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Radiative corrections to the neutrino-deuteron reactions

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The $O(\alpha)$ QED and electroweak radiative corrections to neutrino-deuteron reactions is investigated with particular emphasis given to the constant terms, which have not been treated properly in the literature. This problem is related to the definition of the axial-vector coupling constant g_A as to the inclusion of radiative corrections. After proper calculations of the constants for the Fermi and Gamow-Teller transitions, we find the radiative correction to the neutral current induced reaction, with the usually adopted definition of g_A, is 1.017 for the Higgs boson mass $m_H = 1.5m_Z$. This value is close to that given by Kurylov *et al.*, but this is due to an accidental cancellation of the errors, between those caused by putative identification of constant terms for the Fermi and Gamow-Teller transitions for the charged current reactions and minor errors in their treatment of the constant terms for the neutral current induced reactions.

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I. INTRODUCTION

We consider radiative corrections to neutrino reactions off deuterons,

$$\nu_e + d \to e^- + p + p, \tag{1}$$

$$\nu_e + d \to \nu_e + p + n. \tag{2}$$

These reactions have been used for the solar neutrino measurement at Sudbury Neutrino Observatory (SNO), with the accuracy now reached the level that radiative corrections are non-negligible [1]. This measurement has played a crucial role to fully resolve the long-standing solar neutrino problem [2].

The first attempt to calculate radiative corrections for these processes was made by Towner [3]. Some subtleties associated with the integral of the soft photon emission, as questioned in [4], was remedied by Kurylov *et al.* [5], with a careful treatment of the energy-dependence of the wave function overlap between the initial and final states. As remarked by the latter authors, however, there still remains the problem as to the constant terms of radiative corrections. These authors evaluated the corrections for the charged current process by implicitly assuming that the inner correction constant to the Gamow-Teller part is the same as that to the Fermi transition.

The $O(\alpha)$ radiative correction for the charged current induced reactions is generally written as

$$A(\beta) = (1 + \delta_{\rm out}(\beta))[f_V^2(1 + \delta_{\rm in}^{\rm F})\langle 1\rangle^2 + g_A^2(1 + \delta_{\rm in}^{\rm GT})\langle \boldsymbol{\sigma}\rangle^2], \tag{3}$$

where $\langle 1\rangle$ and $\langle \boldsymbol{\sigma}\rangle$ are the Fermi and Gamow-Teller matrix elements, f_V $(=1)$ and g_A are vector and axial-vector coupling constants; radiative corrections are divided into the outer part $\delta_{\rm out}$ that depends on electron velocity β, hence is process dependent, and the inner parts $\delta_{\rm in}^{\rm F}$ and $\delta_{\rm in}^{\rm GT}$, which are universal, irrespective of the process considered [6]. The outer correction is common to both Fermi and Gamow-Teller parts, and thus factored out. Specifically for the reaction (1), only the Gamow-Teller part contributes. The radiative correction to the neutral current reaction for the axial-vector induced reaction (2) is due to electroweak interactions and is written

$$B(\beta) = (1 + \Delta_{\rm in}^{\rm GT})g_A^2\langle \boldsymbol{\sigma}\rangle^2. \tag{4}$$

Among the radiative correction factors, $\delta_{\rm out}(\beta) \equiv (\alpha/2\pi)g(\beta)$ has been known from early times [7] and $\delta_{\rm in}^{\rm F}$ was calculated by Marciano and Sirlin [8]. The correction for the Gamow-Teller transition $\delta_{\rm in}^{\rm GT}$ was calculated only recently [9]. In the absence of the calculation of $\delta_{\rm in}^{\rm GT}$, the axial-vector coupling constant g_A is extracted from neutron beta decay using

$$A(\beta) = (1 + \delta_{\rm out}(\beta))(1 + \delta_{\rm in}^{\rm F})[\langle 1\rangle^2 f_V^2 + \langle \boldsymbol{\sigma}\rangle^2 \tilde{g}_A^2]. \tag{5}$$

Therefore, the axial-vector coupling usually quoted in the literature is $\tilde{g}_A$ and not the bare g_A, nor $(1 + \delta_{\rm in}^{\rm GT})^{1/2} g_A$ which is the form with proper inclusion of the radiative correction. This does not cause practical problems, however, in so far as one deals only with charged current processes, since the inner correction constant is universal [6]. This is also true for the process involving polarization, from which the axial-vector coupling constant is extracted [10]. The extracted $\tilde{g}_A$ differs from g_A that appears in Lagrangian by

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$$g_A^2 = \frac{1+\delta_{\rm in}^{\rm F}}{1+\delta_{\rm in}^{\rm GT}}\tilde{g}_A^2, \quad (6)$$

so that the effect is absorbed into the redefinition of the axial-vector coupling constant.

This does not apply, however, to the correction for the neutral current process. The radiative correction to the neutral current Gamow-Teller reaction $\Delta_{\rm in}^{\rm GT}$ can be obtained from the general expression given by Marciano and Sirlin [11]. To unfold g_A, however, we need the knowlege of $\delta_{\rm in}^{\rm GT}$. In the work of Kurylov *et al.* [5] $g_A = \tilde{g}_A$ is assumed. The purpose of this paper is to provide complete constant terms of the $O(\alpha)$ radiative corrections for both charged and neutral current induced neutrino-deuteron reactions, (1) and (2).

II. THE CONSTANT TERM OF RADIATIVE CORRECTIONS

The inner radiative corrections that appear in (3) are given by

$$\delta_{\rm in}^{\rm F} = \frac{e^2}{8\pi^2}\left\{\frac{3}{2}\log\left(\frac{m_Z^2}{m_p^2}\right) + 3\bar{Q}\log\left(\frac{m_Z^2}{M^2}\right) + C^{\rm F}\right\}, \quad (7)$$

$$\delta_{\rm in}^{\rm GT} = \frac{e^2}{8\pi^2}\left\{\frac{3}{2}\log\left(\frac{m_Z^2}{m_p^2}\right) + 1 + 3\bar{Q}\log\left(\frac{m_Z^2}{M^2}\right) + C^{\rm GT}\right\}, \quad (8)$$

where m_p and m_Z are the proton and Z boson masses, and M is the lower energy cutoff that represents the scale of the onset of asymptotic behavior of the electroweak theory and is taken to be of the order of 1 GeV. The terms that contain $\bar{Q}$ (=1/6 for the standard quark model) and the constant terms $C^{\rm F}$ and $C^{\rm GT}$ depend on the structure of hadrons and hence are model dependent. The expression (7) and its numerical evaluation were given in [8] and those for (8) were obtained in [9] in a manner parallel to [8]. The constant terms were evaluated as

$$C^{\rm F} = 2.160, \qquad C^{\rm GT} = 3.281. \quad (9)$$

With these numbers we find for $M \approx 1$ GeV,

$$\delta_{\rm in}^{\rm F} = 0.0237, \qquad \delta_{\rm in}^{\rm GT} = 0.0262, \quad (10)$$

where a dominant contribution to $\delta_{\rm in}^{\rm GT}$ comes from weak magnetism [9].

The electroweak radiative corrections to the hadronic matrix elements of the neutral current have been worked out in [11],

$$\begin{aligned}\mathcal{M}_{\rm eff}^{\mu} &= \frac{2im_Z^2}{q^2-m_Z^2}\frac{G_\mu}{\sqrt{2}}\rho_{\rm NC}^{(\nu;h)}(q^2)\langle f|\left\{\bar{\psi}I_3\gamma^\mu\frac{1-\gamma^5}{2}\psi\right.\\ &\quad \left. - \kappa^{(\nu;h)}(q^2)\sin^2\theta_W\bar{\psi}\gamma^\mu Q\psi\right\}|i\rangle\\ &\quad + \frac{i(g^2+g'^2)}{q^2-m_Z^2}\frac{e^2}{32\pi^2}\frac{1}{\sin^2\theta_W\cos^2\theta_W}\\ &\quad \times\langle f|\{a_{\beta_L}J_{\beta_L}^\mu + a_{\beta_R}J_{\beta_R}^\mu\}|i\rangle, \end{aligned} \quad (11)$$

where I_3 and Q are isospin and electric charge, and $\rho_{\rm NC}^{(\nu;h)}(q^2)$ and $\kappa^{(\nu;h)}(q^2)$ include the $O(\alpha)$ electroweak corrections; the momentum transfer q is set equal to zero for our purpose. The last two terms, induced as $O(\alpha)$ corrections, are given by Eq. (19) of Ref. [11], but we only need to know that the interference between the first two and the last two terms takes the form

$$\begin{aligned}&\langle u_L|J_Z^\mu|u_L\rangle\langle u_L|J_{\beta_L}^\nu|u_L\rangle + \langle d_L|J_Z^\mu|d_L\rangle\langle d_L|J_{\beta_L}^\nu|d_L\rangle\\ &\quad = \frac{1}{2}\left(1-\frac{4}{3}\sin^2\theta_W\right)\left(1-\frac{2}{3}\sin^2\theta_W\right)\\ &\qquad \times(\bar{u}_L\gamma^\mu u_L\cdot\bar{u}_L\gamma^\nu u_L - \bar{d}_L\gamma^\mu d_L\cdot\bar{d}_L\gamma^\nu d_L), \end{aligned} \quad (12)$$

$$\begin{aligned}&\langle u_R|J_Z^\mu|u_R\rangle\langle u_R|J_{\beta_R}^\nu|u_R\rangle + \langle d_R|J_Z^\mu|d_R\rangle\langle d_R|J_{\beta_R}^\nu|d_R\rangle\\ &\quad = \frac{2}{9}\sin^2\theta_W(\bar{u}_R\gamma^\mu u_R\cdot\bar{u}_R\gamma^\nu u_R - \bar{d}_R\gamma^\mu d_R\cdot\bar{d}_R\gamma^\nu d_R), \end{aligned} \quad (13)$$

when matrix elements are evaluated with left- and right-handed nonstrange quarks. This vanishes for isosiglet targets, so that the last two terms do not contribute for deuteron reactions. For deuterons only the axial current among the first two terms of (11) contributes, so that the electroweak radiative correction gives rise to the axial coupling g_A renormalized as

$$g_A \to \rho_{\rm NC}^{(\nu;h)}(0)g_A. \quad (14)$$

This means the correction of (4)[1]

$$1 + \Delta_{\rm in}^{\rm GT} = \rho_{\rm NC}^{(\nu;h)}(0)^2. \quad (15)$$

The calculation of Marciano and Sirlin [11] gives

$$\begin{aligned}\Delta_{\rm in}^{\rm GT} &= \frac{e^2}{8\pi^2}\left\{\frac{3\log(\cos^2\theta_W)}{4\sin^4\theta_W} - \frac{7}{4\sin^2\theta_W} + \frac{2a_Z}{\sin^2\theta_W\cos^2\theta_W}\right.\\ &\quad \left. + G(\xi^2,\cos^2\theta_W) + \frac{3}{4\sin^2\theta_W}\cdot\frac{m_t^2}{m_W^2}\right\}, \end{aligned} \quad (16)$$

where m_t is the top quark mass, $\sin^2\theta_W$ is the weak mixing angle in the on-shell scheme, and

$$a_Z = \frac{1}{2\cos^2\theta_W}\left[\frac{5}{2} - \frac{15}{4}\sin^2\theta_W - \frac{1}{5}\sin^4\theta_W + \frac{14}{9}\sin^6\theta_W\right], \quad (17)$$

[1] The expression given by Kurylov *et al.* [5] retains some contributions from the last two terms of (11). It is obvious from symmetry that these terms ought to vanish for deuterons.

$$G(\xi^2, \cos^2\theta_W) = \frac{3\xi^2}{4\sin^2\theta_W}\left\{\frac{\log(\cos^2\theta_W/\xi^2)}{\cos^2\theta_W - \xi^2} + \frac{1}{\cos^2\theta_W}\cdot\frac{\log\xi^2}{1-\xi^2}\right\}, \tag{18}$$

with $\xi = m_H/m_Z$, m_H being the Higgs boson mass. Numerically, the $\rho_{\rm NC}^{(\nu;h)}(0) - 1$ factor is represented by

$$\rho_{\rm NC}^{(\nu;h)}(0) - 1 = 0.010164 - 0.0004628\xi + 3.708 \times 10^{-4}\xi^2 - 1.332\times 10^{-6}\xi^3 + 0.00960\left\{\left(\frac{m_t}{178{\rm GeV}}\right)^2 - 1\right\}, \tag{19}$$

which is correct with the error up to 0.06% for the range $1 < m_H/m_Z < 10$.

The radiative-corrected cross section of (2) is given by multiplying $1 + \Delta_{\rm in}^{\rm GT}$ on the tree value. For $m_t =$ 178 GeV,

$$\rho_{\rm NC}^{(\nu;h)} = 1.00955, \quad \Delta_{\rm in}^{\rm GT} = 0.0192 \quad \text{for} \quad m_H = 1.5m_Z, \tag{20}$$

$$\rho_{\rm NC}^{(\nu;h)} = 1.00862, \quad \Delta_{\rm in}^{\rm GT} = 0.0173 \quad \text{for} \quad m_H = 5.0m_Z. \tag{21}$$

This, together with (7), (8), and (10) gives the complete set of the constants for the $O(\alpha)$ radiative corrections to the neutrino-deuteron reactions. In the usual applications, however, the axial coupling used is $\tilde{g}_A$ derived using (6) rather than g_A ($g_A = 0.9988\tilde{g}_A$). With the use of $\tilde{g}_A$, the cross section for the neutral current induced reaction receives the extra factor $(1 + \delta_{\rm in}^{\rm F})/(1 + \delta_{\rm in}^{\rm GT})$, so that the correction factor for the cross section reads

$$(1 + \Delta_{\rm in}^{\rm GT})\left(\frac{1 + \delta_{\rm in}^{\rm F}}{1 + \delta_{\rm in}^{\rm GT}}\right) = 1.017, \tag{22}$$

for example, for $m_H = 1.5m_Z$. (This value will be 1.015 for $m_H = 5m_Z$).

This number happens to be close to that given by Kurylov *et al.* [5], but it is due to an accidental compensation of the error arising from a neglect of the difference between $\delta_{\rm in}^{\rm GT}$ and $\delta_{\rm in}^{\rm F}$ by their incorrect treatment of the constant term in the radiative correction to the neutral current induced reaction (see footnote above). The results of the SNO experiment [1] using [5], therefore, remain virtually unchanged.

We note as a final remark that the constant term for the radiative correction to the ratio of neutral to charged current reaction (after the usual outer correction [5,7] for the charged current reaction) is -0.6%, which may be compared with the claimed error (0.5%) of nuclear calculations for the ratio of tree level cross sections [12].

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