Fermionic backgrounds and condensation of supergravity fields in the type IIB matrix model

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In a previous paper [1] we constructed wave functions of a D-instanton and vertex operators in type IIB matrix model by expanding supersymmetric Wilson line operators. They describe couplings of a D-instanton and type IIB matrix model to the massless closed string fields, respectively, and form a multiplet of $D = 10$ $\mathcal{N} = 2$ supersymmetries. In this paper we consider fermionic backgrounds and condensation of supergravity fields in IIB matrix model by using these wave functions. We start from the type IIB matrix model in a flat background whose matrix size is $(N + 1) \times (N + 1)$, or equivalently the effective action for $(N + 1)$ D-instantons. We then calculate an effective action for N D-instantons by integrating out 1 D-instanton (which we call a mean-field D-instanton) with an appropriate wave function and show that various terms can be induced corresponding to the choice of the wave functions. In particular, a Chern-Simons-like term is induced when the mean-field D-instanton has a wave function of the antisymmetric tensor field. A fuzzy sphere becomes a classical solution to the equation of motion for the effective action. We also give an interpretation of the above wave functions in the superstring theory side as overlaps of the D-instanton boundary state with the closed string massless states in the Green-Schwarz formalism.

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I. INTRODUCTION

Type IIB (IKKT) matrix model has been proposed as a nonperturbative formulation of superstring theory of type IIB [2]. As an evidence for the nonperturbative formulation, the Schwinger-Dyson equation of Wilson lines is shown to describe the string field equation of motion of type IIB superstring in the light-cone gauge under some plausible assumptions about the continuum limit [3]. Although there are still many issues to be resolved, the model has an advantage to other formulations of superstrings that we can discuss dynamics of space-time more directly [4–7]. The action of the model is given by

$$S_{\mathrm{IKKT}} = -\frac{1}{4}\,\mathrm{tr}[A_\mu, A_\nu]^2 - \frac{1}{2}\,\mathrm{tr}\bar{\psi}\Gamma^\mu[A_\mu, \psi], \qquad (1.1)$$

where A^μ ($\mu = 0, \cdots, 9$) and ten-dimensional Majorana-Weyl fermion ψ are $N \times N$ bosonic and fermionic Hermitian matrices. The action was originally derived from the Schild action for the type IIB superstring by regularizing the world-sheet coordinates by matrices. It is interesting that the same action describes the effective action for N D-instantons [8]. This suggests a possibility that D-instantons can be considered as fundamental objects to generate the space-time itself as well as the dynamical fields (or strings) on the space-time. The bosonic matrices represent noncommutative coordinates of D-instantons and the distribution of eigenvalues of A_μ is interpreted to form space-time. The fermionic coordinates ψ are collective coordinates associated with broken supersymmetries of D-branes but in the matrix model interpretation they describe internal structures of our space-time.

If we take the above interpretation that the space-time is constructed by distribution of D-instantons, how can we interpret the SO(9,1) rotational symmetry of the matrix model action? This symmetry can be interpreted in the sense of mean-field. Namely we can consider that the system of N D-instantons is embedded in larger size $(N + M) \times (N + M)$ matrices as

$$\begin{pmatrix} ND(-1) & \\ & MD(-1) \text{ as background for } ND(-1) \end{pmatrix}, \qquad (1.2)$$

and consider the action (1.1) as an effective action in the background where the rest, M eigenvalues, distribute uniformly in 10 dimensions. If the M eigenvalues distribute inhomogeneously, we may expect that the effective action for N D-instantons is modified so that they live in a curved space-time. This is analogous to a thermodynamic system. In a canonical ensemble, a subsystem in a heat bath is characterized by several thermodynamic quantities like temperature and pressure. Similarly a subsystem of N D-instantons in a ‘‘matrix bath’’ can be considered to be characterized by several ‘‘thermodynamic quantities’’ in a certain large N limit.

Since the matrix model has the $\mathcal{N} = 2$ type IIB supersymmetry

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$$\begin{cases} \delta A_\mu = i\bar{\epsilon}_1 \Gamma_\mu \psi, \\ \delta\psi = -\frac{i}{2}[A_\mu, A_\nu]\Gamma^{\mu\nu}\epsilon_1 + \epsilon_2 1_N, \end{cases} \tag{1.3}$$

we can expect that the configuration of the M background D-instantons describes condensation of massless fields of the type IIB supergravity and the thermodynamic quantities of the matrix bath are characterized by the values of the condensation.

In the following, we consider the simplest case that the background is represented by a wave function of 1 D-instanton (namely $M = 1$ with an appropriate wave function introduced). This simplification can be considered as a mean-field approximation that the configuration of M D-instantons is represented by a mean-field wave function described by a single D-instanton. We call this extra D-instanton a *mean-field D-instanton*. This kind of idea was first discussed by Yoneya in [9]. In the previous paper [1], we constructed a set of wave functions for the mean-field D-instanton. This wave functions have a stringy interpretation, namely, as we see in this paper, they can be interpreted as overlaps of D-instanton boundary states with closed string massless states.

In this paper, we calculate the effect of the mean-field D-instanton on the N D-instantons. We first start from the IIB matrix model with a size $(N+1)\times(N+1)$, or equivalently a system of $(N+1)$ D-instantons, and integrate the mean-field D-instanton with an appropriate wave function. This corresponds to condensation of supergravity fields and the effective action for the N D-instantons is modified. We particularly consider two types of wave functions, namely, those describing an antisymmetric tensor field or a graviton field. In the former case, a Chern-Simons like term is induced in the leading order of the perturbation. (This term vanishes if we assume that the N D-instantons satisfy the equation of motion for the original IIB action.) With this term, a fuzzy sphere becomes a solution to the equation of motion. This phenomenon is similar to the Myers effect [10]. In both cases for the antisymmetric tensor field and the graviton, if we assume that the configuration satisfies the classical equation of motion, the modification of the effective action is given by a vertex operator for each supergravity field.

The content of the paper is as follows. In the next section, we review the results of the previous paper [1] on the wave functions of a D-instanton and the vertex operators for closed string massless states in IIB matrix model. In Sec. III, we give a stringy interpretation of the D-instanton wave functions as overlaps of D-instanton boundary states with massless states of the closed string. In Sec. IV, we calculate the one-loop effective action in general fermionic backgrounds. In Sec. V, we apply this calculation to a system of $(N+1)$ D-instantons and integrate the mean-field D-instanton to obtain an effective action for the rest N D-instantons. We particularly consider the wave functions of the antisymmetric tensor field and the graviton field. We summarize our results in Sec. VI. In the appendix, we review the boundary states in the Green-Schwarz formalism.

II. WAVE FUNCTIONS AND VERTEX OPERATORS

In this section we summarize our previous results on wave functions for a D-instanton and vertex operators in type IIB matrix model. Wave functions are functions of a $d = 10$ coordinate (or its conjugate momentum) and $d = 10$ Majorana-Weyl spinor and contain information on the couplings of a D-instanton to the closed string massless states. Their physical meaning in superstring theories will be clarified in the next section. On the other hand, in the matrix model, interactions corresponding to the supergravity modes are induced as quantum effects and their couplings to these modes are described through the vertex operators.

A. Supersymmetric Wilson line operator

The degrees of freedom of a D-instanton are described by its coordinate, a ten-dimensional vector y_μ and a Majorana-Weyl fermion λ, and thus information of its state is encoded in functions of y_μ and λ, that is, wave functions. Here we give wave functions corresponding to the supergravity modes in the form of $f_A(\lambda)e^{-ik\cdot y}$ with a momentum k, where the index A specifies each mode. Wave functions are defined to form a multiplet of the following $d = 10$ $\mathcal{N} = 2$ supersymmetry transformations

$$\delta^{(1)} f(\lambda) = [\bar{\epsilon}_1 q_1, f(\lambda)] = \epsilon_1 \frac{\partial}{\partial\lambda} f(\lambda), \tag{2.1}$$

$$\delta^{(2)} f(\lambda) = [\bar{\epsilon}_2 q_2, f(\lambda)] = (\bar{\epsilon}_2 \not{k}\lambda) f(\lambda), \tag{2.2}$$

where $\epsilon_i (i = 1, 2)$ are the Majorana-Weyl spinors.

The Majorana-Weyl fermion λ contains 16 degrees of freedom and there are 2^{16} independent wave functions for λ. To reduce the number, we impose the massless condition for the momentum $\not{k}$; $k^2 = 0$. Then, since $\not{k}\lambda$ in (2.2) has only 8 independent degrees of freedom, the supersymmetry can generate only $2^8 = 256$ independent wave functions for λ. They form a massless type IIB supergravity multiplet containing a complex dilaton Φ, a complex dilatino $\tilde{\Phi}$, a complex antisymmetric tensor $B_{\mu\nu}$, a complex gravitino Ψ_μ, a real graviton $h_{\mu\nu}$ and a real 4th-rank self-dual antisymmetric tensor $A_{\mu\nu\rho\sigma}$. A physical meaning of the wave functions in string theories is given by using boundary states of a D-instanton in Sec. III.

Vertex operators $V_A(A^\mu, \psi; k)$ covariantly transform under the following $\mathcal{N} = 2$ supersymmetry of the IIB matrix model,

$$\begin{cases} \delta^{(1)}A_\mu = i\bar{\epsilon}_1\Gamma_\mu\psi, \\ \delta^{(1)}\psi = -\frac{i}{2}[A_\mu, A_\nu]\Gamma^{\mu\nu}\epsilon_1, \end{cases} \quad \begin{cases} \delta^{(2)}A_\mu = 0, \\ \delta^{(2)}\psi = \epsilon_2 1_N. \end{cases} \tag{2.3}$$

We denote the generator of $\delta^{(i)}(i = 1, 2)$ as $Q_i(i = 1, 2)$, respectively. Since the $\mathcal{N} = 2$ supersymmetry algebra closes only on shell, in this section we assume that the $N \times N$ matrices A_μ and ψ satisfy the equations of motion for the IKKT action (1.1),

$$[A^\nu, [A_\mu, A_\nu]] - \frac{1}{2}(\Gamma_0\Gamma_\mu)_{\alpha\beta}\{\psi_\alpha, \psi_\beta\} = 0, \tag{2.4}$$

$$\Gamma^\mu[A_\mu, \psi] = 0. \tag{2.5}$$

In order to construct vertex operators systematically, we start from a supersymmetric Wilson line operator first introduced in [11] for the IIB matrix model;

$$\omega(C) = \mathrm{tr}\prod_j e^{\bar{\lambda}_j Q_1} e^{-i\epsilon k_j^\mu A_\mu} e^{-\bar{\lambda}_j Q_1}. \tag{2.6}$$

Since we are interested in the massless multiplet, we here consider the simplest straight Wilson line operator with a global momentum k;

$$\omega(\lambda, k) = e^{\bar{\lambda}Q_1}\,\mathrm{tr} e^{ik\cdot A} e^{-\bar{\lambda}Q_1}. \tag{2.7}$$

Here, though the Majorana-Weyl spinor λ is a parameter of the supersymmetry transformation, it is eventually interpreted as a fermionic collective coordinate of a D-instanton.

This supersymmetric Wilson line operator $\omega(\lambda, k)$ is invariant under simultaneous supersymmetry transformations for $N \times N$ matrices A^μ, ψ and the parameters (λ, k) as

$$[\bar{\epsilon}_1 Q_1, \omega(\lambda, k)] - [\bar{\epsilon}_1 q_1, \omega(\lambda, k)] = 0, \tag{2.8}$$

$$[\bar{\epsilon}_2 Q_2, \omega(\lambda, k)] - [\bar{\epsilon}_2 q_2, \omega(\lambda, k)] = 0. \tag{2.9}$$

By expanding $\omega(\lambda, k)$ in terms of the wave functions for λ, which are constructed in the manner stated above, as

$$\omega(\lambda, k) = \sum_A f_A(\lambda) V_A(A_\mu, \psi; k), \tag{2.10}$$

it is understood from Eqs. (2.8) and (2.9) that $V_A(A_\mu, \psi; k)$ correctly transform under the $\mathcal{N} = 2$ supersymmetry. Therefore $V_A(A_\mu, \psi; k)$ can be regarded as candidates for the vertex operators. Indeed it will be shown explicitly in Sec. V that a system of N D-instantons couples to the supergravity modes through these vertex operators.

B. Wave functions of a D-instanton

We here summarize our results of the wave functions for the massless multiplet and their supersymmetry transformations. In constructing wave functions which transform covariantly under the supersymmetries, we first assume that the dilaton wave function is proportional to $\exp(-ik \cdot y)$, namely $f_A(\lambda) = 1$. It is annihilated by the supersymmetry transformation q_1. Then the other wave functions can be determined by supersymmetry transformations. For more details, see [1].

By defining a fermion bilinear as $b_{\mu\nu} = k_\sigma \bar{\lambda}\Gamma_{\mu\nu\sigma}\lambda$, the supersymmetry multiplet of the wave functions is given as follows:

(a) dilaton

$$\Phi(\lambda, k) = 1, \tag{2.11}$$

(b) dilatino

$$\tilde{\Phi}(\lambda, k) = k\lambda, \tag{2.12}$$

(c) antisymmetric tensor field

$$B_{\mu\nu}(\lambda, k) = -\frac{1}{2}b_{\mu\nu}(\lambda), \tag{2.13}$$

(d) gravitino

$$\Psi_\mu(\lambda, k) = -\frac{i}{24}(k_\sigma\Gamma^{\nu\sigma}\lambda)b_{\mu\nu}(\lambda), \tag{2.14}$$

(e) graviton

$$h_{\mu\nu}(\lambda, k) = \frac{1}{96}b^\rho_\mu b_{\rho\nu}(\lambda), \tag{2.15}$$

(f) 4th rank self-dual antisymmetric tensor field

$$A_{\mu\nu\rho\sigma}(\lambda, k) = -\frac{i}{32(4!)^2}b_{[\mu\nu}b_{\rho\sigma]}(\lambda), \tag{2.16}$$

(g) gravitino (charge conjugation of (2.14))

$$\Psi^c_\mu(\lambda, k) = -\frac{i}{4\cdot 5!}k^\rho\Gamma_{\rho\lambda}\lambda b^{\lambda\sigma}b_{\sigma\mu}(\lambda), \tag{2.17}$$

(h) antisymmetric tensor field (charge conjugation of (2.13))

$$B^c_{\mu\nu}(\lambda, k) = -\frac{1}{6!}b_{\mu\rho}b^{\rho\sigma}b_{\sigma\nu}(\lambda), \tag{2.18}$$

(i) dilatino (charge conjugation of (2.12))

$$\tilde{\Phi}^c(\lambda, k) = \frac{1}{8!}k_\alpha\Gamma^{\mu\nu\alpha}\lambda b_{\nu\rho}b^{\rho\sigma}b_{\sigma\mu}(\lambda), \tag{2.19}$$

(j) dilaton (charge conjugation of (2.11))

$$\Phi^c(\lambda, k) = \frac{1}{8\cdot 8!}b^\nu_\mu b^\lambda_\nu b^\sigma_\lambda b^\mu_\sigma(\lambda). \tag{2.20}$$

In these expressions we have chosen a specific gauge for each wave function. These wave functions can be interpreted as overlaps of D-instanton boundary states and closed string massless states as we will see in the next section. In the usual convention of superstrings, the first dilaton (2.11) corresponds to a wave function of (dilaton $+i$ axion) and the second one (2.20) corresponds to (dilaton $-i$ axion). The other complex fields also have the same structure.

The supersymmetry transformations (2.1) and (2.2) lead to the following transformations between these wave functions:

$$\begin{aligned}
&\delta\Phi = \bar{\epsilon}_2\tilde{\Phi}, \qquad \delta\tilde{\Phi} = k\epsilon_1\Phi - \frac{i}{24}\Gamma^{\mu\nu\rho}\epsilon_2 H_{\mu\nu\rho}, \qquad \delta B_{\mu\nu} = -\bar{\epsilon}_1\Gamma_{\mu\nu}\tilde{\Phi} + 2i(\bar{\epsilon}_2\Gamma_{[\mu}\Psi_{\nu]} + k_{[\mu}\Lambda_{\nu]}),\\
&\delta\Psi_\mu = \frac{1}{24\cdot 4}[9\Gamma^{\nu\rho}\epsilon_1 H_{\mu\nu\rho} - \Gamma_{\mu\nu\rho\sigma}\epsilon_1 H^{\nu\rho\sigma}] + \frac{i}{2}\Gamma^{\nu\rho}k_\rho h_{\mu\nu}\epsilon_2 + \frac{i}{4\cdot 5!}\Gamma^{\rho_1\cdots\rho_5}\Gamma_\mu\epsilon_2 F_{\rho_1\cdots\rho_5} + k_\mu\xi, \delta h_{\mu\nu}\\
&\quad = -\frac{i}{2}\bar{\epsilon}_1\Gamma_{(\mu}\Psi_{\nu)} - \frac{i}{2}\bar{\epsilon}_2\Gamma_{(\mu}\Psi^c_{\nu)} + k_{(\mu}\xi_{\nu)},\\
&\delta A_{\mu\nu\rho\sigma} = -\frac{1}{(4!)^2}\bar{\epsilon}_1\Gamma_{[\mu\nu\rho}\Psi_{\sigma]} - \frac{1}{(4!)^2}\bar{\epsilon}_2\Gamma_{[\mu\nu\rho}\Psi^c_{\sigma]} + k_{[\mu}\xi_{\nu\rho\sigma]},\\
&\delta\Psi^c_\mu = \frac{i}{2}\Gamma^{\nu\rho}k_\rho h_{\mu\nu}\epsilon_1 + \frac{i}{4\cdot 5!}\Gamma^{\rho_1\cdots\rho_5}\Gamma_\mu\epsilon_1 F_{\rho_1\cdots\rho_5} + \frac{1}{24\cdot 4}[9\Gamma^{\nu\rho}\epsilon_2 H^c_{\mu\nu\rho} - \Gamma_\mu^{\nu\rho\sigma}\epsilon_2 H^c_{\nu\rho\sigma}] + k_\mu\xi^c,\\
&\delta B^c_{\mu\nu} = 2i(\bar{\epsilon}_1\Gamma_{[\mu}\Psi^c_{\nu]} + k_{[\mu}\Lambda^c_{\nu]}) - \bar{\epsilon}_2\Gamma_{\mu\nu}\tilde{\Phi}^c, \qquad \delta\tilde{\Phi}^c = -\frac{i}{24}\Gamma^{\mu\nu\rho}\epsilon_1 H^c_{\mu\nu\rho} + k\epsilon_2\Phi^c, \qquad \delta\Phi^c = \bar{\epsilon}_1\tilde{\Phi}^c,
\end{aligned} \tag{2.21}$$

where ξ, ξ^c, ξ_μ, $\xi_{\mu\nu\rho}$, Λ_μ and Λ^c_μ are gauge parameters. $H_{\mu\nu\rho}$, $H^c_{\mu\nu\rho}$ and $F_{\rho_1\cdots\rho_5}$ are the field strengths of $B_{\mu\nu}$, $B^c_{\mu\nu}$ and $A_{\mu\nu\rho\sigma}$, respectively. This supersymmetry transformation is the same as that in [12] up to normalizations.

C. Vertex operators

Construction of the vertex operators can be done systematically by expanding the supersymmetric Wilson line operator in terms of the wave functions $f_A(\lambda)$ given in the previous subsection. In Sec. V, we will show that these vertex operators indeed describe couplings of type IIB matrix model to the supergravity modes. The derivation itself is systematic but the complete calculation is cumbersome. Partial results were obtained in [13]. More complete analysis was given in [1][1]. The results are as follows.

(a) dilaton

$$V^\Phi = \mathrm{tr} e^{ik\cdot A}, \tag{2.22}$$

(b) dilatino

$$V^{\tilde{\Phi}} = \mathrm{tr} e^{ik\cdot A}\bar{\psi}, \tag{2.23}$$

(c) antisymmetric tensor field

$$V^B_{\mu\nu} = \mathrm{Str}\, e^{ik\cdot A}\left(\frac{1}{16}k^\rho(\bar{\psi}\cdot\Gamma_{\mu\nu\rho}\psi) - \frac{i}{2}[A_\mu, A_\nu]\right), \tag{2.24}$$

(d) gravitino

$$V^\Psi_\mu = \mathrm{Str}\, e^{ik\cdot A}\left(-\frac{i}{12}k^\rho(\bar{\psi}\cdot\Gamma_{\mu\nu\rho}\psi) - 2[A_\mu, A_\nu]\right) \cdot\bar{\psi}\Gamma^\nu, \tag{2.25}$$

(e) graviton

$$\begin{aligned} V^h_{\mu\nu} = 2\mathrm{Str}\, e^{ik\cdot A}\Big\{&[A_\mu, A^\rho]\cdot[A_\nu, A_\rho] + \frac{1}{4}\bar{\psi} \cdot\Gamma_{(\mu}[A_{\nu)}, \psi] - \frac{i}{8}k^\rho\bar{\psi}\cdot\Gamma_{\rho\sigma(\mu}\psi\cdot[A_{\nu)}, A^\sigma]\\ &- \frac{1}{8\cdot 4!}k^\lambda k^\tau(\bar{\psi}\cdot\Gamma^\sigma_{\mu\lambda}\psi)\cdot(\bar{\psi}\cdot\Gamma_{\nu\tau\sigma}\psi)\Big\},\end{aligned} \tag{2.26}$$

(f) 4-th rank self-dual antisymmetric tensor field

$$\begin{aligned} V^A_{\mu\nu\rho\sigma} = -i\mathrm{Str}\, e^{ik\cdot A}\Big\{&F_{[\mu\nu}\cdot F_{\rho\sigma]} + c\bar{\psi} \cdot\Gamma_{[\mu\nu\rho}[A_{\sigma]}, \psi] - \frac{3i}{4}ck^\lambda\bar{\psi}\cdot\Gamma_{\lambda[\mu\nu}\psi\cdot F_{\rho\sigma]}\\ &- \frac{1}{8\cdot 4!}k^\lambda k^\tau(\bar{\psi}\cdot\Gamma_{\lambda[\mu\nu}\psi)\cdot(\bar{\psi}\cdot\Gamma_{\rho\sigma]\tau}\psi)\Big\},\end{aligned} \tag{2.27}$$

where $c = -1/3$. We fixed the value of c by another calculation (See Sec. IV E in [1]).

Hereafter we write down only the leading order terms of vertex operators.

(a) charge conjugation of gravitino

$$V^{\Psi^c}_\mu = \mathrm{Str}\, e^{ik\cdot A}\left([A_\mu, A_\nu]\cdot[A_\rho, A_\sigma]\cdot\Gamma^{\rho\sigma}\Gamma^\nu\psi + \frac{2}{3}\bar{\psi}\cdot\Gamma_\nu[A_\mu, \psi]\cdot\Gamma^\nu\psi\right), \tag{2.28}$$

(b) charge conjugation of antisymmetric tensor field

$$\mathrm{Str}\, e^{ik\cdot A}\left([A_\mu, A_\rho]\cdot[A^\rho, A^\sigma]\cdot[A_\sigma, A_\nu] - \frac{1}{4}[A_\mu, A_\nu]\cdot[A^\rho, A^\sigma]\cdot[A_\sigma, A_\rho]\right), \tag{2.29}$$

[1]Similar calculations were performed in the Banks, Fischler, Shenker, Susskind (BFSS) matrix model in [14,15].

(c) charge conjugation of dilatino

$$V^{\tilde{\Phi}^c} = \mathrm{Str}\, e^{ik\cdot A}\bigg\{\bigg([A_\mu, A_\rho]\cdot[A^\rho, A^\sigma]\cdot[A_\sigma, A_\nu] - \frac{1}{4}[A_\mu, A_\nu]\cdot[A^\rho, A^\sigma]\cdot[A_\sigma, A_\rho]\bigg)\cdot\Gamma^{\mu\nu}\psi + \frac{1}{24}[A_\mu, A_\nu]\cdot[A_\rho, A_\sigma]\cdot[A_\lambda, A_\tau]\cdot\Gamma^{\mu\nu\rho\sigma\lambda\tau}\psi\bigg\}, \quad (2.30)$$

(d) charge conjugation of dilaton

$$V^{\Phi^c} = \mathrm{Str}\, e^{ik\cdot A}\bigg\{[A_\mu, A_\nu]\cdot[A^\nu, A^\rho]\cdot[A_\rho, A_\sigma]\cdot[A^\sigma, A^\mu] - \frac{1}{4}[A_\mu, A_\nu]\cdot[A^\nu, A^\mu]\cdot[A_\rho, A_\sigma]\cdot[A^\sigma, A^\rho] + [A_\sigma, A_\mu]\cdot[A_\nu, A_\rho]\cdot\bar{\psi}\Gamma^\mu\Gamma^{\nu\rho}\cdot[A_\sigma, \psi]\bigg\}. \quad (2.31)$$

Str means a symmetrized trace which is defined by

$$\mathrm{Str}\, e^{ik\cdot A}B_1\cdot B_2\cdots B_n = \int_0^1 dt_1\int_{t_1}^1 dt_2\cdots\int_{t_{n-2}}^1 dt_{n-1} \times \mathrm{tr}\, e^{ik\cdot At_1}B_1 e^{ik\cdot A(t_2-t_1)}B_2\cdots \times e^{ik\cdot A(t_{n-1}-t_{n-2})}B_{n-1}e^{ik\cdot A(1-t_{n-1})}B_n + (\text{permutations of } \mathrm{B_i'}\mathrm{s} \times (i=2,3,\cdots,n)). \quad (2.32)$$

The center-dot on the left hand side means that the operators B_i are symmetrized. In the first term in (2.26), for example, B_1 and B_2 correspond to $[A_\mu, A^\rho]$ and $[A_\nu, A_\rho]$, respectively. See the appendix of [1] for various properties of the symmetrized trace. For notational simplicity we sometimes use Str also for a single operator like $\mathrm{Str}\,(e^{ik\cdot A}B)$ which is equivalent to the ordinary trace.

III. STRINGY INTERPRETATION OF WAVE FUNCTIONS

In this section we show that the wave functions obtained in the previous section can be interpreted as overlaps of D-instanton boundary states and closed string massless states in the Green-Schwarz formalism of type IIB superstring. The ordinary D-instanton is known to be coupled only with the dilaton and the axion states [16] and becomes a source for these closed string modes only. But the D-instanton is a half-BPS state and breaks a half of the supersymmetries and we can construct a supersymmetry multiplet by acting broken supersymmetry generators successively on the simplest D-instanton boundary state. Namely the D-instanton has an internal structure and these multiplet states are coupled also to the other closed string massless states such as gravitons or antisymmetric tensor fields. Hence they become a source for these fields, although the couplings contain higher derivatives. Such internal structures of D-branes were discussed in various papers [14,17–23]. In the following, we show that the wave functions in the previous section are nothing but overlaps of such D-instanton boundary states with the closed string massless states.

We adopt the Green-Schwarz formalism of type IIB superstring and take the light-cone gauge. Our notations and brief summaries of a construction of boundary states in the Green-Schwarz formalism are given in the appendix. Definitions of the supercharges and a boundary state for the D-instanton are obtained by setting

$$\eta = +1, \qquad M_{ij} = \delta_{ij}, \qquad M_{ab} = \delta_{ab}, \qquad M_{\dot{a}\dot{b}} = \delta_{\dot{a}\dot{b}}, \quad (3.1)$$

in the corresponding equations in the appendix (A45)–(A65).

The type IIB superstring has $\mathcal{N} = 2$ supersymmetries with 32 supercharges. A boundary state for the D-instanton is defined by the boundary conditions in Eqs. (A45)–(A47) with (3.1),

$$\partial_\sigma X^i|B\rangle = 0, \qquad Q^{+a}|B\rangle = 0, \qquad Q^{+\dot{a}}|B\rangle = 0, \quad (3.2)$$

and a solution of these conditions is given in Eq. (A60) as

$$|B\rangle = e^{\sum_{n>0}(\frac{1}{n}\alpha^i_{-n}\tilde{\alpha}^i_{-n} - iS^a_{-n}\tilde{S}^a_{-n})}|B_0\rangle, \quad (3.3)$$

where S^a_n and $\tilde{S}^a_n$ are fermionic modes and α^i_n and $\tilde{\alpha}^i_n$ are bosonic modes of the type IIB superstring. From Eq. (A61), the zero-mode part becomes

$$|B_0\rangle = C(|i\rangle|i\rangle - i|\dot{a}\rangle|\dot{a}\rangle), \quad (3.4)$$

where C is a normalization constant. The D-instanton boundary state preserves a half of supersymmetries Q^{+a} and $Q^{+\dot{a}}$, and breaks the other half Q^{-a} and $Q^{-\dot{a}}$ which are defined in (A64) and (A65). The broken and unbroken supercharges satisfy the algebra

$$\{Q^{+a}, Q^{-b}\} = 4p^+\delta_{ab}, \qquad \{Q^{+a}, Q^{-\dot{b}}\} = 2\sqrt{2}\gamma^i_{a\dot{b}}p^i,$$
$$\{Q^{+\dot{a}}, Q^{-b}\} = 2\sqrt{2}\gamma^i_{\dot{a}b}p^i, \quad (3.5)$$
$$\{Q^{+\dot{a}}, Q^{-\dot{b}}\} = 2(P^- + \tilde{P}^-)\delta_{\dot{a}\dot{b}}.$$

The other anticommutators vanish.

A. Coupling of D-instanton boundary states to supergravity modes

States obtained by acting the broken generators $Q^{-a}, Q^{-\dot{a}}$ on the D-instanton boundary states couple to the supergravity modes. Here we concentrate on massless modes and ignore massive excitations.

The zero-mode part of the boundary state of the D-instanton is given by

$$|D(-1)\rangle = \frac{1}{\sqrt{2}}(|i\rangle|i\rangle - i|\dot{a}\rangle|\dot{a}\rangle), \quad (3.6)$$

where we set the normalization constant $C = 1/\sqrt{2}$ for simplicity. This state couples to a linear combination of the dilaton and axion,

$$|\Phi\rangle \equiv \frac{1}{\sqrt{2}}(|i\rangle|i\rangle - i|\dot{a}\rangle|\dot{a}\rangle). \quad (3.7)$$

The coupling is given by

$$\langle\Phi|D(-1)\rangle = 1. \quad (3.8)$$

Acting the broken charge $\lambda^a Q^{-a}$ on $|D(-1)\rangle$, we obtain the fermionic state

$$\lambda^a Q^{-a}|D(-1)\rangle = \sqrt{2p^+}\gamma^i_{a\dot{a}}\lambda^a(|\dot{a}\rangle|i\rangle - i|i\rangle|\dot{a}\rangle). \quad (3.9)$$

This couples to the following linear combination of dilatino states

$$|\tilde{\Phi}_a\rangle \sim \sqrt{p^+}\gamma^i_{a\dot{a}}(|\dot{a}\rangle|i\rangle - i|i\rangle|\dot{a}\rangle), \quad (3.10)$$

and the coupling is given by

$$\langle\tilde{\Phi}_a|\lambda^b Q^{-b}|D(-1)\rangle \sim p^+\lambda^a. \quad (3.11)$$

The normalizations of states for the supergravity modes are fixed so that the supersymmetry transformations of them satisfy Eq. (2.21).

By further acting the broken supersymmetry charges, we can construct the following state

$$\lambda^{a_1}\lambda^{a_2}Q^{-a_1}Q^{-a_2}|D(-1)\rangle = 2\sqrt{2}p^+\gamma^{ij}_{a_1a_2}\lambda^{a_1}\lambda^{a_2}|i\rangle|j\rangle - \sqrt{2}ip^+(\gamma^i_{a_1\dot{a}}\gamma^i_{a_2\dot{b}} - \gamma^i_{a_2\dot{a}}\gamma^i_{a_1\dot{b}})\lambda^{a_1}\lambda^{a_2}|\dot{a}\rangle|\dot{b}\rangle. \quad (3.12)$$

This state couples to the antisymmetric tensor field $B_{\mu\nu}$,

$$|B_{ij}\rangle \sim |i\rangle|j\rangle - |j\rangle|i\rangle - \frac{i}{2}\gamma^{ij}_{\dot{a}\dot{b}}|\dot{a}\rangle|\dot{b}\rangle. \quad (3.13)$$

The coupling between these states is given by

$$\langle B_{ij}|\lambda^{a_1}\lambda^{a_2}Q^{-a_1}Q^{-a_2}|D(-1)\rangle \sim p^+(\gamma^{ij}_{a_1a_2}\lambda^{a_1}\lambda^{a_2}). \quad (3.14)$$

Since the coupling contains momentum p^+, the boundary state (3.12) has a derivative-coupling to the antisymmetric tensor field.

The state multiplied by three broken charges is given by

$$\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}Q^{-a_1}Q^{-a_2}Q^{-a_3}|D(-1)\rangle = (2p^+)^{3/2}\left[\gamma^j_{a_1\dot{a}}\gamma^{ji}_{a_2a_3} + \frac{1}{2}\gamma^i_{a_1\dot{b}}(\gamma^j_{a_2\dot{a}}\gamma^j_{a_3\dot{b}} - \gamma^j_{a_3\dot{a}}\gamma^j_{a_2\dot{b}})\right] \times \lambda^{a_1}\lambda^{a_2}\lambda^{a_3}(|\dot{a}\rangle|i\rangle + i|i\rangle|\dot{a}\rangle). \quad (3.15)$$

This state couples to a linear combination of gravitino states

$$|\Psi^{\dot{a}}_i\rangle \sim \sqrt{p^+}\left[|\dot{a}\rangle|i\rangle + i|i\rangle|\dot{a}\rangle - \frac{1}{8}\gamma^i_{\dot{a}b}\gamma^j_{b\dot{b}}(|\dot{b}\rangle|j\rangle + i|j\rangle|\dot{b}\rangle)\right]. \quad (3.16)$$

Hence the coupling between the boundary state (3.15) and the gravitino state (3.16) becomes

$$\langle\Psi^{\dot{a}}_i|\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}Q^{-a_1}Q^{-a_2}Q^{-a_3}|D(-1)\rangle \sim (p^+)^2\gamma^j_{\dot{a}a_1}\lambda^{a_1}(\gamma^{ji}_{a_2a_3}\lambda^{a_2}\lambda^{a_3}). \quad (3.17)$$

A boundary state which is obtained by acting four broken generators on $|D(-1)\rangle$ becomes

$$\lambda^{a_1}\cdots\lambda^{a_4}Q^{-a_1}\cdots Q^{-a_4}|D(-1)\rangle = 8\sqrt{2}(p^+)^2[(\gamma^{ik}_{a_1a_2}\lambda^{a_1}\lambda^{a_2})(\gamma^{kj}_{a_3a_4}\lambda^{a_3}\lambda^{a_4})|i\rangle|j\rangle - i(\gamma^{ij}_{a_3a_4}\lambda^{a_3}\lambda^{a_4})(\gamma^i_{a_1\dot{a}}\gamma^j_{a_2\dot{b}}\lambda^{a_1}\lambda^{a_2})|\dot{a}\rangle|\dot{b}\rangle]. \quad (3.18)$$

This state couples to the graviton state

$$|h_{ij}\rangle \sim |i\rangle|j\rangle + |i\rangle|j\rangle - \frac{1}{4}\delta_{ij}|k\rangle|k\rangle, \quad (3.19)$$

and its coupling is given by

$$\langle h_{ij}|\lambda^{a_1}\cdots\lambda^{a_4}Q^{-a_1}\cdots Q^{-a_4}|D(-1)\rangle \sim (p^+)^2 \times(\gamma^{ik}_{a_1a_2}\lambda^{a_1}\lambda^{a_2})(\gamma^{kj}_{a_3a_4}\lambda^{a_3}\lambda^{a_4}). \quad (3.20)$$

The coupling contains two derivatives.

We can similarly construct states by acting more broken supersymmetry generators. They couple to the other massless states of type IIB closed string through derivative couplings[2].

B. Wave functions with light-cone momentum

In order to compare the wave functions of the mean-field D-instanton in Sec. II B to the results in the previous subsection, we take the light-cone momentum and rewrite the wave functions in Sec. II B.

Let us take the frame where the momentum is represented as

$$k^\mu = (E, 0, \cdots, 0, E), \quad (3.21)$$

[2]Note that both of Q^{-a} and $Q^{-\dot{a}}$ have the same structure $S^a_0 - i\tilde{S}^a_0$ in the zero-mode part. Hence as long as the massless closed string states are concerned, it is sufficient to consider only one of those two generators, namely $\lambda^a Q^{-a}$.

namely, only the k^+ component is nonvanishing. Then the following relations hold:

$$\not{k} = E(\Gamma_0 + \Gamma_9) = -E(\Gamma^0 - \Gamma^9) = -\sqrt{2}E\Gamma^-,$$
$$\not{k}\lambda = 2iE(\lambda^a, 0, -\lambda^a, 0)^T, \qquad b_{ij} = 4E(\gamma^{ij}_{ab}\lambda^a\lambda^b),$$
$$b_{i-} = 4\sqrt{2}E(\gamma^i_{\dot{a}a}\lambda^{\dot{a}}\lambda^a), \qquad b_{i+} = 0.$$

By using these relations, transverse components of the wave functions in section II B become

$$\Phi = 1, \qquad \tilde{\Phi} = 2iE(\lambda^a, 0, -\lambda^a, 0)^T, \quad (\Gamma_{11}\tilde{\Phi} = +\tilde{\Phi})$$
$$B_{ij} = -2E(\gamma^{ij}_{ab}\lambda^a\lambda^b),$$
$$\Psi_i = 4iE^2(\gamma^{ij}_{bc}\lambda^b\lambda^c)(0, -\gamma^j_{\dot{a}a}\lambda^a, 0, \gamma^j_{\dot{a}a}\lambda^a)^T, \tag{3.22}$$
$$h_{ij} = \frac{1}{6}E^2(\gamma^{ik}_{ab}\lambda^a\lambda^b)(\gamma^{kj}_{cd}\lambda^c\lambda^d),$$

They are the same as the overlaps in the previous subsection with the identification $p^+ = \sqrt{2}E$, up to normalizations. Hence we have shown that the wave functions of the mean-field D-instanton represent couplings of a supersymmetry multiplet of a D-instanton to closed string massless states.

C. Fermionic coherent state of D-instanton

So far we have constructed boundary states by acting a fixed number of broken supersymmetry generators on $|D(-1)\rangle$ so that they form an ordinary set of a supersymmetry multiplet. In order to see the above interpretation more systematically, we construct a fermionic coherent state by acting the unitary operator $\exp(-\lambda^a Q^{-a})$ on $|D(-1)\rangle$;

$$|\lambda\rangle = \exp(-\lambda^a Q^{-a})|D(-1)\rangle. \tag{3.23}$$

Because of the commutation relations (3.5), this state satisfies modified boundary conditions

$$Q^{+a}|\lambda\rangle = 4p^+\lambda^a|\lambda\rangle \tag{3.24}$$

$$Q^{+\dot{a}}|\lambda\rangle = 2\sqrt{2}p^i\gamma^i_{\dot{a}a}\lambda^a|\lambda\rangle. \tag{3.25}$$

In the IIB matrix model, the bosonic coordinates are interpreted as the coordinates of space-time. From the consideration here, the fermionic coordinates can be interpreted as the fermionic parameters which bestow an internal structure on the space-time constructed from the bosonic coordinates.

The wave functions for the mean-field D-instanton can be written as

$$f_A(\lambda) = \langle A|\lambda\rangle, \tag{3.26}$$

for each supergravity state A. In the previous subsection, this relation has been shown separately for each state $|A\rangle$ up to normalization. It can be understood more directly as follows. When the momentum k is taken as (3.21), the supercharges q_1 and q_2 for a D-instanton, (2.1) and (2.2), have the following forms,

$$q_1^a = -i\frac{\partial}{\partial\lambda^a}, \qquad q_2^a = 2iE\lambda^a, \tag{3.27}$$

and satisfy the algebra

$$\{q_1^a, q_2^b\} = 2E\delta_{ab}, \qquad \text{others} = 0. \tag{3.28}$$

On the other hand, as far as massless states are concerned, this algebra is equivalent to the ones among the supercharges $Q^{\pm a}$ and $Q^{\pm\dot{a}}$ with $p^i = P^- = \tilde{P}^- = 0$, Eq. (3.5). Actually actions of q_i^a on the wave functions (3.26) can be regarded as insertions of Q^{-a} and Q^{+a} which act on the massless state of the supergravity modes $|A\rangle$ as follows,

$$q_1^a f_A(\lambda) = -i\frac{\partial}{\partial\lambda}f_A(\lambda) = i(\langle A|Q^{-a})|\lambda\rangle,$$
$$q_2^a f_A(\lambda) = 2iE\lambda^a f_A(\lambda) = \frac{i}{2\sqrt{2}}(\langle A|Q^{+a})|\lambda\rangle,$$

where we have used Eqs. (3.24) and (3.25). Hence a construction of the supergravity multiplet by acting $Q^{\pm a}$ on the closed string massless state $\langle A|$ corresponds to the one by acting q_i^a on wave functions $f_A(\lambda)$ and the wave functions we constructed describe the (derivative) couplings between a D-instanton and various supergravity modes.

IV. ONE-LOOP EFFECTIVE ACTION

In the latter half of the paper, we discuss condensation of massless supergravity fields in type IIB matrix model. We consider a matrix model of size $(N+1)\times(N+1)$ and integrate over 1 D-instanton with the wave functions given in Sec. II. In this way, we can obtain a modified effective action in a weak supergravity background of N D-instantons.

In this section we first give a systematic evaluation of the one-loop effective action with general fermionic backgrounds. The results were partly given in [24,25]. Similar calculations were performed in the BFSS matrix model in [14]. Since we are interested in condensation, we do not use the matrix model equation of motion in this section.

We start from the type IIB matrix model with a size $(N+1)\times(N+1)$ and write $(N+1)\times(N+1)$ bosonic and fermionic Hermitian matrices as $A'_\mu(\mu = 0, \cdots, 9)$ and ψ'. We then decompose them into backgrounds (X_μ, Φ) and fluctuations (a_μ, φ) around them as

$$A'_\mu = X_\mu + a_\mu, \qquad \psi' = \Phi + \varphi. \tag{4.1}$$

In order to perform perturbative calculations, we fix a gauge and add the following terms to the action (1.1),

$$S_{\text{g.f.+ghost}} = -\operatorname{tr}\Big(\frac{1}{2}[X^\mu, a_\mu][X^\nu, a_\nu] + [X_\mu, b][A'^\mu, c]\Big), \tag{4.2}$$

where c and b are ghost and antighost fields, respectively. Substituting the decompositions (4.1) into the action (1.1) and (4.2), we obtain the following expression up to the 2nd order of the fluctuations,

$$\begin{aligned} S_{\rm IKKT} + S_{\rm g.f.+ghost} &= S_{\rm IKKT}(X,\Phi) - \frac{1}{2}\,{\rm tr}\,(a^\mu[X^\nu,[X_\nu,a_\mu]] + 2a_\mu[[X^\mu,X^\nu],a_\nu]) \\ &\quad - \frac{1}{2}\,{\rm tr}\bar{\varphi}\Gamma^\mu[X_\mu,\varphi] - {\rm tr}\bar{\Phi}\Gamma^\mu[a_\mu,\varphi] + {\rm tr}b[X_\mu,[X^\mu,c]] \\ &= S_{\rm IKKT}(X,\Phi) + \frac{1}{2}\,{\rm tr}\,a_\mu\left(\delta_{\mu\nu}\tilde{X}^2 + 2\tilde{F}_{\mu\nu} + \tilde{\bar{\Phi}}\Gamma_\mu\frac{1}{\Gamma\cdot\tilde{X}}\Gamma_\nu\tilde{\Phi}\right)a_\nu \\ &\quad - \frac{1}{2}\,{\rm tr}\left(\bar{\varphi} + [\bar{\Phi},a_\mu]\Gamma_\mu\frac{1}{\Gamma\cdot\tilde{X}}\right)(\Gamma\cdot\tilde{X})\left(\varphi + \frac{1}{\Gamma\cdot\tilde{X}}\Gamma_\nu[a_\nu,\Phi]\right) + {\rm tr}\,b\tilde{X}^2c + \text{higher orders,} \end{aligned} \tag{4.3}$$

where we defined $F_{\mu\nu} = [X_\mu, X_\nu]$, $\Gamma\cdot X \equiv \Gamma_\mu X_\mu$ and denoted the adjoint action of a general operator O as $\tilde{O}S \equiv [O,S]$. Then the one-loop partition function of the IIB matrix model becomes

$$\begin{aligned} Z(X,\Phi) &= \int da_\mu d\varphi dbdc e^{-(S_{\rm IKKT}+S_{\rm g.f.+ghost})} \\ &\sim e^{-S_{\rm IKKT}(X,\Phi)}\det{}^{-1/2}\Bigg(\delta_{\mu\nu}\tilde{X}^2 + 2\tilde{F}_{\mu\nu} \\ &\quad + \tilde{\bar{\Phi}}\Gamma_\mu\frac{1}{\Gamma\cdot\tilde{X}}\Gamma_\nu\tilde{\Phi}\Bigg)\times\det{}^{1/4}\Bigg(\left(\tilde{X}^2 + \frac{1}{2}\Gamma_{\mu\nu}\tilde{F}_{\mu\nu}\right) \\ &\quad \times\frac{1+\Gamma_{11}}{2}\Bigg)\det(\tilde{X}^2). \end{aligned} \tag{4.4}$$

Thus the free energy is given by

$$F(X,\Phi) = -\ln Z(X,\Phi) = S_{\rm IKKT}(X,\Phi) + F_b + F_f, \tag{4.5}$$

$$\begin{aligned} F_b &= \frac{1}{2}\mathcal{T}r\ln(\delta_{\mu\nu}\tilde{X}^2 + 2\tilde{F}_{\mu\nu}) \\ &\quad - \frac{1}{4}\mathcal{T}r\ln\left(\left(\tilde{X}^2 + \frac{1}{2}\Gamma_{\mu\nu}\tilde{F}_{\mu\nu}\right)\frac{1+\Gamma_{11}}{2}\right) - \mathcal{T}r\ln\tilde{X}^2, \end{aligned} \tag{4.6}$$

$$F_f = \frac{1}{2}\mathcal{T}r\ln\left[\delta_{\mu\nu} + \left(\frac{1}{\tilde{X}^2 + 2\tilde{F}}\right)_{\mu\rho}\tilde{\bar{\Phi}}\Gamma_\rho\frac{1}{\Gamma\cdot\tilde{X}}\Gamma_\nu\tilde{\Phi}\right], \tag{4.7}$$

where $\mathcal{T}r$ is the trace of the adjoint operators.

We first expand F_f formally with respect to the inverse powers of $\tilde{X}$. To this end we use the following formulas:

$$\frac{1}{\tilde{X}^2 + 2\tilde{F}} = \frac{1}{1+\frac{2}{\tilde{X}^2}\tilde{F}}\frac{1}{\tilde{X}^2}, \tag{4.8}$$

$$\begin{aligned} \frac{1}{\Gamma\cdot\tilde{X}} &= \frac{1}{1+\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}}\frac{1}{\tilde{X}^2}\Gamma\cdot\tilde{X} \\ &= \frac{1}{2}\frac{1}{1+\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}}\frac{1}{\tilde{X}^2}\Gamma\cdot\tilde{X} + \frac{1}{2}\Gamma\cdot\tilde{X}\frac{1}{1+\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}}\frac{1}{\tilde{X}^2}, \end{aligned} \tag{4.9, 4.10}$$

where

$$(\Gamma\cdot\tilde{X})^2 = \tilde{X}^2 + \frac{1}{2}\Gamma\cdot\tilde{F}, \tag{4.11}$$

and $\Gamma\cdot\tilde{F} \equiv \Gamma_{\mu\nu}\tilde{F}_{\mu\nu}$. In the following we expand the free energy with respect to $1/\tilde{X}$. Since the leading part of $\tilde{X}$ is a distance between N D-instantons and a single D-instanton, this expansion is valid when the single D-instanton is far separated from the other N D-instantons.

A. Second order terms of Φ

First let us focus on the terms with two fermions. As is seen in Sec. V, these terms are relevant for condensation of the antisymmetric tensor field. After using Eqs. (4.8) and (4.10), the second order terms of the fermionic background Φ are given by

$$\begin{aligned} F_f|_{\Phi^2} &= \frac{1}{4}\mathcal{T}r\Bigg[\left(\frac{1}{1+\frac{2}{\tilde{X}^2}\tilde{F}}\right)_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{\bar{\Phi}}\Gamma_\nu\frac{1}{1+\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}} \\ &\quad \times\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi} + \left(\frac{1}{1+\frac{2}{\tilde{X}^2}\tilde{F}}\right)_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{\bar{\Phi}}\Gamma_\nu(\Gamma\cdot\tilde{X}) \\ &\quad \times\frac{1}{1+\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}}\frac{1}{\tilde{X}^2}\Gamma_\mu\tilde{\Phi}\Bigg]. \end{aligned} \tag{4.12}$$

We now expand the effective action with two Φ's (4.12) with respect to $1/\tilde{X}$.

1. $\tilde{X}^{-3}$

The leading order starts from $1/\tilde{X}^3$ and is given by

$$\begin{aligned} &\frac{1}{4}\mathcal{T}r\left[\frac{1}{\tilde{X}^2}\tilde{\bar{\Phi}}\frac{1}{\tilde{X}^2}\Gamma_\mu(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi} + \frac{1}{\tilde{X}^2}\tilde{\bar{\Phi}}\Gamma_\mu(\Gamma\cdot\tilde{X})\Gamma_\mu\frac{1}{\tilde{X}^2}\tilde{\Phi}\right] \\ &\qquad = -2\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{\bar{\Phi}}\frac{1}{\tilde{X}^2}\Gamma_\mu[\tilde{X}_\mu,\tilde{\Phi}]. \end{aligned} \tag{4.13}$$

This is proportional to the equation of motion for the fermion.

2. $\tilde{X}^{-5}$

The next-to-leading order is proportional to $1/\tilde{X}^5$. At this order we have the following terms,

$$
\begin{aligned}
&\frac{1}{4}\mathcal{T}r\bigg[\bigg(-\frac{2}{\tilde{X}^2}\tilde{F}_{\mu\nu}\bigg)\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}\\
&\quad+\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\mu\bigg(-\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}\bigg)\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}\\
&\quad+\bigg(-\frac{2}{\tilde{X}^2}\tilde{F}_{\mu\nu}\bigg)\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu(\Gamma\cdot\tilde{X})\frac{1}{\tilde{X}^2}\Gamma_\mu\tilde{\Phi}\\
&\quad+\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\mu(\Gamma\cdot\tilde{X})\bigg(-\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}\bigg)\frac{1}{\tilde{X}^2}\Gamma_\mu\tilde{\Phi}\bigg].
\end{aligned}
\tag{4.14}
$$

After some calculations, these terms are rewritten as

$$
\begin{aligned}
&-\frac{1}{2}\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\frac{1}{\tilde{X}^2}\Gamma_{\mu\nu}\cdot\Gamma_\rho[\tilde{X}_\rho,\tilde{\Phi}]\\
&\quad-\frac{1}{2}\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}[\bar{\tilde{\Phi}},\tilde{X}_\rho]\Gamma_\rho\cdot\Gamma_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{\Phi}\\
&\quad-\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\frac{1}{\tilde{X}^2}\Gamma_\nu\tilde{X}_\mu\tilde{\Phi}\\
&\quad-\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\frac{1}{\tilde{X}^2}\Gamma_\nu\tilde{\Phi}\tilde{X}_\mu\\
&\quad+\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{X}_\mu\bar{\tilde{\Phi}}\frac{1}{\tilde{X}^2}\Gamma_\nu\tilde{\Phi}\\
&\quad+\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\tilde{X}_\mu\frac{1}{\tilde{X}^2}\Gamma_\nu\tilde{\Phi}.
\end{aligned}
\tag{4.15}
$$

The first two terms are proportional to the equation of motion. It is noted that the terms in Eq. (4.15) vanish if $\tilde{X}_\mu$ is replaced with d_μ. Here d_μ is a vector directed to the center of the N D-instantons from the single D-instanton. Therefore these terms are actually $\mathcal{O}(d^{-6})$ in the $1/d$ expansions.

3. $\tilde{X}^{-7}$

The terms of the order $\tilde{X}^{-7}$ are given by

$$
\begin{aligned}
&\frac{1}{4}\mathcal{T}r\bigg[\bigg(\frac{2}{\tilde{X}^2}\tilde{F}_{\mu\nu}\bigg)\bigg(\frac{2}{\tilde{X}^2}\tilde{F}_{\nu\rho}\bigg)\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\rho\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}\\
&\quad+\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\mu\bigg(\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}\bigg)\bigg(\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}\bigg)\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}\\
&\quad+\bigg(\frac{2}{\tilde{X}^2}\tilde{F}_{\mu\nu}\bigg)\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu\bigg(\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}\bigg)\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}\\
&\quad+\bigg(\frac{2}{\tilde{X}^2}\tilde{F}_{\mu\nu}\bigg)\bigg(\frac{2}{\tilde{X}^2}\tilde{F}_{\nu\rho}\bigg)\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\rho(\Gamma\cdot\tilde{X})\frac{1}{\tilde{X}^2}\Gamma_\mu\tilde{\Phi}\\
&\quad+\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\mu(\Gamma\cdot\tilde{X})\bigg(\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}\bigg)\bigg(\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}\bigg)\frac{1}{\tilde{X}^2}\Gamma_\mu\tilde{\Phi}\\
&\quad+\bigg(\frac{2}{\tilde{X}^2}\tilde{F}_{\mu\nu}\bigg)\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu(\Gamma\cdot\tilde{X})\bigg(\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}\bigg)\frac{1}{\tilde{X}^2}\Gamma_\mu\tilde{\Phi}\bigg].
\end{aligned}
\tag{4.16}
$$

These are rewritten as

$$
\begin{aligned}
&\frac{1}{4}\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2}\Gamma_{\mu\nu\rho\sigma}\Gamma_\lambda[\tilde{X}_\lambda,\tilde{\Phi}]+\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\frac{1}{\tilde{X}^2}\Gamma_{\mu\rho}\Gamma_\sigma[\tilde{X}_\sigma,\tilde{\Phi}]\\
&\quad+\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}[\bar{\tilde{\Phi}},\tilde{X}_\sigma]\Gamma_\sigma\Gamma_{\mu\rho}\frac{1}{\tilde{X}^2}\tilde{\Phi}-\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\mu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\frac{1}{\tilde{X}^2}\Gamma_\rho[\tilde{X}_\rho,\tilde{\Phi}]-\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\mu}\frac{1}{\tilde{X}^2}\\
&\quad\times[\bar{\tilde{\Phi}},\tilde{X}_\rho]\Gamma_\rho\frac{1}{\tilde{X}^2}\tilde{\Phi}-\frac{1}{2}\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\mu}\frac{1}{\tilde{X}^2}\Gamma_\rho[\tilde{X}_\rho,\tilde{\Phi}]-\frac{1}{2}\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2}\tilde{X}_\nu\bar{\tilde{\Phi}}\Gamma_{\mu\rho\sigma}\frac{1}{\tilde{X}^2}\tilde{\Phi}\\
&\quad-\frac{1}{2}\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2}\tilde{X}_\sigma\bar{\tilde{\Phi}}\Gamma_{\mu\nu\rho}\frac{1}{\tilde{X}^2}\tilde{\Phi}-\frac{1}{2}\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_{\mu\nu\sigma}\frac{1}{\tilde{X}^2}\tilde{\Phi}\tilde{X}_\rho\\
&\quad-\frac{1}{2}\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_{\nu\rho\sigma}\frac{1}{\tilde{X}^2}\tilde{\Phi}\tilde{X}_\mu-\frac{1}{2}\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2}\tilde{X}_\nu\Gamma_{\mu\rho\sigma}\tilde{\Phi}\\
&\quad+\frac{1}{2}\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\tilde{X}_\nu\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2}\Gamma_{\mu\rho\sigma}\tilde{\Phi}+2\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\frac{1}{\tilde{X}^2}\Gamma_\rho\tilde{X}_\mu\tilde{\Phi}\\
&\quad+2\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\tilde{X}_\rho\frac{1}{\tilde{X}^2}\Gamma_\mu\tilde{\Phi}-\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\mu\frac{1}{\tilde{X}^2}\tilde{\Phi}\tilde{X}_\rho+\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\rho\frac{1}{\tilde{X}^2}\tilde{\Phi}\tilde{X}_\mu\\
&\quad-\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\tilde{X}_\mu\bar{\tilde{\Phi}}\Gamma_\rho\frac{1}{\tilde{X}^2}\tilde{\Phi}+\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\tilde{X}_\rho\bar{\tilde{\Phi}}\Gamma_\mu\frac{1}{\tilde{X}^2}\tilde{\Phi}+\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\tilde{X}_\mu\Gamma_\rho\tilde{\Phi}\\
&\quad+\mathcal{T}r\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\tilde{X}_\mu\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\Gamma_\rho\tilde{\Phi}.
\end{aligned}
\tag{4.17}
$$

The first six terms vanish if the fermionic background satisfies the equation of motion.

4. $\tilde{X}^{-9}$

The terms of the order $\tilde{X}^{-9}$ are given by

$$\mathcal{T}r\Big[-2\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\sigma\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}-\frac{1}{32}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\mu\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{F})\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{F})\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{F})\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}$$
$$-\frac{1}{2}\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\rho\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{F})\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}-\frac{1}{8}\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{F})\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{F})\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}$$
$$-2\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\sigma(\Gamma\cdot\tilde{X})\frac{1}{\tilde{X}^2}\Gamma_\mu\tilde{\Phi}-\frac{1}{32}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\mu(\Gamma\cdot\tilde{X})\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{F})\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{F})\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{F})\frac{1}{\tilde{X}^2}\Gamma_\mu\tilde{\Phi}$$
$$-\frac{1}{2}\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\rho(\Gamma\cdot\tilde{X})\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{F})\frac{1}{\tilde{X}^2}\Gamma_\mu\tilde{\Phi}-\frac{1}{8}\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu(\Gamma\cdot\tilde{X})\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{F})\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{F})\frac{1}{\tilde{X}^2}\Gamma_\mu\tilde{\Phi}\Big]. \quad (4.18)$$

B. Fourth order terms of Φ

Now let us consider four-fermion terms. These terms are relevant for condensation of gravitons. The fourth order terms of Φ are given by

$$F_f|_{\Phi^4}=-\frac{1}{4}\mathcal{T}r\Big(\frac{1}{1+\frac{2}{\tilde{X}^2}\tilde{F}}\Big)_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu\frac{1}{1+\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}}\times\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\rho\tilde{\Phi}\Big(\frac{1}{1+\frac{2}{\tilde{X}^2}\tilde{F}}\Big)_{\rho\sigma}\times\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\sigma\frac{1}{1+\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}}\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}. \quad (4.19)$$

1. $\tilde{X}^{-6}$

The leading order term is proportional to $1/\tilde{X}^6$. The term of this order is given by

$$-\frac{1}{4}\mathcal{T}r\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\mu\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\nu\tilde{\Phi}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}. \quad (4.20)$$

2. $\tilde{X}^{-8}$

The next order terms are proportional to $1/\tilde{X}^8$ and given by

$$-\frac{1}{2}\mathcal{T}r\Big[\Big(-\frac{2}{\tilde{X}^2}\tilde{F}_{\mu\nu}\Big)\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\rho\tilde{\Phi}\times\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\rho\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}+\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\mu\times\Big(-\frac{1}{2\tilde{X}^2}\Gamma\cdot\tilde{F}\Big)\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\nu\tilde{\Phi}\times\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}\Big]. \quad (4.21)$$

3. $\tilde{X}^{-10}$

The terms of the order $1/\tilde{X}^{10}$ become

$$-\mathcal{T}r\Big[2\frac{1}{\tilde{X}^2}\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\nu\rho}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\rho\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\sigma\tilde{\Phi}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\sigma\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}+\frac{1}{8}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\mu\frac{1}{\tilde{X}^2}\Gamma_{\lambda\rho}\tilde{F}_{\lambda\rho}\frac{1}{\tilde{X}^2}\Gamma_{\sigma\tau}\tilde{F}_{\sigma\tau}$$
$$\times\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\nu\tilde{\Phi}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}+\frac{1}{2}\frac{1}{\tilde{X}^2}F_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu\frac{1}{\tilde{X}^2}\Gamma_{\sigma\tau}\tilde{F}_{\sigma\tau}\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\rho\tilde{\Phi}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\rho\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}$$
$$+\frac{1}{2}\frac{1}{\tilde{X}^2}F_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\rho\tilde{\Phi}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\rho\frac{1}{\tilde{X}^2}\Gamma_{\sigma\tau}\tilde{F}_{\sigma\tau}\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}+\frac{1}{\tilde{X}^2}F_{\mu\nu}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\rho\tilde{\Phi}$$
$$\times\frac{1}{\tilde{X}^2}F_{\rho\sigma}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\sigma\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}+\frac{1}{16}\bar{\tilde{\Phi}}\Gamma_\mu\frac{1}{\tilde{X}^2}\Gamma_{\lambda\rho}\tilde{F}_{\lambda\rho}\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\nu\tilde{\Phi}\frac{1}{\tilde{X}^2}\bar{\tilde{\Phi}}\Gamma_\nu\frac{1}{\tilde{X}^2}\Gamma_{\sigma\tau}\tilde{F}_{\sigma\tau}\frac{1}{\tilde{X}^2}(\Gamma\cdot\tilde{X})\Gamma_\mu\tilde{\Phi}\Big]. \quad (4.22)$$

V. CONDENSATION OF THE SUPERGRAVITY MODES

In this section, we discuss modifications of the effective actions for the IIB matrix model by condensation of D-instantons with appropriate wave functions. They correspond to condensation of massless type IIB supergravity fields. We here consider backgrounds produced by a mean-field D-instanton. As we saw in Secs. II and III, a D-instanton forms a supersymmetry multiplet by acting broken supersymmetry generators on the ordinary D-instanton boundary state. These states couple to the closed string massless states through derivative couplings and become a source for these fields.

Now we write $(N+1)\times(N+1)$ matrices in the decomposition (4.1) as follows:

$$X_\mu=\begin{pmatrix}x_\mu 1_N+A_\mu & 0\\ 0 & y_\mu\end{pmatrix},\qquad \Phi=\begin{pmatrix}\psi & 0\\ 0 & \xi\end{pmatrix}, \quad (5.1)$$

$$a_\mu=\begin{pmatrix}0 & \alpha_\mu\\ \alpha_\mu^\dagger & 0\end{pmatrix},\qquad \varphi=\begin{pmatrix}0 & \phi\\ \phi^\dagger & 0\end{pmatrix}. \quad (5.2)$$

A_μ and ψ are $N \times N$ traceless matrices. y_μ is a bosonic coordinate of a (mean-field) D-instanton. ξ is a fermionic coordinate and a Majorana-Weyl spinor. They represent degrees of freedom of the mean-field D-instanton. Off-diagonal components α_μ and ϕ are N-vectors corresponding to interactions between the diagonal blocks, which we have integrated out at the one-loop order in the previous section. Hence the free energy (4.5) is a function of these diagonal components, $F(X, \Phi) = F(A, x, \psi; y, \xi)$. By choosing wave functions $f_k(y, \xi)$ for the mean-field D-instanton and integrating over y, ξ, we can obtain a modified effective action $S_{\rm eff}(A, x, \psi; f_k)$ by condensation of the massless modes;

$$e^{-S_{\rm eff}(A,x,\psi;f_k)} = \int dy d\xi e^{-F(A,x,\psi;y,\xi)} f_k(y, \xi). \tag{5.3}$$

In what follows, we mainly look at terms without fermionic matrices ψ and replace all fermionic variables by the D-instanton fermionic coordinate ξ.

A. Condensation of the dilaton

In order to express condensation of dilaton in terms of the wave function of the mean-field D-instanton, we put $f_k(y, \xi)$ as

$$f_D(y, \xi) = \int d^{10}k e^{ik\cdot y} \tilde{f}_D(k, \xi) = \int d^{10}k e^{ik\cdot y} f(k) \left(\prod_{\gamma=1}^{16} \xi_\gamma \right). \tag{5.4}$$

Then the ξ integration is already saturated by the wave function. The leading contribution to the effective action is easily shown to be proportional to (the charge conjugation of) the dilaton vertex operator (2.31).

B. Condensation of the antisymmetric tensor $\mathbf{B}_{\mu\nu}$

We now calculate the effective action with an insertion of a wave function describing the antisymmetric tensor field $B_{\mu\nu}$. In the present calculation, the supersymmetry multiplet starts from the dilaton wave function (5.4) and the other functions in the multiplet can be constructed by acting a derivative operator $\partial/\partial\xi$.

By replacing λ in Eq. (2.13) with $\partial/\partial\xi$ and applying the differential operator on $\left(\prod_{\gamma=1}^{16} \xi_\gamma\right)$, we obtain the wave function for the antisymmetric tensor field;

$$\begin{aligned} f_B(y, \xi) &= \int d^{10}k e^{ik\cdot y} \tilde{f}_B(k, \xi) \\ &= \int d^{10}k e^{ik\cdot y} (\zeta_{\mu\nu}(k) k_\rho + \zeta_{\nu\rho}(k) k_\mu + \zeta_{\rho\mu}(k) k_\nu) \\ &\quad \times (\Gamma_{\mu\nu\rho}\Gamma_0)_{\alpha\beta} \frac{\partial}{\partial \xi_\alpha} \frac{\partial}{\partial \xi_\beta} \left(\prod_{\gamma=1}^{16} \xi_\gamma \right), \end{aligned} \tag{5.5}$$

where $\zeta_{\mu\nu}(k)$ is a polarization tensor, $\zeta_{\mu\nu}(k) = -\zeta_{\nu\mu}(k)$. Since our SUSY algebra closes only on shell, $\tilde{f}_D(k, \xi)$ and $\tilde{f}_B(k, \xi)$ fall into the supergravity multiplet for the case $k^2 = 0$. Hereafter we, however, formally extend the wave functions to the off-shell and integrate over the whole momentum region.

1. Contribution at $\mathcal{O}(1/\mathbf{d}^8)$ and Myers-like effect

Let us first look at contributions from the second order terms of Φ. In these terms we can simply replace Φ with ξ and thus the terms of the order $\tilde{X}^{-3}$ and $\tilde{X}^{-5}$ vanish. The terms of the order $\tilde{X}^{-7}$, Eq. (4.17), become

$$\begin{aligned} \frac{1}{2}(\bar{\xi}\Gamma_{\mu\rho\sigma}\xi)\mathcal{T}r\bigg[&-\tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2}\tilde{X}_\nu\left(\frac{1}{\tilde{X}^2}\right)^2 \\ &- \tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{X}_\nu\left(\frac{1}{\tilde{X}^2}\right)^2\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2} + \tilde{F}_{\mu\nu}\left(\frac{1}{\tilde{X}^2}\right)^2\tilde{X}_\nu\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2} \\ &+ \tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\left(\frac{1}{\tilde{X}^2}\right)^2\tilde{X}_\nu\frac{1}{\tilde{X}^2} - \tilde{F}_{\mu\nu}\left(\frac{1}{\tilde{X}^2}\right)^2\tilde{F}_{\rho\sigma}\frac{1}{\tilde{X}^2}\tilde{X}_\nu\frac{1}{\tilde{X}^2} \\ &+ \tilde{F}_{\mu\nu}\frac{1}{\tilde{X}^2}\tilde{X}_\nu\frac{1}{\tilde{X}^2}\tilde{F}_{\rho\sigma}\left(\frac{1}{\tilde{X}^2}\right)^2\bigg]. \end{aligned} \tag{5.6}$$

We then expand these terms with respect to the inverse powers of $d_\mu \equiv x_\mu - y_\mu$. For example, $1/\tilde{X}^2$ is expanded as follows,

$$\frac{1}{\tilde{X}^2} = \frac{1}{d^2}\left(1 - 2\frac{d\cdot A}{d^2}\right) + \mathcal{O}\left(\frac{1}{d^4}\right). \tag{5.7}$$

It is easily realized that the leading terms with $1/d^7$ vanish. The $1/d^8$ term has the following simple form

$$-\frac{1}{2d^8}(\bar{\xi}\Gamma_{\mu\rho\sigma}\xi)\,{\rm tr}\,[A_\nu, F_{\mu\nu}]F_{\rho\sigma}. \tag{5.8}$$

The $1/d^8$ dependence of the term indicates that the interaction is induced by an exchange of massless antisymmetric field.

We then integrate over y_μ and ξ with the wave function (5.5) in order to derive the effective action under condensation of the antisymmetric tensor field. In this calculation, we take our wave function (5.5) such that it damps at the infrared region where $|y - x| \to \infty$. Such a choice of wave function is natural from the view point of the dynamics of the eigenvalues in the matrix model. It was indeed shown that the distributions of the eigenvalues of A_μ are bounded in a finite region dynamically [4]. It is therefore natural to consider that the wave function damps far from the D-instantons. The size of the eigenvalue distribution is a function of N. If the eigenvalues are distributed on d-dim hypersurface uniformly, it is proportional to $N^{1/d}$. The natural scale of the infrared cutoff of the wave function depends on the dynamics of the matrix models, which we do not discuss in the present paper.

The integration over ξ and y can be easily performed as

$$\int d^{10}y d^{16}\xi f_B(y,\xi)\frac{-1}{2(x-y)^8}\bar{\xi}\Gamma_{\mu\rho\sigma}\xi \,\mathrm{tr}[A_\nu, F_{\mu\nu}]F_{\rho\sigma}$$
$$= \int d^{10}y d^{10}k e^{ik\cdot y}(\zeta_{\mu\nu}(k)k_\rho + \zeta_{\nu\rho}(k)k_\mu + \zeta_{\rho\mu}(k)k_\nu) \times \frac{-1}{2(x-y)^8}\mathrm{tr}[A_\sigma, F_{\mu\sigma}]F_{\rho\nu}$$
$$= -\frac{\pi^5}{3}\int d^{10}k\frac{e^{ik\cdot x}}{k^2}(\zeta_{\mu\nu}(k)k_\rho + \zeta_{\nu\rho}(k)k_\mu + \zeta_{\rho\mu}(k)k_\nu)\,\mathrm{tr}[A_\sigma, F_{\mu\sigma}]F_{\rho\nu}. \tag{5.9}$$

Because of our choice of the wave function, $\zeta_{\mu\nu}(k)$ damps at small k.

We therefore obtain the following effective action

$$S_{\mathrm{eff}}(A,x,\psi;f_B) = S_{\mathrm{IKKT}} - i\int d^{10}k f_{\mu\nu\rho}(k)e^{ik\cdot x}\,\mathrm{tr}\,[A_\sigma, F_{\mu\sigma}]F_{\nu\rho}, \tag{5.10}$$

where $f_{\mu\nu\rho}(k) = -\frac{i\pi^5}{3}(k_\mu\zeta_{\nu\rho} + k_\nu\zeta_{\rho\mu} + k_\rho\zeta_{\mu\nu})/k^2$.

This effective action shows that the Chern-Simons-like term is induced by an effect of condensation of the antisymmetric tensor. This phenomenon is similar to the Myers effect [10], but there is a difference. In the case of the Myers effect for D0-branes, a cubic term of bosonic matrices is induced in the Ramond-Ramond variational principle (RR) three-form background. This term can be interpreted as a vertex operator for the RR potential. In our case, however, the leading order of the induced term in Eq. (5.10) is different from the expected vertex operator for the charge conjugation of the antisymmetric tensor field (2.29). Such a term appears at the next order in the $1/d$ expansion as shown in the next subsection. The reason can be understood as follows. If we also calculate the fermionic term containing ψ, we would expect to obtain a term like $\mathrm{tr}\,(\bar{\psi}\Gamma_\mu\psi)F_{\nu\rho}$ and the leading order term in (5.10) with this fermionic term would be cancelled by using the equation of motion (2.4) of the original IKKT action. This kind of terms can not be seen in the vertex operators since we have assumed the equation of motion (2.4) and (2.5) in their construction. Here, since we are interested in investigating the effective actions under condensation of the antisymmetric tensor fields, we do *not* want to use the equations of motion of the original IKKT action and the term in (5.10) should *not* be omitted.

Let us see an effect of the induced term in (5.10) for a particular form of the polarization tensor. Assuming that the coefficient $\int d^{10}k f_{\mu\nu\rho}(k)e^{ik\cdot x}$ is proportional to ϵ_{ijk} with a specific direction $(i,j,k) = (1,2,3)$ and that the region $k\sim 0$ is dominant in the k-integration, the modified matrix model action becomes

$$S_{\mathrm{eff}}(A,x,\psi;f_B) = S_{\mathrm{IKKT}} - i\alpha\epsilon_{ijk}\,\mathrm{tr}\,[A_\nu, F_{i\nu}]F_{jk}, \tag{5.11}$$

with a constant coefficient α. This action has a fuzzy sphere classical solution;

$$A_i = \frac{1}{10\alpha}L_i, \qquad (i=1,2,3)$$
$$A_a = 0, \qquad (\text{for the other directions}), \tag{5.12}$$
$$\psi = 0.$$

The radius of the fuzzy sphere is in inverse proportion to the coefficient α and in the $\alpha\to 0$ limit the fuzzy sphere is expanded and becomes a flat plane. It contrasts with matrix models with the ordinary cubic Chern-Simons term (see, for example [26]) where the radius of the fuzzy sphere is proportional to the coefficient of the Chern-Simons term.

In addition to the fuzzy sphere solution, flat D-branes

$$[A_\mu, A_\nu] = i\theta_{\mu\nu}1_N, \qquad (\theta_{\mu\nu} = -\theta_{\nu\mu}). \tag{5.13}$$

with a constant $\theta_{\mu\nu}$ are also classical solutions of the effective action (for an infinite N). It will be interesting to compare stabilities of these solutions to the fuzzy sphere solution by calculating loop corrections around them.

2. Contribution at $\mathcal{O}(1/\mathbf{d}^9)$ and $B_{\mu\nu}$ vertex operator

The induced term in the previous subsection vanishes if we use the equation of motion for the configuration A_μ. Then the next order $\mathcal{O}(1/d^9)$ term becomes the leading order. From the dimensional analysis, it is expected that the vertex operator corresponding to the charge conjugation of the antisymmetric tensor field (2.29) would appear at the order of $1/d^9$.

Expanding the $\mathcal{O}(\tilde{X}^{-7})$ term (5.6) with respect to $1/d$, we obtain $\mathcal{O}(1/d^9)$ terms

$$\frac{2d_\lambda}{d^{10}}(\bar{\xi}\Gamma_{\mu\rho\sigma}\xi)\,\mathrm{tr}\,(F_{\mu\nu}F_{\rho\sigma}F_{\nu\lambda} + F_{\mu\nu}F_{\nu\lambda}F_{\rho\sigma}) + \frac{2}{d^{10}}(\bar{\xi}\Gamma_{\mu\rho\sigma}\xi)\,\mathrm{tr}\,([A_\nu, F_{\mu\nu}](d\cdot A)F_{\rho\sigma} + [A_\nu, F_{\mu\nu}]F_{\rho\sigma}(d\cdot A)). \tag{5.14}$$

The same order terms with $\mathcal{O}(1/d^9)$ can be obtained also from Eq. (4.18) as

$$-12\frac{d_\lambda}{d^{10}}(\bar{\xi}\Gamma_{\mu\nu\lambda}\xi)\,\mathrm{tr}\left(F_{\mu\rho}F_{\rho\sigma}F_{\sigma\nu} - \frac{1}{4}F_{\mu\nu}F_{\rho\sigma}F_{\sigma\rho}\right) - 2\frac{d_\lambda}{d^{10}}(\bar{\xi}\Gamma_{\mu\rho\sigma}\xi)\,\mathrm{tr}\,(F_{\mu\nu}F_{\rho\sigma}F_{\nu\lambda} + F_{\mu\nu}F_{\nu\lambda}F_{\rho\sigma}). \tag{5.15}$$

Therefore the interaction terms between the mean-field D-instanton and the $N\times N$ block are given at this order by

$$-12\frac{d_\lambda}{d^{10}}(\bar{\xi}\Gamma_{\mu\nu\lambda}\xi)\,\mathrm{tr}\left(F_{\mu\rho}F_{\rho\sigma}F_{\sigma\nu} - \frac{1}{4}F_{\mu\nu}F_{\rho\sigma}F_{\sigma\rho}\right) + \frac{2}{d^{10}}(\bar{\xi}\Gamma_{\mu\rho\sigma}\xi)\,\mathrm{tr}([A_\nu, F_{\mu\nu}](d\cdot A)F_{\rho\sigma} + [A_\nu, F_{\mu\nu}]F_{\rho\sigma}(d\cdot A)). \tag{5.16}$$

The first line represents an interaction through the vertex operator for (the charge conjugation of) the antisymmetric tensor field (2.29). The second term is similar to the Eq. (5.8) except for the insertion of $d \cdot A$. By integrating over y_μ and ξ with the wave function (5.5), the following terms are added to the effective action,

$$-i \int d^{10}k f_{\mu\nu\rho}(k) e^{ik\cdot x}\Big\{-3ik_\rho \operatorname{tr}\Big(F_{\mu\sigma}F_{\sigma\lambda}F_{\lambda\nu} - \frac{1}{4}F_{\mu\nu}F_{\sigma\lambda}F_{\lambda\sigma}\Big) + \frac{1}{2}\operatorname{tr}([A_\sigma, F_{\mu\sigma}](ik\cdot A)F_{\nu\rho} + [A_\sigma, F_{\mu\sigma}]F_{\nu\rho}(ik\cdot A))\Big\}. \quad (5.17)$$

The first term represents a derivative coupling of D-instantons to the vertex operator of the antisymmetric tensor field. The second term can be combined with Eq. (5.10) into a form

$$-i \int d^{10}k f_{\mu\nu\rho}(k) e^{ik\cdot x} \operatorname{Str} e^{ik\cdot A}[A_\sigma, F_{\mu\sigma}]\cdot F_{\nu\rho}. \quad (5.18)$$

If we calculate higher order terms in the $1/d$ expansion, we would expect to obtain higher order terms of (5.18) with respect to the number of bosonic fields A_μ.

C. Condensation of the graviton

Effects of the condensation of gravitons can be seen from the fourth order terms of ξ. The term Eq. (4.20) vanishes by substituting ξ for Φ because of the identity for the Majorana-Weyl spinor, $(\bar\xi\Gamma_{\mu\nu\rho}\xi)\Gamma^{\nu\rho}\xi = 0$. Therefore the leading contribution in the $1/d$ expansion comes from the $\tilde X^{-8}$ terms, Eq. (4.21) by replacing Φ with ξ as

$$(\bar\xi\Gamma_{\nu\lambda\rho}\xi)(\bar\xi\Gamma_{\mu\rho\sigma}\xi)\mathcal{T}r\frac{1}{\tilde X^2}\tilde F_{\mu\nu}\Big(\frac{1}{\tilde X^2}\Big)^2\tilde X_\lambda\Big(\frac{1}{\tilde X^2}\Big)^2\tilde X_\sigma. \quad (5.19)$$

1. Contribution at $\mathcal{O}(1/d^8)$ *and* $\mathcal{O}(1/d^9)$

Order $\mathcal{O}(1/d^8)$ terms vanish

$$\frac{d_\lambda d_\sigma}{d^{10}}(\bar\xi\Gamma_{\nu\lambda\rho}\xi)(\bar\xi\Gamma_{\mu\rho\sigma}\xi)\operatorname{tr}F_{\mu\nu} = 0, \quad (5.20)$$

since $d_\lambda d_\sigma(\bar\xi\Gamma_{\nu\lambda\rho}\xi)(\bar\xi\Gamma_{\mu\rho\sigma}\xi)$ is symmetric under an exchange of (μ, ν).

Similarly order $\mathcal{O}(1/d^9)$ terms also vanish

$$\frac{1}{d^{10}}(\bar\xi\Gamma_{\nu\lambda\rho}\xi)(\bar\xi\Gamma_{\mu\rho\sigma}\xi)\operatorname{tr}(d_\lambda F_{\mu\nu}A_\sigma + d_\sigma F_{\mu\nu}A_\lambda) = 0. \quad (5.21)$$

2. Contribution at $\mathcal{O}(1/d^{10})$

Hence the leading order terms start from $\mathcal{O}(1/d^{10})$ terms. Contributions from the above $\tilde X^{-8}$ term (5.19) are given by

$$\frac{1}{2d^{10}}(\bar\xi\Gamma_{\mu\rho\lambda}\xi)(\bar\xi\Gamma_{\nu\sigma\lambda}\xi)\operatorname{tr}F_{\mu\nu}F_{\rho\sigma} + \frac{4}{d^{12}}d_\lambda(\bar\xi\Gamma_{\mu\rho\sigma}\xi)c_{\mu\nu}(\xi)\operatorname{tr}F_{\nu\rho}F_{\sigma\lambda}, \quad (5.22)$$

where $c_{\mu\nu}(\xi) \equiv d_\rho(\bar\xi\Gamma_{\mu\nu\rho}\xi)$. The same order terms are also obtained from the $\tilde X^{-10}$ terms (4.22) as

$$-\frac{1}{8d^{10}}(\bar\xi\Gamma_{\mu\nu\lambda}\xi)(\bar\xi\Gamma_{\rho\sigma\lambda}\xi)\operatorname{tr}F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2d^{12}}c_{\mu\rho}c_{\rho\nu}\operatorname{tr}F_{\mu\sigma}F_{\sigma\nu} + \frac{3}{2d^{12}}c_{\mu\nu}c_{\rho\sigma}\operatorname{tr}F_{\mu\nu}F_{\rho\sigma} - \frac{9}{2d^{12}}c_{\mu\rho}c_{\nu\sigma}\operatorname{tr}F_{\mu\nu}F_{\rho\sigma} + \frac{3}{2d^{12}}d_\lambda c_{\rho\sigma}(\bar\xi\Gamma_{\mu\nu\rho}\xi)\operatorname{tr}F_{\mu\nu}F_{\sigma\lambda}. \quad (5.23)$$

By using the following Fierz identity,

$$c_{\mu\nu}c_{\rho\sigma} = \frac{1}{3}(c_{\mu\nu}c_{\rho\sigma} + c_{\mu\rho}c_{\sigma\nu} + c_{\mu\sigma}c_{\nu\rho}) - \frac{1}{6}(g_{\mu\rho}c_{\nu\lambda}c_{\lambda\sigma} - g_{\mu\sigma}c_{\nu\lambda}c_{\lambda\rho} - g_{\nu\rho}c_{\mu\lambda}c_{\lambda\sigma} + g_{\nu\sigma}c_{\mu\lambda}c_{\lambda\rho}) + \frac{1}{6}[d_\mu c_{\lambda\nu}(\bar\xi\Gamma_{\rho\sigma\lambda}\xi) - d_\nu c_{\lambda\mu}(\bar\xi\Gamma_{\rho\sigma\lambda}\xi) + d_\rho c_{\lambda\sigma}(\bar\xi\Gamma_{\mu\nu\lambda}\xi) - d_\sigma c_{\lambda\rho}(\bar\xi\Gamma_{\mu\nu\lambda}\xi)] + \frac{d^2}{6}(\bar\xi\Gamma_{\mu\nu\lambda}\xi)(\bar\xi\Gamma_{\rho\sigma\lambda}\xi). \quad (5.24)$$

the sum of these two terms, Eqs. (5.22) and (5.23), can be simplified and depends on ξ only in the form of $c_{\mu\nu}(\xi)$ as

$$(5.22)\text{ and }(5.23) = -\frac{1}{d^{12}}c_{\mu\rho}c_{\rho\nu}\operatorname{tr}F_{\mu\sigma}F_{\sigma\nu}. \quad (5.25)$$

It represents a derivative coupling of a single D-instanton to the graviton vertex operator constructed from the N D-instantons. If we insert the graviton wave function and integrate over the single D-instanton coordinates, we can obtain the graviton vertex operator as an induced term in the effective action.

Similarly interactions mediated by the 4th rank self-dual antisymmetric tensor field would appear, but such terms vanish in the leading order because of the cyclic property of the trace and the Jacobi identity,

$$c_{\mu\nu}c_{\rho\sigma}\operatorname{tr}(F_{\mu\nu}F_{\rho\sigma} + F_{\mu\rho}F_{\sigma\nu} + F_{\mu\sigma}F_{\nu\rho}) = c_{\mu\nu}c_{\rho\sigma}\operatorname{tr}A_\mu([A_\nu, [A_\rho, A_\sigma]] + [A_\rho, [A_\sigma, A_\nu]] + [A_\sigma, [A_\nu, A_\rho]]) = 0. \quad (5.26)$$

If we calculate higher order terms, we would expect to obtain the terms which can be produced by expanding the exponential in

$$\frac{1}{d^{12}} c_{\mu\nu} c_{\rho\sigma} \mathrm{Str}\, e^{ik\cdot A}(F_{\mu\nu}\cdot F_{\rho\sigma} + F_{\mu\rho}\cdot F_{\sigma\nu} + F_{\mu\sigma}\cdot F_{\nu\rho}). \tag{5.27}$$

VI. CONCLUSION

In this paper, we have considered fermionic backgrounds and condensation of supergravity fields in the IIB matrix model. We start from the type IIB matrix model in a flat background with the size $(N+1)\times(N+1)$, namely, a system of $(N+1)$ D-instantons. We then integrate 1 D-instanton (which we call a mean-field D-instanton) and obtain an effective action for N D-instantons by assuming particular forms of wave functions of the mean-field D-instanton. If we assume that the configurations of N D-instantons satisfy the equation of motion, we show that vertex operators obtained in our previous paper [1] are induced in the effective action as leading contributions. If we do not assume it, extra terms also appear. In particular if we take the wave function as that of the antisymmetric tensor field, a Chern-Simons like term is induced in the leading order of perturbations. Though this term is quintic with respect to the field A_μ, a fuzzy sphere becomes a solution to the equation of motion. In this sense, this is a similar mechanism to the Myers effect.

We have also given a stringy interpretation of the wave functions of the mean-field D-instanton as overlaps of the D-instanton boundary state with closed string massless states. The ordinary D-instanton only couples with the dilaton and the axion states. But since a D-instanton is a half-BPS state and breaks one half of the supersymmetries, we can obtain other states by acting broken supersymmetry generators on the ordinary D-instanton state. They couple to other supergravity fields through derivative couplings and form a supersymmetry multiplet in type IIB supergravity. We showed that the wave functions are nothing but the overlaps of these D-instanton boundary states with massless closed string states.

It is interesting to investigate effective actions under condensation of every massless closed string mode systematically, besides the charge conjugation of the antisymmetric tensor field and graviton we studied in this paper. Though it is expected from the analysis of the string theory side that each mode couples to the vertex operator of the N D-instanton system through an appropriate derivative coupling, other types of couplings like the quintic term derived here can also appear. We think that such studies clarify how the IIB matrix model contains dynamics of closed strings.

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APPENDIX

In this appendix, we briefly review the boundary states in the Green-Schwarz formalism of type IIB superstrings in the light-cone gauge.

We first summarize our notations:

(a) *Space-time quantities (in* $(9+1)$*-dimensions)*
Metric:

$$\eta_{\mu\nu} = \mathrm{diag}\,(-1, +1, \cdots, +1), \tag{A1}$$

Gamma matrices (in Majorana representation):

$$\{\Gamma^\mu, \Gamma^\nu\} = -2\eta^{\mu\nu}, \tag{A2}$$

$$\Gamma^0 = \sigma_2 \otimes 1_{16}, \tag{A3}$$

$$\Gamma^i = i\sigma_1 \otimes \gamma^i, \qquad (i = 1, 2, \cdots, 8), \tag{A4}$$

$$\Gamma^9 = i\sigma_3 \otimes 1_{16}, \tag{A5}$$

$$\Gamma_{11} = \Gamma^0\Gamma^1\cdots\Gamma^9 = -\sigma_1 \otimes \begin{pmatrix} 1_8 & 0 \\ 0 & -1_8 \end{pmatrix}, \tag{A6}$$

$$\gamma^i = \begin{pmatrix} 0 & \gamma^i_{a\dot{a}} \\ \gamma^i_{\dot{a}a} & 0 \end{pmatrix}, \qquad \gamma^i_{a\dot{a}} = \gamma^i_{\dot{a}a}, \tag{A7}$$

$$\gamma^i_{a\dot{a}}\gamma^j_{\dot{a}b} + \gamma^j_{a\dot{a}}\gamma^i_{\dot{a}b} = 2\delta^{ij}\delta_{ab}, \tag{A8}$$

$$\gamma^i_{a\dot{a}}\gamma^i_{b\dot{b}} + \gamma^i_{b\dot{a}}\gamma^i_{a\dot{b}} = 2\delta_{ab}\delta_{\dot{a}\dot{b}}, \tag{A9}$$

Spinors:

$$\theta = (\theta^a_1, \theta^{\dot{a}}_1, \theta^a_2, \theta^{\dot{a}}_2)^T, \tag{A10}$$

Weyl spinors:

$$\Gamma_{11}\theta = \theta \longrightarrow \theta = (\theta^a, \theta^{\dot{a}}, -\theta^a, \theta^{\dot{a}})^T, \tag{A11}$$

$$\Gamma_{11}\theta = -\theta \longrightarrow \theta = (\theta^a, \theta^{\dot{a}}, \theta^a, -\theta^{\dot{a}})^T, \tag{A12}$$

(b) *World-sheet quantities*
Metric:

$$\eta_{\alpha\beta} = \mathrm{diag}\,(-1, +1). \tag{A13}$$

Gamma matrices:

$$\{\rho^\alpha, \rho^\beta\} = -2\eta^{\alpha\beta}, \tag{A14}$$

$$\rho^0 = \sigma_2, \qquad \rho^1 = i\sigma_1. \tag{A15}$$

Antisymmetric tensor $\epsilon^{\alpha\beta}$:

$$\epsilon^{01} = +1. \tag{A16}$$

In the Green-Schwarz formalism, the IIB superstring theory is described by ten real bosons $X^\mu(\mu = 0, 1, \cdots, 9)$ and two Majorana-Weyl fermions $\theta^A(A = 1, 2)$ with the same chirality $\Gamma_{11}\theta^A = -\theta^A$. Here we take the light-cone gauge;

$$\Gamma^+\theta^A = 0 \longrightarrow \theta^A = (\theta^{Aa}, 0, \theta^{Aa}, 0), \tag{A17}$$

$$X^+ = x^+ + p^+\tau. \tag{A18}$$

The light-cone components are defined as

$$\Gamma^\pm = \frac{1}{\sqrt{2}}(\Gamma^0 \pm \Gamma^9), \tag{A19}$$

$$X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^9). \tag{A20}$$

The explicit forms of $\Gamma^\pm$ are

$$\Gamma^+ = \frac{i}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \otimes 1_{16}, \tag{A21}$$

$$\Gamma^- = \frac{i}{\sqrt{2}}\begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} \otimes 1_{16}. \tag{A22}$$

The world-sheet action in the light-cone gauge is given by

$$\begin{aligned} S_{\text{l.c.}} &= -\frac{1}{4\pi}\int d^2\sigma(\partial_\alpha X^i\partial^\alpha X^i - i\bar{S}^a\rho^\alpha\partial_\alpha S^a) \\ &= -\frac{1}{4\pi}\int d^2\sigma[-(\partial_\tau X^i)^2 + (\partial_\sigma X^i)^2 \\ &\quad - iS^{1a}(\partial_\tau + \partial_\sigma)S^{1a} - iS^{2a}(\partial_\tau - \partial_\sigma)S^{2a}], \end{aligned} \tag{A23}$$

where S^{Aa} are proportional to θ^{Aa}; $S^{Aa} \propto \sqrt{p^+}\theta^{Aa}$. The coordinates are expanded with respect to the Fourier modes as

$$X^i = x^i + p^i\tau + \frac{i}{\sqrt{2}}\sum_{n\neq 0}\frac{1}{n}(\alpha_n^i e^{-in(\tau-\sigma)} + \tilde{\alpha}_n^i e^{-in(\tau+\sigma)}), \tag{A24}$$

$$S^{1a} = \sum_n S_n^a e^{-in(\tau-\sigma)}, \tag{A25}$$

$$S^{2a} = \sum_n \tilde{S}_n^a e^{-in(\tau+\sigma)}. \tag{A26}$$

Under the quantization, the mode operators satisfy the hermiticity conditions

$$\begin{aligned} \alpha_n^\dagger &= \alpha_{-n}, & \tilde{\alpha}_n^\dagger &= \tilde{\alpha}_{-n}, \\ (S_n^a)^\dagger &= S_{-n}^a, & (\tilde{S}_n^a)^\dagger &= \tilde{S}_{-n}^a. \end{aligned} \tag{A27}$$

Also the commutation relations among them are given by

$$\begin{gathered} [x^i, p^j] = i\delta^{ij}, \qquad [\alpha_m^i, \alpha_n^j] = m\delta^{ij}\delta_{m+n,0}, \\ [\tilde{\alpha}_m^i, \tilde{\alpha}_n^j] = m\delta^{ij}\delta_{m+n,0}, \end{gathered} \tag{A28}$$

$$\{S_m^a, S_n^b\} = \delta^{ab}\delta_{m+n,0}, \qquad \{\tilde{S}_m^a, \tilde{S}_n^b\} = \delta^{ab}\delta_{m+n,0}. \tag{A29}$$

The action (A23) has the $\mathcal{N} = 2$ supersymmetry consisting of the kinematical SUSY

$$\delta S^{Aa} = \sqrt{2p^+}\epsilon^{Aa}, \tag{A30}$$

$$\delta X^i = 0, \tag{A31}$$

and the dynamical SUSY

$$\delta S^{1a} = \frac{1}{\sqrt{p^+}}(\partial_\tau - \partial_\sigma)X^i\gamma^i_{a\dot{a}}\epsilon^{1\dot{a}}, \tag{A32}$$

$$\delta S^{2a} = \frac{1}{\sqrt{p^+}}(\partial_\tau + \partial_\sigma)X^i\gamma^i_{a\dot{a}}\epsilon^{2\dot{a}}, \tag{A33}$$

$$\delta X^i = -\frac{i}{\sqrt{p^+}}\epsilon^{A\dot{a}}\gamma^i_{\dot{a}a}S^{Aa}. \tag{A34}$$

These transformations are generated by the following supercharges

$$Q^{1a} = \int_0^{2\pi}\frac{d\sigma}{2\pi}\sqrt{2p^+}S^{1a} = \sqrt{2p^+}S_0^a, \tag{A35}$$

$$Q^{2a} = \int_0^{2\pi}\frac{d\sigma}{2\pi}\sqrt{2p^+}S^{2a} = \sqrt{2p^+}\tilde{S}_0^a, \tag{A36}$$

$$\begin{aligned} Q^{1\dot{a}} &= \int_0^{2\pi}\frac{d\sigma}{2\pi}\frac{1}{\sqrt{p^+}}(\partial_\tau - \partial_\sigma)X^i\gamma^i_{\dot{a}a}S^{1a} \\ &= \frac{1}{\sqrt{p^+}}\gamma^i_{\dot{a}a}\Big(p^iS_0^a + \sqrt{2}\sum_{n\neq 0}\alpha_n^iS_{-n}^a\Big), \end{aligned} \tag{A37}$$

$$\begin{aligned} Q^{2\dot{a}} &= \int_0^{2\pi}\frac{d\sigma}{2\pi}\frac{1}{\sqrt{p^+}}(\partial_\tau + \partial_\sigma)X^i\gamma^i_{\dot{a}a}S^{2a} \\ &= \frac{1}{\sqrt{p^+}}\gamma^i_{\dot{a}a}\Big(p^i\tilde{S}_0^a + \sqrt{2}\sum_{n\neq 0}\tilde{\alpha}_n^i\tilde{S}_{-n}^a\Big), \end{aligned} \tag{A38}$$

which satisfy the algebra

$$\{Q^{Aa}, Q^{Bb}\} = 2p^+\delta^{AB}\delta^{ab}, \tag{A39}$$

$$\{Q^{1\dot{a}}, Q^{1\dot{b}}\} = 2P^-\delta_{\dot{a}\dot{b}}, \tag{A40}$$

$$\{Q^{2\dot{a}}, Q^{2\dot{b}}\} = 2\tilde{P}^-\delta_{\dot{a}\dot{b}}, \tag{A41}$$

$$\{Q^{Aa}, Q^{B\dot{a}}\} = \sqrt{2}\gamma^i_{a\dot{a}}p^i\delta^{AB}, \tag{A42}$$

with

$$P^- = \frac{1}{p^+}\Big[\frac{p^ip^i}{2} + \sum_{n\neq 0}(nS_{-n}^aS_n^a + \alpha_{-n}^i\alpha_n^i)\Big], \tag{A43}$$

$$\tilde{P}^- = \frac{1}{p^+}\Big[\frac{p^ip^i}{2} + \sum_{n\neq 0}(n\tilde{S}_{-n}^a\tilde{S}_n^a + \tilde{\alpha}_{-n}^i\tilde{\alpha}_n^i)\Big]. \tag{A44}$$

A boundary state is usually defined by a set of the boundary conditions on a constant τ surface:

$$[(\partial_\tau - \partial_\sigma)X^i - M_{ij}(\partial_\tau + \partial_\sigma)X^j]|B, \eta\rangle = 0, \tag{A45}$$

$$Q_\eta^{+a}|B, \eta\rangle \equiv (Q^{1a} + i\eta M_{ab}Q^{2b})|B, \eta\rangle = 0, \tag{A46}$$

$$Q_\eta^{+\dot{a}}|B, \eta\rangle \equiv (Q^{1\dot{a}} + i\eta M_{\dot{a}\dot{b}}Q^{2\dot{b}})|B, \eta\rangle = 0, \tag{A47}$$

where η is a parameter ($\eta^2 = 1$), and M_{ij} is an element of SO(8). For the Neumann directions $M_{ij} = -\delta_{ij}$ and for the Dirichlet directions $M_{ij} = \delta_{ij}$. M_{ab} and $M_{\dot{a}\dot{b}}$ are determined by consistency requirements as follows. Taking the surface $\tau = 0$, the conditions (A45)–(A47) are written in terms of the mode operators as

$$(p^i - M_{ij}p^j)|B, \eta\rangle = 0, \tag{A48}$$

$$(\alpha_n^i - M_{ij}\tilde{\alpha}_{-n}^j)|B, \eta\rangle = 0, \tag{A49}$$

$$(S_0^a + i\eta M_{ab}\tilde{S}_0^b)|B, \eta\rangle = 0, \tag{A50}$$

$$\Big[\gamma^i_{\dot{a}a}p^iS_0^a + i\eta M_{\dot{a}\dot{b}}\gamma^i_{\dot{b}a}\tilde{S}_0^a + \sqrt{2}\sum_{n\neq 0}(\gamma^i_{\dot{a}a}\alpha_n^iS_{-n}^a + i\eta M_{\dot{a}\dot{b}}\gamma^i_{\dot{b}a}\tilde{\alpha}_n^i\tilde{S}_{-n}^a)\Big]|B, \eta\rangle = 0. \tag{A51}$$

Let us determine M_{ab} and $M_{\dot{a}\dot{b}}$. From $\{Q^{+a}, Q^{+b}\}|B, \eta\rangle = 0$, we find

$$M_{ac}M_{bc} = \delta_{ab}, \tag{A52}$$

meaning that M_{ab} is an orthogonal matrix. Next, $\{Q^{+a}, Q^{+\dot{a}}\}|B, \eta\rangle = 0$ leads to

$$(\gamma^i_{a\dot{a}}p^i - M_{ab}M_{\dot{a}\dot{b}}\gamma^i_{b\dot{b}}p^i)|B, \eta\rangle = 0. \tag{A53}$$

Comparing this with (A48), we have

$$\gamma^i_{a\dot{a}}M_{ij} - M_{ab}M_{\dot{a}\dot{b}}\gamma^j_{b\dot{b}} = 0. \tag{A54}$$

The consistency between Eq. (A45) and Eq. (A47) requires

$$(\gamma^i_{\dot{a}a}S_n^a + i\eta M_{ij}M_{\dot{a}\dot{b}}\gamma^j_{\dot{b}b}\tilde{S}_{-n}^b)|B, \eta\rangle = 0 \quad \text{for} \quad n \neq 0, \tag{A55}$$

by using (A54), which are rewritten as

$$\gamma^i_{\dot{a}a}(S_n^a + i\eta M_{ab}\tilde{S}_{-n}^b)|B, \eta\rangle = 0 \quad \text{for} \quad n \neq 0. \tag{A56}$$

Since M_{ij} is an element of SO(8), it can be written as $M_{ij} = (e^{\Omega_{kl}\Sigma^{kl}})_{ij}$ with $(\Sigma^{kl})_{ij} = \delta_i^k\delta_j^l - \delta_i^l\delta_j^k$ being generators of SO(8). Equation (A54) can be solved in terms of Ω_{ij} as

$$M_{ab} = (e^{(1/2)\Omega_{ij}\gamma^{ij}})_{ab}, \tag{A57}$$

$$M_{\dot{a}\dot{b}} = (e^{(1/2)\Omega_{ij}\tilde{\gamma}^{ij}})_{\dot{a}\dot{b}}, \tag{A58}$$

where

$$\gamma^{ij}_{ab} = \frac{1}{2}(\gamma^i_{a\dot{a}}\gamma^j_{\dot{a}b} - \gamma^j_{a\dot{a}}\gamma^i_{\dot{a}b}), \quad \tilde{\gamma}^{ij}_{\dot{a}\dot{b}} = \frac{1}{2}(\gamma^i_{\dot{a}a}\gamma^j_{a\dot{b}} - \gamma^j_{\dot{a}a}\gamma^i_{a\dot{b}}). \tag{A59}$$

The boundary state $|B, \eta\rangle$ can be expressed in the form

$$|B, \eta\rangle = e^{\sum_{n>0}((1/n)M_{ij}\alpha^i_{-n}\tilde{\alpha}^j_{-n} - i\eta M_{ab}S^a_{-n}\tilde{S}^b_{-n})}|B_0, \eta\rangle \tag{A60}$$

with the zero-mode part

$$|B_0, \eta\rangle = C(M_{ij}|i\rangle|j\rangle - i\eta M_{\dot{a}\dot{b}}|\dot{a}\rangle|\dot{b}\rangle). \tag{A61}$$

C is a normalization constant, and the ground states $|i\rangle$ and $|\dot{a}\rangle$ are defined by

$$\alpha_n^j|i\rangle = S_n^a|i\rangle = \alpha_n^i|\dot{a}\rangle = S_n^a|\dot{a}\rangle = 0, \qquad (\text{for } n > 0), \tag{A62}$$

$$S_0^a|i\rangle = \frac{\gamma^i_{a\dot{a}}}{\sqrt{2}}|\dot{a}\rangle, \qquad S_0^a|\dot{a}\rangle = \frac{\gamma^i_{a\dot{a}}}{\sqrt{2}}|i\rangle. \tag{A63}$$

Broken supercharges are given by

$$Q_\eta^{-a} \equiv Q^{1a} - i\eta M_{ab}Q^{2b}, \tag{A64}$$

$$Q_\eta^{-\dot{a}} \equiv Q^{1\dot{a}} - i\eta M_{\dot{a}\dot{b}}Q^{2\dot{b}}, \tag{A65}$$

and the algebra of broken and unbroken supercharges becomes

$$\{Q_\eta^{+a}, Q_\eta^{-b}\} = 4p^+\delta_{ab}, \tag{A66}$$

$$\{Q_\eta^{+a}, Q_\eta^{-\dot{b}}\} = \sqrt{2}\gamma^i_{a\dot{b}}(p^i + M_{ij}p^j), \tag{A67}$$

$$\{Q_\eta^{+\dot{a}}, Q_\eta^{-b}\} = \sqrt{2}\gamma^i_{\dot{a}b}(p^i + M_{ij}p^j), \tag{A68}$$

$$\{Q_\eta^{+\dot{a}}, Q_\eta^{-\dot{b}}\} = 2(P^- + \tilde{P}^-)\delta_{\dot{a}\dot{b}} = 2P^-_{cl}\delta_{\dot{a}\dot{b}}, \tag{A69}$$

and the other anticommutators vanish.

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