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# Mesons and diquarks in the color neutral superconducting phase of dense cold quark matter

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The spectrum of meson and diquark excitations of dense color neutral cold quark matter is investigated in the framework of a two-flavored Nambu–Jona-Lasinio–type model, including a quark $\mu$- and color $\mu_8$-chemical potential. It was found that, in the color superconducting (2SC) phase, i.e. at $\mu > \mu_c = 342$ MeV, $\mu_8$ acquires rather small values $\sim$10 MeV in order to ensure the color neutrality. In this phase the $\pi$ and $\sigma$ meson masses are evaluated around $\sim$330 MeV. The spectrum of scalar diquarks in the color neutral 2SC phase consists of a heavy [$SU_c(2)$-singlet] resonance with mass $\sim$1100 MeV, four light diquarks with mass $3|\mu_8|$, and one Nambu-Goldstone boson, which is in accordance with the Goldstone theorem. Moreover, in the 2SC phase there are five light stable particles as well as a heavy resonance in the spectrum of pseudoscalar diquarks. In the color symmetric phase, i.e. for $\mu < \mu_c$, a mass splitting of scalar diquarks and antidiquarks is shown to arise if $\mu \neq 0$, contrary to the case of $\mu = 0$, where the masses of scalar antidiquarks and diquarks are degenerate at the value $\sim$700 MeV. If the coupling strength in the pseudoscalar diquark channel is the same as in the scalar diquark one (as for QCD-inspired Nambu–Jona-Lasinio models), then in the color symmetric phase pseudoscalar diquarks are not allowed to exist as stable particles.

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## I. INTRODUCTION

Recent investigations, performed in the framework of perturbative QCD, show that, at low temperatures and asymptotically high values of the quark chemical potential $\mu$, the dense baryonic matter is a color superconductor [1]. Evidently, at rather small values of $\mu$, a more adequate description of this phenomenon can be done with the help of different effective models, such as Nambu–Jona-Lasinio (NJL)-type field theories with four-fermionic interaction [2], etc. In such a way, on the basis of NJL-type models with two quark flavors, it was shown that the color superconducting (2SC) phase might be yet present at rather small values of $\mu \sim 350$ MeV, i.e. at baryon densities only several times larger than the density of ordinary nuclear matter (see reviews [3,4]). (For simplicity, throughout the paper we will study quark matter, composed from two quark flavors, i.e. up and down quarks, only.) This is just the density of compact star cores. So color superconductivity, which is eventually existing inside compact stars, might influence different observable astrophysical processes and, thus, deserves to be studied in more detail.

In the early NJL approach to color superconductivity [5,6], the density of the color charge $Q_8$ was not a constraint to zero in the 2SC ground state (the densities of other color charges $Q_i$, where $i = 1, \dots, 7$, are zeros in this phase [7]), leading to a nonvanishing difference between the densities of red/green quarks and blue ones. In this case, the mesonic and diquark excitations of dense quark matter were considered in the framework of a simple two-flavored NJL model with a single quark chemical potential $\mu$ [8,9]. In particular, it was shown that in the color asymmetric 2SC phase of this model does arise an abnormal number of three Nambu-Goldstone (NG) bosons instead of the expected five,[1] and $\pi$ mesons are stable excitations of its ground state with masses $\sim$300 MeV. Besides, there are two light stable scalar diquark modes, whose masses are proportional to $\langle Q_8 \rangle$, as well as one heavy scalar diquark resonance in the 2SC phase.

In reality, however, i.e. possibly in compact star cores or in relativistic heavy-ion experiments, there are several physical constraints on the quark matter. The most evident one is its color neutrality, which means a vanishing of a bulk $Q_i$ color charges ($i = 1, \dots, 8$). Indeed, since the lump of quark matter, which might be created after heavy-ion collisions, originated from color neutral objects

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[1] Recall that, in the 2SC phase of the two-flavored NJL model, the initial $SU_c(3)$ color symmetry is spontaneously broken down to the $SU_c(2)$ one, so one might expect naively five massless bosons. However, due to Lorentz noninvariance of the system, there are indeed only three NG bosons.

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and is surrounded by a color neutral medium, it is expected to be globally color neutral. To fulfill this requirement, usually the local color neutrality constraint is imposed by introducing several new chemical potentials, $\mu_3$, $\mu_8$, etc., into a NJL model [4,10,11]. Otherwise, there will be produced a chromoelectric field resulting in the flow of color charges, so a homogeneous and color conducting quark medium with nonzero color charge densities is not allowed to be a stable one [10]. (Recall that there is no need to add new chemical potentials into QCD. The point is that in the QCD 2SC ground state a nonzero value of the eighth gluon field component might be generated. Effectively, it is the $\mu_8$-chemical potential, so color neutrality is fulfilled automatically in the QCD approach [12].)

In the present paper, in contrast to our previous investigations [8,9], we study the mesonic and diquark excitations of color neutral quark matter[2] in the framework of a simple two-flavor NJL model at zero temperature. We consider both the case of rather small values of the quark chemical potential $\mu$ [the $SU_c(3)$ color symmetric (normal) phase] and the case of $\mu$ values, corresponding to the 2SC phase of the model. In addition, the properties of pseudoscalar diquarks are also included into the consideration.

The paper is organized as follows. In Sec. II, the thermodynamic potential as well as the effective action of the NJL model, extended by an additional color $\mu_8$-chemical potential term, is obtained in the one-quark loop approximation. Further, in Sec. III, the gap equations and the phase structure of the model are investigated under the color neutrality constraint. Here the values of $\mu_8$ are obtained at which the 2SC phase is a color neutral one. Then, in Secs. IV, V, and VI, the masses of the $\pi$ and $\sigma$ mesons, scalar diquarks, and pseudoscalar diquarks are considered, respectively. Finally, in the appendix, the influence of the mixing between $\sigma$ meson and scalar diquarks on the $\sigma$ mass is discussed.

## II. THE MODEL AND ITS EFFECTIVE ACTION

In the original version of the NJL model [2], the four-fermionic interaction of a proton $p$ and neutron $n$ doublet was considered, and the principle of dynamical chiral symmetry breaking was demonstrated. Later, the $(p, n)$ doublet was replaced by a doublet of colored up $u$ and down $d$ quarks, in order to describe phenomenologically the physics of light mesons [13–16], diquarks [17,18], as well as meson-baryon interactions [19,20]. In this sense, the NJL model may be thought of as an effective theory for low-energy QCD.[3] (Of course, it is necessary to remember that in the NJL model, in contrast with QCD, quarks are not confined in the hadronic phase, which is a shortcoming of the model.) At the present time, the phenomenon of dynamical (chiral) symmetry breaking is one of the cornerstones of modern physics. So this effect was studied in the framework of NJL-type models under the influence of external magnetic fields [21], in curved space-times [22], in spaces with nontrivial topology [23], etc. Formally, as was mentioned above, quarks are presented in the mass spectrum of the model. So it is very suitable for the description of normal hot and/or dense quark matter [15,24–26] in which, as is known, quarks are not confined. NJL-type models still remain a simple but useful instrument for studying color superconducting quark matter at moderate densities [3–6], where analytical and lattice computations in the framework of QCD are impossible.

We start from the following two-flavor NJL Lagrangian, called for the description of interactions in the quark-antiquark, scalar diquark, as well as pseudoscalar diquark channels at low and moderate energies and baryon densities (the consideration is performed in Minkovski space-time notation):

$$
\begin{aligned}
L_q = {} & \bar{q}[\gamma^\nu i\partial_\nu - m_0 + \mu\gamma^0]q + G\{(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2\} \\
& + \sum_{A=2,5,7}\{H_s[\bar{q}^C i\gamma^5\tau_2\lambda_A q][\bar{q}i\gamma^5\tau_2\lambda_A q^C] \\
& + H_p[\bar{q}^C\tau_2\lambda_A q][\bar{q}\tau_2\lambda_A q^C]\},
\end{aligned} \tag{1}
$$

where $\mu > 0$ is the quark chemical potential, the quark field $q \equiv q_{i\alpha}$ is a flavor doublet and color triplet as well as a four-component Dirac spinor, where $i = 1, 2$ (or $i = u, d$) and $\alpha = 1, 2, 3$ (or $\alpha = r, g, b$). $q^C = C\bar{q}^t$, $\bar{q}^C = q^t C$ are charge-conjugated spinors, and $C = i\gamma^2\gamma^0$ is the charge conjugation matrix ($t$ denotes the transposition operation). It is supposed that up and down quarks have equal current (bare) mass $m_0$. Furthermore, we use the notations $\tau_a$ for Pauli matrices and $\lambda_A$ for antisymmetric Gell-Mann matrices in flavor and color space, respectively. Clearly, the Lagrangian $L_q$ is invariant under transformations from the color $SU_c(3)$ as well as baryon $U_B(1)$ groups. In addition, at $m_0 = 0$ this Lagrangian is symmetric under the chiral $SU(2)_L \times SU(2)_R$ group (chiral transformations act on the flavor indices of quark fields only). Moreover, since $Q = I_3 + B/2$, where $I_3 = \tau_3/2$ is the

[2]Note that, in compact star cores, one should consider the electrically neutral quark matter in beta equilibrium, whereas in heavy-ion experiments the isospin and strangeness are conserved quantities. For simplicity, these additional constraints on the quark matter are ignored in the present consideration.

[3]Indeed, consider two-flavor QCD, symmetric under the color group $SU_c(3)$. By integrating in the generating functional of QCD over gluons and further "approximating" the nonperturbative gluon propagator by a $\delta$ function, one arrives at an effective local chiral four-quark interaction of the NJL type describing low-energy hadron physics. Moreover, by performing a Fierz transformation of the interaction terms, it is possible to obtain a NJL-type Lagrangian describing the interaction of quarks in the scalar and pseudoscalar ($\bar{q}q$) as well as scalar and pseudoscalar diquark ($qq$) channels [see, e.g., the Lagrangian (1) below].

generator of the third isospin component, $Q$ is the generator of the electric charge, and $B$ is the baryon charge generator, our system is symmetric under the electromagnetic group $U_Q(1)$ as well. If the Lagrangian (1) is obtained from the QCD one-gluon exchange approximation, then $G{:}H_s{:}H_p = 4{:}3{:}3$. However, in the present consideration, we will not fix relations between coupling constants, so they are treated as free parameters. It is necessary to note also that at $\mu = 0$ the Lagrangian (1) is invariant under the charge conjugation symmetry ($q \to q^C \equiv C\bar{q}^t$, $\bar{q} \to \bar{q}^C \equiv q^t C$) that is, however, spoiled by the chemical potential term.

Furthermore, the temperature is chosen to be zero throughout the paper. In this case, there is a critical value $\tilde{\mu}_c$ of the chemical potential, such that at $\mu < \tilde{\mu}_c$ the color symmetric (normal) phase is realized in the system (evidently, the ground state of this phase is a color singlet). However, at $\mu > \tilde{\mu}_c$ one obtains a 2SC phase in which $SU_c(3)$ is spontaneously broken down to $SU_c(2)$. Only two quark colors, say red and green, participate in the gap formation in the 2SC phase; the blue quarks stay gapless. So the densities of red/green quarks, $n_{r,g}$, are equal in this phase; however, the density $n_b$ of blue quarks is not equal to $n_{r,g}$; i.e. local color neutrality is broken.[4] To restore local color neutrality in the 2SC phase of the model (1), usually an additional chemical potential term $\mu_8 Q_8$ is introduced into the considerations [4], where $Q_8 = \bar{q}\gamma^0 T_8 q$, and $T_8 = \mathrm{diag}(1, 1, -2) = \sqrt{3}\lambda_8$ is the matrix in the color space. Hence,

$$L_q \to L = L_q + \mu_8 Q_8. \tag{2}$$

If $\mu_8$ is an independent model parameter, then Lagrangian (2) is symmetric under the color $SU_c(2) \times U_{\lambda_8}(1)$ group. However, if local color neutrality is imposed, then the chemical potential $\mu_8$ is not an independent parameter. Its value must be chosen in such a way that the ground state expectation value $\langle Q_8 \rangle$ is identically equal to zero. Hence, $\mu_8$ depends on $\mu$, etc. For example, in the color symmetric phase, i.e. at $\mu < \mu_c$ (it will be shown in Sec. III that in the general case $\mu_c \neq \tilde{\mu}_c$), where $\langle Q_8 \rangle \equiv 0$ even in the theory (1), we have to put $\mu_8 \equiv 0$. However, $\mu_8$ has a nontrivial $\mu$ dependence at $\mu > \mu_c$ in order to supply the zero value of $\langle Q_8 \rangle$. As a consequence, we see that the color symmetry group of the model (2) depends on $\mu$: at $\mu < \mu_c$ it is $SU_c(3)$, whereas at $\mu > \mu_c$ it is the $SU_c(2) \times U_{\lambda_8}(1)$ subgroup of $SU_c(3)$, which is just the color symmetry group of the term $\mu_8 Q_8$.

In the present paper, we are going to study both the ground state properties of the system with Lagrangian $L$ and its mass spectrum in the quark, meson, and diquark sectors. So we have to obtain the thermodynamic potential $\Omega$ as well as the effective action of the model up to a second order in the bosonic degrees of freedom. To begin with, let us introduce the linearized version of Lagrangian $L$ that contains auxiliary bosonic fields:

$$\begin{aligned}\tilde{L} = {}& \bar{q}[\gamma^\nu i\partial_\nu + \mu\gamma^0 + \mu_8 T_8 - \sigma - m_0 - i\gamma^5\pi_a\tau_a]q \\ & - \frac{1}{4G}[\sigma\sigma + \pi_a\pi_a] - \frac{1}{4H_s}\Delta_A^{s*}\Delta_A^s - \frac{1}{4H_p}\Delta_{A'}^{p*}\Delta_{A'}^p \\ & - \frac{\Delta_A^{s*}}{2}[\bar{q}^C i\gamma^5\tau_2\lambda_A q] - \frac{\Delta_A^s}{2}[\bar{q} i\gamma^5\tau_2\lambda_A q^C] \\ & - \frac{\Delta_{A'}^{p*}}{2}[\bar{q}^C\tau_2\lambda_{A'} q] - \frac{\Delta_{A'}^p}{2}[\bar{q}\tau_2\lambda_{A'} q^C],\end{aligned} \tag{3}$$

where, as well as in the following, the summation over repeated indices $a = 1, 2, 3$ and $A, A' = 2, 5, 7$ is implied. Lagrangians $L$ and $\tilde{L}$ are equivalent on the equations of motion for bosonic fields, from which it follows that

$$\begin{aligned}\sigma(x) &= -2G(\bar{q}q), \\ \Delta_A^s(x) &= -2H_s(\bar{q}^C i\gamma^5\tau_2\lambda_A q), \\ \Delta_A^{s*}(x) &= -2H_s(\bar{q} i\gamma^5\tau_2\lambda_A q^C), \\ \pi_a(x) &= -2G(\bar{q} i\gamma^5\tau_a q), \\ \Delta_{A'}^p(x) &= -2H_p(\bar{q}^C\tau_2\lambda_{A'} q), \\ \Delta_{A'}^{p*}(x) &= -2H_p(\bar{q}\tau_2\lambda_{A'} q^C).\end{aligned} \tag{4}$$

One can easily see from (4) that mesonic fields $\sigma, \pi_a$ are real quantities, i.e. $(\sigma(x))^\dagger = \sigma(x)$, $(\pi_a(x))^\dagger = \pi_a(x)$ (the superscript symbol $\dagger$ denotes the Hermitian conjugation), but all diquark fields $\Delta_A^{s,p}$ are complex ones, so

$$(\Delta_A^s(x))^\dagger = \Delta_A^{s*}(x), \qquad (\Delta_{A'}^p(x))^\dagger = \Delta_{A'}^{p*}(x).$$

Moreover, $\Delta_A^s$ and $\Delta_{A'}^p$ are scalars and pseudoscalars, correspondingly. Clearly, the real $\sigma$ and $\pi_a$ fields are color singlets; all scalar diquarks $\Delta_A^s$ form a color antitriplet $\bar{3}_c$ of the $SU_c(3)$ group. The same is true for pseudoscalar diquarks $\Delta_{A'}^p$ which are also the components of an $\bar{3}_c$-multiplet of the color group. Evidently, in the $SU_c(3)$-color symmetric phase (at $\mu < \mu_c$) all diquark fields must have zero ground state expectation values, i.e. $\langle \Delta_A^s \rangle = 0$ and $\langle \Delta_{A'}^p \rangle = 0$. Otherwise, we have an indication that the ground state of the system is no more an $SU_c(3)$-invariant one.

Lagrangian (3) provides us with a common footing for obtaining both the thermodynamic potential and the mass spectrum for bosonic excitations. Indeed, in the one-fermion loop approximation, the effective action for the boson fields is expressed through the path integral over quark fields:

[4]Thus, $\langle Q_8 \rangle \neq 0$. However, the color charge $Q_3 = \bar{q}\gamma^0 T_3 q$, where $T_3 = \mathrm{diag}(1, -1, 0)$ is the matrix in the color space, vanishes in the 2SC phase. Other color charges $Q_i$ ($i \neq 8$) are also zeros in this phase [7].

$$\exp(iS_{\rm eff}(\sigma, \pi_a, \Delta_A^{s,p}, \Delta_{A'}^{s,p*})) = N' \int [d\bar{q}][dq] \times \exp\left(i \int \tilde{L} d^4x\right),$$

where

$$S_{\rm eff}(\sigma, \pi_a, \Delta_A^{s,p}, \Delta_{A'}^{s,p*}) = -\int d^4x \left[\frac{\sigma^2 + \pi_a^2}{4G} + \frac{\Delta_A^s \Delta_A^{s*}}{4H_s} + \frac{\Delta_{A'}^p \Delta_{A'}^{p*}}{4H_p}\right] + \tilde{S}_{\rm eff}, \tag{5}$$

$N'$ is a normalization constant. The quark contribution term $\tilde{S}_{\rm eff}$ is here given by:

$$\exp(i\tilde{S}_{\rm eff}) = N' \int [d\bar{q}][dq] \exp\left(\frac{i}{2} \int [\bar{q} D^+ q + \bar{q}^C D^- q^C - \bar{q} K q^C - \bar{q}^C K^* q] d^4x\right). \tag{6}$$

In (6) we have used the following notations:

$$\begin{aligned} D^+ &= i\gamma^\nu \partial_\nu - m_0 + \hat{\mu}\gamma^0 - \Sigma, \\ D^- &= i\gamma^\nu \partial_\nu - m_0 - \hat{\mu}\gamma^0 - \Sigma^t, \\ \Sigma &= \sigma + i\gamma^5 \pi_a \tau_a, \\ \Sigma^t &= \sigma + i\gamma^5 \pi_a \tau_a^t, \\ K^* &= (\Delta_A^{p*} + i\Delta_A^{s*}\gamma^5)\tau_2 \lambda_A, \\ K &= (\Delta_A^p + i\Delta_A^s \gamma^5)\tau_2 \lambda_A, \end{aligned} \tag{7}$$

where $D^\pm$ are nontrivial operators in the coordinate, spinor, color, and flavor spaces.[5] In the framework of the Nambu-Gorkov formalism, where quarks are composed into a bispinor $\Psi = \binom{q}{q^C}$, it is possible to integrate in (6) over quark fields and obtain

$$\tilde{S}_{\rm eff}(\sigma, \pi_a, \Delta_A^{s,p}, \Delta_{A'}^{s,p*}) = \frac{1}{2i} {\rm Tr}_{\{{\rm NG}sfcx\}} \ln \begin{pmatrix} D^+, & -K \\ -K^*, & D^- \end{pmatrix} \equiv \frac{1}{2i} {\rm Tr}_{\{{\rm NG}sfcx\}} \ln Z. \tag{8}$$

Besides an evident trace in the two-dimensional Nambu-Gorkov (NG) space, the Tr operation in (8) stands for calculating the trace in spinor ($s$), flavor ($f$), color ($c$), as well as four-dimensional coordinate ($x$) spaces, correspondingly.

Starting from (5)–(8), it is possible to define the thermodynamic potential $\Omega(\sigma, \pi_a, \Delta_A^{s,p}, \Delta_{A'}^{s,p*})$ of the model:

$$S_{\rm eff}|_{\sigma,\pi_a,\Delta_A^{s,p},\Delta_{A'}^{s,p*}={\rm const}} = -\Omega(\sigma, \pi_a, \Delta_A^{s,p}, \Delta_{A'}^{s,p*}) \int d^4x, \tag{9}$$

where, in the spirit of the mean-field approximation, all boson fields are supposed to be $x$ independent. It is well known that ground state expectation values $\langle\sigma(x)\rangle \equiv \sigma^o$, $\langle\pi_a(x)\rangle \equiv \pi_a^o$, $\langle\Delta_A^{s,p}(x)\rangle \equiv \Delta_A^{s,po}$, $\langle\Delta_{A'}^{s,p*}(x)\rangle \equiv \Delta_{A'}^{s,p*o}$ are coordinates of the global minimum point of the thermodynamic potential $\Omega$; i.e. they form a solution of the gap equations (evidently, in our approach all ground state expectation values do not depend on coordinates $x$):

$$\frac{\partial\Omega}{\partial\pi_a} = 0, \qquad \frac{\partial\Omega}{\partial\sigma} = 0, \qquad \frac{\partial\Omega}{\partial\Delta_A^{s,p}} = 0, \qquad \frac{\partial\Omega}{\partial\Delta_{A'}^{s,p*}} = 0. \tag{10}$$

Let us make the following shift of bosonic fields in $S_{\rm eff}$: $\sigma(x) \to \sigma(x) + \sigma^o$, $\pi_a(x) \to \pi_a(x) + \pi_a^o$, $\Delta_A^{s,p*}(x) \to \Delta_A^{s,p*}(x) + \Delta_A^{s,p*o}$, $\Delta_A^{s,p}(x) \to \Delta_A^{s,p}(x) + \Delta_A^{s,po}$, where $\sigma^o$, $\pi_a^o$, $\Delta_A^{s,po}$, $\Delta_A^{s,p*o}$ have no coordinate dependency. In this case, the matrix $Z$ from (8) is transformed in the following way:

$$Z \to \begin{pmatrix} D_o^+, & -K_o \\ -K_o^*, & D_o^- \end{pmatrix} - \begin{pmatrix} \Sigma, & K \\ K^*, & \Sigma^t \end{pmatrix} \equiv S_0^{-1} - \begin{pmatrix} \Sigma, & K \\ K^*, & \Sigma^t \end{pmatrix}, \tag{11}$$

where $S_0$ is the quark propagator matrix in the Nambu-Gorkov representation, and

$$(K_o, K_o^*, D_o^\pm, \Sigma_o, \Sigma_o^t) = (K, K^*, D^\pm, \Sigma, \Sigma^t)|_{\sigma=\sigma^o,\pi_a=\pi_a^o,\ldots}.$$

Then, expanding the expression (5) up to a second order over the meson and diquark fields, we have

$$S_{\rm eff}(\sigma, \pi_a, \Delta_A^{s,p}, \Delta_{A'}^{s,p*}) = S_{\rm eff}^{(0)} + S_{\rm eff}^{(2)}(\sigma, \pi_a, \Delta_A^{s,p}, \Delta_{A'}^{s,p*}) + \cdots, \tag{12}$$

where [due to the gap equations, the term linear over meson and diquark fields is absent in (12)]

[5] In order to bring the quark sector of the Lagrangian (3) to the expression, given in the square brackets of (6), we have also used the following well-known relations: $\partial_\nu^t = -\partial_\nu$, $C\gamma^\nu C^{-1} = -(\gamma^\nu)^t$, $C\gamma^5 C^{-1} = (\gamma^5)^t = \gamma^5$, $\tau^2\vec{\tau}\tau^2 = -(\vec{\tau})^t$,

$$\tau^2 = \begin{pmatrix} 0, & -i \\ i, & 0 \end{pmatrix}.$$

$$S_{\rm eff}^{(0)} = -\int d^4x\left[\frac{\sigma^o\sigma^o + \pi_a^o\pi_a^o}{4G} + \frac{\Delta_A^{so}\Delta_A^{s*o}}{4H_s} + \frac{\Delta_{A'}^{po}\Delta_{A'}^{p*o}}{4H_p}\right] - \frac{i}{2}\,{\rm Tr}_{\{{\rm NG}scfx\}}\ln(S_0^{-1}) \equiv -\Omega(\sigma^o, \pi_a^o, \Delta_A^{s,po}, \Delta_{A'}^{s,p*o})\int d^4x, \tag{13}$$

$$S_{\rm eff}^{(2)}(\sigma, \pi_a, \Delta_A^{s,p}, \Delta_{A'}^{s,p*}) = -\int d^4x\left[\frac{\sigma^2 + \pi_a^2}{4G} + \frac{\Delta_A^s\Delta_A^{s*}}{4H_s} + \frac{\Delta_{A'}^p\Delta_{A'}^{p*}}{4H_p}\right] + \frac{i}{4}\,{\rm Tr}_{\{{\rm NG}scfx\}}\left\{S_0\begin{pmatrix}\Sigma, & K\\ K^*, & \Sigma^t\end{pmatrix}S_0\begin{pmatrix}\Sigma, & K\\ K^*, & \Sigma^t\end{pmatrix}\right\}. \tag{14}$$

In the following, on the basis of the effective action $S_{\rm eff}^{(2)}$, we will study the spectrum of meson/diquark excitations in the color superconducting phase of the NJL model under consideration. So it is convenient to present the effective action (14) in the following form:

$$S_{\rm eff}^{(2)} = S_{\rm mesons}^{(2)} + S_{\rm diquarks}^{(2)} + S_{\rm mixed}^{(2)}, \tag{15}$$

where

$$S_{\rm mesons}^{(2)} = -\int d^4x\frac{\sigma^2 + \pi_a^2}{4G} + \frac{i}{4}\,{\rm Tr}_{scfx}\{S_{11}\Sigma S_{11}\Sigma + 2S_{12}\Sigma^t S_{21}\Sigma + S_{22}\Sigma^t S_{22}\Sigma^t\}, \tag{16}$$

$$S_{\rm diquarks}^{(2)} = -\int d^4x\left[\frac{\Delta_A^s\Delta_A^{s*}}{4H_s} + \frac{\Delta_{A'}^p\Delta_{A'}^{p*}}{4H_p}\right] + \frac{i}{4}\,{\rm Tr}_{scfx}\{S_{12}K^*S_{12}K^* + 2S_{11}KS_{22}K^* + S_{21}KS_{21}K\}, \tag{17}$$

$$S_{\rm mixed}^{(2)} = \frac{i}{2}\,{\rm Tr}_{scfx}\{S_{11}\Sigma S_{12}K^* + S_{21}\Sigma S_{11}K + S_{12}\Sigma^t S_{22}K^* + S_{21}KS_{22}\Sigma^t\}, \tag{18}$$

and $S_{ij}$ are the matrix elements of the quark propagator matrix $S_0$, given in (11).

## III. GAP EQUATIONS AND COLOR NEUTRALITY CONDITION

Let us for a moment assume that in (2) the chemical potential $\mu_8$ is an independent parameter ( $\neq 0$). Then Lagrangian $L$ is invariant under the color ${\rm SU}_c(2) \times {\rm U}_{\lambda_8}(1)$ symmetry group. Recall that the phase structure of any theory is defined by a competition of its order parameters. In our case, the order parameters (ground state expectation values) $\langle\sigma(x)\rangle$, $\langle\pi_a(x)\rangle$, $\langle\Delta_A^s(x)\rangle$, $\langle\Delta_{A'}^p(x)\rangle$, are obtained from a solution of the gap equations (see the previous section). Since the consideration of the model (2) with a total set of order parameters is a very hard task, we shall assume that parity is conserved, i.e. $\langle\pi_a(x)\rangle = 0$, $\langle\Delta_{A'}^p(x)\rangle = 0$ (in Sec. VI A some arguments are presented, however, that at sufficiently high values of pseudoscalar coupling $H_p$ parity might be spontaneously broken down), thus having to deal only with $\langle\sigma(x)\rangle$ and $\langle\Delta_A^s(x)\rangle$. In this case, three different phases might exist in the model (2): (i) In the first one, the normal phase, $\langle\Delta_A^s(x)\rangle = 0$ for all $A = 2, 5, 7$. In this phase the initial color symmetry remains intact. (ii) The second one is a well-known 2SC phase with $\langle\Delta_2^s(x)\rangle \neq 0$ and $\langle\Delta_{5,7}^s(x)\rangle = 0$. The ground state of this phase is invariant under ${\rm SU}_c(2)$-color symmetry. (iii) Finally, there might exist a phase with $\langle\Delta_2^s(x)\rangle \neq 0$, $\langle\Delta_5^s(x)\rangle \neq 0$ and $\langle\Delta_7^s(x)\rangle = 0$. [Note, the two phases (ii) and (iii) are not unitarily equivalent, since there are no color transformations from ${\rm SU}_c(2) \times {\rm U}_{\lambda_8}(1)$ that connect the corresponding ground state expectation values.] However, since color neutrality cannot be achieved in the ground state of the form (iii) (see [7]), throughout of our paper only two order parameters, $\langle\sigma(x)\rangle$ and $\langle\Delta_2^s(x)\rangle \equiv \Delta$, will be taken into account, whereas other ones will be supposed to have zero expectation values, i.e. $\langle\pi_a(x)\rangle = 0$, $\langle\Delta_{A'}^p(x)\rangle = 0$, $\langle\Delta_{5,7}^s(x)\rangle = 0$. So, below, only the competition between the normal phase ($\Delta = 0$) and the 2SC one ($\Delta \neq 0$) will be considered. As a result, one may deal with a thermodynamic potential $\Omega$, which depends on two variables $\Delta, \langle\sigma\rangle$ or, equivalently, $\Delta, m \equiv m_0 + \langle\sigma\rangle$ only. Then the expression for the thermodynamic potential $\Omega$ can be calculated with the help of (13) (see also, e.g., [11]):

$$\Omega(m, \Delta) = \frac{(m - m_0)^2}{4G} + \frac{|\Delta|^2}{4H_s} - 4\sum_{\pm}\int\frac{d^3q}{(2\pi)^3}|E_\Delta^\pm| - 2\sum_{\pm}\int\frac{d^3q}{(2\pi)^3}|\breve{E}^\pm|, \tag{19}$$

where $E_\Delta^\pm = \sqrt{(E^\pm)^2 + |\Delta|^2}$, $E^\pm = E \pm \bar{\mu}$, $\breve{E}^\pm = E \pm \breve{\mu}$, $\bar{\mu} = \mu + \mu_8$, $\breve{\mu} = \mu - 2\mu_8$, $E = \sqrt{\vec{q}^2 + m^2}$. Since the integrals in the right-hand side of (19) are ultraviolet divergent, we regularize them and the other divergent integrals below by implementing a three-dimensional cutoff $\Lambda$. The resulting gap equations look like:

$$\frac{\Delta}{4H_s} = 4i\Delta \int \frac{d^4q}{(2\pi)^4} \left\{ \frac{1}{q_0^2 - (E_\Delta^+)^2} + \frac{1}{q_0^2 - (E_\Delta^-)^2} \right\} = 2\Delta \int \frac{d^3q}{(2\pi)^3} \left\{ \frac{1}{E_\Delta^+} + \frac{1}{E_\Delta^-} \right\}, \tag{20}$$

$$\frac{m - m_0}{2G} = 4im \sum_\pm \int \frac{d^4q}{(2\pi)^4} \frac{1}{E} \left\{ \frac{\breve{E}^\pm}{q_0^2 - (\breve{E}^\pm)^2} \right\} + 8im \sum_\pm \int \frac{d^4q}{(2\pi)^4} \frac{1}{E} \left\{ \frac{E^\pm}{q_0^2 - (E_\Delta^\pm)^2} \right\}. \tag{21}$$

In (20) and (21) as well as in other expressions containing four-dimensional momentum integrals, $q_0$ is shorthand for $q_0 + i\varepsilon \cdot \mathrm{sgn}(q_0)$, where $\varepsilon \to 0_+$. This prescription correctly implements the roles of $\mu$, $\mu_8$ as chemical potentials and preserves the causality of the theory (see, e.g., [27]).

Now let us impose the local color neutrality requirement on the ground state of the model. It means that the quantity $\mu_8$ takes such values that the density of the 8-color charge $\langle Q_8 \rangle \equiv -\partial\Omega/\partial\mu_8$ equals zero for arbitrary fixed values of other model parameters. So we have the following local color neutrality constraint:

$$\langle Q_8 \rangle = -\frac{\partial \Omega}{\partial \mu_8} \equiv 4 \int \frac{d^3q}{(2\pi)^3} \left\{ \frac{E^+}{E_\Delta^+} - \frac{E^-}{E_\Delta^-} - \frac{\breve{E}^+}{|\breve{E}^+|} + \frac{\breve{E}^-}{|\breve{E}^-|} \right\} = 0. \tag{22}$$

The system of Eqs. (20)–(22) has two different solutions. As we have already discussed after (2), the first one (with $\Delta = 0$, $\mu_8 = 0$) corresponds to the $SU(3)_c$-symmetric phase of the model (normal phase), the second one (with $\Delta \neq 0$ and $\mu_8 \neq 0$) to the 2SC phase. As usual, solutions of these equations give local extrema of the thermodynamic potential $\Omega(m, \Delta)$ (19), so one should also check which of them corresponds to the absolute minimum of $\Omega$. Having found the solution corresponding to the stable state of quark matter (the absolute minimum of $\Omega$), we obtained the behavior of the gaps $m$, $\Delta$ as well as the $\mu_8$ vs the quark chemical potential $\mu$ (see Figs. 1 and 2). [Note that, in all numerical calculations of the paper, we use the parameter set

$$G = 5.86\ \mathrm{GeV}^{-2}, \qquad \Lambda = 618\ \mathrm{MeV}, \qquad m_0 = 5.67\ \mathrm{MeV}, \qquad H_s = 3G/4 \tag{23}$$

that leads in the framework of the NJL model to the well-known vacuum phenomenological values of the pion weak-decay constant $F_\pi = 92.4$ MeV, pion mass $M_\pi = 140$ MeV, and chiral quark condensate $\langle \bar{q}q \rangle = -(245\ \mathrm{MeV})^3$. The relation between $H_s$ and $G$ in (23) is induced, e.g., by the structure of QCD four-quark vertices in the one-gluon exchange approximation.] The region $\mu < \mu_c = 342$ MeV is the domain of $SU_c(3)$-color symmetric quark matter because $\Omega$ in this case is minimized by $m \neq 0$ and $\Delta = 0$, $\mu_8 = 0$. For $\mu > \mu_c$, the solution with $m \neq 0$, $\Delta \neq 0$, and $\mu_8 \neq 0$, corresponding to the 2SC phase, gives the global minimum of $\Omega$, and thereby the color superconducting phase is favored. The transition between these two phases is of the first order, which is characterized by a discontinuity in the behavior of $m$ and $\Delta$ vs $\mu$ (see Fig. 1). Finally, remark that the critical value $\tilde{\mu}_c$ of the quark chemical potential was calculated in the framework of the model (1) as well. In terms of the parameter set (23), we have $\tilde{\mu}_c = 350$ MeV [8,9]; i.e. in the color neutral quark matter the transition to the 2SC phase occurred at slightly lower values of the quark chemical potentials.

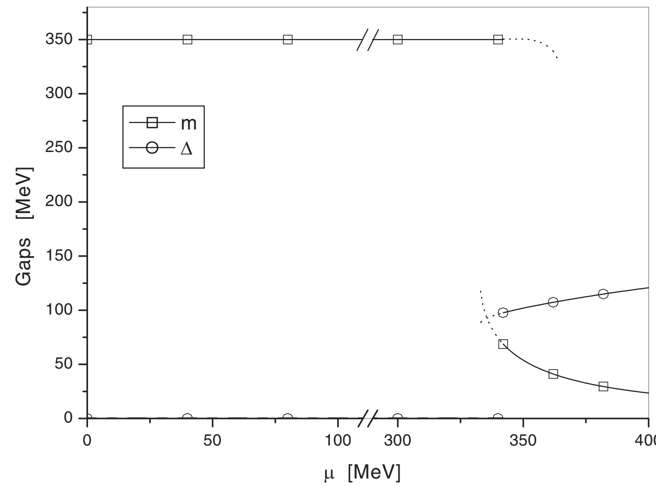


FIG. 1. The constituent quark mass $m$ vs $\mu$ and color gap $\Delta$ vs $\mu$ under local color neutrality constraint.

Similarly to Ref. [28], it is possible to find the following expressions for the matrix elements $S_{ij}$ of the quark propagator matrix $S_0$:

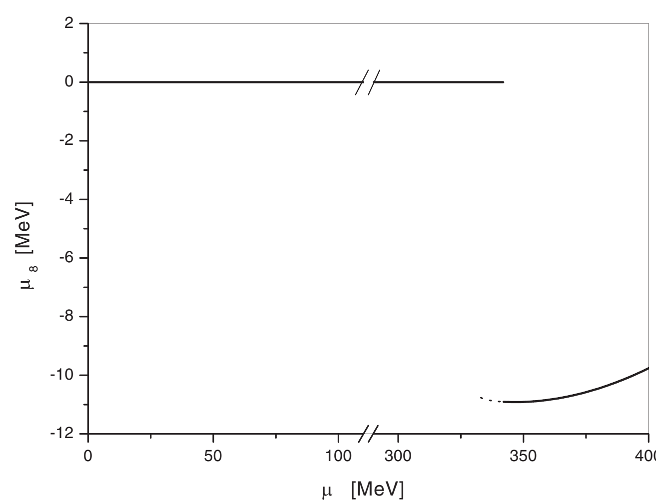


FIG. 2. The behavior of $\mu_8$ vs $\mu$ in the model (2) when local color neutrality is imposed.

$$S_{11} = \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{q_0 - E^+}{q_0^2 - (E_\Delta^+)^2} \gamma^0 \bar{\Lambda}_+ + \frac{q_0 + E^-}{q_0^2 - (E_\Delta^-)^2} \gamma^0 \bar{\Lambda}_- \right\} P_{12}^{(c)} + \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{\gamma^0 \bar{\Lambda}_+}{q_0 + \breve{E}^+} + \frac{\gamma^0 \bar{\Lambda}_-}{q_0 - \breve{E}^-} \right\} P_3^{(c)}, \tag{24}$$

$$S_{22} = \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{q_0 - E^-}{q_0^2 - (E_\Delta^-)^2} \gamma^0 \bar{\Lambda}_+ + \frac{q_0 + E^+}{q_0^2 - (E_\Delta^+)^2} \gamma^0 \bar{\Lambda}_- \right\} P_{12}^{(c)} + \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{\gamma^0 \bar{\Lambda}_+}{q_0 + \breve{E}^-} + \frac{\gamma^0 \bar{\Lambda}_-}{q_0 - \breve{E}^+} \right\} P_3^{(c)}, \tag{25}$$

$$S_{21} = -i\Delta^* \tau_2 \lambda_2 \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{\gamma^5 \bar{\Lambda}_+}{q_0^2 - (E_\Delta^+)^2} + \frac{\gamma^5 \bar{\Lambda}_-}{q_0^2 - (E_\Delta^-)^2} \right\}, \tag{26}$$

$$S_{12} = -i\Delta \tau_2 \lambda_2 \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \left\{ \frac{\gamma^5 \bar{\Lambda}_+}{q_0^2 - (E_\Delta^-)^2} + \frac{\gamma^5 \bar{\Lambda}_-}{q_0^2 - (E_\Delta^+)^2} \right\}, \tag{27}$$

where $\bar{\Lambda}_\pm = \frac{1}{2}\{1 \pm [\gamma^0(\vec{\gamma}\vec{q} - m)/E]\}$, and $P_{12}^{(c)} = \mathrm{diag}(1, 1, 0)$, $P_3^{(c)} = \mathrm{diag}(0, 0, 1)$ are the projectors on the red/green and blue directions in the color space, correspondingly.

The poles of the matrix elements (24)–(27) of the quark propagator give the dispersion laws, i.e. the momentum dependence of energy, for quarks in a medium. Thus, we have $E_\Delta^-$ for the energy of red/green quarks and $E_\Delta^+$ for the energy of red/green antiquarks. Moreover, the energy of blue quarks (antiquarks) is equal to $\breve{E}^-$ ($\breve{E}^+$). It is clear from Figs. 1 and 2 that in the 2SC phase, i.e. at $\mu > \mu_c$, we have $(\mu + \mu_8) > m$, $(\mu - 2\mu_8) > m$. Then in this phase the quantity $E$ can reach both the value $(\mu + \mu_8)$ (the Fermi energy for red/green quarks) and the value $(\mu - 2\mu_8)$ (the Fermi energy for blue quarks). In this case, in order to create a red/green quark in the 2SC phase, a minimal amount of energy (the gap) equal to $|\Delta|$ at the Fermi level ($E = \mu + \mu_8$) is required. Similarly, there is no energy cost to create a blue quark at its Fermi level $E = \mu - 2\mu_8$; i.e. blue quarks are gapless in the 2SC phase. In the normal phase of the model, i.e. at $\mu < \mu_c$, the minimal energy $(m - \mu)$ is needed for the creation of a quark of any color. Note that, in both the 2SC and normal phases of the model, the minimal energy required for a quark creation differs from the minimal energy required for the creation of an antiquark of the same color. This fact reflects the breaking of charge conjugation symmetry in the presence of chemical potentials.

Without loss of generality, we will assume throughout the paper that $\Delta$ is a real non-negative quantity. Given the explicit expression for the quark propagator $S_0$, in the next sections we will calculate two-point (unnormalized) correlators of meson and diquark fluctuations over the ground state in the one-loop (mean-field) approximation and find their masses.

## IV. MASSES OF THE $\pi$ AND $\sigma$ MESONS

It turns out that the $S^{(2)}_{\text{mixed}}$ part (18) of the effective action is composed from $\sigma(x)$, $\Delta_2^s(x)$, and $\Delta_2^{s*}(x)$ fields only. So it provides us with nondiagonal matrix elements $\Gamma_{\sigma X}$ ($X = \Delta_2^{s*}, \Delta_2^s$) of the inverse propagator matrix of $\sigma$, $\Delta_2^{s*}$, and $\Delta_2^s$. Moreover, each term in $S^{(2)}_{\text{mixed}}$ is proportional to $\Delta$ or $\Delta^*$ as well as to the constituent quark mass $m$ (see the appendix). Hence, in the color symmetric phase ($\Delta = 0$) there is no mixing between $\sigma$ meson and diquark $\Delta_2^s$ at all. The parameter $m$ is small (or even equals zero if $m_0 = 0$) in the 2SC phase, so we ignore for simplicity the $\sigma$-$\Delta_2^s$ mixing effect in this phase, too. As a result, in order to get the masses of mesons we use only the effective action (16), which has the form $S^{(2)}_{\text{mesons}} = S^{(2)}_{\sigma\sigma} + S^{(2)}_{\pi\pi}$, where

$$S^{(2)}_{\sigma\sigma} = -\int d^4x \frac{\sigma^2}{4G} + \frac{i}{4} \mathrm{Tr}_{scfx}\{S_{11}\sigma S_{11}\sigma + 2S_{12}\sigma S_{21}\sigma + S_{22}\sigma S_{22}\sigma\} \equiv -\frac{1}{2}\int d^4u d^4v \sigma(u)\Gamma(u-v)\sigma(v), \tag{28}$$

$$S^{(2)}_{\pi\pi} = -\int d^4x \frac{\pi_a^2}{4G} + \frac{i}{4} \mathrm{Tr}_{scfx}\{S_{11}(i\gamma^5\pi_a\tau_a)S_{11}(i\gamma^5\pi_b\tau_b) + 2S_{12}(i\gamma^5\pi_a\tau_a^t)S_{21}(i\gamma^5\pi_b\tau_b) + S_{22}(i\gamma^5\pi_a\tau_a^t)S_{22}(i\gamma^5\pi_b\tau_b^t)\} \equiv -\frac{1}{2}\int d^4u d^4v \pi_k(u)\Pi_{kl}(u-v)\pi_l(v). \tag{29}$$

In these formulas $\Gamma(x-y)$ is the inverse propagator of $\sigma$ mesons, and $\Pi_{ab}(x-y)$ is the (diagonal) matrix of the inverse $\pi$-meson propagator. Evidently,

$$\Gamma(x-y) = -\frac{\delta^2 S^{(2)}_{\sigma\sigma}}{\delta\sigma(y)\delta\sigma(x)}, \qquad \Pi_{ab}(x-y) = -\frac{\delta^2 S^{(2)}_{\pi\pi}}{\delta\pi_b(y)\delta\pi_a(x)}. \tag{30}$$

Next, using in (28) and (29) the expressions (24)–(27) for the matrix elements $S_{ij}$, it is possible to obtain with the help of relations (30) the functions $\Gamma(x-y)$, $\Pi_{ab}(x-y)$, and then their momentum space representations, $\Gamma(p)$, $\Pi_{ab}(p)$, correspondingly.[6] The zeros of these functions determine the particle and antiparticle dispersion laws, i.e. the relations between their energy and three-momenta. In the present paper, we are interested mainly in the investigation of the modification of meson and diquark masses in dense and cold color neutral quark matter. In this case, the particle mass is defined as the value of its energy in the rest frame, $\vec{p}=0$ (see, e.g., [9,29]), where the calculation of inverse propagators for $\sigma$ and $\pi$ mesons is significantly simplified. Indeed, in the rest frame it is possible to get:

$$\Pi_{ab}(p_0) = \frac{\delta_{ab}}{2G} - 8\delta_{ab}\int\frac{d^3q}{(2\pi)^3}\frac{E_\Delta^+E_\Delta^- + E^+E^- + \Delta^2}{E_\Delta^+E_\Delta^-} \times\frac{E_\Delta^+ + E_\Delta^-}{(E_\Delta^+ + E_\Delta^-)^2 - p_0^2} - 16\delta_{ab}\int\frac{d^3q}{(2\pi)^3}\frac{\theta(E-\breve{\mu})E}{4E^2-p_0^2} \equiv \delta_{ab}\Pi(p_0), \tag{31}$$

$$\Gamma(p_0) = \Gamma_0(p_0^2) + \Gamma_1(p_0^2), \tag{32}$$

$$\Gamma_0(p_0^2) = \frac{1}{2G} - 8\int\frac{d^3q}{(2\pi)^3}\frac{\vec{q}^2}{E^2}\frac{E_\Delta^+E_\Delta^- + E^+E^- + \Delta^2}{E_\Delta^+E_\Delta^-} \times\frac{E_\Delta^+ + E_\Delta^-}{(E_\Delta^+ + E_\Delta^-)^2 - p_0^2} - 16\int\frac{d^3q}{(2\pi)^3}\frac{\vec{q}^2}{E}\frac{\theta(E-\breve{\mu})}{4E^2-p_0^2}, \tag{33}$$

$$\Gamma_1(p_0^2) = 16\Delta^2m^2\int\frac{d^3q}{(2\pi)^3E^2}\left\{\frac{1}{E_\Delta^+[p_0^2-4(E_\Delta^+)^2]} + \frac{1}{E_\Delta^-[p_0^2-4(E_\Delta^-)^2]}\right\}, \tag{34}$$

where the same notations as in (19) were used. The zeros of the functions (31) and (32) give us the masses of $\pi$ and $\sigma$ mesons, respectively (see Fig. 3).[7] We see that in the 2SC phase the masses of the sigma and pi mesons are about 300 MeV. Moreover, the pion is a stable particle in this phase (only electroweak decay channels are allowed). This conclusion is supported by the following arguments. It is clear that the first and the second integrals in (31) are analytical functions in the whole complex $p_0^2$ plane, except the cuts $E_{\min}^2 < p_0^2 < \infty$ and $(2\breve{\mu})^2 < p_0^2 < \infty$, respectively [here $E_{\min} = \sqrt{(\bar{\mu}-m)^2+|\Delta|^2} + \sqrt{(\bar{\mu}+m)^2+|\Delta|^2}$ is the minimum of the expression $E_\Delta^- + E_\Delta^+$, which is taken at the point $|\vec{p}|=0$]. Evidently, $E_{\min}$ corresponds to the threshold for the pion decay into a red-green quark-antiquark pair, whereas $2\breve{\mu}$ corresponds to the threshold for the pion decay into a blue quark-antiquark pair. It is easily seen from Fig. 3 that in the 2SC phase the pion mass is less than the values of these two thresholds. Because there are no other singularities in (31) corresponding to different channels of the pion decay, we can conclude that in the 2SC phase the pion is a stable particle.

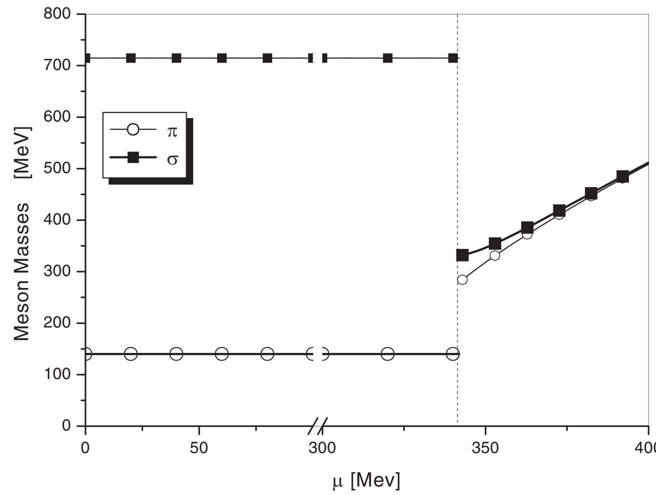


FIG. 3. The masses of the $\sigma$ meson and pion as functions of $\mu$, when mixing of $\sigma$ and $\Delta_2^s$ is neglected.

The neglect of the mixing between the $\sigma$ meson and $\Delta_2^s$-scalar diquark also results in a stable $\sigma$ meson. However, if the mixing is taken into account, then in the 2SC phase the $\sigma$ meson is a resonance, decaying into a pair of quarks, whose width is a rather small quantity, i.e. about 30 MeV (see the appendix).

## V. MASSES OF SCALAR DIQUARKS

As in the previous section, we will ignore for simplicity the term $S_{\rm mixed}^{(2)}$ (18), which mixes $\sigma$ and $\Delta_2^{s*}$, $\Delta_2^s$ diquarks, in the effective action (15). In this case, in order to obtain the masses of diquarks, we need to analyze the term $S_{\rm diquarks}^{(2)}$ (17) only. It can be easily presented in the following form:

$$S_{\rm diquarks}^{(2)} = \sum_{A=2,5,7}\{S_{sAA}^{(2)} + S_{pAA}^{(2)}\}, \tag{35}$$

where labels $s$, $p$ denote the contributions from scalar and pseudoscalar diquark fields, correspondingly, and

[6] One should not be confused by the coordinate or momentum space representation which is used for the inverse propagators and other quantities, since this is clear from the arguments of these functions or the context.

[7] In our numerical investigations of the 2SC phase, presented in Fig. 3, we have ignored in (32) the term $\Gamma_1(p_0^2)$ proportional to $\Delta^2m^2$, since it is comparable (or even less) in magnitude with nondiagonal elements $\Gamma_{\sigma X}(p_0)$ ($X=\Delta_2^{s*},\Delta_2^s$) of the full inverse propagator matrix of $\sigma$, $\Delta_2^{s*}$, and $\Delta_2^s$ (see the appendix), which are not taken into account in the above consideration.

$$S^{(2)}_{sAA} = -\int d^4x \frac{\Delta^s_A \Delta^{s*}_A}{4H_s} + \frac{i}{2}\,\mathrm{Tr}_{scfx}\{S_{11} i\Delta^s_A \gamma^5 \tau_2 \lambda_A S_{22} i\Delta^{s*}_A \gamma^5 \tau_2 \lambda_A\}, \tag{36}$$

$$S^{(2)}_{pAA} = -\int d^4x \frac{\Delta^p_A \Delta^{p*}_A}{4H_p} + \frac{i}{2}\,\mathrm{Tr}_{scfx}\{S_{11} \Delta^p_A \tau_2 \lambda_A S_{22} \Delta^{p*}_A \tau_2 \lambda_A\}, \tag{37}$$

for fixed $A = 5, 7$, and

$$\begin{aligned} S^{(2)}_{s22} = &-\int d^4x \frac{\Delta^s_2 \Delta^{s*}_2}{4H_s} + \frac{i}{4}\,\mathrm{Tr}_{scfx}\{S_{12} i\Delta^{s*}_2 \gamma^5 \tau_2 \lambda_2 S_{12} i\Delta^{s*}_2 \gamma^5 \tau_2 \lambda_2 \\ &+ 2S_{11} i\Delta^s_2 \gamma^5 \tau_2 \lambda_2 S_{22} i\Delta^{s*}_2 \gamma^5 \tau_2 \lambda_2 \\ &+ S_{21} i\Delta^s_2 \gamma^5 \tau_2 \lambda_2 S_{21} i\Delta^s_2 \gamma^5 \tau_2 \lambda_2\}, \end{aligned} \tag{38}$$

$$\begin{aligned} S^{(2)}_{p22} = &-\int d^4x \frac{\Delta^p_2 \Delta^{p*}_2}{4H_p} + \frac{i}{4}\,\mathrm{Tr}_{scfx}\{S_{12} \Delta^{p*}_2 \tau_2 \lambda_2 S_{12} \Delta^{p*}_2 \tau_2 \lambda_2 \\ &+ 2S_{11} \Delta^p_2 \tau_2 \lambda_2 S_{22} \Delta^{p*}_2 \tau_2 \lambda_2 \\ &+ S_{21} \Delta^p_2 \tau_2 \lambda_2 S_{21} \Delta^p_2 \tau_2 \lambda_2\}. \end{aligned} \tag{39}$$

It follows from (35)–(39) that there is no mixing between scalar and pseudoscalar diquarks. Moreover, scalar diquarks (or pseudoscalar ones), as such, are not mixed to one another. Starting from the above formulas, we will find the inverse propagators of diquarks which might be introduced by the following way ($A = 2, 5, 7$):

$$S^{(2)}_{sAA} = -\frac{1}{2}\sum_{X,Y} \int d^4u d^4v X(u) \Gamma^{As}_{XY}(u - v) Y(v), \tag{40}$$

$$S^{(2)}_{pAA} = -\frac{1}{2}\sum_{P,Q} \int d^4u d^4v P(u) \Gamma^{Ap}_{PQ}(u - v) Q(v), \tag{41}$$

where (for each fixed value of $A$) $X(x), Y(x) = \Delta^s_A(x), \Delta^{s*}_A(x)$, $P(x), Q(x) = \Delta^p_A(x), \Delta^{p*}_A(x)$, and $\Gamma^{As}_{XY}(z)$ or $\Gamma^{Ap}_{PQ}(z)$ are matrix elements of the $2 \times 2$ inverse propagator matrix for $\Delta^s_A(x), \Delta^{s*}_A(x)$ or $\Delta^p_A(x), \Delta^{p*}_A(x)$ fields, respectively. Given diquark propagators, it is possible then to obtain the masses of diquarks.

### A. Scalar diquarks in the 2SC phase ($\Delta \neq 0$, $\mu_8 \neq 0$)

In the present section, using (40) for different $A = 2, 5, 7$, we will study step by step the masses of scalar diquark excitations in the 2SC phase of the model (2), when the color neutrality condition is taken into account.

Let us begin with the $\Delta^s_5$-$\Delta^{s*}_5$ diquark sector. It follows from (40) at $A = 5$ that

$$\Gamma^{5s}_{XY}(x - y) = -\frac{\delta^2 S^{(2)}_{s55}}{\delta Y(y) \delta X(x)} \tag{42}$$

[recall that $X, Y = \Delta^s_5(x), \Delta^{s*}_5(x)$ in the case under consideration]. Note also that $\Gamma^{5s}(z)$ is a symmetric matrix, i.e. $\Gamma^{5s}_{XY}(z) = \Gamma^{5s}_{YX}(-z)$. It is clear from (40) and (42) that $\Gamma^{5s}_{\Delta^s_5\Delta^s_5}(z) = \Gamma^{5s}_{\Delta^{s*}_5\Delta^{s*}_5}(z) = 0$, and this matrix has nonzero elements of the form (in the momentum space representation):

$$\Gamma^{5s}_{\Delta^{s*}_5\Delta^s_5}(p) = \frac{1}{4H_s} - i\,\mathrm{Tr}_{sc} \int \frac{d^4q}{(2\pi)^4} \{S_{11}(q + p) i\gamma^5 \lambda_5 \times S_{22}(q) i\gamma^5 \lambda_5\}, \tag{43}$$

$$\Gamma^{5s}_{\Delta^s_5\Delta^{s*}_5}(p) = \frac{1}{4H_s} - i\,\mathrm{Tr}_{sc} \int \frac{d^4q}{(2\pi)^4} \{S_{22}(q + p) i\gamma^5 \lambda_5 \times S_{11}(q) i\gamma^5 \lambda_5\}, \tag{44}$$

where the Fourier-transformed expressions $S_{11}(q)$, $S_{22}(q)$ can be easily determined from (24) and (25). It follows from (43) and (44) that $\Gamma^{5s}_{\Delta^{s*}_5\Delta^s_5}(-p) = \Gamma^{5s}_{\Delta^s_5\Delta^{s*}_5}(p)$. Since we are interested in diquark masses, it is necessary to use the rest frame in (43) and (44), i.e. $p = (p_0, 0, 0, 0)$ (see also [9,29]). In this case, the calculation of matrix elements (43) and (44) is greatly simplified, and mass excitations are connected with the zeros of the quantity $\det\Gamma^{5s}(p_0)$ in the $p_0^2$ plane.[8] So we have

$$\Gamma^{5s}_{\Delta^{s*}_5\Delta^s_5}(p_0) = \frac{1}{4H_s} - 4i \int \frac{d^4q}{(2\pi)^4} \times \left\{ \frac{q_0 + E^+}{(p_0 + q_0 + \breve{E}^+)(q_0^2 - (E^+_\Delta)^2)} + \frac{q_0 - E^-}{(p_0 + q_0 - \breve{E}^-)(q_0^2 - (E^-_\Delta)^2)} \right\}. \tag{45}$$

The $p_0$, $q_0$ dependency in the integrand of (45) is presented in an evident form. Other quantities in (45), such as $E^\pm$, etc., depend on $|\vec{q}|$ only. The expression (45) is valid for both $\Delta = 0$ and $\Delta \neq 0$. For the case $\Delta \neq 0$, i.e. in the color superconducting phase, it is possible to use the gap equation (20) in order to eliminate the coupling constant $H_s$ from this formula. In this way we find:

[8] Recently, the Bethe-Salpeter equation approach has been used to obtain diquark masses in the 2SC phase of cold dense QCD at asymptotically large values of the chemical potential [30]. There, the mass of the diquark was defined as the energy of a bound state of two virtual quarks in the center of mass frame, i.e. as in our approach, in the rest frame for the whole diquark.

$$\begin{aligned}\Gamma^{5s}_{\Delta_5^{s*}\Delta_5^s}(p_0) &= 4i(p_0-3\mu_8)\int\frac{d^4q}{(2\pi)^4}\\ &\quad\times\left\{\frac{1}{(p_0+q_0+\check{E}^+)(q_0^2-(E_\Delta^+)^2)}\right.\\ &\quad\left.+\frac{1}{(p_0+q_0-\check{E}^-)(q_0^2-(E_\Delta^-)^2)}\right\}\\ &\equiv 2(p_0-3\mu_8)H(p_0),\\ \Gamma^{5s}_{\Delta_5^s\Delta_5^{s*}}(p_0) &= -2(p_0+3\mu_8)H(-p_0).\end{aligned} \tag{46}$$

The last equation in (46) is due to the relation $\Gamma^{5s}_{\Delta_5^{s*}\Delta_5^s}(-p_0) = \Gamma^{5s}_{\Delta_5^s\Delta_5^{s*}}(p_0)$. Recall that in (46) $q_0+p_0$ and $q_0$ are shorthand for $(p_0+q_0)+i\varepsilon\cdot\mathrm{sgn}(p_0+q_0)$ and $q_0+i\varepsilon\cdot\mathrm{sgn}(q_0)$, where $\varepsilon\to 0_+$ [see also comment after formula (21)]. Performing in (46) the $q_0$ integration, one obtains

$$\begin{aligned}H(p_0) = \int\frac{d^3q}{(2\pi)^3}&\left\{\frac{\theta(\check{E}^+)}{(p_0+\check{E}^++E_\Delta^+)E_\Delta^+}\right.\\ &+\frac{\theta(-\check{E}^+)}{(p_0+\check{E}^+-E_\Delta^+)E_\Delta^+}+\frac{\theta(\check{E}^-)}{(p_0-\check{E}^--E_\Delta^-)E_\Delta^-}\\ &\left.+\frac{\theta(-\check{E}^-)}{(p_0-\check{E}^-+E_\Delta^-)E_\Delta^-}\right\}.\end{aligned} \tag{47}$$

Now with the help of (46) one easily gets the expression for the determinant of the inverse propagator matrix $\Gamma^{5s}(p_0)$ in the rest frame:

$$\begin{aligned}\det\Gamma^{5s}(p_0) &= \Gamma^{5s}_{\Delta_5^{s*}\Delta_5^s}(p_0)\Gamma^{5s}_{\Delta_5^s\Delta_5^{s*}}(p_0)\\ &\equiv -4(p_0^2-9\mu_8^2)H(p_0)H(-p_0).\end{aligned} \tag{48}$$

The diquark squared mass spectrum in the $\Delta_5^s$ sector of the theory is defined by zeros of the $\det\Gamma^{5s}(p_0)$ in the $p_0^2$ plane. Evidently, the point $p_0^2 = 9\mu_8^2$ is the solution of the equation $\det\Gamma^{5s}(p_0) = 0$. Let us suppose that some nonzero point $p_0 = -M_D^s$ is the zero of the function $H(p_0)$, i.e. $H(-M_D^s) = 0$. Then at $p_0 = M_D^s$ the determinant (48) is also equal to zero, so the point $p_0^2 = (M_D^s)^2$ is another zero of $\det\overline{\Gamma}^{5s}(p_0)$ in the $p_0^2$ plane, and the second bosonic excitation of this sector has nonzero mass $M_D^s$. It follows from (22) and (47) that $H(p_0)$ is proportional to $\langle Q_8\rangle$ at the point $p_0 = 3\mu_8$. Namely, $H(3\mu_8) \equiv -\langle Q_8\rangle/(4\Delta^2)$. Since $\langle Q_8\rangle$ is zero due to the constraint (22), we may conclude that $M_D^s = 3|\mu_8|$. Hence, in the $\Delta_5^s$ sector of the model there are two real bosonic excitations with equal masses $M_D^s \equiv 3|\mu_8|$.

The similar is true for the $\Delta_7^s$ sector of the model, so, in the whole $\Delta_5^s$, $\Delta_7^s$ sector of the NJL model that is under the color neutrality constraint, there are four massive scalar excitations with equal masses $M_D^s \equiv 3|\mu_8|$. These particles form two real antidoublets of the $\mathrm{SU}_c(2)$ group or one complex antidoublet.

Consider now the diquark excitations of the 2SC ground state in the $\Delta_2^s$-$\Delta_2^{s*}$ sector of the model. In this case, the matrix $\Gamma^{2s}(p_0)$ (the momentum representation for the inverse propagator matrix at $\vec{p} = 0$, i.e. in the rest frame) has the following structure:

$$\begin{aligned}\Gamma^{2s}_{\Delta_2^s\Delta_2^s}(p_0) &= \Gamma^{2s}_{\Delta_2^{s*}\Delta_2^{s*}}(p_0) = 4\Delta^2 I_0(p_0^2),\\ \Gamma^{2s}_{\Delta_2^s\Delta_2^{s*}}(p_0) &= \Gamma^{2s}_{\Delta_2^{s*}\Delta_2^s}(-p_0)\\ &= (4\Delta^2-2p_0^2)I_0(p_0^2)+4p_0I_1(p_0^2),\end{aligned} \tag{49}$$

where

$$I_0(p_0^2) = \int\frac{d^3q}{(2\pi)^3}\frac{1}{E_\Delta^+[4(E_\Delta^+)^2-p_0^2]} + \int\frac{d^3q}{(2\pi)^3}\frac{1}{E_\Delta^-[4(E_\Delta^-)^2-p_0^2]}, \tag{50}$$

$$I_1(p_0^2) = \int\frac{d^3q}{(2\pi)^3}\frac{E^+}{E_\Delta^+[4(E_\Delta^+)^2-p_0^2]} - \int\frac{d^3q}{(2\pi)^3}\frac{E^-}{E_\Delta^-[4(E_\Delta^-)^2-p_0^2]}. \tag{51}$$

The mass spectrum is defined by the equation

$$\det\Gamma^{2s}(p_0) \equiv 4p_0^2\{(p_0^2-4\Delta^2)I_0^2(p_0^2)-4I_1^2(p_0^2)\} = 0. \tag{52}$$

In the $p_0^2$ plane this equation has an evident zero, corresponding to a Nambu-Goldstone boson, $p_0^2 = 0$. Detailed

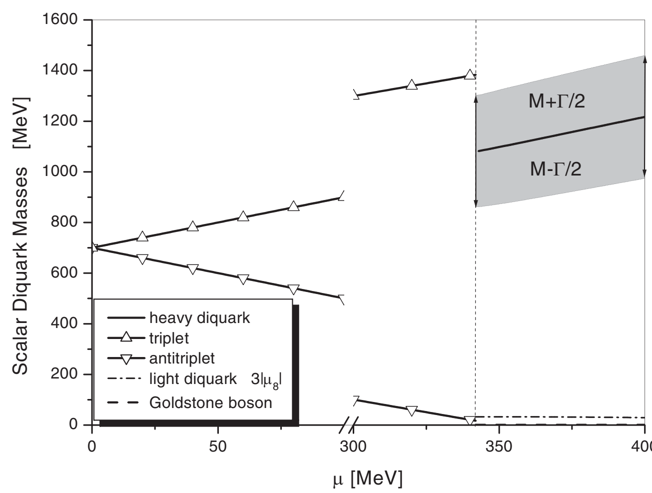


FIG. 4. The masses of diquarks. At $\mu < \mu_c = 342$ MeV, six diquark states are split into a (color) triplet of heavy states with mass $M_{\Delta^*}$ and an antitriplet of light states with mass $M_\Delta$ [see (55)]. In the 2SC phase ($\mu > \mu_c$), one observes the Nambu-Goldstone boson, four light diquarks with the mass $3|\mu_8|$, and a heavy singlet state with the mass $M$ (solid line). The shaded rectangle displays the width $\Gamma$ of the heavy singlet resonance; its upper border is a half-width higher than the mass, and the bottom border is a half-width lower.

investigation, similar to that from Ref. [9], shows that on the second Riemann sheet of $p_0^2$ there is another zero of (52) which corresponds to a heavy resonance. Its mass $M$ and width $\Gamma$ are depicted in Fig. 4.

As a result, we conclude that in the 2SC phase there are four light real scalar diquark excitations with mass $3|\mu_8|$ and one Nambu-Goldstone boson, which appears due to a spontaneous breaking of the $SU_c(2) \times U_{\lambda_8}(1)$ color symmetry down to $SU_c(2)$. Moreover, a heavy scalar diquark resonance, which is an $SU_c(2)$ singlet, is also presented in the mass spectrum of the model at $\mu > \mu_c = 342$ MeV (see Fig. 4).

### B. Scalar diquarks in the normal phase ($\Delta = 0$, $\mu_8 = 0$)

In the $SU_c(3)$-symmetric phase ($\mu < \mu_c$), the diquark gap $\Delta$ is zero and the three complex diquark fields $\Delta_A^s(x)$ ($A = 2, 5, 7$) are not mixed with other fields in the second order effective action of the model. Moreover, at $\Delta = 0$, as is easily seen from the color neutrality constraint (22), we have $\mu_8 = 0$. So, in order to study the diquark masses, it is enough to consider, e.g., the $\Delta_5^s$-diquark sector. In this phase the determinant of the inverse propagator matrix $\Gamma^{5s}(p_0)$ looks like (we use the rest frame, where $\vec{p} = 0$)

$$\det\Gamma^{5s}(p_0) = \Gamma^{5s}_{\Delta_5^{s*}\Delta_5^s}(p_0)\Gamma^{5s}_{\Delta_5^s\Delta_5^{s*}}(p_0) = \Gamma^{5s}_{\Delta_5^{s*}\Delta_5^s}(p_0)\Gamma^{5s}_{\Delta_5^{s*}\Delta_5^s}(-p_0), \quad (53)$$

where $\Gamma^{5s}_{\Delta_5^{s*}\Delta_5}(p_0)$ is presented in (45). [Note that the last equality in formula (48) for $\det\Gamma^{5s}(p_0)$ in the 2SC phase is based on the usage of a nontrivial solution $\Delta \neq 0$ of the gap equation (20). So, in the normal $SU_c(3)$-symmetric phase, where $\Delta = 0$, it is not valid.] Taking into account the relation $m > \mu$ that is realized in the normal phase only (see Fig. 1), we see that $E \equiv \sqrt{\vec{q}^2 + m^2} > \mu$ and, as a consequence, $E^\pm > 0$ are fulfilled in this phase. So one can easily integrate in (45) over $q_0$ and obtain the following expression that is suitable only for $SU_c(3)$-symmetric phase:

$$\Gamma^{5s}_{\Delta_5^{s*}\Delta_5^s}(p_0) = \frac{1}{4H_s} - 16\int \frac{d^3q}{(2\pi)^3}\frac{E}{4E^2 - (p_0 + 2\mu)^2} \equiv \frac{1}{4H_s} - F_s(\epsilon), \quad (54)$$

where $\epsilon = (p_0 + 2\mu)^2$. Clearly, the diquark mass spectrum is defined by the equation $\det\Gamma^{5s}(p_0) = 0$ or by zeros of (54), where the function $F_s(\epsilon)$ is analytical in the whole complex $\epsilon$ plane, except for the cut $4m^2 < \epsilon$ along the real axis. (In general, $F_s(\epsilon)$ is defined on a complex Riemann surface which is to be described by several sheets. However, a direct numerical computation based on Eq. (54) gives its values on the first sheet only [we use the parameter set (23)]. To find a value on the rest of the Riemann surface, a special procedure of continuation is needed.) The numerical analysis of (54) on the first Riemann sheet shows that the equation $\Gamma^{5s}_{\Delta_5^{s*}\Delta_5}(p_0) = 0$ has a root ($\epsilon_0$) on the real axis ($0 < \epsilon_0 < 4m^2$), providing us with the following massive diquark modes which are the solutions of Eq. (53):

$$(M_\Delta^s)^2 = (1.998m - 2\mu)^2, \quad (M_{\Delta^*}^s)^2 = (1.998m + 2\mu)^2. \quad (55)$$

We relate $M_\Delta^s$ in (55) to the mass of the diquark with the baryon number $B = 2/3$ and $M_{\Delta^*}^s$ to the mass of the antidiquark with $B = -2/3$. (Qualitatively, a similar behavior of diquark and antidiquark masses vs $\mu$ was obtained in Ref. [31] in the NJL model with two-colored quarks.) The difference between diquark and antidiquark masses in (55) is explained by the absence of a charge conjugation symmetry in the presence of a chemical potential.

Finally, due to the underlying color $SU(3)_c$ symmetry, the previous statement is valid also for $\Delta_5^*$, $\Delta_5$ and $\Delta_7^*$, $\Delta_7$. As a result, we have a color antitriplet of diquarks with the mass $M_\Delta^s$ (55) as well as a color triplet of antidiquarks with the mass $M_{\Delta^*}^s$. The results of numerical computations are presented in Fig. 4 for $\mu < \mu_c = 342$ MeV.

Recall that, in our analysis, we have used the constraint $H_s = 3G/4$, thereby fixing the constant $H_s$ through $G$. It is useful, however, to discuss now the influence of $H_s$ on diquark masses. Indeed, it is clear from (54) that the root $\epsilon_0$ lies inside the interval $0 < \epsilon_0 < 4m^2$ only if $H_s^* < H_s < H_s^{**}$, where $H_s^*$ and $H_s^{**}$ are defined by

$$H_s^* \equiv \frac{1}{4F_s(4m^2)} = \frac{\pi^2}{4[\Lambda\sqrt{m^2 + \Lambda^2} + m^2\ln((\Lambda + \sqrt{m^2 + \Lambda^2})/m)]},$$
$$H_s^{**} \equiv \frac{1}{4F_s(0)} = \frac{\pi^2}{4[\Lambda\sqrt{m^2 + \Lambda^2} - m^2\ln((\Lambda + \sqrt{m^2 + \Lambda^2})/m)]} = \frac{3mG}{2(m - m_0)}. \quad (56)$$

In this case, there are stable diquarks and antidiquarks in the color symmetric phase. The behavior of their masses qualitatively resembles that given by Eqs. (55). For a rather weak interaction in the diquark channel ($H_s < H_s^*$), $\epsilon_0$ runs onto the second Riemann sheet, and unstable diquark modes (resonances) appear. Unlike this, a sufficiently strong interaction in the diquark channel ($H_s > H_s^{**}$) pushes $\epsilon_0$ towards the negative semiaxis, i.e. $(p_0 + 2\mu)^2 < 0$. The latter indicates a tachyon singularity in the diquark propagator, evidencing that the $SU_c(3)$-color symmetric ground state is not stable. Indeed, at a very large $H_s$, as has been shown in Ref. [32] at $H_p = 0$, the color symmetry is spontaneously broken even at a vanishing chemical potential. We guess that this result remains true at rather small values of $H_p$ ($H_p < H_s$) as well, justifying the above mentioned tachyon singularity of the diquark propagator.

## VI. MASSES OF PSEUDOSCALAR DIQUARKS

It is clear from (35) that in the framework of the NJL model (2) pseudoscalar diquarks are not mixing with each other as well as with meson and scalar diquark fields. So, to get their masses, we will start from the general expression (41) for the inverse propagator matrices of pseudoscalar diquarks.

### A. Pseudoscalar diquarks in the 2SC phase ($\Delta \neq 0$, $\mu_8 \neq 0$)

In the 2SC phase there is an $SU(2)_c$ symmetry between $\Delta_5^p$-$\Delta_5^{p*}$ and $\Delta_7^p$-$\Delta_7^{p*}$ sectors, so it is enough to study an inverse propagator, e.g., in one of these sectors. For $A = 5$ it follows from (41) that

$$\Gamma^{5p}_{PQ}(x-y) = -\frac{\delta^2 S^{(2)}_{p55}}{\delta Q(y)\delta P(x)}, \tag{57}$$

where $P(x), Q(x) = \Delta_5^p(x), \Delta_5^{p*}(x)$. Further, using the evident expression (37) for $S^{(2)}_{p55}$, one can obtain the matrix elements (57) in the rest frame, $\vec{p} = 0$, of the momentum space representation

$$\begin{aligned}\Gamma^{5p}_{\Delta_5^{p*}\Delta_5^p}(p_0) = \frac{1}{4H_p} + 4\int\frac{d^3q}{(2\pi)^3}\Big\{\frac{m^2}{E^2}\Big[\frac{(\breve{E}^- - p_0 + E^+)\theta(-\breve{E}^-)}{(p_0 - \breve{E}^-)^2 - (E^+_\Delta)^2} - \frac{E^+ - E^+_\Delta}{2(p_0 - \breve{E}^- - E^+_\Delta)E^+_\Delta} - \frac{(\breve{E}^+ + p_0 + E^-)\theta(\breve{E}^+)}{(p_0 + \breve{E}^+)^2 - (E^-_\Delta)^2}\\ + \frac{E^- + E^-_\Delta}{2(p_0 + \breve{E}^+ - E^-_\Delta)E^-_\Delta}\Big] + \frac{\vec{q}^2}{E^2}\Big[\frac{(E^+ - \breve{E}^+ - p_0)\theta(\breve{E}^+)}{(p_0 + \breve{E}^+)^2 - (E^+_\Delta)^2} - \frac{E^+ - E^+_\Delta}{2(p_0 + \breve{E}^+ - E^+_\Delta)E^+_\Delta}\\ + \frac{(\breve{E}^- - p_0 - E^-)\theta(-\breve{E}^-)}{(p_0 - \breve{E}^-)^2 - (E^-_\Delta)^2} + \frac{E^- + E^-_\Delta}{2(p_0 - \breve{E}^- - E^-_\Delta)E^-_\Delta}\Big]\Big\},\end{aligned} \tag{58}$$

$\Gamma^{5p}_{\Delta_5^p\Delta_5^{p*}}(p) = \Gamma^{5p}_{\Delta_5^{p*}\Delta_5^p}(-p)$, and $\Gamma^{5p}_{\Delta_5^p\Delta_5^p}(p) = \Gamma^{5p}_{\Delta_5^{p*}\Delta_5^{p*}}(p) = 0$. Let $1/4H_p = (1/4H_s) + \eta$. Then, using for $1/4H_s$ the gap equation (20) (recall that in the 2SC phase $\Delta \neq 0$), we obtain from (58) [transforming the multiplier before the second square bracket in (58) as $\vec{q}^2/E^2 = 1 - m^2/E^2$]:

$$\Gamma^{5p}_{\Delta_5^{p*}\Delta_5^p}(p_0) = \eta + 2(p_0 - 3\mu_8)H(p_0) + m^2\tilde{H}(p_0), \tag{59}$$

where $H(p_0)$ is defined in (47). Since in the 2SC phase $m \ll \Delta$ (or $m$ even a zero if $m_0 = 0$), we ignore the last term in (59) [due to this reason, an explicit form of $\tilde{H}(p_0)$ is not presented here] and obtain in this way the following expression for the determinant of the matrix $\Gamma^{5p}(p_0)$:

$$\begin{aligned}\det\Gamma^{5p}(p_0) &= \Gamma^{5p}_{\Delta_5^{p*}\Delta_5^p}(p_0)\Gamma^{5p}_{\Delta_5^p\Delta_5^{p*}}(p_0) = \Gamma^{5p}_{\Delta_5^{p*}\Delta_5^p}(p_0)\Gamma^{5p}_{\Delta_5^{p*}\Delta_5^p}(-p_0) = [\eta + 2(p_0 - 3\mu_8)H(p_0)][\eta - 2(p_0 + 3\mu_8)H(-p_0)]\\ &\approx [\eta + 2(p_0 - 3\mu_8)^2\beta][\eta + 2(p_0 + 3\mu_8)^2\beta],\end{aligned} \tag{60}$$

where $\beta = dH(p_0)/dp_0|_{p_0=3\mu_8}$. In the last relation in (60), we have expanded the function $H(p_0)$ into a Taylor series of $p_0$ at the point $p_0 = 3\mu_8$ and took into account that $H(3\mu_8)$ equals zero under the color neutrality constraint (see the end of Sec. VA). Solving in this approximation the equation $\det\Gamma^{5p}(p_0) = 0$, it is possible to find pseudoscalar diquark excitations with two different masses:

$$M^p_{D1} = |3\mu_8 + \sqrt{-\eta/(2\beta)}|, \qquad M^p_{D2} = \sqrt{-\eta/(2\beta)} - 3\mu_8. \tag{61}$$

Since $\beta < 0$ [as is easily seen from (47)], we conclude from formulas (61) that at $\eta > 0$ both $M^p_{D1}$ and $M^p_{D2}$ are real (positive) quantities, suggesting that these pseudoscalars are stable particles at $H_p \leq H_s$. [The case $\eta < 0$, which corresponds to an unstable 2SC ground state, will be discussed in detail below, after (66).] The similar is true in the $\Delta_7^p$-$\Delta_7^{p*}$ sector of the model, so in the whole $\Delta_5^p$-$\Delta_5^{p*}$, $\Delta_7^p$-$\Delta_7^{p*}$ sector of the NJL model which is under the color neutrality constraint, there are four massive pseudoscalar excitations: two of them form an $SU_c(2)$ antidoublet with mass $M^p_{D1}$; another two particles form an $SU_c(2)$ antidoublet with mass $M^p_{D2}$.

Now let us consider the 2SC ground state excitations in the $\Delta_2^p$-$\Delta_2^{p*}$ sector. In this case, starting from the effective action (39), it is possible to obtain the inverse propagator matrix $\Gamma^{2p}$ which is defined by the relation (41). In the rest frame, where $p = (p_0, 0, 0, 0)$, its Fourier-transformed matrix elements look like

$$\Gamma^{2p}_{\Delta_2^{p*}\Delta_2^p}(p_0) = \frac{1}{4H_p} - 4i\int\frac{d^4q}{(2\pi)^4}\left\{\frac{2m^2}{E^2}\cdot\frac{p_0+q_0-E^+}{(p_0+q_0)^2-(E_\Delta^+)^2}\cdot\frac{q_0-E^-}{q_0^2-(E_\Delta^-)^2}+\frac{\vec{q}^2}{E^2}\left[\frac{p_0+q_0-E^+}{(p_0+q_0)^2-(E_\Delta^+)^2}\cdot\frac{q_0+E^+}{q_0^2-(E_\Delta^+)^2}\right.\right.$$
$$\left.\left.+\frac{p_0+q_0+E^-}{(p_0+q_0)^2-(E_\Delta^-)^2}\cdot\frac{q_0-E^-}{q_0^2-(E_\Delta^-)^2}\right]\right\}, \tag{62}$$

$$\Gamma^{2p}_{\Delta_2^p\Delta_2^{p*}}(p_0) = \Gamma^{2p}_{\Delta_2^{p*}\Delta_2^p}(-p_0), \qquad \Gamma^{2p}_{\Delta_2^p\Delta_2^p}(p_0) = \Gamma^{2p}_{\Delta_2^{p*}\Delta_2^{p*}}(p_0) = -4\Delta^2\mathbb{P}(p_0),$$
$$\mathbb{P}(p_0) = \int\frac{d^4q}{i(2\pi)^4}\left\{\frac{2m^2}{E^2}\cdot\frac{1}{(p_0+q_0)^2-(E_\Delta^+)^2}\cdot\frac{1}{q_0^2-(E_\Delta^-)^2}+\frac{\vec{q}^2}{E^2}\left[\frac{1}{(p_0+q_0)^2-(E_\Delta^+)^2}\cdot\frac{1}{q_0^2-(E_\Delta^+)^2}\right.\right.$$
$$\left.\left.+\frac{1}{(p_0+q_0)^2-(E_\Delta^-)^2}\cdot\frac{1}{q_0^2-(E_\Delta^-)^2}\right]\right\}. \tag{63}$$

Using in (62) and (63) the substitution $1/4H_p = (1/4H_s) + \eta$ and then eliminating the coupling constant $H_s$ in favor of another model parameter [with the help of the gap equation (20)], we obtain after $q_0$ integrations:

$$\mathbb{P}(p_0) = I_0(p_0^2) + m^2A(p_0),$$
$$\Gamma^{2p}_{\Delta_2^{p*}\Delta_2^p}(p_0) = \eta + (4\Delta^2 - 2p_0^2)I_0(p_0^2) + 4p_0I_1(p_0^2) + m^2B(p_0), \tag{64}$$

where $I_0(p_0^2)$ and $I_1(p_0^2)$ are presented in (50) and (51), respectively. The last terms in each of expressions (64) are proportional to $m^2$. In the 2SC phase the constituent quark mass $m$ is a vanishingly small quantity (or it is exactly zero if the current quark mass vanishes, $m_0 = 0$) as compared with $\Delta$, etc. (see Fig. 1), so we will ignore the contributions of these terms in the matrix elements (62) and (63). Thus, there is no need to have explicit expressions for the functions $A(p_0)$ and $B(p_0)$ from (64). In this approximation the determinant of the inverse propagator matrix $\Gamma^{2p}(p_0)$ looks like:

$$\begin{aligned}\det\Gamma^{2p}(p_0) &= \Gamma^{2p}_{\Delta_2^{p*}\Delta_2^p}(p_0)\Gamma^{2p}_{\Delta_2^p\Delta_2^{p*}}(p_0) - \Gamma^{2p}_{\Delta_2^p\Delta_2^p}(p_0)\Gamma^{2p}_{\Delta_2^{p*}\Delta_2^{p*}}(p_0)\\ &= [\eta - 2p_0^2I_0(p_0^2)][\eta - 2(p_0^2 - 4\Delta^2)I_0(p_0^2)] - 16p_0^2I_1^2(p_0^2).\end{aligned} \tag{65}$$

Obviously, at $\eta = 0$ the equation $\det\overline{\Gamma^{2p}}(p_0) = 0$ coincides with Eq. (52) and has the same solutions. The first one, $p_0^2 = 0$, corresponds to a stable massless pseudoscalar excitation; the second one lies in the second Riemann sheet of the variable $p_0^2$. So it is a heavy pseudoscalar resonance and its mass and width are represented in Fig. 4. At small nonzero values of $\eta$, it is reasonable to suppose that the equation $\det\overline{\Gamma^{2p}}(p_0) = 0$ has a root lying on the second Riemann sheet of $p_0^2$ as well. It might be considered as a weak disturbance of the resonance solution of this equation at $\eta = 0$, so its mass and width behavior vs $\mu$ are qualitatively the same as in Fig. 4. Another solution of this equation, $p_0^2 = (M^p_{D3})^2$, should not be significantly different from the solution $p_0^2 = 0$ at $\eta = 0$. So, in searching for $(M^p_{D3})^2$, one can expand the expression (65) into a Taylor series of $p_0^2 = 0$:

$$(M^p_{D3})^2 = \frac{\eta(\eta + 8\Delta^2a)}{16b^2 + 2a(\eta + 8\Delta^2a) + 2\eta(a - 4\Delta^2a')}, \tag{66}$$

where $a = I_0(0)$, $b = I_1(0)$, $a' = I_0'(0)$. Note that both the heavy resonance and stable excitation with mass squared (66) in the pseudoscalar diquark channel are singlets with respect to $SU_c(2)$. Since $a > 0$, the expression (66) is a positive one at rather small and positive values of $\eta$. However, at sufficiently small but negative values of $\eta$, it is a negative quantity; i.e. a tachyonic pseudodiquark excitation appeared in the model. This fact indicates the instability of the 2SC ground state with $\langle\Delta_2^s(x)\rangle = \Delta$, $\langle\Delta_5^s(x)\rangle = 0$, $\langle\Delta_7^s(x)\rangle = 0$, and $\langle\Delta_A^p(x)\rangle = 0$. Perhaps, in this case, i.e. at $H_p > H_s$, the phase with nonzero ground state expectation values of pseudoscalar diquarks, $\langle\Delta_A^p(x)\rangle \neq 0$, should be realized.

As a result, we have shown that in the 2SC phase of the NJL model (2) there are five stable diquark excitations in the pseudoscalar channel. They form a singlet as well as two antidoublets (in the case $H_p < H_s$) of the $SU_c(2)$ group with masses presented in (66) and (61), correspondingly. Moreover, there is also a heavy resonance that is $SU_c(2)$ singlet with mass about 1100 MeV in this channel.

### B. The case of normal phase ($\Delta = 0$, $\mu_8 = 0$, $m \neq 0$)

Suppose that we are in the $SU_c(3)$-symmetric (normal) phase of our model, where $\Delta = 0$. As is easily seen from the color neutrality constraint (22), in this phase $\mu_8 = 0$. Moreover, here $\breve{E}^\pm = E^\pm = E \pm \mu > 0$, since in this phase $m > \mu$ (see Fig. 1 at $\mu < \mu_c$). Then, without loss of generality, it is sufficient to study the mass spectrum, e.g., in the sector of $\Delta_5^p$-$\Delta_5^{p*}$ diquarks. For the normal phase we have from (58):

$$\Gamma^{5p}_{\Delta_5^{p*}\Delta_5^p}(p_0) = \frac{1}{4H_p} - 16\int \frac{d^3q}{(2\pi)^3}\frac{\vec{q}^2}{E}\frac{1}{4E^2 - (p_0 + 2\mu)^2} \equiv \frac{1}{4H_p} - F_p(\epsilon), \quad (67)$$

where $\epsilon = (p_0 + 2\mu)^2$ and the function $F_p(\epsilon)$ is increasing on the interval $(-\infty, 4m^2)$. Moreover, it is analytical in the whole complex $\epsilon$ plane, except for the cut $4m^2 < \epsilon$ along the real axis. The masses of pseudoscalar diquarks are defined by the equation

$$\det\Gamma^{5p}(p_0) = \Gamma^{5ps}_{\Delta_5^{p*}\Delta_5^p}(p_0)\Gamma^{5p}_{\Delta_5^p\Delta_5^{p*}}(p_0) = \Gamma^{5p}_{\Delta_5^{p*}\Delta_5^p}(p_0)\Gamma^{5p}_{\Delta_5^{p*}\Delta_5^p}(-p_0) = 0, \quad (68)$$

i.e. by zeros $\epsilon_0$ of the matrix element (67) such that $0 < \epsilon_0 < 4m^2$ or lying in the second Riemann sheet of the complex variable $\epsilon$. (The first one corresponds to masses of stable excitations, the second one to masses of resonances.) In the present consideration, we restrict ourselves to looking only for stable pseudoscalar diquarks. It is clear that there is a single zero $\epsilon_0$ of (67), obeying the condition $0 < \epsilon_0 < 4m^2$, if and only if the coupling constant $H_p$ is constrained by the relation $H_p^* < H_p < H_p^{**}$, where

$$H_p^* = \frac{1}{4F_p(4m^2)} = \frac{\pi^2}{4[\Lambda\sqrt{m^2+\Lambda^2} - m^2\ln((\Lambda + \sqrt{m^2+\Lambda^2})/m)]},$$
$$H_p^{**} = \frac{1}{4F_p(0)} = \frac{\pi^2\Lambda\sqrt{m^2+\Lambda^2}}{4[3m^2\Lambda^2 + \Lambda^4 - 3m^2\Lambda\sqrt{m^2+\Lambda^2}\ln((\Lambda + \sqrt{m^2+\Lambda^2})/m)]}. \quad (69)$$

[Note that $H_p^* = H_s^{**}$ from (56).] In this case, the masses of stable pseudoscalar diquarks and antidiquarks are the following:

$$(M_\Delta^p)^2 = (\sqrt{\epsilon_0} - 2\mu)^2, \qquad (M_{\Delta^*}^p)^2 = (\sqrt{\epsilon_0} + 2\mu)^2, \quad (70)$$

respectively. [The mass splitting in (70) is again explained by the absence of a charge conjugation symmetry in the presence of a chemical potential.] It follows from the underlying color $SU(3)_c$ symmetry of the normal phase that, at $H_p^* < H_p < H_p^{**}$, there is indeed a color antitriplet of pseudoscalar diquarks with the mass $M_\Delta^p$ as well as a color triplet of pseudoscalar antidiquarks with the mass $M_{\Delta^*}^p$ [see (70)]. For other regions of the $H_p$ values, stable pseudoscalar diquark excitations of the $SU(3)_c$-color symmetric ground state are forbidden. For a rather weak interaction in this channel ($H_p < H_p^*$), $\epsilon_0$ runs onto the second Riemann sheet, and unstable pseudoscalar diquark modes (resonances) appear. Unlike this, a sufficiently strong interaction in this channel ($H_p > H_p^{**}$) pushes $\epsilon_0$ towards the negative semiaxis, i.e. $(p_0 + 2\mu)^2 < 0$. The later indicates a tachyonic singularity in the pseudoscalar diquark propagator, evidencing that the $SU(3)_c$ color symmetric ground state is not stable. In this case we guess that a parity-breaking color superconducting phase is realized in which the ground state expectation values of pseudoscalar diquarks are not zero, $\langle\Delta_A^p(x)\rangle \neq 0$.

Finally, a few words about diquarks in the particular case (recall that $\mu < \mu_c$) $H_s = H_p \equiv H$, which corresponds to a NJL model inspired by a one-gluon exchange approximation in QCD. In this case we see that, if $0 < H < H_s^*$, then both scalar and pseudoscalar diquarks are resonances. If $H_s^* < H < H_s^{**}$, then in the normal phase, including the case $\mu = 0$, only scalar diquarks are stable, but pseudoscalar ones are unstable particles. For larger values of $H$, the normal phase is unstable in itself, since either or both the scalar diquark propagator (at $H_s^{**} = H_p^* < H < H_p^{**}$) and the pseudoscalar one (at $H_p^{**} < H$) have tachyonic singularities. Hence, one can conclude that at $H_s = H_p \equiv H$ scalar diquarks are allowed to exist, at $H_s^* < H < H_s^{**}$, as stable excitations of the normal phase. Pseudoscalar diquarks in this phase are always unstable particles (resonances).

## VII. SUMMARY AND DISCUSSION

In our previous papers [8,9], the masses of mesons and diquarks, surrounded by moderately dense quark matter, were investigated in the framework of NJL model (1) at $H_p = 0$, and the color neutrality constraint was missed, for simplicity. In the present paper, we have calculated the mass spectrum of meson and diquark excitations in the color neutral cold dense quark matter. We started from a low-energy Nambu–Jona-Lasinio–type effective model (2) for quarks of two flavors, with a quark chemical potential $\mu$, and extended by including the chemical potential $\mu_8$ of the 8th color charge. Moreover, the interaction in the pseudoscalar diquark channel was taken into account, in addition. We considered only the interplay between normal and 2SC phases. This is a quite reasonable assumption in the framework of the model (1). Then it was shown that, in the presence of color neutrality, the transition to the 2SC phase occurred at a somewhat smaller value of the quark chemical potential ($\mu_c = 342$ MeV) than without this constraint ($\mu_c = 350$ MeV).

It was proved in the present paper that in both models (1) and (2), i.e. with or without a color neutrality constraint, the $\sigma$ meson is mixed with the scalar diquark $\Delta_2^s$ in the 2SC phase. In the previous paper [9], this mixing was ignored in the consideration of the $\sigma$-meson mass. At first, we have found that, if $\sigma$-$\Delta_2^s$ mixing is ignored as in Ref. [9], then the color neutrality requirement does not change qualitatively the properties of $\pi$ and $\sigma$ mesons, obtained in the framework of NJL model (1) without the $\mu_8$ term. This is an expected result, since both models (1) and (2) have an identical chiral symmetry. Hence, at small values of $\mu_8$ (see Fig. 2) the meson masses acquire small corrections as well (compare Fig. 2 from Ref. [9] and Fig. 3 of our present paper). It follows from our consideration that in this case (without mixing) both $\sigma$ and $\pi$ mesons are stable particles in the 2SC phase with masses of about 340 Mev (Fig. 3). However, if mixing is taken into account, then in the 2SC phase the $\sigma$ meson is a resonance, decaying into a pair of quarks with a rather small width $\sim$30 MeV (see the appendix). As far as we know, the properties of $\pi$ and $\sigma$ mesons in the 2SC phase have not been discussed in the literature before.

Moreover, the properties of scalar diquarks in the 2SC phase are changed drastically, when the color neutrality condition is imposed. Indeed, for the model (1) we have found in the 2SC phase an anomalous number of three Nambu-Goldstone bosons, the $\mathrm{SU}_c(2)$ antidoublet of light diquarks, and a heavy resonance that is an $\mathrm{SU}_c(2)$ singlet [8,9]. Contrarily, our present investigation shows that in the model (2) the scalar diquark sector of the 2SC phase contains two real $\mathrm{SU}_c(2)$ antidoublets of light excitations with the same mass $3|\mu_8|$, one Nambu-Goldstone boson, and a heavy resonance with mass about 1100 MeV (see Fig. 4). To understand such a sharp difference in scalar diquark masses, predicted by these two models, it is necessary to compare their color symmetries. The first model, Lagrangian (1), is invariant under $\mathrm{SU}_c(3)$. However, in the second model (2) this symmetry is broken *explicitly*, due to the presence of the $\mu_8$ term, to the subgroup $\mathrm{SU}_c(2) \times \mathrm{U}_{\lambda_8}(1)$. Then, in the 2SC phase, where the ground state is an $\mathrm{SU}_c(2)$ invariant for both models [here $\langle \Delta_2^s(x) \rangle \neq 0$, $\langle \Delta_{5,7}^s(x) \rangle = 0$], we have spontaneous breaking of the above mentioned symmetries. As a consequence, there are five broken symmetry generators and an abnormal number of three NG bosons for the model (1) (the explanation of this fact is presented in detail in Ref. [8]). On the other hand, for model (2) we have in the 2SC phase only one broken $U_{\lambda_8}$-symmetry generator, resulting in a single NG boson.

The properties of the pseudoscalar diquarks in the 2SC phase depend essentially on the relation between coupling constants $H_s$ and $H_p$. First, note that at $H_p > H_s$ the 2SC phase is an unstable one due to a negative mass squared (66) of an $\mathrm{SU}_c(2)$-singlet pseudoscalar mode (tachyonic instability). At $H_p < H_s$ the pseudoscalar excitations of this channel form in the 2SC phase two real stable $\mathrm{SU}_c(2)$ antidoublets with different masses (61), as well as the stable light $\mathrm{SU}_c(2)$ singlet with mass (66) and a heavy resonance with mass $\sim$1100 MeV.

We have also found that the antidiquark masses exceed those of the diquarks in the normal $\mathrm{SU}_c(3)$ symmetric phase (for $\mu < \mu_c \approx 342$ MeV). This splitting of the masses is explained by the violation of $C$ parity (charge conjugation) in the presence of a chemical potential. In contrast, at $\mu = 0$ the model is $C$-invariant and all diquarks and antidiquarks of the same parity have an identical mass. It follows from our investigation that stable quark pair formation occurs in the scalar channel at a weaker coupling strength than in the pseudoscalar one. Indeed, if the parameter set of the model is fixed by the relations (23), then in the normal phase scalar diquarks are stable particles (with masses about 700 MeV at $\mu = 0$), since in this case $H_s^* < H_s = 3G/4 < H_s^{**}$ [see (56)]. However, only at sufficiently high values of $H_p$, i.e. at $H_s^{**} = H_p^* < H_p < H_p^{**}$ [see (69)], might stable pseudoscalar diquarks exist in the normal phase. On the other hand, if $H_p < H_s = 3G/4$, then in the normal phase pseudoscalar diquarks are resonances, decaying into two quarks. If $\mu$ exceeds the critical value $\mu_c$ and the system passes to the 2SC phase at $H_p < H_s = 3G/4$, then five of six pseudoscalar diquarks acquire stability. Really, in the 2SC phase, in contrast to the normal one, all particles move inside the medium, so their decay might be prohibited by a Pauli blocking principle (Mott effect). Suppose that $H_p = H_s \equiv H$ (in this case the NJL model is inspired by a one-gluon exchange approximation in QCD). Then at $H < H_s^{**}$ the normal phase is a stable one, but at $H > H_s^{**}$ a tachyonic instability appears, so the normal phase is destroyed (see Sec. V B). We have shown for this particular case that in the normal phase there might exist stable scalar diquarks; however, pseudoscalar diquark modes are always unstable excitations.

Of course, all observable particles render themselves as colorless objects in the hadronic phase, and the diquarks are expected to be confined, as they are no $\mathrm{SU}(3)_c$ color singlets. Nevertheless, one may look at our and other related results on diquark masses as an indication of the existence of rather strong quark-quark correlations inside baryons, which might help to explain baryon dynamics. Some lattice simulations reveal strong attraction in the diquark channel [33] with a diquark mass $\sim$600 MeV. Recently, in Ref. [34], the mass and extremely narrow width, as well as other properties, of the pentaquark $\Theta^+$ were explained just on the assumption that it is composed of an antiquark and two highly correlated *ud* pairs. At the present time, the nature of the mechanism which may entail strong attraction of quarks in diquark channels is actively discussed both in the nonperturbative QCD and in other models (see, e.g., [35], and references therein).

For simplicity, we have studied the masses of the one-particle excitations of the color neutral quark matter. Similar investigations can be performed in both the color neutral and the $\beta$-equilibrated two-flavor NJL models, where electric charge neutrality is required in addition, so one more (electric charge) chemical potential $\mu_Q$ must be introduced. In this case, for some range of the coupling constant $H_s$ the gapless 2SC phase (g2SC) is realized [see, e.g., [4], where the particular case $H_p = 0$ of the NJL model (1) was considered]. In contrast to the ordinary 2SC phase, two additional gapless quark excitations then appeared in the g2SC phase, but its meson and diquark mass spectrum, presumably, will not change qualitatively. Our belief is based on the structure of the ground state expectation values of scalar diquarks in the g2SC phase, i.e. $\langle\Delta_2^s(x)\rangle \equiv \Delta \neq 0$, $\langle\Delta_{5,7}^s(x)\rangle = 0$. This is expected to be identical to that of our present consideration when only the color neutrality of the quark matter is taken into account.

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## APPENDIX: CORRECTION TO THE $\sigma$-MESON MASS

In Sec. IV the numerical values for the $\sigma$-meson mass $M_\sigma$ was obtained in the assumption that the mixing of $\sigma$ with $\Delta_2^s$ is absent. Moreover, we have shown in this approach that the $\sigma$ meson is a stable particle in the 2SC phase. Now let us prove that a deeper investigation of the $\sigma$-meson mass, based on the inclusion of the mixing between $\sigma$ and $\Delta_2^s$, results in the conclusion that in the 2SC phase $\sigma$ is a resonance, having a small decay width into a quark pair.

Indeed, starting from the total number of effective actions (18), (28), and (38) it is possible to find the $3\times 3$ inverse propagator matrix $\mathcal{G}^{-1}$ of $\sigma$, $\Delta_2^s$, and $\Delta_2^{s*}$ fields. In the center of mass frame of the momentum space representation ($\vec{p}=0$), it has the following form in the 2SC phase:

$$\mathcal{G}^{-1}(p_0) = \begin{pmatrix} \Gamma(p_0), & \Gamma_{\sigma\Delta_2^s}(p_0), & \Gamma_{\sigma\Delta_2^{s*}}(p_0) \\ \Gamma_{\Delta_2^s\sigma}(p_0), & \Gamma^{2s}_{\Delta_2^s\Delta_2^s}(p_0), & \Gamma^{2s}_{\Delta_2^s\Delta_2^{s*}}(p_0) \\ \Gamma_{\Delta_2^{s*}\sigma}(p_0), & \Gamma^{2s}_{\Delta_2^{s*}\Delta_2^s}(p_0), & \Gamma^{2s}_{\Delta_2^{s*}\Delta_2^{s*}}(p_0) \end{pmatrix}, \tag{A1}$$

where

$$\Gamma_{\sigma\Delta_2^s}(p_0) = \Gamma_{\Delta_2^{s*}\sigma}(p_0) = \Gamma_{\sigma\Delta_2^{s*}}(-p_0) = \Gamma_{\Delta_2^s\sigma}(-p_0) = 4m\Delta\int\frac{d^3q}{(2\pi)^3}\bigg\{\frac{2E^+ + p_0}{EE_\Delta^+[4(E_\Delta^+)^2 - p_0^2]} + \frac{2E^- - p_0}{EE_\Delta^-[4(E_\Delta^-)^2 - p_0^2]}\bigg\}. \tag{A2}$$

Other matrix elements from (A1) are presented in (32) and (49). Note that the matrix elements (A2), mixing $\sigma$ with $\Delta_2^s$ and $\Delta_2^{s*}$, are proportional to the dynamical quark mass $m$. So in the 2SC phase both $m$ and these matrix elements may be considered as small quantities (see Fig. 1). The mass spectrum is defined by the equation $\det(\mathcal{G}^{-1}(p_0)) = 0$, which has a rather complicated form [note that $\det(\mathcal{G}^{-1}(p_0))$ is an even function vs $p_0$, i.e. it depends on $p_0^2$]:

$$F(p_0^2)\Gamma_0(p_0^2) = m^2\Delta^2 f(p_0^2), \tag{A3}$$

where

$$\begin{aligned} F(p_0^2) &= \Gamma^{2s}_{\Delta_2^s\Delta_2^s}\Gamma^{2s}_{\Delta_2^{s*}\Delta_2^{s*}} - \Gamma^{2s}_{\Delta_2^{s*}\Delta_2^s}\Gamma^{2s}_{\Delta_2^s\Delta_2^{s*}}, \\ m^2\Delta^2 f(p_0^2) &= \Gamma_{\Delta_2\sigma}\Gamma_{\sigma\Delta_2}\Gamma^{2s}_{\Delta_2^*\Delta_2^*} + \Gamma_{\sigma\Delta_2^*}\Gamma_{\Delta_2^*\sigma}\Gamma^{2s}_{\Delta_2\Delta_2} \\ &\quad - \Gamma_{\sigma\Delta_2}\Gamma_{\Delta_2^*\sigma}\Gamma^{2s}_{\Delta_2\Delta_2^*} - \Gamma_{\sigma\Delta_2^*}\Gamma_{\Delta_2\sigma}\Gamma^{2s}_{\Delta_2^*\Delta_2} \\ &\quad - F(p_0^2)\Gamma_1(p_0^2), \end{aligned} \tag{A4}$$

and the functions $\Gamma_0$, $\Gamma_1$ are defined in (33) and (34). In the $p_0^2$ plane Eq. (A3) has three solutions. One of them corresponds to a Nambu-Goldstone boson, $p_0^2 = 0$. The other two we denote as $(p_0^2)_\Delta$ and $(p_0^2)_\sigma$. Since $m$ is a small quantity, the right-hand side (RHS) of (A3) can be considered as a small perturbation. Then the solution $(p_0^2)_\sigma$ [as well as $(p_0^2)_\Delta$] can be constructed in the framework of a perturbative expansion, based on the smallness of the RHS of (A3). In the zeroth order we have $F(p_0^2)\Gamma_0(p_0^2) = 0$, so $(p_0^2)_\sigma^{(0)} = M_\sigma^2$, where $M_\sigma$ is the zero of the function $\Gamma_0(p_0^2)$ and is graphically represented in Fig. 3. One can easily obtain the first perturbative correction

$$(p_0^2)_\sigma = M_\sigma^2 + \frac{m^2\Delta^2 f(M_\sigma^2)}{F(M_\sigma^2)\Gamma_0'(M_\sigma^2)} + \cdots \tag{A5}$$

and so on. Now let us put our attention on the fact that $f(p_0^2)$ and $F(p_0^2)$ are analytical functions in the whole complex $p_0^2$ plane except the cut, composed from all real points such that $4\Delta^2 < p_0^2 < \infty$. [In the rest of the real $p_0^2$ axis, $f(p_0^2)$ and $F(p_0^2)$ take real values.] Through the cut, these functions can be analytically continued to the second Riemann sheet. In our case, $M_\sigma \approx 350$ MeV and $\Delta \sim 100$ MeV, so $M_\sigma^2 \in (4\Delta^2, \infty)$, i.e. it lies on the cut. Hence, both $f(M_\sigma^2)$ and $f(M_\sigma^2)$ have imaginary parts, and the solution (A5) lies in the second Riemann sheet. It means that $(p_0^2)_\sigma$ corresponds to a resonance in the mass spectrum. Numerical estimates show that the width of this resonance is a rather small quantity, less than 30 MeV.

Similar corrections can be easily performed for the heavy scalar diquark resonance mass, corresponding to another solution $(p_0^2)_\Delta$ of Eq. (A3).


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