Comment on "Glueball spectrum from a potential model"

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In a recent article, W.-S. Hou and G.-G. Wong [Phys. Rev. D **67**, 034 003 (2003)] have investigated the spectrum of two-gluon glueballs below 3 GeV in a potential model with a dynamical gluon mass. We point out that, among the 18 states calculated by the authors, only three are physical. The other states either are spurious or possess a finite mass only due to an arbitrary restriction of the variational parameter.

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In Ref. [1], the authors have studied the bound states of confined gluons, known as glueballs, with a potential model proposed two decades ago by Cornwall and Soni [2,3]. The motivation was to cast some light on the glueballs properties with a simple model, which allows the calculation of the spectrum of glueballs composed of two, three or more gluons. This article [1] was an extension of a previous article [4] where only a vanishing angular momentum was considered.

The potential model used by Hou and Wong is potentially interesting, but the interaction between two gluons used to calculate the glueball spectrum have two kind of singularities. The potential between two gluons reads

$$\begin{aligned} V_{2g}(r) = -\lambda\Bigg\{ & \Bigg[\Bigg[\frac{1}{4} + \frac{1}{3}\mathbf{S}^2 + \frac{3}{2m^2}(\mathbf{L}\cdot\mathbf{S})\frac{1}{r}\frac{\partial}{\partial r} \\ & - \frac{1}{2m^2}\bigg((\mathbf{S}\cdot\nabla)^2 - \frac{1}{3}\mathbf{S}^2\nabla^2\bigg)\Bigg]\frac{e^{-mr}}{r} \\ & + \bigg(1 - \frac{5}{6}\mathbf{S}^2\bigg)\frac{\pi}{m^2}\delta^3(\mathbf{r})\Bigg\} + 2m(1 - e^{-\beta m r}), \end{aligned} \quad (1)$$

where m is the effective mass of the gluon.

The first kind of singularity comes from the delta function which is attractive for spin zero states. This singularity has been correctly regularized by the authors with a standard method: the singular delta function is replaced by a Gaussian function (a Yukawa function is another possibility)

$$\delta^3(\mathbf{r}) \rightarrow \frac{k^3 m^3}{\pi^{3/2}} e^{-(kmr)^2}. \quad (2)$$

Consequently, the 0^{++} ($L = S = 0$) and 2^{++} ($L = 0$, $S = 2$) states, for which the spin-orbit and the diagonal tensor interaction do not contribute, have been correctly considered in the articles [1,4]. However, due to inaccurate numerical calculations, the values of the masses obtained in Ref. [1] are questionable. The values of the parameters found in this article are $\lambda = 1.5$, $\beta = 0.5$ and $m \cong 670$ MeV (the gluon mass was not precisely determined). We have calculated the masses of the lightest 0^{++} and 2^{++} glueballs with the variational method used in Ref. [1], and we have obtained an agreement with the values published for $k = 4.183$ (the value of this last parameter was not given in the article). With these values of the four parameters, we have performed an accurate calculation using two different numerical methods: the Lagrange Mesh method [5] and a variational method using an harmonic oscillator basis up to 16 quanta. The masses of the 0^{++} and 2^{++} glueballs are found to be 282 MeV and 2238 MeV, respectively, which shows that the method used in Ref. [1] is not accurate enough to reach convergence. Obviously, small variations of the gluon mass cannot save the situation.

This defect cannot be totally compensated with another value of the parameter k (which carries less physical meaning). To reproduce the masses of the 0^{++} and 2^{++} glueballs, both k and m must be modified. With $m = 705$ MeV and $k = 3.298$, these masses are correctly reproduced (0^{++} mass is 1731 MeV and 2^{++} mass is 2402 MeV). Even if the values of the parameters seem little changed, it is worth noting that the wave functions, and consequently the associated observables, are quite different. For instance, with these new parameters, the root mean square radius of the lightest 0^{++} glueball is found to be 0.28 fm, whereas the published value is 0.1 fm. Fortunately, the smearing of the delta function is only necessary for the lightest scalar glueballs. Other states are not concerned by the values of k.

Until now, we just pointed out some inaccuracies in the calculation, which yields some difficulties in the interpretation of the parameters and yields large errors for the value of the masses of glueballs predicted by the models. Now we discuss more serious problems of the model as treated in Ref. [1].

The second kind of singularity of the potential (1) appears in the spin-orbit and in the tensor interactions (which were not considered in the article [4]). Indeed, these interactions feature an unpleasant r^{-3} singularity at the origin. Thus, when the spin-orbit or the tensor terms, which comes from relativistic corrections, are attractive the system collapses and has an infinite negative binding energy. The standard approach to solve this problem is to estimate the

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mass corrections by using the wave functions computed without the relativistic corrections. This procedure was not used in Ref. [1]. The authors prefer to treat the singularity of the spin-orbit and tensor terms by arbitrarily restricting the values of their variational parameter a. Too large values are forbidden in order to avoid the collapse. No real justification can be done for this restriction. Consequently, the physical meaning of most states presented in this article is questionable.

This last remark implies that the states which can be described without any problems by the potential (1) are 0^{++}, 2^{++} ($L = 0$, $S = 2$), 2^{++} ($L = 2$, $S = 0$) (for these three states, the spin-orbit and the diagonal tensor interactions are vanishing), 3^{++} ($L = 2$, $S = 2$) (the spin-orbit interaction is zero while the diagonal tensor term is repulsive), and 2^{-+} ($L = 1$, $S = 1$) (the repulsive spin-orbit counteracts the attractive diagonal tensor interactions).

At last, from the expression (1) for the interaction between two gluons, it is easy to remark that the maximal value of the binding energy is equal to $2m$, because the confining potential is characterized by a saturation for $r \rightarrow \infty$. Thus the maximal value for the mass of the glueball in this potential model is equal to $4m$. Since the constituent gluon mass has been found to be (or very close to) 670 MeV in Ref. [1], the heaviest glueball has a mass equal to 2680 MeV. All the states having a mass greater than 2680 MeV are then spurious.

To conclude, all the states presented by Hou and Wong are spurious or characterized by an infinite negative binding energy, except three states, the lightest 0^{++}, 2^{++}, 2^{-+}.

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