Relativistic effects in the processes of heavy quark fragmentation

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In the framework based on the quasipotential method and relativistic quark model a new covariant expression for the heavy quark fragmentation amplitude to fragment into the pseudoscalar and vector S-wave heavy mesons is obtained. It contains all possible relativistic corrections including the terms connected with the transformation law of the bound state wave function to the reference frame of the moving meson. Relativistic corrections of order $\mathbf{p}^2/m^2$ to the heavy quark fragmentation distributions into $(\bar{c}c)$, $(\bar{b}c)$, $(\bar{b}b)$ states are calculated as functions of the longitudinal momentum fraction z and the transverse momentum p_T relative to the jet axis.

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I. INTRODUCTION

The decay and production processes of the bound states with heavy quarks have been investigated with greater intensity in the last few years. The research aims of many experiments (ALEPH, DELPHI, SLD, CLEO, Belle, SELEX, LHC-b) are directed towards the growth of experimental accuracy in the derivation of the static characteristics of heavy hadrons, and their production and decay rates in different reactions [1–6]. The production of heavy mesons and baryons via heavy quark fragmentation in the e^+e^- annihilation represents one of the possible mechanisms for the formation of heavy hadrons with two heavy quarks. The fragmentation cross sections for the production of heavy hadrons can be calculated in an analytical form using the factorization hypothesis. The heavy quark production amplitude can be calculated on the basis of perturbative QCD. The characteristic quark virtualities of heavy quarks in the hard production are of the order of their masses while the quark virtualities in the bound state are much less than their masses due to the nonrelativistic motion. So, the total amplitude can be represented as a convolution of the hard transition amplitude with a nonperturbative factor (the wave function) determining the transition of free heavy quarks into a bound state [7]. The fragmentation mechanism was used for the study of the production processes of heavy mesons and baryons in e^+e^- annihilation in Refs. [8–15] (a more complete list of references can be found in Refs. [2,6,13]). The growth of theoretical accuracy for the calculation of corresponding production cross sections can be reached in two ways. First, it is necessary to take into account radiative corrections to the perturbative amplitude describing the production of free heavy quarks via heavy quark fragmentation. Second, we must consistently consider the relativistic corrections in the fragmentation amplitude connected with the relative motion of heavy quarks forming heavy hadrons. From the point of view of nonrelativistic QCD (NRQCD) both effects are caused by the matrix elements as a function of the typical heavy quark velocity in the bound state rest frame of orders $O(v_Q)$ and $O(v_Q^2)$, respectively [11]. The experimental data indicate that the calculations of different production probabilities for heavy quarkonium and double heavy baryons should be improved by a systematic account of relativistic corrections. Such effects as the relative motion of heavy quarks forming heavy quarkonia and diquarks and the diquark structure effects in the calculation of the fragmentation functions of heavy diquarks should be considered. The role of relativistic effects was studied already in the processes of c-quark fragmentation into J/Ψ, η_c in Ref. [16] on the basis of the Bethe-Salpeter approach, in the gluon fragmentation into S-wave quarkonium in Ref. [17], and in the inclusive production of polarized J/Ψ from b-quark decay in Ref. [18]. The consideration of the intrinsic motion of quarks forming the heavy mesons can explain the discrepancy between theoretical predictions and experimental data for the cross sections of the process $e^+e^- \to \Psi\eta_c$ [19,20]. The aim of the present work is to get a systematically improved description of the relativistic effects in the processes of the heavy quark fragmentation in the quasipotential approach [21]. Our goal also consists of the calculation of the relativistic corrections in heavy quark b and c fragmentation functions into pseudoscalar and vector heavy mesons $(Q_1\bar{Q}_2)$ on the basis of the relativistic quark model used earlier in the calculation of mass spectra of heavy mesons and baryons and their decay rates in different reactions [22–24]. In particular, we investigate double distribution probabilities for the heavy quark fragmentation over longitudinal meson momentum z and transverse meson momentum p_T. Analytical expressions for the fragmentation probabilities as functions of transverse momentum of heavy mesons $(Q_1\bar{Q}_2)$ are obtained.

II. GENERAL FORMALISM

The heavy meson production through the heavy quark fragmentation is shown in Fig. 1. In the first stage the Z^0 boson decays into a quark-antiquark pair with four-

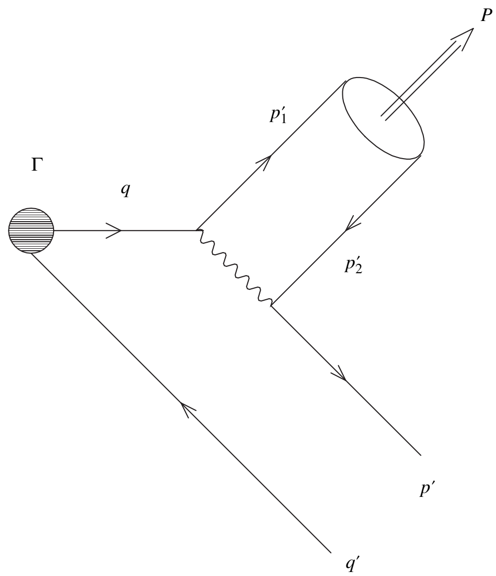

FIG. 1. The Feynman diagram for the fragmentation of a heavy quark Q_1 with a four-momentum q to a heavy meson $(Q_1\bar{Q}_2)$ with a four-momentum P. Γ is the vertex function determining the production of the quark-antiquark pair in the Z^0 decay.

momenta q and q', respectively. After that, one heavy quark with the four-momentum q fragments to the heavy quarkonium. In the quasipotential approach the invariant transition amplitude of a heavy quark b or c into a heavy meson can be expressed as a simple convolution of a perturbative production amplitude $T(p_1', p_2', p', q')$ of free quarks and the quasipotential wave function of the bound state $(Q_1\bar{Q}_2)$ $\Psi_P(\mathbf{p})$ [21,25]:

$$M(q, P, p', q') = \int \frac{d\mathbf{p}}{(2\pi)3} \bar{\Psi}_P(\mathbf{p}) T(p_1', p_2', p', q'), \quad (1)$$

where four-momenta of fragmenting quarks (b, c) and spectator antiquarks $(\bar{b}, \bar{c})$ forming the heavy meson are defined as follows:

$$p_1' = \eta_1 P + p, \qquad p_2' = \eta_2 P - p, \quad (2)$$

where p' is the four-momentum of a free spectator quark b or c and P is the four-momentum of the heavy meson. The coefficients $\eta_{1,2}$ in the definition (2) are taken in such a way that the following orthogonality condition is fulfilled:

$$(p \cdot P) = 0, \qquad \eta_{1,2} = \frac{M^2 - m_{2,1}^2 + m_{1,2}^2}{2M^2}. \quad (3)$$

$M = (m_1 + m_2 + W)$ is the bound state mass.

The transition of the pair of a heavy quark and antiquark into color-singlet mesons can be envisioned as a complicated process in which the colors and spins of the heavy quark and antiquark play an important role. Different color-spin nonperturbative factors entering the amplitude $T(p_1, p_2, p', q')$ control the production of the heavy quark bound states. In this process the gluon virtuality $k^2 \gg \Lambda_{QCD}^2$ and the strong coupling constant $\alpha_s(k^2) \ll 1$. Then the hard part of the fragmentation amplitude (1) in the leading order over α_s takes the form

$$T(p_1', p_2', p', q') = \frac{4\alpha_s}{3\sqrt{3}} \frac{D_{\lambda\sigma}(k)}{(s - m_1^2)} \bar{u}_1(p_1')\gamma_\lambda(\hat{q} + m_1) \times \Gamma v_1(q')\bar{u}_2(p')\gamma_\sigma v_2(p_2'), \quad (4)$$

where Γ is the vertex function for the transition of the Z^0 boson into the quark-antiquark pair; the gluon propagator is taken in the axial gauge with four-vector $n = (1, 0, 0, -1)$:

$$D_{\lambda\sigma}(k) = \frac{1}{k^2 + i\epsilon}\left[-g_{\lambda\sigma} + \frac{k_\sigma n_\lambda + k_\lambda n_\sigma}{k \cdot n}\right], \quad (5)$$

$s = q^2$, and $k = (q - \eta_1 P - p) = (\eta_2 P - p + p')$ is the gluon four-momentum. The color factor $(T^a)_{il} \times (T^a)_{mj}\delta_{ij}/\sqrt{3} = 4\delta_{ml}/3\sqrt{3}$ was already extracted in the amplitude (4). The transformation law of the bound state wave functions from the rest frame to the moving one with four-momenta P is given by [23–25]

$$\Psi_P^{\rho\omega}(\mathbf{p}) = D_1^{1/2,\rho\alpha}(R_{L_P}^W) D_2^{1/2,\omega\beta}(R_{L_P}^W)\Psi_0^{\alpha\beta}(\mathbf{p}),$$
$$\bar{\Psi}_P^{\lambda\sigma}(\mathbf{p}) = \bar{\Psi}_0^{\varepsilon\tau}(\mathbf{p}) D_1^{+1/2,\varepsilon\lambda}(R_{L_P}^W) D_2^{+1/2,\tau\sigma}(R_{L_P}^W), \quad (6)$$

where R^W is the Wigner rotation, L_P is the Lorentz boost from the meson rest frame to a moving one, and the rotation matrix $D^{1/2}(R)$ is defined by

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D_{1,2}^{1/2}(R_{L_P}^W) = S^{-1}(\mathbf{p}_{1,2}) S(\mathbf{P}) S(\mathbf{p}), \quad (7)$$

where

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p) + m}{2m}}\left(1 + \frac{(\boldsymbol{\alpha}\mathbf{p})}{\epsilon(p) + m}\right)$$

is the usual Lorentz transformation matrix of the four-spinor. For further transformations of the amplitude (4) the following relations are useful:

$$S_{\alpha\beta}(\Lambda) u_\beta^\lambda(p) = \sum_{\sigma=\pm 1/2} u_\alpha^\sigma(\Lambda p) D_{\sigma\lambda}^{1/2}(R_{\Lambda p}^W),$$
$$\bar{u}_\beta^\lambda(p) S_{\beta\alpha}^{-1}(\Lambda) = \sum_{\sigma=\pm 1/2} D_{\lambda\sigma}^{+1/2}(R_{\Lambda p}^W)\bar{u}_\alpha^\sigma(\Lambda p). \quad (8)$$

Using also the transformation law of the Dirac bispinors to the rest frame

$$\bar{u}_1(\mathbf{p}) = \bar{u}_1(0)\frac{(\hat{p}_1 + m_1)}{\sqrt{2\epsilon_1(\mathbf{p})(\epsilon_1(\mathbf{p}) + m_1)}}, \quad p_1 = (\epsilon_1, \mathbf{p}),$$
$$v_2(-\mathbf{p}) = \frac{(\hat{p}_2 - m_2)}{\sqrt{2\epsilon_2(\mathbf{p})(\epsilon_2(\mathbf{p}) + m_2)}} v_2(0), \quad p_2 = (\epsilon_2, -\mathbf{p}), \quad (9)$$

we can introduce the projection operators $\hat{\Pi}^{P,V}$ onto the states $(Q_1\bar{Q}_2)$ in the meson with total spin 0 and 1 as follows:

$$\hat{\Pi}^{P,V} = [v_2(0)\bar{u}_1(0)]_{S=0,1} = \gamma_5(\hat{\epsilon}^*)\frac{1+\gamma^0}{2\sqrt{2}}. \tag{10}$$

As a result the heavy quark $b(c)$ fragmentation amplitude into the mesons $(b\bar{c})$, $(b\bar{b})$, or $(c\bar{c})$ takes the form

$$M(q,P,p',q') = \frac{2\alpha_s\sqrt{2M}}{3\sqrt{6}}\int\frac{D_{\lambda\sigma}(k)}{(s-m_1^2)}\frac{d\mathbf{p}}{(2\pi)^3}\bar{\Psi}_0(\mathbf{p})\bar{u}_2(p')\gamma_\sigma\frac{(\hat{\tilde{p}}_2-m_2)}{\sqrt{2\epsilon_2(\mathbf{p})(\epsilon_2(\mathbf{p})+m_2)}}\hat{\tilde{\epsilon}}^*(v)(\hat{v}+1)$$
$$\times\frac{(\hat{\tilde{p}}_1+m_1)}{\sqrt{2\epsilon_1(\mathbf{p})(\epsilon_1(\mathbf{p})+m_1)}}\gamma_\lambda(\hat{q}+m_1)\Gamma_\alpha v_1(q'), \tag{11}$$

where the four-vectors $\tilde{\epsilon}$, $\tilde{p}_{1,2}$ are given by

$$\tilde{\epsilon} = L_P(0,\boldsymbol{\epsilon}) = \left(\boldsymbol{\epsilon}\mathbf{v}, \boldsymbol{\epsilon}+\frac{(\boldsymbol{\epsilon}\mathbf{v})\mathbf{v}}{1+v^0}\right),\qquad \hat{\tilde{p}}_{1,2} = S(L_P)\hat{p}_{1,2}S^{-1}(L_P),\qquad S(L_P)(1\pm\gamma^0)S^{-1}(L_P) = \pm(\hat{v}\pm1),\quad \hat{v}=\frac{\hat{P}}{M}. \tag{12}$$

Transforming the bispinor contractions in the numerator of the expression (11) we can find the following expression for the heavy quark fragmentation amplitude including the effects of relative motion of the heavy quarks:

$$M(q,P,p',q') = \frac{2\alpha_s\sqrt{2M}}{3\sqrt{6}}\int\frac{\bar{\Psi}_0(\mathbf{p})}{\sqrt{\frac{\epsilon_1(\mathbf{p})}{m_1}\frac{(\epsilon_1(\mathbf{p})+m_1)}{2m_1}}\sqrt{\frac{\epsilon_2(\mathbf{p})}{m_2}\frac{(\epsilon_2(\mathbf{p})+m_2)}{2m_2}}}\frac{D_{\lambda\sigma}(k)}{(s-m_1^2)}\frac{d\mathbf{p}}{(2\pi)^3}\bar{u}_2(p')\gamma_\sigma\left[\frac{\hat{v}-1}{2}+\hat{v}\frac{\mathbf{p}^2}{2m_2(\epsilon_2+m_2)}-\frac{\hat{\tilde{p}}}{2m_2}\right]$$
$$\times\hat{\tilde{\epsilon}}^*(v)(\hat{v}+1)\left[\frac{\hat{v}+1}{2}-\hat{v}\frac{\mathbf{p}^2}{2m_1(\epsilon_1+m_1)}+\frac{\hat{\tilde{p}}}{2m_1}\right]\gamma_\lambda(\hat{q}+m_1)\Gamma_\alpha v_1(q'). \tag{13}$$

The fragmentation amplitude (13) keeps at least two sources of relativistic corrections. The corrections of the first group appear from the quark-antiquark interaction operator. They can be taken into account by means of the numerical solution of the Schroedinger equation with the relevant potential. The second part of these corrections is determined by several functions depending on the momenta of the relative motion of quarks $\mathbf{p}$. In the limit of zero relative momentum $\mathbf{p}$ the amplitude $M(q,P,p',q')$ was studied in Refs. [8–10]. At last, there exist the one-loop corrections to the fragmentation amplitude which we have not considered here.

Heavy quarkonium can be characterized by the hard momentum scale m (the mass m of the heavy quarks), the soft momentum scale mv_Q, and the ultrasoft momentum scale mv_Q^2. We assume that the heavy quarkonium is a nonrelativistic system. This implies that the ratio $\mathbf{p}^2/m^2 \sim v_Q^2 \ll 1$. So, we introduce the expansion of all factors in Eq. (13) over relative momentum $\mathbf{p}$ up to terms of the second order:

$$\frac{1}{k^2} = \frac{1}{k_0^2}+\frac{1}{k_0^4}[2qp-p^2]+\frac{4}{k_0^6}(qp)^2,\quad k_0^2 = \eta_2 s+\eta_1 m_2^2-\eta_1\eta_2 M^2,$$
$$\frac{1}{kn} = \frac{1}{(qn-\eta_1 Pn)}+\frac{pn}{(qn-\eta_1 Pn)^2}+\frac{(pn)^2}{(qn-\eta_1 Pn)^3},$$
$$(\hat{\tilde{p}}_1+m_1) = m_1(\hat{v}+1)+\hat{v}\frac{\mathbf{p}^2}{2m_1}+\hat{\tilde{p}},\qquad (\hat{\tilde{p}}_2-m_2) = m_2(\hat{v}-1)+\hat{v}\frac{\mathbf{p}^2}{2m_2}-\hat{\tilde{p}},$$
$$\tilde{p} = L_P(0,\mathbf{p}) = \left(\mathbf{p}\mathbf{v}, \mathbf{p}+\frac{\mathbf{v}(\mathbf{p}\mathbf{v})}{v^0+1}\right). \tag{14}$$

Substituting the expansions (14) into Eq. (13) we obtain

$$M(q,P,p',q') = \frac{2\alpha_s\sqrt{2M}}{3\sqrt{6}}\int\bar{\Psi}_0(\mathbf{p})\left[1-\frac{3}{8}\frac{\mathbf{p}^2}{8m_1^2}-\frac{3}{8}\frac{\mathbf{p}^2}{8m_2^2}\right]\frac{1}{(s-m_1^2)}\frac{d\mathbf{p}}{(2\pi)^3}\bar{u}_2(p')\gamma_\sigma\left[\frac{\hat{v}-1}{2}+\hat{v}\frac{\mathbf{p}^2}{4m_2^2}-\frac{\hat{\tilde{p}}}{2m_2}\right]$$
$$\times\hat{\tilde{\epsilon}}^*(v)(\hat{v}+1)\left[\frac{\hat{v}+1}{2}+\hat{v}\frac{\mathbf{p}^2}{4m_1^2}+\frac{\hat{\tilde{p}}}{2m_1}\right]\gamma_\lambda(\hat{q}+m_1)\Gamma_\alpha v_1(q')\left[\frac{1}{k_0^2}+\frac{1}{k_0^4}[2qp-p^2]+\frac{4}{k_0^6}(qp)^2\right]$$
$$\times\left\{-g_{\lambda\sigma}+(k_\sigma n_\lambda+k_\lambda n_\sigma)\left[\frac{1}{(qn-\eta_1Pn)}+\frac{pn}{(qn-\eta_1Pn)^2}+\frac{(pn)^2}{(qn-\eta_1Pn)^3}\right]\right\}. \quad (15)$$

Let us emphasize that for the system of two heavy quarks, relative motion corrections entering in the gluon propagator or heavy quark propagators have the same order $O(v_Q^2)$ contrary to the system including heavy and light quarks. In the last case, leading order relativistic corrections are determined by the relativistic factors belonging to the light quark but other terms contain an additional small ratio m_q/m_Q. The obtained relation (15) which has the form of a three-dimensional integral in the momentum space is valid when the integration is restricted to the soft momentum region, where the wave function has significant support. Otherwise it would diverge at high momenta. Moreover, our aim consists of preserving here only the terms of the second order over $|\mathbf{p}|/m$, omitting corrections of higher order.

III. HEAVY QUARK FRAGMENTATION FUNCTIONS INTO P AND V MESONS

We use the NRQCD factorization approach to the calculation of the fragmentation reactions which was developed in Refs. [8,10]. The fragmentation function of the heavy quark Q_1 to produce $^1S_0$ or $^3S_1$ $(Q_1\bar{Q}_2)$ meson states is determined by the following expression:

$$D_{Q_1\to V(Q_1\bar{Q}_2)}(z) = \frac{1}{16\pi^2}\int ds\cdot\theta\left(s-\frac{M^2}{z}-\frac{m_2^2}{1-z}\right)\times\lim_{q_0\to\infty}\frac{|M|^2}{|M_0|^2}, \quad (16)$$

where q_0 is the energy of the fragmentating quark: $q_\mu=(q_0,0,0,\sqrt{q_0^2-s})$; $M_0=\bar{u}_1(q)\Gamma v_1(q')$ is the amplitude of free quark Q_1 production on the mass shell and z is the meson longitudinal momentum fraction relative to the fragmenting heavy quark. Let us consider the fragmentation production of the vector meson. Omitting the momentum of the relative motion of heavy quarks $\mathbf{p}$ in Eq. (14) we obtain the fragmentation amplitude which contains the leading order contribution and the correction due to the quark bound state energy W $(M=m_1+m_2+W)$:

$$M_1=\frac{2\sqrt{2M}\alpha_s\bar{\Psi}(0)}{3\sqrt{6}}\frac{1}{(s-m_1^2)(\eta_2s+\eta_1m_2^2-\eta_1\eta_2M^2)}\left[\bar{u}_2(p')2\hat{\tilde{\epsilon}}^*(v)(\hat{q}+m_1)\Gamma_\alpha v_1(q')\right.$$
$$+\frac{(s+\eta_2m_1M-\eta_1M^2)}{(nq-\eta_1nP)}\bar{u}_2(p')\hat{n}\hat{\tilde{\epsilon}}^*(v)(\hat{v}+1)\Gamma_\alpha v_1(q')+\frac{(m_1-\eta_1M)}{(nq-\eta_1nP)}\bar{u}_2(p')\hat{n}\hat{\tilde{\epsilon}}^*(v)(\hat{v}+1)\hat{p}'\Gamma_\alpha v_1(q')$$
$$\left.+\frac{(m_2-\eta_2M)}{(nq-\eta_1nP)}\bar{u}_2(p')\hat{n}\hat{\tilde{\epsilon}}^*(v)(\hat{v}+1)(\hat{q}+m_1)\Gamma_\alpha v_1(q')\right]. \quad (17)$$

Taking into account linear terms in the binding energy W in the expansion of Eq. (16) we next perform an averaging and summation over the meson spins in initial and final states in the square modulus $|M|^2$,

$$\frac{1}{3}\sum_{\text{spin}}\tilde{\epsilon}_\alpha^*(v)\tilde{\epsilon}_\beta(v)=\frac{1}{3}(-g_{\alpha\beta}+v_\alpha v_\beta), \quad (18)$$

and then consider the limit $q_0\to\infty$ in the obtained expression. In this limit $\hat{P}$ and $\hat{q}$ have the order of the M_Z mass and the coefficients in corresponding expressions are determined by the heavy quark masses of the fragmenting quark m_1 and the spectator quark m_2. In the leading order we can substitute $P=zq$, and the trace in $|M|^2$ over the Dirac indices is proportional to $Tr(\Gamma_\alpha\hat{q}'\Gamma_\beta\hat{q}')$. It will disappear in the ratio $|M|^2/|M_0|^2$. Then the fragmentation probability (16) can be written as a sum of two terms:

$$D_{Q_1\to V(Q_1\bar{Q}_2)}(z)=\frac{8\alpha_s^2|\Psi(0)|^2}{27m_2^3}\frac{rz(1-z)^2}{[1-(1-r)z]^6}(v_0+v_1), \quad (19)$$

$$v_0=2-2(3-2r)z+3(3-2r+4r^2)z^2-2(1-r)(4-r+2r^2)z^3+(1-r)^2(3-2r+2r^2)z^4, \quad (20)$$

$$v_1 = \frac{W}{3m_2[1-(1-r)z]^2}[-12 + 6r + (60 - 84r + 48r^2)z + (-138 + 259r - 296r^2 + 102r^3)z^2 + (192 - 420r + 552r^2 - 536r^3 + 330r^4)z^3 + (-168 + 446r - 614r^2 + 746r^3 - 612r^4 + 232r^5)z^4 + (84 - 274r + 476r^2 - 650r^3 + 574r^4 - 272r^5 + 62r^6)z^5 + (-18 + 57r - 126r^2 + 282r^3 - 412r^4 + 333r^5 - 144r^6 + 28r^7)z^6 + r(10 - 40r + 56r^2 - 24r^3 - 14r^4 + 16r^5 - 4r^6)z^7]. \tag{21}$$

As mentioned above there exist several sources of relativistic corrections $|\mathbf{p}|^2/m_{1,2}^2$ in the expression (15). The first part of terms appears from the expansion of the gluon propagator and relativistic factors in the Dirac bispinors. The structure of the spinor matrix element in this case is the same as in Eq. (17):

$$M_{21}(q,P,p',q') = \frac{2\alpha_s\sqrt{2M}}{3\sqrt{6}}\int \bar{\Psi}_{(\bar{Q}_1Q_2),0}(\mathbf{p})\frac{d\mathbf{p}}{(2\pi)^3}\frac{(m_1+m_2)}{m_2(s-m_1^2)^2}\left\{\left[1-\frac{p^2}{k_0^2}+\frac{4(qp)^2}{k_0^4}-\frac{\mathbf{p}^2}{8m_1^2}-\frac{\mathbf{p}^2}{8m_2^2}\right]\bar{u}_2(p')2\hat{\tilde{\epsilon}}^*(v)(\hat{q}+m_1)\times\Gamma_\alpha v_1(q') + \left[1-\frac{p^2}{k_0^2}+\frac{4(qp)^2}{k_0^4}-\frac{\mathbf{p}^2}{8m_1^2}-\frac{\mathbf{p}^2}{8m_2^2}+\frac{2(qp)(pn)}{k_0^2(nq-\eta_1 nP)}+\frac{(pn)^2}{(nq-\eta_1 nP)^2}\right]\times\frac{(s-m_1^2)}{(nq-\eta_1 nP)}\bar{u}_2(p')\hat{n}\hat{\tilde{\epsilon}}^*(v)(\hat{v}+1)\Gamma_\alpha v_1(q')\right\}. \tag{22}$$

Another part of corrections is determined both by the gluon propagator terms and relativistic addenda $\hat{\tilde{p}}$ in the square brackets of Eq. (15):

$$M_{22}(q,P,p',q') = -\frac{2\alpha_s\sqrt{2M}}{3\sqrt{6}}\int \bar{\Psi}_{(\bar{Q}_1Q_2),0}(\mathbf{p})\frac{d\mathbf{p}}{(2\pi)^3}\frac{(m_1+m_2)}{m_2(s-m_1^2)^2}\left[\frac{(np)}{(nq-\eta_1 nP)}+\frac{2qp}{k_0^2}\right]\times\frac{(p_\sigma n_\lambda + p_\lambda n_\sigma)}{(nq-\eta_1 nP)}\bar{u}_2(p')\gamma_\sigma\hat{\tilde{\epsilon}}^*(v)(\hat{v}+1)\gamma_\lambda(\hat{q}+m_1)\Gamma_\alpha v_1(q'), \tag{23}$$

$$M_{23}(q,P,p',q') = -\frac{2\alpha_s\sqrt{2M}}{3\sqrt{6}}\int \bar{\Psi}_{(\bar{Q}_1Q_2),0}(\mathbf{p})\frac{d\mathbf{p}}{(2\pi)^3}\frac{(m_1+m_2)}{m_2(s-m_1^2)^2}\frac{(p_\sigma n_\lambda + p_\lambda n_\sigma)}{(nq-\eta_1 nP)}\bar{u}_2(p')\gamma_\sigma\times\left[\frac{\hat{\tilde{p}}}{2m_2}\hat{\tilde{\epsilon}}^*(v)(\hat{v}+1)+\hat{\tilde{\epsilon}}^*(v)(\hat{v}+1)\frac{\hat{\tilde{p}}}{2m_1}\right]\gamma_\lambda(\hat{q}+m_1)\Gamma_\alpha v_1(q'), \tag{24}$$

$$M_{24}(q,P,p',q') = -\frac{2\alpha_s\sqrt{2M}}{3\sqrt{6}}\int \bar{\Psi}_{(\bar{Q}_1Q_2),0}(\mathbf{p})\frac{d\mathbf{p}}{(2\pi)^3}\frac{(m_1+m_2)}{m_2(s-m_1^2)^2}\left[g_{\lambda\sigma}-\frac{(k_\sigma n_\lambda + k_\lambda n_\sigma)}{(nq-\eta_1 nP)}\right]\bar{u}_2(p')\gamma_\sigma\frac{\hat{\tilde{p}}}{2m_2}\times\hat{\tilde{\epsilon}}^*(v)(\hat{v}+1)\frac{\hat{\tilde{p}}}{2m_1}\gamma_\lambda(\hat{q}+m_1)\Gamma_\alpha v_1(q'). \tag{25}$$

Further transformations of the interference terms $(M_{21}M_{2i}^* + M_{2i}M_{21}^*)$ $(i = 2, 3, 4)$ in the square modulus $|M|^2$ are performed by means of the system REDUCE [26]. The relation

$$\int d\Omega_{\mathbf{p}} p_\mu p_\nu = (g_{\mu\nu} - v_\mu v_\nu)\frac{\mathbf{P}^2 - 3P^{02}}{9P^{02}}\int d\Omega_{\mathbf{p}}\mathbf{p}^2, \tag{26}$$

which accounts the orthogonality condition (3), is used for the integration over the angle variables in the relative momentum space. Omitting numerous intermediate expressions appearing in the calculation $|M|^2$ which have sufficiently cumbersome forms, we present the final result for the relativistic correction in the fragmentation function (16):

$$D^{\rm rel}_{Q_1\to V(Q_1\bar{Q}_2)}(z) = \frac{8\alpha_s^2|\Psi(0)|^2}{27m_2^3}\frac{rz(1-z)^2}{[1-(1-r)z]^6}v_2, \tag{27}$$

$$v_2(z)=\frac{\langle\mathbf{p}^2\rangle}{36[1-(1-r)z]^2(1-r)^2m_2^2}[-2+16r(2-3r)+2(1-2r)(5-44r+30r^2)z+(-23+18r-206r^2+508r^3-432r^4)z^2+8(1-r)(4+40r-11r^2-50r^3+66r^4)z^3-2(1-r)^2(14+214r+47r^2-248r^3+120r^4)z^4+2(1-r)^3(7+99r+120r^2-230r^3+108r^4)z^5-(1-r)^4(3+26r+82r^2-84r^3+48r^4)z^6]. \tag{28}$$

Here $\langle\mathbf{p}^2\rangle$ is the special parameter determining the numerical value of relativistic effects [see the discussion before Eq. (60)]. Integrating expressions (19) and (27) over z we obtain the total fragmentation probability:

$$\Omega_V=\int_0^1 D_{Q_1\to V(Q_1\bar{Q}_2)}(z)dz=\frac{8\alpha_s^2|\Psi(0)|^2}{405m_2^3(1-r)^6}\left(f_{0v}(r)+\frac{W}{m_2}f_{1v}(r)+\frac{\langle\mathbf{p}^2\rangle}{m_2^2}f_{2v}(r)\right), \tag{29}$$

$$f_{0v}(r)=24+85r-235r^2+300r^3-85r^4-89r^5+15r(7-4r+3r^2+10r^3+2r^4)\ln r, \tag{30}$$

$$f_{1v}(r)=\frac{1}{42(1-r)^2}[-2607+7185r+23\,576r^2-116\,018r^3+159\,670r^4-170\,373r^5+153\,860r^6-34\,906r^7-20\,387r^8+210r(-42+223r-388r^2+236r^3-268r^4-43r^5+212r^6+28r^7)\ln r], \tag{31}$$

$$f_{2v}(r)=\frac{1}{252(1-r)^2}[488-3017r+48\,979r^2-201\,740r^3+136\,955r^4+23\,597r^5-20\,958r^6+15\,696r^7-105r(7-200r+347r^2+1194r^3-222r^4+156r^5+48r^6)\ln r]. \tag{32}$$

The parameter $r=m_2/(m_1+m_2)$ is the ratio of the constituent mass of a spectator quark to the mass of two quarks composing the heavy meson. The contributions to the fragmentation functions (20), (21), and (28) give the distributions in the longitudinal momentum z of the heavy mesons. The z dependence of relativistic and bound state corrections to the fragmentation functions is shown in Fig. 2 for the vector heavy mesons and in Fig. 3 for the pseudoscalar heavy mesons [see Eq. (42)]. We find it convenient to present here the fragmentation functions of antiquarks for the comparison with experimental data. To extend the present calculations to the distributions in the transverse momentum p_T of the heavy meson, the following relation between the invariant mass s of the fragmenting heavy quark, the transverse momentum p_T, and longitudinal momentum z is taken into account [27]:

$$s(z,t)=\frac{M^2+p_T^2}{z}+\frac{m_2^2+p_T^2}{1-z}. \tag{33}$$

Introducing further the dimensionless variable $t=p_T/(m_1+m_2)$, we can determine the p_T-dependent relativistic corrections to the fragmentation probability $D_{Q_1\to V(Q_1\bar{Q}_2)}(t)$ [28]:

$$D_{Q_1\to V(Q_1\bar{Q}_2)}(t)=\int_0^1 dz\frac{2M^2t}{z(1-z)}D_{Q_1\to V(Q_1\bar{Q}_2)}(z,s(z,t))=\frac{4\alpha_s^2|\Psi(0)|^2r}{27m_2^3(1-r)^6t^6}\left[D_{0v}(t)+\frac{W}{m_2}D_{1v}(t)+\frac{\langle\mathbf{p}^2\rangle}{m_2^2}D_{2v}(t)\right], \tag{34}$$

$$D_{0v}(t)=\Big\{(-30r^3+30r^4)t+(61r-33r^2-48r^3+20r^4)t^3+(-5+13r-16r^2+4r^3+4r^4)t^5+\arctan\frac{t(1-r)}{r+t^2}\times[30r^4+(-99r^2-66r^3+30r^4)t^2+(9+20r+r^2+22r^3+8r^4)t^4+(9-12r+4r^2+8r^3)t^6]+\ln(r)\times[-96r^3t+(48r+56r^2+16r^3)t^3]+\ln\frac{(1+t^2)}{(r^2+t^2)}[-48r^3t+(24r+28r^2+8r^3)t^3+(-4+8r-4r^2)t^7]\Big\}, \tag{35}$$

$$D_{1v}(t) = \frac{1}{3t^2(1-r)^2}\left\{-\frac{24rt(1-r)^6}{1+t^2} + (24r - 144r^2 + 360r^3 - 480r^4 + 45r^5 + 486r^6 - 291r^7)t \right.$$
$$+ (-24r + 144r^2 - 633r^3 + 2115r^4 - 1233r^5 - 1266r^6 + 897r^7)t^3 + (-18r + 1130r^2 - 4129r^3 + 3938r^4$$
$$+ 595r^5 - 1994r^6 + 478r^7)t^5 + (30 - 215r + 581r^2 - 742r^3 + 738r^4 - 596r^5 + 148r^6 + 56r^7)t^7$$
$$+ (60r - 255r^2 + 360r^3 - 195r^4 + 30r^5)t^9] + \arctan\frac{t(1-r)}{(r+t^2)}[315r^6 - 315r^7 + (90r^4 - 1650r^5 - 345r^6$$
$$+ 960r^7)t^2 + (-63r^2 - 1800r^3 + 5646r^4 - 714r^5 - 3495r^6 + 741r^7)t^4 + (-54 + 237r - 33r^2 - 807r^3$$
$$+ 642r^4 - 621r^5 + 423r^6 + 150r^7)t^6 + (-54 + 297r - 507r^2 + 216r^3 - 252r^4 + 147r^5 + 126r^6)t^8$$
$$+ (-60r + 195r^2 - 165r^3 + 30r^4)t^{10}] + \ln(r)[(-576r^5 + 288r^7)t + (-432r^3 + 48r^4 + 5856r^5 - 4416r^6$$
$$- 48r^7)t^3 + (-144r + 1296r^2 - 2136r^3 - 768r^4 + 1320r^5 + 432r^6)t^5] + \ln\frac{(1+t^2)}{(r^2+t^2)}[(-288r^5 + 144r^7)t$$
$$+ (-216r^3 + 24r^4 + 2928r^5 - 2208r^6 - 24r^7)t^3 + (-72r + 648r^2 - 1068r^3 - 384r^4 + 660r^5 + 216r^6)t^5$$
$$\left. + (24 - 156r + 312r^2 - 192r^3 + 72r^4 - 60r^5)t^9]\right\}, \quad (36)$$

$$D_{2v}(t) = \frac{1}{36t^2(1-r)^2}\left\{-420r^5(1-r)t + (6074r^3 - 20\,646r^4 + 11\,976r^5 + 2596r^6)t^3\right.$$
$$+ (-573r + 2309r^2 + 5008r^3 - 5180r^4 - 1636r^5 + 72r^6)t^5 + (5 - 189r + 1124r^2 - 960r^3 + 68r^4 + 48r^5$$
$$- 96r^6)t^7 + \arctan\frac{t(1-r)}{r+t^2}[420r^6 + (-7470r^4 + 15\,480r^5 + 5040r^6)t^2 + (1887r^2 - 1758r^3 - 20\,334r^4$$
$$- 6936r^5 + 900r^6)t^4 + (-9 + 100r + 1303r^2 - 1614r^3 - 632r^4 + 216r^5 - 168r^6)t^6$$
$$+ (-9 + 108r - 1336r^2 - 96r^3 + 64r^4 - 192r^5)t^8] + \ln(r)[(-3072r^5 + 4608r^6)t$$
$$+ (6240r^3 - 10\,688r^4 - 24256r^5 + 768r^6)t^3 + (-240r + 712r^2 + 5232r^3 + 3392r^4 + 384r^5)t^5]$$
$$+ \ln\frac{(r^2+t^2)}{(t^2+1)}[(-1536r^5 + 2304r^6)t + (3120r^3 - 5344r^4 - 12\,128r^5 + 384r^6)t^3$$
$$\left. + (-120r + 356r^2 + 2616r^3 + 1696r^4 + 192r^5)t^5 + (4 + 72r + 324r^2 - 208r^3 + 96r^4)t^9]\right\}. \quad (37)$$

The leading order contribution (35) coincides with the result of Ref. [28] and the two other terms (36) and (37) determine the relativistic and bound state corrections. The functions (35)–(37) are plotted in Fig. 4.

Let us next consider the calculation of the fragmentation functions into pseudoscalar heavy mesons η_c, B_c, η_b. The general expression of the fragmentation amplitude consists of three terms:

$$M_3 = \frac{2\sqrt{2M}\alpha_s|\Psi(0)|}{3\sqrt{6}}\frac{(m_1+m_2)}{m_2(s-m_1^2)^2}[a_1\bar{u}_2(p')\gamma_5(\hat{q}+m_1)\Gamma_\alpha v_1(q') + a_2\bar{u}_2(p')\gamma_5\Gamma_\alpha v_1(q') + a_3\bar{u}_2(p')\gamma_5\hat{n}(\hat{q}+m_1)$$
$$\times \Gamma_\alpha v_1(q')], \quad (38)$$

[the fourth possible term $\bar{u}_2(p')\gamma_5\hat{n}\Gamma_\alpha v_1(q')$ does not contribute to the fragmentation function] where the coefficients a_i ($i = 1, 2, 3$) contain the leading order contribution and corrections proportional to $\langle \mathbf{p}^2\rangle$ and W:

$$a_1 = 1 - \frac{2}{9}\frac{\langle\mathbf{p}^2\rangle}{(s-m_1^2)r} + \frac{\langle\mathbf{p}^2\rangle}{m_1^2}\left[\frac{5}{24} + \frac{2}{9}r - \frac{5}{9}r^2 + \frac{1}{1-z(1-r)}\left(\frac{1}{2}zr^3 - \frac{2}{3}zr^2 - \frac{1}{6}r^2 - \frac{1}{18}rz + \frac{7}{18}r + \frac{2}{9}z - \frac{2}{9}\right)\right]$$
$$+ \frac{2(m_1+m_2)W}{(s-m_1^2)}(1-r) + \frac{W}{(m_1+m_2)}\left(2 - \frac{1}{r} - \frac{z}{1-z(1-r)} + \frac{2rz}{1-z(1-r)}\right), \quad (39)$$

$$a_2 = 1 + \frac{\langle \mathbf{p}^2\rangle}{m_1^2}\left(-\frac{1}{6}r^2 + \frac{5}{18}r - \frac{17}{12}\right) + \frac{2}{9}\frac{\langle \mathbf{p}^2\rangle}{m_2^2}\frac{r^2z^2}{[1-z(1-r)]^2} + \frac{\langle \mathbf{p}^2\rangle}{m_1^2}\frac{1}{1-z(1-r)} \times\left(-\frac{2}{9}r^2z + \frac{2}{9}r^2 + \frac{1}{3}rz - \frac{1}{3}r - \frac{1}{9}z + \frac{1}{9}\right) + \frac{\langle \mathbf{p}^2\rangle}{m_1^2}\frac{m_2^2}{(s-m_1^2)}\left(\frac{5}{9}r - \frac{5}{9rz} + \frac{1}{r} + \frac{2}{9r^2z} - \frac{2}{9r^2} + \frac{1}{3z} - \frac{4}{3}\right) - \frac{4}{9r}\frac{\langle \mathbf{p}^2\rangle}{(s-m_1^2)} + \frac{2(m_1+m_2)W}{(s-m_1^2)}(1-r) + \frac{W}{(m_1+m_2)}\left(2 - \frac{1}{r}\right), \quad (40)$$

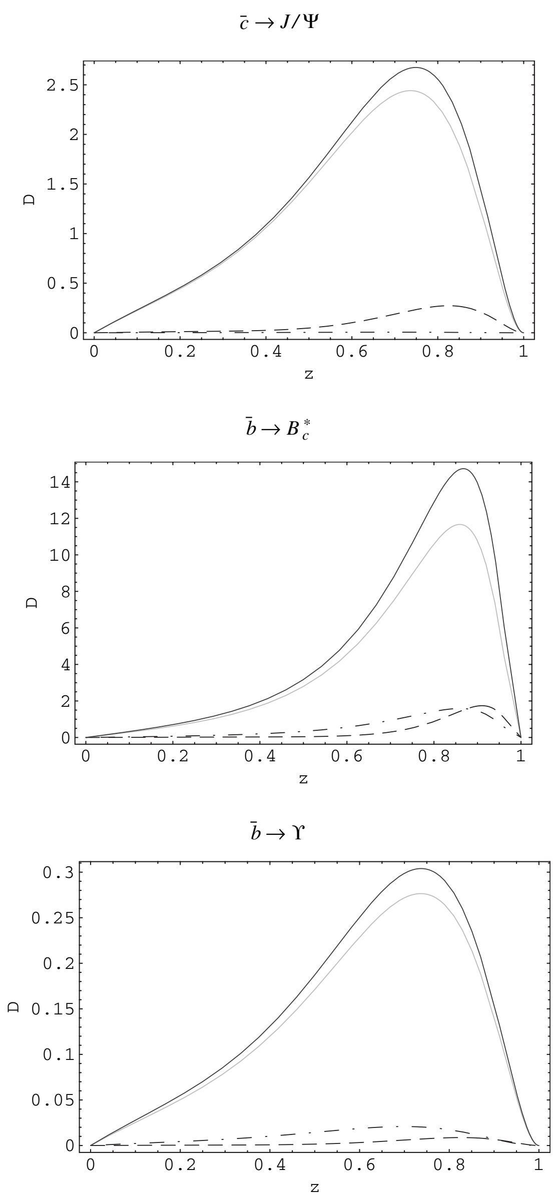

FIG. 2. The contributions to the fragmentation functions $D(\bar{c} \to J/\Psi)(z)$, $D(\bar{b} \to B_c^*)(z)$, $D(\bar{b} \to \Upsilon)(z)$. The thick solid line shows the total fragmentation function, the dashed line shows the relativistic correction (27), and the dot-dashed line shows the correction proportional to the binding energy W (21). The thin solid line corresponds to the distributions without corrections. All functions have been multiplied by a factor of 10^4.

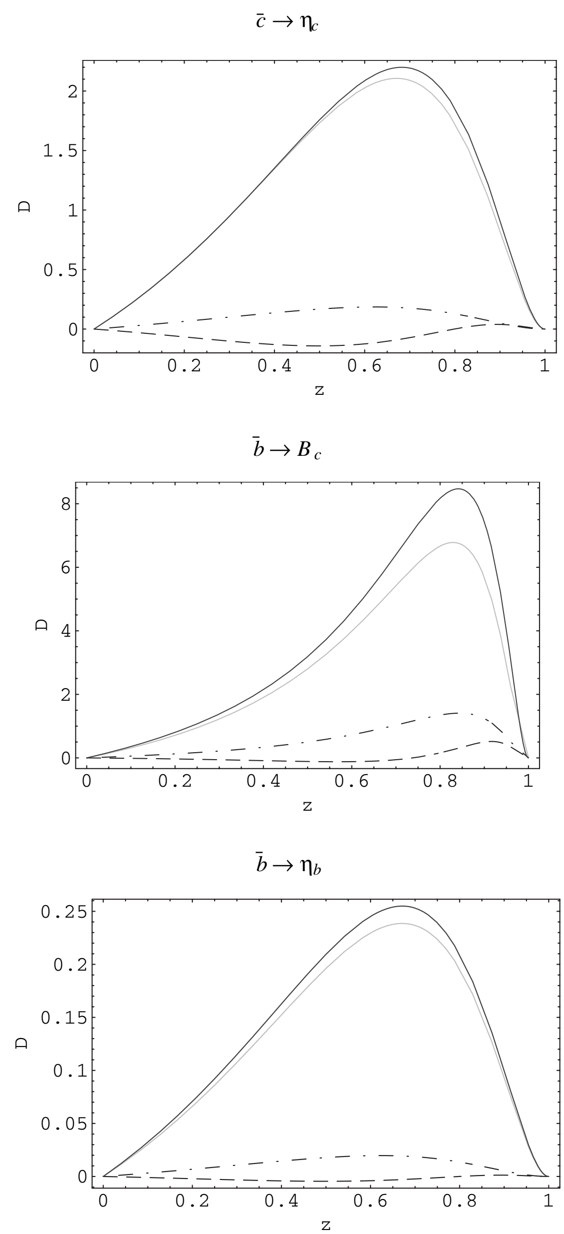

FIG. 3. The contributions to the fragmentation functions $D(\bar{c} \to \eta_c)(z)$, $D(\bar{b} \to B_c)(z)$, $D(\bar{b} \to \eta_b)(z)$. The thick solid line shows the total fragmentation function, the dashed line shows the relativistic correction (45), and the dot-dashed line shows the correction proportional to the binding energy W (44). The thin solid line corresponds to the distributions without corrections. All functions have been multiplied by a factor of 10^4.

$$a_3 = 1 - \frac{\langle \mathbf{p}^2 \rangle}{9m_1^2}\left(r + \frac{1}{8}\right) + \frac{\langle \mathbf{p}^2 \rangle}{m_1 m_2}\frac{rz}{1 - z(1 - r)} + \frac{2r^2z^2}{9}\frac{\langle \mathbf{p}^2 \rangle}{m_2^2} - \frac{2}{9r}\frac{\langle \mathbf{p}^2 \rangle}{(s - m_1^2)} + \frac{2(m_1 + m_2)W}{(s - m_1^2)}(1 - r) + \frac{W}{(m_1 + m_2)}\left(2 - \frac{1}{r}\right). \tag{41}$$

The exact expressions (39)–(41) are obtained on the basis of Eq. (15) ($\tilde{\epsilon} \to \gamma_5$) keeping the terms of the second order over relative momentum p and binding energy corrections. The squared modulus amplitude $|M_3|^2$ leads by using the definition (16) after the integration over s to the following fragmentation distribution for the production of the pseudoscalar heavy mesons:

$$D_{Q_1 \to P(Q_1\bar{Q}_2)}(z) = \frac{8\alpha_s^2|\Psi(0)|^2}{81m_2^3}\frac{rz(1 - z)^2}{[1 - (1 - r)z]^6}(p_0 + p_1 + p_2), \tag{42}$$

$$p_0 = 6 + 18(2r - 1)z + (21 - 74r + 68r^2)z^2 - 2(1 - r)(6 - 19r + 18r^2)z^3 + 3(1 - r)^2(1 - 2r + 2r^2)z^4, \tag{43}$$

$$\begin{aligned} p_1 = \frac{W}{m_2[1 - (1 - r)z]^2}\{&-12 + 6r + 60z + 24r(-5 + 2r)z + [-126 + r(425 - 482r + 218r^2)]z^2 \\ &+ 2[72 + r(-329 + 2r(298 + r(-283 + 131r)))]z^3 - 2(1 - r)[48 + r(-219 + r(410 + 7r(-61 + 33r))]z^4 \\ &+ 2(1 - r)^2[18 + r(-79 + 2r(69 + 13r(-5 + 3r)))]z^5 + (r - 1)^3[6 + r(-25 + r(35 + 6r(-4 + 3r)))]z^6\}, \end{aligned} \tag{44}$$

$$\begin{aligned} p_2 = \frac{\langle \mathbf{p}^2 \rangle}{36(1 - r)^2m_2^2[1 - (1 - r)z]^2}\{&-6 - 48r - 6[-5 + 2r(-3 + r(-3 + 22r))]z - [63 + 2r(-191 \\ &+ r(431 + 4r(-197 + 146r)))]z^2 + 8(1 - r)[9 + r(-108 + r(217 + 5r(-42 + 29r)))]z^3 \\ &- 2(1 - r)^2[24 + r(-390 + r(697 + 4r(-53 + 5r)))]z^4 + 2(r - 1)^3[-9 + r(155 + 2r(-122 + r(-7 + 36r)))]z^5 \\ &+ (1 - r)^4(-3 + 42r - 58r^2 + 24r^4)z^6\}. \end{aligned} \tag{45}$$

Integrating expressions (43)–(45) over z we obtain the total fragmentation probability for the pseudoscalar mesons:

$$\Omega_P = \int_0^1 D_{Q_1 \to P(Q_1\bar{Q}_2)}(z)dz = \frac{8\alpha_s^2|\Psi(0)|^2}{405m_2^3(1 - r)^6}\left(f_{0p}(r) + \frac{W}{m_2}f_{1p}(r) + \frac{\langle \mathbf{p}^2 \rangle}{m_2^2}f_{2p}(r)\right), \tag{46}$$

$$f_{0p}(r) = 8 + 5r + 215r^2 - 440r^3 + 265r^4 - 53r^5 + 15r(1 + 8r + r^2 - 6r^3 + 2r^4)\ln r, \tag{47}$$

$$\begin{aligned} f_{1p}(r) = \frac{1}{42(1 - r)}[&897 - 3640r + 8400r^2 - 59\,850r^3 + 147\,105r^4 - 132\,762r^5 + 46\,830r^6 - 6980r^7 \\ &+ 210r(6 - 3r - 129r^2 + 115r^3 + 123r^4 - 84r^5 + 18r^6)\ln r], \end{aligned} \tag{48}$$

$$\begin{aligned} f_{2p}(r) = \frac{1}{756(1 - r)^2}[&614 - 9051r - 9681r^2 + 106\,470r^3 - 73\,815r^4 - 45\,129r^5 + 41\,146r^6 - 10\,554r^7 \\ &+ 105r(-3 - 204r + 281r^2 + 798r^3 - 402r^4 + 48r^5 + 24r^6)\ln r]. \end{aligned} \tag{49}$$

The transverse momentum fragmentation functions for the production of pseudoscalar mesons η_c, B_c, and η_b can be derived in a similar way as for the vector mesons. The corresponding expressions are given as follows:

$$D_{Q_1 \to P(Q_1\bar{Q}_2)}(t) = \int_0^1 dz\frac{2M^2t}{z(1 - z)}D_{Q_1 \to P(Q_1\bar{Q}_2)}(z, s(z, t)) = \frac{4\alpha_s^2|\Psi(0)|^2r}{81m_2^3(1 - r)^6t^6}\left[D_{0p}(t) + \frac{W}{m_2}D_{1p}(t) + \frac{\langle \mathbf{p}^2 \rangle}{m_2^2}D_{2p}(t)\right], \tag{50}$$

$$D_{0p}(t) = \Big\{-(1-r)t[30r^3 - r(61-20r+28r^2)t^2 - (3-48r+48r^2-12r^3)t^4] + 3\arctan\frac{t(1-r)}{r+t^2}[10r^4 - 3r^2(11+2r+2r^2)t^2 + (3+4r+19r^2-6r^3)t^4 + (3+12r-20r^2+8r^3)t^6)] - 24rt\ln(r)[4r^2 - (2+r+2r^2)t^2] - 12t\ln\frac{(1+t^2)}{(r^2+t^2)}[4r^3 - r(2+r+2r^2)t^2 + (1-r)^2t^6]\Big\}, \quad (51)$$

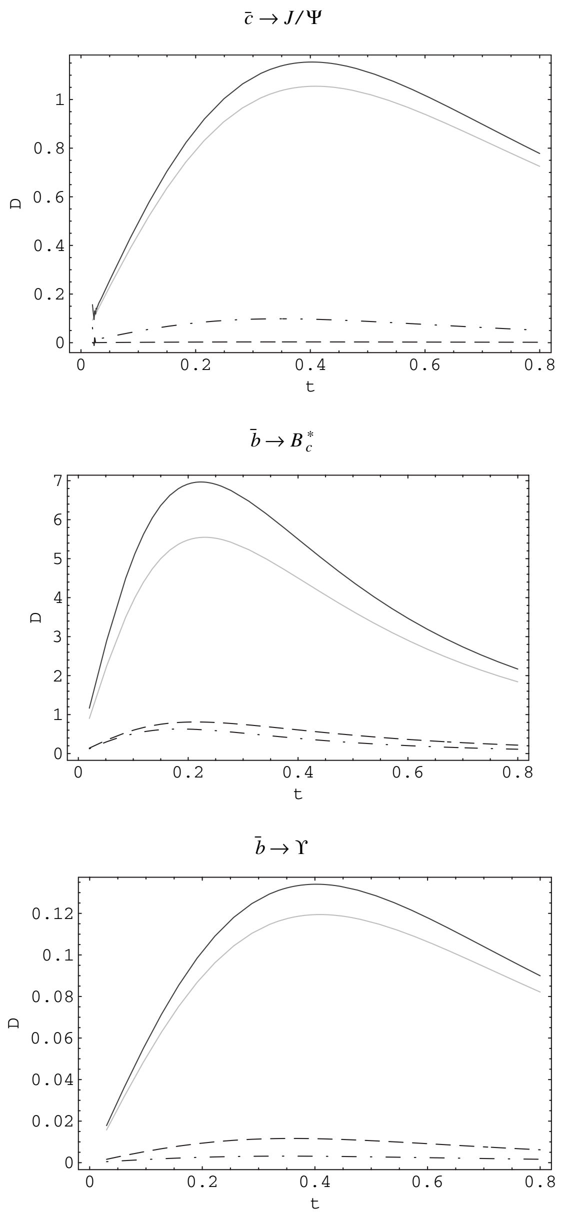

FIG. 4. The contributions to the transverse distributions $D(\bar{c} \to J/\Psi)(t)$, $D(\bar{b} \to B_c^*)(t)$, $D(\bar{b} \to \Upsilon)(t)$ relative to the heavy quark fragmentation axis. The thick solid line shows the total fragmentation function, the dot-dashed line shows the relativistic correction (37), and the dashed line shows the correction proportional to the binding energy W (36). The thin solid line corresponds to the distributions without corrections. All functions have been multiplied by a factor of 10^4.

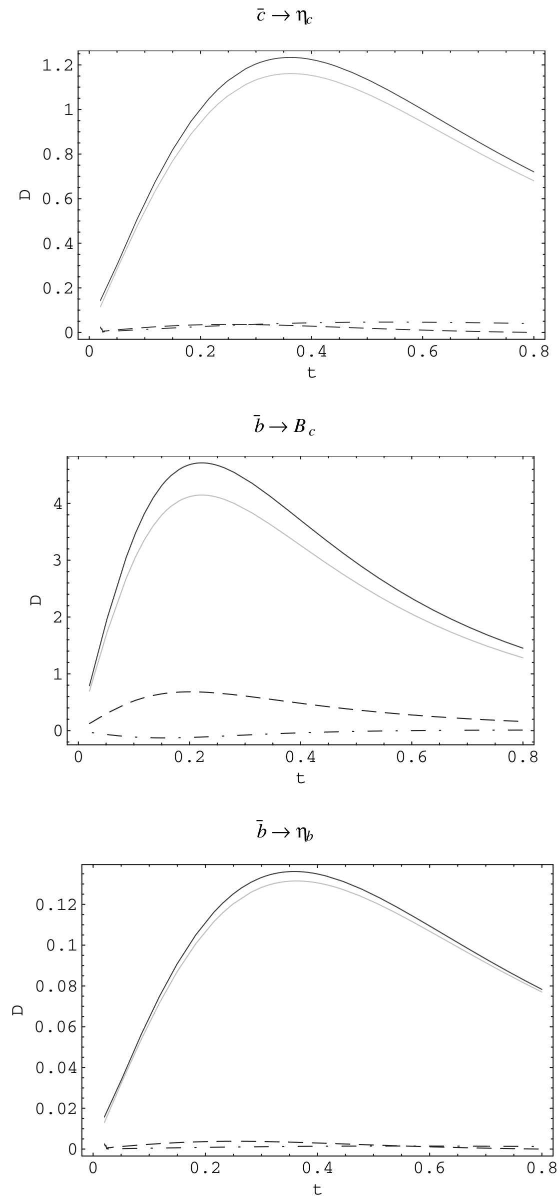

FIG. 5. The contributions to the transverse distributions $D(\bar{c} \to \eta_c)(t)$, $D(\bar{b} \to B_c)(t)$, $D(\bar{b} \to \eta_b)(t)$ relative to the heavy quark fragmentation axis. The thick solid line shows the total fragmentation function, the dot-dashed line shows the relativistic correction (53), and the dashed line shows the correction proportional to the binding energy W (52). The thin solid line corresponds to the distributions without corrections. All functions have been multiplied by a factor of 10^4.

$$D_{1p}(t) = \frac{1}{t^2(1-r)}\left\{\frac{24rt(1-r)^4(2r-1)}{1+t^2} + 3r(8-48r+112r^2-128r^3+37r^4+19r^5)t\right.$$
$$+ r(-24+144r-691r^2+1032r^3+84r^4-545r^5)t^3 + r(10+360r-1180r^2+1037r^3-495r^4+268r^5)t^5$$
$$- 3(2-19r+117r^2-300r^3+320r^4-140r^5+20r^6)t^7 + 3\arctan\frac{t(1-r)}{(r+t^2)}[35r^6 - 10r^4(-13+15r$$
$$+ 25r^2)t^2 + r^2(-15-281r+501r^2+96r^3+79r^4)t^4 + (-6+31r-7r^2+79r^3-245r^4+42r^5+8r^6)t^6$$
$$- (6-11r-68r^2+208r^3-160r^4+40r^5)t^8] + 24rt\ln(r)[-16r^5 + 2r^2(-7-10r+38r^2+4r^3)t^2$$
$$+ (-2+20r-21r^2-5r^3-22r^4)t^4] + 12t\ln\frac{(r^2+t^2)}{1+t^2}[16r^6 - 2r^3(-7-10r+38r^2+4r^3)t^2$$
$$\left. + r(2-20r+21r^2+5r^3+22r^4)t^4 - (1-r)^2(2-5r+5r^2)t^8]\right\}, \qquad (52)$$

$$D_{2p}(t) = \frac{1}{36t^2(1-r)^2}\left\{-420r^5(1-r)t + 2r^3(1957-7323r+4368r^2+998r^3)t^3 - r(429-2629r+152r^2-396r^3\right.$$
$$+ 1684r^4+760r^5)t^5 + 3(-1+65r-308r^2+360r^3-108r^4-24r^5+16r^6)t^7$$
$$+ \arctan\frac{t(1-r)}{r+t^2}[(420r^6 + (-5310r^4+11\,640r^5+4440r^6)t^2 + (1023r^2-4158r^3-9486r^4-4080r^5$$
$$- 1764r^6)t^4 + (-9-324r-9r^2+1674r^3+1632r^4-360r^5+72r^6)t^6 + (-9-36r+744r^2-552r^3$$
$$- 24r^4+96r^5)t^8] + 24rt\ln(r)[-128r^4 + 192r^5 - 4r^2(-35+110r+110r^2+26r^3)t^2 + (-10-r+150r^2$$
$$+ 152r^3+100r^4+4r^5)t^4] + 12t\ln\frac{(1+t^2)}{(r^2+t^2)}[-128r^5 + 192r^6 - 4r^3(-35+110r+110r^2+26r^3)t^2$$
$$\left. + r(-10-r+150r^2+152r^3+100r^4+4r^5)t^4 - (-1+14r-13r^2-4r^3+r^4)t^8)]\right\}. \qquad (53)$$

The leading order function (51) coincides with the result of Ref. [28], and the two other functions (52) and (53) determine the relativistic and binding energy corrections. Functions (51)–(53) are plotted in Fig. 5. The obtained results for the transverse momentum distributions are valid for the transverse momentum p_T up to values of order of the meson mass. The further growth of the momentum p_T demands the consideration of the omitted corrections of order $O(p_T/M_Z)$.

IV. QUASIPOTENTIAL QUARK MODEL

To estimate numerical values of the investigated effects in the heavy quark fragmentation we used the relativistic quark model. In the quasipotential approach the bound state of a quark and antiquark is described by the Schrödinger type equation [29]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_0(\mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p},\mathbf{q},M)\Psi_0(\mathbf{q}), \qquad (54)$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1E_2}{E_1+E_2} = \frac{M^4-(m_1^2-m_2^2)^2}{4M^3}, \qquad (55)$$

and the particle energies E_1, E_2 are given by

$$E_1 = \frac{M^2-m_2^2+m_1^2}{2M}, \qquad E_2 = \frac{M^2-m_1^2+m_2^2}{2M}. \qquad (56)$$

Here $M = E_1 + E_2$ is the bound state mass, $m_{1,2}$ are the masses of heavy quarks (Q_1 and Q_2) which form the meson, and $\mathbf{p}$ is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}. \qquad (57)$$

The kernel $V(\mathbf{p},\mathbf{q},M)$ in Eq. (54) is the quasipotential operator of the quark-antiquark interaction. Within an effective field theory (NRQCD) the quark-antiquark potential was constructed in Refs. [30,31] by the perturbation theory improved by the renormalization group resummation of large logarithms. In the quasipotential quark model the kernel $V(\mathbf{p},\mathbf{q},M)$ is constructed phenomenologically with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. The heavy quark-antiquark potential with the account of retardation effects and the one-loop radiative corrections can be presented in the form of a sum of spin-independent and spin-dependent parts. Explicit expressions for it are given in Refs. [23,24].

Taking into account the accuracy of the calculation of relativistic corrections to the fragmentation probabilities, we can use for the description of the bound system $(Q_1\bar{Q}_2)$ the following simplified interaction operator in the coordinate representation:

$$\tilde{V}(r) = -\frac{4}{3}\frac{\bar{\alpha}_V(\mu^2)}{r} + Ar + B, \quad (58)$$

where the parameters of the linear potential $A = 0.18\ \mathrm{GeV}^2$, $B = -0.16$ GeV,

$$\bar{\alpha}_V(\mu^2) = \alpha_s(\mu^2)\left[1 + \left(\frac{a_1}{4} + \frac{\gamma_E\beta_0}{2}\right)\frac{\alpha_s(\mu^2)}{\pi}\right], \quad a_1 = \frac{31}{3} - \frac{10}{9}n_f, \quad \beta_0 = 11 - \frac{2}{3}n_f. \quad (59)$$

Here $n_f = 3$ is the number of flavors and μ is a renormalization scale. All the parameters of the model, like quark masses, parameters of the linear confining potential A and B, mixing coefficient ε, and anomalous chromomagnetic quark moment κ entering in the quasipotential $V(\mathbf{p}, \mathbf{q}, M)$ were fixed from the analysis of heavy quarkonium masses [22–24] and radiative decays [23]. The heavy quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV and the parameters of the linear potential $A = 0.18\ \mathrm{GeV}^2$ and $B = -0.16$ GeV have standard values of the quark models. Solving the Schrödinger-like quasipotential equation we obtain an initial expression for the bound state wave functions in the case of $(c\bar{c})$, $(\bar{b}c)$, and $(\bar{b}b)$ systems. For numerical estimations of relativistic effects in the production of heavy mesons via heavy quark fragmentation we need the values of the wave functions at the origin, the bound state energy and the parameter of relativistic effects: $\int \mathbf{p}^2\bar{\Psi}_0(\mathbf{p})d\mathbf{p}/(2\pi)^3$. Note that this integral would diverge in the high momentum region. Obviously, the reason for this divergence is connected with the used expansion of the integral function in the basic equation (13) over the ratio $\mathbf{p}^2/m^2$. In the coordinate representation this divergence would appear when we set $\mathbf{r} = 0$ in the Coulomb part of the potential (58). Different regularization prescriptions are commonly used in this case [32–35]. An approach to the calculation of this integral was formulated in Ref. [36] for solving the orthopositronium decay problem in quantum electrodynamics. Their prescription is in agreement with the calculations carried out in the effective field theories [37]. Unfortunately, in the investigation of the bound states of heavy quarks we cannot use it because the valid wave function asymptotics at $p \to \infty$ is not Coulomb like. So, to fix the value of the relativistic correction we explore the dimensional regularization scheme where the scaleless momentum integral $\int V(\mathbf{p} - \mathbf{q})\Psi(\mathbf{q})[(d^d\mathbf{q})/(2\pi)^d][(d^d\mathbf{p})/(2\pi)^d]$ related to the problem vanishes [32,33,38]. Then we can express the necessary quantity in the form

$$\langle\mathbf{p}^2\rangle \equiv \frac{1}{\Psi(0)}\int\frac{d^d\mathbf{p}}{(2\pi)^d}\mathbf{p}^2\bar{\Psi}_0(\mathbf{p}) = 2\mu_R\tilde{W} + 2\mu_R|B|. \quad (60)$$

The solutions of the Schrödinger-like equation (54) with the potential (58) determine the energy spectrum $\tilde{W}$ of the heavy quark system and lead to the numerical values of the parameter (60) for the bound states $(\bar{c}c)$, $(\bar{b}b)$, and $(\bar{b}c)$ which are presented in Table I. They are in qualitative agreement with the other possible approach for the estimation of the value (60) based on the natural regularization directly connected with the relativistic structure factors entering in the fragmentation amplitude (13). Heavy quark symmetry predicts that the wave functions of the vector and pseudoscalar states are different due to corrections of order v_Q^2. The analogous statement is valid for the parameter $\langle\mathbf{p}^2\rangle$. Nevertheless, in this study we neglect this difference and write in Table I equal values for $\Psi(0)$ and $\langle\mathbf{p}^2\rangle$ for V and P mesons. Our value $\langle\mathbf{p}^2\rangle = 0.5\ \mathrm{GeV}^2$ for $(c\bar{c})$ states is slightly smaller than $\langle\mathbf{p}^2\rangle = 0.7\ \mathrm{GeV}^2$ used in Ref. [16] where it was fixed from the analysis of the quarkonium decay rates. The theoretical uncertainty of the obtained values $\langle\mathbf{p}^2\rangle$ in Table I is determined by perturbative and nonperturbative corrections to the quasipotential [22,23] and does not exceed 30%.

V. DISCUSSION AND CONCLUSIONS

As mentioned above, the problem of heavy hadron production in e^+e^- and $p\bar{p}$ collisions became very urgent in the last few years. The experimental investigations carried out in this field allowed one to measure the b quark

TABLE I. Basic parameters of the relativistic quark model and the total fragmentation probabilities Ω for the reactions $\bar{c} \to \eta_c, J/\Psi$, $\bar{b} \to B_c, B_c^*$, $\bar{b} \to \eta_b, \Upsilon$.

State $n^{2S+1}L_J$	Particle	Mass, GeV [39]	$\bar{\alpha}_V$	Bound state energy, GeV	$\Psi(0)$, $\mathrm{GeV}^{3/2}$	$\langle\mathbf{p}^2\rangle$, GeV^2	Ω
1^1S_0	η_c	2.980	0.451	−0.120	0.27	0.5	1.49×10^{-4}
1^3S_1	J/Ψ	3.097	0.451	−0.003	0.27	0.5	1.28×10^{-4}
1^1S_0	B_c	6.270[a]	0.361	−0.160	0.33	0.7	3.49×10^{-4}
1^3S_1	B_c^*	6.332[a]	0.361	−0.098	0.33	0.7	4.89×10^{-4}
1^1S_0	η_b	9.400[a]	0.267	−0.360	0.46	1.4	0.14×10^{-4}
1^3S_1	Υ	9.460	0.267	−0.300	0.46	1.4	0.15×10^{-4}

[a]This value was obtained in Ref. [22].

fragmentation function in Z^0 decays [5]. The study of the fragmentation processes is important as a tool to reveal the features of nonperturbative quantum chromodynamics. There appear experimental data indicating essential differences between the theoretical predictions and experiment [1,2,6,19,20]. In the present study we investigated the role of relativistic and bound state corrections in the heavy quark b, c fragmentation processes. The amplitude of heavy quark fragmentation is obtained in a new form (13) which accounts for all possible relativistic factors for the calculation of relativistic corrections to the fragmentation functions. Let us summarize several peculiarities related to the calculation performed above.

1. We obtain the heavy quark fragmentation functions for both heavy quarks b and c which fragment to pseudoscalar and vector heavy mesons starting with the meson production amplitude (1).

2. All possible sources of relativistic corrections including the transformation factors for the two quark bound state wave function have been taken into account.

3. We investigated the role of relativistic effects in the fragmentation probabilities over two variables: the longitudinal momentum z and transverse momentum p_T of the heavy meson.

Analyzing the obtained analytical expressions for the fragmentation functions both in longitudinal and transverse momentum we can point out that the calculated corrections for all vector and pseudoscalar mesons do not exceed 20% of the leading order contribution. The numerical value of the binding energy correction is dependent on the initial choice of the heavy quark masses because $W = (M - m_1 - m_2)$. So, for example, the binding energy corrections are extremely small for the charmonium production in our model where $W_{J/\Psi} = -0.003$ GeV. The relativistic correction is essentially more important in this case. Earlier the binding energy and relativistic corrections were studied in Ref. [16] for the fragmentation functions of a charm quark to decay into η_c and J/Ψ. The comparison of our numerical results with the calculation in Ref. [16] shows that they are in agreement for the production of the η_c meson. In the case of the reaction $\bar{c} \rightarrow J/\Psi$ the binding corrections are numerically close, but relativistic corrections connected with the expression (60) are essentially different both in the sign and numerical value. Our relativistic correction coincides in the sign with the leading order contribution and is numerically 3 times smaller than in Ref. [16]. In our opinion the reason for the difference between our calculation and the result of Ref. [16] consists of the distinction of initial models which are used for the calculation of the fragmentation functions. In Ref. [16] the calculation of the heavy quark fragmentation function is constructed on the basis of its definition in terms of matrix elements of field operators at light cone. Moreover, the quark bound state is described by the four-dimensional Bethe-Salpeter amplitude. It enters the quarkonium production amplitude in the form corresponding to the rest frame of the bound state [see Eq. (3) in Ref. [16]], despite the fact that the bound state moves with four-momentum P in the considered reference frame. Let us note that the transformation law of the Bethe-Salpeter wave function in the transition from c.m. frame to an arbitrary reference frame was discussed by many authors (see, for example, Ref. [25]). The quasipotential method, used in this work, is a three-dimensional approach to the bound state problem. The quasipotential wave function depends on the quark three-momentum of relative motion. As initial expression for the calculation of the fragmentation functions we take Eq. (1) for the fragmentation amplitude. It contains the quark-antiquark wave function in the reference frame moving with the momentum P (the four-momentum of a double heavy meson). So, in our calculation all relativistic factors connected with the transformation of the Dirac bispinors and two-particle bound state wave function to the rest frame are taken into consideration. Moreover, our numerical estimations of analytical relations are based on a different numerical value for the expression (60). In the paper [16] the parameter $\langle \mathbf{p}^2 \rangle$ was fixed (using the mass of c quark $m_c = 1.43$ GeV) by means of a condition analogous to our equation (60) without the addendum proportional to the parameter B entering in the confinement part of the potential. The results obtained in the present study evidently show that the relativistic plus bound state corrections lead to the systematic increase of the fragmentation probabilities for the pseudoscalar and vector mesons. Our total fragmentation functions for the decays $\bar{c} \rightarrow (c\bar{c})$, $\bar{b} \rightarrow (\bar{b}c)$, $\bar{b} \rightarrow (b\bar{b})$ retain the initial shape of the leading order contribution. In the production of vector mesons both corrections proportional to W and $\langle \mathbf{p}^2 \rangle$ have the same sign giving us a more essential modification of the leading order contribution in comparison with the pseudoscalar meson production. Nevertheless, it should be particularly emphasized that the obtained modification of the fragmentation functions due to relativistic and bound state effects improves slightly the compatibility of theoretical results with the experimental data. But new fragmentation functions cannot explain the discrepancy between the fragmentation model and experiment occurring at the present time. Indeed, comparing the CDF Collaboration data for the differential cross section for the production of J/Ψ mesons as a function of transverse momentum p_T with the calculations of the fragmentation contribution on the basis of the color-singlet model, we observe that the discrepancy amounts to a factor of order 10 in the region of small transverse momenta p_T [1–4]. So, new expressions of the fragmentation probabilities (19), (27), and (34), derived here, do not lead to the coincidence of the theoretical predictions and experiment.

The fragmentation functions (19), (27), and (42) depend not only on z but also on the factorization scale μ. They should be considered at a scale μ of the order of the heavy quark masses. The evolution of the fragmentation functions to the scale $\mu = M_Z/2$ is determined by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [40]:

$$\mu^2 \frac{\partial}{\partial \mu^2} D_{Q\to H}(z, \mu^2) = \int_z^1 \frac{dy}{y} P_{Q\to Q}\left(\frac{z}{y}, \mu\right) D_{Q\to H}(y, \mu^2), \tag{61}$$

where $P_{Q\to Q}(x)$ is the quark splitting function [41]. The modification of the z-shape of the fragmentation functions is shown in Fig. 6. The average values of the momentum fraction for the production of different heavy mesons at the scale $\mu = M_Z/2$ are the following: $\langle z\rangle = 0.50$ (J/Ψ), $\langle z\rangle = 0.46$ (η_c), $\langle z\rangle = 0.63$ (B_c^*), $\langle z\rangle = 0.59$ (B_c), $\langle z\rangle = 0.56$ (Υ), $\langle z\rangle = 0.52$ (η_b).

In the case of $(\bar{c}c)$ or $(\bar{b}b)$ mesons Eqs. (29) and (46) acquire a more simple form:

$$\begin{aligned}\Omega_V &= \int_0^1 D_{\bar{Q}\to V(\bar{Q}Q)}(z)dz \\ &= \frac{32\alpha_s^2|\Psi(0)|^2}{27m_2^3}\left[\frac{1189}{30} - 57\ln 2\right. \\ &\quad + \frac{W}{m_2}\left(134\ln 2 - \frac{78\,149}{840}\right) \\ &\quad \left. + \frac{\langle \mathbf{p}^2\rangle}{m_2^2}\left(\frac{1078}{9}\ln 2 - \frac{78\,416}{945}\right)\right].\end{aligned} \tag{62}$$

$$\begin{aligned}\Omega_P &= \int_0^1 D_{\bar{Q}\to P(\bar{Q}Q)}(z)dz \\ &= \frac{8\alpha_s^2|\Psi(0)|^2}{81m_2^3}\left[\frac{1546}{5} - 444\ln 2\right. \\ &\quad \left. + \frac{W}{m_2}\left(-104\ln 2 + \frac{15\,581}{240}\right) + \frac{\langle \mathbf{p}^2\rangle}{m_2^2}\left(\frac{16}{9}\ln 2 - \frac{139}{48}\right)\right].\end{aligned} \tag{63}$$

Numerical values of the total fragmentation probabilities are presented in Table I. The evolution conserves the integral probabilities Ω_V and Ω_P of the fragmentation. Using expressions (29) and (46) we obtain the significant experimental ratio

$$\eta(r) = \frac{\Omega_V}{\Omega_V + \Omega_P}, \tag{64}$$

which predicts the relative number of vector and pseudoscalar mesons. For the $(\bar{c}c)$, $(\bar{b}c)$, and $(\bar{b}b)$ mesons the ratio (64) gives the following numbers: 0.46, 0.58, 0.52. The obtained results for the relativistic and bound state corrections to the different heavy quark fragmentation functions are shown in Figs. 2–5. Relative order contributions of relativistic and binding corrections are the biggest for the $(\bar{b}c)$ mesons because of the little growth of the parameter (60) and bound state energy W as compared to $(c\bar{c})$ states. The decrease of these corrections in the bottomonium is explained by the increase of heavy quark mass ($m_c \longrightarrow m_b$). All considered effects in the production of $(\bar{b}c)$, $(\bar{b}b)$, and $(\bar{c}c)$ mesons are computed to the leading order in α_s in the color-singlet model. For S states like J/Ψ, η_c, B_c, B_c^*, η_b, and Υ the color-octet terms are suppressed relative to the color-singlet terms by a factor of the second order over the relative velocity v_Q [1]. Our results should be useful for the comparison with more accurate $(\bar{c}c)$, $(\bar{b}b)$, and $(\bar{b}c)$ meson production measurements in Z^0 decays or in $p\bar{p}$ collisions at the Tevatron.

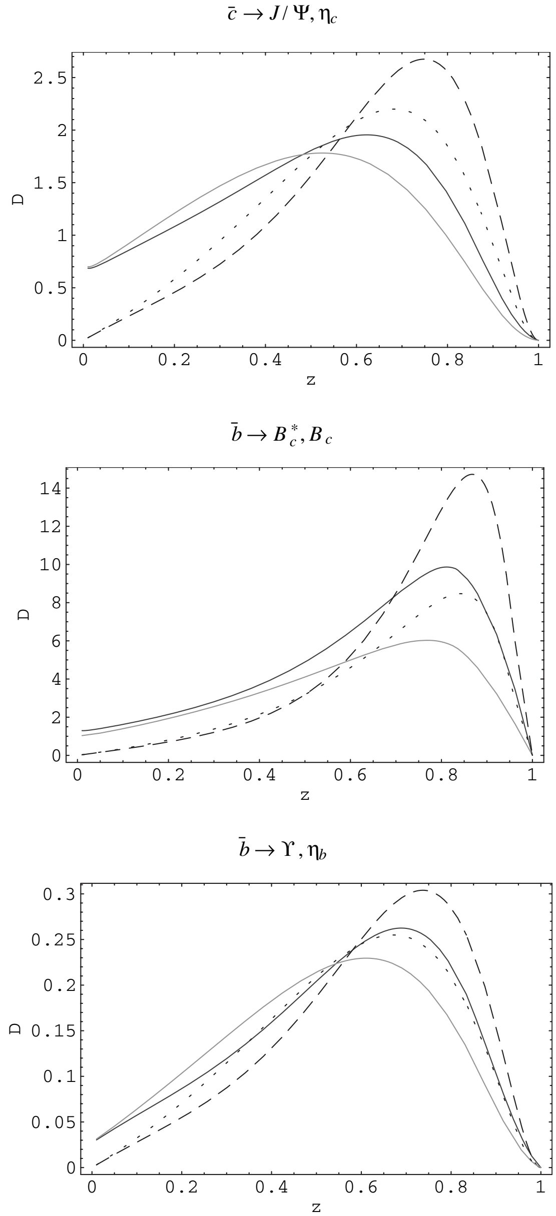

FIG. 6. The total fragmentation functions for the production of vector and pseudoscalar heavy mesons as a function of z for $\mu = m_1 + 2m_2$ (dashed lines) and $\mu = M_Z/2$ (solid lines). The thick and thin solid lines correspond to the vector and pseudoscalar mesons, respectively. All functions have been multiplied by a factor of 10^4.

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