PHYSICAL REVIEW D **72,** 076001 (2005)

Soft gluon radiation and energy dependence of total hadronic cross sections

R. M. Godbole*

Center for High Energy Physics, Indian Institute of Science, Bangalore, 560 012, India

A. Grau†

Departamento de Fisica Teorica y del Cosmos, Universidad de Granada, Spain

G. Pancheri‡

INFN, Laboratori Nazionali di Frascati, P.O. Box 13, I-00044 Frascati, Italy

Y. N. Srivastava§

Physics Department and INFN, University of Perugia, Perugia, Italy

(Received 27 September 2004; published 7 October 2005)

An impact parameter representation for soft gluon radiation is applied to obtain both the initial decrease of the total cross section ($\sigma_{\rm tot}$) for proton-proton collisions as well as the later rise of $\sigma_{\rm tot}$ with energy for both pp and $p\bar{p}$. The nonperturbative soft part of the eikonal includes only limited low energy gluon emission and leads to the initial decrease in the proton-proton cross section. On the other hand, the rapid rise in the hard, perturbative jet part of the eikonal is tamed into the experimentally observed mild increase by soft gluon radiation whose maximum energy rises slowly with energy.

DOI: 10.1103/PhysRevD.72.076001 PACS numbers: 11.30.Er, 13.20.Eb, 13.20.Jf, 29.40.Gx

I. INTRODUCTION

In this paper, we extend our approach to the role played by soft gluon radiation in determining the energy dependence of total cross sections [1] by including new low energy features. The experimental information on the total cross section is now sufficient to allow for definite progress towards its description through QCD. Thanks to the recent measurements at HERA [2–5] and LEP [6,7] providing knowledge of total hadronic cross sections involving photons in the energy interval $\sqrt{s} = 1$–100 GeV, we now possess a complete set of processes to study in detail and in depth, namely, pp, $p\bar{p}$, γp, and $\gamma\gamma$. The purely hadronic processes are well measured over an extended range, up to cosmic ray energies [8], leading to quite precise parametrizations [9], while the other two allow for probes of the hadronic content of the photon versus that of the proton. They also allow for checks of the Gribov-Froissart factorization hypothesis [10,11].

The three basic features which the data exhibit and which require a theoretical explanation and understanding are (i) the normalization of the cross section, (ii) an initial decrease, and (iii) a subsequent rise with energy. All three are reasonably well described in the Regge trajectory language, which suggests a parametrization of all total cross sections [12] as a sum of powers of the square of the c.m. energy s. The oldest and simplest of these parametrizations is

$$\sigma_{\rm tot}(s) = Xs^{\epsilon} + Ys^{-\eta}. \qquad (1)$$

In this model, the initial decrease reflects the disappearance (with increasing energy) of a Regge trajectory exchanged in the t channel, with $\eta = 1 - \alpha_\rho(0)$ where the intercept (at $t = 0$) of the leading Regge trajectory is $\alpha_\rho(0) \approx 0.5$. At the same time, the rise in the cross section is attributed to the exchange of a trajectory (the Pomeron) with the quantum numbers of the vacuum, such that $\alpha_P(0) = 1 + \epsilon$. ϵ is expected to be a small number so as not to defy too much the Froissart bound, and, phenomenologically, $\epsilon = 0.08$–0.12 [13,14].

The two terms in which the cross sections are split in Eq. (1) describe well the main features of the observed total cross-section data, with an apparently constant behavior at $\sqrt{s} \approx \frac{\eta}{\epsilon}\frac{Y}{X}$. For proton-proton and proton-antiproton scattering, this occurs between 15 and 25 GeV, where the cross section is about 40 mb. At large energies, the Regge term disappears, while the first term is important everywhere and dominates asymptotically. Thus, the decrease is due to the Regge term, the rise described by the Pomeron with some taming due to the Regge term, and the normalization is determined by both the Regge and the Pomeron terms. More recent analyses add further terms such as $\log^2(s/s_0)$ terms [15].

In the following, we shall discuss these features one by one within the context of QCD, using the simple but useful parametrization of Eq. (1) as reference. We shall modify it and through it develop further the model of Ref. [1] to increase our understanding of the energy dependence within QCD. In Sec. II, we discuss the QCD origin of the

*Electronic address: rohini@cts.iisc.ernet.in
†Electronic address: igrau@ugr.es
‡Electronic address: pancheri@lnf.infn.it
§Electronic address: srivastava@pg.infn.it

two component model of Eq. (1). In Sec. III, we summarize how the Born term of the QCD contribution, the minijets, is embedded in the eikonal formalism, which provides a unitarized description of the total cross section. In Sec. IV, we discuss the analyticity requirements upon the impact parameter amplitudes of the eikonal formalism. In Sec. V, we review the soft gluon transverse momentum distribution on which our model for the b distribution of partons is based. In Sec. VI, we discuss the main features of the model [1] for the energy dependence of the impact parameter distribution, induced by soft gluon emission. In Sec. VII, through soft gluon emission, we obtain the transverse parton overlap function for the nonperturbative part of the cross section. In Sec. VIII, we introduce the normalization of the cross section. We present our model for $\sigma_{\rm tot}$ for pp and $p\bar{p}$ and compare them with currently available data. In Sec. IX, we present an interesting feature of our model: In a proton or antiproton collision, the (average) distance between the scattering centers (i.e., the constituents) in the transverse space decreases as the energy increases. Similar results have been previously obtained in an analysis of the hadronic events at the Tevatron, through an impact parameter picture. We note again that our model of the energy dependent impact parameter distribution offers a reasonable understanding of both the initial fall and later rise with energy of the $\sigma_{\rm tot}$, whereas for the normalization there is still further work to do.

Before beginning the detailed discussion, we indicate below briefly and qualitatively why we believe that it is both important and necessary to include effects of soft gluon emission in the minijet formalism [16–18]. We recall here that in the minijet picture the rise in cross sections is driven by the increasing number of low-x gluon-gluon collisions and that the predicted rate of the rise is generally found to be uncomfortably large. In Sec. VI, we give the details of the mechanism by which the effect of soft gluon emissions can reduce the rate of this rise, at any given c.m. energy. Below, we summarize the effect qualitatively.

In a calculation in the QCD improved parton model, the effect of gluon radiation on the longitudinal momentum carried by the partons is included, at least in part, in the Dokshitzer, Gribov, Lipatov, Altarelli, and Parisi [19] evolved parton distribution. Here we crucially look at the effect of initial state gluon radiation on the transverse momentum distribution of partons. In our model, the soft gluon resummed transverse momentum distribution of partons in the hadrons and the parton distribution in impact parameter space are Fourier transforms of each other. The larger the transverse momentum, the larger is the acollinearity of the two colliding partons, leading to a reduction in parton luminosity and, hence, to a reduction of the cross section. The higher the c.m. energy of the parent hadron, the more energetic are the parent valence quarks emitting gluons and the more is the acollinearity of the two partons involved in the parton subprocess. Hence, we expect the effect to be dependent on the c.m. energy.

II. THE TWO COMPONENT MODEL

Before entering into the technical details about unitarization, it is good to ask (i) where the "two component" structure of Eq. (1) comes from and (ii) why the difference in the two powers (in s) is approximately one-half.

Let us first remember that the two terms of Eq. (1) reflect the well known duality between resonance and Regge pole exchange on the one hand and background and Pomeron exchange on the other, established in the late 1960s through finite energy sum rules [20]. This correspondence meant that, while at low energy the cross section could be written as due to a background term and a sum of resonances, at higher energy it could be written as a sum of Regge trajectory exchanges and a Pomeron exchange.

Our present knowledge of QCD description of hadronic phenomena gives further insight into the nature of these two terms. We shall start answering the above two questions through considerations about the bound state nature of hadrons, which necessarily transcends perturbative QCD. For hadrons made of light quarks (q) and glue (g), the two terms arise from $q\bar{q}$ and gg excitations. For these, the energy is given by a sum of three terms: (i) the rotational energy, (ii) the Coulomb energy, and (iii) the "confining" energy. If we accept the Wilson area conjecture in QCD, (iii) reduces to the linear potential [21,22]. Explicitly, in the c.m. frame of two massless particles, either a $q\bar{q}$ or a gg pair separated by a relative distance r with relative angular momentum J, the energy is given by

$$E_i(J, r) = \frac{2J}{r} - \frac{C_i \bar{\alpha}}{r} + C_i \tau r, \tag{2}$$

where $i = 1$ refers to $q\bar{q}$, $i = 2$ refers to gg, τ is the "string tension," and the Casimir's are $C_1 = C_F = 4/3$, $C_2 = C_G = 3$. $\bar{\alpha}$ is the QCD coupling constant evaluated at some average value of r and whose value will disappear in the ratio to be considered. The hadronic rest mass for a state of angular momentum J is then computed through minimizing the above energy

$$M_i(J) = \mathrm{Min}_r \left[\frac{2J}{r} - \frac{C_i \bar{\alpha}}{r} + C_i \tau r \right], \tag{3}$$

which gives

$$M_i(J) = 2\sqrt{(C_i \tau)[2J - C_i \bar{\alpha}]}. \tag{4}$$

The result may then be inverted to obtain the two sets of linear Regge trajectories $\alpha_i(s)$

$$\alpha_i(s) = \frac{C_i \bar{\alpha}}{2} + \left(\frac{1}{8 C_i \tau} \right) s = \alpha_i(0) + \alpha_i' s. \tag{5}$$

Thus, the ratio of the intercepts is given by

$$\frac{\alpha_{gg}(0)}{\alpha_{q\bar{q}}(0)} = C_G/C_F = \frac{9}{4}. \tag{6}$$

Using our present understanding that resonances are $q\bar{q}$ bound states while the background, dual to the Pomeron, is provided by gluon-gluon exchanges [23], the above equation can be rewritten as

$$\frac{\alpha_P(0)}{\alpha_R(0)} = C_G/C_F = \frac{9}{4}. \tag{7}$$

If we restrict our attention to the leading Regge trajectory, namely, the degenerate $\rho - \omega - \phi$ trajectory, then $\alpha_R(0) = \eta \approx 0.48$–$0.5$, and we obtain for $\epsilon \approx 0.08$–0.12, a rather satisfactory value. The same argument for the slopes gives

$$\frac{\alpha'_{gg}}{\alpha'_{q\bar{q}}} = C_F/C_G = \frac{4}{9}, \tag{8}$$

so that, if we take for the Regge slope $\alpha'_R \approx 0.88$–0.90, we get $\alpha'_P \approx 0.39$–0.40, in fair agreement with lattice estimates [24].

We now have good reasons for a breakup of the amplitude into two components. To proceed further, it is necessary to realize that precisely because massless hadrons do not exist, Eq. (1) violates the Froissart bound and, thus, must be unitarized. To begin this task, let us first rewrite Eq. (1) by putting in the "correct" dimensions

$$\bar{\sigma}_{\text{tot}}(s) = \sigma_1(s/\bar{s})^\epsilon + \sigma_2(\bar{s}/s)^{1/2}, \tag{9}$$

where we have imposed the nominal value $\eta = 1/2$. In the following, we shall obtain rough estimates for the size of the parameters in Eq. (9).

A minimum occurs in $\bar{\sigma}_{\text{tot}}(s)$ at $s = \bar{s}$, for $\sigma_2 = 2\epsilon\sigma_1$. If we make this choice, then Eq. (9) has one less parameter and it reduces to

$$\bar{\sigma}_{\text{tot}}(s) = \sigma_1[(s/\bar{s})^\epsilon + 2\epsilon(\bar{s}/s)^{1/2}]. \tag{10}$$

We can isolate the rising part of the cross section by rewriting the above as

$$\bar{\sigma}_{\text{tot}}(s) = \sigma_1[1 + 2\epsilon(\bar{s}/s)^{1/2}] + \sigma_1[(s/\bar{s})^\epsilon - 1]. \tag{11}$$

Equation (11) separates cleanly the cross section into two parts: The first part is a "soft" piece which shows a saturation to a constant value (but which contains no rise), and the second is a "hard" piece which has all the rise. Moreover, $\bar{s}$ naturally provides the scale beyond which the cross sections would begin to rise. Thus, our "Born" term assumes the generic form

$$\sigma^B_{\text{tot}}(s) = \sigma_{\text{soft}}(s) + \vartheta(s - \bar{s})\sigma_{\text{hard}}(s), \tag{12}$$

with σ_{soft} containing a constant [the "old" Pomeron with $\alpha_P(0) = 1$] plus a (Regge) term decreasing as $1/\sqrt{s}$ and with an estimate for their relative magnitudes ($\sigma_2/\sigma_1 \sim 2\epsilon$). We shall assume that the rising part of the cross section σ_{hard} is provided by jets which are calculable by perturbative QCD, obviating (at least in principle) the need of an arbitrary parameter ϵ.

An estimate of σ_1 may also be obtained through the hadronic string picture. Equation (3) gives us the mean distance between quarks or the "size" of a hadronic excitation of angular momentum J in terms of the string tension

$$\bar{r}(J)^2 = \frac{2J - C_F\bar{\alpha}}{\tau}. \tag{13}$$

So the size R_1 of the lowest hadron [which in this Regge string picture has $J = 1$, since $\alpha_R(0) = 1/2$] is given by

$$R_1^2 = \frac{1}{\tau} = 8\alpha'. \tag{14}$$

If two hadrons each of size R_1 collide, their effective radius for scattering would be given by

$$R_{\text{eff}} = \sqrt{R_1^2 + R_1^2} = \sqrt{2}R_1, \tag{15}$$

and the constant cross section may be estimated (semiclassically) to be roughly

$$\sigma_1 = 2\pi R^2_{\text{eff}} = 4\pi R_1^2 \approx \frac{4\pi}{\tau} = 32\pi\alpha', \tag{16}$$

which is about 40 mb, a reasonable value. In the later sections, for the soft cross section we shall take a value of this order of magnitude as the nominal value.

The unitarization now proceeds very simply by eikonal exponentiation in impact parameter space, as described in detail in the sections to follow.

III. THE RISE WITH ENERGY

QCD offers an elegant explanation of the rise in the minijet formalism, as has been pointed out by several authors in the past [16–18]. The original suggestion was that the rise of σ_{tot} with energy is driven by the rapid rise with energy of the inclusive jet cross section

$$\sigma^{ab}_{\text{jet}}(s) = \int_{p_{t\min}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^1 dx_1 \int_{4p_t^2/(x_1 s)}^1 dx_2 \times \sum_{i,j,k,l} f_{i|a}(x_1) f_{j|b}(x_2) \frac{d\hat{\sigma}_{ij\to kl}(\hat{s})}{dp_t}, \tag{17}$$

where subscripts a and b denote particles $(\gamma, p, \ldots)$, i, j, k, l are partons, and x_1, x_2 the fractions of the parent particle momentum carried by the parton. $\hat{s} = x_1 x_2 s$ and $\hat{\sigma}$ are hard partonic scattering cross sections. Note that $d\hat{\sigma}/dp_t \propto p_t^{-3}$; the cross section defined in Eq. (17) therefore depends very sensitively on $p_{t\min}$, which is supposed to parametrize the transition from perturbative to nonperturbative QCD. The rise of the inclusive jet cross section with energy is understood in terms of the increasing number of hard partons, which gives rise to an increasing probability for the occurrence of hard scattering processes.

Quantitatively, factorization of QCD allows us to use the currently available parametrizations of the scale dependent parton densities and calculate the energy dependence of the resulting jet cross section, by convoluting the parton densities with the subprocess cross section determined by perturbative QCD.

For any fixed $p_{t\,\min}$, typically 1–2 GeV, one finds that this cross section is a steeply rising function of energy. If $\sqrt{s} \gg p_{t\,\min}$, the integral in Eq. (17) receives its dominant contribution from $x_{1,2} \ll 1$. The relevant parton densities can then be approximated by a simple power law, $f \propto x^{-J}$. In case of pp or $\bar{p}p$ scattering, $a = b$ and the cross section asymptotically scales like [25]

$$\sigma_{\rm jet} \propto \frac{1}{p_{t\,\min}^2}\left(\frac{s}{4p_{t\,\min}^2}\right)^{J-1} \log\frac{s}{4p_{t\,\min}^2}, \tag{18}$$

if $J > 1$. For $J \simeq 1.3$, as measured by HERA, the jet cross section will therefore grow much faster than the total $\bar{p}p$ cross section, which grows only $\propto \log^2 s$ (Froissart bound [26]), or, phenomenologically [12] for $\sqrt{s} \leq 2$ TeV, $\propto s^{0.08}$. Eventually, the jet cross section (17) will therefore exceed the total $\bar{p}p$ cross section. In fact, this rise is far more violent than the experimentally observed gentle rise of the total cross section [18,27]. This has led to various phenomenological strategies (among them the eikonal formalism [28]) directed towards softening this rise. The apparent paradox is solved by the observation that, by definition, inclusive cross sections include a multiplicity factor. Since a hard partonic scattering always produces a pair of (mini)jets, we can write

$$\sigma_{\rm jet}^{ab} = \langle n_{\rm jet\ pair}\rangle \sigma_{\rm inel}^{ab}, \tag{19}$$

where $\langle n_{\rm jet\ pair}\rangle$ is the average number of (mini)jet pairs per inelastic collision. $\sigma_{ab}^{\rm jet} > \sigma_{\rm inel}^{ab}$ then implies $\langle n_{\rm jet\ pair}\rangle > 1$, which means that, on average, each inelastic event contains more than one hard partonic scatter. The simplest possible assumption about these multiple partonic interactions is that they occur completely independently of each other, in which case $n_{\rm jet\ pair}$ obeys a Poisson distribution. At a slightly higher level of sophistication, one assumes these interactions to be independent only at fixed impact parameter b; indeed, it seems natural to assume that events with small b usually have larger $n_{\rm jet\ pair}$. This leads to the eikonal formalism mentioned above.

Convenient and elegant, the eikonal method reduces the rise of this cross section and allows one to enforce the requirement of s-channel unitarity. Here one obtains the total cross section through the eikonal formula

$$\sigma_{\rm tot} = 2\int d^2\vec{b}[1 - e^{-{\rm Im}\chi(b,s)} \cos {\rm Re}\chi(b,s)], \tag{20}$$

and one introduces the jet cross section as the term which drives the rise in the eikonal function. This can be done unambiguously by defining the inelastic cross section [29] given in the eikonal formulation by

$$\sigma_{\rm inel} = \int d^2\vec{b}[1 - e^{-2\,{\rm Im}\chi(b,s)}]. \tag{21}$$

This expression can also be obtained upon summing multiple collisions which are Poisson distributed with an average number $n(b,s) = 2\,{\rm Im}\chi(b,s)$. Making the approximation ${\rm Re}\chi = 0$, one obtains a very simple expression

$$\sigma_{\rm tot} = 2\int d^2\vec{b}[1 - e^{-n(b,s)/2}]. \tag{22}$$

To proceed further, one needs to introduce the soft processes, which cannot be described by perturbative QCD. Following the separation shown in Sec. II, one can approximate $n(b,s)$ by introducing a separation between soft and hard processes as:

$$n(b,s) = n_{\rm soft} + n_{\rm hard} = A_{\rm soft}(b)\sigma_{\rm soft}(s) + A_{\rm jet}(b)\sigma_{\rm jet}(s). \tag{23}$$

The separation between hard and soft processes is, of course, approximate and so is the factorization into energy and transverse dimension dependence. However, it is useful as it allows one to break down the calculation into building blocks, which can be separately understood and put together again later as part of the whole structure.

The procedure of Eq. (20) reduces the fast rise due to the minijet cross section, but the extent to which this softens the rise is highly dependent on the impact parameter (b) dependence of $n(b,s)$. The simplest [28] ansatz which introduces a minimum number of parameters is to assume that the b dependence is the same for both the soft and the jet component and, further, that it is given by the Fourier transform of the electromagnetic form factor of the colliding hadron, $\mathcal{F}(q)$ [28]. Thus, for protons and antiprotons one will have

$$A_{\rm soft}(b) = A_{\rm jet}(b) = \frac{1}{(2\pi)^2}\int d^2\vec{b} e^{iq\cdot b}[\mathcal{F}_p(q)]^2 = \frac{1}{(2\pi)^2}\int d^2\vec{b} e^{iq\cdot b}\left[\frac{\nu^2}{q^2+\nu^2}\right]^4. \tag{24}$$

It is well known [30] that these eikonal minijet models (EMM) are unable to properly reproduce the experimentally observed, complete rise of the cross section from the beginning up to the asymptotia, without introducing further parameters. As an example, results obtained using Eq. (24) with $\nu = 0.71$ GeV^2 and current Glück, Reya, and Vogt parton densities for the proton [31] to calculate the jet cross sections in Eq. (17) are shown in Fig. 1. One can see from the figure that a $p_{t\,\min} \approx 2$ GeV is needed, in order to obtain a numerical value of the total proton cross sections in the 80 mb range, at Tevatron energies. However, for such $p_{t\,\min} = 2$ GeV, the rise does not begin until $\sqrt{s}$ is already in the 100 GeV region. On the other hand, a smaller value

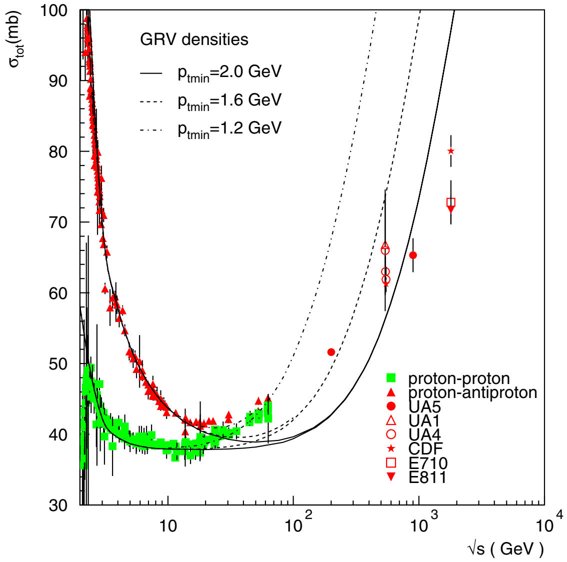

FIG. 1 (color online). Comparison between data [8,32–37] and the EMM (see text) for different minimum jet transverse momentum.

for the regulator $p_{t\,\mathrm{min}}$, typically just above 1 GeV, would allow for the beginning of the rise around 20–30 GeV, as the data indicate, but then the cross section rises too rapidly in comparison with the Tevatron data. In Fig. 1, in all jet cross sections computed using Eq. (17), we use the strong coupling constant α_s at scale p_t. Even at $p_t = p_{t\,\mathrm{min}}$, this value for the scale, albeit low, is still in a range where the asymptotic freedom expression is approximately valid. For the low energy behavior, all the curves shown in Fig. 1 are obtained with the same phenomenological fit as in Ref. [1].

Different solutions to the above problems have been adopted, including energy dependence of the cutoff parameter $p_{t\,\mathrm{min}}$, adding more terms in Eq. (23), or using Eq. (24) [28,30] with different constants for the low and the high energy part.

We have taken the approach to keep fixed $p_{t\,\mathrm{min}}$ (for a given beam and target combination) and have an energy dependent overlap function, the energy dependence being modeled by a QCD motivated calculation. The QCD motivated model gives a transverse momentum distribution of the partons which is energy dependent, and this, in turn, makes our overlap functions energy dependent. We shall see in the next sections that in this model soft gluon emission has an effect very similar to that of an energy dependent $p_{t\,\mathrm{min}}$.

IV. ANALYTICITY REQUIREMENTS ON THE IMPACT PARAMETER DISTRIBUTION

Within the context of QCD, the problem described in the previous section, sometimes referred to as the soft and hard Pomeron problem, can have (at least) two different origins. If complete factorization between the dependence on the energy $\sqrt{s}$ and the impact parameter $\vec{b}$ holds, then the above mentioned soft and hard Pomeron problem may simply be taken to be indicative that the s dependence generated through gluon densities is wrong. On the other hand, given the fact that gluon densities are measured in deep inelastic scattering experiments, one may think of an alternative explanation of the problem. This second, complementary explanation is that not all the s dependence is due to the jet cross sections and there is further energy dependence in the impact parameter distribution. Some general analyticity arguments can be invoked to shed light on the above and, thus, limit the arbitrariness in the choice of the function $\chi(b, s)$.

Consider the elastic scattering amplitude $T_{\mathrm{el}}(s, t)$ (with s and t the usual Mandelstam variables) normalized so that the total cross section is given by

$$\sigma_{\mathrm{tot}} = 2\int d^2\vec{b}[1 - e^{-\mathrm{Im}\chi(b,s)} \cos \mathrm{Re}\chi(b, s)] = \left(\frac{2}{s}\right)\mathrm{Im}T_{\mathrm{el}}(s, t = 0), \tag{25}$$

and the elastic differential cross section by

$$\frac{d^2\sigma_{\mathrm{el}}}{d^2\vec{q}} = \frac{1}{4\pi^2 s^2}|T_{\mathrm{el}}|^2. \tag{26}$$

Consistently with Eq. (25), one then has

$$T_{\mathrm{el}}(s, t) = is\int d^2\vec{b}e^{i\vec{q}\cdot\vec{b}}[1 - e^{i\chi(b,s)}]. \tag{27}$$

For complete absorption, i.e., $\mathrm{Re}\chi = 0$, we get

$$T_{\mathrm{el}}(s, t) = is\int d^2\vec{b}e^{i\vec{q}\cdot\vec{b}}[1 - e^{-n(b,s)/2}]. \tag{28}$$

Let us briefly examine restrictions imposed on the large b behavior of the function $n(b, s)$ by the requirements of analyticity. Consider the Fourier transform of the elastic scattering amplitude,

$$\mathcal{A}(s, b) = \frac{-i}{4\pi s}\int_{-\infty}^{0} dt T_{\mathrm{el}}(s, t)J_0(b\sqrt{-t}), \tag{29}$$

where $t = -\vec{q}^2$, with $\vec{q}$ the transverse momentum variable. Equivalently, we have

$$T_{\mathrm{el}}(s, t) = \pi is\int_0^\infty db^2 J_0(b\sqrt{-t})\mathcal{A}(s, b). \tag{30}$$

The finite range of hadronic interactions implies that the partial wave expansion converges beyond the physical region, i.e., throughout the Lehmann ellipse. This requires that $T_{\mathrm{el}}(s, t)$ be analytic in t up to $t = 4\mu^2$, where μ is the pion mass. For positive t, we continue the above expression for $\sqrt{-t} = iW$, with W real and positive. In this "unphysical" region, we have

$$T_{\mathrm{el}}(s, W^2) = \pi is\int db^2 I_0(bW)\mathcal{A}(s, b^2). \tag{31}$$

For large b, $I_0(bW) \sim e^{bW}/\sqrt{2\pi}bW$, so that for the integral to converge, one needs

$$|\mathcal{A}(s, b^2)| < e^{-bW_o} \quad \text{with } W_o \simeq 2\mu. \tag{32}$$

In the purely absorptive model ($\mathrm{Re}\chi = 0$), thus,

$$1 - e^{-n(b,s)/2} < e^{-bW_o/2}, \tag{33}$$

and we see that $n(b, s)$ must be bounded at least by an exponential. Stronger (but model dependent) constraints arise provided one imposes that the elastic differential cross section exhibit a "diffraction peak." That is,

$$T_{\mathrm{el}}(s, t) \simeq f(s)e^{\hat{b}(s)t}, \tag{34}$$

where $\hat{b}(s)$ is the so-called width of the diffraction peak, which has an observed (approximately) logarithmic s dependence. Then Eq. (29) gives

$$\begin{aligned}\hat{\mathcal{A}}(s, b) &\simeq \frac{if(s)}{4\pi s}\int dq^2 J_0(bq)e^{-\hat{b}(s)q^2}\\ &= \frac{if(s)}{2\pi s}\left[\frac{1}{2\hat{b}(s)}\right]e^{-[b^2/4\hat{b}(s)]},\end{aligned} \tag{35}$$

which requires a Gaussian falloff of the amplitude in the impact parameter b, with its scale determined by the width of the diffraction peak. In the Regge pole description,

$$\hat{b}(s) \sim \alpha' \ln(s/s_0), \qquad f(s) \sim -i\beta(s/s_o)^{1+\epsilon} \tag{36}$$

and

$$\mathcal{A}\,(s, b) \simeq \frac{\beta(s/s_o)^{\epsilon}}{4\pi(\alpha' s_o)\ln(s/s_0)}\, e^{-[b^2/4\alpha' \ln(s/s_0)]}. \tag{37}$$

In the EMM, where the impact parameter distribution is given by the Fourier transform of the proton form factor, a model we refer to as the form factor (FF) model, one has

$$n(b, s) = \frac{\nu^2}{96\pi}(\nu b)^3 K_3(\nu b)[\sigma_{\mathrm{soft}} + \sigma_{\mathrm{jet}}]. \tag{38}$$

The modified Bessel functions of the third kind $K_\mu(z)$ are bounded by an exponential at large values of the argument, i.e., $K_\mu(z) \sim \sqrt{\pi/2z}e^{-z}\{1 + \mathcal{O}(\frac{1}{z})\}$. We see then that the EMM in the FF formulation does satisfy the requirements of analyticity in the Lehmann ellipse. But in the EMM, the observed shrinking of the diffraction peak, corresponding to an energy dependent near Gaussian falloff, is not present. Even if one were to introduce an *ad hoc* energy dependence (as is often the practice) instead of the constant scale parameter ν, as in the FF model, still one would be nowhere near the stronger Gaussian decrease at large impact parameter values. This is one reason why the FF model in the eikonal formulation, where jet cross sections drive the rise, fails to provide an adequate description of the overall energy dependence of the total cross section, without introducing an *ad hoc* modification of the scale parameters.

V. THE SOFT GLUON TRANSVERSE MOMENTUM DISTRIBUTION

The model for the total cross section presented in this paper is based on the ansatz that QCD provides the main processes at work leading to the observed energy rise of all measured total cross sections. An important motivation of this model is to make quantitative calculations based on current QCD phenomenology, namely, current parton densities with their energy momentum dependence, running behavior of the coupling constant, and soft gluon resummation techniques. Of all these, at present soft gluon resummation is the one which presents the toughest technical challenge.

Let us begin by considering the well known function which describes soft gluon emission from a parton-parton pair, namely, the soft gluon transverse momentum distribution [38,39]

$$\frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} \equiv \Pi(K_\perp) = \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\mathbf{K}_\perp\cdot\mathbf{b} - h(b)}, \tag{39}$$

with

$$\begin{aligned}h(b) &= \int d^3\bar{n}_g(k)[1 - e^{-i\mathbf{k}_\perp\cdot\mathbf{b}}]\\ &= \int \frac{d^3k}{2k_0}\sum_{i,j=\mathrm{colors}} |j^{\mu,i}(k)j_{\mu,j}(k)|[1 - e^{-i\mathbf{k}_\perp\cdot\mathbf{b}}],\end{aligned} \tag{40}$$

where $d^3\bar{n}_g(k)$ is the distribution for single gluon emission in a scattering process, and $j^{\mu,i}$ is the QCD current responsible for emission. The above expression has been widely used to study the initial state transverse momentum distribution in Drell-Yan processes [40,41] as well as W production [42,43].

In the limit of large $k_\perp b$, one can neglect the exponential term, and the above expression reduces to the well known Sudakov form factor [44], namely,

$$h(b) \approx S(b) = \int d^3\bar{n}_g(k) = \int \frac{d^3k}{2k_0}\sum_{\mathrm{colors}} |j^{\mu,i}(k)j_{\mu,j}(k)|. \tag{41}$$

Introducing the running coupling constant and its asymptotic freedom expression, the integration from $1/b$ up to an upper limit Q gives [45]

$$h(b) \approx \frac{4C_F}{(11 - 2n_f/3)} \ln\frac{Q^2}{\Lambda^2} \ln\frac{\ln(Q^2/\Lambda^2)}{\ln(1/b^2\Lambda^2)}. \tag{42}$$

For small momenta, however, the above expression is not sufficient to reproduce the observed transverse momentum distribution in various hadronic processes. Thus, an intrinsic transverse momentum (of Drell-Yan pairs or W boson or partons, depending on the physical process under consideration) has to be introduced. The function now reads [46]

$$h(b) = b^2 p_{\perp\mathrm{int}}^2 + S(b), \tag{43}$$

where the intrinsic transverse momentum $p_{\perp\mathrm{int}}$ is a constant, of the order of a few hundred MeV, parametrized according to the process under consideration.

In our model, the function in Eq. (40), in addition to describing various hadronic transverse momentum effects, also plays a major role in total cross-section calculations. Our model for the hadronic transverse momentum distributions due to soft gluon resummation differs in two major points from what we have just described. Basically, we focus on the lower and higher limits of integration in Eq. (40). At the lower limit, since this expression refers to soft gluons, we suggest that the correct use of this equation requires one to integrate the gluon momentum down to $k_\perp = 0$ and avoid the introduction of *ad hoc* quantities such as the intrinsic transverse momentum. At the upper limit, one needs to specify, for each given process, how the maximum soft gluon energy is defined. However, in order to use the above expression for a believable calculation, a number of points need to be clarified, namely,

(i) whether it makes sense to use a parton picture when the gluon momenta become close to zero, with the related question of what the behavior is of the strong coupling constant α_s when one integrates the gluon momentum down to zero;
(ii) whether the emitting particles are quarks or gluons;
(iii) what the constraints are from kinematics upon the maximum gluon momentum.

We shall discuss some of these issues in detail in the sections to come; here we comment briefly on these three points.

Concerning the parton picture, while we use it for the minijet contribution to the cross section, soft gluon Bloch-Nordsieck resummation factors out of the LO basic scattering process and is thus independent of parton densities, involving only the momenta and the QCD coupling between soft gluons and the emitting partons. On the other hand, when $k_\perp \to 0$, this coupling is not an observable quantity, since it refers to a single soft gluon emission, and one soft gluon is not an observable quantity (only its integrated spectrum is). This has two effects: first, one needs to use a nonperturbative expression for the QCD coupling constant, since the momenta are so small, and, second, only the *integral* of moments of α_s will matter. The infrared behavior of α_s is a matter of speculation. We propose our own model, whose justification rests upon a Regge description and on the Richardson type potential for quarkonium. As will be described in the next sections, a specific form for α_s is chosen, singular in the infrared limit but integrable [1].

Another issue to address relates to the effect of emission from quarks, valence and sea, and from gluons in the parton processes. In this paper, we deal only with emission from the initial valence quarks and use the relevant kinematics with their averages. This approximation is justified by the fact that, relative to the emission from the initial valence quarks, emission from the gluon legs is to be considered as emission from internal legs, thus subleading in infrared terms. A complete calculation should, of course, include also emission from partons other than the valence quarks and, hence, mostly the low-x gluons. We expect this inclusion may eventually increase the softening effect.

To describe the softening effect quantitatively, one needs to focus on the maximum transverse momentum allowed to single gluon emission. This quantity is energy dependent, as one can easily see using the kinematics of single gluon emission in parton-parton scattering of initial c.m. energy $\sqrt{\hat{s}}$. For the process

$$p_i + p_j \to \text{gluon} + X, \tag{44}$$

where $\hat{s} = (p_i + p_j)^2$ and X is a final state of given momentum Q, the maximum transverse momentum of the gluon is given by

$$q_{\max}(\hat{s}) = \frac{\sqrt{\hat{s}}}{2}\left(1 - \frac{Q^2}{\hat{s}}\right). \tag{45}$$

If we consider the state X to be the final (mini)jet-jet system in the inelastic collision contributing to the cross section, then the above expression depends on the parton subenergies and on the final state momentum of the jet-jet system, characterized by a transverse momentum $p_t \geq p_{t\min}$, where $p_{t\min}$ is a scale chosen to separate hard and soft processes. In principle, for each subprocess of given $\hat{s}$ and Q^2, one should evaluate the function $h(b)$ with the above $q_{\max}$. In practice, we use a value for the maximum transverse momentum allowed to single gluons, which is averaged over the initial and final parton momenta. This is shown more explicitly in Ref. [1] and in Sec. VI, but the result is that, for a given $p_{t\min}$ cutoff in the minijet distribution, for the valence quarks, the scale $q_{\max}$ increases with the c.m. energy of the hadron-hadron system. This can be qualitatively understood by considering that the valence quarks will, on the average, carry a larger energy and can then shed more soft gluons. Thus, in the picture we present for hadron-hadron collisions, as the overall energy increases, we have more parton-parton collisions for the same $p_{t\min}$ (since the number of low x gluons increases) but also more energy available to soft gluons both from initial valence quarks and from all the hard partons in general, and, thus, more of a reduction. It is the balancing of these two effects which we believe to be responsible for the observed softer rise of total cross sections.

VI. SOFT GLUON EMISSION AND ENERGY DEPENDENCE IN THE IMPACT PARAMETER DISTRIBUTION

In our previous work, we have advocated that a cure for the difficulty in obtaining the early dramatic rise and the softer asymptotic behavior *simultaneously* lies within QCD itself; viz., the ubiquitous soft gluon emission accompanying all QCD scatterings which can slow down any abrupt rise in the cross section. To make this quantitative, we put forward a model for the impact parameter distribution of partons in the hadrons, based on the Fourier transform of the transverse momentum distribution of the soft gluons emitted in the collisions, as described in the Block-Nordsieck (BN) summation procedure. This distribution is energy dependent simply because the maximum energy allowed to each single emitted soft gluon, in turn, depends on the energy of the colliding partons. In detail, we have a picture of parton-parton collisions at all admissible subenergy values and with a given transverse momentum due to initial state radiation. In our model the soft gluon resummed transverse momentum distribution of partons in the hadrons and the parton distribution in impact parameter space are Fourier transforms of each other. In principle, this formalism could be used to obtain impact parameter dependent parton densities, but our aim in the present paper is to obtain a prediction for the total cross section based on currently used QCD functions and parameters such as parton densities and $\Lambda_{\rm QCD}$. We thus follow our previous proposal [1,47] to average out the behavior of partons in their transverse momentum variable and arrive at the following expression:

$$\begin{aligned} 2\,{\rm Im}\chi(b,s) &= n(b,s;q_{\max},p_{t\min}) = n_{\rm soft}+n_{\rm jet}\\ &= A_{\rm soft}(b)\sigma_{\rm soft}+A_{\rm BN}(b,q_{\max})\sigma_{\rm jet}. \end{aligned} \tag{46}$$

As mentioned earlier, the eikonal formulation provides a natural framework, in which different contributions to the total cross section can be resolved into their various structural elements: The rise is incorporated in $n_{\rm jet}$, and the decrease and normalization in $n_{\rm soft}$. In our previous work, we had parametrized phenomenologically the soft part and used perturbative QCD for the jet part. In this paper, we study whether soft gluon summation can describe the (experimentally observed) initial decrease in proton-proton scattering.

For this purpose, we write

$$n(b,s) = A_{\rm BN}^{\rm soft}\sigma_{\rm soft} + A_{\rm BN}^{\rm jet}\sigma_{\rm jet}, \tag{47}$$

with

$$A_{\rm BN} = \frac{e^{-h(b,s)}}{\int d^2\vec{b}\,e^{-h(b,s)}}, \tag{48}$$

where from Eq. (40) we have

$$\begin{aligned} h(b,s) = \frac{8}{3\pi}\int_0^{q_{\max}}\frac{dk}{k}\alpha_s(k^2)\ln\left(\frac{q_{\max}+\sqrt{q_{\max}^2-k^2}}{q_{\max}-\sqrt{q_{\max}^2-k^2}}\right)\\ \times[1-J_0(kb)], \end{aligned} \tag{49}$$

and $q_{\max}$ depends on energy and the kinematics of the process [41]. From Eq. (45), the following average expression for $q_{\max}$ was proposed in our previous paper [1]:

$$M \equiv \langle q_{\max}(s)\rangle = \frac{\sqrt{s}}{2}\frac{\sum_{i,j}\int\frac{dx_1}{x_1}f_{i/a}(x_1)\int\frac{dx_2}{x_2}f_{j/b}(x_2)\sqrt{x_1x_2}\int_{z_{\min}}^1 dz(1-z)}{\sum_{i,j}\int\frac{dx_1}{x_1}f_{i/a}(x_1)\int\frac{dx_2}{x_2}f_{j/b}(x_2)\int_{z_{\min}}^1(dz)}, \tag{50}$$

with $z_{\min} = 4p_{t\min}^2/(sx_1x_2)$ and $f_{i/a}$ the valence quark densities used in the jet cross-section calculation.

M establishes the scale which, on the average, regulates soft gluon emission in the collisions, whereas $p_{t\min}$ provides the scale which characterizes the onset of hard parton-parton scattering. For any parton-parton subprocess characterized by a $p_{t\min} \approx 1$–2 GeV, M has a logarithmic increase at reasonably low energy and an almost constant behavior at high energy [1]. The eikonal formalism which we use to describe the total cross section incorporates multiple parton-parton collisions, accompanied by soft gluon emission from the initial valence quarks, to leading order. Notice that, in this model, we consider emissions only from the external quark legs. In the impulse approximation on which the parton model itself is based, the valence quarks are free, external particles. In this picture, emission of soft gluons from the gluons involved in the hard scattering is nonleading. As the energy increases, more and more hard gluons are emitted, but there is also a larger and larger probability of soft gluon emission: The overall effect is a rise of the cross section, tempered by the soft emission; i.e., the violent minijet rise due to semihard gluon-gluon collisions is tamed by soft gluons. Crucial in this model are the scale and the behavior of the strong coupling constant which is present in the integral over the soft gluon spectrum. While in the jet cross section, α_s never plunges into the infrared region, as the scattering partons are by construction semihard, in the soft gluon spectrum the opposite is true and a regularization is mandatory. We notice, however, that here, as in other problems of soft hadron physics [48], what matters most is not the value of $\alpha_s(0)$ but rather its integral. Thus, all that we need to demand is that α_s be integrable, even if singular [49]. We employ the same phenomenological expression for α_s as used in our previous works, namely,

$$\alpha_s(k_\perp) = \frac{12\pi}{(33-2N_f)}\frac{p}{\ln[1+p(\frac{k_\perp}{\Lambda})^{2p}]}. \tag{51}$$

Through the above, we were able to reproduce the effect of the phenomenologically introduced intrinsic transverse momentum of hadrons [49] and, more recently, obtained a very good description of the entire region where the total cross section rises [1]. This expression for α_s coincides with the usual one-loop expression for large values of $k_\perp$, while going to a singular limit for small $k_\perp$. For $p = 1$ this expression corresponds to the Richardson potential [50] used in bound state problems. We see from Eq. (49) that $p = 1$ leads to a divergent integral and, thus, cannot be used. Notice that, presently, in the expression for $h(b, s)$, the masses of the emitting particles are put to zero as is usual in perturbative QCD. Thus, for a convergent integral, one requires $p < 1$, and the successful phenomenology indicated in Ref. [1] gave $p = 3/4$. However, more study is needed, especially in the full utilization of the Bloch-Nordsieck description, before one can completely define an expression for α_s in the infrared limit. A different possibility is to use a so-called frozen α_s model, for which $\alpha_s(k_\perp^2) = 12\pi/27 \ln[(k_\perp^2 + a^2\Lambda^2)/\Lambda^2)]$. These two expressions lead to very different large-b behavior of the function $n(b, s)$ and, in light of the above discussion concerning the shrinking of the diffraction peak, give quite a different s dependence in the rising region of the total proton cross section.

In Ref. [1], we presented analytic approximations for the function $h(b, s)$, obtaining in the frozen α_s case

$$\lim_{b\to\infty} h(b, M, \Lambda) = \frac{2c_F\bar{\alpha}_s}{\pi}\left[\frac{1}{4}\ln(2Mb) + 2\ln(Mb) \times \ln(a\Lambda b) - \ln^2(a\Lambda b)\right], \tag{52}$$

while for the singular case

$$\lim_{b\to\infty} h(b, M, \Lambda) = \frac{2c_F\bar{b}}{\pi}(b^2\Lambda^2)^p\left[\frac{1}{8(1-p)}\left(2\ln(2Mb) + \frac{1}{1-p}\right) + \frac{1}{2p}\left(2\ln(Mb) - \frac{1}{p}\right)\right],$$

where $M \equiv q_{\max}$, $\bar{\alpha}_s = 12\pi/27 \ln a^2$, $\bar{b} = 12\pi/(33 - 2N_f)$. From the above, one can see that an approximately Gaussian limit results in the singular α_s case but not in the frozen case. Indeed, for the singular case, one has, to the lowest order, $\lim_{b\to\infty} n(b, s) \sim e^{-b^{2p}}$, i.e., an exact Gaussian limit for the Richardson potential, which corresponds to $p = 1$. This provides the theoretical reason why the entire region where the total cross section rises is well described by perturbative QCD (jet cross section) combined with Bloch-Nordsieck summation with a singular α_s in the infrared region. In contrast, neither the frozen α_s model nor the FF model are successful there.

Other models for the behavior of α_s in the infrared region and studies of the range of variability of the parameters used in Eq. (51) will be presented in a forthcoming publication.

VII. THE DECREASE PRIOR TO THE ONSET OF MINIJETS

Let us now address the question of the lower energy range, prior to the rise, using the same phenomenological and theoretical tools of Ref. [1] but abandoning the FF and the frozen α_s models, which appear to be inadequate. To study the low energy region, we apply our procedure to proton-proton scattering, where the absence of resonances in the s channel and leading Regge poles in the t channel make the picture remarkably simple. At low energies, say, before 10 GeV in the proton-proton c.m. system, one observes a very soft decrease, which converts in a rise at an energy of ≈ 15 GeV in the c.m. In this low energy region, we know that gluon densities are still very small and that (almost) all hard parton-parton scattering takes place among valence quarks: $\sigma_{\rm jet}$, as defined through Eq. (17), is a few thousandths of the observed $\sigma_{\rm tot}$. In the breakup of Imχ into a soft and a hard part, the parameter $p_{t\min}$ separates hard and soft processes, namely, for $p_t^{\rm parton} \ge p_{t\min}$, one counts parton-parton processes as part of the jet cross section, whereas for $p_t^{\rm parton} \le p_{t\min}$ the process can be counted as part of $\sigma_{\rm soft}$. Thus, in this region, we can study the contribution of valence quark scattering without complications from inelastic gluon-gluon collisions.

This region then exhibits the effect of soft gluon emission accompanying gluon exchanges among the valence quarks. At higher energies, these soft interactions still take place and are a substantial part of the cross section, but they will be shielded by the more dramatic behavior of the perturbative QCD processes, since as the energy increases, smaller and smaller x values of the gluon densities are probed and gluon exchanges among gluons start becoming important. Thus, we must build a piece of the total cross section which will survive at high energies but which does not contribute to the rise. To begin with, we start with a very simple ansatz: that for proton-proton the cross section $\sigma_{\rm soft}$ is a constant and the slight decrease comes from the straggling, acollinearity effect of soft gluon emission. We therefore propose, in the first instance, the following expression for the average number of soft collisions:

$$n_{\rm soft}(b, s) = A_{\rm BN}^{\rm soft}\sigma_0, \tag{53}$$

with $A_{\rm BN}^{\rm soft}$ calculated through Eqs. (48) and (49), and investigate whether it is possible to find a constant σ_0 and a set of parameters ($q_{\max}$) which can describe pp scattering at low energy. For the soft part, the scale $q_{\max}$ corresponds to the maximum energy allowed to soft gluons accompanying scattering with a final parton transverse momentum smaller than $p_{t\min}$. We are dealing with soft emission (for hard gluons $p_t > p_{t\min}$), and, thus, we expect $q_{\max}$ not to be larger than 10%–20% of $p_{t\min}$. This provides an upper bound for $q_{\max}$ for soft processes.

The observation is then that, for processes contributing to $n_{\rm soft}$, a soft gluon will always carry away less energy than for those contributing to $n_{\rm hard}$. The question is how much lower the allowed energy is. We have proceeded phenomenologically and found a set of values which, as will be shown in the last section, can give an acceptable

TABLE I. Average q_{max} values used for the impact parameter distribution of the soft part of the eikonal.

$\sqrt{s}$ (GeV)	q_{max}^{soft} (GeV)
5	0.19
6	0.21
7	0.22
8	0.23
9	0.235
10	0.24
50	0.24
100	0.24

description for σ_{pp} before the rise. These values are shown in Table I. Notice that, in order to reduce the number of free parameters, we assume that, at low energies, there is only one value of q_{max}, for both hard and soft processes. However, as the energy increases, the scale characterizing the soft processes does not grow as the one for the hard case; the latter is obtained through the kinematics of jet production in a hard parton-parton scattering. Here we should expect the scale to start as a slowly increasing function of s but to become a constant as soon as hard processes become substantial for $\sqrt{s} \geq 10$ GeV. This is necessary; i.e., q_{max} does not increase indefinitely, because as the energy available to soft gluons increases, at a certain point the soft gluons will become hard and then undergo scattering among themselves.

Clearly, a soft scale not larger than 240 MeV is consistent with our understanding of how a proton is structured, if we attribute σ_{soft}, the soft component, as the cross section when the scattering protons (and antiprotons) manifest themselves as a quark and a spin zero diquark. The point

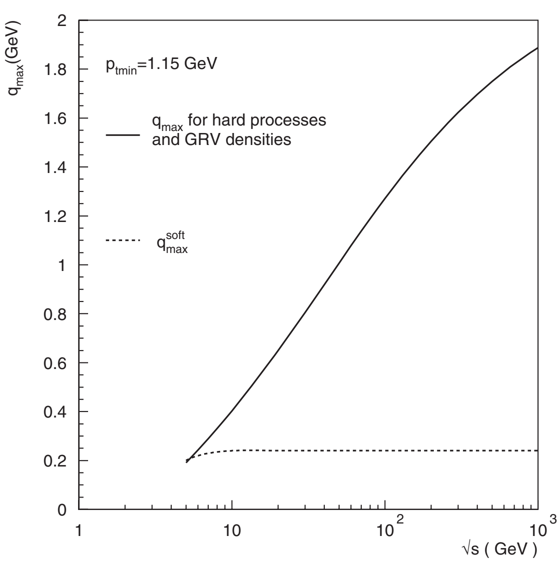

FIG. 2. The maximum transverse momentum allowed (on the average) for single soft gluon emission as a function of the c.m. energy of scattering hadrons.

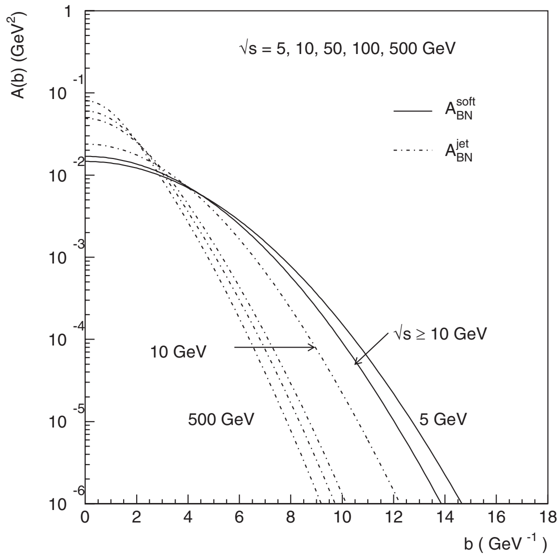

FIG. 3. The impact parameter distribution function calculated for the soft gluon summation model, using the q_{max} values described in the text, for various energy values. Solid lines are for the soft term.

is that the slopes for meson and baryon Regge trajectories are justifiably equal only if a baryon is pictured as a quark/diquark system similar to a meson as a quark/antiquark system (thus making the string tensions equal). This mode for the nucleon is soft and diffusely spread over about a Fermi [51]. Consequently, the soft gluon radiation distribution must be limited (lest it break the system). Thus, we estimate for the soft process $q_{max} \approx 1/(1 \text{ Fermi}) = 0.2$ GeV.

With the value of $p_{t\,min}$ which gave a smooth description of the total cross section in Ref. [1], we plot in Fig. 2 the behavior of q_{max} as a function of energy, where the upper curve is the one obtained using Eq. (50), for $p_{t\,min} = 1.15$ GeV, whereas the soft q_{max} starts with the same value obtained for the jet term and then is made to become a constant when it reaches 240 MeV.

With these values for q_{max} we can now calculate A_{BN} for both the hard and the soft terms in the eikonal as a function of the impact parameter b. Both soft and hard A_{BN} are shown in Fig. 3 for a set of representative c.m. energies, $\sqrt{s} = 5, 10, 50, 100, 500$ GeV. Notice that A_{BN}^{soft} does not change for $\sqrt{s} \geq 10$ GeV, since q_{max} remains constant.

VIII. NORMALIZATION AND TOTAL CROSS SECTIONS

In order to obtain the average number of collisions and thus the total cross sections, the overall normalization given by σ_{soft} has to be determined.

As mentioned in the introduction, we are assuming that the entire rise is due to σ_{jet}. For the proton-proton cross section, one needs only one further parameter for the non-perturbative region, namely, a constant σ_0 which gives the normalization of the cross section. As far as the proton-antiproton cross section is concerned, the rapid decrease after the resonances is interpreted as dual to the Regge trajectory exchange, and it should be described by a power $s^{\alpha_R(0)-1} \approx 1/\sqrt{s}$. Neglecting the real part of the eikonal, our model is now complete and reads as follows:

$$\sigma_{\rm tot} = 2\int d^2\vec{b}[1 - e^{-{\rm Im}\chi(b,s)}], \tag{54}$$

with

$$2\,{\rm Im}\chi(b,s) = A_{\rm BN}(b, q_{\rm max}^{\rm soft})\sigma_{\rm soft}^{pp,\bar{p}} + A_{\rm BN}(b, q_{\rm max}^{\rm jet})\sigma_{\rm jet}(s; p_{t\,{\rm min}}). \tag{55}$$

We also have

$$\sigma_{\rm soft}^{pp} = \sigma_0, \qquad \sigma_{\rm soft}^{p\bar{p}} = \sigma_0\left(1 + \frac{2}{\sqrt{s}}\right). \tag{56}$$

We find that, in order to properly reproduce the normalization of the cross section, we need a value $\sigma_0 = 48$ mb, in good agreement with the considerations of Sec. II. We now show in Fig. 4 the average number of collisions as a function of b, distinguishing between hard and soft contributions and using the values of $q_{\rm max}$ shown in Fig. 2. We show only the low energy region $\sqrt{s} = 10$–100 GeV, where the transition between soft and hard processes occurs.

Finally, in Fig. 5 we show the results of our model, putting all the pieces together, for the total cross section for proton-proton and proton-antiproton collisions. We see that the model gives an overall satisfactory description of the energy behavior of available data.

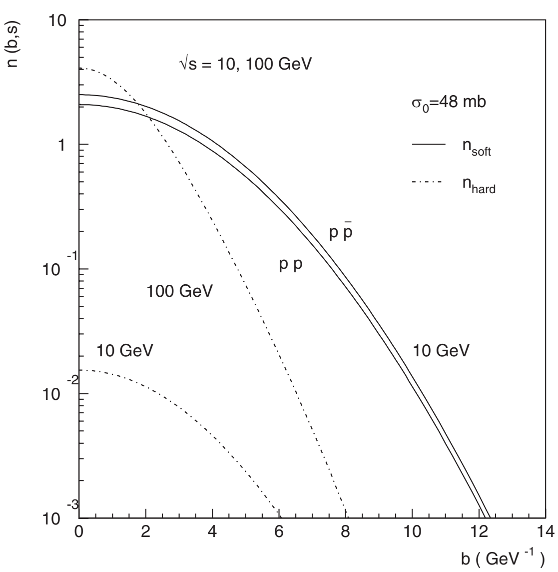

FIG. 4. The average number of collisions at $\sqrt{s} = 10, 100$ GeV is plotted for the soft gluon summation model, using the $q_{\rm max}$ values described in the text. The dotted-dashed line corresponds to the jet contribution at $\sqrt{s} = 10, 100$ GeV.

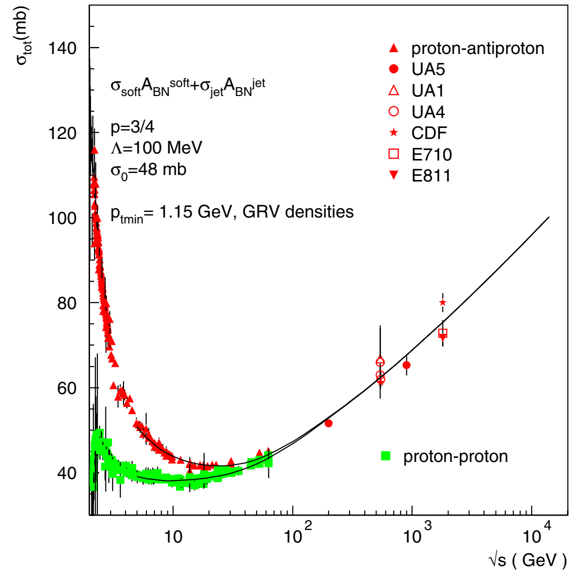

FIG. 5 (color online). Comparison of pp and $p\bar{p}$ total cross section data [8,32–37] with results from the Bloch-Nordsieck model described in the text.

IX. ENERGY DEPENDENT $\langle b^2 \rangle$ IN THE BLOCH-NORDSIECK MODEL

The energy dependent transverse overlap function $A_{\rm BN}$, discussed in the previous sections, can be used to estimate the energy dependence of the average distance among partons in the transverse space during a scattering process, namely,

$$\langle b^2 \rangle = \frac{\int d^2\vec{b}b^2[A_{\rm BN}(b, q_{\rm max}^{\rm soft}) + A_{\rm BN}(b, q_{\rm max}^{\rm jet})]}{\int d^2\vec{b}[A_{\rm BN}(b, q_{\rm max}^{\rm soft}) + A_{\rm BN}(b, q_{\rm max}^{\rm jet})]}. \tag{57}$$

The energy dependence of the average rms distance between partons so defined is shown in Fig. 6.

One can see from this figure that, for the hard part of the eikonal in the Bloch-Nordsieck model (dotted curve), the mean distance between the scattering partons does decrease as the energy increases, thereby increasing the shadowing and taming the rise, as opposed to the form factor model where $\langle b^2 \rangle$ is a constant. This is then further reflected in a more modest high energy rise for the BN model as seen in Fig. 5. It is also pleasing to note the following self-consistency. That is, at low values of $\sqrt{s}$, values of $\langle b^2 \rangle$ are the same for both the soft and the hard part of the eikonal in both models, as they must, since at low energy the transverse overlap function from the BN model is very similar to that from the FF model.

Observations about the need of a shrinkage in the radius of the proton have been made in Refs. [52,53], where multiparticle production in hadron-hadron interactions has been studied in detail in an eikonal Monte Carlo model. They find that, in the hard multiparton model, a good fit to the CDF data is obtained if the proton radius is decreased [54] by about a factor of 1.7, as compared to the form

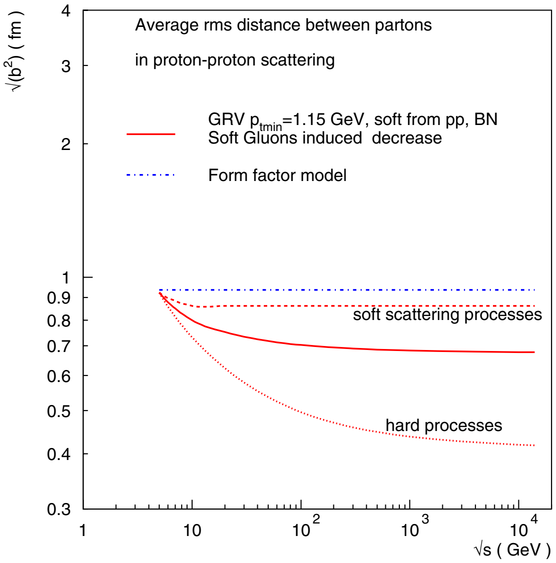

FIG. 6 (color online). The average distance between partons in the Bloch-Nordsieck model (solid line) and the form factor model (dotted-dashed line) is shown as a function of energy. For the former case, the contribution to this distance from the purely soft processes (dashed line) with $p_t \leq p_{t\,\mathrm{min}}$, and purely hard ones (dotted line) with $p_t \geq p_{t\,\mathrm{min}}$, is also shown.

factor model. Now the observation has been extended to photoproduction as well [55].

X. CONCLUSION

In conclusion, we have shown that standard QCD processes such as hard parton-parton scattering and soft gluon emission from valence quarks can account for two salient features of the total proton-proton cross section, the rise at high energy and the very gentle decrease at low energy. An important characteristic of this treatment is that, as the minijet cross section rises with energy, soft gluon emission produces an acollinearity of the partons and reduces the probability of collisions. This affects the cross sections in two ways: At low energy it produces a very soft decrease in $\sigma_{\mathrm{tot}}^{pp}$ and contributes to the faster decrease in $\sigma_{\mathrm{tot}}^{p\bar{p}}$; at high energy it tames the rise due to σ_{jet}. It is then possible to have a very small $p_{t\,\mathrm{min}}$ to see the onset of the rise around 10–20 GeV, without encountering too large a cross section when the energy climbs into the TeV range and beyond. We stress that the above behavior is obtained from leading soft gluon emission from the valence quarks. Subleading emission from internal gluon legs is not considered here. It is to be emphasized that singular α_s appears necessary for this purpose, thereby implying that confinement plays a crucial role in the energy dependence of the total cross section.

Further input is needed to understand the scale or the normalization, which plays a dominant role in the early decrease of the proton-antiproton cross section.

ACKNOWLEDGMENTS

This work was supported in part through EU RTN Contract No. CT2002-0311. R. G. acknowledges the partial support of the Department of Science and Technology, India, under Project No. SP/S2/K-01/2000-II. A. G. acknowledges support from MCYT under Project No. FPA2003-09298-c02-01. G. P. is thankful for the hospitality of Boston University Theoretical Physics group where this work was completed.

[1] A. Grau, G. Pancheri, and Y. N. Srivastava, Phys. Rev. D **60**, 114020 (1999).
[2] M. Derrick *et al.* (ZEUS Collaboration), Phys. Lett. B **293**, 465 (1992); Z. Phys. C **63**, 391 (1994); S. Chekanov *et al.* (ZEUS Collaboration), Nucl. Phys. **B627**, 3 (2002).
[3] S. Aid *et al.* (H1 Collaboration), Z. Phys. C **69**, 27 (1995).
[4] J. Breitweg *et al.* (ZEUS Collaboration), Phys. Lett. B **487**, 53 (2000).
[5] C. Ginsburg *et al.* (ZEUS Collaboration), in *Proceedings of the 8th International Workshop on Deep Inelastic Scattering, Liverpool, 2000*, edited by J. A. Gracey and T. Greenshaw (World Scientific, Singapore, 2001).
[6] M. Acciarri *et al.* (L3 Collaboration), Phys. Lett. B **408**, 450 (1997); **519**, 33 (2001); A. Csilling *et al.* (L3 Collaboration), Nucl. Phys. B Proc. Suppl. **82**, 239 (2000).
[7] G. Abbiendi *et al.* (OPAL Collaboration), Eur. Phys. J. C **14**, 199 (2000); F. Waeckerle, Nucl. Phys. B Proc. Suppl. **71**, 381 (1999); Stefan Söldner-Rembold, hep-ex/9810011.
[8] S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).
[9] M. M. Block, F. Halzen, and T. Stanev, Phys. Rev. D **62**, 077501 (2000); M. M. Block, F. Halzen, G. Pancheri, and T. Stanev, hep-ph/0003226.
[10] A. Martin and F. Cheung, *Analyticity Properties and Bounds on Scattering Amplitudes* (Gordon and Breach, New York, 1970).
[11] M. M. Block and A. B. Kaidalov, Phys. Rev. D **64**, 076002 (2001).
[12] A. Donnachie and P. V. Landshoff, Phys. Lett. B **296**, 227 (1992).
[13] K. Hagiwara *et al.* (Particle Data Group), Phys. Rev. D **66**, 010001 (2002).
[14] R. M. Godbole, A. de Roeck, A. Grau, and G. Pancheri, J. High Energy Phys. 06 (2003) 061.
[15] J. R. Cudell *et al.*, in *Proceedings of the XXXII International Symposium on Multiparticle Dynamics, Alushta, Ukraine, 2002* (World Scientific, Singapore, 2002); Phys. Rev. Lett. **89**, 201801 (2002).

[16] D. Cline, F. Halzen, and J. Luthe, Phys. Rev. Lett. **31**, 491 (1973).
[17] G. Pancheri and C. Rubbia, Nucl. Phys. **A418**, 117c (1984).
[18] T. Gaisser and F. Halzen, Phys. Rev. Lett. **54**, 1754 (1985).
[19] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977); V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972); L. N. Lipatov, Sov. J. Nucl. Phys. **20**, 95 (1972); Yu. L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977).
[20] K. Igi and S. Matsuda, Phys. Rev. Lett. **18**, 625 (1967).
[21] Y. Srivastava, S. Pacetti, G. Pancheri, and A. Widom, in *Proceedings of the 2001 e^+e^- Physics at Intermediate Energies Workshop, Stanford, CA, 2001*, edited by D. Bettoni, eConf C010430, T19 (2001).
[22] Y. Srivastava and A. Widom, Phys. Rev. D **63**, 077502 (2001).
[23] P. V. Landshoff, hep-ph/0108156.
[24] H. B. Mayer and M. J. Teper, Phys. Lett. B **605**, 344 (2005).
[25] S. Lomatch, F. I. Olness, and J. C. Collins, Nucl. Phys. **B317**, 617 (1989).
[26] M. Froissart, Phys. Rev. **123**, 1053 (1961); A. Martin, Phys. Rev. **129**, 1432 (1963).
[27] G. Pancheri and Y. N. Srivastava, Phys. Lett. B **182**, 199 (1986).
[28] L. Durand and H. Pi, Phys. Rev. Lett. **58**, 303 (1987); A. Capella, J. Kwiecinsky, and J. Tran Thanh, Phys. Rev. Lett. **58**, 2015 (1987).
[29] A. Corsetti, R. M. Godbole, and G. Pancheri, Phys. Lett. B **435**, 441 (1998).
[30] M. M. Block, F. Halzen, and B. Margolis, Phys. Rev. D **45**, 839 (1992); M. Block, E. Gregores, F. Halzen, and G. Pancheri, Phys. Rev. D **58**, 017503 (1998).
[31] M. Glück, E. Reya, and A. Vogt, Phys. Rev. D **46**, 1973 (1992).
[32] R. Battiston *et al.* (UA4 Collaboration), Phys. Lett. **117B**, 126 (1982); M. Bozzo *et al.* (UA4 Collaboration), Phys. Lett. **147B**, 392 (1984); C. Augier *et al.* (UA4/2 Collaboration), Phys. Lett. B **344**, 451 (1995).
[33] G. Arnison *et al.* (UA1 Collaboration), Phys. Lett. **128B**, 336 (1983).
[34] G. J. Alner *et al.* (UA5 Collaboration), Z. Phys. C **32**, 153 (1986).
[35] N. Amos *et al.* (E710 Collaboration), Phys. Rev. Lett. **68**, 2433 (1992).
[36] F. Abe *et al.* (CDF Collaboration), Phys. Rev. D **50**, 5550 (1994).
[37] C. Avilia *et al.* (E811 Collaboration), Phys. Lett. B **445**, 419 (1999).
[38] G. Pancheri-Srivastava and Y. N. Srivastava, Phys. Rev. D **15**, 2915 (1977).
[39] Y. L. Dokshitzer, D. Diakonov, and S. I. Troian, Phys. Lett. **79B**, 269 (1978).
[40] G. Parisi and R. Petronzio, Nucl. Phys. **B154**, 427 (1979).
[41] P. Chiappetta and M. Greco, Nucl. Phys. **B199**, 77 (1982).
[42] F. Halzen, A. D. Martin, D. M. Scott, and M. P. Tuite, Z. Phys. C **14**, 351 (1982).
[43] G. Altarelli, R. K. Ellis, M. Greco, and G. Martinelli, Nucl. Phys. **B246**, 12 (1984).
[44] V. V. Sudakov, Sov. Phys. JETP **3**, 65 (1956).
[45] J. Botts and G. Sterman, Phys. Lett. B **224**, 201 (1989).
[46] R. Jakob and P. Kroll, Phys. Lett. B **315**, 463 (1993).
[47] A. Corsetti, A. Grau, G. Pancheri, and Y. N. Srivastava, Phys. Lett. B **382**, 282 (1996).
[48] Y. L. Dokshitzer, hep-ph/9812252.
[49] A. Nakamura, G. Pancheri, and Y. Srivastava, Z. Phys. C **21**, 243 (1984).
[50] J. L. Richardson, Phys. Lett. **82B**, 272 (1979).
[51] M. Anselmino, E. Predazzi, S. Ekelin, S. Frederiksson, and D. B. Lichtenberg, Rev. Mod. Phys. **65**, 1199 (1993); B. Z. Kopeliovich, I. K. Potashnikova, B. Povh, and E. Predazzi, Phys. Rev. D **63**, 054001 (2001).
[52] I. Borozan and M. H. Seymour, J. High Energy Phys. 09 (2002) 015.
[53] I. Borozan, Ph.D. thesis, University College, London, 2002, http://www.hep.ucl.ac.uk/theses.shtml.
[54] Note that there is a misprint in Ref. [52], which states that the proton radius has to be increased by a factor of 1.7; M. H. Seymour (private communication).
[55] J. M. Butterworth and S. Butterworth, Comput. Phys. Commun. **153**, 164 (2003); JETWEB: http://jetweb.hep.ucl.ac.uk/.