

On the existence of heavy pentaquarks: The large N_c and heavy quark limits and beyond

Thomas D. Cohen*

Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA

Paul M. Hohler†

Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA

Richard F. Lebed‡

Department of Physics and Astronomy, Arizona State University, Tempe, Arizona 85287-1504, USA

(Received 25 August 2005; published 12 October 2005)

We present a very general argument that the analogue of a heavy pentaquark (a state with the quantum numbers of a baryon combined with an additional light quark and a heavy antiquark \bar{Q}) must exist as a particle stable under strong interactions in the combined heavy quark and large N_c limits of QCD. Moreover, in the combined limit these heavy pentaquark states fill multiplets of $SU(4) \times O(8) \times SU(2)$. We explore the question of whether corrections in the combined $1/N_c$ and $1/m_Q$ expansions are sufficiently small to maintain this qualitative result. Since no model-independent way is known to answer this question, we use a class of realistic hadronic models in which a pentaquark can be formed via nucleon-heavy-meson binding through a pion-exchange potential. These models have the virtue that they necessarily yield the correct behavior in the combined limit, and the long-distance parts of the interactions are model independent. If the long-distance attraction in these models were to predict bound states in a robust way (i.e., largely insensitive to the details of the short-range interaction), then one could safely conclude that heavy pentaquarks do exist. However, in practice the binding does depend very strongly on the details of the short-distance physics, suggesting that the real world is not sufficiently near the combined large N_c, m_Q limit to use it as a reliable guide. Whether stable heavy pentaquarks exist remains an open question.

DOI: [10.1103/PhysRevD.72.074010](https://doi.org/10.1103/PhysRevD.72.074010)

PACS numbers: 12.39.Mk, 11.15.Pg, 12.39.Hg, 12.39.Pn

I. INTRODUCTION

The existence of pentaquarks remains a vexing unresolved experimental question. Ten groups performing a variety of experiments [1–3] have reported the appearance of the pentaquark state now called Θ^+ , a resonance with baryon number +1, strangeness +1, and a mass in the vicinity of 1540 MeV. However, these experiments were all performed with relatively limited statistics and significant cuts, raising the possibility that the reported resonance is due to nothing more than statistical fluctuations. One ground for skepticism arises from a series of experiments that did not find a Θ^+ resonance [4,5]. Of course, it is unclear whether some of the experiments with negative results should be sensitive to such an observation, since there is no reliable theoretical framework for predicting the Θ^+ production rate. The Θ^+ width generates another source of doubt: $\Gamma(\Theta^+)$ must be exceedingly narrow (in the range of 1–2 MeV or smaller), or it would have been detected long ago [6], and to many it strains credulity that such a narrow state exists in this kinematic range.

One common thread in these early reports of detection (or nondetection) of the Θ^+ is the dependence of the experimental analysis upon revisited old data, and the

appearance of the signal only after the imposition of various cuts. Given the limited size of these old data sets, all of the studies yielded spectra with very limited statistics, creating the possibility of narrow peaks due to statistical fluctuations. The need for high-statistics experiments became very clear. Special-purpose experiments designed to look for pentaquarks with high statistics have been performed at Jefferson Lab; the CLAS Collaboration has analyzed the high-statistics data from photons on both a proton target [7] and a deuterium target [8], and finds no evidence for a Θ^+ peak. While these experiments alone do not rule out the Θ^+ , they show that at least two of the previous claims of evidence for the state, the SAPHIR γp result [2] and the CLAS γd result [3], were indeed statistical fluctuations. The prospect that other claims of evidence for the Θ^+ may also evaporate weighs heavily on the field. The initial observation [9] of Ξ pentaquark states appears to be headed for a similar fate [5].

The theoretical landscape for pentaquarks has been just as murky. A paper by Diakonov, Petrov, and Polyakov [10] was seminal in focusing attention on the pentaquark, in that it predicted a narrow state at almost exactly the mass where the Θ^+ was later reported. However, this paper is based upon an approximation later shown to be inconsistent with the large N_c assumptions implicit in the model [11]. After the experimental claims of pentaquarks appeared, a vast literature of models for the Θ^+ followed. In all of these models the existence of the Θ^+ depends upon *ad hoc*

*Electronic address: cohen@physics.umd.edu†Electronic address: pmhohler@physics.umd.edu‡Electronic address: Richard.Lebed@asu.edu

assumptions; thus they cannot be used reliably to predict the existence of the state, and accordingly are not reviewed here. Ultimately one may hope for lattice QCD eventually to resolve the theoretical question of whether the state exists. However, current lattice simulations for both heavy and light pentaquarks [12], while always improving, still remain inconclusive.

Given this morass, it is sensible to ask whether one can find a regime in which the question of the pentaquark's existence is more tractable. It has been noted previously in the context of various models [13,14] that heavy pentaquarks, states in which the \bar{s} quark in Θ^+ is replaced by a \bar{c} or a \bar{b} quark, is more likely to be bound than the \bar{s} type. The principal purpose of this paper is to explore the possible existence of heavy pentaquarks. We show in a particular limit of QCD, the combined large N_c and heavy quark limits, that heavy pentaquarks must exist, that they are stable under strong interactions, and that they fall into multiplets of $SU(4) \times O(8) \times SU(2)$. Here, the $SU(4)$ is the large N_c spin-flavor symmetry of the light u and d quarks [15–17], the $O(8)$ is a dynamical symmetry associated with collective vibrations of the heavy antiquark \bar{Q} (mass m_Q) relative to the remainder of the system [18], and the $SU(2)$ is the symmetry of separate rotations of the \bar{Q} spin. We then explore the critical question of whether $1/N_c$ and $1/m_Q$ corrections are sufficiently small for this qualitative result to survive in the physical world. There are no known analytic methods starting directly from QCD to answer this last question; thus, we investigate the question in the context of models.

We employ models that treat the heavy pentaquark as a bound state of a heavy meson and a nucleon interacting via pion exchange. Although similar models have been considered previously [19], the present work expands on them and is done in the context of the combined heavy quark and large N_c limits. Such models have two principal virtues: First, as we show below, the combined large N_c and m_Q limit mandates the existence of bound pentaquarks. Indeed, our demonstration is based on the fact that QCD in the combined limit can be reduced to a model of this form. Second, the long-distance behavior of the model is well known empirically (up to experimental uncertainties in the pion-heavy-meson coupling constant). If the long-distance attraction due to pion exchange were sufficient to bind the pentaquark for any reasonable choice of short-distance dynamics (as happens in the combined limit) then one would have a robust prediction that heavy pentaquarks exist. Unfortunately, we find that this is not the case.

Before proceeding it is useful to clarify a semantic point. Our discussion relies heavily on the large N_c limit of QCD; as N_c becomes large, the minimum number of quarks in a baryon containing a heavy antiquark is not 5, but rather $N_c + 2$. Nonetheless, we still denote such states as “pentaquarks,” to make the obvious connection to the $N_c = 3$ world.

This paper is organized as follows. In Sec. II, we provide a brief background on heavy pentaquarks. Section III presents a rigorous argument for the existence of heavy pentaquarks in the combined large N_c and large m_Q limits. In Sec. IV, we discuss the symmetry structure of heavy pentaquarks in the combined limit, and, in particular, the fact that they fall into multiplets of $SU(4) \times O(8) \times SU(2)$. Then we explore in Sec. V the question of whether this qualitative result survives in the real world of $N_c = 3$ and finite m_Q by studying simple models based on a pion exchange between nucleons and heavy mesons. Finally, Sec. VI presents a brief discussion of the implications of this work and concludes.

II. HEAVY PENTAQUARKS: BACKGROUND

The experimental situation involving reports of heavy pentaquarks remains murky. The H1 Collaboration at HERA has reported [20] a narrow resonance Θ_c appearing in $D^{*-}p$ [$(\bar{c}d)(uud)$] and $D^{*+}\bar{p}$ [$(c\bar{d})(\bar{u}\bar{u}\bar{d})$] states produced in inelastic ep collisions, with a mass of $3099 \pm 3 \pm 5$ MeV and a width of 12 ± 3 MeV. We note that the Θ_c , even if it withstands further experimental scrutiny, is *not* the type of heavy pentaquark discussed in this paper, since it is a resonance unstable against strong decay. Moreover, subsequent evidence argues against its existence: The FOCUS Collaboration [21], using a method similar to that of H1 but with greater statistics, finds no evidence for Θ_c . The experimental situation for heavy pentaquarks remains in a state as unsatisfactory as for their lighter cousins.

On the theoretical side, much of the heavy pentaquark research to date has been performed in the context of different variants of the quark model [13,22,23]. Our purpose here is not to review this work in any detail, but to stress one of its key points: Heavy pentaquarks occur far more naturally than light pentaquarks in such models, simply because a heavy quark is drawn more closely than a lighter quark to the bottom of any potential well. At the time much of the theoretical analysis was performed, many researchers assumed that light pentaquarks were experimentally firmly established, and so such models seemed to make rather robust predictions of stable pentaquarks. Now that the existence of the light pentaquarks has become more questionable, the reliability of heavy pentaquark predictions can also be questioned. Nevertheless, the tendency of heavy pentaquarks to bind more tightly than light ones remains generically true, a simple fact that continues to play a crucial role in the analysis of this paper.

Stewart, Wessling, and Wise [13] also raise a critical issue in the context of a diquark type model, namely, whether heavy pentaquarks could prove stable against strong decays. They argue that negative-parity heavy pentaquarks should have the lowest energy (in contrast to the positive-parity Θ^+ of the Jaffe-Wilczek model [24]) since this involves s-wave interactions between the diquarks.

They suggest that the additional attraction in such negative-parity states might be sufficient to render the states stable against strong decays. In this paper we argue that pentaquarks do in fact exist, at least in the combined large N_c and large m_Q limits of QCD.

Since the large N_c limit plays a critical role in our argument, it is useful to remark upon previous work on heavy pentaquarks as $N_c \rightarrow \infty$. References [22,25,26] impose large N_c counting rules in the context of a quark picture as a way to implement large N_c QCD. Such a picture suggests a Hamiltonian and asymptotically stable eigenstates. However, *generic* excited baryons at large N_c are broad resonances with $O(N_c^0)$ widths and require an approach respecting their nature as poles occurring at complex values in scattering amplitudes. Two of this work's authors have developed just such a "scattering picture" [27]. While obtainable through a generalization of the large N_c treatment for the stable ground-state band of baryons [17], the scattering approach naturally allows a proper treatment of resonant behavior such as large configuration mixing between resonances of identical quantum numbers [28]. Even for pentaquarks of $O(N_c^0)$ widths, the scattering approach predicts multiplets degenerate in both mass and width [29]. But this technology, while generally true, is not required in the current work; as we now show, the heavy pentaquarks discussed in this paper are stable against strong decay, at least in the combined formal limit $N_c \rightarrow \infty$, $m_Q \rightarrow \infty$.

III. THE EXISTENCE OF HEAVY PENTAQUARKS

We now show that heavy pentaquarks exist in the combined large N_c and large m_Q limits: They are stable against strong decay. We must first choose an appropriate parameter to describe the limiting procedure. Here, the natural choice is the λ expansion, where

$$\lambda \sim 1/N_c, \Lambda_{\text{QCD}}/m_Q, \quad (1)$$

Λ_{QCD} is the hadronic scale, and m_Q is the mass of the heavy quark. We note that the natural expansion turns out to be in powers of $\lambda^{1/2}$ [18], instead of λ^1 for a pure $1/N_c$ expansion.

Consider the states in the QCD Hilbert space that have energy less than $M_N + M_H + m_\pi$ (M_H is the mass of the lightest hadron containing heavy antiquark \bar{Q}), and have baryon number +1 and heavy quark Q number -1. These conditions exactly describe potentially narrow heavy pentaquarks Θ_Q (assuming no symmetry forbids the one-pion decay). Now consider further states with energy less than $M_N + M_H$; any pentaquark state appearing here must be a bound state as no hadronic decay can occur. However, scattering states clearly occur between the nucleon and the heavy meson that have the appropriate quantum numbers and have low enough energies. Therefore states that can be labeled Θ_Q exist.

Yet is it possible to describe such a state as bound in some realistic potential? First note that momenta in the scattering states scale as λ^0 . Therefore, since the N, H reduced mass μ scales as λ^{-1} , the kinetic energy scales as λ^1 , which is much smaller than $m_\pi = O(\lambda^0)$. One may therefore construct an effective theory in which all scatterings with >2 final-state hadrons are integrated out.

However, these states naively appear nonlocal, which would prevent the construction of a local potential. The range of the nonlocality scales as the inverse of momenta p associated with the smallest kinetic energy T one integrates out. In this case, $T \sim m_\pi$. Therefore, the range scales like $1/p = (2\mu m_\pi)^{-1/2} \sim \lambda^{1/2} \rightarrow 0$ as $\lambda \rightarrow 0$: The nonlocality disappears.

Next, one must ensure that the potential that binds the pentaquark does not vanish in the combined limit. From Witten's original N_c counting [30], one finds that indeed $V(\vec{r}) \sim \lambda^0$, preventing its disappearance relative to the kinetic energy. Noting that the heavy quark coupling scales as $g_s \sim N_c^{-1/2}$, the nucleon coupling is of order $g_A/f_\pi \sim N_c^{1/2}$, and the pion propagator is of order $m_\pi \sim N_c^0$, one combines these ingredients to find the desired λ^0 scaling for the potential.

We can now easily prove the existence of stable heavy pentaquarks. Having established the locality and scaling of the potential between heavy hadrons, we have successfully reduced a quantum field theory problem to one of non-relativistic quantum mechanics. It is well understood in this context that a potential with an attractive region has an infinite number of bound states as $\mu \rightarrow \infty$ (see Appendix A for details). In the present case, $\mu \sim \lambda^{-1} \rightarrow \infty$, while $V(\vec{r}) \sim \lambda^0$. Thus, proving the existence of heavy pentaquarks in the combined limit requires only that $V(\vec{r})$ is attractive in at least some region. Fortunately, we know the form of $V(\vec{r})$ at large distances: It is given by a one-pion-exchange potential (OPEP), because π is the lightest hadron that can be exchanged between H and N . It is moreover known that, regardless of the relative signs of the coupling constants, attractive channels appear in the OPEP. Thus, $V(\vec{r})$ necessarily has attractive regions, serving to bind the heavy pentaquark.

IV. SYMMETRIES OF HEAVY PENTAQUARKS

We now show that, in the combined large N_c and large m_Q limit, the pentaquark states form a multiplet of the group $\text{SU}(4) \times \text{O}(8) \times \text{SU}(2)$, which is an emergent symmetry of QCD. The $\text{SU}(4)$ group is a spin-flavor symmetry of the light quarks similar to that in Refs. [15–17]. The argument [15,17] that $\text{SU}(2)_{\text{spin}} \times \text{SU}(N_f)_{\text{flavor}}$ combine to form a contracted $\text{SU}(2N_f)$ is completely applicable to the case of heavy pentaquarks, where here we restrict to $N_f = 2$.

The $\text{O}(8)$ group is the symmetry associated with the configuration of the heavy quark relative to the light de-

degrees of freedom. For nonexotic baryons, the origin of this symmetry is explained in Ref. [18]. Since the reason for such a symmetry may not be so familiar, we provide further details here. Consider an attractive potential $V(\vec{r})$ of the sort described in Sec. III. Such a $V(\vec{r})$ has a minimum, near which it can be approximated as harmonic. In the large N_c and large m_Q limits, the wave function is localized near this minimum, creating an emergent U(3) simple harmonic oscillator symmetry. This U(3) symmetry is generated by $T_{ij} \equiv a_i^\dagger a_j$ ($i, j = 1, 2, 3$), where a_j is the annihilation operator in the j th coordinate direction. The generators satisfy U(3) commutation relations:

$$[T_{ij}, T_{kl}] = \delta_{kj}T_{il} - \delta_{il}T_{kj}. \quad (2)$$

Additionally, as $N_c \rightarrow \infty$ the creation and annihilation operators also become generators of the emergent symmetry with the commutation relations

$$\begin{aligned} [a_j, T_{kl}] &= \delta_{kj}a_l, & [a_i^\dagger, T_{kl}] &= -\delta_{il}a_k^\dagger, \\ [a_i, a_j^\dagger] &= \delta_{ij}\mathbb{1}, \end{aligned} \quad (3)$$

where $\mathbb{1}$ is the identity operator. The 16 generators $\{T_{ij}, a_l, a_k^\dagger, \mathbb{1}\}$ form the *minimal spectrum-generating algebra* for the U(3) harmonic oscillator. It is related to the U(4) algebra generated by T_{ij} ($i, j = 1, 2, 3, 4$) and satisfying commutation relations Eq. (2) by the limiting procedure

$$a_j = \lim_{R \rightarrow \infty} T_{4j}/R, \quad a_i^\dagger = \lim_{R \rightarrow \infty} T_{i4}/R, \quad \mathbb{1} = \lim_{R \rightarrow \infty} T_{44}/R^2. \quad (4)$$

Such a procedure is called a *group contraction*. Hence the group generated by $\{T_{ij}, a_l, a_k^\dagger, \mathbb{1}\}$ is called a contracted U(4) group.

The generating algebra of the contracted U(4) group can be expanded by including the operators $S_{ij} = a_i a_j$ and $S_{ij}^\dagger = a_i^\dagger a_j^\dagger$ ($i, j = 1, 2, 3$) with the following commutation relations:

$$\begin{aligned} [S_{ij}, S_{kl}] &= [S_{ik}, a_l] = 0, & [S_{ij}, a_k^\dagger] &= \delta_{jk}a_i + \delta_{ik}a_j, \\ [S_{ij}, T_{kl}] &= \delta_{jk}S_{il} + \delta_{ik}S_{jl}, \\ [S_{ij}, S_{kl}^\dagger] &= (\delta_{ki}\delta_{lj} + \delta_{il}\delta_{kj})\mathbb{1} + \delta_{ki}T_{lj} + \delta_{lj}T_{ki} \\ &\quad + \delta_{il}T_{kj} + \delta_{kj}T_{li}, \end{aligned} \quad (5)$$

while the commutation relations for S_{ij}^\dagger can be obtained through Hermitian conjugation. This set of 28 generators $\{S_{ij}, S_{ij}^\dagger, T_{ij}, a_l, a_k^\dagger, \mathbb{1}\}$ forms a closed operator algebra, which is a contracted O(8).

Reference [18] continues by showing that this emergent O(8) is also an emergent symmetry of QCD. Extension to the present case is straightforward. The argument for the presence of the contracted O(8) emergent symmetry relies on one's ability to approximate the bottom of $V(\vec{r})$ as a harmonic oscillator potential. As we have seen, the large

N_c and large m_Q limits ensure this feature by leading to $\mu \rightarrow \infty$. These conditions remain just as true for a heavy antiquark; thus the argument from [18] applies to heavy pentaquarks.

The SU(2) is simply the symmetry of invariance under spin rotations of the heavy quark: In the heavy quark limit, states with any alignment of the heavy quark spin are degenerate.

V. BOUND STATES AND THE ONE-PION-EXCHANGE POTENTIAL

Now that we have shown stable heavy pentaquarks exist in the combined large N_c large m_Q limit, the critical question becomes whether they also occur in our $N_c = 3$ finite m_Q world. To our knowledge, this question cannot be answered in a model-independent way without solving QCD, and so we resort to models for enlightenment.

We focus here on effective potential models based upon one-pion-exchange at long distance. As discussed in Sec. III, such models are clearly useful not only because they represent physically correct phenomenology, but also guarantee stable pentaquarks in the combined limit. But we also note that the argument does not depend upon the particular short-distance behavior of the effective potential. If the real world is sufficiently close to the combined-limit world for the argument to remain valid, all models of this sort must yield (multiple) stable pentaquarks. Note that the masses of the various pentaquark states can depend sensitively upon the details of the short-distance interaction, but their existence cannot. The question then becomes whether models of this type predict bound pentaquarks in a robust way, independent of the details of the short-distance physics. If so, one has a strong reason to believe that the states are, in fact, bound in nature.

We construct a ‘‘realistic’’ potential that has the correct long-distance behavior (OPEP) and an *ad hoc* short-distance part constrained only by the natural scales of strong interaction physics. Our potential acts between a nucleon and a heavy meson (D or B). The nucleon-pion analogue is well understood; its interaction Lagrangian reads

$$\mathcal{L}_{NN\pi} = -\frac{g_A}{f_\pi\sqrt{2}}\bar{N}\tau^a\gamma_\nu\gamma_5N\partial^\nu\pi^a, \quad (6)$$

where the axial coupling constant $g_A \simeq 1.27$, and the pion decay constant $f_\pi \simeq 131$ MeV.

However, the heavy-meson-pion interaction is not as straightforward. We use a formalism similar to that outlined by Manohar and Wise [31] to encode the heavy quark symmetry. In the limit of N_Q heavy quark flavors, QCD develops an emergent SU($2N_Q$) symmetry [32]. As a consequence of this symmetry, physical states do not depend on the spin of the heavy quark; thus the $D(B)$ and $D^*(B^*)$ mesons form a degenerate multiplet in the $m_c(m_b) \rightarrow \infty$ limit. The heavy-meson-pion interactions can involve

transformations between pseudoscalar and vector states ($B \leftrightarrow B^*$ or $D \leftrightarrow D^*$). Using heavy quark symmetry, one combines them into a single field:

$$H \equiv \frac{(1 + \not{v})}{2} [P_\mu^* \gamma^\mu - P \gamma_5], \quad (7)$$

where v^μ is the four-velocity, and the pseudoscalar and vector heavy-meson fields are P and P_μ^* , respectively. This combination allows the interaction Lagrangian to be written in a manner similar to that of the nucleon interaction,

$$\mathcal{L}_{\text{int}} = -\frac{g_H}{f_\pi \sqrt{2}} \text{Tr} \bar{H} \tau^a \gamma_\mu \gamma_5 H \partial^\mu \pi^a. \quad (8)$$

Of course, the pseudoscalar and vector mesons are *not* degenerate in the real world, due to $1/m_Q$ corrections. The mass difference must be included in realistic models.

Both the nucleon and heavy-meson interactions with the pion can be expressed in terms of the spin and isospin of the particles:

$$\mathcal{L}_{NN\pi} = \frac{2\sqrt{2}g_A}{f_\pi} (\vec{S}_N \cdot \vec{\partial} \pi^a) I_N^a, \quad (9)$$

$$\mathcal{L}_{\text{int}} = \frac{2\sqrt{2}g_H}{f_\pi} (\vec{S}_l \cdot \vec{\partial} \pi^a) I_H^a, \quad (10)$$

where \vec{S}_N and \vec{I}_N are the spin and isospin of the nucleon, \vec{S}_l is the spin of the light quark in H , and \vec{I}_H is the isospin of the H field. Combining Eqs. (9) and (10), treating the nucleon and heavy meson in the static limit (i.e., ignoring recoil, which is suppressed in the combined limit) and Fourier transforming yields the OPEP in position space:

$$\begin{aligned} V_\pi(\vec{r}) &= \vec{I}_N \cdot \vec{I}_H [2S_{12} V_T(r) + 4\vec{S}_N \cdot \vec{S}_l V_c(r)] \\ &= (I^2 - I_N^2 - I_H^2) [S_{12} V_T(r) \\ &\quad + (K^2 - S_N^2 - S_l^2) V_c(r)], \end{aligned} \quad (11)$$

where the central part of the potential (r measured in units of $1/m_\pi$) is

$$V_c(r) = \frac{g_A g_H}{2\pi f_\pi^2} \frac{e^{-r}}{3r}, \quad (12)$$

and the tensor part is

$$V_T(r) = \frac{g_A g_H}{2\pi f_\pi^2} \frac{e^{-r}}{6r} \left(\frac{3}{r^2} + \frac{3}{r} + 1 \right). \quad (13)$$

I is the total isospin of the combined system, while $\vec{K} \equiv \vec{S}_N + \vec{S}_l$, and

$$S_{12} \equiv 4[3(\vec{S}_N \cdot \hat{r})(\vec{S}_l \cdot \hat{r}) - \vec{S}_N \cdot \vec{S}_l]. \quad (14)$$

It remains unknown whether g_A and g_H are of the same sign or of different signs, so the potential could have an additional overall negative sign.

Clearly, the OPEP dominates the interaction at large r since π is the lightest hadron. At shorter ranges the OPEP is no longer dominant and the effective potential is qualitatively different. The value of r at which the OPEP ceases to dominate the effective potential is presumably of order $1/\Lambda_{\text{QCD}} \sim 1$ fm, the characteristic range in strong interactions. Therefore, for distances less than some cutoff value $r_0 \sim 1$ fm, we use a purely phenomenological potential. Note that we do not simply add such a short-range potential to the OPEP at short distances, but entirely replace the OPEP by this new potential: The $1/r^3$ behavior of the tensor part of the OPEP at short ranges is unphysical and would completely dominate the potential if not removed. The short-distance potentials used are taken to be either (central) constants or quadratic functions, and their strengths are allowed to vary. If the logic of our underlying argument based upon the combined limit also holds for realistic m_Q values and $N_c = 3$, then the precise details of the potentials should be irrelevant to whether the pentaquark states bind.

We use the OPEP of Eq. (11) in a nonrelativistic Schrödinger equation and solve for bound states. Since the tensor term in the potential allows mixing between L states, L is not a good quantum number. However, S_{12} commutes with the parity operator, making P a good quantum number. Therefore, states labeled by J, S (total spin $\vec{S} \equiv \vec{S}_Q + \vec{K}$), and P are used as eigenstates. Treating states mixed under L requires a coupled-channel calculation; we obtain the coupled equations by including all possible states labeled by L and K that are consistent with a given set of J, S , and P .

Lastly, since this potential is intended to be realistic, in principle B - B^* and D - D^* mass differences can affect the results. Of course, these differences are $1/m_Q$ effects and vanish in the heavy quark limit. Since the principal reason for the model calculation is to test qualitatively whether we live in the regime of validity of the combined $1/N_c$ and $1/m_Q$ expansion, it makes sense to include this difference. However, in practice the effect of this mass difference is entirely repulsive, making the states less likely to bind. Thus, if the states do not bind in the equal-mass case, they do not bind at all. Accordingly, we use equal masses and only investigate the effect of the mass splitting in cases where binding occurs.

We attempt to make our model as realistic as possible, given the rather simple forms assumed for the short-distance potential. To this end, we choose for the heavy-meson coupling constant $g_H \approx \pm 0.59$ (extracted from $D^* \rightarrow D\pi$ decay [33], see below) and collect values for other observables [34] in Table I. As an initial guess, we also constrain the parameters of the short-range potential such that this potential combined with a OPEP between nucleons gives the correct 2.2 MeV deuteron binding energy. This choice is not necessary, but it has the virtue of ensuring that the potential parameters are not completely

TABLE I. Constants used in bound-state calculations for heavy pentaquarks.

Quantity Name	Quantity Value
g_A	1.27
f_π	131 MeV
g_H	± 0.59
m_π	138 MeV
m_N	938.92 MeV
m_B	5279 MeV
m_D	1867 MeV
Δ_B	46 MeV
Δ_D	141 MeV

unreasonable from the point of view of hadronic physics. We summarize the potentials in Eq. (B1). Ultimately, we vary many of the parameters in order to probe the robustness of the qualitative results.

We then solve coupled differential equations using standard numerical methods. We seek bound-state solutions for all $J = \frac{1}{2}$ and $J = \frac{3}{2}$ states using both a constant and a quadratic form for the short-distance potential, for $I = 0$ and $I = 1$, and with either sign of g_H relative to g_A . Initially (as discussed above), we assume no mass splitting between the pseudoscalar and vector mesons. A complete set of tables of bound states thus obtained appears in Tables II and III. Here we focus on describing some key features of these results.

For constant and quadratic potentials constrained by matching to the deuteron energy, bound states of the pentaquark are quite sparse. No channel supports a bound state

with a D meson. The B meson is able to bind weakly in the channels with negative parity, but only with $I = 0$. Binding in these states is relatively weak, around 1.3 MeV for the constant potential and around 3.9 MeV for the quadratic potential, and binding energies are consistently the same between these channels (Table II, Cols. A and B). It should be noted that both [13] and our calculations have the negative-parity states being more stable. The greater binding for the quadratic (versus the constant) potential is natural since it is significantly deeper.

We also analyze the case in which the short-distance potential is simply set to zero. For this case, the OPEP does not bind a pentaquark for any channel. In order for this potential to bind without the aid of short-distance potential, g_H would need to be raised to unreasonably high levels, near 1 (approximately double the extracted value), and in some cases larger than 2. When realistic mass differences between the vector and pseudoscalar mesons are introduced, binding becomes weaker. This mass splitting eliminates binding for all channels with either type of potential we consider.

The heavy-meson coupling constant g_H used in our analysis is motivated by the results of a recent experiment by the CLEO Collaboration that measured [33] the width of the $D^{*\pm} \rightarrow D^0 \pi^\pm$ decay. The value of g_H is extracted from the width and found to be $\pm 0.59 \pm 0.07$. The analogous decay process is energetically forbidden in the B sector, preventing a direct extraction; therefore, we exploited heavy quark symmetry and used the same value of g_H for the B sector. Note, however, the uncertainty in the bottom sector due to possible $1/m_Q$ corrections. Accordingly, we also investigated using a range of

 TABLE II. B meson bound-state energies for each channel, where + and - refer to relative sign of g_A and g_H . All energies in MeV. Column A: constant potential, $V_0 = -62.79$ MeV and $r_0 = 1$ fm; B: quadratic potential; C: constant potential, $V_0 = -276$ MeV and $r_0 = 1$ fm; D: constant potential, $V_0 = -62.79$ MeV and $r_0 = 1.5$ fm.

Channel J	S	P	I	A		B		C		D	
				+	-	+	-	+	-	+	-
$\frac{1}{2}$	$\frac{1}{2}$	-	0	1.30	1.35	3.89	1.92, 3.62	139.38, 142.14	—	14.49, 16.01	15.46, 16.15
			1	—	—	0.35	0.27	—	139.38, 140.76	15.32, 15.60	15.04, 15.46
$\frac{1}{2}$	$\frac{1}{2}$	+	0	—	—	—	—	14.9, 32.39	4, 19.32, 46.5	—	—
			1	—	—	—	—	12.72, 18.22, 26.91	9.45	—	—
$\frac{1}{2}$	$\frac{3}{2}$	-	0	1.30	1.31	3.89	3.67	140.76	140.76	15.87	15.32
			1	—	—	—	0.26	140.76	140.76	15.04	15.32
$\frac{1}{2}$	$\frac{3}{2}$	+	0	—	—	—	—	32.15	3.35, 45.95	—	—
			1	—	—	—	—	12.12, 27.19	8.36, 22.08	—	—
$\frac{3}{2}$	$\frac{1}{2}$	-	0	1.42	1.31	3.89	3.67	140.76	140.76	15.87	15.32
			1	—	—	—	0.26	140.76	140.76	15.04	15.32
$\frac{3}{2}$	$\frac{1}{2}$	+	0	—	—	—	—	15.32, 18.49, 32.43	4.65	—	—
			1	—	—	—	—	12.80	17.25, 17.66, 22.91	—	—
$\frac{3}{2}$	$\frac{3}{2}$	-	0	1.42	1.25	3.89	3.67	140.76	140.76	15.87	15.32
			1	—	—	—	0.20	140.76	140.76	15.04	15.32
$\frac{3}{2}$	$\frac{3}{2}$	+	0	—	—	—	—	18.22, 32.29	—	—	—
			1	—	—	—	—	4.18, 23.18	—	—	—

TABLE III. D meson bound-state energies for each channel, where + and - refer to the relative sign of g_A and g_H . All energies in MeV. Column A: constant potential, $V_0 = -276$ MeV and $r_0 = 1$ fm; B: constant potential, $V_0 = -62.79$ MeV and $r_0 = 1.5$ fm.

Channel		I	A		B		
J	S		+	-	+	-	
$\frac{1}{2}$	$\frac{1}{2}$	-	0	113.99, 110.4	—	7.36, 9.00	8.45, 9.27
			1	—	114.82, 115.78	8.40, 8.79	8.16, 8.63
$\frac{1}{2}$	$\frac{1}{2}$	+	0	2.91	16	—	—
			1	—	—	—	—
$\frac{1}{2}$	$\frac{3}{2}$	-	0	117.3	116.2	9.00	8.45
			1	115.23	115.23	8.45	8.45
$\frac{1}{2}$	$\frac{3}{2}$	+	0	2.10	15.87	—	—
			1	—	—	—	—
$\frac{3}{2}$	$\frac{1}{2}$	-	0	117.3	116.20	9.00	8.45
			1	115.37	115.78	8.45	8.45
$\frac{3}{2}$	$\frac{1}{2}$	+	0	2.91	—	—	—
			1	—	—	—	—
$\frac{3}{2}$	$\frac{3}{2}$	-	0	117.3	116.20	9.00	8.45
			1	115.09	115.09	8.45	8.45
$\frac{3}{2}$	$\frac{3}{2}$	+	0	2.53	—	—	—
			1	—	—	—	—

heavy-meson couplings and find the same qualitative results.

These results depend upon the strength of the short-distance potential. Clearly, as these potentials become more strongly attractive, the states are more likely to bind. As the potential needed to bind deuterium may be anomalously small, a deeper constant potential was also considered. Table II, Col. C and Table III, Col. A show the results when the constant potential is decreased from the depth needed to bind deuterium, -62.79 MeV, to about 4 times as deep, -276 MeV. The deeper well both produces more bound states and causes previously unbound states to bind (In particular, the D meson can form a bound state in the deeper potential).

The choice of OPEP cutoff at $r = 1$ fm is arbitrary. One does not expect the OPEP to be valid for $r < 1$ fm, but the effective cutoff might occur at somewhat larger r . Table II, Col. D and Table III, Col. B present the binding of states with a cutoff of 1.5 fm (the potential depth is -62.79 MeV). The negative-parity states remain the only bound ones, but the binding is now stronger, and the D meson binds. These fluctuations in strength of binding indicate the importance of the short-distance physics to the heavy pentaquark formation.

VI. DISCUSSION

Despite our general argument using the large N_c and large m_Q combined limit that the long-range OPEP is sufficient to bind pentaquarks, we find in our class of models that, if a heavy pentaquark binds at all due to one-pion exchange, it does so weakly in a few channels and depends in a nontrivial way upon the details of the

short-range interaction. The main implication is obvious: In the real world, $1/N_c$ and $1/m_Q$ corrections can be substantial. Indeed, they are large enough to render unreliable even qualitative predictions about heavy pentaquarks based upon the combined limit.

Given this somewhat unhappy result, the most important question is whether or not heavy pentaquarks do in fact bind to form stable states under strong interactions, and if so, whether only very weakly-bound states occur, such as the ones seen here. Both of these questions remain open. We simply do not know enough about the short-distance part of the effective potential to provide a definitive answer. An optimistic view is that the short-distance interaction between the heavy meson and the nucleon is likely to be more attractive than that between nucleons, which has a strong repulsive core. This argument is particularly plausible if one views at least part of the repulsive core between nucleons to arise due to the Pauli principle between overlapping nucleon wave functions; this effect is greatly reduced in the interaction between a nucleon and a heavy meson. If it is true that the short-range effective potential between the heavy meson and the nucleon is significantly more attractive than the analogous nucleon-nucleon case, then it is quite likely that heavy pentaquarks form stable, tightly-bound states.

Finally, we address the question of why the qualitative prediction of the combined large N_c and large m_Q limits is insufficient. At first sight this may seem surprising, since both the $1/N_c$ and $1/m_Q$ expansions have proven to be predictive in many situations. One must remember, however, that the quality of a systematic expansion depends on coefficients as well as the expansion parameter, and the size of these coefficients depends on the observable being

studied. If some observable has “unnaturally” large coefficients, then the expansion can easily fail unless the expansion parameter is extremely small. This view is echoed in [35]. The relevant question is whether one ought to expect unnaturally large corrections to the leading behavior.

In retrospect, it is perhaps not so surprising that combined expansion is insufficient here. One can make an analogous argument, based entirely upon $1/N_c$ counting, that both the deuteron and the 1S_0 two-nucleon channel ought to be deeply bound and have a large number of bound states: Both the effective interaction between nucleons and the masses of the two nucleons grow as N_c^1 . However, as has been stressed elsewhere [36], this argument fails for smaller values of N_c . Similarly, numerous doubly-heavy strongly-bound tetraquarks ought to exist in the heavy quark limit: The effective interaction between heavy mesons is independent of the heavy quark mass and scales as $1/(N_c m_Q)$. However, as discussed in Ref. [31] and based upon models similar to those studied here, it is questionable whether they are bound for finite m_Q . Evidently, the coefficients describing interactions between hadrons can in some qualitative way be sufficient to weaken significantly results one would naively expect directly from the $1/N_c$ or $1/m_Q$ expansions, yielding very large corrections to the leading-order results for real-world parameters. Why this is so is one of QCD’s more intriguing mysteries.

In conclusion, we showed that heavy pentaquarks must exist in combined large N_c and large m_Q limit, and that they form multiplets of $SU(4) \times O(8) \times SU(2)$. We constructed a one-pion-exchange potential between a nucleon and a heavy meson, and solved coupled nonrelativistic Schrödinger equations, obtaining bound states. Some weakly-bound states do exist in some models, but their existence depends upon unknown short-distance physics. The lack of binding emphasizes that the real world is too far from the idealized world of large N_c and large m_Q to render the expansions robust for these observables. In order to deduce whether or not heavy pentaquarks exist requires a more complete understanding of the short-distance physics than is presently known.

ACKNOWLEDGMENTS

T. D. C. and P. M. H. were supported by the D.O.E. through grant DE-FGO2-93ER-40762; R. F. L. was supported by the N.S.F. through grant PHY-0140362.

APPENDIX A: BOUND STATES IN QUANTUM MECHANICS

Consider a smoothly varying potential $V(r)$ that vanishes as $r \rightarrow \infty$. If $V(r)$ is nonsingular and has an attractive region, it must possess a minimum at some r_0 . In the neighborhood of r_0 the potential is approximately harmonic [i.e., $V(r) \approx \frac{k}{2}(r - r_0)^2$]. Therefore, if the wave function is for some reason localized near the minimum, then the system can be approximated as a harmonic oscillator. For large reduced mass μ the kinetic energy operator is small, and minimizing the wave function’s curvature forces its localization near $r = r_0$, as desired. The harmonic oscillator potential has an infinite number of bound states, separated by multiples of $\omega = \sqrt{k/\mu}$. Thus we see that multiple bound states must exist for sufficiently large μ . If the potential is singular (but not more singular than $1/r^2$, so that a ground state exists), the large size of μ localizes the wave function deep in the potential near the singularity, again allowing plenty of room for bound states.

APPENDIX B: TABLES OF RESULTS

This appendix focuses on our numerical results. Table I lists the parameters used in the calculation. Equation (B1) summarizes the potentials that were used. Table II presents the energies of bound states for a B meson binding with a nucleon, while Table III presents the same for a D meson.

Equation (B1) shows potentials used in heavy pentaquark calculations. The labels are: total isospin I , nucleon isospin I_N , heavy-meson isospin I_H , tensor force S_{12} , tensor potential $V_T(r)$, nucleon spin S_N , light quark in heavy meson spin S_l , sum of nucleon spin and light quark spin K , central potential $V_c(r)$. Numerical values are such that potentials are measured in MeV, distances in MeV^{-1} , unless noted otherwise. Both $V_1(r)$ and $V_2(r)$ are central potentials. The parameters in $V_2(r)$ were fixed by making the potential differentiable at r_0 and bind deuterium with the appropriate energy.

$$\begin{aligned}
 V_\pi &= \begin{cases} (I^2 - I_N^2 - I_H^2)[S_{12}V_T(r) + (K^2S_N^2 - S_H^2)V_c(r)] & r > r_0 \\ V_1(r) \text{ or } V_2(r) & r < r_0 \end{cases} \\
 V_c(r) &= \frac{g_A g_H}{2\pi f_\pi^2} \frac{e^{-m_\pi r}}{3r} m_\pi^2, \\
 V_T(r) &= \frac{g_A g_H}{2\pi f_\pi^2} \frac{e^{-m_\pi r}}{6r} \left(\frac{3}{m_\pi^2 r^2} + \frac{3}{m_\pi r} + 1 \right) m_\pi^2, \\
 V_1(r) &= V_0 (V_0 = -62.79 \text{ MeV or } -276 \text{ MeV}), \\
 V_2(r) &= -252.659 \frac{\text{MeV}}{\text{fm}^2} r^2 + 541.321 \frac{\text{MeV}}{\text{fm}} r - 309.822 \text{ MeV}. \tag{B1}
 \end{aligned}$$

- [1] T. Nakano *et al.* (LEPS Collaboration), Phys. Rev. Lett. **91**, 012002 (2003); V.V. Barmin *et al.* (DIANA Collaboration), Phys. At. Nucl. **66**, 1715 (2003); [Yad. Fiz. **66**, 527 (2003)]; A.E. Asratyan, A.G. Dolgolenko, and M.A. Kubantsev, Phys. At. Nucl. **67**, 682 (2004); [Yad. Fiz. **67**, 704 (2004)]; V. Kubarovsky *et al.* (CLAS Collaboration), Phys. Rev. Lett. **92**, 032001 (2004); **92**, 049902(E) (2004); A. Airapetian *et al.* (HERMES Collaboration), Phys. Lett. B **585**, 213 (2004); S. Chekanov *et al.* (ZEUS Collaboration), *ibid.* **591**, 7 (2004); M. Abdel-Bary *et al.* (COSY-TOF Collaboration), *ibid.* **595**, 127 (2004); A. Aleev *et al.* (SVD Collaboration), hep-ex/0401024.
- [2] J. Barth *et al.* (SAPHIR Collaboration), Phys. Lett. B **572**, 127 (2003).
- [3] S. Stepanyan *et al.* (CLAS Collaboration), Phys. Rev. Lett. **91**, 252001 (2003).
- [4] J.Z. Bai *et al.* (BES Collaboration), Phys. Rev. D **70**, 012004 (2004); B. Aubert *et al.* (BABAR Collaboration), hep-ex/0408064; K. Abe *et al.* (Belle Collaboration), hep-ex/0409010; S.R. Armstrong, Nucl. Phys. B Proc. Suppl. **142**, 364 (2005); S. Schael *et al.* (ALEPH Collaboration), Phys. Lett. B **599**, 1 (2004); Yu.M. Antipov *et al.* (SPHINX Collaboration), Eur. Phys. J. A **21**, 455 (2004); M.J. Longo *et al.* (HyperCP Collaboration), Phys. Rev. D **70**, 111101 (2004); D.O. Litvintsev *et al.* (CDF Collaboration), Nucl. Phys. B Proc. Suppl. **142**, 374 (2005); K. Stenson *et al.* (FOCUS Collaboration), Int. J. Mod. Phys. A **20**, 3745 (2005); R. Mizuk *et al.* (Belle Collaboration), hep-ex/0411005; C. Pinkenburg, (PHENIX Collaboration), J. Phys. G **30**, S1201 (2004).
- [5] I. Abt *et al.* (HERA-B Collaboration), Phys. Rev. Lett. **93**, 212003 (2004).
- [6] S. Nussinov, hep-ph/0307357; R.A. Arndt, I.I. Strakovsky, and R.L. Workman, Phys. Rev. C **68**, 042201 (2003); **69**, 019901(E) (2004); J. Haidenbauer and G. Krein, Phys. Rev. C **68**, 052201 (2003); R.N. Cahn and G.H. Trilling, Phys. Rev. D **69**, 011501 (2004); A. Sibirtsev, J. Haidenbauer, S. Krewald, and U.-G. Meissner, Phys. Lett. B **599**, 230 (2004); W.R. Gibbs, Phys. Rev. C **70**, 045208 (2004).
- [7] R. De Vita, invited talk at the APS APR05 Meeting, Tampa, Florida, <http://meetings.aps.org/Meeting/APR05/Event/31944>.
- [8] K. Hicks, plenary talk at the 3rd Asia Pacific Few Body Conference, <http://physics3.sut.ac.th>.
- [9] C. Alt *et al.* (NA49 Collaboration), Phys. Rev. Lett. **92**, 042003 (2004).
- [10] D. Diakonov, V. Petrov, and M.V. Polyakov, Z. Phys. A **359**, 305 (1997).
- [11] T.D. Cohen, Phys. Lett. B **581**, 175 (2004); hep-ph/0312191; D. Diakonov and V. Petrov, Phys. Rev. D **69**, 056002 (2004); N. Itzhaki, I.R. Klebanov, P. Ouyang, and L. Rastelli, Nucl. Phys. **B684**, 264 (2004); P. Pobylitsa, Phys. Rev. D **69**, 074030 (2004); M. Praszalowicz, Phys. Lett. B **583**, 96 (2004); Acta Phys. Pol. B **35**, 1625 (2004); J. Ellis, M. Karliner, and M. Praszalowicz, J. High Energy Phys. 05 (2004) 002; P. Schweitzer, Eur. Phys. J. A **22**, 89 (2004); R.L. Jaffe, Eur. Phys. J. C **35**, 221 (2004).
- [12] F. Csikor *et al.*, J. High Energy Phys. 11 (2003) 070; S. Sasaki, Phys. Rev. Lett. **93**, 152001 (2004); N. Mathur *et al.*, Phys. Rev. D **70**, 074508 (2004); N. Ishii *et al. ibid.* **71**, 034001 (2005); T.T. Takahashi *et al. ibid.* **71**, 114509 (2005); B.G. Lasscock *et al. ibid.* **72**, 014502 (2005); T.-W. Chiu *et al. ibid.* **72**, 034505 (2005); F. Csikor *et al.*, hep-lat/050312; C. Alexandrou and A. Tsapalis, hep-lat/0503013; K. Holland and K.J. Juge, hep-lat/0504007.
- [13] I.W. Stewart, M.E. Wessling, and M.B. Wise, Phys. Lett. B **590**, 185 (2004).
- [14] M. Karliner and H.J. Lipkin, hep-ph/0307343; Phys. Lett. B **575**, 249 (2003).
- [15] J.-L. Gervais and B. Sakita, Phys. Rev. Lett. **52**, 87 (1984); Phys. Rev. D **30**, 1795 (1984).
- [16] C.D. Carone, H. Georgi, and S. Osofsky, Phys. Lett. B **322**, 227 (1994); M. Luty and J. March-Russell, Nucl. Phys. **B426**, 71 (1994).
- [17] R. Dashen, E. Jenkins, and A.V. Manohar, Phys. Rev. D **49**, 4713 (1994); **51**, 2489(E) (1995).
- [18] C.-K. Chow and T.D. Cohen, Nucl. Phys. **A688**, 842 (2001); Phys. Rev. Lett. **84**, 5474 (2000).
- [19] M. Shmatikov, Phys. Lett. B **349**, 411 (1995).
- [20] A. Aktas *et al.* (H1 Collaboration), Phys. Lett. B **588**, 17 (2004).
- [21] J.M. Link *et al.* (FOCUS Collaboration), hep-ex/0506013.
- [22] M.E. Wessling, Phys. Lett. B **603**, 152 (2004); **618**, 269 (2005); Ph.D. thesis, hep-ph/0505213.
- [23] C. Gignoux, B. Silvestre-Brac, and J.M. Richard, Phys. Lett. B **193**, 323 (1987); H. Lipkin, Phys. Lett. B **195**, 484 (1987); F. Stancu, Phys. Rev. D **58**, 111501 (1998); M. Genovese, J.M. Richard, F. Stancu, and S. Pepin, Phys. Lett. B **425**, 171 (1998).
- [24] R.L. Jaffe and F. Wilczek, Phys. Rev. Lett. **91**, 232003 (2003); Phys. Rev. D **69**, 114017 (2004).
- [25] E. Jenkins and A.V. Manohar, J. High Energy Phys. 06 (2004) 039.
- [26] D. Pirjol and C. Schat, Phys. Rev. D **71**, 036004 (2005).
- [27] T.D. Cohen and R.F. Lebed, Phys. Rev. Lett. **91**, 012001 (2003); Phys. Rev. D **67**, 096008 (2003); **68**, 056003 (2003); **70**, 096015 (2004); hep-ph/0507267; T.D. Cohen, D.C. Dakin, A. Nellore, and R.F. Lebed, Phys. Rev. D **70**, 056004 (2004); T.D. Cohen, D.C. Dakin, R.F. Lebed, and D.R. Martin, Phys. Rev. D **71**, 076010 (2005).
- [28] T.D. Cohen, D.C. Dakin, A. Nellore, and R.F. Lebed, Phys. Rev. D **69**, 056001 (2004).
- [29] T.D. Cohen and R.F. Lebed, Phys. Lett. B **578**, 150 (2004); **619**, 115 (2005).
- [30] E. Witten, Nucl. Phys. **B160**, 57 (1979).
- [31] A.V. Manohar and M.B. Wise, Nucl. Phys. **B399**, 17 (1993).
- [32] N. Isgur and M.B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990); Nucl. Phys. **B348**, 276 (1991); H. Georgi, Nucl. Phys. **B348**, 293 (1991); T. Mannel, W. Roberts, and Z. Ryzak, Nucl. Phys. **B355**, 38 (1991); F. Hussain, J.G. Körner, M. Krämer, and G. Thompson, Z. Phys. C **51**, 321 (1991).
- [33] A. Anastassov *et al.* (CLEO Collaboration), Phys. Rev. D **65**, 032003 (2002).
- [34] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).
- [35] V.B. Kopeliovich, hep-ph/0507028.
- [36] T.D. Cohen and D.C. Dakin, Phys. Rev. C **68**, 017001 (2003).