \qquad \Pi^+: \mathcal{R} - \mathcal{R}_\pi(T) = 0. \tag{48}$$

Substituting (48) in (45) and (46) we get the metric on the hypersurface as

$$S_\Pi^-: ds_-^2 = -e^{2\nu(t,r_\pi)}dt^2 + R^2(t, r_\pi)d\Omega^2, \tag{49}$$

$$S_\Pi^+: ds_+^2 = -\mathcal{D}(\mathcal{R}_\pi)dT^2 + \mathcal{D}^{-1}(\mathcal{R}_\pi)d\mathcal{R}_\pi^2 + \mathcal{R}_\pi^2 d\Omega^2. \tag{50}$$

The Israel-Darmois conditions to match the interior spacetime with the exterior require that the first and second fundamental forms of the boundary hypersurface match. The first fundamental form is given by

$$g_{\mu\nu}d\zeta^\mu d\zeta^\nu, \tag{51}$$

where $\zeta$ parametrizes the hypersurface ($\zeta^\iota$: $\tau, \theta, \phi$). The matching of the first fundamental form gives, from Eqs. (49) and (50),

$$R(t, r_\pi) = \mathcal{R}_\pi, \tag{52}$$

$$d\tau = e^{\nu(t,r_\pi)}dt, \tag{53}$$

$$d\tau = \left[\mathcal{D}_\pi - \frac{\mathcal{R}_{\pi,T}^2}{\mathcal{D}_\pi}\right]^{1/2}dT. \tag{54}$$

The above three conditions must be satisfied for a smooth matching of the collapsing interior to the exterior spacetime. The next set of conditions will be given by the matching of the second fundamental forms.

The second fundamental form is given by

$$K_{\mu\nu} d\zeta^\mu d\zeta^\nu, \tag{55}$$

where

$$K^{\pm}_{\mu\nu} = -n^{\pm}_{\sigma} x^{\sigma}_{,\zeta^\mu,\zeta^\nu} - n^{\pm}_{\sigma}\Gamma^{\sigma}_{\beta\gamma} x^{\beta}_{,\zeta^\mu} x^{\gamma}_{,\zeta^\nu} \tag{56}$$

is the extrinsic curvature [21]. The (,) denotes partial differentiation. $n_\sigma$ is the normal to the hypersurface which is given by

$$n_\sigma = \frac{f_{,\sigma}}{[g^{\mu\nu} f_{,\mu} f_{,\nu}]^{1/2}}, \tag{57}$$

where $f = 0$ is the equation of the boundary hypersurface. Direct calculation gives

$$n^{-}_{\sigma} = (0, e^{\psi(t,r_\pi)}, 0, 0), \tag{58}$$

$$n^{+}_{\sigma} = (-\mathcal{R}^{\pi}_{,\tau}, T_{,\tau}, 0, 0). \tag{59}$$

The $K_{\theta\theta}$ extrinsic curvatures are calculated as

$$K^{+}_{\theta\theta} = (\mathcal{D}\mathcal{R}T_{,\tau})_\pi, \qquad K^{-}_{\theta\theta} = (RR_{,r}e^{-\psi})_\pi. \tag{60}$$

Now using the fact that the second fundamental forms match (i.e. $[K^{+}_{\theta\theta} - K^{-}_{\theta\theta}]_\pi = 0$), we get using Eqs. (52)–(54) and (60) after simplification,

$$R^2_{\pi,r} e^{-2\psi} - R^2_{\pi,\tau} e^{-2\nu} = 1 - \frac{2M}{\mathcal{R}_\pi} \mp \frac{|\Lambda|}{3}\mathcal{R}^2_\pi. \tag{61}$$

This is identical to the Cahill and McVittie definitions for the mass function ([16,22]), and hence from Eq. (9) we may by comparison take

$$F_\pi = 2M \pm \frac{|\Lambda|}{3}\mathcal{R}^3_\pi. \tag{62}$$

This expression suggests that for a smooth matching of the interior and exterior spacetimes the interior mass function at the surface must equal the generalized Schwarzschild mass.

As we can see there is a contribution to the mass function from the cosmological constant. A positive cosmological constant has an additive contribution and a negative cosmological constant has a deductive contribution to the mass function $F_\pi$. The matching of a collapsing dust interior with exterior SdS/SAdS would give similar results [7]. The $K_{\tau\tau}$ and $K_{\phi\phi}$ components of the extrinsic curvature may similarly be calculated. The condition $[K^{+}_{\tau\tau} - K^{-}_{\tau\tau}]_\pi = 0$ gives no new information since the radial pressure is zero. Because of spherical symmetry, $[K^{+}_{\phi\phi} - K^{-}_{\phi\phi}]_\pi = 0$ gives the same result as Eq. (62). All the above conditions (52)–(54) and (62) must be satisfied for the smooth matching of spacetimes across the boundary.

## V. HORIZONS

Apparent horizons ($\mathcal{H}$) are the boundaries of trapped regions [14] in the spacetime. We discuss below the effect of a nonzero $\Lambda$ term on the apparent horizons of the tangential pressure fluid collapse models considered here briefly. In general, the equation of $\mathcal{H}$ can be written as

$$\mathcal{H}: g^{\mu\nu} R_{,\mu} R_{,\nu} = 0. \tag{63}$$

Substituting (45) in (63) we get

$$R^2_{,r} e^{-2\psi} - R^2_{,\tau} e^{-2\nu} = 0. \tag{64}$$

From the definition of the mass function Eqs. (9) and (64) we therefore have

$$1 - \frac{F}{R} = 0. \tag{65}$$

Finally, from (13) and (65),

$$\mathcal{H}: (3 - \Lambda R^2)R = 3r^3\mathcal{M}. \tag{66}$$

When $\Lambda = 0$ there is necessarily only one apparent horizon given by

$$R = r^3\mathcal{M} \tag{67}$$

which is the Schwarzschild horizon in case we are considering that geometry. The same equation also defines the horizon within the collapsing cloud, where $R(t, r)$ is one of the metric functions. For the case when $\Lambda > 0$, Eq. (66) is a cubic equation with at least one positive and one negative root when all roots are real. For $\Lambda > 0$ the various cases are given as below.

(1) For

$$3r^3\mathcal{M} < \frac{2}{\sqrt{\Lambda}} \tag{68}$$

there are two positive roots for (66) and hence there are *two* apparent horizons. These horizons are given by

$$R_c(r) = \frac{2}{\sqrt{\Lambda}}\cos\left[\frac{1}{3}\cos^{-1}\left(-\frac{3}{2}r^3\mathcal{M}\sqrt{\Lambda}\right)\right], \tag{69}$$

$$R_b(r) = \frac{2}{\sqrt{\Lambda}}\cos\left[\frac{4\pi}{3} + \frac{1}{3}\cos^{-1}\left(-\frac{3}{2}r^3\mathcal{M}\sqrt{\Lambda}\right)\right]. \tag{70}$$

These have been at times called the cosmological, and the black hole horizons [9].

(2) For

$$3r^3\mathcal{M} = \frac{2}{\sqrt{\Lambda}}, \tag{71}$$

there is only one positive root for (66), given by

$$R_{bc}(r) = \frac{1}{\sqrt{\Lambda}}. \tag{72}$$

This corresponds to a single apparent horizon.

(3) For

$$3r^3\mathcal{M} > \frac{2}{\sqrt{\Lambda}}, \tag{73}$$

there are no positive roots and hence there are no apparent horizons.

The case (73) also shows that the mass of the black hole is bounded above by $F = 1/\sqrt{\Lambda}$ and attains the largest proper area $4\pi/\Lambda$. Some general results exist in the literature [9] showing that in spacetimes with $\Lambda > 0$ and matter satisfying the strong energy condition, the area of a black hole cannot exceed $4\pi/\Lambda$. A detailed treatment of apparent horizons in the LTB dust collapse models with a nonzero $\Lambda$ term is given by Cissoko *et al.* [7].

From Eq. (19), the time of the apparent horizon formation in the fluid model is given by

$$t_{ah}(r) = t_s(r) - t_1(r), \tag{74}$$

where $t_1(r)$ is defined as

$$t_1(r) = \int_0^{v_{ah}(r)} \frac{\sqrt{v}dv}{\sqrt{e^{4\nu_0}vb_0 + e^{2\nu_0}(v^3 h(rv) + \mathcal{M} + \frac{\Lambda}{3}v^3)}}. \tag{75}$$

Also $rv_{ah} = R_b$ or $R_c$, $h(R) = (e^{2\nu_0(R)} - 1)/R^2$ and $t_s(r)$ is the time of singularity formation. For shells close to the center of the cloud ($r \ll r_\pi$) the expression becomes

$$t_{ah}(r) = t_s(r) - \int_0^{v_{ah}(r)} \frac{\sqrt{3v}dv}{\sqrt{(\Lambda + 6p_{\theta_2})v^3 + 3b_0(r)v + 3\mathcal{M}}}. \tag{76}$$

It is observed that the cosmological constant modifies the time of the formation of horizons and also the time lag between horizon formation and singularity formation in the fluid model.

## VI. NATURE OF THE CENTRAL SINGULARITY

The final end state of gravitational collapse and the nature of the resulting singularity continue to be among the most outstanding problems in gravitation theory and relativistic astrophysics today. As pointed out earlier, the hypothesis that such a collapse leading to a singularity, under physically realistic conditions must end in the formation of a black hole, and that the eventual singularity must be hidden below the event horizons of gravity is the cosmic censorship conjecture. Despite numerous attempts, this conjecture as such remains a major unsolved problem lying at the foundation of black hole physics today.

From such a perspective, we need to examine the nature of the singularity, in terms of its visibility or otherwise for outside observers, when it develops within the context of the models considered here. This should tell us how the presence of the $\Lambda$ term modifies these considerations, because we already know that in the case of a tangential pressure present, but a vanishing cosmological constant, both black holes and naked singularities do develop as final collapse end states depending on the nature of the initial data (see e.g. [10]).

We have already seen above that the cosmological constant modifies the time of formation of trapped surfaces. If the formation of the horizon precedes the formation of the central singularity then the singularity will be necessarily covered, i.e. it is a black hole. If on the other hand, the horizon formation occurs after the singularity formation, there may be future directed nonspacelike geodesics that end in the past at the singularity. Then the final end state would be a naked singularity. Thus we need to find whether there exist future directed null geodesics that end at the singularity in the past.

Towards analyzing this issue, let us define a function $h(R)$ as

$$h(R) = \frac{e^{2\nu_0(R)} - 1}{R^2} = 2g(R) + \mathcal{O}(R^2). \tag{77}$$

Using Eq. (77) in Eq. (19), we get after simplification,

$$\sqrt{v}\dot{v} = -\sqrt{e^{4\nu_0}vb_0 + e^{2\nu_0}\left(\left(\frac{\Lambda}{3} + h(rv)\right)v^3 + \mathcal{M}\right)}. \tag{78}$$

Integrating the above equation, we get

$$t(v, r) = \int_v^1 \frac{\sqrt{v}dv}{\sqrt{e^{4\nu_0}vb_0 + e^{2\nu_0}(v^3(h(rv) + \frac{\Lambda}{3}) + \mathcal{M})}}. \tag{79}$$

The time of formation of a shell-focusing singularity, for a specific shell, is obtained by taking the limits of integration in the above as (0,1). The shells collapse consecutively, one after the other to the center as there are no shell crossings ($R' > 0$). We are interested in the central shell (i.e. the singularity forming at $r = 0$), since we will see that all $r > 0$ shells are necessarily covered on becoming singular. Taylor expanding the above function around $r = 0$, we get

$$t(v,r) = t(v,0) + r\frac{dt(v,r)}{dr}\bigg|_{r=0} + \frac{r^2}{2!}\frac{d^2t(v,r)}{d^2r^2}\bigg|_{r=0} + \cdots. \tag{80}$$

Let us denote

$$\mathcal{X}_n(v) = \frac{d^n t(v,r)}{dr^n}\bigg|_{r=0}. \tag{81}$$

The initial data are taken to be smooth (i.e. with only even powers of $r$ allowed). Because of this choice, the first derivative of the functions appearing in the above equation vanish at $r = 0$. Hence we have

$$\mathcal{X}_1(v) = 0. \tag{82}$$

Defining $B_f(r, v) = e^{4\nu_0} v b_0 + e^{2\nu_0}((\frac{\Lambda}{3} + h)v^3 + \mathcal{M})$ we may write the Taylor expansion about $r = 0$ as

$$t(v, r) = t(v, 0) - \frac{r^2}{4} \int_v^1 \frac{B_f''(0, v)\sqrt{v}dv}{B_f(0, v)^{3/2}} + \cdots, \tag{83}$$

and we have

$$\mathcal{X}_2(v) = -\int_v^1 \frac{B_f''(0, v)\sqrt{v}dv}{B_f(0, v)^{3/2}}. \tag{84}$$

In order to consider the possibility of the existence of null geodesic families which end at the singularity in the past, and to examine the nature of the singularity occurring at $R = 0$, $r = 0$ in this model, let us consider the outgoing null geodesic equation which is given by

$$\frac{dt}{dr} = e^{\psi - \nu}. \tag{85}$$

We use a method which is similar to that given in [23]. The singularity curve is given by $v(t_s(r), r) = 0$, which corresponds to $R(t_s(r), r) = 0$. Therefore, if we have any future directed outgoing null geodesics terminating in the past at the singularity, we must have $R \to 0$ as $t \to t_s$ along the same. Now writing Eq. (85) explicitly in the terms of the variables $(u = r^\alpha, R)$, we have

$$\frac{dR}{du} = \frac{1}{\alpha} r^{-(\alpha-1)} R' \left[1 - \sqrt{\frac{be^{2\nu_0} + \frac{r^3\mathcal{M}}{R} + \frac{\Lambda R^2}{3} - 1}{be^{2\nu_0}}}\right]. \tag{86}$$

Equation (86) is required to be finite and positive, for the existence of a naked singularity [23]. In order to get the tangent to the null geodesic in the $(R, u)$ plane, we choose a particular value of $\alpha$ such that the geodesic equation is expressed only in terms of $(\frac{R}{u})$. A specific value of $\alpha$ is to be chosen which enables us to calculate the proper limits at the central singularity. In the tangential pressure collapse model discussed in the previous section we have $\mathcal{X}_1(0) = 0$, and hence we choose $\alpha = \frac{7}{3}$ so that when the limit $r \to 0$, $t \to t_s$ is taken we get the value of tangent to null geodesic in the $(R, u)$ plane as

$$\frac{dR}{du} = \frac{3}{7}\left(\frac{R}{u} + \frac{\sqrt{M_0}\mathcal{X}_2(0)}{\sqrt{\frac{R}{u}}}\right) \frac{(1 - \frac{F}{R})}{\sqrt{G}(\sqrt{G} + \sqrt{H})}. \tag{87}$$

Now note that for any point with $r > 0$ on the singularity curve $t_s(r)$ we have $R \to 0$, whereas $F$ (interpreted as the gravitational mass within the comoving radius $r$) tends to a finite positive value once the energy conditions are satisfied. Under the situation, the term $F/R$ diverges in the above equation, and all such points on the singularity curve will be covered as there will be no outgoing null geodesics from such points.

We hence need to examine the central singularity at $r = 0$, $R = 0$ to determine if it is visible or not. That is, we need to determine if there are any solutions existing to the outgoing null geodesic equation, which terminate in the past at the singularity and in the future go to a distant observer in the spacetime, and if so under what conditions these exist. Let $x_0$ be the tangent to the null geodesics in the $(R, u)$ plane, at the central singularity, then it is given by

$$x_0 = \lim_{t\to t_s} \lim_{r\to 0} \frac{R}{u} = \frac{dR}{du}\bigg|_{t\to t_s; r\to 0}. \tag{88}$$

Using Eq. (87), we get

$$x_0^{3/2} = \frac{7}{4}\sqrt{\mathcal{M}_0}\mathcal{X}_2(0). \tag{89}$$

In the $(R, u)$ plane, the null geodesic equation will be

$$R = x_0 u, \tag{90}$$

while in the $(t, r)$ plane, the null geodesic equation near the singularity will be

$$t - t_s(0) = x_0 r^{7/3}. \tag{91}$$

It follows that if $\mathcal{X}_2(0) > 0$, then that implies that $x_0 > 0$, and we then have radially outgoing null geodesics coming out from the singularity, making the central singularity locally visible. On the other hand, if $\mathcal{X}_2(0) < 0$, we will have a black hole solution. We have, however, already seen in Eq. (84) that the value of $\mathcal{X}_2(0)$ entirely depends upon the initial data and the cosmological term $\Lambda$. Given any $\Lambda$, the initial data can always be chosen such that the end state of the collapse would be either a naked singularity or a black hole. Hence, it follows that for both positive and negative cosmological constants, a naked singularity can occur as the final end state of gravitational collapse.

We noted here earlier that dust collapse models have been analyzed in the presence of a cosmological constant. The nature of the final singularity in that case has been analyzed using the so-called "roots" method (Deshingkar *et al.* [7]). We show below that the different treatment we have used above to deal with the collapsing clouds with pressure included arrives at similar conclusions when specialized to the dust case.

The Einstein equations for dust may be obtained by putting $p_\theta = 0$ in the tangential pressure fluid model above. They take the form,

$$\rho(r, t) = \frac{\mathcal{M}_d'(r)}{R^2 R'}, \tag{92}$$

$$\dot{R}^2 = \frac{\mathcal{M}_d(r)}{R} + f(r) + \frac{\Lambda R^2}{3}, \tag{93}$$

where $f(r) = r^2 b_0(r)$ is the dust energy free function. Since we are interested in collapse we must have $\dot{R} \leq 0$. Then we have from the above equations,

$$\sqrt{R}\dot{R} = -\sqrt{\mathcal{M}_d(r) + f(r)R + \frac{\Lambda R^3}{3}}. \tag{94}$$

Using the scaling freedom we may write

$$\mathcal{M}_d(r) = \int_0^r \rho(0, r) r^2 dr = \rho^{\mathrm{avg}}(r) r^3. \tag{95}$$

Rewriting Eq. (94) in terms of the scale factor $v$,

$$\sqrt{v}\dot{v} = -\sqrt{\rho^{\mathrm{avg}}(r) + \frac{f(r)v}{r^2} + \frac{\Lambda v^3}{3}}. \tag{96}$$

Define $f(r)v/r^2$ as $L(r, v)$ which is at least a $C^2$ function of its arguments. Then $t_s(r, v)$ is readily calculated from (96) to be

$$t_s(r, v) = \int_v^1 \frac{\sqrt{v} dv}{\sqrt{\rho^{\mathrm{avg}}(r) + L(v, r) + \frac{\Lambda v^3}{3}}}. \tag{97}$$

The shell-focusing singularity $R = 0$ occurs first for the comoving coordinate $r = 0$. The time of its formation is

$$t_s(0) = \int_0^1 \frac{\sqrt{v} dv}{\sqrt{\rho^{\mathrm{avg}}(0) + L(v, 0) + \frac{\Lambda v^3}{3}}}. \tag{98}$$

As we did for the fluid model, we Taylor expand $t_s(r)$ near $r = 0$ to get

$$t_s(r) = t_s(0) + r\mathcal{X}_1(0) + r^2\mathcal{X}_2(0) + \mathcal{O}(r^3), \tag{99}$$

where $\mathcal{X}_1(v) = 0$ (assuming smooth initial data), and

$$\mathcal{X}_2(v) = -\int_v^1 \frac{B_d''(0, v)\sqrt{v} dv}{B_d(0, v)^{3/2}} \tag{100}$$

with $B_d(r, v) = \rho^{\mathrm{avg}}(r) + L(v, r) + (\Lambda v^3/3)$. Again, it needs to be analyzed whether there exists future directed null geodesics that end at the singularity in the past. Consider the marginally bound case ($f(r) = 0$). Then the equation of the null geodesic is

$$\frac{dt}{dr} = R'. \tag{101}$$

In terms of $(u, r)$ it is written as

$$\frac{dR}{du} = \frac{R'}{\alpha r^{\alpha - 1}}\left(1 - \sqrt{\frac{\mathcal{M}_d(r)}{R} + \frac{\Lambda R^2}{3}}\right). \tag{102}$$

Choosing $\alpha = \frac{7}{3}$,

$$\frac{dR}{du} = \frac{3}{7}\left(\frac{\sqrt{v}v'}{\sqrt{\frac{R}{u}}} + \frac{R}{u}\right)\left(1 - \sqrt{\frac{\mathcal{M}_d(r)}{R} + \frac{\Lambda R^2}{3}}\right). \tag{103}$$

Taking the limits as before, the final expression becomes

$$\frac{4}{7}x_0^{3/2} = \sqrt{\rho_0^{\mathrm{avg}}}\mathcal{X}_2(0). \tag{104}$$

It follows that for both positive and negative values of $\Lambda$, there exist initial data that may give $\mathcal{X}_2(0) > 0$. Thus the dust central singularity may be locally visible, even when there is a nonzero cosmological constant present. It may thus be claimed that the dust central naked singularity is not precluded by $\Lambda$.

## VII. CONCLUDING REMARKS

The currently observed accelerated expansion of the universe, and the AdS/CFT conjecture in string theories have generated considerable interest in spacetime scenarios with a nonzero cosmological constant. One may have expected $\Lambda$ to have mainly long range effects without any influence on local gravitational collapse. But the present study as well as earlier studies ([7–9]) show that the cosmological term plays an important role in the dynamics of gravitational collapse. We studied here the collapse of a tangential pressure fluid model with a nonvanishing $\Lambda$. Various aspects of gravitational collapse with $\Lambda$ have been considered which include collapse dynamics, junction conditions, apparent horizons, and nature of the singularity. The question of cosmic censorship in the collapse models is discussed, and it is also shown that when the pressures are put to zero these models reduce to the dust case.

We find here that the cosmological term plays an important role in the dynamics of gravitational collapse. This is in terms of a continual collapse or a rebounce as specified by conditions we have given, and by its effects on the geometry of trapped surfaces. The $\Lambda$ term also affects the junction conditions during the matching of the interior and exterior spacetimes using Israel-Darmois criteria. Further, the apparent horizon structure is affected by $\Lambda$ as we pointed out. It is seen that the cosmological term modifies the time of formation of singularities and the time lag between singularity formation and horizon formation. Some specific remarks and conclusions are as below.

(1) It is seen that the value of $\Lambda$ relative to the other physical quantities decides emphatically the final outcome of collapse, that is, whether it goes to a singularity or undergoes a bounce. In the collapse model with $\Lambda$ we studied, one observes that when conditions (27) and (28) or (29) are satisfied by the physical parameters, the collapse of the particular shell always proceeds to a singularity. When a shell satisfies condition (30), two distinct possibilities arise. If (31) is satisfied further to (30) the final outcome is again always a singularity. But the most interesting case is when the physical parameters are such that conditions (30), (32), and (36) are satisfied. In this scenario there is a bounce and there is no singularity formation. In all the above situations one observes that the cosmological term plays an important role. As discussed in the text it is seen that in this fluid model, it is $\Lambda + 6p_{\theta_2}b(r)$ along with $b_0(r)$ that determines the evolution of the shell. This is in contrast to the dust models with a nonzero $\Lambda$, where solely the cosmological constant decided the evolution of a shell for a given energy function, in terms of bounce or formation of a singularity. Also, here unlike dust collapse models there is the possibility of bounce for both positive and negative values of $\Lambda$, owing to the contribution from the tangential pressures.

(2) Matching of the collapsing interior to an exterior which is Schwarzschild–de Sitter or anti–de Sitter, using the Israel-Darmois criteria leads to the conclusion that the necessary and sufficient conditions for smooth matching of the interior and exterior spacetimes in the presence of a cosmological term are (52)–(54) and (62). Specifically, from Eq. (62) we see that the mass function at the cloud boundary (denoted by $r = r_\pi$) must be equal to the generalized Schwarzschild mass. The contribution to the mass function at $r = r_\pi$ from the cosmological term is positive or negative depending on the sign of $\Lambda$. This is consistent with the interpretation of the mass function as representing the total gravitational mass within a shell of comoving radius $r$.

(3) For a vanishing cosmological term there is only one apparent horizon, the Schwarzschild horizon. But if $\Lambda$ is positive satisfying (68), there are two horizons in the collapse model and for (71) there is again only a single horizon. Interestingly, it is seen that when condition (73) is satisfied by the physical parameters there are no horizons in the model. The apparent horizon formation in the fluid model, with vanishing radial pressures, crucially depends on the value of the cosmological term. Another aspect of $\Lambda$ relevant to collapse studies and especially to cosmic censorship is that it modifies the time lag between horizon formation and singularity formation. The relevant expressions are given in Eqs. (75) and (76).

(4) The final outcome of gravitational collapse is one of the most important open problems in gravitation theory, and the study of the fluid with vanishing radial pressure and also the dust collapse models here indicate that the presence of a nonvanishing $\Lambda$ term cannot, in any conclusive manner, act as a cosmic censor. Although the values of the null geodesic tangents are modified by the presence of $\Lambda$, there are still cases where the singularity is locally visible depending on the initial data. The global visibility of such a singularity (null rays from the singularity reaching an asymptotic observer) which is locally naked, will depend on the overall behavior of the various functions concerned which appear in the analysis ([12]). We also note that studies pertaining to radiation collapse (of a type II matter field [14]) in spacetimes with $\Lambda$ exist in the literature [24]. Such studies also support the conclusion that both collapse end states, namely, a black hole or a naked singularity, are possible in the presence of a nonzero $\Lambda$.

(5) It follows, in particular, that both black holes and naked singularities do develop as collapse end states in the presence of a $\Lambda$ term, in spacetimes which are asymptotically de Sitter or anti–de Sitter.

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