PHYSICAL REVIEW D **72,** 074001 (2005)

Coherent study of $\chi_{c0,2} \to VV$, PP and SS

Qiang Zhao*

[1]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing, 100049, People's Republic of China*
[2]*Department of Physics, University of Surrey, Guildford, GU2 7XH, United Kingdom*
(Received 6 August 2005; published 4 October 2005)

We investigate the decays of $\chi_{c0,2}$ into vector meson pairs (VV), pseudoscalar pairs (PP), and scalar pairs (SS) in a general factorization scheme. The purpose is to clarify the role played by the OZI-rule violations and SU(3) flavour breakings in the decay transitions, and their correlations with the final-state meson wavefunctions. For $\chi_{c0,2} \to VV$ and PP, we obtain an overall self-contained description of the experimental data. Applying this factorization to $\chi_{c0} \to f_0^i f_0^j$, where $i, j = 1, 2, 3$ denotes $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$, respectively, we find that specific patterns will arise from the model predictions for the decay branching ratio magnitudes, and useful information about the structure of those three scalars can be abstracted.

DOI: 10.1103/PhysRevD.72.074001 PACS numbers: 13.25.Gv, 12.39.Mk, 12.39.St

Charmonium decays provide a great opportunity for studying the gluon dynamics at relatively low energies. In line with the lattice QCD predictions for the existence of the lowest glueball state (0^{++}) at $1.45 \sim 1.75$ GeV [1,2], one of the most important motivations to study charmonium decays is to search for glueball states and fill in the missing link of QCD. During the past few years, enriched information from experiments reveals a rather crowded scalar-meson spectrum at $1 \sim 2$ GeV. For example, three f_0 states, $f_0^{1,2,3}$ ($= f_0(1710), f_0(1500), f_0(1370)$), are observed in both pp scattering and $p\bar{p}$ annihilations with different decay modes into pseudoscalar pairs [3–6]. They are also confirmed by the BES data in $J/\psi \to \omega f_0^i$ and ϕf_0^i [7–9]. Moreover, another scalar $f_0(1790)$, which is distinguished from $f_0(1710)$, is also seen at BES [10]. In contrast with the $f_0(1710)$, $f_0(1790)$ does not obviously couple to $K\bar{K}$, while its coupling to $\pi\pi$ is quite strong. Since all these states cannot be simply accommodated into a $Q\bar{Q}$ meson configuration, observation of such a rich scalar-meson spectrum could be signals for exotic states beyond simple $Q\bar{Q}$ configurations [11,12]. The fast-growing database also allows us to develop QCD phenomenologies from which we can gain insights into the glueball production mechanism [13–17].

So far, it is still a mystery that the glueball states have remained hidden from experiments for such a long time [12,18]. One possibility could be that a pure glueball state cannot survive a long enough lifetime before it transits into $Q\bar{Q}$ pairs. Because of this, a glueball would be more likely to mix with nearby $Q\bar{Q}$. As a result, it should be more sensible to look for a glueball-$Q\bar{Q}$ mixing state, instead of a pure glueball. Such a mixing scheme, on the one hand, indeed highlights some characters of the meson properties, and fits in the experimental data quite well [5,14,16]. It seems also promising in accounting for the crowded scalar-meson spectrum at $1 \sim 2$ GeV [16]. On the other hand, it raises questions about the role played by the OZI rule [19] in the glueball production mechanism, which needs understanding in a much broader context. In Ref. [20], it is shown that the idea of intermediate virtual meson exchanges [21–23] can provide a dynamic explanation for the large OZI-rule violations in $J/\psi \to \omega f_0(1710)$ and $\phi f_0(1710)$. It implies that correlations of OZI-rule violations could be an important dynamic process in charmonium decays. Such correlations should not be exclusively restricted to the scalar-meson productions in J/ψ decays. It could also contribute to other processes [24], such as $\chi_{c0,2}$ hadronic decays. Therefore, a systematic investigation of $\chi_{c0,2} \to VV$, PP and SS is necessarily useful for clarifying the role played by the OZI-rule violations.

The other relevant issue in the charmonium decays into light hadrons is the SU(3) flavour symmetry breakings. In many cases, the flavour-blind assumption turns out to be a good approximation for the gluon-$Q\bar{Q}$ couplings. However, due to the presence of OZI-violations, an explicit consideration of the SU(3) flavour symmetry breaking will be useful for disentangling correlations from different mechanisms.

In this work, we will present a factorization scheme for $\chi_{c0,2} \to VV$, PP, and SS. Our purpose is to disentangle the roles played the OZI-rule violations and SU(3) flavour symmetry breakings, which will correlate with the final-state meson wavefunctions in $\chi_{c0,2}$ decays. By this study, we hope to abstract additional information about the structure of the scalar mesons f_0^i. It can be reflected by specific patterns arising from the decay branching ratio magnitudes due to such correlations.

To proceed, we first distinguish the singly OZI disconnected (SOZI) processes and doubly OZI disconnected (DOZI) ones by considering transitions illustrated by Fig. 1. The SOZI transition [Fig. 1(a)] hence can be expressed as

$$\langle q_1\bar{q}_2 q_3\bar{q}_4 | V_0 | gg\rangle = g_{\langle 13\rangle} g_{\langle 24\rangle}, \tag{1}$$

*e-mail: qiang.zhao@surrey.ac.uk

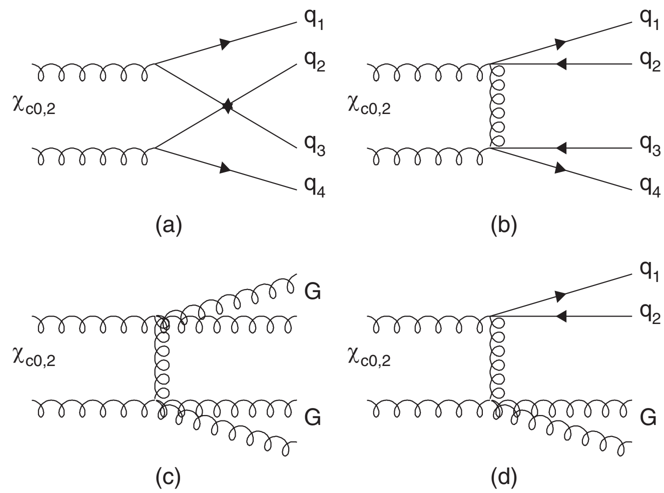

FIG. 1. Schematic pictures for the decays of $\chi_{c0,2}$ into meson pairs via the production of different components.

where V_0 is the interaction potential, and $|gg\rangle$ denotes the minimum two-gluon radiations in $c\bar{c}$ annihilation; g_{13} and g_{24} are the coupling strengths. For the nonstrange coupling, we denote $g_{13} = g_{24} = g_0$. To include the SU(3) flavour symmetry breaking effects, we introduce $R \equiv \langle s\bar{s}|V_0|g\rangle/\langle u\bar{u}|V_0|g\rangle = \langle s\bar{s}|V_0|g\rangle/\langle d\bar{d}|V_0|g\rangle$, for which $R = 1$ refers to the SU(3) flavour symmetry limit.

The DOZI process of Fig. 1(b) describes the transition that the two $q\bar{q}$ pairs are created and recoil without quark exchanges. The amplitude can be expressed as

$$\langle q_1\bar{q}_2q_3\bar{q}_4|V_1|gg\rangle = g'_{\langle 12\rangle}g'_{\langle 34\rangle} = rg_{\langle 12\rangle}g_{\langle 34\rangle}, \tag{2}$$

where V_1 denotes the interaction potential, and parameter r is defined as the relative strength between the SOZI and DOZI transition amplitudes. It is introduced to take into account the DOZI effects.

The two gluons from the $c\bar{c}$ annihilation can also couple to the glueball components via Fig. 1(c). The transition amplitude can be expressed as

$$\langle GG|V_2|gg\rangle = g''g'' = tg_0^2, \tag{3}$$

where V_2 is the interaction potential and g'' is the gluon coupling to the scalar glueball; Parameter t is introduced to account for its coupling strength relative to the SOZI process.

Another possible process in the decay of $\chi_{c0,2}$ is via Fig. 1(d), where the two gluons will couple to $q\bar{q}$ and glueball, respectively. According to Eqs. (2) and (3), its relative coupling strength to the SOZI process can be written as

$$\langle q_1\bar{q}_2G|V_3|gg\rangle = \sqrt{r}tg_0^2. \tag{4}$$

With the above factorization scheme, we consider a transition of $\chi_{c0,2} \to M^iM^j$, where M^i and M^j denote two final-state quarkonia such as VV, PP, or $f_0^if_0^j$. Firstly, in a general basis we assume a wavefunction for $I = 0$ M^i:

$$M^i = x_i|G\rangle + y_i|s\bar{s}\rangle + z_i|n\bar{n}\rangle, \tag{5}$$

where $|G\rangle$, $|s\bar{s}\rangle$, and $|n\bar{n}\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$ are the pure glueball, and flavor singlet $q\bar{q}$ components, respectively; Coefficients x_i, y_i and z_i denote the mixing angles of those components, and satisfy the unitarity condition.

The transition amplitude for $\chi_{c0,2} \to M^iM^j$ can be written as

$$\langle M^iM^j|(V_0 + V_1 + V_2 + V_3)|gg\rangle = \langle(x_iG + y_is\bar{s} + z_in\bar{n})(x_jG + y_js\bar{s} + z_jn\bar{n})|(V_0 + V_1 + V_2 + V_3)|gg\rangle. \tag{6}$$

Notice that the potential V_0 will only allow the transitions to $I = 0$ meson pairs, i.e. transitions of $\langle n\bar{n}s\bar{s}|V_1|gg\rangle = \langle s\bar{s}n\bar{n}|V_1|gg\rangle = 0$. We can then decompose the above transition into

$$\begin{aligned}\langle M^iM^j|(V_0 + V_1 + V_2 + V_3)|gg\rangle = g_0^2[&x_i(tx_j + R\sqrt{rt}y_j + \sqrt{2rt}z_j) + Ry_i(\sqrt{rt}x_j + (1 + r)Ry_j + \sqrt{2}rz_j)\\ &+ z_i(\sqrt{2rt}x_j + \sqrt{2}rRy_j + (1 + 2r)z_j)].\end{aligned} \tag{7}$$

For $\chi_{c0,2}$ decays into other octet meson pairs with $I \neq 0$, e.g. $K\bar{K}$, $\pi\pi$, etc, we can see that only V_0 transition is allowed, and all the others are forbidden. This selection rule immediately requires that decay channels with $I \neq 0$ will not suffer from the OZI-rule violations. As a consequence, the OZI violation effects and SU(3) flavour symmetry breaking can be separated out. The explicit reduction of Eq. (7) for $\chi_{c0,2} \to VV$ and PP will be given later.

In the calculation of the partial decay widths, a commonly used form factor is applied:

$$\mathcal{F}^2(\mathbf{p}) = p^{2l}\exp(-\mathbf{p}^2/8\beta^2), \tag{8}$$

where $\mathbf{p}$ and l are the three momentum and relative angular momentum of the final-state mesons, respectively, in the $\chi_{c0,2}$ rest frame. We adopt $\beta = 0.5$ GeV, which is the same as in Refs. [13,15,16]. Such a form factor will largely account for the size effects from the spatial wavefunctions of the initial and final-state mesons. After taking into account this, we assume that parameter g_0 will be an overall factor for a given initial charmonium.

(i) $\chi_{c0,2} \to VV$

For $\chi_{c0,2} \to \phi\phi$, $\omega\omega$, and $\omega\phi$, the transition amplitude reduces to simple forms due to their ideally mixed flavor wavefunctions, i.e. $\phi = s\bar{s}$, and $\omega = n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$.

This means that for ϕ meson, $x_i = z_i = 0$ and $y_i = 1$, and for ω meson, $x_j = y_j = 0$ and $z_i = 1$. Therefore, we have

$$\begin{aligned} \langle \phi\phi|\hat{V}|gg\rangle &= g_0^2 R^2(1+r) \\ \langle \omega\omega|\hat{V}|gg\rangle &= g_0^2(1+2r) \\ \langle \omega\phi|\hat{V}|gg\rangle &= g_0^2 rR\sqrt{2}, \end{aligned} \tag{9}$$

where we compactly write $V_0 + V_1 + V_2 + V_3$ as $\hat{V}$.

For $\chi_{c0,2} \to K^*\bar{K}^*$ and $\rho\rho$, we have

$$\langle K^{*+}K^{*-}|\hat{V}|gg\rangle = g_0^2 R^2 \qquad \langle \rho^+\rho^-|\hat{V}|gg\rangle = g_0^2. \tag{10}$$

It can be shown that the amplitudes for other charge combinations are the same.

Interesting correlations arise from those decay amplitudes. Equation (10) shows that the decay of $\chi_{c0,2} \to \rho\rho$ is free of interferences from the DOZI processes. Nevertheless, it does not suffer from the possible SU(3) flavour symmetry breaking. Ideally, a measurement of this branching ratio will be useful for us to determine g_0. Unfortunately, experimental data for this channel are not available. The decay of $\chi_{c0,2} \to K^*\bar{K}^*$ also merits great advantages. It is also free of DOZI interferences, and the SU(3) breaking effects have simple correlation with the basic amplitude g_0^2. In contrast, Eq. (9) involves correlations from both OZI-rule violations and SU(3) breakings.

So far, three channels have been measured for χ_{c0} decays by BES collaboration [25–27]: $BR_{\chi_{c0}\to\phi\phi} = (1.0 \pm 0.6) \times 10^{-3}$, $BR_{\chi_{c0}\to\omega\omega} = (2.29 \pm 0.58 \pm 0.41) \times 10^{-3}$, and $BR_{\chi_{c0}\to K^{*0}\bar{K}^{*0}} = (1.78 \pm 0.34 \pm 0.34) \times 10^{-3}$. This allows us to determine the parameters with very small χ^2:

$$\begin{aligned} r = 0.45 \pm 0.48, \qquad R = 0.90 \pm 0.22, \\ g_0 = 0.25 \pm 0.06\ \mathrm{GeV}^{1/2}. \end{aligned} \tag{11}$$

We note that the central values can be obtained by directly solving the equations for these three channels. Fitting the data can provide an estimate of the parameter uncertainties due to the experimental error bars. The results show that large errors of the data can lead to large uncertainties for the parameters, especially for r. To see more clearly their correlations, we take the fraction between the branching ratios for $\omega\omega$ and $\phi\phi$ to derive:

$$r = \frac{R^2 C_0 - 1}{2 - R^2 C_0}, \tag{12}$$

where $C_0 \equiv [p_\phi BR_{\chi_{c0}\to\omega\omega}\mathcal{F}^2(\mathbf{p}_\phi)/p_\omega BR_{\chi_{c0}\to\phi\phi} \times \mathcal{F}^2(\mathbf{p}_\omega)]^{1/2} = 1.6$. It shows that for a range of $1/C_0 \le R^2 < 2/C_0$, the above relation leads to $0 \le r < +\infty$. In other words, the combined uncertainties from the SU(3) breaking and experimental uncertainties can result in rather significant changes to r. Therefore, improved measurements are still needed for the determination of r. Meanwhile, the value for R suggests that the SU(3) breaking is about 20%. This seems to be consistent with the success of SU(3) flavour symmetry in many circumstances.

Adopting the central values for the parameters, we can predict the branching ratios for $\chi_{c0} \to \omega\phi$ and $\rho\rho$, and the results are listed in Table I. We find a relatively smaller branching ratio for $\chi_{c0} \to \omega\phi$, i.e. $BR_{\chi_{c0}\to\omega\phi} = 0.45 \times 10^{-3}$. This channel should be forbidden if the OZI-rule is respected. However, with the errors from Eq. (11), the estimate of the root mean square error gives 1.5×10^{-3}, which is the same order of magnitude as $\phi\phi$ and $\omega\omega$ channels. This implies that the OZI-rule violations could be significant. A direct measurement of $\omega\phi$ channel will put a better constraint on the OZI-rule violation effects.

The decay of $\chi_{c0} \to \rho\rho$ does not suffer the OZI rule, and its branching ratio is predicted to be sizeable. Notice that χ_{c0} has large branching ratios for $2(\pi^+\pi^-)$ and $\rho^0\pi^+\pi^-$ [28], the predicted value, $BR_{\chi_{c0}\to\rho\rho} = 1.88 \times 10^{-3}$, appears to be reasonable. A precise measurement of this channel is strongly desired for the determination of parameter g_0. Note that g_0 appears in the branching ratio with a power of 4. The error for g_0 as in Eq. (11) can still lead to large uncertainties of 92% for $BR_{\chi_{c0}\to\rho\rho}$.

For $\chi_{c2} \to VV$, so far, branching ratios for three channels have been available from BES [25–27]: $BR_{\chi_{c2}\to\phi\phi} = (2.00 \pm 0.55 \pm 0.61) \times 10^{-3}$, $BR_{\chi_{c2}\to\omega\omega} = (1.77 \pm 0.47 \pm 0.36) \times 10^{-3}$, and $BR_{\chi_{c2}\to K^*\bar{K}^*} = (4.86 \pm 0.56 \pm 0.88) \times 10^{-3}$. This will allow us to make a parallel analysis as the χ_{c0} decays. By fitting the data, we obtain

$$\begin{aligned} r = 0.24 \pm 0.29, \qquad R = 1.09 \pm 0.21, \\ g_0 = 0.26 \pm 0.06\ \mathrm{GeV}^{1/2}. \end{aligned} \tag{13}$$

Again, the central values can be obtained by solving the corresponding equations, and large uncertainties for the fitting results highlight the effects from the experimental errors. It is interesting that the parameters for χ_{c0} and χ_{c2} are not dramatically different from each other. Instead, they are quite consistent. In particular, in contrast with χ_{c0} decays, the calculation shows that the OZI-rule is better respected in χ_{c2} decays. We also note that with a factor 3 reduction of the data errors in both cases, the

TABLE I. The branching ratios for $\chi_{c0,2} \to VV$. The first three rows ($\phi\phi$, $\omega\omega$ and $K^*\bar{K}^*$) are experimental data from BES [25–27], while the last two rows, highlighted by bold faces, are the model predictions for $\omega\phi$ and $\rho\rho$ channels. The numbers in the brackets are the root mean square errors estimated with Eqs. (11) and (13), respectively.

Decay channels	$BR_{\chi_{c0}\to VV}(\times 10^{-3})$	$BR_{\chi_{c2}\to VV}(\times 10^{-3})$
$\phi\phi$	1.0 ± 0.6	2.00 ± 0.82
$\omega\omega$	2.29 ± 0.71	1.77 ± 0.59
$K^*\bar{K}^*$	1.78 ± 0.48	4.86 ± 1.04
$\omega\phi$	**0.45 (1.07)**	**0.24 (0.65)**
$\rho\rho$	**1.88 (1.80)**	**2.41 (2.22)**

uncertainty of r can be improved to the second decimal place.

With the central values of the fitted parameters, we predict the branching ratios for $\omega\phi$ and $\rho\rho$ channels. Similar to χ_{c0}, the channel $\omega\phi$ is relatively suppressed with $BR_{\chi_{c2}\to\omega\phi} = 0.24 \times 10^{-3}$. The branching ratio for $\rho\rho$ is found sizeable with a value of $BR_{\chi_{c2}\to\rho\rho} = 2.41 \times 10^{-3}$. We also present the estimated errors for these two channels in Table I.

(ii) $\chi_{c0,2} \to PP$

The same analysis can be applied to the $\chi_{c0,2}$ decays into pseudoscalar meson pairs. We will adopt the same notations as Part (i) for the parameters. But one should keep in mind that they do not necessarily have the same values. For $I = 0$ channels, i.e. $\chi_{c0,2} \to \eta\eta$, $\eta'\eta'$, and $\eta\eta'$, the mixing of η and η' are defined as

$$\eta = \cos\alpha n\bar{n} - \sin\alpha s\bar{s}, \qquad \eta' = \sin\alpha n\bar{n} + \cos\alpha s\bar{s}, \tag{14}$$

where $\alpha = 54.7° + \theta_P$ and θ_P is the octet-singlet mixing angle in the SU(3) flavour basis. We do not consider possible glueball components mixed within the η and η' wavefunctions at this moment though they can be included. Quite directly, the transition amplitude of Eq. (7) can be reduced by requiring $x_i = x_j = 0$ and $t = 0$:

$$\begin{aligned}
\langle\eta\eta|\hat{V}|gg\rangle &= g_0^2[\cos^2\alpha + R^2\sin^2\alpha \\
&\quad + r(\sqrt{2}\cos\alpha - R\sin\alpha)^2] \\
\langle\eta'\eta'|\hat{V}|gg\rangle &= g_0^2[\sin^2\alpha + R^2\cos^2\alpha \\
&\quad + r(\sqrt{2}\sin\alpha + R\cos\alpha)^2] \\
\langle\eta\eta'|\hat{V}|gg\rangle &= g_0^2 r[(1 - R^2/2)\sin 2\alpha + \sqrt{2}R\cos 2\alpha] \\
\langle K^+K^-|\hat{V}|gg\rangle &= g_0^2 R \\
\langle\pi^+\pi^-|\hat{V}|gg\rangle &= g_0^2,
\end{aligned} \tag{15}$$

where the transition potential $\hat{V}$ is a compact form for $V_0 + V_1 + V_2 + V_3$.

Experimental data for $\eta\eta$, K^+K^- and $\pi\pi$ are available [28–30], which will allow us to constrain the parameters and examine the model. We first determine g_0^2 in $\chi_{c0,2} \to \pi\pi$. Then, by taking the ratio between K^+K^- and $\pi\pi$, we determine the SU(3) breaking parameter R:

$$R = \left[\frac{3p_\pi\mathcal{F}^2(p_\pi)\Gamma_{K^+K^-}}{2p_K\mathcal{F}^2(p_K)\Gamma_{\pi\pi}}\right]^{1/2}, \tag{16}$$

where factors 2 and 3 are the weighting factors for the charged kaon and pion decay channels. Substituting $\Gamma_{K^+K^-} = 6.0 \times 10^{-3}\Gamma_{tot}$ and $\Gamma_{\pi\pi} = 7.4 \times 10^{-3}\Gamma_{\text{tot}}$ [28,29] into the above equation, we have $R = 1.06$, which is in good agreement with the range found for $\chi_{c0} \to VV$.

With the data for $\chi_{c0} \to \eta\eta$, we, in principle, can determine r via the first equation in Eq. (15). However, we find that within the large uncertainties of the present data, $BR_{\chi_{c0}\to\eta\eta} = (2.1 \pm 1.1) \times 10^{-3}$, the determination of parameter r also possesses large uncertainties. To show this problem, we plot the branching ratios for $\chi_{c0} \to \eta\eta$, $\eta'\eta'$ and $\eta\eta'$ in terms of r in Fig. 2(a). The solid line is for $BR_{\chi_{c0}\to\eta\eta}$, which exhibits a slow change with r. The arrow acrosses the central value of the experimental data, and marks the error bars. Therefore, the slow variation of $BR_{\chi_{c0}\to\eta\eta}$ makes it difficult to determine r. Interestingly, it shows that $BR_{\chi_{c0}\to\eta'\eta'}$ and $BR_{\chi_{c0}\to\eta\eta'}$ are both fast changing functions of r. In other words, one can determine parameter r by measuring these two branching ratios, and matching the pattern of the curves in a narrow region of r. Meanwhile, an improved measurement of $\eta\eta$ channel is also strongly recommended.

For $\chi_{c2} \to PP$, there are also experimental data available for three channels: $BR_{\chi_{c2}\to K^+K^-} = (9.4 \pm 2.1) \times 10^{-4}$, $BR_{\chi_{c2}\to\pi^+\pi^-} = (1.77 \pm 0.27) \times 10^{-3}$, and $BR_{\chi_{c2}\to\eta\eta} < 1.5 \times 10^{-3}$. However, notice that the branching ratio for $\eta\eta$ is only an upper limit and quite large uncertainties are with the data, the parameters cannot be well-constrained. Similar to the case of $\chi_{c0} \to PP$, we can determine R and g_0 with the data for $\pi\pi$ and $K\bar{K}$: $R = 0.70$, $g_0 = 0.30$. In comparison with the previous cases, it shows large SU(3) symmetry breakings. A possible reason could be due to the poor status of the data.

Similar to χ_{c0} decays, we plot the change of branching ratios of $\eta\eta$, $\eta'\eta'$ in terms of a range of r. The results are presented in Fig. 2(b). Taking the upper limit of $\eta\eta$ branching ratio as a constraint, the OZI-rule violation parameter r at least has a value of -0.16. Note that the relative branching ratio magnitudes among these three channels vary fast. Additional experimental information about $\eta\eta'$ or $\eta'\eta'$ will provide better constraint on r.

(iii) $\chi_{c0,2} \to SS$

For $\chi_{c0,2} \to SS$, where S denotes scalars, a direct $gg \to GG$ coupling will also contribute to the $I = 0$ decay amplitudes, and an additional parameter t has to be included (see Eq. (7)). Therefore, new features may arise from the

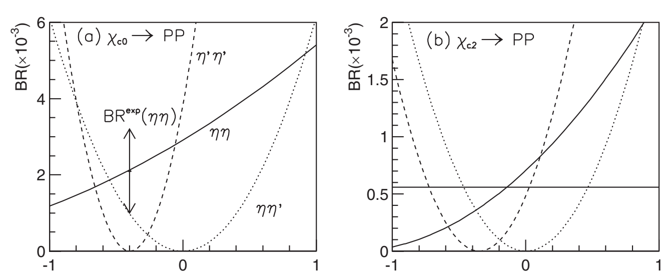

FIG. 2. Branching ratios for the decays of χ_{c0} (a) and χ_{c2} (b) into $\eta\eta$ (solid curve), $\eta'\eta'$ (dashed curve), and $\eta\eta'$ (dotted curve) in terms of the SU(3) parameter r. The two-direction arrow in (a) denotes the central value of $BR^{\text{exp}}_{\chi_{c0}\to\eta\eta} = (2.1 \pm 1.1) \times 10^{-3}$, while the length marks the error bars. The thin straight line in (b) labels the upper limit $BR^{\text{exp}}_{\chi_{c2}\to\eta\eta} < 1.5 \times 10^{-3}$ [30].

$I = 0$ decay channels. In particular, it should be interesting to investigate the sensitivity of the decay branching ratios to the structure of the $I = 0$ scalars, i.e. f_0^i. Specific patterns arising from these channels will be useful for providing evidence for the presence of glueball components in the scalar-meson wavefunctions.

In contrast, the decays of $\chi_{c0,2} \to a_0(1450)a_0(1450)$, and $K_0^*(1430)\bar{K}_0^*(1430)$ will be free of the interferences from the $gg \to GG$ transitions due to the isospin conservation.

To proceed, we will focus on the decays of $\chi_{c0} \to f_0^i f_0^j$. In principle, all the decay channels into any two of these scalars are allowed except for $f_0^1 f_0^1$, of which the threshold is slightly above the χ_{c0} mass. We will adopt the wavefunctions based on the glueball-$Q\bar{Q}$ mixings [13,15], which are examined in the J/ψ hadronic decays [16]. As shown in Ref. [16], the branching ratios for $J/\psi \to V f_0^i$ exhibit specific patterns, which cannot be explained by simple $Q\bar{Q}$ mixings within the scalars. It is also found in Ref. [16] that strong OZI-rule violation effects could contribute to the transition amplitudes.

Because of lack of data, the parameters cannot be explicitly determined. However, based on our experiences from the first two parts, we can assume that the SU(3) flavour symmetry breaking is negligible. Thus, we fix $R = 1$ in the calculations as an approximation. Also, for $gg \to GG$ coupling, we assume that it has the same strength as the SOZI transitions, i.e. $t = 1$. We are then left with parameters g_0 and r for which, in principle, experimental constraints are needed. But we can still proceed one step further to consider the branching ratio fractions, where parameter g_0 cancels.

Taking $\chi_{c0} \to f_0^1 f_0^3$ as reference, we plot in Fig. 3 the branching ratio fractions of $BR_{f_0^i f_0^j}/BR_{f_0^1 f_0^3}$ in terms of r in a range of $0 < r < 3$. Interestingly, it shows that the branching ratio fractions are separated into two distinguished regions. In the limit of $r \to 0$, the branching ratio of $\chi_{c0} \to f_0^1 f_0^3$ becomes the smallest one, and $\chi_{c0} \to f_0^2 f_0^2$ is the largest. However, at the region of $r > 1$, the branching ratio of $\chi_{c0} \to f_0^1 f_0^3$ becomes the largest.

In Refs. [16,20], large OZI-rule violations are found in association with the scalar-meson production in J/ψ decays. Therefore, we conjecture that large contributions from the DOZI processes will be present in $\chi_{c0} \to f_0^i f_0^j$. Based on this, the branching ratio fractions at $r > 1$ strongly suggest that $\chi_{c0} \to f_0^1 f_0^3$ will be the largest decay channel. Notice that f_0^1 couples strongly to $K\bar{K}$, while f_0^3 strongly to $\pi\pi$, one might be able to see rather clear evidence for these two states in e.g. $\chi_{c0} \to f_0^1 f_0^3 \to K^+K^-\pi^+\pi^-$ at BES [31].

Because of lack of data for $\chi_{c2} \to SS$, we do not discuss this channel in this work.

To summarize, by distinguishing the SOZI and DOZI processes and including a direct glueball production mechanism in the transition amplitudes, the overall available data can be accounted for. Since these basic transition processes have been factorized out, we can study their correlations among all the decay channels. We find that the OZI-rule violations play quite different roles in the production of VV, PP, and SS in the $\chi_{c0,2}$ decays. This may explain that pQCD approaches systematically underestimate the data [32–34][1]. On the other hand, a recent study based on the $^3P_0$ quark pair creation model shows that the nonperturbative mechanism still plays important roles in $\chi_{c0} \to \phi\phi$ [35]. In this approach, a coherent description of both pQCD favored and unfavored mechanisms allows us not only to isolate their contributions, but also to investigate their correlated interferences. Such correlations produce specific patterns for the branching ratio magnitudes of different channels, and can be highlighted by experimental data.

For $\chi_{c0,2} \to VV$, we find that the branching ratio of $\chi_{c0} \to \omega\phi$ is crucial for determine the role of the OZI-rule violations, and for $\chi_{c0,2} \to PP$, an improved measurement of $BR_{\chi_{c0}\to\eta\eta}$ will clarify the correlations from the OZI violations. For $\chi_{c0} \to f_0^i f_0^j$, we find that large OZI-rule violations will lead to suppressions on most of the branching ratios except $f_0^1 f_0^3 = f_0(1710)f_0(1370)$. With an analogue to the large OZI-rule violations in the f_0 scalar productions in $J/\psi \to V f_0$, we would expect that large

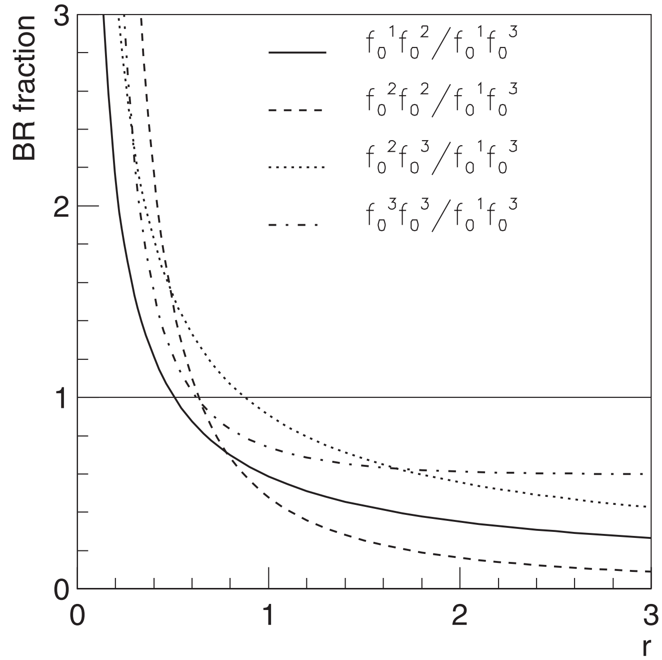

FIG. 3. Branching ratio fractions for χ_{c0} decays into f_0 pairs in terms of the OZI-rule violation parameter r. The thin straight line denotes ratio unity and separates the two regions for the ratios greater/smaller than 1.

[1] We notice that in hep-ph/0506293, an improved result is obtained by Luchinsky at leading twist

OZI-rule violations would occur in $\chi_{c0} \to f_0^i f_0^j$. Therefore, an observation of sizeable branching ratio for $f_0^1 f_0^3 = f_0(1710) f_0(1370)$ will be a strong signal for the glueball-$Q\bar{Q}$ mixings within the scalars. In contrast, small OZI violations will lead to relatively smaller branching ratios for $f_0^1 f_0^3$. If this occurs, we will then have difficulty to understand the large OZI violations in $J/\psi \to V f_0^i$ [16,20]. Experimental data from BES Collaboration can provide an important test of this approach, and also put a more stringent constraint on the model parameters. We expect that a confirmation of this will also provide an additional evidence for the glueball-$Q\bar{Q}$ mixings within those scalars.

The author thanks F. E Close and B. S. Zou for useful comments on this work, and Z. J. Guo and C. Z. Yuan for useful communications about the BES experiment. This work is supported, in part, by grants from the U.K. Engineering and Physical Sciences Research Council (Grant No. GR/S99433/01), and the Institute of High Energy, Chinese Academy of Sciences.

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