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Deconfinement and color superconductivity in cold neutron stars

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We study the deconfinement transition of hadronic matter into quark matter in neutron star conditions in the light of color superconductivity. Deconfinement is considered to be a first order phase transition that conserves color and flavor. It gives a short-lived ($\tau \sim \tau_{\rm weak}$) transitory colorless-quark-phase that is *not* in β-equilibrium. We deduce the equations governing deconfinement when quark pairing is allowed and find the regions of the parameter space (pairing gap Δ versus bag constant B) where deconfinement is possible inside cold neutron stars. We show that for a wide region of (B, Δ) a pairing pattern is reachable within a strong interaction timescale, and the resulting "2SC-like" phase is preferred energetically to the unpaired phase. We also show that although β-stable hybrid star configurations are known to be possible for a wide region of the (B, Δ)-space, many of these configurations could not form in practice because deconfinement is forbidden, i.e. the here studied non-β-stable *intermediate* state cannot be reached.

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I. INTRODUCTION

A general feature of degenerate Fermi systems is that they become unstable if there exist any attractive interaction at the Fermi surface. As recognized by Bardeen, Cooper and Schrieffer (BCS) [1] this instability leads to the formation of a condensate of Cooper pairs and the appearance of superconductivity. In QCD any attractive quark-quark interaction will lead to pairing and color superconductivity, a subject already addressed in the late 1970s and early 1980s [2,3] which came back a few years ago since the realization that the typical superconducting gaps in quark matter may be larger than those predicted in these early works (Δ as high as $\sim$100 MeV) [4]. The phase diagram of QCD has been analyzed in the light of color superconductivity and model calculations suggest that the phase structure is very rich at high densities. Depending on the number of flavors, the quark masses, the interaction channels, and other variables many possible β-stable color-superconducting phases of quark matter are possible [5–9].

There is at present some indication that the quark-gluon plasma might have been produced in laboratory [10]. However, it is not yet established whether deconfinement happens in nature in the high-density, low-temperature regime that is relevant for neutron stars. Unfortunately, first principle calculations are not available in this region of the QCD phase diagram. In turn we shall base our analysis on phenomenological considerations which could delineate at least a broad brush picture of the physics involved. Matter in compact stars should be electrically neutral and colorless in bulk. Also, any equilibrium configuration of such matter should remain in β-equilibrium. Satisfying these requirements impose nontrivial relations between the chemical potentials of different quarks. Moreover, such relations substantially influence the pairing dynamics between quarks, for instance, by suppressing some color-superconducting phases and by favoring others [11].

At not very large densities, the appearance of strange quarks is suppressed because of their finite mass and quark matter is composed almost completely by quarks u and d. Pairing between them would be possible if their Fermi momenta are not very different. The resulting pairing pattern is the so-called two-flavor color superconductor (2SC) in which the up-and-down quarks form Cooper pairs in the color-antitriplet, flavor-singlet, spin-zero channel. In the conventional picture of the 2SC phase it is assumed that pairs are formed by up-red (u_r) and down-green (d_g) quarks, as well as by up-green (u_g) and down-red (d_r) quarks. The other two quarks (u_b and d_b) do not participate in pairing. A more recent analysis shows that the ground state of dense up-and-down quark matter under local and global charge neutrality conditions with β-equilibrium has at least four possibilities: normal, regular 2SC, gapless 2SC phases, and mixed phase composed of 2SC phase and normal components [6]. A new interesting feature of some of these phases is that pairing is allowed between particles even in the case $\delta\mu > \Delta$ [6].

At sufficiently large densities the value of the chemical potential exceeds the mass of the strange quark m_s. Strange quarks appear in the mixture, and Cooper pairing can happen between up, down and strange quarks. Pairing involving strange quarks is expected to exist if the resulting gaps (Δ_{us} and Δ_{ds}) are larger than $\sim m_s^2/(2\mu)$, the difference between the u and s Fermi momenta in the absence of pairing. Depending on the value of the strange quark mass, as well as other parameters in the theory, many different paired configurations are possible. At very high densities it seems clear that the color-flavor locked phase (CFL) is the ground state. However, at not very large densities, it is

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possible that up-and-down quarks form 2SC matter, while the strange quarks do not participate in pairing, eventually forming a <ss> condensate with a much smaller gap. Finally, even more exotic crystalline phases of 1SC quark matter have been analyzed.

To the best of our knowledge, all previous works about color superconductivity in compact stars have dealt with matter in β-equilibrium. This is the situation expected to appear in strange stars or hybrid stars as soon as they settle in a stable configuration. However, during the deconfinement transition in neutron stars, matter is beac out of equilibrium with respect to weak interactions. In fact, the transition from β-stable hadron matter to quark matter in cold neutron stars should occur trough a quantum nucleation process [12–17]. Quantum fluctuations could form both virtual drops of unpaired quark matter (hereafter the $Q^*_{\rm unp}$ phase) or virtual drops of color-superconducting quark matter (Q^*_Δ phase). In both cases, the flavor content of the quark matter virtual drop must be equal to that of the confined β-stable hadronic phase at the same pressure (the central pressure of the hadronic star). In fact, since quark deconfinement and quark-quark pairing are due to the strong interaction, the oscillation time ν_0^{-1} of a virtual quark droplet in the potential energy barrier separating the hadronic from the quark phase, is of the same order of the strong interaction characteristic time ($\tau_{\rm strong} \sim 10^{-23}$ s). The latter is many orders of magnitude smaller than the weak interaction characteristic time ($\tau_{\rm weak} \sim 10^{-8}$ s). Thus, quark flavor must be conserved forming a virtual drop of quark matter [16–20]. Which one of the two kind of droplets ($Q^*_{\rm unp}$ or Q^*_Δ) will nucleate depends on the value of the corresponding Gibbs free energy per baryon ($g_{\rm unp}$, g_Δ). In fact, the latter quantity enters in the expression of the volume term of the energy barrier separating the confined and deconfined phases (see e.g. Eq. (7) in [17], where the Gibbs free energy per baryon is denoted by μ_i, $i = Q^*, H$). Clearly, when $g_\Delta < g_{\rm unp}$ the nucleation of a Q^*_Δ drop will be realized.

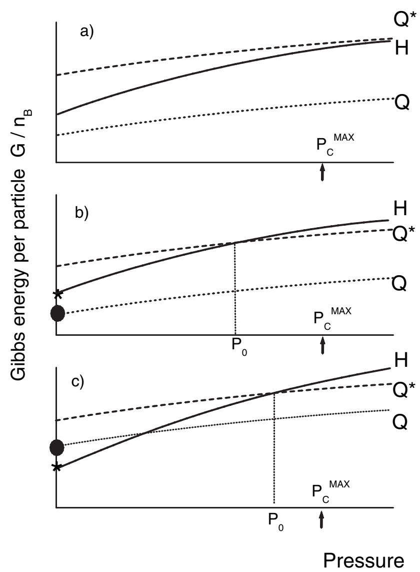

FIG. 1. Schematic comparison of the free energy of hadronic matter (H), non-β-stable "just-deconfined" matter (Q^*), and β-stable quark matter (Q) for different cases. In the case of panel a) the transition can never occur inside neutron stars in spite of the final state (Q) having a lower energy per baryon. As explained in the text, a direct transition to Q is strongly suppressed. Since Q^* has a larger energy per baryon than H for all pressures below the central pressure of the maximum-mass hadronic star ($P_c^{\rm max}$), deconfinement cannot occur even if the Q phase has a lower energy. In panels b) and c) the phase Q^* has a lower energy per baryon than H for some pressures below ($P_c^{\rm max}$). Therefore, deconfinement is possible if pressures between P_0 and $P_c^{\rm max}$ are reached inside a given neutron star. The difference between panels b) and c) is the energy per particle at zero pressure (indicated with a dot for Q and with an asterisk for H). In b) quark stars are the so-called strange stars, because they can be made up of quark matter from the center up to the surface ($P = 0$). In case c) they are hybrid stars, because at zero pressure H has a lower energy than Q. For a fixed hadronic equation of state, these possibilities correspond to different values of the parameters of the quark model (the vacuum energy density B and the superconducting gap Δ).

The direct formation by quantum fluctuations of a drop of β-stable quark matter (Q phase) is also possible in principle. However, it is strongly suppressed with respect to the formation of the non β-stable drop by a factor $\sim G_{\rm Fermi}^{2N/3}$ being N the number of particles in the critical size quark drop. This is so because the formation of a β-stable drop will imply the almost simultaneous conversion of $\sim N/3$ up-and-down quarks into strange quarks. For a critical size β-stable nugget at the center of a neutron star it is found $N \sim 100 - 1000$, and therefore the factor is actually tiny. This is the same reason that impedes that an iron nucleus converts into a drop of strange quark matter, even in the case in which strange quark matter had a lower energy per baryon (Bodmer-Witten-Terazawa hypothesis). Because of this reason it is assumed that a direct transition to β-stable quark matter is not possible.[1] Therefore, the β-stable state Q could be reached only after the β-decay of the intermediate state Q^*. This is in agreement with many other previous works, see e.g. [16–20].

[1] Notice that the nucleation of an initial quark droplet might be induced in principle by external influences such as high energy cosmic rays or neutrinos [13]. However, estimates of the production rates of quark droplets by neutrino sparking [21] show that this mechanism is not likely to drive a neutron to quark conversion for realistic values of the minimum center of mass energy necessary to produce a quark-gluon plasma in heavy ion collisions. Ultra high energy neutrinos would be also harmless because the outer crust acts as a shield due to the huge cross section [21]. In this paper we are assuming that the conversion must proceed trough an intermediate "two-flavor" phase, but other possibilities cannot be definitely excluded.

In this context, one question addressed by the present work is whether the system settles in a paired or in an unpaired state just after the deconfinement. On the other hand, notice that although the above mentioned non-β-stable quark phase is very short-lived, it constitutes an unavoidable intermediate state that must be reached before arriving to the final β-stable configuration, e.g. CFL quark matter (c.f. [22,23]). The second question we shall address is whether this intermediate phase can eventually preclude the transition to the final β-stable state in spite of the latter having a lower energy. This is because the Q-phase can be formed only after the nucleation of a real (i.e critical size) drop of $Q^*_{\rm unp}$ or Q^*_Δ matter, and its subsequent "long term" ($t \sim \tau_{\rm weak} \sim 10^{-8}$ s) weak decay process.

II. DECONFINEMENT OF HADRONIC MATTER INTO COLOR-SUPERCONDUCTING QUARK MATTER

Given the uncertainties in the nature of matter at high densities, the analysis is based on the extrapolation to higher densities of an hadronic model valid around the nuclear saturation density ρ_0, and the extrapolation to ρ_0 of a quark model that is expected to be valid only for $\rho \rightarrow \infty$. Within this kind of analysis the (in general) different functional form of both EOSs, induces the phase transition to be first order. Notice that from lattice QCD calculations there are indications that the transition is actually first order in the high-density and low-temperature regime, although this calculations involve temperatures that are still larger than those in neutron stars, and do not include the effect of color superconductivity [24].

Deconfinement is analyzed here as a first order phase transition that conserves the flavor abundances in both phases. Therefore, it gives a transitory colorless-quark-phase that is *not* in β-equilibrium.

For describing the just-deconfined quark phase we shall model it as a free Fermi mixture of quarks and leptons and we will subtract the pairing and the vacuum energy. The thermodynamic potential can be written as

$$\Omega = \Omega_Q^{\rm free} + \Omega_Q^{\rm gap} + \Omega_L + B, \tag{1}$$

with

$$\Omega_Q^{\rm free} = \sum_{f,c} \frac{1}{\pi^2} \int_0^{k_{fc}} [E_{cf} - \mu_{fc}] p^2 dp, \tag{2}$$

where $E_{cf} = (p^2 + m_{fc}^2)^{1/2}$, and the sum is to be made over all colors and flavors of the quark mixture, being $f = u, d, s$ the flavor index, and $c = r, g, b$ the color index. Confinement is introduced in Eq. (1) by means of the bag constant B, and Ω_L is the contribution of leptons. Pairing is included through the term

$$\Omega_Q^{\rm gap} = -\frac{\mathcal{A}}{\pi^2} \Delta^2 \bar{\mu}^2, \tag{3}$$

which results from $E_{cf} = \sqrt{p^2 + m^2 \pm \Delta^2}$, by expanding in Eq. (2) to $\mathcal{O}(\Delta^2/\mu^2)$. For simplicity, all particles that pair are assumed to have the same gap Δ. The mean chemical potential in Eq. (3) is defined through $\bar{\mu} = N^{-1} \sum_{f,c} \mu_{fc}$ where the sum is to be made only over particles that participate of pairing, and N is the number of different quarks that pair. The Fermi momenta are $k_{fc} = (\mu_{fc}^2 - m_{fc}^2)^{1/2}$. The binding energy of the diquark condensate is included by subtracting $\Delta^2 \bar{\mu}^2/(4\pi^2)$ for every quasiparticle with gap Δ [25]. As we shall see below, the relevant pairing pattern for the deconfinement transition in neutron stars is "2SC-like", in which d_r and u_g quarks pair yielding two quasiparticles with gap Δ, and u_r and d_g quarks pair, yielding two quasiparticles with gap Δ (c.f. [25]). Therefore, we shall set $\mathcal{A} = 1$ along this work. Note that for color-flavor-locked quark matter it is $\mathcal{A} = 3$.

As already emphasized by Rajagopal and Wilczeck [7], the exact nature of the interaction that generates Δ is not relevant to the order we are working. This means that Ω is given by the above prescription regardless of whether the pairing is due to a point like four-Fermi interaction, as in Nambu-Jona-Lasinio models [26], or due to the exchange of a gluon, as in QCD at asymptotically high energies. Of course, the strength and form of the interaction determine the value of Δ, and also its dependence with the density. Lacking of an accurate calculation for Δ, which may as large as $\sim$100 MeV (and even larger in the presence of an external magnetic field [27]), we shall keep it as a free constant parameter.

The thermodynamic quantities are straightforwardly derived from the standard expressions: the pressure is $P = -\Omega$ and the energy density at zero temperature is given by $\varepsilon = \sum_{Q,L} \mu_i n_i + \Omega$, where the sum is to be carried over all quarks and leptons. In particular, the particle number densities $n_{fc} = -\partial\Omega/\partial\mu_{fc}$ are given by

$$n_{fc} = \frac{k_{fc}^3}{3\pi^2} + \frac{2\mathcal{A}}{N\pi^2} \Delta^2 \bar{\mu}, \tag{4}$$

for the quarks that participate in pairing, and by

$$n_{fc} = \frac{k_{fc}^3}{3\pi^2}, \tag{5}$$

for quarks that do not pair. The number density of each flavor in the quark phase is given by

$$n_f = \sum_c n_{fc}, \tag{6}$$

and the baryon number density n_B by

$$n_B = \frac{1}{3} \sum_{fc} n_{fc} = \frac{1}{3} \sum_c n_c = \frac{1}{3} \sum_f n_f. \tag{7}$$

In order to close the above equations we need to impose additional physical conditions describing the composition of the mixture (e.g. a set of conditions on the chemical potentials). One possibility, extensively employed in the literature, is β-equilibrium of the quark phase. This condition describes matter a sufficiently large time after deconfinement ($\tau \gg \tau_{\mathrm{weak}} \sim 10^{-8}$ s). However, for studying deconfinement, the relevant timescale is $\tau \ll \tau_{\mathrm{weak}}$. As already emphasized in [16–20] the appropriate condition for $\tau \ll \tau_{\mathrm{weak}}$ is flavor conservation between hadronic and deconfined quark matter. This can be written as

$$Y_f^H = Y_f^Q \qquad f = u, d, s, L \tag{8}$$

being $Y_f^H \equiv n_f^H/n_B^H$ and $Y_i^Q \equiv n_f^Q/n_B^Q$ the abundances of each particle in the hadron and quark phase, respectively, (we shall omit the superindexes H and Q in the following). In other words, the just-deconfined quark phase must have the same "flavor" composition than the β-stable hadronic phase from which it originated.

Additionally, the deconfined phase must be locally colorless; therefore, it must be composed by an equal number or red, green and blue quarks:

$$n_r = n_g = n_b \tag{9}$$

being n_r, n_g and n_b the number densities of red, green and blue quarks, respectively, given by:

$$n_c = \sum_f n_{fc}. \tag{10}$$

Color neutrality can be automatically fulfilled by imposing that each flavor must be colorless separately, i.e. $n_{ur} = n_{ug} = n_{ub}$, $n_{dr} = n_{dg} = n_{db}$, and $n_{sr} = n_{sg} = n_{sb}$. But in general this configuration will not allow pairing with a significant gap. As already stated, for quarks having different color and flavor the pairing gap may be as large as 100 MeV, while for particles having the same flavor the gap is found to be about 2 orders of magnitude smaller (see [25] and references therein). Pairing is allowed even in the case $\delta\mu > \Delta$ but the corresponding gaps are small [6,28]. Therefore, in order to allow pairing between quarks with a non negligible gap, the Fermi momenta of at least u_r and d_g quarks must be equal (the choice of these two particular colors and flavors is just a convention). This implies the equality of the corresponding number densities

$$n_{ur} = n_{dg}. \tag{11}$$

The above condition represents a state that fulfills all the physical requirements of the deconfined phase (e.g. is color and electrically neutral), and should be the actual state (for $\tau \ll \tau_{\mathrm{weak}}$) if it has the lowest free energy per baryon. Energy must be paid in order to equal at least two Fermi seas, but in compensation the pairing energy is recovered. The gained energy depends on the value of the pairing gap Δ and, at least for sufficiently large Δ, is expected to be larger than the energy invested to force a pairing pattern. Also, notice that color conversion of quarks allows the adjustment of the Fermi seas within a given flavor in a very short timescale ($\sim \tau_{\mathrm{strong}}$), i.e. several orders of magnitude faster than β-equilibration ($\tau_{\mathrm{strong}} \ll \tau_{\mathrm{weak}}$).

We emphasize that this phase is *not* in flavor equilibrium. After a weak interaction timescale this transitory pairing pattern will be abandoned by the system in favor of the lowest-energy β-stable configuration. Depending on the density, the lowest energy state may be LOFF, gapless 2SC, gapless CFL, standard CFL, (to name just some possibilities) as extensively discussed in the literature.

III. APPLICATION TO SIMPLIFIED EQUATIONS OF STATE

A. Deconfinement of pure neutron matter

A simple solution can be found in the case of the deconfinement of pure neutron matter, since strange quarks and electrons are not present in the hadronic gas. First, we apply the colorless conditions and flavor conservation introduced in the previous section, in order determine the abundances of each quark species. In order to allow pairing of at least two different quarks species in the just-deconfined phase, we impose the condition of Eq. (11), i.e. $n_{ur} = n_{dg}$. Using one of the colorless conditions ($n_r = n_g$) it is found that $n_{dr} = n_{ug}$, implying that these quarks can also pair with a significant gap. From the remaining colorless condition ($n_r = n_b$) it is found $n_{ur} + n_{dr} = n_{ub} + n_{db}$. The condition of flavor conservation states $n_d = 2n_u$; therefore, $n_{dr} + n_{dg} + n_{db} = 2(n_{ur} + n_{ug} + n_{ub})$. Introducing the ratio $x = n_{ug}/n_{ur}$ we find from the above equations:

$$n_{ub} = 0 \tag{12}$$

$$\frac{n_{db}}{n_{ur}} = 1 + x. \tag{13}$$

Therefore, for massless particles at $T = 0$ the chemical potentials are related by:

$$\mu_{ug} = \mu_{dr} = x^{1/3}\mu_{dg} = x^{1/3}\mu_{ur} \tag{14}$$

$$\mu_{ub} = 0 \tag{15}$$

$$\mu_{db} = (1 + x)^{1/3}\mu_{ur}. \tag{16}$$

With this configuration, the pressure P_Δ and the Gibbs free energy per baryon $g_\Delta = (\sum_{fc} n_{fc}\mu_{fc})/n_B$ take the simple form

$$P_\Delta = \frac{[2(1 + x^{4/3}) + (1 + x)^{4/3}]\mu_{ur}^4}{12\pi^2} + \frac{(1 + x^{2/3})\Delta^2\mu_{ur}^2}{2\pi^2} - B, \tag{17}$$

$$g_\Delta = \frac{[2(1 + x^{4/3}) + (1 + x)^{4/3}]\mu_{ur}}{1 + x}. \tag{18}$$

Minimizing $g_\Delta = g_\Delta(P_\Delta, x)$ with respect to x it is found that the minimum correspond to $x = 1$. Therefore, quarks $u_r - d_g$ and $u_g - d_r$ pair in a "2SC-like" pattern like the one shown in Fig. 2.

In order to determine whether the system settles in a paired or in an unpaired configuration we compare the Gibbs free energy per baryon of the above configuration with the Gibbs free energy per baryon of an unpaired quark gas (both evaluated at the same pressure, and with the same flavor composition). For unpaired quark matter the ground state of the colorless mixture (compatible with flavor conservation) is shown in Fig. 2, and is described by $n_{dr} = n_{dg} = n_{db} = 2n_{ur} = 2n_{ug} = 2n_{ub} = \frac{2}{3}n_B$. The Gibbs free energy per baryon reads

$$g_{\rm unp} = [4\pi^2(1 + 2^{4/3})^3(P + B)]^{1/4}. \tag{19}$$

The results are shown in Fig. 2 where the difference $g_\Delta - g_{\rm unp}$ is plotted as a function of pressure for different values of the parameter Δ. For large enough values of Δ the energy of the pairing gap is able to compensate the increase of the free energy resulting from equating the Fermi seas of the particles that pair. It is also worth noting that since we are comparing the Gibbs energy per baryon of different phases at equal pressures (alternatively, we could compare the pressure at fixed Gibbs energy per baryon), the results depend on the bag constant B. This dependence is not found in the comparisons for β-stable quark matter made in [25] because the bag constant B cancels out when comparing ΔF at fixed μ. Note that ΔF is not the relevant thermodynamic function for the analysis of the deconfinement transition in neutron stars performed here.

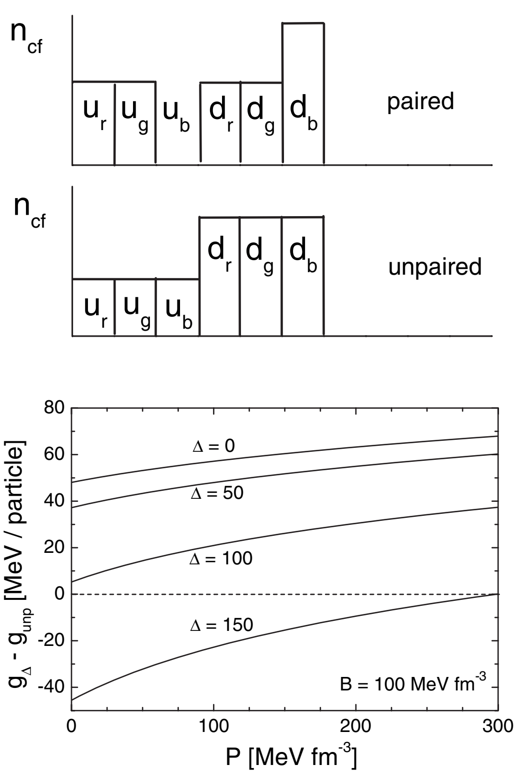

FIG. 2. Deconfinement of pure neutron matter. *Upper panel*: Sketch of the lowest energy configuration of the paired and unpaired phases just after the deconfinement. *Lower panel*: The difference in the Gibbs energy per particle between paired and unpaired quark matter. For positive values of $g_\Delta - g_{\rm unp}$ the preferred phase just after the deconfinement is the unpaired one while for negative values it is the paired one.

B. Deconfinement of n-p-e^- gas

Next we consider the deconfinement of a charge neutral uniform system of neutrons, protons and electrons (n_n, n_p and n_e are the associated number densities). As discussed above, we impose $n_{ur} = n_{dg}$ in order to allow pairing. Then, color charge neutrality imposes $n_{dr} = n_{ug}$ and $n_{ur} + n_{dr} = n_{ub} + n_{db}$. The resulting configuration allows pairing of quarks u_r and d_g with a gap Δ, and of u_g and d_r with a gap that is assumed to have the same value.

On the other hand, we can put the condition of *flavor conservation* in the following simple form

$$n_d = \xi n_u, \tag{20}$$

being $\xi \equiv Y_d^H/Y_u^H$ (c.f. Eq. (8)). In the case of the n-p-e^- gas, the parameter ξ can be expressed in terms of the proton fraction $Y_p = n_p/n_B$ of nuclear matter as $\xi = (2 - Y_p)/(1 + Y_p)$. It is easy to check that $\xi = 2$ corresponds to the deconfinement of pure neutron matter, $\xi = 1$ to symmetric nuclear matter and $\xi = 0.5$ to the unrealistic case of pure proton matter. We emphasize that in the case of β-stable n-p-e^- system, ξ is a function of density (or pressure) that depends only on the state of the *hadronic* matter that deconfines.

Therefore, flavor conservation states that $n_{dr} + n_{dg} + n_{db} = \xi(n_{ur} + n_{ug} + n_{ub})$. In addition, the condition of Eq. (8) applied to electrons yields:

$$3n_e = 2n_u - n_d, \tag{21}$$

which confirms that flavor conservation automatically guarantees electric charge conservation. Finally, we impose that 1) $n_{dr} = n_{ug}$ in order to allow for paring between quarks d_r with u_g, and 2) $n_{ur} = n_{dg}$ in order to allow for paring between quarks u_r with d_g.

Introducing the ratio $x = n_{ug}/n_{ur}$, and using Eqs. (20) and (21) we find the particle number densities of each flavor and color in the paired phase as a function of one of the particle densities (e.g. n_{ur}), the parameter ξ that depends only of the state of the hadronic phase, and the free parameter x:

$$n_{ug} = x n_{ur} \tag{22}$$

$$n_{ub} = (1 + x) \frac{2 - \xi}{1 + \xi} n_{ur} \tag{23}$$

$$n_{dr} = x n_{ur} \tag{24}$$

$$n_{dg} = n_{ur} \tag{25}$$

$$n_{db} = (1 + x) \frac{2\xi - 1}{1 + \xi} n_{ur}. \tag{26}$$

Note that the free parameter x that can be eliminated by minimizing the Gibbs energy per baryon $g_\Delta = (\sum_{fc} n_{fc} \mu_{fc} + \mu_e n_e)/n_B$ with respect to x at constant pressure P_Δ. The minimization gives $x = 1$, which means that the configuration of the paired phase that is energetically preferred is the one having $n_{dr} = n_{ug} = n_{ur} = n_{dg}$, as sketched in the upper panel of Fig. 2. In Fig. 3 we compare the Gibbs energy per baryon of the "2SC-like" paired phase and unpaired matter for different values of the bag constant B and the pairing gap Δ. Comparing with the pure neutron matter case of Fig. 2 it can be noticed that an increase in the proton fraction of the hadronic phase favors the formation of a paired quark phase after deconfinement.

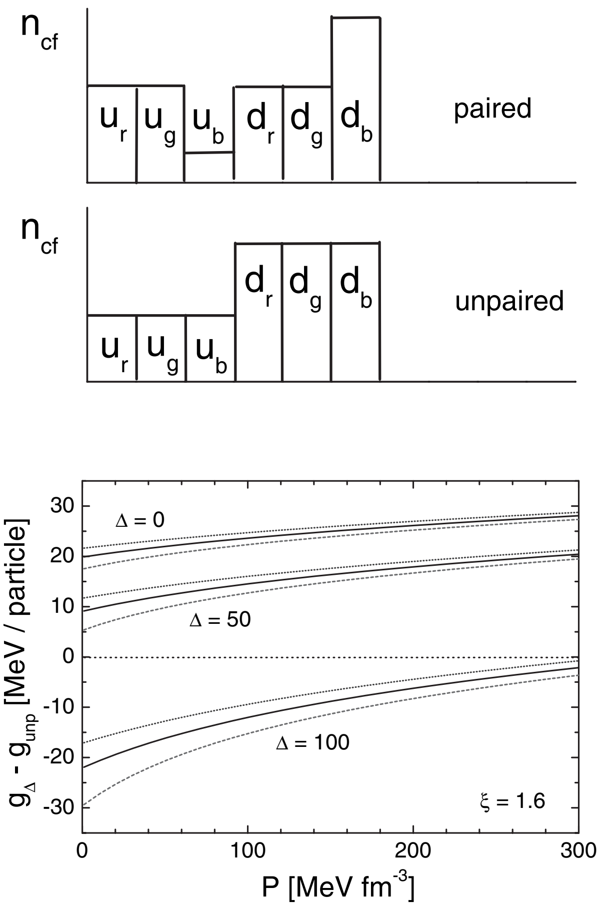

FIG. 3. Deconfinement of n-p-e^- gas. *Upper panel*: Sketch of the lowest energy configuration of the *just-deconfined* paired and unpaired phases . *Lower panel*: The difference in the Gibbs energy per baryon between both phases at $\xi = 1.6$. For positive values of $g_\Delta - g_{\rm unp}$ the preferred phase just after the deconfinement is the unpaired one while for negative values it is the paired one. The results are shown for $B = 60$ MeV fm^{-3} (dashed line), $B = 100$ MeV fm^{-3} (solid line), and $B = 140$ MeV fm^{-3} (dotted line).

IV. DECONFINEMENT OF COLD HADRONIC MATTER

In the following we analyze the deconfinement of a general hadronic system including strange hadrons and then we apply the results to a realistic EOS in order to study deconfinement inside cold neutron stars.

A. Deconfinement of a general hadronic equation of state

(i) *Flavor conservation:* After deconfinement the particle densities of quarks u, d and s are the same as in the hadronic phase and can be determined by Eqs. (8). Another equivalent way of expressing the flavor conservation condition is in terms of two parameters ξ and η:

$$n_d = \xi n_u. \tag{27}$$

$$n_s = \eta n_u. \tag{28}$$

where $\xi \equiv Y_d^H / Y_u^H$ and $\eta \equiv Y_s^H / Y_u^H$ depend only on the composition of the hadronic phase. These expressions are valid for *any* hadronic EOS. For hadronic matter containing n, p, Λ, Σ^+, Σ^0, Σ^-, Ξ^-, and Ξ^0, we have

$$\xi = \frac{n_p + 2n_n + n_\Lambda + n_{\Sigma^0} + 2n_{\Sigma^-} + n_{\Xi^-}}{2n_p + n_n + n_\Lambda + 2n_{\Sigma^+} + n_{\Sigma^0} + n_{\Xi^0}}, \tag{29}$$

$$\eta = \frac{n_\Lambda + n_{\Sigma^+} + n_{\Sigma^0} + n_{\Sigma^-} + 2n_{\Xi^0} + 2n_{\Xi^-}}{2n_p + n_n + n_\Lambda + 2n_{\Sigma^+} + n_{\Sigma^0} + n_{\Xi^0}}. \tag{30}$$

As typical values, we notice that $\eta = 0$ corresponds to zero strangeness, and that at the center of the maximum-mass star (calculated with the hadronic equation of state of Glendenning and Moszkowski GM1 [29]) we have $\xi = 1.15$ and $\eta = 0.85$. Notice that ξ and η determine univocally the number of electrons present in the system through electric charge neutrality of the deconfined phase:

$$3n_e = 2n_u - n_d - n_s. \tag{31}$$

(ii) *Pairing condition:* As made in the previous section for the n-p-e^- gas, we impose that 1) $n_{dr} = n_{ug}$ in order to allow for paring between quarks d_r with u_g, and 2) $n_{ur} = n_{dg}$ in order to allow for paring between quarks u_r with d_g.

(iii) *Color neutrality:* The condition $n_r = n_g$ leads immediately to $n_{sr} = n_{sg}$. Also, $n_r = n_b$ leads to $2n_{ur} + n_{sr} = n_{ub} + n_{db} + n_{sb}$.

The above conditions lead to the pairing pattern schematically shown in the upper panel of Fig. 4. Note that these conditions still leave a degree of freedom that can be fixed by introducing an additional parameter h relating the particle number densities of two arbitrary quark species. Therefore, it is possible to impose the equality of two arbitrary Fermi seas in order to allow pairing between them. We have analyzed the 10 possible combinations and verified that 6 of them lead to a negative value of the particle number density of at least one quark species. The other 4 possibilities allow pairing of particles that do not have different color and flavor, allowing pairing with a negligible gap. For this reason, it is more convenient to introduce $h \equiv n_{sb}/n_{sr}$, and minimize the free energy with respect to h.

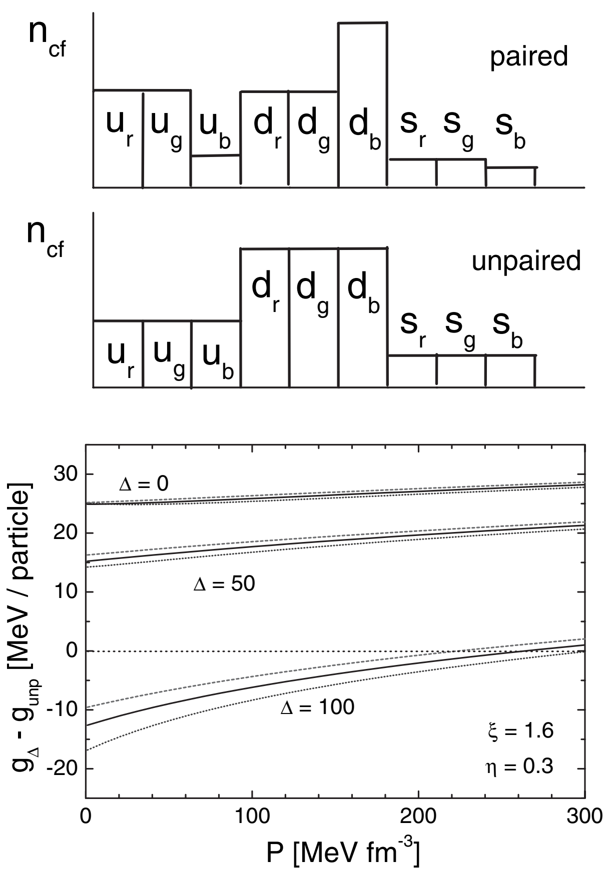

FIG. 4. Deconfinement of β-stable hadronic matter including strange hadrons. *Upper panel:* Sketch of the particle number configuration just after deconfinement. The most general paired configuration compatible with the condition $n_{ur} = n_{dg}$, $n_{ug} = n_{dr}$, flavor conservation, and charge neutrality is the one sketched here. *Lower panel:* The difference in the Gibbs energy per baryon between both phases. For positive values of $g_\Delta - g_{\rm unp}$ the preferred phase just after the deconfinement is the unpaired one while for negative values it is the paired one. The curves correspond to $B = 60$ MeV fm^{-3} (dashed line), $B = 100$ MeV fm^{-3} (solid line), and $B = 140$ MeV fm^{-3} (dotted line). We employed $m_s = 150$ MeV.

Using the above Eqs. we find the following linear set of equations:

$$2n_{ur} + n_{sr} = n_{ub} + n_{db} + n_{sb} \tag{32}$$

$$2n_{ur} + n_{db} = \xi(2n_{ur} + n_{ub}) \tag{33}$$

$$2n_{sr} + hn_{sr} = \eta(2n_{ur} + n_{ub}), \tag{34}$$

from which we obtain the number densities of each quark species in the paired phase as functions of only four quantities:

$$n_{ub} = 2\frac{4 + \eta + 2h - \eta h - 2\xi - h\xi}{2 - \eta + h + \eta h + 2\xi + h\xi} n_{ur} \tag{35}$$

$$n_{db} = 2\frac{-2 + \eta - h - \eta h + 4\xi + 2h\xi}{2 - \eta + h + \eta h + 2\xi + h\xi} n_{ur} \tag{36}$$

$$n_{sb} = \frac{6\eta h}{2 - \eta + h + \eta h + 2\xi + h\xi} n_{ur} \tag{37}$$

$$n_{sr} = \frac{6\eta}{2 - \eta + h + \eta h + 2\xi + h\xi} n_{ur} \tag{38}$$

$$n_e = \frac{2(2 + h)(2 - \eta - \xi)}{2 - \eta + h + \eta h + 2\xi + h\xi} n_{ur}. \tag{39}$$

Remember that the other particle densities are given by $n_{ug} = n_{dr} = n_{dg} = n_{ur}$, $n_{sg} = n_{sr}$, and $n_{sb} = hn_{sr}$.

The pressure and Gibbs energy per baryon of the paired deconfined phase can also be written in terms of the same parameters:

$$P_\Delta = \sum_{fc} \frac{k_{fc}^4}{12\pi^2} + \frac{\mu_e^4}{12\pi^2} + \frac{1}{\pi^2}\bar{\mu}^2\Delta^2 - B, \tag{40}$$

$$g_\Delta = \sum_{fc} \frac{n_{fc}\mu_{fc}}{n_B} + \frac{\mu_e n_e}{n_B}, \tag{41}$$

where $k_{fc} = (\mu_{fc}^2 - m_{fc}^2)^{1/2}$, $\bar{\mu} = \mu_{ur}$, the chemical potentials μ_{fc} are obtained from

$$n_{fc} = \frac{\mu_{fc}^3}{3\pi^2} + \frac{2\Delta^2\bar{\mu}}{\pi^2} \qquad f_c = u_r, u_g, d_r, d_g \tag{42}$$

$$\mu_{fc} = (3\pi^2 n_{fc})^{1/3} \qquad f_c = u_b, d_b \tag{43}$$

$$\mu_{fc} = [(3\pi^2 n_{fc})^{2/3} + m_s^2]^{1/2} \qquad f_c = s_r, s_g, s_b. \tag{44}$$

The minimization of g_Δ with respect to h gives $h = 1$ and therefore the number densities are given by the following equations:

$$n_{ub} = \frac{4 - 2\xi}{1 + \xi} n_{ur} \tag{45}$$

$$n_{db} = \frac{4\xi - 2}{1 + \xi} n_{ur} \tag{46}$$

$$n_{sb} = \frac{2\eta}{1 + \xi} n_{ur} \tag{47}$$

$$n_e = \frac{2(2 - \eta - \xi)}{1 + \xi} n_{ur}. \tag{48}$$

with $n_{ug} = n_{dr} = n_{dg} = n_{ur}$ and $n_{sg} = n_{sr} = n_{sb}$. Eqs. (40)–(48) constitute the equations of state for just-deconfined quark matter. In the lower panel of Fig. 4 we show Δg for particular values of the parameters ($\xi = 1.6$, $\eta = 0.3$).

B. Deconfinement transition in neutron stars

The above conditions allow us to study the regions of the parameter space Δ versus B where the deconfinement transition is possible inside neutron stars (see Figs. 5 and 6). In this analysis we shall employ the equations of state of Glendenning and Moszkowski GM1 for the hadronic phase [29]. Depending on the value of B and Δ there are three possibilities: 1) deconfinement is not possible at the center of the neutron star, 2) deconfinement to an unpaired phase is preferred, and 3) deconfinement to a paired phase is preferred. These regions are limited by the following curves:

(i) A curve along which the three phases have the same state, that is, $g_H(P, \eta, \xi) = g_{\rm unp}(P, \eta, \xi) = g_\Delta(P, \eta, \xi)$ evaluated at the same pressure $P_H = P_{\rm unp} = P_\Delta$ and with the same "quark composition", i.e. $\eta_H = \eta_{\rm unp} = \eta_\Delta$ and $\xi_H = \xi_{\rm unp} = \xi_\Delta$. This gives the solid line in Figs. 5 and 6 separating the gray and white regions.

(ii) A curve along which g, P, ξ and η of quark matter has the same value than at the center of the hadronic neutron star (in Figs. 5 and 6: dashed curve for paired quark matter and solid vertical line for unpaired quark matter). The position of this curve depends on the mass of the neutron star. In Fig. 5 we show the results for the maximum-mass hadronic star within the GM1 EOS (1.8 $M_\odot$), and in Fig. 6 for a neutron star with 1.6 $M_\odot$.

The regions meet at a point of coordinates (B_*, Δ_*) indicated with an asterisk in Figs. 5 and 6. The position of this point (that characterizes rather well the size of the regions) depends on the assumed value of the strange quark mass m_s, and on the mass of the neutron star. In Table I we show the dependence of (B_*, Δ_*) on the assumed value of the strange quark mass m_s for the 1.8 $M_\odot$ neutron star of Fig. 5. We also indicate with a dot the maximum of the gray region (which is the same in Figs. 5 and 6) and give the corresponding value $\Delta_{\rm max}$ in Table I.

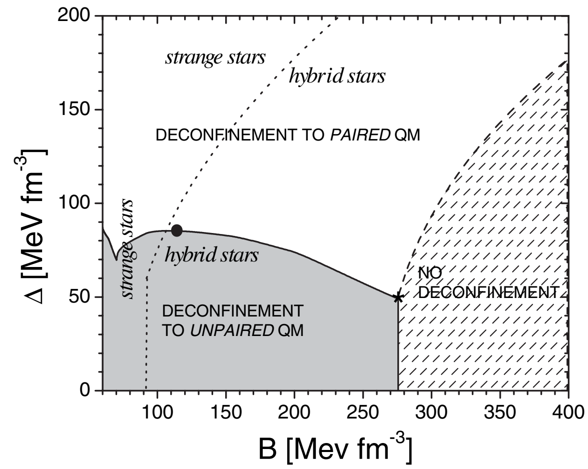

FIG. 5. The parameter space Δ vs B indicating the regions for which deconfinement is possible inside the maximum-mass neutron star with the GM1 EOS [29] ($M_{\rm max} = 1.8 M_\odot$). We also indicate whether the final state reached after β-equilibration of the just-deconfined phase has energy per baryon less or greater than the neutron mass (i.e. leads to the formation of strange stars or hybrid stars, respectively). We adopted $m_s = 150$ MeV for the strange quark mass. If (B, Δ) fall inside the dashed region, deconfinement is not possible even at the center of the maximum-mass star with this EOS. For (B, Δ) inside the gray region the just-deconfined unpaired phase has always less energy per baryon than the just-deconfined paired phase. For (B, Δ) in the white region the just-deconfined phase is always paired quark matter. The regions met at a point of coordinates (B_*, Δ_*) indicated with an asterisk and shown in Table I for different values of the strange quark mass m_s. The maximum of the gray region is indicated with a dot, and the corresponding value $\Delta_{\rm max}$ is shown in Table I for different m_s.

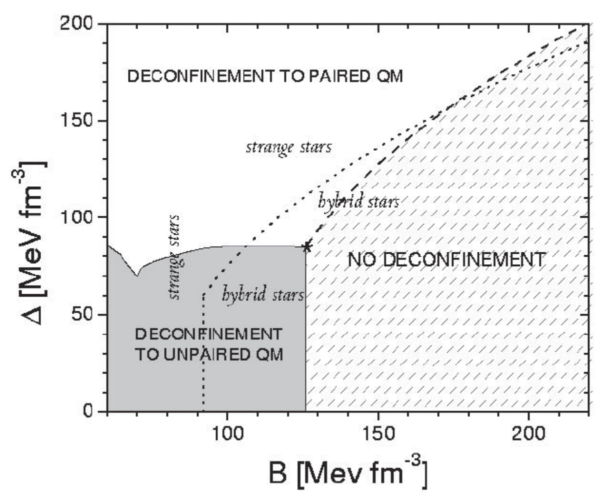

FIG. 6. The same as Fig. 5 but for a 1.6 $M_\odot$ neutron star. The solid curve is the same as in Fig. 5 but the dashed curve is strongly shifted to the left, i.e., the region of the parameter space that allows deconfinement for a 1.6 $M_\odot$ neutron star is much smaller than for a 1.8 $M_\odot$ neutron star. Notice that the larger part of the available parameter space would lead to the formation of strange stars rather than hybrid stars.

TABLE I. We show the coordinates of the point (B_*, Δ_*) (indicated with an asterisk in Fig. 5) for different values of the strange quark mass m_s. We also list the coordinate $\Delta_{\max}$ of the point indicated with a dot in Fig. 5. The qualitative shape of Fig. 5 remains the same in all cases, but the size of each region changes according with the representative points given here.

m_s (MeV)	B_* (MeV fm^{-3})	Δ_* (MeV)	$\Delta_{\max}$ (MeV)
0	323	34	78
100	300	37	80
150	275	49	85
200	241	68	97

We have also included in the parameter space the curve separating the regions in which β-stable quark matter has an energy per baryon smaller than the neutron mass from the region in which $\epsilon/n_B(P=0) > m_n$ (for simplicity, paired β-stable quark matter is assumed in all cases to be CFL). To the left of this curve the final state after β-equilibration is absolutely stable quark matter leading to the formation of strange stars. To the right, β-stable quark matter is restricted to the core of neutron stars (hybrid stars). The position of this curve also depends on the value of m_s. In Figs. 5 and 6 it is shown for $m_s = 150$ MeV (for more details the reader is referred to [30]).

V. DISCUSSION

In this paper we have analyzed the deconfinement transition from hadronic matter to quark matter, and investigated the role of color superconductivity in this process. We have deduced the equations governing deconfinement when quark pairing is allowed and, employing a realistic equation of state for hadronic matter, we have found the regions of the parameter space B versus Δ where the deconfinement transition is possible inside neutron stars. The main results are shown in Figs. 5 and 6 and were explained in the last section. In the following we discuss some implications for neutron star structure.

Stars containing quark phases fall into two main classes: hybrid stars (where quark matter is restricted to the core) and strange stars (made up completely by quark matter). This structural characteristic depends on whether the energy per baryon of β-equilibrated quark matter at zero pressure and zero temperature is less than the neutron mass (the so-called ‘‘absolute stability’’ condition). In the absence of pairing, quark matter in β-equilibrium has an energy per baryon (at $P = 0$) smaller than the neutron mass only if B is in the range $57\ \text{MeV fm}^{-3} \lesssim B \lesssim 90\ \text{MeV fm}^{-3}$. Within this range of B, unpaired β-stable quark matter is the so-called strange quark matter, and it is possible the existence of stars made up entirely by the quark phase. For $B \gtrsim 90\ \text{MeV fm}^{-3}$ unpaired β-stable quark matter at $P = 0$ and $T = 0$ decays into hadrons, and therefore it can be present only in the core of neutron stars. The size of the core (if any) depends on the value of B: the larger the value of B, the smaller the size of the quark matter core (for a given neutron star mass).

Pairing enlarges substantially the region of the parameter space where β-stable quark matter has an energy per baryon smaller than the neutron mass [30,31]. Although the gap effect does not dominate the energetics, being of the order $(\Delta/\mu)^2\sim$ a few percent, the effect is substantially large near the zero-pressure point (which determines the stability and also the properties of the outer layers and surface of the star). As a consequence, a ‘‘CFL strange matter’’ is allowed for the same parameters that would otherwise produce unbound strange matter without pairing [30]. The line separating strange matter from nonabsolutely stable quark matter is shown in dotted line in Figs. 5 and 6, according to [30].

Concerning just-deconfined quark matter (i.e. not in β-equilibrium) it has been already shown that the transition to *unpaired* quark matter is not possible in a 1.6 $M_\odot$ neutron star if the Bag constant is $B \gtrsim 126\ \text{MeV fm}^{-3}$, because the transition pressure is never reached inside the star, even in the proto-neutron star phase [20]. The results when pairing is allowed have been shown in the previous section, where we have shown the ‘‘deconfinement’’ parameter space for the maximum-mass neutron star with the GM1 EOS (1.8 $M_\odot$), and for a 1.6 $M_\odot$ neutron star. As it is evident from Figs. 5 and 6, deconfinement is facilitated for large Δ (i.e. it is possible for a larger range of B). This result can be roughly understood if we think paired matter as unpaired matter with an effective bag constant depending on the chemical potential (or on density): $B_{\text{eff}}(\Delta, \mu) = B - \frac{\mathcal{A}}{\pi^2}\Delta^2\bar{\mu}^2$. The minus sign of the condensation term in the previous expression allows deconfinement for larger values of B. Nevertheless, notice that this simple interpretation is not strictly correct because the chemical equilibrium is different in paired and unpaired phases. In fact, due to the more convenient chemical equilibrium, the unpaired phase is energetically preferred for small Δ.

For a large part of the deconfined parameter space Δ vs. B, β-stable quark matter has (at any pressure) an energy per baryon smaller than the neutron mass. The situation corresponds to the one sketched in Fig. 1(b). Therefore, if quark stars are formed, they would be made up of quark matter from the center up to the surface. In Figs. 5 and 6 this corresponds to the part of the gray and the white regions to the left of the dotted line.

To the right of the dotted line of Figs. 5 and 6 stars containing quark phases are hybrid. The situation corresponds to the one sketched in Fig. 1(c), i.e. the hadronic phase is preferred at low pressures. From the point of view of the structural properties, stable hybrid stars have been found to be possible in a wide region of the parameter space [32]. Nevertheless, notice that these configurations could not form in practice if they fall in the region where deconfinement is forbidden (dashed region of Figs. 5 and 6). That is, for a given hadronic star, there exist a stable hybrid star with the same baryonic mass that has a lower energy (gravitational mass). Nevertheless, the hadronic star cannot deconfine because the here studied non-β-stable *intermediate* state has a larger energy per baryon, as shown schematically in Fig. 1(a). This is the case, for example, if $B = 153$ MeV fm^{-3} ($B^{1/4} =$ 185 MeV) as can be seen by comparing with the results of Fig. 5 of the paper by Alford and Reddy [32]: for $m_s =$ 150 MeV, $B = 153$ MeV fm^{-3} and any reasonable value of Δ, stable hybrid configurations with maximum masses up to 1.6 $M_\odot$ are found in [32]. However, in these cases the pairs (B, Δ) fall comfortably inside the dashed region of Fig. 6 where deconfinement is not allowed. For the same value of B, heavier stars ($\sim 1.8\ M_\odot$) could deconfine, since (B, Δ) would be inside the gray region of Fig. 5, but the resulting configuration would be not structurally stable and would form a black hole (c.f. [32]). Although the EOSs are different in [32] and in the present work, this should not affect this generic trend. Notice that qualitatively similar results have been found in [17,20] for unpaired quark matter.

A stated in the Introduction, the transition from nuclear matter to quark matter proceeds by bubble nucleation. However, notice that for large B the results with typical surface tension $\sigma = 10$–30 MeV fm^{-2} do not differ much from the case in bulk [16,17]. This means that we do not expect that the dashed region of Figs. 5 and 6 will change significantly when including surface effects. Anyway, even if the surface tension were very large, the here presented bulk case is still relevant because it gives a lower limit for the transition: i.e., if deconfinement is not possible in bulk, it will be even more difficult when including surface effects. In other words, the dashed line of Figs. 5 and 6 could move to the left in a more refined study, but not to the right. A complete study of the astrophysical implications is in progress and will be published elsewhere.

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