

# Multigauge-boson vertices and chiral Lagrangian parameters in Higgsless models with ideal fermion delocalization

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Higgsless models with fermions whose  $SU(2)$  properties are “ideally delocalized,” such that the fermion’s probability distribution is appropriately related to the  $W$  boson wave function, have been shown to minimize deviations in precision electroweak parameters. As contributions to the  $S$  parameter vanish to leading order, current constraints on these models arise from limits on deviations in multi-gauge-boson vertices. We compute the form of the triple and quartic gauge-boson vertices in these models and show that these constraints provide lower bounds only of the order of a few hundred GeV on the masses of the lightest  $KK$  resonances. Higgsless models with ideal fermion delocalization provide an example of extended electroweak gauge interactions with suppressed couplings of fermions to extra gauge bosons, and these are the only models for which triple-gauge-vertex measurements provide meaningful constraints. We relate the multigauge couplings to parameters of the electroweak chiral Lagrangian, and the parameters obtained in these  $SU(2) \times SU(2)$  models apply equally to the corresponding five-dimensional gauge theory models of QCD. We also discuss the collider phenomenology of the  $KK$  resonances in models with ideal delocalization. These resonances are found to be fermiophobic, therefore traditional direct collider searches are not sensitive to them and measurements of gauge-boson scattering will be needed to find them.

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## I. INTRODUCTION

Higgsless models [1] do just what their name suggests: they provide electroweak symmetry breaking, including unitarization of the scattering of longitudinal  $W$  and  $Z$  bosons, without employing a scalar Higgs [2] boson. In a class of well-studied models [3,4] based on a five-dimensional  $SU(2) \times SU(2) \times U(1)$  gauge theory in a slice of anti-de Sitter space, electroweak symmetry breaking is encoded in the boundary conditions of the gauge fields. In addition to a massless photon and near-standard  $W$  and  $Z$  bosons, the spectrum includes an infinite tower of additional massive vector bosons (the higher Kaluza-Klein or  $KK$  excitations), whose exchange is responsible for unitarizing longitudinal  $W$  and  $Z$  boson scattering [5–8]. The electroweak properties and collider phenomenology of many such models have been discussed in the literature [9–16].

An alternative approach to analyzing the properties of Higgsless models [17–24] is to use deconstruction [25,26] and to compute the electroweak parameters  $\alpha S$  and  $\alpha T$  [27–29] in a related linear moose model [30]. We have shown [24] how to compute all four of the leading zero-momentum electroweak parameters defined by Barbieri *et al.* [15] in a very general class of linear moose models. Using these techniques, we showed [24] that a Higgsless model whose fermions are localized (i.e., derive their electroweak properties from a single site on the deconstructed lattice) cannot simultaneously satisfy unitarity bounds and precision electroweak constraints unless the model includes extra light vector bosons with masses comparable to those of the  $W$  or  $Z$ .

It has recently been proposed [31–33] that the size of corrections to electroweak processes may be reduced by including delocalized fermions. In deconstruction, a delocalized fermion is realized as a fermion whose  $SU(2)$  properties arise from several sites on the deconstructed lattice [34,35]. We examined the case of a fermion whose  $SU(2)$  properties arise from two adjacent sites [34], and confirmed that (even in that simple case) it is possible to minimize the electroweak parameter  $\alpha S$  by choosing a suitable amount of fermion delocalization. In subsequent work [36], we studied the properties of deconstructed

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Higgsless models with fermions whose  $SU(2)$  properties arise from delocalization over many sites of the deconstructed lattice. In an arbitrary Higgsless model we showed that if the probability distribution of the delocalized fermions is related to the  $W$  wave function (a condition we call “ideal” delocalization), in flat space and  $AdS_5$  for example (assuming the absence of brane kinetic energy terms for the gauge fields)

$$|\psi(z)_{\text{ideal}}|^2 \propto \frac{1}{g_5^2} \chi_W(z), \quad (\text{flat space})$$

$$|\psi(z)_{\text{ideal}}|^2 \propto \frac{1}{z g_5^2} \chi_W(z), \quad (AdS_5)$$

then deviations in precision electroweak parameters are minimized.<sup>1</sup> In particular, three  $(\hat{S}, \hat{T}, W)$  of the four leading zero-momentum precision electroweak parameters defined by Barbieri *et al.* [15] vanish at tree level.

This paper extends our analysis of ideal fermion delocalization in several ways. We compute the form of the triple and quartic gauge-boson vertices in these models and relate them to the parameters of the electroweak chiral Lagrangian. As the symmetry structure of the models discussed is  $SU(2) \times SU(2)$ , the chiral Lagrangian parameters obtained apply equally to the corresponding five-dimensional gauge theory models of QCD [18,37–43]. It is also notable that Higgsless models with ideal delocalization provide an example of a theory in which the chiral parameters  $\alpha_2$  (or  $L_{9R}$ ) and  $\alpha_3$  (or  $L_{9L}$ ) are not equal. We discuss the collider phenomenology of the  $KK$  resonances in models with ideal delocalization in order to determine how experiments can constrain them in the absence of bounds from precision electroweak measurements.

To begin, we provide additional details of the flat-space and warped-space  $SU(2)_A \times SU(2)_B$  models with brane kinetic terms that we will study. We then discuss the wave functions, masses, and couplings of the  $\gamma$ ,  $W$ , and  $Z$  gauge bosons in the model; we also lay the groundwork for comparing the behavior of brane-localized and ideally delocalized fermions. We note that the  $KK$  resonances lying above the  $W$  and  $Z$  bosons are fermiophobic.

In Secs. IV and V we discuss the three-point and four-point gauge-boson vertices which are crucial to the phenomenology of models with ideal fermion delocalization. We first calculate the Hagiwara-Peccei-Zeppenfeld-Hikasa parameters [44] to facilitate comparison with experiment. We then cast our results into the language of the electroweak chiral Lagrangian and find the values of the Longhitano parameters [45–49], and the corresponding parameters as defined by Gasser and Leutwyler [50]. We also provide results for models based on a bulk  $SU(2)$

gauge theory, and discuss how these are related to those for an  $SU(2)_A \times SU(2)_B$  theory.

We then apply our results to analyze the unusual phenomenology of models with ideally delocalized fermions. We show that current measurements of triple-gauge-boson vertices allow the lightest  $KK$  resonances above the observed  $W$  and  $Z$  bosons to have masses of only a few hundred GeV. Higgsless models with ideal fermion delocalization provide an example of extended electroweak gauge interactions with suppressed couplings of fermions to extra gauge bosons, and these are the only models for which triple-gauge-vertex measurements provide meaningful constraints.<sup>2</sup> Moreover, triple-gauge-vertex measurements are the only current source of constraints on Higgsless models with ideal fermion delocalization—as mentioned earlier, these models are not constrained by precision electroweak tests. Nor do existing direct searches for  $W'$  and  $Z'$  states constrain our models: such searches assume that the  $W'$  and  $Z'$  are produced and/or decay through their couplings to fermions—but the  $KK$  resonances in models with ideal delocalization are fermiophobic.<sup>3</sup> Studies of gauge-boson rescattering [52] will be needed in order to probe the  $KK$  resonances of these models in more detail. Our conclusions are summarized in Sec. VII.

## II. $SU(2)_A \times SU(2)_B$ HIGGSLESS MODELS

In this section we describe the five-dimensional  $SU(2)_A \otimes SU(2)_B$  gauge theories, both in flat and warped spacetime, which give rise to the Higgsless models discussed in this paper. The moose diagram [30] for the deconstruction corresponding to these models is shown in Fig. 1. The fermions derive their  $SU(2)_W$  properties from a linear combination<sup>4</sup> of all of the  $SU(2)$  groups—in a manner related to the  $W$  boson profile as required for ideal delocalization [36]. For the purposes of the analyses presented here, we will take the  $U(1)$  properties of the fermions to arise from the single  $U(1)$  group. As drawn, this model does not have an interpretation as a local higher-dimensional gauge theory; however, the results of this model agree (up to corrections suppressed by  $M_W^4/M_{W_1}^4$ ) with a consistent higher-dimensional theory based on an  $SU(2) \times SU(2) \times U(1)$  gauge group in the bulk [53].

### A. Flat

We begin by considering a five-dimensional  $SU(2)_A \otimes SU(2)_B$  gauge theory in flat space, in which the fifth

<sup>1</sup>The vanishing of  $\alpha S$  to leading order for a “flat” fermion wave function in  $AdS_5$ , which is approximately related to the  $W$  wave function in that model, was noted also in [3,31].

<sup>2</sup>Another interesting example is given in [51].

<sup>3</sup>The weakened coupling of fermions to  $KK$  modes in the case of a flat fermion wave function in  $AdS_5$  is noted in [31].

<sup>4</sup>More precisely, we associate each ordinary fermion with the lightest (chiral) mode of a corresponding five-dimensional fermion field. Fermion delocalization corresponds to a wave function of the lightest mode which extends into the bulk.

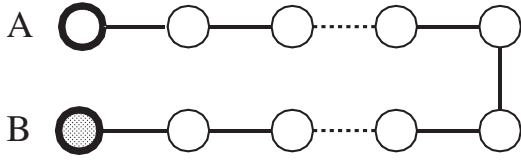


FIG. 1. Moose diagram [30] for the deconstruction corresponding to the  $SU(2)_A \times SU(2)_B$  models analyzed in this paper.  $SU(2)$  gauge groups are shown as open circles;  $U(1)$  gauge group is shown as a shaded circle. The brane kinetic energy terms are indicated by the thick circles. The fermions couple to a linear combination of all of the  $SU(2)$  groups, as well as to the single  $U(1)$  group. As drawn, this model does not have an interpretation as a local higher-dimensional gauge theory; however, the results of this model agree (up to corrections suppressed by  $M_W^4/M_{W_1}^4$ ) with a consistent higher-dimensional theory based on an  $SU(2) \times SU(2) \times U(1)$  gauge group in the bulk [53].

dimension (denoted by the coordinate  $z$ ) is compactified on an interval of length  $\pi R$ . The action of the gauge theory is given by

$$S_{5D} = \int_0^{\pi R} dz \int d^4x \left[ \frac{1}{g_{5A}^2} \left( -\frac{1}{4} W_{\mu\nu}^{Aa} W_{\alpha\beta}^{Aa} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} W_{\mu z}^{Aa} W_{\nu z}^{Aa} \eta^{\mu\nu} \right) + \frac{1}{g_{5B}^2} \left( -\frac{1}{4} W_{\mu\nu}^{Ba} W_{\alpha\beta}^{Ba} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} W_{\mu z}^{Ba} W_{\nu z}^{Ba} \eta^{\mu\nu} \right) \right], \quad (1)$$

where  $a = 1, 2, 3$ . The boundary conditions for  $W_{\mu}^{Aa}$  and  $W_{\mu}^{Ba}$  are taken as

$$\partial_z W_{\mu}^{Aa}(x, z)|_{z=0} = 0, \quad W_{\mu}^{B1,2}(x, z)|_{z=0} = 0, \quad (2)$$

$$\partial_z W_{\mu}^{B3}(x, z)|_{z=0} = 0,$$

$$\partial_z \left( \frac{1}{g_{5A}^2} W_{\mu}^{Aa}(x, z) + \frac{1}{g_{5B}^2} W_{\mu}^{Ba}(x, z) \right) \Big|_{z=\pi R} = 0, \quad (3)$$

$$(W_{\mu}^{Aa}(x, z) - W_{\mu}^{Ba}(x, z)) \Big|_{z=\pi R} = 0.$$

The boundary conditions at  $z = 0$  explicitly break  $SU(2)_A \otimes SU(2)_B$  down to  $SU(2)_W \otimes U(1)_Y$ , where we identify  $SU(2)_W$  with  $SU(2)_A$  and hypercharge with the  $T^3$  component of  $SU(2)_B$ . The boundary conditions at  $z = \pi R$  break  $SU(2)_A \otimes SU(2)_B$  to their diagonal subgroup; collectively the boundary conditions leave only electromagnetism unbroken. We further introduce  $SU(2)_A$  and  $U(1)_Y$  kinetic terms on the  $z = 0$  brane:

$$S_{z=0} = \int_0^{\pi R} dz \int d^4x \left[ -\delta(z - \epsilon) \frac{1}{4g_0^2} W_{\mu\nu}^{Aa} W_{\alpha\beta}^{Aa} \eta^{\mu\alpha} \eta^{\nu\beta} - \delta(z - \epsilon) \frac{1}{4g_Y^2} W_{\mu\nu}^{B3} W_{\alpha\beta}^{B3} \eta^{\mu\alpha} \eta^{\nu\beta} \right] \quad (\epsilon \rightarrow 0+). \quad (4)$$

As the only  $U(1)$  gauge symmetry exists on the  $z = 0$  brane, the hypercharge of the fermions arises from cou-

plings localized at this brane (for a discussion of a consistent higher-dimensional gauge theory allowing for ideal fermion delocalization, see [53]).

The 5D fields  $W_{\mu}^{Aa}(x, z)$  and  $W_{\mu}^{Ba}(x, z)$  can be decomposed into  $KK$ -modes,

$$W_{\mu}^{A1,2}(x, z) = \sum_n W_{\mu}^{(n)1,2}(x) \chi_{W(n)}^A(z),$$

$$W_{\mu}^{B1,2}(x, z) = \sum_n W_{\mu}^{(n)1,2}(x) \chi_{W(n)}^B(z), \quad (5)$$

$$W_{\mu}^{A3}(x, z) = \gamma_{\mu}(x) \chi_{\gamma}^A(z) + \sum_n Z_{\mu}^{(n)}(x) \chi_{Z(n)}^A(z),$$

$$W_{\mu}^{B3}(x, z) = \gamma_{\mu}(x) \chi_{\gamma}^B(z) + \sum_n Z_{\mu}^{(n)}(x) \chi_{Z(n)}^B(z).$$

Here  $\gamma_{\mu}(x)$  is the photon, and  $W_{\mu}^{(n)1,2}(x)$  and  $Z_{\mu}^{(n)}(x)$  are the  $KK$  towers of the massive  $W$  and  $Z$  bosons, the lowest of which correspond to the observed  $W$  and  $Z$  bosons. The mode functions  $\chi_{W(n)}^{A,B}(z)$ ,  $\chi_{Z(n)}^{A,B}(z)$ , and  $\chi_{\gamma}^{A,B}(z)$  obey the differential equations

$$\frac{d^2}{dz^2} \chi_{W(n)}^{A,B} + M_{W(n)}^2 \chi_{W(n)}^{A,B} = 0, \quad (6)$$

$$\frac{d^2}{dz^2} \chi_{Z(n)}^{A,B} + M_{Z(n)}^2 \chi_{Z(n)}^{A,B} = 0, \quad (7)$$

$$\frac{d^2}{dz^2} \chi_{\gamma}^{A,B} = 0, \quad (8)$$

and the boundary conditions

$$\chi_{W(n)}^B(z)|_{z=0} = \partial_z \chi_{\gamma}^B(z)|_{z=0} = \partial_z \chi_{\gamma}^A(z)|_{z=0} = 0,$$

$$\partial_z \chi_{W(n)}^A(z)|_{z=0} = -\frac{g_{5A}^2}{g_0^2} M_{W(n)}^2 \chi_{W(n)}^A(0), \quad (9)$$

$$\partial_z \chi_{Z(n)}^A(z)|_{z=0} = -\frac{g_{5A}^2}{g_0^2} M_{Z(n)}^2 \chi_{Z(n)}^A(0),$$

$$\partial_z \chi_{Z(n)}^B(z)|_{z=0} = -\frac{g_{5B}^2}{g_Y^2} M_{Z(n)}^2 \chi_{Z(n)}^B(0),$$

and

$$\partial_z \left( \frac{1}{g_{5A}^2} \chi_{W(n)}^A(z) + \frac{1}{g_{5B}^2} \chi_{W(n)}^B(z) \right) \Big|_{z=\pi R} = 0,$$

$$(\chi_{W(n)}^A(z) - \chi_{W(n)}^B(z)) \Big|_{z=\pi R} = 0,$$

$$\partial_z \left( \frac{1}{g_{5A}^2} \chi_{Z(n)}^A(z) + \frac{1}{g_{5B}^2} \chi_{Z(n)}^B(z) \right) \Big|_{z=\pi R} = 0, \quad (10)$$

$$(\chi_{Z(n)}^A(z) - \chi_{Z(n)}^B(z)) \Big|_{z=\pi R} = 0,$$

$$\partial_z \left( \frac{1}{g_{5A}^2} \chi_{\gamma}^A(z) + \frac{1}{g_{5B}^2} \chi_{\gamma}^B(z) \right) \Big|_{z=\pi R} = 0,$$

$$(\chi_{\gamma}^A(z) - \chi_{\gamma}^B(z)) \Big|_{z=\pi R} = 0.$$

Note that the boundary conditions at  $z = 0$  reflect the presence of the  $SU(2)_A \times U(1)_Y$  brane kinetic term Eq. (4).

Substituting Eq. (5) into  $S_{5D} + S_{z=0}$  and requiring that the 4D fields  $W_\mu^{(n)}(x)$ ,  $Z_\mu^{(n)}(x)$ ,  $\gamma_\mu(x)$  possess canonically normalized kinetic terms, we see the mode functions are normalized as

$$1 = \int_0^{\pi R} dz \left\{ \frac{1}{g_{5A}^2} |\chi_{W^{(n)}}^A(z)|^2 + \frac{1}{g_{5B}^2} |\chi_{W^{(n)}}^B(z)|^2 \right\} + \frac{1}{g_0^2} |\chi_{W^{(n)}}^A(0)|^2, \quad (11)$$

$$1 = \int_0^{\pi R} dz \left\{ \frac{1}{g_{5A}^2} |\chi_\gamma^A(z)|^2 + \frac{1}{g_{5B}^2} |\chi_\gamma^B(z)|^2 \right\} + \frac{1}{g_0^2} |\chi_\gamma^A(0)|^2 + \frac{1}{g_Y^2} |\chi_\gamma^B(0)|^2, \quad (12)$$

$$1 = \int_0^{\pi R} dz \left\{ \frac{1}{g_{5A}^2} |\chi_{Z^{(n)}}^A(z)|^2 + \frac{1}{g_{5B}^2} |\chi_{Z^{(n)}}^B(z)|^2 \right\} + \frac{1}{g_0^2} |\chi_{Z^{(n)}}^A(0)|^2 + \frac{1}{g_Y^2} |\chi_{Z^{(n)}}^B(0)|^2. \quad (13)$$

Since the lightest massive  $KK$ -modes ( $n = 0$ ) are identified as the observed  $W$  and  $Z$  bosons, we will write

$$M_W^2 \equiv M_{W^{(0)}}^2, \quad M_Z^2 \equiv M_{Z^{(0)}}^2, \quad (14)$$

and

$$\chi_W^{A,B} \equiv \chi_{W^{(0)}}^{A,B}, \quad \chi_Z^{A,B} \equiv \chi_{Z^{(0)}}^{A,B}. \quad (15)$$

## B. Warped

We will also consider a related model based on  $SU(2)_A \otimes SU(2)_B$  in a truncated anti-de Sitter space. We adopt the conformally flat metric

$$ds^2 = \left(\frac{R}{z}\right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad (16)$$

for the  $AdS_5$  space, and require that the coordinate  $z$  be restricted to the interval

$$R (\equiv e^{-b/2} R') \leq z \leq R'. \quad (17)$$

The end point  $z = R$  is known as the ‘‘Planck’’ brane, while  $z = R'$  is referred as the ‘‘TeV’’ brane. We will assume a large hierarchy between  $R$  and  $R'$  ( $b \gg 1$ ).

The 5D action of an  $SU(2) \otimes SU(2)$  gauge theory in this warped space is

$$S_{5D} = \int_R^{R'} dz \frac{R}{z} \int d^4x \left[ \frac{1}{g_{5A}^2} \left( -\frac{1}{4} W_{\mu\nu}^{Aa} W_{\alpha\beta}^{Aa} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} W_{\mu z}^{Aa} W_{\nu z}^{Aa} \eta^{\mu\nu} \right) + \frac{1}{g_{5B}^2} \left( -\frac{1}{4} W_{\mu\nu}^{Ba} W_{\alpha\beta}^{Ba} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} W_{\mu z}^{Ba} W_{\nu z}^{Ba} \eta^{\mu\nu} \right) \right], \quad (18)$$

where  $a = 1, 2, 3$ . In order to arrange for nontrivial weak mixing angle, we further introduce a  $U(1)_Y$  kinetic term on the Planck brane ( $z = R$ ),

$$S_{\text{Planck}} = \int_R^{R'} dz \delta(z - R - \epsilon) \times \int d^4x \frac{1}{g_Y^2} \left[ -\frac{1}{4} W_{\mu\nu}^{B3} W_{\alpha\beta}^{B3} \eta^{\mu\alpha} \eta^{\nu\beta} \right] \quad (\epsilon \rightarrow 0+). \quad (19)$$

As before, the  $U(1)_Y$  gauge symmetry exists only on the left-hand (Planck) brane, and the hypercharge couplings of the fermions therefore arise from couplings localized to that brane.

The 5D fields  $W_\mu^{Aa}(x, z)$  and  $W_\mu^{Ba}(x, z)$  can be decomposed into  $KK$ -modes, exactly as before [see Eqs. (5)]. However, the mode functions  $\chi_{W^{(n)}}^{A,B}(z)$ ,  $\chi_{Z^{(n)}}^{A,B}(z)$ , and  $\chi_\gamma^{A,B}(z)$  now obey modified differential equations:

$$0 = z \partial_z \left( \frac{1}{z} \partial_z \chi_{W^{(n)}}^{A,B} \right) + M_{W^{(n)}}^2 \chi_{W^{(n)}}^{A,B}, \quad (20)$$

$$0 = z \partial_z \left( \frac{1}{z} \partial_z \chi_{Z^{(n)}}^{A,B} \right) + M_{Z^{(n)}}^2 \chi_{Z^{(n)}}^{A,B}, \quad (21)$$

$$0 = z \partial_z \left( \frac{1}{z} \partial_z \chi_\gamma^{A,B} \right). \quad (22)$$

The form of the boundary conditions at the TeV brane,  $z = R'$ , is the same as the form of those at  $z = \pi R$  in the flat case [Eqs. (10)]. The boundary conditions at the Planck brane,  $z = R$ , are modified from those at  $z = 0$  in the flat case:

$$\begin{aligned} \partial_z \chi_{W^{(n)}}^A(z)|_{z=R} &= \partial_z \chi_{Z^{(n)}}^A(z)|_{z=R} = \partial_z \chi_\gamma^A(z)|_{z=R} = 0, \\ \chi_{W^{(n)}}^B(z)|_{z=R} &= \partial_z \chi_\gamma^B(z)|_{z=R} = 0, \\ \partial_z \chi_{Z^{(n)}}^B(z)|_{z=R} &= -\frac{g_{5B}^2}{g_Y^2} M_{Z^{(n)}}^2 \chi_{Z^{(n)}}^B(R). \end{aligned} \quad (23)$$

Note that the boundary condition at  $z = R$  reflects the presence of the  $U(1)_Y$  brane kinetic term Eq. (19).

Substituting Eq. (5) into  $S_{5D} + S_{\text{Planck}}$  and requiring that the 4D fields  $W_\mu^{(n)}(x)$ ,  $Z_\mu^{(n)}(x)$ , and  $\gamma_\mu(x)$  possess canonically normalized kinetic terms, we see the mode functions are normalized as

$$1 = \int_R^{R'} dz \left( \frac{R}{z} \right) \left\{ \frac{1}{g_{5A}^2} |\chi_{W^{(n)}}^A(z)|^2 + \frac{1}{g_{5B}^2} |\chi_{W^{(n)}}^B(z)|^2 \right\}, \quad (24)$$

$$1 = \int_R^{R'} dz \left( \frac{R}{z} \right) \left\{ \frac{1}{g_{5A}^2} |\chi_\gamma^A(z)|^2 + \frac{1}{g_{5B}^2} |\chi_\gamma^B(z)|^2 \right\} + \frac{1}{g_Y^2} |\chi_\gamma^B(z=R)|^2, \quad (25)$$

$$1 = \int_R^{R'} dz \left( \frac{R}{z} \right) \left\{ \frac{1}{g_{5A}^2} |\chi_{Z(n)}^A(z)|^2 + \frac{1}{g_{5B}^2} |\chi_{Z(n)}^B(z)|^2 \right\} + \frac{1}{g_Y^2} |\chi_{Z(n)}^B(z=R)|^2. \quad (26)$$

The lightest massive  $KK$ -modes ( $n = 0$ ) are, again, identified as the observed  $W$  and  $Z$  bosons.

### III. COUPLINGS, MASSES, AND WAVE FUNCTIONS

In this section we derive expressions for the couplings, masses, and wave functions of the photon,  $W$  and  $Z$ -bosons for the Higgsless models described above.

#### A. The photon and the bare weak angles

Let us start with the zero-mode  $\chi_\gamma^{A,B}$ , which is identified as the photon. The solution of the differential equation Eq. (8) obeying boundary conditions Eqs. (9) and (10) in the flat case is

$$\chi_\gamma^A(z) = \chi_\gamma^B(z) = \text{constant}. \quad (27)$$

The identical result is obtained in the warped case.

From the normalization conditions in Eqs. (12) and (25), we find

$$\chi_\gamma^A(z) = \chi_\gamma^B(z) = \left[ \pi R \left( \frac{1}{g_{5A}^2} + \frac{1}{g_{5B}^2} \right) + \frac{1}{g_0^2} + \frac{1}{g_Y^2} \right]^{-1/2} \quad (\text{flat}), \quad (28)$$

$$\chi_\gamma^A(z) = \chi_\gamma^B(z) = \left[ \frac{bR}{2} \left( \frac{1}{g_{5A}^2} + \frac{1}{g_{5B}^2} \right) + \frac{1}{g_Y^2} \right]^{-1/2} \quad (\text{AdS}_5). \quad (29)$$

Since  $\chi_\gamma^{A,B}(z)$  is the mode function of the photon, these expressions yield the electromagnetic coupling  $e$ :

$$\frac{1}{e^2} = \pi R \left( \frac{1}{g_{5A}^2} + \frac{1}{g_{5B}^2} \right) + \frac{1}{g_0^2} + \frac{1}{g_Y^2} \quad (\text{flat}), \quad (30)$$

$$\frac{1}{e^2} = \frac{bR}{2} \left( \frac{1}{g_{5A}^2} + \frac{1}{g_{5B}^2} \right) + \frac{1}{g_Y^2} \quad (\text{AdS}_5). \quad (31)$$

We can immediately identify quantities  $c_0, s_0$  that satisfy the relation

$$s_0^2 + c_0^2 = 1, \quad (32)$$

and may be interpreted as the cosine and sine of the bare weak mixing angle:

$$c_0^2 = \frac{g_0^2}{g_0^2 + g_Y^2}, \quad s_0^2 = \frac{g_Y^2}{g_0^2 + g_Y^2} \quad (\text{flat}), \quad (33)$$

$$\frac{c_0^2}{e^2} = \left( \frac{bR}{2} \right) \frac{1}{g_{5B}^2} + \frac{1}{g_Y^2}, \quad \frac{s_0^2}{e^2} = \left( \frac{bR}{2} \right) \frac{1}{g_{5A}^2} \quad (\text{AdS}_5). \quad (34)$$

The ratio of the left-handed and right-handed squared couplings will be useful in our later analysis

$$\kappa \equiv \frac{g_{5B}^2}{g_{5A}^2}, \quad (35)$$

of both the flat and warped cases. For the flat-space model, we will also find it convenient to define the dimensionless quantities:

$$\tilde{g}_{5A}^2 \equiv \frac{g_{5A}^2}{\pi R}, \quad \tilde{g}_{5B}^2 \equiv \frac{g_{5B}^2}{\pi R}, \quad (36)$$

$$\tilde{M}_W \equiv \pi R M_W, \quad \tilde{M}_Z \equiv \pi R M_Z, \quad \tilde{z} \equiv \frac{z}{\pi R}, \quad (37)$$

and the ratio of couplings:

$$\lambda \equiv \frac{g_0^2}{\tilde{g}_{5A}^2 + \tilde{g}_{5B}^2}. \quad (38)$$

#### B. The $W$ and $Z$

From the differential equations (6) and (7), boundary conditions (9) and (10), and normalization conditions (11) and (13), we can determine the masses and wave functions of the  $W$  and  $Z$  bosons in the flat-space model:

$$\tilde{M}_W^2 = \lambda - \left( \frac{1 + 3\kappa}{3} \right) \lambda^2 + O(\lambda^3), \quad (39)$$

$$\chi_W^A(z) = C_W \left[ 1 - \frac{1}{(1 + \kappa)} \tilde{z} + \tilde{M}_W^2 \left\{ \frac{1}{3} \left( \frac{1 + 3\kappa}{1 + \kappa} \right) \tilde{z} - \frac{1}{2} \tilde{z}^2 + \frac{1}{6(1 + \kappa)} \tilde{z}^3 \right\} + O(\tilde{M}_W^4) \right], \quad (40)$$

$$\chi_W^B(z) = C_W \left( \frac{\kappa}{1 + \kappa} \right) \left[ \tilde{z} + \tilde{M}_W^2 \left\{ \frac{2}{3} \tilde{z} - \frac{1}{6} \tilde{z}^3 \right\} + O(\tilde{M}_W^4) \right], \quad (41)$$

$$C_W = g_0 \left[ 1 - \frac{1 + 3\kappa}{6} \tilde{M}_W^2 + O(\tilde{M}_W^4) \right], \quad (42)$$

$$\tilde{M}_Z^2 = \frac{\lambda}{c_0^2} - \left( \frac{\lambda}{c^2} \right)^2 \frac{s^4(3 + \kappa) - 4s^2c^2\kappa + c^4(\kappa + 3\kappa^2)}{3\kappa} + O(\lambda^3), \quad (43)$$

$$\chi_Z^A(z) = C_Z \left[ 1 - \frac{1}{c_0^2(1+\kappa)} \tilde{z} + \tilde{M}_Z^2 \left\{ \frac{s^4(3+\kappa) - 4s^2c^2\kappa + c^4(\kappa+3\kappa^2)}{3c^2\kappa(1+\kappa)} \tilde{z} - \frac{1}{2} \tilde{z}^2 + \frac{1}{6c^2(1+\kappa)} \tilde{z}^3 \right\} + O(\tilde{M}_Z^4) \right], \quad (44)$$

$$\begin{aligned} \chi_Z^B(z) = & -C_Z \frac{s_0^2}{c_0^2} \left[ 1 - \tilde{M}_Z^2 \frac{s^4(1+\kappa) + s^2c^2(1-\kappa^2) - c^4(\kappa+\kappa^2)}{\kappa} \right] \\ & \times \left[ 1 - \frac{\kappa}{s_0^2(1+\kappa)} \tilde{z} + \tilde{M}_Z^2 \left\{ \frac{s^4(3+\kappa) - 4s^2c^2\kappa + c^4(\kappa+3\kappa^2)}{3s^2(1+\kappa)} \tilde{z} - \frac{1}{2} \tilde{z}^2 + \frac{\kappa}{6s^2(1+\kappa)} \tilde{z}^3 \right\} + O(\tilde{M}_Z^4) \right], \quad (45) \end{aligned}$$

$$C_Z = g_0 c_0 \left[ 1 + \tilde{M}_Z^2 \frac{s^4(3+5\kappa) - s^2c^2(2\kappa+6\kappa^2) - c^4(\kappa+3\kappa^2)}{6\kappa} + O(\tilde{M}_Z^4) \right]. \quad (46)$$

Employing the analogous equations (20)–(26) for the warped-space model yields

$$(M_W R')^2 = \left(\frac{4}{b}\right) \frac{1}{(1+\kappa)} \left\{ 1 + \frac{3}{2b} \frac{1}{(1+\kappa)} \right\} + O\left(\frac{1}{b^3}\right), \quad (47)$$

$$\begin{aligned} \chi_W^A(z) = & \frac{e}{s_0} \left\{ 1 + \frac{3}{8} \frac{1}{(1+\kappa)} \left(\frac{2}{b}\right) \right\} \left[ 1 - \frac{1}{2} \left( \ln \frac{z}{R} - \frac{1}{2} \right) (M_W z)^2 \right. \\ & \left. + \frac{1}{16} \left( \ln \frac{z}{R} - \frac{5}{4} \right) (M_W z)^4 + \dots \right], \quad (48) \end{aligned}$$

$$\begin{aligned} \chi_W^B(z) = & \frac{e}{s_0} \kappa \left\{ 1 + \frac{3}{8} \frac{1}{(1+\kappa)} \left(\frac{2}{b}\right) \right\} \left[ \frac{1}{2} (M_W z)^2 \left(\frac{b}{2}\right) \right. \\ & \left. - \frac{1}{16} (M_W z)^4 \left(\frac{b}{2}\right) + \dots \right], \quad (49) \end{aligned}$$

$$(M_Z R')^2 = \frac{1}{c_0^2} \left(\frac{4}{b}\right) \frac{1}{(1+\kappa)} \left\{ 1 + \frac{3}{2b} \frac{1}{c_0^2} \frac{1}{(1+\kappa)} \right\} + O\left(\frac{1}{b^3}\right), \quad (50)$$

$$\begin{aligned} \chi_Z^A(z) = & e \frac{c_0}{s_0} \left\{ 1 + \frac{3}{8c_0^2} \frac{1}{(1+\kappa)} \left(\frac{2}{b}\right) \right\} \left[ 1 - \frac{1}{2} \left( \ln \frac{z}{R} - \frac{1}{2} \right) \right. \\ & \left. \times (M_Z z)^2 + \frac{1}{16} \left( \ln \frac{z}{R} - \frac{5}{4} \right) (M_Z z)^4 + \dots \right], \quad (51) \end{aligned}$$

$$\begin{aligned} \chi_Z^B(z) = & e \frac{s_0}{c_0} \left\{ 1 + \frac{3}{8c_0^2} \frac{1}{(1+\kappa)} \left(\frac{2}{b}\right) \right\} \left[ -1 + \frac{1}{2} \left( \ln \frac{z}{R} - \frac{1}{2} \right) \right. \\ & \left. + \frac{e^2}{s_0^2} \frac{\kappa}{g_Y^2} \left(\frac{b}{2}\right) (M_Z z)^2 - \frac{1}{16} \left( \ln \frac{z}{R} - \frac{5}{4} + \frac{e^2}{s_0^2} \frac{\kappa}{g_Y^2} \right) \right. \\ & \left. \times \left(\frac{b}{2}\right) (M_Z z)^4 + \dots \right]. \quad (52) \end{aligned}$$

### C. Fermion profiles

It will also be necessary to introduce the wave function assumed for the left-handed components of the ordinary fermions (the right-handed components are assumed to be localized on the same brane as the localized hypercharge

kinetic energy term). We explore two possibilities, brane-localized fermions (which are phenomenologically disfavored [24] because this predicts sizeable precision electroweak corrections) and ideally delocalized fermions as defined in [36] (which are favored because they yield small electroweak corrections).

The profile of the brane-localized fermion is expressed as (note that we include any relevant metric factors in our definition of  $|\psi|^2$ —see Appendix A)

$$|\psi_{\text{localized}}^A(z)|^2 = \delta(z) \quad (\text{flat}), \quad (53)$$

$$|\psi_{\text{localized}}^A(z)|^2 = \delta(z-R) \quad (\text{AdS}_5), \quad (54)$$

$$|\psi_{\text{localized}}^B(z)|^2 = 0 \quad (\text{flat}) \text{ or } (\text{AdS}_5). \quad (55)$$

In contrast, the wave function of an ideally delocalized fermion is specifically related to the  $W$  wave function. For a flat-space model, the relationship is

$$|\psi_{\text{ideal}}^A(z)|^2 = C_{\text{ideal}} \left( \frac{1}{g_0^2} \delta(z) + \frac{1}{g_{5A}^2} \right) \chi_W^A(z), \quad (56)$$

$$|\psi_{\text{ideal}}^B(z)|^2 = C_{\text{ideal}} \frac{1}{g_{5B}^2} \chi_W^B(z), \quad (57)$$

while in an  $\text{AdS}_5$  model, it is

$$|\psi_{\text{ideal}}^A(z)|^2 = C_{\text{ideal}} \left( \frac{R}{z} \right) \frac{1}{g_{5A}^2} \chi_W^A(z), \quad (58)$$

$$|\psi_{\text{ideal}}^B(z)|^2 = C_{\text{ideal}} \left( \frac{R}{z} \right) \frac{1}{g_{5B}^2} \chi_W^B(z). \quad (59)$$

The constant,  $C_{\text{ideal}}$  is determined from the fermion normalization condition (see Appendix A)

$$\int_a^b dz \{ |\psi_{\text{ideal}}^A(z)|^2 + |\psi_{\text{ideal}}^B(z)|^2 \} = 1, \quad (60)$$

where the limits of integration  $[a, b]$  are  $[0, \pi R]$  for flat-space models and  $[R, R']$  for warped-space models, to be

$$C_{\text{ideal}} = g_0 \left[ 1 - \frac{\lambda}{6} (5 + 3\kappa) \right] \quad (\text{flat}), \quad (61)$$

$$C_{\text{ideal}} = \frac{e}{s_0} \left\{ 1 - \frac{3}{8} \frac{1}{(1 + \kappa)} \left( \frac{2}{b} \right) \right\} \quad (\text{AdS}_5). \quad (62)$$

### D. $G_F$ , $s_z^2$ , and $c_z^2$

The coupling between the  $W$  boson and fermion  $\psi$  is measured by the overlap integral

$$g_W = \int_a^b dz \{ |\psi^A(z)|^2 \chi_W^A(z) + |\psi^B(z)|^2 \chi_W^B(z) \}, \quad (63)$$

where the limits of integration  $[a, b]$  are  $[0, \pi R]$  for flat-space models and  $[R, R']$  for warped-space models. Given the forms of the fermion and  $W$ -boson wave functions above, one finds directly that the couplings are as follows:

$$g_W = \chi_W^A(z = a) \quad (\text{brane localized}), \quad (64)$$

$$g_W = C_{\text{ideal}} \quad (\text{ideal}), \quad (65)$$

where  $a$  is the  $z$ -coordinate of the brane on which the fermion is localized ( $0$  for flat space;  $R$  for  $\text{AdS}_5$ ) and  $C_{\text{ideal}}$  is the fermion normalization constant from Sec. III C.

The contributions to  $G_F$  arising from the exchange of higher-mass  $KK$  modes are suppressed in these models (the  $KK$  wave functions are suppressed precisely where the fermion wave functions are concentrated), and therefore we can express the Fermi constant to this order as

$$4\sqrt{2} G_F = \frac{g_W^2}{M_W^2}. \quad (66)$$

Drawing on the values of  $g_W$  and  $M_W$  from our earlier discussion, we deduce that

$$4\sqrt{2} G_F = \frac{g_0^2}{\lambda} (\pi R)^2 \{ 1 + O(\lambda^2) \} \quad (67)$$

(flat, brane localized),

$$4\sqrt{2} G_F = \frac{g_0^2}{\lambda} (\pi R)^2 \left[ 1 - \frac{4}{3} \lambda \right] \quad (\text{flat, ideal}), \quad (68)$$

$$4\sqrt{2} G_F = \frac{e^2}{s_0^2} (R')^2 \frac{b}{4} (1 + \kappa) \left\{ 1 + O\left(\frac{1}{b^2}\right) \right\} \quad (69)$$

( $\text{AdS}_5$ , brane localized),

$$4\sqrt{2} G_F = \frac{e^2}{s_0^2} (R')^2 \frac{b}{4} (1 + \kappa) \left\{ 1 - \frac{3}{2} \frac{1}{(1 + \kappa)} \left( \frac{2}{b} \right) \right\} \quad (70)$$

( $\text{AdS}_5$ , ideal).

Note that in the expressions for brane-localized fermions, the leading order correction vanishes; this is the  $\lambda^1$  term for

the flat-space model and the  $b^{-1}$  term for the warped-space model.

The standard definition of the weak mixing angle is given by

$$s_Z^2 c_Z^2 = \frac{e^2}{4\sqrt{2} G_F M_Z^2}. \quad (71)$$

The deviation of  $s_Z^2$  from its bare value  $s_0^2$  is parametrized by

$$s_Z^2 = s_0^2 + \Delta, \quad c_Z^2 = c_0^2 - \Delta. \quad (72)$$

Since we already have calculated  $M_Z$  and  $G_F$ , we are able to evaluate  $\Delta$  for each model directly:

$$\Delta = \frac{\lambda}{3} \left( \frac{s^2}{c^2 - s^2} \right) \left\{ 3c^2(c^2 - s^2)\kappa + (c^4 - 10c^2s^2 + s^4) - 3(c^2 - s^2)s^2 \frac{1}{\kappa} \right\} \quad (\text{flat, brane localized}), \quad (73)$$

$$\Delta = \frac{\lambda}{3} \left( \frac{s^2}{c^2 - s^2} \right) \left\{ 3c^2(c^2 - s^2)\kappa + (5c^4 - 6c^2s^2 + s^4) - 3(c^2 - s^2)s^2 \frac{1}{\kappa} \right\} \quad (\text{flat, ideal}). \quad (74)$$

$$\Delta = -\frac{3}{4} \frac{s_0^2}{c_0^2 - s_0^2} \frac{1}{(1 + \kappa)} \left( \frac{2}{b} \right) \quad (75)$$

( $\text{AdS}_5$ , brane localized),

$$\Delta = \frac{3}{4} s_0^2 \frac{1}{(1 + \kappa)} \left( \frac{2}{b} \right) \quad (\text{AdS}_5, \text{ ideal}). \quad (76)$$

### E. $\alpha S$

Ideal fermion delocalization takes its name from the fact that it is constructed in such a way as to minimize the size of precision electroweak corrections, as discussed in [36]. For example, the  $S$  parameter can be calculated from the  $T_3$ - $Y$  correlation function [22,24] arising from  $Z$ -exchange, and we therefore find

$$1 + \frac{\alpha S}{4s_Z^2 c_Z^2} = -\frac{1}{e^2} \chi_Z^B(z = a) \int_a^b dz \{ |\psi^A(z)|^2 \chi_Z^A(z) + |\psi^B(z)|^2 \chi_Z^B(z) \}, \quad (77)$$

where the limits of integration  $[a, b]$  are  $[0, \pi R]$  for flat space and  $[R, R']$  for warped space. Using the forms of the wave functions from Secs. III B and III C, we calculate

$$\alpha S = \frac{8}{3} \lambda s^2 \quad (\text{flat, brane localized}), \quad (78)$$

$$\alpha S = 0 + O(\lambda^2) \quad (\text{flat, ideal}), \quad (79)$$

$$\alpha S = 3s^2 \frac{1}{(1+\kappa)} \left(\frac{2}{b}\right) \quad (\text{AdS}_5, \text{ brane localized}), \quad (80)$$

$$\alpha S = 0 + O\left(\frac{1}{b^2}\right) \quad (\text{AdS}_5, \text{ ideal}). \quad (81)$$

Note that for ideal delocalization,  $\alpha S$  vanishes to leading order.

#### IV. MULTI-GAUGE-BOSON VERTICES

##### A. Triple-gauge-boson interactions: Notation

To leading order, in the absence of  $CP$  violation, the triple-gauge-boson vertices may be written in the Hagiwara-Peccei-Zeppenfeld-Hikasa triple-gauge-vertex notation [44]:

$$\begin{aligned} \mathcal{L}_{TGV} = & -ie \frac{c_Z}{s_Z} [1 + \Delta \kappa_Z] W_\mu^+ W_\nu^- Z^{\mu\nu} \\ & - ie [1 + \Delta \kappa_\gamma] W_\mu^+ W_\nu^- A^{\mu\nu} - ie \frac{c_Z}{s_Z} [1 + \Delta g_1^Z] \\ & \times (W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) Z_\nu \\ & - ie (W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) A_\nu, \end{aligned} \quad (82)$$

where the two-index tensors denote the Lorentz field-strength tensor of the corresponding field. In the standard model,  $\Delta \kappa_Z = \Delta \kappa_\gamma = \Delta g_1^Z \equiv 0$ .

In any vector-resonance model, such as the Higgsless models considered here, the interactions (82) come from reexpressing the non-Abelian couplings in the kinetic energy terms in the original Lagrangian [e.g., Eqs. (1) and (4) or (18) and (19)] in terms of the mass-eigenstate fields. In this case, one obtains equal contributions to the deviations of the first and third terms, and the second and fourth terms in Eq. (82). In addition the coefficient of the fourth term is fixed by electromagnetic gauge invariance, and therefore in these models we find

$$\Delta \kappa_\gamma \equiv 0 \quad \Delta \kappa_Z \equiv \Delta g_1^Z. \quad (83)$$

##### B. Triple-gauge-boson vertices

We are now ready to calculate the triple-gauge-boson vertices. In the flat-space case, the relevant integral (using  $V$  to stand for either  $\gamma$  or  $Z$ ) is

$$\begin{aligned} g_{VWW} = & \int_0^{\pi R} dz \left\{ \frac{1}{g_{5A}^2} \chi_V^A(z) |\chi_W^A(z)|^2 + \frac{1}{g_{5B}^2} \chi_V^B(z) |\chi_W^B(z)|^2 \right\} \\ & + \frac{1}{g_0^2} \chi_V^A(0) |\chi_W^A(0)|^2 \end{aligned} \quad (84)$$

while in the warped-space case, one writes instead

$$\begin{aligned} g_{VWW} = & \int_R^{R'} dz \left(\frac{R}{z}\right) \left\{ \frac{1}{g_{5A}^2} \chi_V^A(z) |\chi_W^A(z)|^2 \right. \\ & \left. + \frac{1}{g_{5B}^2} \chi_V^B(z) |\chi_W^B(z)|^2 \right\}. \end{aligned} \quad (85)$$

By inserting the forms of the gauge-boson wave functions from Secs. III A and III B, we verify that

$$g_{\gamma WW} = e \quad (86)$$

in all cases, and find that the  $ZWW$  coupling has the forms

$$g_{ZWW} = e \frac{c_0}{s_0} \left[ 1 + \lambda \frac{2 + 5\kappa + \kappa^2 - 2c^2(1+\kappa)^3}{4c^2\kappa(1+\kappa)} \right] \quad (\text{flat}) \quad (87)$$

$$g_{ZWW} = e \frac{c_0}{s_0} \left\{ 1 + \frac{5}{24} \frac{1}{c_0^2} \frac{(1-\kappa)}{(1+\kappa)^2} \left(\frac{2}{b}\right) \right\} \quad (\text{AdS}_5). \quad (88)$$

We use the values of  $\Delta$  from Sec. III D to rewrite these expressions in terms of the weak mixing angle  $s_Z$  rather than the bare angle  $s_0$ :

$$g_{ZWW} = e \frac{c_Z}{s_Z} \left[ 1 - \lambda \frac{(7+\kappa) - 6c^2(1-\kappa)}{12c^2(c^2 - s^2)(1+\kappa)} \right] \quad (\text{flat, brane localized}), \quad (89)$$

$$g_{ZWW} = e \frac{c_Z}{s_Z} \left[ 1 + \frac{\lambda}{12c^2} \frac{7+\kappa}{1+\kappa} \right] \quad (\text{flat, ideal}), \quad (90)$$

$$\begin{aligned} g_{ZWW} = & e \frac{c_Z}{s_Z} \left[ 1 + \frac{1}{24} \frac{1}{c^2(c^2 - s^2)} \right. \\ & \left. \times \frac{(10c^2 - 14) - \kappa(10c^2 + 4)}{(1+\kappa)^2} \left(\frac{2}{b}\right) \right] \end{aligned} \quad (91)$$

(AdS<sub>5</sub>, brane localized),

$$g_{ZWW} = e \frac{c_Z}{s_Z} \left[ 1 + \frac{1}{12c^2} \frac{7+2\kappa}{(1+\kappa)^2} \left(\frac{2}{b}\right) \right] \quad (\text{AdS}_5, \text{ ideal}). \quad (92)$$

The more compact notation  $c$  or  $s$  is used in an expression where the difference between employing the bare and corrected weak angles would cause only higher-order corrections.

Comparing these results with the form of Eq. (82) in Sec. IV A, we see immediately that

$$\begin{aligned} \Delta g_1^Z = \Delta \kappa_Z = & - \frac{\lambda}{12c^2} \frac{1}{(c^2 - s^2)} \frac{(7+\kappa) - 6c^2(1-\kappa)}{(1+\kappa)} \\ & (\text{flat, brane localized}), \end{aligned} \quad (93)$$



$$\Delta g_1^Z = \Delta \kappa_Z = \frac{\lambda}{12c^2} \frac{7 + \kappa}{1 + \kappa} \quad (\text{flat, ideal}), \quad (94)$$

$$\Delta g_1^Z = \Delta \kappa_Z = \frac{1}{24} \frac{1}{c^2(c^2 - s^2)} \frac{(10c^2 - 14) - \kappa(10c^2 + 4)}{(1 + \kappa)^2} \left(\frac{2}{b}\right) \quad (\text{AdS}_5, \text{brane localized}), \quad (95)$$

$$\Delta g_1^Z = \Delta \kappa_Z = \frac{1}{12c^2} \frac{7 + 2\kappa}{(1 + \kappa)^2} \left(\frac{2}{b}\right) \quad (\text{AdS}_5, \text{ideal}). \quad (96)$$

We will relate these results to the values of chiral Lagrangian parameters in Sec. V and will comment on the implications of experimental limits on  $\Delta g_1^Z$  and  $\Delta \kappa_Z$  in Sec. VI.

### C. Quartic gauge-boson vertices

The quartic  $W$ -boson coupling  $g_{WWWW}$  is evaluated in flat space by the overlap integral

$$g_{WWWW} = \int_0^{\pi R} dz \left\{ \frac{1}{g_{5A}^2} |\chi_W^A(z)|^4 + \frac{1}{g_{5B}^2} |\chi_W^B(z)|^4 \right\} + \frac{1}{g_0^2} |\chi_W^A(0)|^4 \quad (97)$$

$$g_{WWWW} = \frac{e^2}{s_Z^2} \left[ 1 + \lambda \frac{(-18 + 16c^2) + (-27 + 14c^2)\kappa - (3 + 14c^2)\kappa^2}{15(c^2 - s^2)(1 + \kappa)^2} \right] \quad (\text{flat, brane localized}), \quad (101)$$

$$g_{WWWW} = \frac{e^2}{s_Z^2} \left[ 1 + \frac{\lambda}{5} \frac{6 + 9\kappa + \kappa^2}{(1 + \kappa)^2} \right] \quad (\text{flat, ideal}) \quad (102)$$

$$g_{WWWW} = \frac{e^2}{s_Z^2} \left[ 1 - \frac{3}{4} \frac{1}{(1 + \kappa)} \frac{1}{c^2 - s^2} \left(\frac{2}{b}\right) + \frac{1}{24} \frac{(-9\kappa^3 + 20\kappa + 11)}{(1 + \kappa)^4} \left(\frac{2}{b}\right) \right] \quad (\text{AdS}_5, \text{brane localized}), \quad (103)$$

$$g_{WWWW} = \frac{e^2}{s_Z^2} \left[ 1 + \frac{3}{4} \frac{1}{(1 + \kappa)} \left(\frac{2}{b}\right) + \frac{1}{24} \frac{(-9\kappa^3 + 20\kappa + 11)}{(1 + \kappa)^4} \left(\frac{2}{b}\right) \right] \quad (\text{AdS}_5, \text{ideal}). \quad (104)$$

We will relate these results to the values of chiral Lagrangian parameters in Sec. V and will comment on the phenomenological implications for  $W$ -boson scattering in Sec. VI.

### V. CHIRAL LAGRANGIAN PARAMETERS

In studying the phenomenology of our models, it is useful to make contact with the parametrization afforded

and in warped space using

$$g_{WWWW} = \int_R^{R'} dz \left(\frac{R}{z}\right) \left\{ \frac{1}{g_{5A}^2} |\chi_W^A(z)|^4 + \frac{1}{g_{5B}^2} |\chi_W^B(z)|^4 \right\}. \quad (98)$$

By inserting the appropriate forms of the  $W$  boson wave functions from Sec. III B, we find

$$g_{WWWW} = \frac{e^2}{s_0^2} \left[ 1 + \lambda \left\{ s^2 \frac{(1 + \kappa)^2}{\kappa} - \frac{7 + 38\kappa + 52\kappa^2 + 15\kappa^3}{15(1 + \kappa)^2} \right\} \right] \quad (\text{flat}) \quad (99)$$

$$g_{WWWW} = \frac{e^2}{s_0^2} \left[ 1 + \frac{1}{24} \frac{(-9\kappa^3 + 20\kappa + 11)}{(1 + \kappa)^4} \left(\frac{2}{b}\right) \right] \quad (\text{AdS}_5). \quad (100)$$

We use the values of  $\Delta$  from Sec. III D to rewrite these expressions in terms of the weak mixing angle  $s_Z$  rather than the bare angle  $s_0$ :

by the effective electroweak chiral Lagrangian. Of the complete set of 12  $CP$ -conserving operators written down by Longhitano [45–48,54,55] and Appelquist and Wu [49], those which apply to our Higgsless models<sup>5</sup> are the following:

$$\mathcal{L}_1 \equiv \frac{1}{2} \alpha_1 g_W g_Y B_{\mu\nu} \text{Tr}(TW^{\mu\nu}) \quad (105)$$

$$\mathcal{L}_2 \equiv \frac{1}{2} i \alpha_2 g_Y B_{\mu\nu} \text{Tr}(T[V^\mu, V^\nu]) \quad (106)$$

$$\mathcal{L}_3 \equiv i \alpha_3 g_W \text{Tr}(W_{\mu\nu}[V^\mu, V^\nu]) \quad (107)$$

$$\mathcal{L}_4 \equiv \alpha_4 [\text{Tr}(V^\mu V^\nu)]^2 \quad (108)$$

$$\mathcal{L}_5 \equiv \alpha_5 [\text{Tr}(V_\mu V^\mu)]^2. \quad (109)$$

Here  $W_{\mu\nu}, B_{\mu\nu}, T \equiv U\tau_3 U^\dagger$ , and  $V_\mu \equiv (D_\mu U)U^\dagger$ , with  $U$  being the nonlinear sigma-model field<sup>6</sup> arising from

<sup>5</sup>In our models, weak isospin violation arises only through hypercharge and vanishes as  $g_Y \rightarrow 0$ . Hence, the coefficients  $\beta_1$  and  $\alpha_{6\dots 11}$  of the chiral Lagrangian operators that include weak isospin violation which persists in the limit  $g_Y \rightarrow 0$  must vanish at tree level.

<sup>6</sup> $SU(2)_W \equiv SU(2)_L$  and  $U(1)_Y$  is identified with the  $T_3$  part of  $SU(2)_R$ .

$SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$ , are the  $SU(2)_W$ -covariant and  $U(1)_Y$ -invariant building blocks of the expansion. An alternative parametrization by Gasser and Leutwyler [50] gives a different set of names to the coefficients of these operators:

$$\alpha_1 = L_{10}, \quad \alpha_2 = -\frac{1}{2}L_{9R}, \quad \alpha_3 = -\frac{1}{2}L_{9L}, \quad (110)$$

$$\alpha_4 = L_2, \quad \alpha_5 = L_1. \quad (111)$$

The chiral Lagrangian coefficients are related to  $\alpha S$  and the Hagiwara-Peccei-Zeppenfeld-Hikasa [44] triple-gauge-vertex parameters and the quartic  $W$  boson vertex as follows:

$$\alpha S = -16\pi\alpha(\alpha_1), \quad (112)$$

$$\Delta g_1^Z = \frac{1}{c^2(c^2 - s^2)} e^2 \alpha_1 + \frac{1}{s^2 c^2} e^2 \alpha_3, \quad (113)$$

$$\Delta \kappa_Z = \frac{2}{c^2 - s^2} e^2 \alpha_1 - \frac{1}{c^2} e^2 \alpha_2 + \frac{1}{s^2} e^2 \alpha_3, \quad (114)$$

$$\Delta \kappa_\gamma = \frac{1}{s^2} (-e^2 \alpha_1 + e^2 \alpha_2 + e^2 \alpha_3), \quad (115)$$

$$g_{WWWW} = \frac{e^2}{s^2 Z} \left[ 1 + \frac{2}{c^2 - s^2} e^2 \alpha_1 + \frac{2}{s^2} e^2 \alpha_3 + \frac{1}{s^2} e^2 \alpha_4 \right]. \quad (116)$$

Again, in any vector-resonance model all multigauge vertices arise from reexpressing the non-Abelian couplings in the kinetic energy terms in the original Lagrangian in terms of the mass-eigenstate fields. In this case, we find that  $\alpha_4 \equiv -\alpha_5$ . The leading corrections to  $WW$  and  $WZ$  elastic scattering arise from  $\alpha_{4,5}$  [56].

Using these relationships and the values previously derived for  $\alpha S$ ,  $\Delta g_1^Z$ ,  $\Delta \kappa_Z$ ,  $\Delta \kappa_\gamma$ , and  $g_{WWWW}$ , we arrive at values for the  $\alpha_i$  in each of the  $SU(2)_A \times SU(2)_B$  Higgsless models we have been considering. The values are given in Tables I and II. These values are consistent

TABLE I. Longhitano's parameters in  $SU(2) \otimes SU(2)$  flat Higgsless models for the cases of brane-localized fermions and ideally delocalized fermions.

Flat $SU(2) \times SU(2)$		
Longhitano parameters	Brane localized	Ideally delocalized
$e^2 \alpha_1$	$-\frac{2}{3} \lambda s^2$	0
$e^2 \alpha_2$	$-\frac{1}{12} \frac{(7+\kappa)}{(1+\kappa)} \lambda s^2$	$-\frac{1}{12} \frac{(7+\kappa)}{(1+\kappa)} \lambda s^2$
$e^2 \alpha_3$	$-\frac{1}{12} \frac{(1+7\kappa)}{(1+\kappa)} \lambda s^2$	$\frac{1}{12} \frac{(7+\kappa)}{(1+\kappa)} \lambda s^2$
$e^2 \alpha_4$	$\frac{1}{30} \frac{(1+14\kappa+\kappa^2)}{(1+\kappa)^2} \lambda s^2$	$\frac{1}{30} \frac{(1+14\kappa+\kappa^2)}{(1+\kappa)^2} \lambda s^2$
$e^2 \alpha_5$	$-\frac{1}{30} \frac{(1+14\kappa+\kappa^2)}{(1+\kappa)^2} \lambda s^2$	$-\frac{1}{30} \frac{(1+14\kappa+\kappa^2)}{(1+\kappa)^2} \lambda s^2$

TABLE II. Longhitano's parameters in  $SU(2) \otimes SU(2)$  warped Higgsless models for the cases of the brane-localized fermions and the ideally delocalized fermions.

AdS <sub>5</sub> $SU(2) \times SU(2)$		
Longhitano parameters	Brane localized	Ideally delocalized
$e^2 \alpha_1$	$-\frac{3s^2}{4} \frac{1}{(1+\kappa)} \left(\frac{2}{b}\right)$	0
$e^2 \alpha_2$	$-\frac{s^2}{12} \frac{7+2\kappa}{(1+\kappa)^2} \left(\frac{2}{b}\right)$	$-\frac{s^2}{12} \frac{7+2\kappa}{(1+\kappa)^2} \left(\frac{2}{b}\right)$
$e^2 \alpha_3$	$-\frac{s^2}{12} \frac{2+7\kappa}{(1+\kappa)^2} \left(\frac{2}{b}\right)$	$\frac{s^2}{12} \frac{7+2\kappa}{(1+\kappa)^2} \left(\frac{2}{b}\right)$
$e^2 \alpha_4$	$\frac{s^2}{24} \frac{1+9\kappa+\kappa^2}{(1+\kappa)^3} \left(\frac{2}{b}\right)$	$\frac{s^2}{24} \frac{1+9\kappa+\kappa^2}{(1+\kappa)^3} \left(\frac{2}{b}\right)$
$e^2 \alpha_5$	$-\frac{s^2}{24} \frac{1+9\kappa+\kappa^2}{(1+\kappa)^3} \left(\frac{2}{b}\right)$	$-\frac{s^2}{24} \frac{1+9\kappa+\kappa^2}{(1+\kappa)^3} \left(\frac{2}{b}\right)$

with several symmetry considerations. First,  $\alpha_2 \equiv -L_{9R}/2$  is the coefficient of an operator that is not related to the  $SU(2)_W$  properties of the model; as such, this coefficient should be unaffected by the degree of delocalization of the  $SU(2)_W$  properties of the fermions. Indeed, we see that  $\alpha_2$  is the same for both the brane-localized and ideal fermions. Conversely, we expect the values of  $\alpha_1$  and  $\alpha_3$  to be sensitive to the  $SU(2)_W$  properties of the fermions and this is observed in our results, yielding examples of theories in which  $\alpha_2 \neq \alpha_3$  (or  $L_{9R} \neq L_{9L}$ ). Third, in the limit where  $\kappa \rightarrow 1$ , the models with brane-localized fermions should display an  $A \leftrightarrow B$  parity; this is consistent with the fact that  $\alpha_2 = \alpha_3$  for  $\kappa = 1$ . Finally, since  $\Delta \kappa_\gamma \equiv 0$ , we find  $\alpha_2 = -\alpha_3$  for the case of ideal delocalization, in which  $\alpha_1 = 0$ .

We have also evaluated the chiral Lagrangian parameters for  $SU(2)$  Higgsless models. A flat-space  $SU(2)$  model defined on the interval  $0 \leq z \leq 2\pi R$  is simply the  $\kappa = 1$  limit of the  $SU(2)_A \times SU(2)_B$  model considered here; the values of the  $\alpha_i$  may be read fairly easily from Table I. The  $\alpha_i$  for a warped-space  $SU(2)$  model are given in Table III. The points where the derivation of results for the warped-space  $SU(2)$  model differs from the analysis of  $SU(2)_A \times SU(2)_B$  are covered in Appendix B.

TABLE III. Longhitano's parameters in  $SU(2)$  warped Higgsless models for the cases of brane-localized fermions and ideally delocalized fermions.

AdS <sub>5</sub> $SU(2)$		
Longhitano parameters	Brane localized	Ideally delocalized
$e^2 \alpha_1$	$-\frac{s^2}{4} \left(\frac{2}{b}\right)$	0
$e^2 \alpha_2$	$-\frac{s^2}{12} \left(\frac{2}{b}\right)$	$-\frac{s^2}{12} \left(\frac{2}{b}\right)$
$e^2 \alpha_3$	$-\frac{s^2}{6} \left(\frac{2}{b}\right)$	$\frac{s^2}{12} \left(\frac{2}{b}\right)$
$e^2 \alpha_4$	$\frac{s^2}{24} \left(\frac{2}{b}\right)$	$\frac{s^2}{24} \left(\frac{2}{b}\right)$
$e^2 \alpha_5$	$-\frac{s^2}{24} \left(\frac{2}{b}\right)$	$-\frac{s^2}{24} \left(\frac{2}{b}\right)$

## VI. COLLIDER PHENOMENOLOGY

### A. Triple-gauge vertices

For a model with brane-localized fermions, the nonzero value of  $\alpha S$  provides a strong constraint on the mass of the lightest  $KK$  resonance [24]. However, in models with ideally delocalized fermions,  $\alpha S = 0$ . In these models, experimental constraints from measurements of the triple-gauge-boson vertices can provide valuable bounds on the  $KK$  masses.

Currently, the strongest bounds on  $\Delta g_1^Z$  and  $\Delta \kappa_Z$  come from LEP II. The 95% C.L. upper limit (recalling that  $\Delta g_1^Z$  is positive in our models) is  $\Delta g_1^Z \leq 0.028$  [57]. We can estimate the degree to which this constrains Higgsless models with ideal delocalization by considering how  $\Delta g_1^Z$  is related to the mass of the lightest  $KK$  resonance.

For an  $SU(2)_A \times SU(2)_B$  flat-space model, the form of  $\Delta g_1^Z$  is shown in Eq. (94) to depend on  $\lambda$ . From Eqs. (37) and (39), we see that  $\lambda \approx (\pi R M_W)^2$ ; furthermore, the mass of the lightest  $KK$  resonance is  $M_{W_1} \approx 1/2R$ , independent<sup>7</sup> of  $\kappa$ , so we find that

$$\Delta g_1^Z = \frac{\pi^2}{12c^2} \left( \frac{M_W}{M_{W_1}} \right)^2 \left[ \frac{1}{4} \cdot \frac{7 + \kappa}{1 + \kappa} \right] \quad (117)$$

where the factor in square brackets equals 1 for  $\kappa = 1$ . In an  $SU(2)_A \times SU(2)_B$  model in  $AdS_5$  space, Eq. (96) shows that  $\Delta g_1^Z$  depends on  $b$ . In this model,  $1/b \approx \frac{1}{4}(1 + \kappa) \times (M_W R')^2$  and  $R' = x_1/M_{W_1}$  (again, independent<sup>7</sup> of  $\kappa$ ) where  $x_1 \approx 2.4$  is the first zero of the Bessel function  $J_0$ . Putting this all together, we have

$$\Delta g_1^Z = \frac{3x_1^2}{16c^2} \left( \frac{M_W}{M_{W_1}} \right)^2 \left[ \frac{2}{9} \cdot \frac{7 + 2\kappa}{1 + \kappa} \right] \quad (118)$$

where the factor in square brackets equals 1 for  $\kappa = 1$ .

Inserting numerical values for  $M_W$ ,  $c$ , and  $x_1$  and denoting the 95% C.L. experimental upper bound on  $\Delta g_1^Z$  as  $\Delta g_{\max}$ , we find the bound

$$M_{W_1} \geq \sqrt{\frac{6900}{\Delta g_{\max}}} \left[ \frac{1}{4} \cdot \frac{7 + \kappa}{1 + \kappa} \right] \text{ GeV} \quad (\text{flat}), \quad (119)$$

$$M_{W_1} \geq \sqrt{\frac{9100}{\Delta g_{\max}}} \left[ \frac{2}{9} \cdot \frac{7 + 2\kappa}{1 + \kappa} \right] \text{ GeV} \quad (AdS_5). \quad (120)$$

The LEP II data therefore implies a 95% C.L. lower bound of 500 GeV (570 GeV) on the first  $KK$  resonance in flat (warped) space models for  $\kappa = 1$  and lower bounds of 250–650 GeV (380–700 GeV) as  $\kappa$  varies from  $\infty$  to 0.

<sup>7</sup>In the limit where  $g_0 = 0$  it is straightforward to see that there are two ways for a  $KK$  resonance profile to satisfy the boundary conditions at  $z = \pi R$  (flat) or  $z = R'$  (warped). Either the profile can vanish at the boundary, or its  $z$ -derivative can vanish there. In neither case does the mass of the resonance depend on  $\kappa$ .

Future experiments will improve the limits on  $\Delta g_1^Z$  and  $\Delta \kappa_Z$  significantly. An analysis of  $WZ$  production at the LHC by Dobbs [58] including both systematic and statistical effects finds that with  $30 \text{ fb}^{-1}$  of integrated luminosity it should be possible to set a 95% C.L. bound of  $-0.0086 < \Delta g_1^Z < 0.011$ . The limit on  $\Delta \kappa_Z$  is expected to be significantly looser, as are likely limits from single electroweak gauge-boson production at LHC [59]. It therefore appears that LHC would be sensitive to  $KK$  resonances up to 790 GeV for a flat-space model and 960 GeV for an  $AdS_5$  space model for  $\kappa = 1$ . Reference [60] finds that a linear electron-positron collider with polarized beams should be sensitive to both  $\Delta g_1^Z$  and  $\Delta \kappa_Z$ . The anticipated  $2\sigma$  upper bounds on  $\Delta g_1^Z$  are 0.0048 (0.0027) for a 500 GeV (800 GeV) collider; for  $\Delta \kappa_Z$  the anticipated limit is 0.00098 (0.00042) for a 500 GeV (800 GeV) collider. Thus, a 500 GeV (800 GeV) linear collider would be sensitive to a  $KK$  resonance of mass up to 2.6 TeV (4 TeV) in a flat-space model and up to 3.2 TeV (4.9 TeV) in a warped-space model for  $\kappa = 1$ .

A similar analysis reveals that neither the LHC nor a linear collider [61] will be able to probe the quartic gauge-boson vertices *per se* because the masses of the  $KK$  resonances required to yield visible values of  $\alpha_4 = -\alpha_5$  are similar in magnitude to the relevant subprocess scattering energies. Rather, one would look directly for resonances in elastic  $WW$  or  $WZ$  scattering (as discussed briefly below).

### B. Direct searches

It is interesting to note Higgsless models with ideally delocalized fermions cannot be constrained by direct collider searches for new gauge bosons that rely on the bosons' couplings to fermions. As we have seen, the higher  $KK$  resonances are fermiophobic to leading order (their coupling to fermions is suppressed by  $M_W^2/M_{W_{(n)}}^2$ ).

Existing searches for  $W'$  bosons [62] assume that the  $W'$  bosons couple to ordinary quarks and leptons (generally with SM strength). All assume that the  $W'$  is produced via these couplings; all but one also assume that the  $W'$  decays only to fermions and that the  $W' \rightarrow WZ$  decay channel is unavailable. However, by construction, the ideally delocalized fermions in our Higgsless models have no coupling to the  $KK$  resonances  $W_{(n \geq 1)}$ . Hence, none of the current searches apply. Proposed future searches via the Drell-Yan process at the LHC, or via  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  [63] and  $e\gamma \rightarrow \nu q + X$  [64] at a linear collider, also rely on  $W'$  couplings to fermions and will not apply either.

Likewise, existing direct searches for  $Z'$  bosons [62] rely on  $Z'f\bar{f}$  couplings and assume that the decay channel  $Z' \rightarrow W^+W^-$  is not available; as such, they do not constrain the  $Z_{(n \geq 1)}$  of our Higgsless models with ideally delocalized fermions. Proposed future searches at LHC and the linear collider that rely on fermionic couplings for production or decay of the  $Z'$  will not apply either.

The only way to perform a direct search for the  $W_{(n \geq 1)}$  and  $Z_{(n \geq 1)}$  states will be to study  $WW$  or  $WZ$  elastic scattering. If no resonances are seen, these processes will also afford the opportunity to constrain the values of the chiral Lagrangian parameters  $\alpha_4$  and  $\alpha_5$ . We will comment further on vector boson scattering in Higgsless models in future work [56].

## VII. CONCLUSIONS

In this paper we have extended the analysis of Higgsless models with ideal delocalization in several ways. We have computed the form of the triple and quartic gauge-boson vertices in these models and related them to the parameters of the electroweak chiral Lagrangian. As contributions to the  $S$  parameter vanish to leading order, current constraints on these models arise from limits on deviations in multi-gauge-boson vertices and these constraints were shown to provide lower bounds of the order of a few hundred GeV on the masses of the lightest  $KK$  resonances above the  $W$  and  $Z$  bosons. We also studied the collider phenomenology of the  $KK$  resonances in models with ideal delocalization. We showed that these resonances are fermiophobic—therefore traditional direct collider searches are not sensitive to them and measurements of gauge-boson scattering will be needed to find them.

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## APPENDIX A: NORMALIZATION OF FERMION WAVE FUNCTION

The 5D action of the  $\partial^\mu \gamma_\mu$  portion of the fermionic kinetic term takes the following form [65]:

$$i \int d^4x dz N^\psi(z) \bar{\Psi}(x, z) \gamma^\mu \partial_\mu \Psi(x, z), \quad (\text{A1})$$

where  $N^\psi(z)$  is the factor which is determined from  $\sqrt{g}$  and the ‘‘fünfbein’’ in the specific metric. [ $N^\psi(z) = (\frac{R}{z})^4$  for the case of warped metric.]

Gauge invariance requires that the coupling of fermionic current to the gauge boson  $W_\mu(x, z)$  takes the form of

$$\int d^4x dz N^\psi(z) \bar{\Psi}(x, z) \gamma^\mu W_\mu(x, z) \Psi(x, z). \quad (\text{A2})$$

The  $KK$ -decomposition of  $W_\mu(x, z)$  and  $\Psi(x, z)$  are ex-

pressed as follows:

$$W_\mu(x, z) = \sum_n W_\mu^{(n)}(x) \chi_{W_{(n)}}(z), \quad (\text{A3})$$

$$\Psi(x, z) = \sum_n \psi^{(n)}(x) \chi_{\psi_{(n)}}(z). \quad (\text{A4})$$

If we require the 4D fermion kinetic term be canonically normalized,  $\chi_{\psi_{(n)}}(z)$  must be normalized by the following condition:

$$\int dz N^\psi(z) |\chi_{\psi_{(n)}}(z)|^2 = 1. \quad (\text{A5})$$

In this case, couplings of the fermion zero-mode  $\psi^{(0)}(x)$  to  $W_\mu^{(n)}$  bosons are expressed as follows:

$$g_{W_{(n)}} = \int dz N^\psi(z) |\chi_{\psi_{(0)}}(z)|^2 \chi_{W_{(n)}}. \quad (\text{A6})$$

Since weight function  $N^\psi(z)$  appears in both Eqs. (A5) and (A6), it is convenient to include  $N^\psi(z)$  in the definition of fermion wave function:

$$|\psi(z)|^2 \equiv N^\psi(z) |\chi_{\psi_{(0)}}(z)|^2. \quad (\text{A7})$$

Then, the normalization condition and couplings are expressed as follows:

$$\int dz |\psi(z)|^2 = 1, \quad (\text{A8})$$

$$g_{W_{(n)}} = \int dz |\psi(z)|^2 \chi_{W_{(n)}}. \quad (\text{A9})$$

The orthonormal conditions for  $\chi_{W_{(n)}}$  are given by

$$\int dz N^W(z) \chi_{W_{(n)}} \chi_{W_{(m)}} = \delta_{n,m}, \quad (\text{A10})$$

therefore the profile of ideally delocalized fermion is expressed as

$$|\psi_{\text{ideal}}(z)|^2 \propto N^W(z) \chi_{W_{(0)}}(z), \quad (\text{A11})$$

where the overall normalization constant is determined from Eq. (A8).

## APPENDIX B: $SU(2)$ MODEL IN $\text{AdS}_5$

This appendix sketches the analysis of the  $SU(2)$  Higgsless modes in warped space, focusing on where it differs from that of the  $SU(2) \times SU(2)$  models discussed in the text. We start from the 5D action of an  $SU(2)$  gauge theory, using the  $\text{AdS}_5$  metric of Eq. (16)

$$S_{5D} = \int_R^{R'} dz \frac{R}{z g_5^2} \int d^4x \left[ -\frac{1}{4} \eta^{\mu\alpha} \eta^{\nu\beta} W_{\mu\nu}^a W_{\alpha\beta}^a + \frac{1}{2} \eta^{\mu\nu} W_{\mu z}^a W_{\nu z}^a \right]. \quad (\text{B1})$$

The boundary conditions are taken as

$$\begin{aligned} \partial_z W_\mu^{1,2,3}(x, z)|_{z=R} = 0, \quad W_\mu^{1,2}(x, z)|_{z=R'} = 0, \\ \partial_z W_\mu^3(x, z)|_{z=R'} = 0, \end{aligned} \quad (\text{B2})$$

in order to make the theory consistent with the standard model symmetry breaking structure  $SU(2) \times U(1) \rightarrow U(1)$ . To ensure a nontrivial weak mixing angle, we further introduce a brane kinetic term at  $z = R'$ ,

$$\begin{aligned} S_{\text{TeV}} = \int_R^{R'} dz \frac{1}{g_Y^2} \delta(z - R' + \epsilon) \\ \times \int d^4x \left[ -\frac{1}{4} \eta^{\mu\alpha} \eta^{\nu\beta} W_{\mu\nu}^3 W_{\alpha\beta}^3 \right], \quad (\epsilon \rightarrow 0+). \end{aligned} \quad (\text{B3})$$

We also assume that the  $U(1)_Y$  fermion coupling is localized at the  $z = R'$  brane.

The 5D field  $W_\mu(x, z)$  can be decomposed into  $KK$ -modes, whose mode functions  $\chi$  obey differential equations and  $z = R$  boundary conditions identical to those of  $\chi_W^A$  in the  $SU(2) \times SU(2)$  model. The  $z = R'$  boundary conditions,

$$\begin{aligned} 0 = \chi_{W(n)}(z)|_{z=R'}, \quad 0 = \partial_z \chi_\gamma(z)|_{z=R'}, \\ 0 = \partial_z \chi_{Z(n)}(z) - \frac{R' g_Y^2}{R g_Y^2} M_{Z(n)}^2 \chi_{Z(n)}(z) \Big|_{z=R'}, \end{aligned} \quad (\text{B4})$$

reflect the hypercharge brane kinetic term.

The expressions for  $e$ ,  $\chi_\gamma$ , and  $s_0$  are obtained just as in the main text, and differ only in the absence of terms involving  $g_{5B}^2$ .

The  $W$  boson mass is found to be

$$(M_W R')^2 = \frac{4}{b} \left( 1 + \frac{3}{2b} \right) + \mathcal{O}\left(\frac{1}{b^3}\right) \quad (\text{B5})$$

and the profile is

$$\begin{aligned} \chi_W(z) = \frac{e}{s_0} \left[ 1 + \frac{3}{8} \frac{2}{b} \right] \left[ 1 - \frac{1}{2} \left( \ln \frac{z}{R} - \frac{1}{2} \right) (M_W z)^2 \right. \\ \left. + \frac{1}{16} \left( \ln \frac{z}{R} \right) (M_W z)^4 \right]. \end{aligned} \quad (\text{B6})$$

Note that, as one would expect, this mass and profile agree with those of  $W^A$  in the  $SU(2) \times SU(2)$  model when one sets  $\kappa = 0$ . The mass and profile of the  $Z$  boson can likewise be obtained:

$$(M_Z R')^2 = \frac{4}{b} \frac{1}{c_0^2} \left[ 1 + \frac{1}{2b} \left( 3 - \frac{s_0^2}{c_0^2} \right) \right] + \mathcal{O}\left(\frac{1}{b^3}\right), \quad (\text{B7})$$

$$\begin{aligned} \chi_Z(z) = e \frac{c_0}{s_0} \left[ 1 + \frac{3}{8c_0^2} \frac{2}{b} \right] \left[ 1 - \frac{1}{2} \left( \ln \frac{z}{R} - \frac{1}{2} \right) (M_Z z)^2 \right. \\ \left. + \frac{1}{16} \left( \ln \frac{z}{R} \right) (M_Z z)^4 \right]. \end{aligned} \quad (\text{B8})$$

These reflect the influence of a hypercharge brane kinetic term at  $z = R'$  rather than at  $z = R$ .

The calculations of the fermion profiles,  $g_W$  and  $G_F$ , follow the route laid out in the main text. For brane-localized fermions, these quantities depend only on  $\chi_W^A$  in the  $SU(2) \times SU(2)$  model and taking the  $\kappa = 0$  limit yields their values in the  $SU(2)$  model. For ideally delocalized fermions, we find

$$C_{\text{ideal}} = g_W = [4\sqrt{2} G_F M_W^2]^{1/2} = \frac{e}{s_0} \left( 1 + \frac{1}{8} \frac{2}{b} \right). \quad (\text{B9})$$

The value of  $\Delta$  relating  $s_0$  to  $s_Z$  is

$$\Delta = -\frac{s^2}{4} \frac{3c^2 - s^2}{c^2 - s^2} \frac{2}{b} \quad (\text{brane localized}), \quad (\text{B10})$$

$$\Delta = -\frac{s^2}{4} \frac{2}{b} \quad (\text{ideal}). \quad (\text{B11})$$

Performing the appropriate integrals yields the multi-gauge-boson vertex couplings

$$g_{\gamma WW}^2 = e^2, \quad (\text{B12})$$

$$g_{Z WW}^2 = \frac{c_0^2}{s_0^2} e^2 \left[ 1 + \frac{5}{12c^2} \frac{2}{b} \right], \quad (\text{B13})$$

$$g_{WWW} = \frac{e^2}{s_0^2} \left[ 1 + \frac{11}{24} \frac{2}{b} \right], \quad (\text{B14})$$

and, after applying Eq. (B10) and (B11), the Hagiwara-Peccei-Zeppenfeld-Hikasa parameters emerge:

$$\Delta \kappa_\gamma = 0, \quad \Delta g_1^Z = \Delta \kappa_Z = -\frac{1 + c^2}{12c^2(c^2 - s^2)} \frac{2}{b} \quad (\text{B15})$$

(brane localized),

$$\Delta \kappa_\gamma = 0, \quad \Delta g_1^Z = \Delta \kappa_Z = \frac{1}{12c^2} \frac{2}{b} \quad (\text{ideal}). \quad (\text{B16})$$

From these, the values of the  $\alpha_i$  listed in Table III are readily derived. Note that  $\alpha_4 = -\alpha_5$  matches the  $\kappa = 0$  value for the warped  $SU(2) \times SU(2)$  model because  $g_{WWW}$  depends only on  $\chi_W^A$  in the  $SU(2) \times SU(2)$  model at  $\kappa = 0$ .

- [1] C. Csaki, C. Grojean, H. Murayama, L. Pilo, and J. Terning, *Phys. Rev. D* **69**, 055006 (2004).
- [2] P. W. Higgs, *Phys. Lett.* **12**, 132 (1964).
- [3] K. Agashe, A. Delgado, M. J. May, and R. Sundrum, *J. High Energy Phys.* 08 (2003) 050.
- [4] C. Csaki, C. Grojean, L. Pilo, and J. Terning, *Phys. Rev. Lett.* **92**, 101802 (2004).
- [5] R. Sekhar Chivukula, D. A. Dicus, and H.-J. He, *Phys. Lett. B* **525**, 175 (2002).
- [6] R. S. Chivukula and H.-J. He, *Phys. Lett. B* **532**, 121 (2002).
- [7] R. S. Chivukula, D. A. Dicus, H.-J. He, and S. Nandi, *Phys. Lett. B* **562**, 109 (2003).
- [8] H.-J. He, *Int. J. Mod. Phys. A* **20**, 3362 (2005).
- [9] G. Cacciapaglia, C. Csaki, C. Grojean, and J. Terning, *Phys. Rev. D* **70**, 075014 (2004).
- [10] Y. Nomura, *J. High Energy Phys.* 11 (2003) 050.
- [11] R. Barbieri, A. Pomarol, and R. Rattazzi, *Phys. Lett. B* **591**, 141 (2004).
- [12] H. Davoudiasl, J. L. Hewett, B. Lillie, and T. G. Rizzo, *Phys. Rev. D* **70**, 015006 (2004).
- [13] G. Burdman and Y. Nomura, *Phys. Rev. D* **69**, 115013 (2004).
- [14] H. Davoudiasl, J. L. Hewett, B. Lillie, and T. G. Rizzo, *J. High Energy Phys.* 05 (2004) 015.
- [15] R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, *Nucl. Phys.* **B703**, 127 (2004).
- [16] J. L. Hewett, B. Lillie, and T. G. Rizzo, *J. High Energy Phys.* 10 (2004) 014.
- [17] R. Foadi, S. Gopalakrishna, and C. Schmidt, *J. High Energy Phys.* 03 (2004) 042.
- [18] J. Hirn and J. Stern, *Eur. Phys. J. C* **34**, 447 (2004).
- [19] R. Casalbuoni, S. De Curtis, and D. Dominici, *Phys. Rev. D* **70**, 055010 (2004).
- [20] R. S. Chivukula, E. H. Simmons, H. J. He, M. Kurachi, and M. Tanabashi, *Phys. Rev. D* **70**, 075008 (2004).
- [21] M. Perelstein, *J. High Energy Phys.* 10 (2004) 010.
- [22] R. S. Chivukula, E. H. Simmons, H.-J. He, M. Kurachi, and M. Tanabashi, *Phys. Lett. B* **603**, 210 (2004).
- [23] H. Georgi, *Phys. Rev. D* **71**, 015016 (2005).
- [24] R. Sekhar Chivukula, E. H. Simmons, H. J. He, M. Kurachi, and M. Tanabashi, *Phys. Rev. D* **71**, 035007 (2005).
- [25] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, *Phys. Rev. Lett.* **86**, 4757 (2001).
- [26] C. T. Hill, S. Pokorski, and J. Wang, *Phys. Rev. D* **64**, 105005 (2001).
- [27] M. E. Peskin and T. Takeuchi, *Phys. Rev. D* **46**, 381 (1992).
- [28] G. Altarelli and R. Barbieri, *Phys. Lett. B* **253**, 161 (1991).
- [29] G. Altarelli, R. Barbieri, and S. Jadach, *Nucl. Phys.* **B369**, 3 (1992).
- [30] H. Georgi, *Nucl. Phys.* **B266**, 274 (1986).
- [31] G. Cacciapaglia, C. Csaki, C. Grojean, and J. Terning, *Phys. Rev. D* **71**, 035015 (2005).
- [32] R. Foadi, S. Gopalakrishna, and C. Schmidt, *Phys. Lett. B* **606**, 157 (2005).
- [33] G. Cacciapaglia, C. Csaki, C. Grojean, M. Reece, and J. Terning, hep-ph/0505001.
- [34] R. S. Chivukula, E. H. Simmons, H. J. He, M. Kurachi, and M. Tanabashi, *Phys. Rev. D* **71**, 115001 (2005).
- [35] R. Casalbuoni, S. De Curtis, D. Dolce, and D. Dominici, *Phys. Rev. D* **71**, 075015 (2005).
- [36] R. Sekhar Chivukula, E. H. Simmons, H. J. He, M. Kurachi, and M. Tanabashi, *Phys. Rev. D* **72**, 015008 (2005).
- [37] D. T. Son and M. A. Stephanov, *Phys. Rev. D* **69**, 065020 (2004).
- [38] R. S. Chivukula, M. Kurachi, and M. Tanabashi, *J. High Energy Phys.* 06 (2004) 004.
- [39] T. Sakai and S. Sugimoto, *Prog. Theor. Phys.* **113**, 843 (2005).
- [40] J. Erlich, E. Katz, D. T. Son, and M. A. Stephanov, hep-ph/0501128.
- [41] L. Da Rold and A. Pomarol, *Nucl. Phys.* **B721**, 79 (2005).
- [42] J. Hirn and V. Sanz, hep-ph/0507049.
- [43] T. Sakai and S. Sugimoto, hep-th/0507073.
- [44] K. Hagiwara, R. D. Peccei, D. Zeppenfeld, and K. Hikasa, *Nucl. Phys.* **B282**, 253 (1987).
- [45] T. Appelquist and C. W. Bernard, *Phys. Rev. D* **22**, 200 (1980).
- [46] A. C. Longhitano, *Nucl. Phys.* **B188**, 118 (1981).
- [47] A. C. Longhitano, *Phys. Rev. D* **22**, 1166 (1980).
- [48] T. Appelquist, Print-80-0832 (Yale), based on lectures presented at the 21st Scottish Universities Summer School in Physics, St. Andrews, Scotland, 1980.
- [49] T. Appelquist and G. H. Wu, *Phys. Rev. D* **48**, 3235 (1993).
- [50] J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* **158**, 142 (1984).
- [51] H. Georgi, hep-ph/0508014.
- [52] A. Birkedal, K. Matchev, and M. Perelstein, *Phys. Rev. Lett.* **94**, 191803 (2005).
- [53] R. S. Chivukula, E. H. Simmons, H. J. He, M. Kurachi, and M. Tanabashi, hep-ph/0509110.
- [54] B. Holdom, *Phys. Lett. B* **258**, 156 (1991).
- [55] A. F. Falk, M. E. Luke, and E. H. Simmons, *Nucl. Phys.* **B365**, 523 (1991).
- [56] R. S. Chivukula, E. H. Simmons, H.-J. He, M. Kurachi, and M. Tanabashi (to be published).
- [57] The LEP Collaborations ALEPH, DELPHI, L3, OPAL and the LEP TGC Working Group. LEPEWWG/TC/2005-01, 2005.
- [58] M. Dobbs, AIP Conf. Proc. **753**, 181 (2005).
- [59] O. J. P. Eboli and M. C. Gonzalez-Garcia, *Phys. Rev. D* **70**, 074011 (2004).
- [60] W. Menges, LC-PHSM-2001-022.
- [61] E. Boos, H. J. He, W. Kilian, A. Pukhov, C. P. Yuan, and P. M. Zerwas, *Phys. Rev. D* **61**, 077901 (2000).
- [62] S. Eidelman *et al.* (Particle Data Group), *Phys. Lett. B* **592**, 1 (2004) and 2005 partial updates for edition 2006 (URL: <http://pdg.lbl.gov>).
- [63] S. Godfrey, P. Kalyniak, B. Kamal, and A. Leike, *Phys. Rev. D* **61**, 113009 (2000).
- [64] S. Godfrey, P. Kalyniak, B. Kamal, M. A. Doncheski, and A. Leike, *Phys. Rev. D* **63**, 053005 (2001).
- [65] C. Csaki, C. Grojean, J. Hubisz, Y. Shirman, and J. Terning, *Phys. Rev. D* **70**, 015012 (2004).