PHYSICAL REVIEW D **72,** 056005 (2005)

Dynamical *CP* violation in the early universe and leptogenesis

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(Received 10 August 2005; published 23 September 2005)

In a recent publication, we suggested a mechanism for obtaining dynamical *CP* violation in the early universe based on the out-of-equilibrium evolution of complex scalar fields. In this paper, we suggest several ways of transferring the *CP* asymmetry from the scalar sector to the leptonic sector. In particular, we point out how a ''transient Maki-Nakagawa-Sakata (Pontecorvo) matrix'' can generate an asymmetry between fermions and antifermions directly.

DOI: 10.1103/PhysRevD.72.056005 PACS numbers: 11.30.Er

I. INTRODUCTION

CP violation is one of the key ingredients required for baryogenesis [1]. In most particle-physics models, *CP* violation is explicit in the Lagrangian. In the standard model (SM), *CP* violation is built in via complex phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2], albeit with a very small amplitude. Since SM *CP* violation is very small, it is not possible to obtain a sufficiently large net baryon to entropy ratio in this scenario. Hence, in extensions of the SM additional *CP* violation is often introduced via *CP*-violating phases in an extended Higgs sector to explain lepto/baryogenesis.

Following earlier ideas of Dolgov [3], we recently proposed an alternative dynamical mechanism of *CP* violation in the early universe [4] (see also [5]). Here we elaborate on this mechanism. In this scenario, the Lagrangian is *CP* symmetric. However, there are complex phases which arise as initial conditions for scalar fields in the early universe. In the framework of inflationary cosmology it is possible that these phases are coherent over the present Hubble patch, as discussed in [4]. These complex phases lead to *CP* violation. The magnitude of *CP* violation decreases as the scalar fields relax to their ground state. Thus, in the framework of dynamical cosmological *CP* violation, large *CP* violation in the early universe leading to the observed baryon to entropy ratio is no longer in conflict with the small magnitude of the presently measured *CP* violation in the laboratory.

In [4], we discussed the general framework of dynamical cosmological *CP* violation in the case of a model with two complex scalar fields. We showed explicitly how the dynamics of the background scalar fields (the ''condensates'') can lead to the generation of a *CP* asymmetry in the scalar field quanta which are generated via the decay of the condensates. We briefly mentioned how this *CP* asymmetry in the scalar sector can be transformed into a *CP* asymmetry of the fermions leading eventually to baryogenesis [6]. In this paper we elaborate on these two issues. Moreover we point out a second channel to produce *CP* violation in the fermionic sector directly from the dynamics of background scalar fields. We observe that nontrivial phases of the evolution of the condensate can induce complex Yukawa terms in the Lagrangian. This results in what we call the ''transient Maki-Nakagawa-Sakata (Pontecorvo) MNS(P) matrix'' [8], and leads to *CP* violation in the leptonic sector.

The outline of this paper is as follows: in Sec. II we review the basic ideas of dynamical cosmological *CP* violation as discussed in [4] and give a brief overview of the ways in which this can result in lepto/baryogenesis. In Sec. III we study the avenues for leptogenesis which make use of an asymmetry in scalar field quanta generated by the evolving background condensate. In Sec. IV we study leptogenesis channels in which the background condensate directly induces the lepton asymmetry via the transient MNS(P) matrix. In Sec. V, we consider a specific leptogenesis channel and evaluate how the resulting lepton asymmetry depends on the parameters and initial conditions of our model. In Sec. VI, we comment on other possibilities of realizing leptogenesis using complex initial conditions. We conclude in Sec. VII with a summary of the main results and a discussion of future directions of research.

II. DYNAMICAL *CP* VIOLATION

In this section, we review the scenario of [4] in which nontrivial phases in the initial conditions of scalar fields lead to dynamical *CP* violation in a theory in which the Lagrangian is symmetric under *CP*. We then explain how this dynamical *CP* violation can be transferred to fermions, leading to a lepton-antilepton asymmetry, and eventually to a baryon-antibaryon asymmetry.

A. Phases from initial conditions

Consider a toy model containing the two complex doublet scalar fields ϕ_1 and ϕ_2 (we take doublets instead of

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singlets only to make the field content of the model look more like that of the standard model). For the moment, we say nothing about the quantum numbers of these two fields, except that they are the same for both ϕ_i, $i = 1, 2$. The scalar potential of the model is taken to be

$$V(\phi_1, \phi_2) = \sum_{i=1,2} m_i^2 \phi_i^\dagger \phi_i + V_4(\phi_1, \phi_2), \qquad (2.1)$$

where it is assumed that $m_1 > 2m_2$, and

$$V_4(\phi_1, \phi_2) = g(\phi_2^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \text{H.c.}, \qquad (2.2)$$

where g is a real coupling constant.

It is natural to assume that only the neutral components of ϕ_i pick up an initial nonvanishing expectation value [9]. From now on the ϕ_i's will denote the neutral components of the respective doublets. Note that their potential will look exactly the same as Eqs. (2.1) and (2.2), with the Hermitian conjugates replaced by complex conjugates of the neutral components.

In the context of hot big bang or inflationary cosmology, it is natural to assume that the neutral scalar fields will start out displaced from their vacuum values. In a noninflationary universe, the spatial gradients of these fields would be large and they would rapidly relax to their ground-state values. However, in the context of inflationary cosmology the situation is very different. If the masses of the neutral scalar fields are smaller than the Hubble constant during inflation, the fields will evolve essentially as free scalar fields. That is, they will respond to quantum fluctuations like a free scalar field, and thus acquire a large root mean square expectation value when averaged over a region of radius H^{-1} (a "Hubble patch"). At the end of inflation, these neutral scalar fields will thus in general be displaced from the minimum of the bare potential energy function by an amount which is large compared to H (see e.g. [11–13] for a review).

After inflation ends, the Hubble constant will gradually decrease, eventually falling below the mass scale of the scalar fields. At this point, these scalar fields will begin to roll towards their ground state. It is this dynamics in the postinflationary universe which will lead to dynamical *CP* violation and resulting leptogenesis. To see this, observe that although one of the two phases of the two neutral complex scalar fields can in general be eliminated by a phase redefinition, the second cannot. If we choose the initial value of ϕ_2 to be real (and denoted by a_2), but take the initial value of ϕ_1 to have a phase α, i.e. to be $a_1 e^{i\alpha}$, then in order to analyze the particle-physics processes one needs to consider fluctuations of the scalar fields around this complex initial condition (background). This effectively introduces nontrivial complex phases in the scalar potential as well as in the Yukawa interactions with the fermions, leading to *CP* violation. In this paper we examine ways in which such *CP* violation can be transferred to the fermion sector, concentrating principally on leptogenesis scenarios

B. Baryogenesis preceded by leptogenesis

In order to produce a baryon-antibaryon asymmetry we first of all need processes which violate the baryon number. Secondly, we require *C* and *CP* violation [1] (for recent reviews on baryogenesis, we refer the reader to [14]). In most baryogenesis scenarios, the required *CP* violation is explicitly introduced into the Lagrangian. However, as discussed earlier, in our approach this is not the case. Rather, it is the initial conditions which generate this asymmetry. This dynamical mechanism of *CP* violation can be coupled to many of the well-known models of baryogenesis. However, in light of current data which point to a nonzero neutrino mass [15], one of the preferred mechanisms for generating the baryon asymmetry involves first generating a lepton asymmetry. This asymmetry then can be converted to a baryon-antibaryon asymmetry via baryon-number-violating sphaleron processes [16]. As we emphasize below, this mechanism does not require any new baryon or lepton-number-violating terms in the Lagrangian. In the following, we shall focus on this possibility. (Note that, in this paper, we do not discuss the possibility of baryogenesis via quarks alone. However, it is straightforward to modify our scenarios to include *CP* violation in the quark sector directly.)

Note that it is not necessary to have more leptons than antileptons in order for the sphalerons to produce more baryons than antibaryons. In fact, an asymmetry in only certain states is sufficient. This observation comes about because the weak interactions—and hence the sphaleron interactions—couple to only left-handed particles and right-handed antiparticles. Since only these states are involved, the production of ν_R or $\bar{\nu}_L$ cannot lead to a baryon-antibaryon asymmetry. While it is true that a ψ_L state can be converted into a ψ_R state, the rate is proportional to the mass of the ψ. For light neutrinos, this conversion rate is sufficiently slow that an asymmetry between ν_L and $\bar{\nu}_R$ survives to the weak scale. Thus, sphalerons can convert an asymmetry between ν_L and $\bar{\nu}_R$ alone into a baryon asymmetry.

On the other hand, this does not hold for electrons since any generated asymmetry is washed out by the mass terms [17,18]. Thus, an asymmetry between e_L and $\bar{e}_R$ cannot be converted into a baryon-antibaryon asymmetry via the standard electroweak sphaleron effects.

The upshot of this discussion is that we can successfully obtain baryogenesis involving sphalerons provided it is preceded by one of two scenarios. Either (A) we have full leptogenesis, i.e. there is an asymmetry between lepton and antilepton number, or (B) an asymmetry between ν_L and $\bar{\nu}_R$ is created. In the former case of course one requires the presence of lepton-number-violating processes, as is required in thermal leptogenesis scenarios [19], for example. The latter scenario is in spirit similar to Dirac leptogenesis [20], but it differs in two aspects. First, there need not be explicit lepton-number violation, and second, *a priori* the Lagrangian is *CP* conserving.

In the following sections we discuss two distinct mechanisms for generating a lepton asymmetry. The first involves *CP*-violating scalar interactions which generate an asymmetry between scalar particles and their antiparticles. This *CP* asymmetry is then transferred to the lepton sector. The second uses what we call the transient MNS(P) matrix. Note that, for either mechanism to work, one needs Yukawa interactions between the rolling scalar fields and the SM fermions. However, the specific applications of these interactions depend on the mechanism, and are slightly different.

III. LEPTON ASYMMETRY FROM AN ASYMMETRY OF SCALAR QUANTA

A. *CP* violation in the scalar sector

We begin by considering mechanisms by which a *CP* asymmetry involving scalars can be transferred to the lepton sector. As discussed in [4] (and making use of the phase conventions discussed in Sec. II) we consider field fluctuations about the initial conditions:

$$\phi_2' = \phi_2 - a_2, \qquad \phi_1' = \phi_1 - a_1 e^{i\alpha}. \tag{3.1}$$

Note that in the context of cosmologically evolving fields, the background fields a_i vary with time. We shall address the dynamical evolution later (in Sec. V). For now, we mention that, in order for the *CP*-violating effects at different times to add up coherently, we work in the adiabatic approximation where we assume that the time dependence of the background fields is slow compared to the interaction time scale of the fluctuations. We will focus on the time dependence of the amplitude of the fields. However, in principle, the phase α can also be time dependent [21].

The relations (3.1) can be inverted and inserted into the scalar potential of (2.1). Expanding the quartic term of (2.2), one finds both quadratic and cubic terms. Dropping the primes, the cubic interaction terms become

$$V_3 = g[a_1 e^{i\alpha}\phi_2^\dagger|\phi_2|^2 + a_2\phi_2^\dagger\phi_2^\dagger\phi_1 + 2a_2|\phi_2|^2\phi_1] + \text{H.c.} \tag{3.2}$$

The induced couplings contribute to the decay $\phi_1 \to \phi_2\phi_2$ [4]. This decay is *CP* violating, as can be seen as follows. There are two types of diagrams: a tree diagram and several loop diagrams. Because these diagrams have relative weak and strong phases, one finds that the number of ϕ_2 states is unequal to the number of $\phi_2^\dagger$ states [22]. This is *CP* violation in the scalar sector. Specifically, the relative asymmetry A_{CP} in the decay rates is

$$A_{CP} \sim \sin 2\alpha. \tag{3.3}$$

This asymmetry in the scalar sector must now be transferred to the leptonic sector. This happens through the Yukawa interactions. As discussed earlier, there are two possibilities. Either we have full leptogenesis, or an asymmetry between ν_L and $\bar{\nu}_R$ alone is created. We refer to these as scenarios A and B, respectively.

B. Transferring asymmetry to fermions

We begin with mechanism A and briefly discuss the generation of a *CP* asymmetry in the leptonic sector via the decay of scalar quanta. In this case one requires lepton-number-violating processes in which the asymmetry in the *CP* values of the scalar excitations, computed above, transfers directly into an equivalent asymmetry in the leptonic sector. In the case of Dirac neutrinos, the generic *CP*-violating rate asymmetry is of the form

$$\Delta\Gamma_{\phi_i\to f_1 f_2\cdots} = \Gamma_{\phi_i\to f_1 f_2\cdots} - \Gamma_{\phi_i^*\to \bar{f}_1\bar{f}_2\cdots} \simeq \Gamma_{\phi_i\to f_1 f_2\cdots}A_{CP}, \tag{3.4}$$

where $\Gamma_{\phi_i\to f_i\cdots}$ is the decay rate of a scalar ϕ_i to a final state involving Dirac fermions f_i. Here, A_{CP} denotes the inherent *CP* asymmetry in the production of scalars ϕ_i and its *CP* conjugated state, ϕ_i^*. Note that an asymmetry cannot be generated for Majorana neutrinos. This is because in this case the final states are not distinguishable as particles and antiparticles, which is required to obtain an asymmetry.

We now consider an example of scenario B, in which one generates only an asymmetry between ν_L and $\bar{\nu}_R$. Consider the following Yukawa interactions between neutrinos and the doublets ϕ_i:

$$\mathcal{L}_y = \frac{1}{2}d_{\nu i}\bar{\nu}(1+\gamma_5)\nu\phi_i + \text{H.c.} \tag{3.5}$$

This leads to the decays (we consider only diagonal couplings here)

$$\phi_2 \to \bar{\nu}_L\nu_L, \qquad \phi_2^\dagger \to \bar{\nu}_R\nu_R. \tag{3.6}$$

The asymmetry in the number of ϕ_2 and $\phi_2^\dagger$ fields will therefore translate into an asymmetry in the number of $\bar{\nu}_L\nu_L$ and $\bar{\nu}_R\nu_R$ final states. As explained above, since only the states ν_L and $\bar{\nu}_R$ interact via the weak interactions, sphalerons will convert these states into baryons. In other words, this will give rise to a baryon asymmetry.

This asymmetry is easily estimated to be

$$Y_B = \frac{22}{79}Y_\nu, \tag{3.7}$$

where we have assumed that only the standard model degrees of freedom are involved in the sphaleron process [23]. The neutrino/lepton asymmetry Y_ν is estimated from the asymmetry in the production rate of the ϕ_2 field. Following (3.3) and (3.4) we have the lepton asymmetry

$$Y_\nu \propto \sin\alpha \Gamma_{\phi_2 \to \bar{\nu}_L \nu_L} \sim \frac{1}{8\pi} \sin\alpha d_{\nu 2}^2 m_2, \tag{3.8}$$

so that

$$Y_B \sim \frac{22}{79} \frac{1}{8\pi} \sin\alpha d_{\nu 2}^2 m_2. \tag{3.9}$$

The requirement that this asymmetry not be washed out is given by the condition [24]

$$\frac{m_2^2 m_\nu^2 \Gamma_{\phi_2 \to \bar{\nu}_L \nu_L}}{2T^4 H(m_2)} \ll 1. \tag{3.10}$$

In (3.10), T denotes the temperature and $H(m_2)$ is the value of the Hubble constant at scale m_2, when the scalar decays to neutrinos of mass m_ν. Note that this condition is obtained by solving for the relevant Boltzmann equations (for details, we refer the reader to [24]) and is easily satisfied because of the smallness of the neutrino mass.

IV. DIRECT LEPTOGENESIS INDUCED BY COMPLEX PHASES

We now turn to the second method of generating a *CP* asymmetry in the leptonic sector, namely, by means of a transient MNS(P) matrix.

Recall first how *CP* violation comes about in the SM. Here one has the Yukawa couplings $\frac{1}{2} Y_{ij} \bar{\psi}_i (1 + \gamma_5) \psi_j \phi$, where ψ can be a quark or a lepton field and ϕ is the ordinary Higgs field. Because there are three generations, the Yukawa couplings Y_{ij} are complex. When ϕ acquires a vacuum expectation value, one develops (complex) mass terms. Their diagonalization leads to the CKM matrix for quarks, or the MNS(P) matrix for leptons.

In the present case, we again have couplings of the form $\frac{1}{2} Y_{ij} \bar{\psi}_i (1 + \gamma_5) \psi_j \phi_{1,2}$, where the $\phi_{1,2}$ are the background scalar fields. Here we assume that *CP* is conserved in the Lagrangian, so that the Y_{ij} are real. However, in dynamical *CP* violation, the initial conditions of the scalar fields have relatively complex values [e.g. see Eq. (3.1)]. These initial conditions effectively lead to complex (nondiagonal) Yukawa couplings, which again lead to mass matrices whose diagonalization includes *CP*-violating phases. However, in contrast to the SM, once the fields relax to their minima, the contributions to the masses are switched off, and the *CP*-violating mixing matrix vanishes. In other words, the mixing matrix is *transient*, and no dynamical *CP* violation remains at late times.

Of particular interest to us is the case where ψ is the neutrino. Here nonzero neutrino masses are generated, effectively leading to a MNS(P)-type matrix. This matrix can asymmetrically produce ν_L and $\bar{\nu}_R$ even when no lepton-number-violating interactions are present in the theory.

A. Transient MNS(P) matrix

The above qualitative description can be made quantitative. As before, we take the background scalar fields to be SU(2) doublets (other representations can be straightforwardly included) and assume only the neutral components acquire background values. The general form for the Yukawa interactions is

$$\mathcal{L}_y = \frac{1}{2} d_e \bar{e}(1 + \gamma_5) e D_b^* + \frac{1}{2} d_\nu \bar{\nu}(1 + \gamma_5) \nu_R D + \text{H.c.}, \tag{4.1}$$

where D represents the neutral components of the doublet fields. Note that d_e and d_ν are 3×3 Yukawa flavor matrices which become complex due to initial conditions.

Let us first consider the case of Dirac fermions. The induced complex Dirac-type masses can then be parametrized as

$$M_{AB} = a_2 d_{AB}^2 + a_1 e^{i\alpha} d_{AB}^1, \tag{4.2}$$

where d_{AB}^i are the Yukawa couplings for the doublets ϕ_i, and A, B are the flavor indices. Since the d_{AB}^2 are different from d_{AB}^1, M_{AB} is a generic arbitrary complex 3×3 matrix that needs to be diagonalized using biunitary transformations. Therefore, for a given leptonic doublet, the charged and neutral fermion mass matrices read as

$$M_e = U_e^\dagger M_e^d V_e \quad \text{and} \quad M_\nu = U_\nu^\dagger M_\nu^d V_\nu, \tag{4.3}$$

where the superscript d denotes diagonal elements. One finds as usual the MNS(P) matrix

$$U_e^\dagger U_\nu, \tag{4.4}$$

which obviously includes complex *CP*-violating interactions.

To see this *CP* violation, we define the matrices K_e and K_ν such that

$$U_e K_e V_e^\dagger = (M_e^d)^2 \quad \text{and} \quad (M_\nu^d)^2 = U_\nu K_\nu^d V_\nu^\dagger. \tag{4.5}$$

The condition for *CP* violation is derived to be [25]

$$C_{ij} = \text{Im}(K_{eij} K_{\nu ij}^*) + \text{Im}(K_{eik} K_{\nu ik}^*) \neq 0; \quad i < j; \quad j \neq k. \tag{4.6}$$

It is convenient to assume that the matrix describing charged-lepton couplings (U_e) is diagonal, so that the mixing matrix comes purely from the neutrino sector: $U_{\text{MNS(P)}} = U_\nu$. The largest contribution to *CP* violation then comes from the case where the charged lepton is a τ. For a specific leptonic flavor (say α), the condition (4.6) translates to

$$C_{\tau\alpha} \approx 2m_\tau^2 \, \text{Im}(K_{\nu\tau\alpha}^*) \neq 0. \tag{4.7}$$

The phase in the neutrino mixing matrix is now responsible for *CP* violation in scalar decays. For illustration, let us consider a flavor specific decay of the type

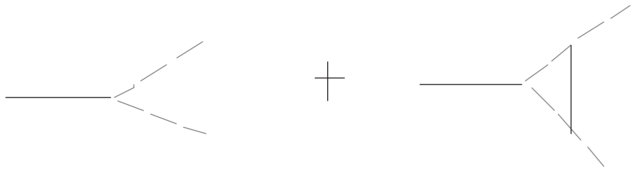

FIG. 1. Feynman diagrams of the tree and loop processes considered for the decay, $\phi_2 \to \bar{\nu}_{\tau R}\nu_{\tau L}$. The dashed lines are for fermions and the solid lines are for scalars.

$$\phi_2 \to \bar{\nu}_{\tau L}\nu_{\tau L}, \qquad \phi_2^\dagger \to \bar{\nu}_{\tau R}\nu_{\tau R}. \tag{4.8}$$

This decay can be *CP* violating because of a phase arising from the one-loop vertex-correction graph. This process is described in Fig. 1. The resulting *CP*-violating asymmetry is

$$Y_\nu \propto \arg(d_{\tau\tau}) - \arg(d_{e\tau}d_{e\mu}d^*_{\mu\tau}) = \sin\delta, \tag{4.9}$$

where δ is the complex (Dirac) phase in U_ν.

Several remarks are in order. First, the asymmetry in (4.9) is generated solely due to mixing in the leptonic sector. This is easily seen by the presence of different flavor-changing Yukawa couplings in the expression for the asymmetry. Second, we require a *CP*-conserving phase in the interference between tree and one-loop diagrams. This comes about due to an absorptive piece in the loop calculation. Finally, the asymmetry is prevented from being washed out due to the same condition as that of (3.10). As before, the lepton-number asymmetry is converted to a baryon-number asymmetry using sphaleron effects.

B. Majorana fermions

We now mention briefly the more complicated case where the neutrinos are Majorana particles. In comparison to the Dirac case, the Majorana fermions (due to the reality of the fields) have two more *CP*-violating phases (for three generations) in the mass matrix. These additional phases give further avenues for *CP* violation. The important feature that distinguishes this from the Dirac case is the possibility that $C_{\alpha\alpha}$ [Eq. (4.6)] can take nonzero values [26].

To illustrate this, consider just two generations. In this case, there is no *CP* violation for Dirac neutrinos, but there can be for Majorana neutrinos. The effective Majorana mass is

$$m_{\alpha\alpha} = m_1 c_1^2 + m_2 s_1^2 e^{2i\gamma}, \tag{4.10}$$

where c_1/s_1 are the cosine/sine of the mixing angle and γ is the *CP*-violating phase. Thus, in this case, one can obtain *CP*-violating lepton asymmetries as above in the decays $\phi_2 \to \bar{\nu}_L\nu_L$ and $\phi_2^\dagger \to \bar{\nu}_R\nu_R$. However, because of their Majorana nature, it is necessary in this case that the final-state neutrinos have different flavors.

Note that, for Majorana neutrinos, we can replace our scalars ϕ_i with those used in the standard thermal leptogenesis scenario [19]; an extension to the nonthermal case [28] is straightforward.

V. ESTIMATING BARYON ASYMMETRY IN A SPECIFIC EXAMPLE

A. Relating α to the MNS(P) phase

Up to this point, we have concentrated exclusively on showing that the *CP* violation in the scalar sector can be successfully transferred to the fermion sector. However, one might wonder whether this *CP* violation is sufficient to obtain the right amount of baryogenesis, i.e. a correct baryon to entropy ratio. In this section we explore this question.

In order to examine this, we need a framework for obtaining *CP* violation in the fermion sector. For this, we use the transient MNS(P) matrix with Dirac neutrinos. In Sec. IV, we showed that *CP* violation occurs with two scalar doublets and three generations. However, it is also possible to obtain a transient MNS(P) matrix with only two generations, if one considers a left-right model, i.e. the gauge symmetry is extended to $G = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. This is the scenario we adopt here.

As discussed earlier, the neutrinos couple to the rolling scalar doublets. We write the background fields in the form

$$\phi_1(t) = a_1(t) \quad \text{and} \quad \phi_2(t) = a_2(t)e^{i\alpha}. \tag{5.1}$$

In the left-right model, these two fields are combined into a single matrix:

$$\Phi = \begin{pmatrix} a_2 e^{i\alpha} & 0 \\ 0 & a_1 \end{pmatrix}. \tag{5.2}$$

We focus only on the leptonic sector. The Yukawa interactions for the lepton doublet L are

$$\mathcal{L}_Y = \frac{1}{2}y_l\bar{L}\Phi(1+\gamma_5)L + \frac{1}{2}y_\nu\bar{L}\,\tilde{\Phi}(1+\gamma_5)L;$$
$$\tilde{\Phi} = \sigma_2\Phi^*\sigma_2. \tag{5.3}$$

The mass matrices for the charged lepton and neutrino fields are therefore

$$M_l = M_l^0 + rM_\nu e^{-i\alpha}, \quad M_\nu = rM_l e^{i\alpha} + M_\nu^0, \quad r = \frac{a_1(t)}{a_2(t)}. \tag{5.4}$$

In the absence of phases which break *CP* explicitly, the mass matrices $M^0_{l,\nu}$, $M_{l,\nu}$ are real and symmetric (this is an advantage of choosing the group G). As a result, the mass matrices can be diagonalized by a single mixing matrix $V_{l,\nu}$ (and the left- and right-handed mixing matrices are related, $U_L = U_R^*$):

$$V_{l,\nu}M_{l,\nu}V^T_{l,\nu} = M^{\mathrm{diag}}_{l,\nu}. \tag{5.5}$$

The corresponding MNS(P) matrix is

$$U = V_l V_\nu^\dagger. \tag{5.6}$$

With two generations, apart from an overall phase, the MNS(P) matrix can be parametrized as

$$U = \begin{pmatrix} e^{i\delta_1}\cos\theta & e^{i\delta_2}\sin\theta \\ -e^{-i\delta_2}\sin\theta & e^{-i\delta_1}\cos\theta \end{pmatrix}. \tag{5.7}$$

Note that, although the above complex mass matrix is written in terms of the two phases $\delta_{1,2}$, only the difference $\delta_1 - \delta_2$ is physical.

For calculational ease we assume that $a_1(t) \ll a_2(t)$. In this case, $r \ll 1$ for any given time, and thus $r(t)$ can be treated as a perturbative parameter. In this limit, the relation between the phase α and the complex phase difference $\delta(\equiv \delta_2 - \delta_1)$ is given approximately by [29]

$$\delta \equiv \delta_1 - \delta_2 \simeq \frac{m_\tau}{m_\nu} r \sin\alpha. \tag{5.8}$$

In (5.8) m_ν is one of the mass eigenvalues of the neutrino sector which we take to be considerably smaller than the tau lepton mass, m_τ. Note that the MNS(P) phase is time dependent as long as r is time dependent.

In the following we estimate this time dependence for r. This will allow us to determine whether the proper baryon to entropy ratio is obtained.

B. Cosmological evolution and the baryon asymmetry

As in all models of leptogenesis, in our scenario the net baryon number is generated via sphalerons from an asymmetry in the left-handed lepton number. As long as sphaleron transitions are thermodynamically allowed, i.e. at energy scales higher than the electroweak scale, they will equilibrate the baryon and left-handed lepton numbers [30]. In our context, this means that a baryon asymmetry of the same order of magnitude as the left-handed lepton asymmetry produced by the processes described in the previous subsection will be generated. In turn, the left-handed lepton asymmetry is generated by the motion of the scalar field ϕ. We will estimate the net baryon to entropy ratio by the ratio of left-handed leptons over photons produced during the time interval when the rolling of the scalar field is most important in the sense discussed below. Note that, once produced, the baryon to entropy ratio does not change in time as long as there are no other phase transitions which generate entropy after the period of leptogenesis. Thus, in the following we will first determine the time interval when most of the left-handed leptons are produced, and then estimate the resulting baryon to entropy ratio following the methods of [5,7].

The equation of motion for any rolling complex scalar field which is homogeneous in space is

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V(\phi_i)}{\partial \phi}, \tag{5.9}$$

where H is the Hubble parameter. The dynamics of ϕ depend on whether $H(t)$ is larger or smaller than m_ϕ, the mass of the field ϕ which is given by the square root of the second derivative of the potential $V(\phi)$.

At early times, while

$$H(t) > m_\phi, \tag{5.10}$$

the evolution of ϕ is overdamped and follows the approximate equation

$$3H\dot{\phi} = -\frac{\partial V(\phi_i)}{\partial \phi} = -m_\phi^2 \phi. \tag{5.11}$$

In the early radiation-dominated universe

$$H(t) = \frac{1}{2t}, \tag{5.12}$$

and thus the solution of (5.11) becomes

$$\phi(t) = \phi(t_0) e^{-(1/3)m_\phi^2(t^2 - t_0^2)}, \tag{5.13}$$

where t_0 is the initial time. Making use of (5.10) and (5.12), we see that the motion of ϕ in this phase is negligible. Hence, no leptogenesis takes place during this phase.

The inequality (5.10) is saturated at a time we denote as t_e. As soon as

$$H(t) \leq m_\phi, \tag{5.14}$$

the scalar field ϕ starts rolling as described by (5.9) and eventually performs damped oscillations about its ground state. The sign of the induced lepton asymmetry depends on the sign of $\dot{\phi}$. Thus, if ϕ were oscillating with constant amplitude, no net lepton number would be generated. The net lepton asymmetry is determined by the overall decrease in the amplitude of ϕ. Most of this decrease happens during the time interval before ϕ crosses zero for the first time, which occurs at a time we denote by t_*. The time interval between t_e and t_* is less than one Hubble expansion time $H(t_e)^{-1}$.

For the baryogenesis channel considered in the above subsection, the net baryon to entropy ratio is given by [5,7]

$$\frac{n_B}{s} \sim \frac{y_\nu^2 Y_\nu (\phi\dot{\phi})(t_e)}{8\pi n_\gamma(t_e)}, \tag{5.15}$$

where y_ν is the typical value of a neutrino Yukawa coupling constant from (5.3), Y_ν is the CP asymmetry per decay from (4.9), n_γ is the number density of photons, s denotes the entropy density, and n_B the net baryon-number density. A way to understand the above equation is as follows: $m_\phi Y_\nu$ is the CP asymmetry in the decay rate of ϕ quanta, $(\phi\dot{\phi})(t_e)$ is the rate of change in the number of ϕ quanta at time t_e, the time when most of the net baryon-number density is generated, and m_ϕ^{-1} gives the time interval during which leptogenesis is effective and must be multiplied with the rate of generation of the baryon asymmetry to obtain the final baryon to entropy ratio.

In turn, the asymmetry factor Y_ν in the above Eq. (5.15) is given by combining (4.9) and (5.8) and inserting the time evolution of the function $r(t)$. However, given the gauge symmetry of our model, the masses of both scalar fields ϕ_1 and ϕ_2 are the same and they will thus begin rolling at the same time with vanishing velocity. Hence, $r(t)$ will be constant in time. Since we are also taking the phase α to be constant, the phase δ and hence Y_ν are independent of time.

To evaluate the order of magnitude of the result (5.15), we must estimate the value of $\dot{\phi}(t_e)$. From the equation of motion (5.9) it follows that

$$\dot{\phi}(t_e) \sim m_\phi \phi(t_e) \tag{5.16}$$

and hence

$$\frac{n_B}{s} \sim \frac{y_\nu^2 Y_\nu m_\phi \phi^2(t_e)}{8\pi n_\gamma(t_e)}. \tag{5.17}$$

Since the time t_e is given by the saturation of (5.10), the photon number density is given by

$$n_\gamma(t_e) \sim m_\phi^{3/2} m_{\rm pl}^{3/2}, \tag{5.18}$$

where $m_{\rm pl}$ is the Planck mass, and we have used the Friedmann equations to relate the temperature of the radiation bath (whose cube yields n_γ) to the Hubble expansion rate at time t_e which in turn is equal to m_ϕ. Thus, (5.17) becomes

$$\frac{n_B}{s} \sim y_\nu^2 Y_\nu \frac{\phi^2(t_e)}{m_\phi^{1/2} m_{\rm pl}^{3/2}}. \tag{5.19}$$

We now add an additional constraint from cosmology: we demand that the energy density in ϕ at time t_e be subdominant, i.e.

$$m_\phi^2 \phi^2(t_e) \ll T^4(t_e) \sim H^2(t_e) m_{\rm pl}^2, \tag{5.20}$$

where $T(t)$ denotes the temperature of radiation, and where in the second step we have again used the Friedmann equations. Recalling that $H(t_e) = m_\phi$, the inequality (5.20) yields an upper bound on $\phi(t_e)$:

$$\phi(t_e) \ll m_{\rm pl}, \tag{5.21}$$

a bound which is quite natural from the point of view of particle physics (we cannot trust the physics we used for field values larger than $m_{\rm pl}$). Inserting the bound (5.21) into (5.19), we obtain a bound on the strength of our baryogenesis scenario of the form

$$\frac{n_B}{s} \ll y_\nu^2 Y_\nu \left(\frac{m_{\rm pl}}{m_\phi}\right)^{1/2}. \tag{5.22}$$

This result demonstrates that our dynamical CP violation scenario can lead, even in the case of small coupling constants, to a large net baryon to entropy ratio.

VI. OTHER CHANNELS FOR LEPTO- AND BARYOGENESIS

There are other ways of transferring a CP asymmetry in the scalar sector to the lepton sector. In this section, we briefly describe some alternative mechanisms.

A. Asymmetry due to scattering

In Sec. III, we considered the possibility that a CP-violating asymmetry in the scalar sector can be generated in the decay $\phi_1 \to \phi_2\phi_2$. This is then converted to a CP asymmetry in the lepton sector via the decay of ϕ_2 scalars into neutrinos. However, a CP asymmetry in the scalar sector can also be created by scattering processes such as $\phi_1\phi_2 \to 2\phi_2$. This process is generated by the quartic interaction V_4 in (2.2) and has a one-loop vertex correction due to the cubic interaction V_3 generated in (3.2). Once again, the asymmetry comes about because the coupling of ϕ_2 to ϕ_1 is complex, but ϕ_2^3 interactions are real. This leads to a phase difference between the tree diagram and the one-loop vertex-correction graph, and contributes to the asymmetry. The relevant Feynman graph for this process is shown in Fig. 2.

The asymmetry which is generated between the cross sections for ϕ_2 and ϕ_2^* production (via the CP-conjugate process) is

$$\sigma - \bar{\sigma} \sim (\text{loop factor}) \times \sigma \sin\alpha; \quad \sigma = \frac{1}{32\pi}\frac{g^2}{s}. \tag{6.1}$$

In (6.1) s is the center of mass energy for the scattering process and g is the relevant quartic coupling in (2.2). Following (6.1), the asymmetry in the rate of ϕ_2 production is

$$\Delta\Gamma_{\phi_2} = (\sigma - \bar{\sigma}) \cdot N_{\phi_2} = \frac{\sin\alpha}{32\pi}\frac{g^2}{s} \cdot N_{\phi_2}, \tag{6.2}$$

where N_{ϕ_2} is the number density of the scattering of ϕ_2 particles [31].

Once again, in this case, the lepton asymmetry is generated by allowing the scalar ϕ_2 to decay via the process $\phi_2 \to \nu_L \bar{\nu}_L$. The lepton-number asymmetry generated in this way is proportional to the Yukawa coupling [(3.9)]:

$$Y_\nu \propto d_{\nu 2}^2 \Delta\Gamma_{\phi_2} = \frac{\sin\alpha}{32\pi}\frac{(g d_{\nu 2})^2}{s} \cdot N_{\phi_2}. \tag{6.3}$$

The interesting aspect of (6.3) is that the asymmetry could

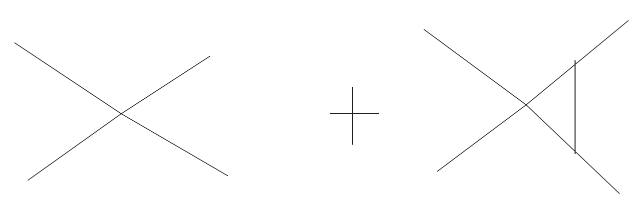

FIG. 2. Feynman diagrams of the tree and loop processes for the reaction $\phi_1\phi_2 \to 2\phi_2$.

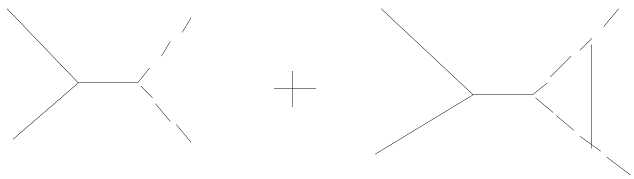

FIG. 3. Feynman diagrams of the tree and loop processes for the reaction $\phi_1\phi_2 \to \nu_L\bar{\nu}_R$.

be large for large values of N_{ϕ_2}, even if the Yukawa coupling is small. Simultaneously, the washout factor is not large because the inverse scattering may be kinematically suppressed if the initial masses are large compared to the final-state masses. In our case, we have $m_1 > m_2$ and we expect $m_i \gg a_2 d_{\nu 2}$ due to small Yukawa couplings.

One can also produce a lepton-number asymmetry directly via the pair production of Dirac fermions in the scattering process $\phi_1\phi_2 \to \nu_L\bar{\nu}_L$. The relevant diagrams are shown in Fig. 3. Taking the final state to be $\nu_{\tau L}\bar{\nu}_{\tau L}$, we obtain an asymmetry which is approximately given by

$$Y_\nu \propto g|d_{\tau\tau}d_{e\tau}d_{e\mu}d_{\mu\tau}|\frac{\sin\delta}{16\pi s}\cdot N_{\phi_2}. \tag{6.4}$$

The asymmetry in (6.4) is smaller than that generated in (6.3) due to a larger Yukawa suppression.

Finally, a word about Majorana neutrinos. The above asymmetry which we have generated is suppressed if we use Majorana neutrinos. This follows due to spin statistics, where we notice that our scalars have to generate final states in the *P*-wave mode and hence the scenario is disfavored if it involves Majorana neutrinos.

B. Exotic fermions

Finally, it is also possible to construct scenarios involving the use of exotic fermions to transfer the *CP* violation to the fermion sector. Although we did not discuss such models, there are many possibilities. For example, the scalar fields ϕ_i can couple to such (neutral or charged) exotic fermion fields, so that the *CP* asymmetry in the ϕ_i is then transferred to the exotic fermion sector. The subsequent decays of exotic fermions into ordinary fermions create the necessary lepton-number asymmetry.

VII. DISCUSSION AND CONCLUSIONS

In this paper we have elaborated on the scenario of dynamical cosmological *CP* violation we proposed in [4] (see also [5]). In this scenario, the *CP*-violating phases are due to cosmological initial conditions for some new scalar fields ϕ_i (which could be some of the moduli fields emerging from new physics at very high energies). Here, we have focused on how to transfer the *CP* asymmetry from the scalar sector to the leptonic sector, and on the connection with baryogenesis.

We mention several ways to achieve the transfer of the *CP* asymmetry to the leptonic sector. One intriguing possibility is that Yukawa couplings between the scalar field ϕ_i and the standard model leptons could generate a transient (in time) MNS(P) matrix which generates a *CP* asymmetry in the leptonic sector, yielding an implementation of leptogenesis. In this model, we estimate the resulting net baryon to entropy ratio. As expected, the result depends on the initial values of the scalar fields. We find, however, that it is easy to generate a sufficiently large baryon asymmetry to explain the data, even if the coupling constants and the value of the *CP* violating phase in the scalar sector are small.

Note that our scenario does not assume any new sources of *CP* and baryon-number violation in the Lagrangian. The *CP* violation comes from the scalar field initial conditions, and the baryon asymmetry is generated via sphalerons from an asymmetry in the left-handed leptons, an asymmetry which is compensated by an asymmetry in the right-handed leptons. Given that the *CP* asymmetry is sourced from initial conditions, it is not possible to predict the baryon asymmetry, even if an underlying theory is known. Nonetheless, as a consequence of this scenario, there should be an asymmetry in the number of right-handed neutrinos commensurate with the observed net baryon to entropy ratio.

ACKNOWLEDGMENTS

This work is supported by NSERC (Canada) and by the Fonds de Recherche sur la Nature et les Technologies du Québec. R. B. is also supported in part by the U.S. Department of Energy under Contract No. DE-FG02-91ER40688, TASK A.

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