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Crossing the $w = -1$ barrier in the D3-brane dark energy model

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We explore a possibility for the Universe to cross the $w = -1$ cosmological constant barrier for the dark energy state parameter. We consider the Universe as a slowly decaying D3-brane. The D3-brane dynamics are approximately described by a nonlocal string tachyon interaction and the backreaction of gravity is incorporated in the closed string tachyon dynamics. In a local effective approximation this model contains one phantom component and one usual field with a simple polynomial interaction. To understand cosmological properties of this system we study toy models with the same scalar fields but with modified interactions. These modifications admit polynomial superpotentials. We find restrictions on these interactions under which it is possible to reach $w = -1$ from below at large time. Explicit solutions with the dark energy state parameter crossing or not crossing the barrier $w = -1$ at large time are presented.

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I. INTRODUCTION

The combined analysis of type Ia supernovae, galaxy cluster measurements, and Wilkinson microwave anisotropy probe (WMAP) data provide evidence for accelerated cosmic expansion [1–3]. The cosmological acceleration strongly indicates that the present day Universe is dominated by a smoothly distributed slowly varying dark energy (DE) component. The modern constraints on the DE state parameter are around the cosmological constant value $w = -1 \pm 0.1$ [3–8] and the possibility that w varies in time is not excluded. From the theoretical point of view there are three essentially different cases: $w > -1$ (quintessence), $w = -1$ (cosmological constant), and $w < -1$ (phantom) (see [9–37] and refs. therein).

Since from the observational point of view there is no barrier between these three possibilities it is worth considering models where these three cases are realized. Under general assumptions it is proved in [38] that within single scalar field models one can realize only one possibility: $w \geq -1$ (usual model), or $w \leq -1$ (phantom model). It is interesting that the interaction with the cold dark matter does not change the situation and does not remove the cosmological constant barrier [38,39]. There are several phenomenological models describing the crossing of the cosmological constant barrier [40–57]. Most of them use more then one scalar field or use a nonminimal coupling with gravity, or modified gravity, in particular, via braneworld scenarios. In two-field models one of these two fields is a phantom and the other one is a usual field and the interaction is nonpolynomial in general.

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It is important to find a model which follows from fundamental principles and describes a crossing of the $w = -1$ barrier.

In this paper we show that such a model may appear within a brane approach when the Universe is considered as a slowly decaying D3-brane and a possibility to cross the barrier comes from taking into account a backreaction of the D3-brane. This DE model [36] assumes that our Universe is a slowly decaying D3-brane and its dynamics are described by the open string tachyon mode and the backreaction of this brane is incorporated in the dynamics of the closed string tachyon. The open string tachyon dynamics are described within a level truncated open string field theory (OSFT). The notable feature of this OSFT description of the tachyon dynamics is a nonlocal polynomial interaction [58–68]. It turns out the open string tachyon behavior is effectively described by a scalar field with a negative kinetic term (phantom) [69–73]. However this model does not suffer from quantum instability, which usually phantom models have, since in the nonlocal theory obtained from OSFT there are no ghosts at all near the nonperturbative vacuum [36].

Level truncated cubic OSFT fixes the form of the interaction of local fields to be a cubic polynomial with nonlocal form factors. Integrating out low lying auxiliary fields one gets a 4th degree polynomial [64,65]. Higher order auxiliary fields may change the coefficients in front of lower terms and produce higher degree polynomials. All these corrections are of higher orders of α'.

The second scalar field comes from the closed string sector, similar to [74] and its effective local description is given by an ordinary kinetic term [75] and, generally speaking, a nonpolynomial self-interaction [76]. An exact form of the open-closed tachyon interaction is not known and we consider the simplest polynomial interaction.

Our goal is to understand the following: is it possible in the two-component polynomial model that w crosses the barrier $w = -1$ at large time and reaches -1 from below at infinity? For this purpose we study special polynomial two-component models. For these models there exist the third order odd superpotentials. The existence of a superpotential puts restrictions on the form of the potential. For polynomial potentials these restrictions give relations among coefficients. In this polynomial case we can estimate the behavior of DE state parameter at large times. We expect that small variations of the coefficients of the potentials obtained from the given superpotential do not change qualitatively the behavior of the system.

The superpotentials under consideration produce potentials which are rather close to the form of the open-closed tachyon potential for a non-BPS brane. Indeed, within the level truncated string field theory description of a non-BPS D3-brane decay both fields have tachyon mass terms and the interaction is polynomial, the 4th order at the lowest levels. A natural deformation of this form of the open-closed string tachyon potential is given by extra 6th order terms.

Corresponding local models in the flat background admit exact solutions. An exact solution of an effective local model describing the pure open sector of a non-BPS D3-brane is given by the kink solution [69,70] and the closed tachyon dynamics under reasonable assumptions is given by a lump solution [75,77]. In a nonflat background there is a deformation of the effective local model describing the pure open sector of a non-BPS D3-brane such that the corresponding Friedmann equations have exact solutions [37]. A more straightforward generalization of the model [37] to the case of two fields gives a model with a kink-lump solution. This solution at late times has a behavior as a quintessence model, i.e. w goes to -1 from the above.

We also construct an exactly solvable stringy DE model with the state parameter which crosses the cosmological constant barrier $w = -1$ at a rather late time from above, reaches its local minimal value that is less than -1, and approaches -1 from below at infinite time. The form of the potential in this case is rather complicated and we cannot construct it from the string field theory yet. The Hubble parameter in this model is a nonmonotonic function of time as is the DE state parameter w.

II. THE MODEL

We consider a model of Einstein gravity interacting with a single phantom scalar field ϕ and one standard scalar field ξ in the spatially flat Friedmann universe. Since these scalar fields are assumed to come from string field theory the string mass M_s and a dimensionless open string coupling constant g_o emerges. In typical cases phantom represents the open string tachyon and the usual scalar field the closed string tachyon [36,75,77]. The action is

$$S = \int d^4x\sqrt{-g}\Big(\frac{M_P^2}{2M_s^2}R + \frac{1}{g_o^2}\Big(+\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\xi\partial_\nu\xi - V(\phi,\xi)\Big)\Big), \tag{1}$$

where M_P is the reduced Planck mass, $g_{\mu\nu}$ is a spatially flat Friedmann metric

$$ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2),$$

and the coordinates (t, x_i) and fields ϕ and ξ are dimensionless. Hereafter we use the dimensionless parameter m_p for short:

$$m_p^2 = \frac{g_o^2 M_P^2}{M_s^2}. \tag{2}$$

If the scalar fields depend only on time then the equations of motion are as follows

$$3H^2 = \frac{1}{m_p^2}\Big(-\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\xi}^2 + V\Big), \tag{3a}$$

$$2\dot{H} = \frac{1}{m_p^2}(\dot{\phi}^2 - \dot{\xi}^2), \tag{3b}$$

$$\ddot{\phi} + 3H\dot{\phi} = \frac{\partial V}{\partial\phi}, \tag{3c}$$

$$\ddot{\xi} + 3H\dot{\xi} = -\frac{\partial V}{\partial\xi}. \tag{3d}$$

Here dot denotes the time derivative and $H \equiv \dot{a}(t)/a(t)$.

The form of the potential is assumed to be given from string field theory within the level truncation scheme. Usually for a finite order truncation the potential is a polynomial and its particular form depends on the string type.

In the present analysis we impose the following restriction on the potential:

(a) potential admits an existence of a polynomial superpotential (see details in [78] and in the next section)
(b) potential is even
(c) $\phi(t)$ has nonzero asymptotics and $\xi(t)$ has zero asymptotics as $t \rightarrow \infty$
(d) potential is not more than 6th degree
(e) coefficient in front of 5th and 6th powers are of order $1/m_p^2$ and the limit $m_p^2 \rightarrow \infty$ gives a nontrivial 4th degree potential.

Particular exact solutions will be found by using more specific ansatzes. We will see that for the solution to be constructed in Section IV the form of the potentials in the limit $m_p^2 \rightarrow \infty$ reproduces the one given by an approximation of the lowest level truncated string field theory.

III. $w = -1$ BARRIER FOR TWO-COMPONENT MODEL WITH POLYNOMIAL SUPERPOTENTIAL

A. Setup

We can assume that $H(t)$ is a function (named a superpotential, see for example [78]) of $\phi(t)$ and $\xi(t)$:

$$H(t) = W(\phi(t), \xi(t)).$$

This allows us to rewrite (3b) as

$$\frac{\partial W}{\partial \phi}\dot{\phi} + \frac{\partial W}{\partial \xi}\dot{\xi} = \frac{1}{2m_p^2}(\dot{\phi}^2 - \dot{\xi}^2). \tag{4}$$

System (3) is certainly solved provided the relations

$$\frac{\partial W}{\partial \phi} = \frac{1}{2m_p^2}\dot{\phi}, \qquad \frac{\partial W}{\partial \xi} = -\frac{1}{2m_p^2}\dot{\xi} \tag{5}$$

are satisfied. If this is the case we have the following relation between the potential V and the superpotential W

$$V = 3m_p^2 W^2 + 2m_p^4\left(\left(\frac{\partial W}{\partial \phi}\right)^2 - \left(\frac{\partial W}{\partial \xi}\right)^2\right). \tag{6}$$

This relation gives the potential in terms of W and its first derivatives with respect to ϕ and ξ. Provided the superpotential is given to find a solution of the dynamical system one has to solve the second order system of ordinary differential Eqs. (5).

B. Construction of the potential

In this subsection we construct the potentials that admit a polynomial superpotential. Recall, that we restrict ourself to have 6th degree even polynomial potential. Then general substitutions for $\dot{\phi}(t)$ and $\dot{\xi}(t)$ are as follows

$$\dot{\phi} = \sum_{m,n=0,1,2} p_{mn}\xi^m\phi^n, \qquad \dot{\xi} = \sum_{m,n=0,1,2} x_{mn}\xi^m\phi^n. \tag{7}$$

Equivalence of second mixed derivatives of W implies

$$x_{12} = -p_{21}, \qquad x_{11} = -2p_{20}, \qquad x_{01} = -p_{10},$$

$$p_{11} = -2x_{02}, \qquad x_{21} = p_{12} = x_{22} = p_{22} = 0.$$

For the potential to be even we have to set constants p_{01}, p_{10}, x_{01}, x_{10} to zero and an integration constant in W should be zero as well. Also in order to have the 6th degree of the interaction potential for ϕ and ξ we have to put $p_{21} = 0$ and $x_{12} = 0$. Substituting expressions (7) into (5) and integrating we get

$$W = \frac{1}{2m_p^2}\left(p_{00}\phi + \frac{1}{3}p_{02}\phi^3 - x_{00}\xi - \frac{1}{3}x_{20}\xi^3 + p_{20}\phi\xi^2 - x_{02}\phi^2\xi\right). \tag{8}$$

One can obtain the potential V from relation (6). However, we postpone this to the next subsection when the asymptotic late time behavior will be specified.

Note that in the case of a single field the superpotential W defines this scalar field, as a solution of the first order differential equation, which always can be trivially solved in quadratures [78] and there is no difference to start with the explicit form of the (phantom) scalar field as a function of time or with the corresponding form of the superpotential. In the case of two fields the superpotential method gives the second order system of differential equations, which may be nonintegrable. In this case it is more preferable to start from the form of the superpotential, which corresponds to the required form of the potential. In Section V we demonstrate that the usual and the phantom scalar fields can have very unusual dependence on time, which cannot be predicted from the consideration of models with single field and a polynomial potential.

C. Time evolution

Differential Eqs. (7) when all relations among p_{mn} and x_{mn} constants are taken into account read as follows

$$\begin{aligned}\dot{\phi}(t) &= p_{00} + p_{02}\phi^2(t) - 2x_{02}\phi(t)\xi(t) + p_{20}\xi^2(t),\\ \dot{\xi}(t) &= x_{00} + x_{02}\phi^2(t) - 2p_{20}\phi(t)\xi(t) + x_{20}\xi^2(t).\end{aligned} \tag{9}$$

To specify the boundary conditions let us recall that we have in mind the following picture. We assume that the phantom field ϕ smoothly rolls from the unstable perturbative vacuum ($\phi = 0$) to a nonperturbative one, say, $\phi = a$ and stops there. The field ξ is expected to asymptotically go to zero in the infinite future. This asymptotic behavior implies $p_{00} = -p_{02}a^2$ and $x_{00} = -x_{02}a^2$ and we have the following system left:

$$\begin{aligned}\dot{\phi}(t) &= p_{02}\phi^2(t) - 2x_{02}\phi(t)\xi(t) + p_{20}\xi^2(t) - p_{02}a^2,\\ \dot{\xi}(t) &= x_{02}\phi^2(t) - 2p_{20}\phi(t)\xi(t) + x_{20}\xi^2(t) - x_{02}a^2.\end{aligned} \tag{10}$$

The superpotential W can be rewritten in the following form

$$W = \frac{1}{6m_p^2}(p_{02}\phi(\phi^2 - 3a^2) + 3p_{20}\phi\xi^2 + 3x_{02}\xi(a^2 - \phi^2) - x_{20}\xi^3). \tag{11}$$

The corresponding potential V is as follows

$$\begin{aligned}V = {}&\frac{1}{2}(p_{02}(\phi^2 - a^2) - 2x_{02}\phi\xi + p_{20}\xi^2)^2\\ &- \frac{1}{2}(x_{02}(\phi^2 - a^2) - 2p_{20}\phi\xi + x_{20}\xi^2)^2\\ &+ \frac{1}{12m_p^2}(p_{02}\phi(3a^2 - \phi^2) - 3x_{02}\xi(a^2 - \phi^2)\\ &- 3p_{20}\phi\xi^2 + x_{20}\xi^3)^2.\end{aligned} \tag{12}$$

D. Cosmological consequences: late time behavior

From the cosmological point of view we address the following questions to our model. What is the behavior of the Hubble parameter H, how do the state parameter w and the deceleration parameter q evolve?

Even without having a time dependence for the fields ϕ and ξ we can answer some of the above questions provided that we know the asymptotic behavior of the fields. Indeed, we assume the field $\phi(t)$ starts from 0 and goes to a finite asymptotic a and its velocity goes to zero in the infinite future. The field $\xi(t)$ and its velocity $\dot{\xi}(t)$ go to zero in the infinite future. Recall, that the function $H(t)$ is restored once we substitute the time dependence $\phi(t)$ and $\xi(t)$ into (11). As the first result we see that $H(t)$ in the infinite future goes asymptotically to the following value

$$H_\infty = -\frac{a^3 p_{02}}{3m_p^2}. \qquad (13)$$

We immediately see that p_{02} should be negative if a is a positive asymptotic value of the field $\phi(t)$. Also it is evident that $\dot{H}(t)$ goes to zero.

Further one can expand functions $\phi(t)$ and $\xi(t)$ for large times as follows

$$\phi(t) = a + f(t) + \ldots, \qquad \xi(t) = g(t) + \ldots, \qquad (14)$$

where $f(t) \ll a$, $g(t) \ll a$, and the ratio $f(t)/g(t)$ is finite. Assuming such an expansion we have the following asymptotic behavior of the Hubble parameter

$$H_{\rm as} = \frac{a}{2m_p^2}(p_{02}f^2(t) - 2x_{02}f(t)g(t) + p_{20}g^2(t)) - \frac{a^3 p_{02}}{3m_p^2}.$$

The eigenvalues of the quadratic form in f and g:

$$\lambda_{H,1} = \frac{1}{2}(p_{20} + p_{02} + \sqrt{(p_{20} - p_{02})^2 + 4x_{02}}),$$
$$\lambda_{H,2} = \frac{1}{2}(p_{20} + p_{02} - \sqrt{(p_{20} - p_{02})^2 + 4x_{02}})$$

determine whether $H(t)$ comes to its asymptotic value from above ($\lambda_{H,1} > 0$ and $\lambda_{H,2} > 0$) or from below ($\lambda_{H,1} < 0$ and $\lambda_{H,2} < 0$). If $\lambda_{H,1}$ and $\lambda_{H,2}$ have opposite signs we need to use more detailed approximation.

Now we turn to the behavior of the state parameter w and the deceleration parameter q. They are related with the Hubble parameter by the following relations

$$w = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}, \qquad q = -1 - \frac{\dot{H}}{H^2}.$$

Since $H(t)$ in our consideration goes asymptotically to a finite constant and its time derivative vanishes both the state and the deceleration parameters go to -1. The question does w approach -1 from above or from below is very important. The first case is the so-called quintessencelike behavior and the second one is the phantomlike behavior. It is convenient to rewrite the relation for the state parameter using the Eq. (3b) as follows

$$w = -1 - \frac{\dot{\phi}^2(t) - \dot{\xi}^2(t)}{3m_p^2 H^2}.$$

Substituting expressions for the $\dot{\phi}(t)$ and $\dot{\xi}(t)$ from (10) we get

$$w = -1 - \frac{\Delta}{3m_p^2 H^2}, \qquad (15)$$

where

$$\Delta = (p_{02}\phi^2(t) - 2x_{02}\phi(t)\xi(t) + p_{20}\xi^2(t) - p_{02}a^2)^2 - (x_{02}\phi^2(t) - 2p_{20}\phi(t)\xi(t) + x_{20}\xi^2(t) - x_{02}a^2)^2. \qquad (16)$$

We employ again the asymptotic expansion (14) to write

$$\Delta_{\rm as} = 4a^2(p_{02}^2 - x_{02}^2)f^2(t) - 4a^2(p_{20}^2 - x_{02}^2)g^2(t) + 8a^2 x_{02}(p_{20} - p_{02})f(t)g(t). \qquad (17)$$

The quadratic form

$$(p_{02}^2 - x_{02}^2)f^2(t) - (p_{20}^2 - x_{02}^2)g^2(t) + 2x_{02}(p_{20} - p_{02})f(t)g(t)$$

has the following eigenvalues

$$\lambda_{\Delta,1} = \frac{1}{2}(p_{02}^2 - p_{20}^2 + \sqrt{(p_{02}^2 + p_{20}^2)^2 + 4x_{02}^2(x_{02}^2 - 2p_{20}p_{02})}),$$
$$\lambda_{\Delta,2} = \frac{1}{2}(p_{02}^2 - p_{20}^2 - \sqrt{(p_{02}^2 + p_{20}^2)^2 + 4x_{02}^2(x_{02}^2 - 2p_{20}p_{02})}). \qquad (18)$$

Therefore, when both $\lambda_{\Delta,1}$ and $\lambda_{\Delta,2}$ are positive we have a phantomlike behavior, when these λs are both negative we have a quintessencelike behavior. When $\lambda_{\Delta,1}$ and $\lambda_{\Delta,2}$ have opposite signs we may have oscillations at large times near the cosmological constant barrier $w = -1$.

In the next sections we consider two special solutions. The first one corresponds to the quintessence behavior and the second one does so to the phantom behavior. Moreover we will see that for these solutions the state parameter crosses the $w = -1$ barrier. Notice that such a crossing is forbidden in single field models [38].

IV. QUINTESSENCE LATE TIME SOLUTION

A. Ansatz and corresponding potential

We are about to construct a solution to the system (10). The system is essentially simplified if we take

$$x_{02} = x_{20} = 0. \qquad (19)$$

The latter is the ansatz we explore in this section.

Substitution of this ansatz into (10) gives

$$\dot{\phi}(t) = p_{02}(\phi^2 - a^2) + p_{20}\xi^2(t),$$
$$\dot{\xi}(t) = -2p_{20}\phi(t)\xi(t). \tag{20}$$

The superpotential W given by (8) under conditions (19) reads as follows

$$W = \frac{1}{6m_p^2}\phi(-p_{02}(3a^2 - \phi^2) + 3p_{20}\xi^2). \tag{21}$$

The corresponding potential V can be found using relation (6) to be

$$V = \frac{1}{2}(p_{02}(\phi^2 - a^2) + p_{20}\xi^2)^2 - 2p_{20}^2\phi^2\xi^2 + \frac{1}{12m_p^2}\phi^2(p_{02}(3a^2 - \phi^2) - 3p_{20}\xi^2)^2. \tag{22}$$

B. Solution

A solution to the system (20) when the field ϕ starts from 0 and goes asymptotically to a and the field ξ asymptotically vanishes is the following

$$\phi = a\tanh(2ap_{20}t) \tag{23}$$

and

$$\xi = \frac{a\sqrt{2+r}}{\cosh(2ap_{20}t)}. \tag{24}$$

Hereafter in this section we denote $r = p_{02}/p_{20}$.

Let us note that one obtains the same solution (23) and (24) for different potentials. Namely, the solution is not violated if we take a potential of the form

$$V_1 = V + \delta V, \tag{25}$$

where V is the potential given by (22) and δV is such that δV, $\partial(\delta V)/\partial\phi$, and $\partial(\delta V)/\partial\xi$ are zero on the solution. For $\phi(t)$ and $\xi(t)$ given by (23) and (24), respectively, the most general even form of δV with the 6th degree interaction is the following

$$\delta V = A\left[\phi^2 + \frac{1}{2+r}\xi^2 - a^2\right]^2 \times (1 + v_1\phi^2 + v_2\xi^2 + v_3\phi\xi). \tag{26}$$

This example shows that the same functions $\phi(t)$, $\xi(t)$ (and consequently the Hubble parameter $H(t)$, state parameter w, and deceleration parameter $q(t)$) can correspond to different potentials $V(\phi, \xi)$.

C. Cosmological properties

Substituting (23) and (24) into (21) we obtain the Hubble parameter

$$H = \frac{a^3}{3m_p^2}\tanh(2ap_{20}t)\left(\frac{p_{02} + 3p_{20}}{\cosh^2(2ap_{20}t)} - p_{02}\right). \tag{27}$$

The function $H(t)$ is not monotonic for general values of the parameters and has an extremum at the point

$$t_c = \frac{\log\left(\sqrt{\frac{3+r}{2+r}} + \sqrt{\frac{1}{2+r}}\right)}{2ap_{20}}.$$

We are certainly interested in the case when this t_c is real, i.e. the argument of the logarithm should be a positive real value. That means that we have to have $r > -2$. Moreover, if $r > -2$ then the argument of the logarithm is greater than 1, and consequently the value of the logarithm is positive. Further we recall that in order to have a positive asymptotic for the $H(t)$ we require $p_{02} < 0$. On the other hand expression for t_c implies that $p_{20} > 0$ if we are interested in positive time semiaxis (the situation is symmetric for the negative time semiaxis). Thus, r turns out to be less than 0. Eventually, we state that

$$-2 < r < 0.$$

The Hubble parameter in the extremum is

$$H_c = \frac{2a^3}{3m_p^2}p_{20}\sqrt{\frac{1}{3+r}}.$$

Recall that at large times the Hubble constant goes to

$$H_\infty = -\frac{a^3}{3m_p^2}p_{02}$$

and the ratio H_c/H_∞ is as follows

$$\frac{H_c}{H_\infty} = -\frac{2}{r}\sqrt{\frac{1}{3+r}} \tag{29}$$

and is determined by the ratio r of parameters p_{02} and p_{20}. It is a matter of a simple algebra to check that for $r > -2$ the ratio (29) is greater than 1. This means that for the specified domain of r the point t_c corresponds to a maximum. The typical plots corresponding to the performed analysis are shown in Fig. 1. In the case $r < -2$ the t_c becomes imaginary and the function $H(t)$ turns out to be monotonic. This situation is close to the single field model. The case of $r > 0$ is implausible because $H(t)$ changes the sign during the evolution and has a negative asymptotic.

The state parameter w is given by the following expression

$$w = -1 + \frac{4p_{20}m_p^2\cosh^4(2ap_{20}t)}{a^2(p_{02}(\cosh^2(2ap_{20}t) - 1) - 3p_{20})} \times \left(\frac{1 - 3\tanh^2(2ap_{20}t)}{\cosh^2(2ap_{20}t) - 1} + \frac{2p_{02}}{p_{02}(\cosh^2(2ap_{20}t) - 1) - 3p_{20}}\right). \tag{30}$$

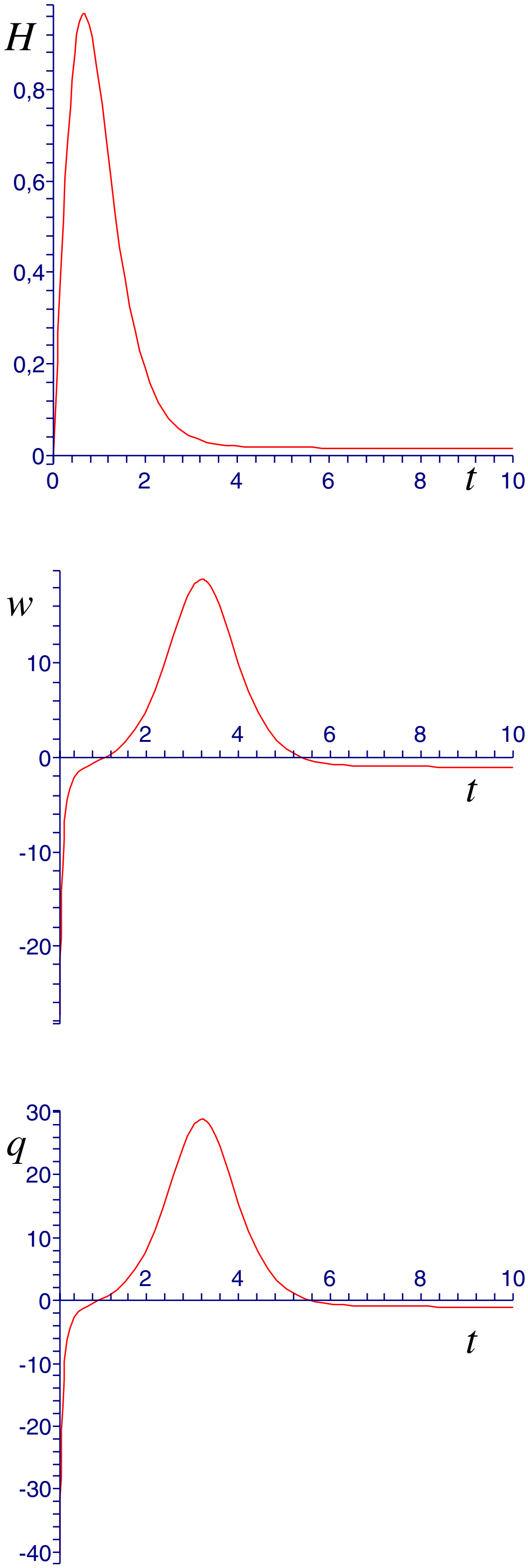

FIG. 1 (color online). The Hubble parameter $H(t)$, the total w, and the total q for $m_p^2 = 0.2$, $a = 1$, $p_{20} = 1/2$, and $p_{02} = -0.01$.

It has a singularity in the origin and behaves as $-1/t^2$. At the point t_c the state parameter w crosses -1 because at this point $\dot{H}(t) = 0$. After t_c for particular parameters (see Fig. 1) there appears a period of deceleration ($q > 0$), however, at late times the Universe returns to the acceleration, w and q for this solution approach -1 from the above. The latter is evident from the expression for Δ (16)

$$\Delta = -4p_{20}^2 a^4 \left(\frac{2 + r}{\cosh^2(2ap_{20}t)} - \frac{3 + r}{\cosh^4(2ap_{20}t)} \right).$$

For the large time the first term in the parentheses dominates over the second one and since in our case it is assumed that $r > -2$ we obtain that $\Delta < 0$, w goes to -1 from the above, and the solutions have the quintessencelike behavior.

D. Connection to superstring field theory (SSFT)

The potential (22) contains mass terms for the fields ϕ and ξ. Their masses are given as follows

$$m_\phi^2 = p_{02}^2 a^2 \left(\frac{3a^2}{2m_p^2} - 2 \right), \qquad m_\xi^2 = -2p_{02}p_{20}a^2. \quad (31)$$

However we have obtained previously the following restrictions: p_{02} should be negative, p_{20} should be positive once the asymptotic a is chosen to be positive in order to have a suitable cosmological behavior. Also it follows from (29) that the ratio r should be small if we want to observe a large ratio of the maximal value of the Hubble parameter and its asymptotic value. These restrictions imply that both m_ϕ and m_ξ are small, the field ϕ is a tachyon in the limit of the large reduced Planck mass, and the field ξ has a positive mass squared.

The situation drastically changes once we make use of the freedom (26). For simplicity we can choose $v_1 = v_2 = v_3 = 0$. In this case the new potential will give new masses for the fields. Indeed,

$$\begin{aligned} M_\phi^2 &= p_{02}^2 a^2 \left(\frac{3a^2}{2m_p^2} - 2 \right) - 4Aa^2, \\ M_\xi^2 &= -2p_{02}p_{20}a^2 - \frac{4Aa^2}{2 + r}. \end{aligned} \quad (32)$$

Provided p_{02}/A is small we effectively change the characteristic of the fields because now if $A > 0$ they are both tachyons. Moreover, in the limit $r \to 0$ the ratio of the masses M_ξ^2/M_ϕ^2 goes to 2 as it should be if we consider ϕ as the open string tachyon and ξ as the closed string tachyon.

The trajectories of the fields ϕ and ξ are presented in Fig. 2.

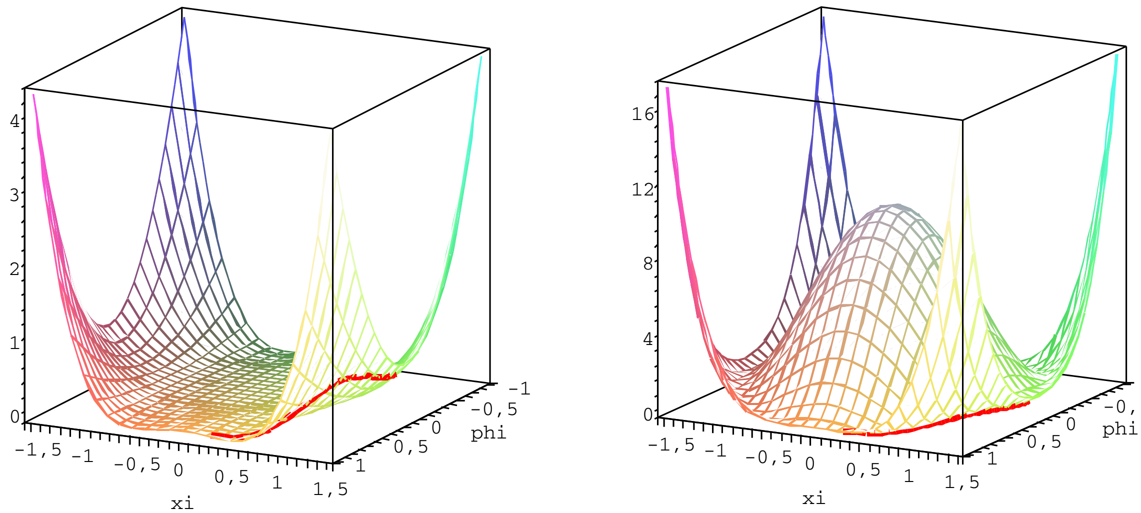

FIG. 2 (color online). Potential (22) for $m_p^2 = 0.2$, $a = 1$, $p_{20} = 1/2$, and $p_{02} = -0.01$ (left) and potential (25) for $\rho = \sigma = \tau = 0$, $A\alpha = 10$, and all other parameters the same as in the left plot (right).

V. PHANTOM LATE TIME SOLUTION

A. Ansatz and corresponding potential

In this section we construct a more complicated solution to system (10) which exhibits a phantomlike late time behavior for particular values of the parameters. Let us assume the following relations among the coefficients of system (10)

$$p_{02} + x_{02} = -(x_{02} + p_{20}) = p_{20} + x_{20} = -c. \qquad (33)$$

Under this assumption we have

$$\begin{aligned}\dot\phi(t) &= p_{02}(\phi^2(t) - a^2) + 2(p_{02} + c)\phi(t)\xi(t) \\ &\quad + (p_{02} + 2c)\xi^2(t), \\ \dot\xi(t) &= (p_{02} + c)(a^2 - \phi^2(t)) - 2(p_{02} + 2c)\phi(t)\xi(t) \\ &\quad - (p_{02} + 3c)\xi^2(t). \qquad (34)\end{aligned}$$

The superpotential under conditions (33) has the form

$$\begin{aligned}W = \frac{1}{6m_p^2}(&-p_{02}\phi(3a^2 - \phi^2) + 3(p_{02} + 2c)\phi\xi^2 \\ &- 3(p_{02} + c)\xi(a^2 - \phi^2) + (p_{02} + 3c)\xi^3) \qquad (35)\end{aligned}$$

and the corresponding potential is

$$\begin{aligned}V = \frac{1}{2}&(p_{02}(\phi^2 - a^2) + 2(p_{02} + c)\phi\xi + (p_{02} + 2c)\xi^2)^2 \\ &- \frac{1}{2}((p_{02} + c)(a^2 - \phi^2) - 2(p_{02} + 2c)\phi\xi \\ &- (p_{02} + 3c)\xi^2)^2 + \frac{1}{12m_p^2}(p_{02}\phi(3a^2 - \phi^2) \\ &+ 3(p_{02} + c)\xi(a^2 - \phi^2) - 3(p_{02} + 2c)\phi\xi^2 \\ &- (p_{02} + 3c)\xi^3)^2. - 3(p_{02} + 2c)\phi\xi^2 \\ &- (p_{02} + 3c)\xi^3)^2.\end{aligned}$$

B. Solution

Combining the equations of system (34) one readily finds

$$\dot\phi(t) + \dot\xi(t) = ca^2 - c(\phi(t) + \xi(t))^2.$$

Therefore

$$\phi(t) + \xi(t) = a\tanh(ac(t + t_0)). \qquad (36)$$

Substituting $\xi(t) = a\tanh(ac(t + t_0)) - \phi(t)$ into (34) one finds

$$\phi = a\tanh(ac(t + t_0)) - \frac{a^2(p_{02} + c)t - \tilde C}{\cosh^2(ac(t + t_0))} \qquad (37)$$

and

$$\xi = \frac{a^2(p_{02} + c)t - \tilde C}{\cosh^2(ac(t + t_0))}. \qquad (38)$$

The behavior of the solution depends on the particular values of parameters a, c, t_0, and $\tilde C$. We adjust $\tilde C$ in such a way that $\phi(0) = 0$. This gives

$$\tilde C = -\frac{a}{2}\sinh(2act_0).$$

The form of trajectories (37) and (38) for the particular values of the parameters is presented in Fig. 3.

C. Cosmological properties

On solutions (37) and (38) the Hubble parameter has the following form

$$\begin{aligned}H = &-\frac{a^3 p_{02}\sinh(ac(t + t_0))(2\cosh^2(ac(t + t_0)) + 1)}{6m_p^2\cosh^3(ac(t + t_0))} \\ &- \frac{a^4 c(p_{02} + c)t + \sinh(2act_0)}{2m_p^2\cosh^4(ac(t + t_0))}. \qquad (39)\end{aligned}$$

An analysis of this function is rather complicated because we come to transcendent equations once we want to find the extrema. The situation is simplified a bit if we use the Eq. (3b) to express the $\dot{H}(t)$. One can write

$$\dot{H}(t) \sim \dot{\phi}^2(t) - \dot{\xi}^2(t) = (\dot{\phi}(t) - \dot{\xi}(t))(\dot{\phi}(t) + \dot{\xi}(t)). \tag{40}$$

The last multiplier in our solution is equal to

$$\frac{a^2 c}{\cosh^2(ac(t + t_0))}.$$

The latter expression has the same sign as c and becomes 0 only in the infinite future. Note that c should be positive if a is taken to be positive. Otherwise the asymptotic value of $H(t)$ will be negative. Thus, the zeros of the $\dot{H}(t)$ are determined by zeros of the first multiplier $\dot{\phi}(t) - \dot{\xi}(t)$. Also, the sign of the first multiplier can determine the late time behavior. Using the exact dependence of the fields one can write

$$\dot{\phi} - \dot{\xi} = \frac{a^2}{\cosh^2(ac(t + t_0))}(2c(2a(p_{02} + c)t + \sinh(2act_0))\tanh(ac(t + t_0)) - c - 2p_{02}). \tag{41}$$

This expression leads to the following consequences. The late time behavior is governed by the sign of the sum $p_{02} + c$. It should be positive if we expect the phantomlike late

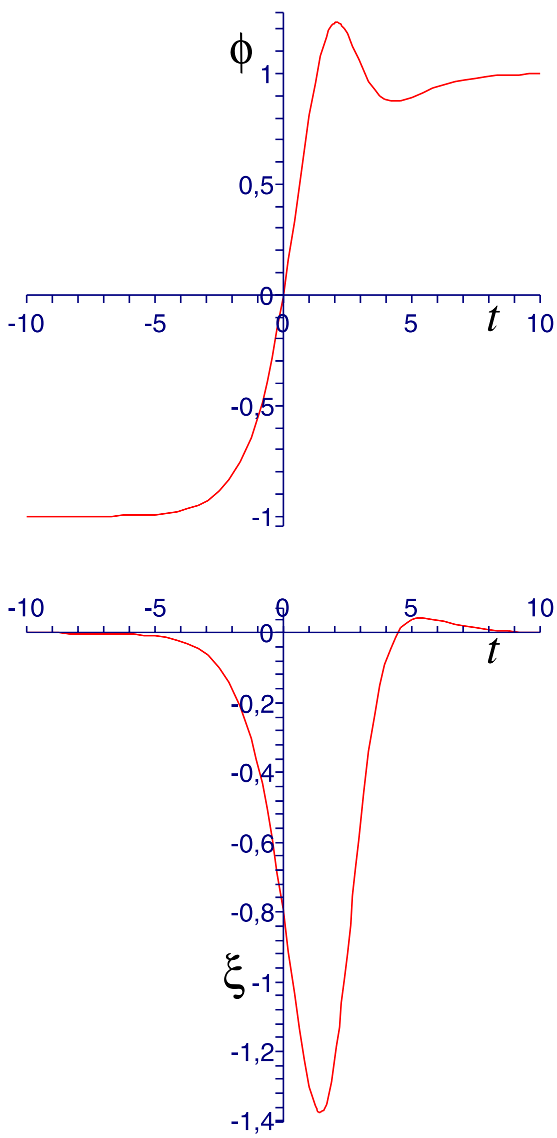

FIG. 3 (color online). $\phi(t)$ and $\xi(t)$ for $a = 1$, $c = 0.55$, $p_{02} = -0.05$, $t_0 = -2$, and $m_p^2 = 0.2$.

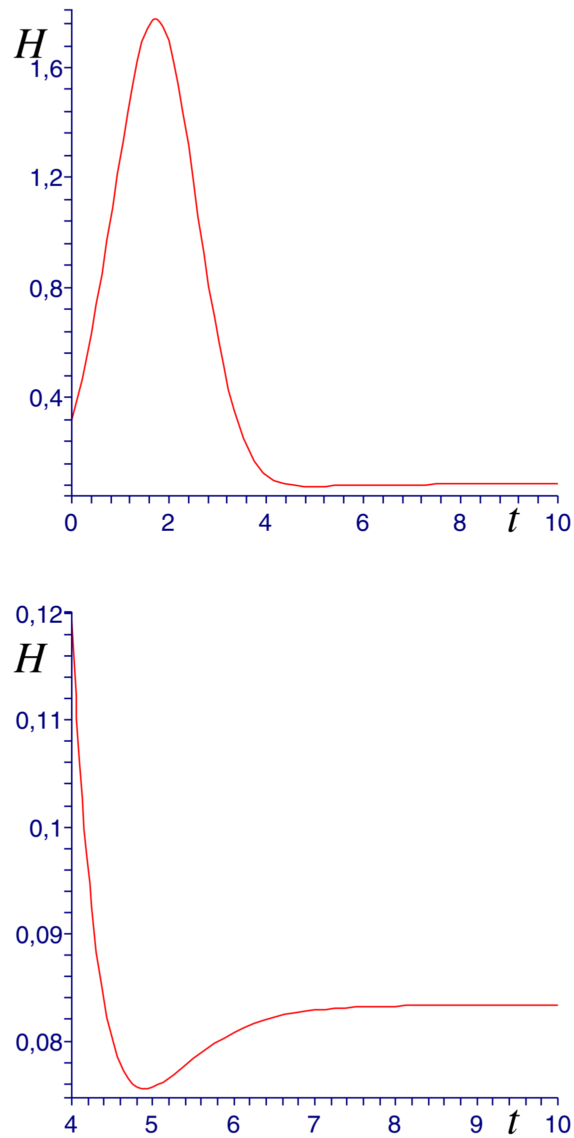

FIG. 4 (color online). The Hubble parameter $H(t)$ (top) and its fine structure (bottom) at $a = 1$, $c = 0.55$, $p_{02} = -0.05$, $t_0 = -2$, and $m_p^2 = 0.2$.

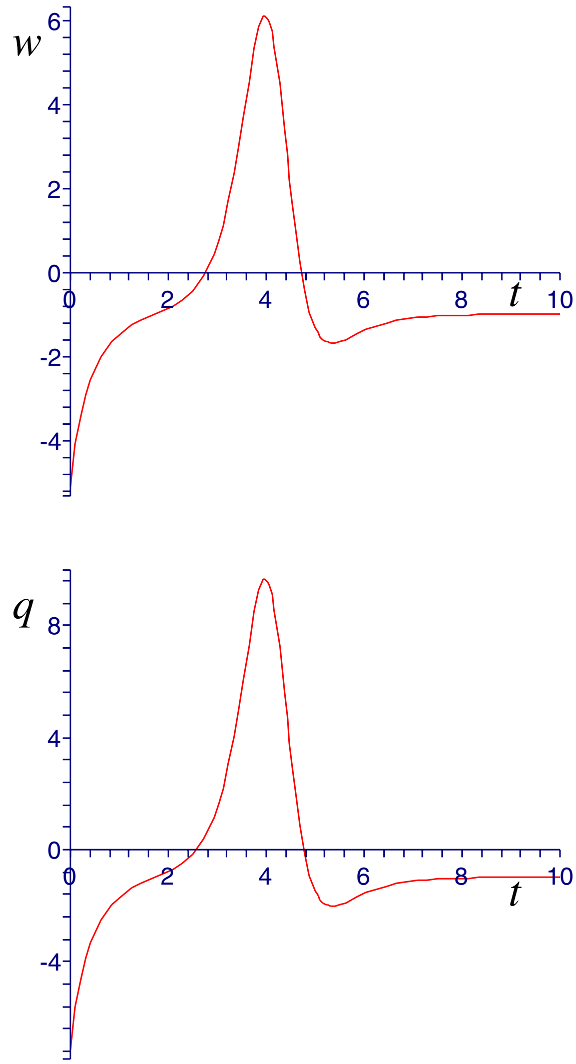

FIG. 5 (color online). The state parameter $w(t)$ and the deceleration parameter $q(t)$ at $a = 1$, $c = 0.55$, $p_{02} = -0.05$, $t_0 = -2$, and $m_p^2 = 0.2$.

time behavior. Also we observe that a natural choice $t_0 = 0$ does not lead to new interesting cosmological properties. Indeed, in this case the expression (41) is governed by the function $t \tanh(act)$ which is monotonic at $t > 0$. Thus, the nominator of (41) has not more than one zero. If so, then we cannot have a large peak for the function $H(t)$ and a phantomlike late time behavior simultaneously (exactly such an evolution is interesting from the cosmological point of view) because to posses these properties $H(t)$ should have a local minimum. To summarize we have to have $a > 0$, $c > 0$, $p_{02} < 0$, and t_0 should be finite if we want to observe new effects and there should be $p_{02} + c > 0$ for the phantomlike late time behavior.

The complete analysis of zeros of the Eq. (41) is very cumbersome. However, it is possible to find particular values of the parameters for which it has two roots. This situation is demonstrated in Figs. 4 and 5.

VI. CONCLUSION AND DISCUSSION

In this paper we have investigated the dynamics of two-component DE models, with one phantom field and one usual field with special polynomial potentials. The main motivation for us was a model of the Universe as a slowly decaying D3-brane whose dynamics are described by a tachyon field [36]. To take into account the backreaction of gravity we consider one more scalar field. This scalar field has a usual kinetic term. The model is close to one discussed in [79] which is also considered in the DE context. The model in [79] is unstable, while stability of our model is ensured by its string origin. Also note that in a closed bosonic string sector an extra phantom appears [80].

Within two-component DE models with a general class of interactions which correspond to polynomial superpotentials we have found conditions that show whether the model is a phantomlike (w goes to -1 from below), or it is a quintessencelike (w goes to -1 from above). In particular, for the simplest model inspired by a D3-brane we have found that an inclusion of the closed string tachyon drastically changes the late time regime so for the two-component model we have $w > -1$ at large time, while in the open string case one has $w < -1$.

The two-component model considered in this paper is interesting also for the following reason. Several attempts to unify early time inflation with late time accelerated Universe (see for example [81–84] and refs. therein) have been performed. Generally speaking it is rather difficult to do this mainly because the ratio of the Hubble parameters in the end of the inflation and during the period of late acceleration should be very large. In our case we have such a possibility just by taking r close to 0 in formula (29).

Let us recall that two scalar fields, both with usual kinetic terms, have been used in hybrid inflation [85], and, in particular, in [86,87] the superpotential has a simple quadratic form.

We have also found superpotentials depending on two components for which we have $w < -1$ at late times. We have presented the explicit solution implementing this possibility. It would be very interesting to study small deformations of the corresponding potential and to clarify if the constructed solution is stable or not under deformations of the form of the potentials and after including the cold dark matter.

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