PHYSICAL REVIEW D **72,** 056001 (2005)

Phenomenology of the baryon resonance 70-plet at large N_c

Thomas D. Cohen*
Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA

Richard F. Lebed†
Department of Physics and Astronomy, Arizona State University, Tempe, Arizona 85287-1504, USA
(Received 26 July 2005; published 16 September 2005)

We examine the multiplet structure and decay channels of baryon resonances in the large N_c QCD generalization of the $N_c = 3$ SU(6) spin-flavor **70**. We show that this "**70**," while a construct of large N_c quark models, actually consists of five model-independent irreducible spin-flavor multiplets in the large N_c limit. The preferred decay modes for these resonances fundamentally depend upon which of the five multiplets to which the resonance belongs. For example, there exists an SU(3) "**8**" of resonances that is η-philic and π-phobic, and an "**8**" that is the reverse. Moreover, resonances with a strong SU(3) "**1**" component prefer to decay via a $\bar{K}$ rather than via a π. Remarkably, available data appears to bear out these conclusions.

DOI: 10.1103/PhysRevD.72.056001 PACS numbers: 11.15.Pg, 14.20.Gk, 14.20.Jn

I. INTRODUCTION

Numerous scattering experiments performed during the past several decades have generated a plethora of data revealing the excitation behavior of baryons. The most striking feature of this data at lower energies is the existence of observable resonant states: the excited baryons. Inasmuch as QCD is the underlying fundamental theory of strong interactions, the entire data set including the resonances should be obtainable directly from QCD. However, despite considerable recent progress in the treatment of excited states in lattice QCD [1], the extraction of resonant state properties *ab initio* from QCD remains a very hard problem. Indeed, first-principles QCD has so far yielded no simple explanation for the mere existence of resonances narrow enough to be resolved. Thus, to a very large extent most of our insight into these resonant states is gleaned from models, such as the constituent quark model, whose connection to full QCD remains obscure. Given this unsatisfactory situation, it is useful to ask whether there are any known systematic approaches to QCD that can give some qualitative or semiquantitative insight into aspects of baryon resonances, independent of models. In a series of papers [2–9] (see Ref. [10] for short reviews), we have argued that large N_c QCD and the $1/N_c$ expansion about this limit provide just such an approach. In this paper we explore the formal and phenomenological implications of this approach for the states which, in the conventional quark model language, are collected into an SU(6) 70-plet.

An important caveat is necessary at the outset: In a hypothetical world where N_c is truly large, the $1/N_c$ expansion is clearly valid and provides very accurate predictions. However, in the real world $N_c = 3$, and $1/N_c$ corrections can be substantial. While for some observables (e.g., masses of stable baryons) the leading-order large N_c predictions give reasonable qualitative and often semiquantitative descriptions of the world, for others (e.g., scalar meson properties) the large N_c predictions are quite poor. Thus the question of whether large N_c analysis is merely an exercise in mathematical physics or a useful phenomenological tool depends on which observables are being studied. While a number of interesting phenomenological predictions of baryon resonance observables have already been obtained from such analyses, the exact extent to which the approach successfully describes this sector remains an open question. In part, this paper addresses the issue by showing how certain qualitative features observed in the decays of 70-plet states can be understood in the context of large N_c QCD.

Two principal ideas underlie the model-independent large N_c approach to excited baryons developed in Refs. [2–9]. First, one must focus from the outset directly on the physical scattering observables from which resonances are ultimately extracted (meson-nucleon, electroproduction, or photoproduction scattering amplitudes) rather than on the resonance positions themselves. Second, such scattering amplitudes can be represented as operators to be evaluated between asymptotic meson-baryon states, and as such are subject to the contracted SU($2N_f$) symmetry (N_f being the number of light quark flavors) known to emerge from a model-independent analysis based upon large N_c consistency relations [11]. Combining these two ideas allows one to derive expressions, true at large N_c, for the scattering amplitude in any given channel to be expressed as a sum of terms consisting of group-theoretical factors multiplied by *reduced amplitudes*. As there are fewer reduced amplitudes than observable scattering amplitudes, the approach has predictive power: the scattering amplitudes in different channels are related at large N_c, and one predicts that various linear combinations of amplitudes are equal at large N_c.

*Electronic address: cohen@physics.umd.edu
†Electronic address: Richard.Lebed@asu.edu

It should be noted that this analysis does not by itself predict the existence of any baryon resonances *ab initio*. Generic large N_c counting gives excited baryon widths of $O(N_c^0)$, which is the same order as the spacing between states. Whether at large N_c such states are sufficiently narrow to resolve is a matter of dynamical detail and not generic large N_c scaling. The spacing of baryon resonances differs sharply from that of excited mesons, which have widths of $O(1/N_c)$ and hence are narrow at large N_c. However, this analysis does make a definitive prediction about resonances: Any resonances that do exist must fall into multiplets that become degenerate in both mass and width (or equivalently, coupling constant) at large N_c. The reason is simple: Resonances are poles in the scattering amplitudes, and at large N_c these amplitudes are entirely determined by the reduced amplitudes. Hence, a resonance in some channel implies a pole in a reduced amplitude. However, each reduced amplitude contributes to multiple physical amplitudes, each of which therefore has a resonance at the same location [up to $O(1/N_c)$ corrections]. The pattern of the degeneracy is fully fixed by the contracted SU($2N_f$) symmetry. For $N_f = 2$ these degenerate multiplets are completely determined by a single quantum number K that emerges from the analysis.

The scheme outlined above is fully model independent and exact at large N_c. If one makes a further assumption about resonances—namely, that decay channels near a resonant energy are dominated by the resonance rather than the continuum—then one can also use the contracted SU($2N_f$) symmetry to deduce selection rules for the decays that hold at large N_c. This additional assumption is needed since the extraction of resonance branching ratios (BR) from scattering data is intrinsically model dependent. However, to the extent that the amplitude is dominated by the resonance (in a limited kinematical region), this model dependence becomes small. As noted above, large N_c analysis alone does not imply that baryon resonances even exist and clearly gives no guidance on the question of whether the resonances are sufficiently prominent for the meaningful extraction of BR. In this work we rely on the phenomenological fact that prominent resonances are known to exist in the region of interest, 1.4–2.0 GeV.

Much of the early work based on this approach [2–7] was limited to nonstrange particles. While the intellectual underpinnings are the same regardless of N_f, the inclusion of strange quarks complicates the analysis in important technical ways, particularly in the limit of SU(3) flavor symmetry. The large N_c analogues to the 2-flavor physical states possess isospin quantum numbers identical to those at $N_c = 3$. Additional representations also arise, but these are dismissed as "large N_c artifacts." However, in the case of three degenerate flavors, *none* of the $N_c = 3$ SU(3) flavor representations for baryons remain the same dimension as their $N_c > 3$ generalizations [11,12].

In fact, *all* of the large N_c baryon multiplets are infinite dimensional as $N_c \to \infty$ [see Eq. (1)]. This raises two issues when one attempts to relate large N_c predictions to the real world. The first is how to associate a given large N_c multiplet with an $N_c = 3$ multiplet. The second is that, since large N_c multiplets are (infinitely) larger than their $N_c = 3$ cousins, one needs to prescribe how to associate states in the analogous $N_c = 3$ representation with states in the large N_c representation. The first issue is easily resolved: There is an obvious association between $N_c = 3$ representations with representations for any arbitrary N_c, which is given explicitly in the following section. One then computes quantities at arbitrary N_c and takes the large N_c limit. To make manifest the connection between the large N_c and $N_c = 3$ representations, we adopt the convention of denoting the large N_c analogue of the baryon **8** as "**8**," the analogue of the **10** as "**10**," and so forth. The second issue is also relatively straightforward to resolve: One considers only those states within a multiplet with the same values of isospin and strangeness as occur for $N_c = 3$.

Since the baryon representations increase in dimension with N_c, the SU(3) Clebsch-Gordan coefficients (CGC) needed for this analysis are not tabulated in standard sources. Instead, one requires the N_c-dependent CGC computed and tabulated in Ref. [8]. As discussed in this paper, the CGC implicitly impose formally and phenomenologically interesting selection rules as N_c becomes large.

The basic analysis of Refs. [2–9] is fully model independent. Another approach to excited baryons at large N_c uses large N_c generalizations of the quark model, or at least a quark "picture" in which the quantum numbers of the N_c quarks are the important degrees of freedom [13–15]. The large N_c quark model has the same emergent symmetries as large N_c QCD. Thus, if one focuses entirely on those properties that are related to the symmetry, the large N_c quark model may be viewed as an efficient way to deduce group-theoretical results. It was shown explicitly in the case of the mixed-symmetry (MS) $N_f = 2$ "**20**"-plet of SU($2N_f$) associated with $\ell = 1$ orbital excitations that the patterns of degeneracy from the large N_c quark model are compatible with the degeneracy patterns among resonances directly deduced from large N_c QCD [3]. One of the purposes of the present paper is to show explicitly that the same compatibility holds for the $N_f = 3$ MS "**70**"-plet states of the large N_c quark model: At leading (N_c^0) order the states fall into multiplets which are compatible with the degeneracy patterns deduced from full large N_c QCD.

The technical advantages of the method based on the large N_c quark model are quite apparent: It is elegant and efficient to classify quark model operators in terms of their N_c scaling behavior. Since many operators connecting states are subleading in $1/N_c$ counting, the approach constrains the possible eigenstates, which in turn generates degenerate multiplets at large N_c.

It is worth noting, however, that the large N_c quark model builds in dynamics beyond the emergent symmetry. All of this dynamics is model dependent and thus cannot be taken as direct predictions of large N_c QCD. The model-dependent aspects include: (i) the existence of the resonances, (ii) the fact that the resonances have negligible widths (i.e., are stable) in the model, with widths only added in via an *ad hoc* prescription, and (iii) assumptions about the detailed nature of the state. The third aspect is particularly important: Models used typically assume that the states fall into unmixed configurations of $SU(2N_f) \times O(3)$; the physical picture behind this symmetry is that there is only a single orbitally excited quark [giving rise to the O(3)] on top of a spherically symmetric core with an $SU(2N_f)$ spin-flavor symmetry. This assumption does not follow from large N_c QCD. As noted in Ref. [4], configuration mixing of states of this type can occur at leading (N_c^0) order. In large N_c QCD the only emergent symmetry is $SU(2N_f)$, which refers to the entire state and is *not* a symmetry of the spin and flavor of individual quarks, as is the case for the excited states in the unmixed quark model. Thus in large N_c QCD it is not meaningful to ask whether the spin-flavor symmetry is in a pure MS state such as the "**70**"-plet of the quark model.

Given these problems, one might simply avoid using quark model language entirely and rely exclusively on the symmetries of large N_c QCD. While we generally advocate this view, it is useful nonetheless to make contact with the quark model picture since this picture informs so much of our intuition about excited baryons. Accordingly, in previous papers [2,3] we identified the states in the excited SU(4) "**20**"-plet in terms of complete multiplets labeled by the K quantum number. In this paper we generalize the analysis to three flavors and extend the analysis to the SU(6) "**70**"-plet. In particular, we find that the ("**70**", 1^-) of $SU(6) \times O(3)$ is a reducible multiplet in large N_c, consisting [in the SU(3) limit] of 5 complete multiplets labeled by K.

This paper has four main purposes. The first is to flesh out the 3-flavor version of the model-independent approach that was briefly described in Ref. [9]; the second is to point out the existence of SU(3)-flavor selection rules that emerge at large N_c. The third is to tie the general scattering approach to the quark model-based approach for the "**70**"-plet states; and the fourth is to apply these methods to describe phenomenologically the decays of the **70**-plet states (or more precisely, the states that are typically assigned to the **70**-plet in quark models).

This paper is organized as follows: In Sec. II we provide essential group-theoretical background and establish notation. Section III presents a salient property of SU(3) Clebsch-Gordan coefficients at large N_c that is useful in obtaining information about processes involving strange resonances. Section IV shows the explicit connection between 3-flavor and 2-flavor scattering amplitude expressions. In Sec. V we show that the "**70**" consists of 5 multiplets labeled by K, and exhibit the connection to quark-picture operators. Section VI provides a number of phenomenological consequences of our results, and Sec. VII concludes.

II. GROUP THEORY PRELIMINARIES

Much of the content of this section appears in Ref. [8] and is presented here for the reader's convenience. An irreducible representation (*irrep*) of SU(3) symmetry may be denoted by its Dynkin weights (p, q), which indicate a Young tableau with $p + q$ boxes in the top row and q boxes in the bottom row. In terms of the maximal value of hypercharge and the isospin of the singly degenerate states of the top row in the SU(3) weight diagram, one finds $Y_{\max} = \frac{1}{3}(p + 2q)$ and $I_{\rm top} = \frac{1}{2}p$.

Mesons at arbitrary N_c still carry the quantum numbers of a single $q\bar{q}$ pair, and hence their SU(3) flavor irreps are unchanged when N_c is changed. The SU(3) irreps may also be denoted as usual by their dimensions, if no ambiguity arises: e.g., $\mathbf{8} = (1, 1)$.

Baryons, on the other hand, carry the quantum numbers of N_c quarks [in order to form an $SU(N_c)$ color singlet from color-fundamental irreps], and therefore the baryon SU(3) flavor irreps grow in size with N_c. The baryon SU(3) irreps R corresponding to large N_c generalizations of those occurring at $N_c = 3$ are taken to be $R = (2I_{\rm top}, \frac{N_c}{2} + \frac{3r}{2} - I_{\rm top})$, which has $Y_{\max} = \frac{N_c}{3} + r$. The quantity $r = O(N_c^0)$ is an integer, as required by quantization of the Wess-Zumino term for arbitrary N_c [16]. The dimension $[R] = \frac{1}{2}(p + 1)(q + 1)(p + q + 2)$ of an arbitrary SU(3) irrep assumes a useful limiting expression for the large N_c baryon irreps:

$$[R] \longrightarrow \frac{N_c^2}{8}[I_{\rm top}] \quad \text{as } N_c \to \infty, \tag{1}$$

where $[I_{\rm top}] = 2I_{\rm top} + 1$ is the isomultiplet dimension. A baryon irrep that generalizes a familiar $N_c = 3$ counterpart may also be denoted by its $N_c = 3$ dimension within quote marks; the ones useful to this work are

$$\begin{aligned} &\text{“}\mathbf{1}\text{”} \equiv [0, (N_c - 3)/2], && \text{“}\mathbf{8}\text{”} \equiv [1, (N_c - 1)/2], \\ &\text{“}\mathbf{10}\text{”} \equiv [3, (N_c - 3)/2], && \text{“}\overline{\mathbf{10}}\text{”} \equiv [0, (N_c + 3)/2], \\ &\text{“}\mathbf{27}\text{”} \equiv [2, (N_c + 1)/2], && \text{“}\mathbf{35}\text{”} \equiv [4, (N_c - 1)/2]. \end{aligned} \tag{2}$$

Other irreps appear only for $N_c > 3$ and are denoted only by their Dynkin weights. An exception that is useful to us in the following is

$$\text{“}\mathbf{S}\text{”} \equiv [2, (N_c - 5)/2], \tag{3}$$

so named because its $Y_{\max} = \frac{N_c}{3} - 1$ isomultiplet has $I = 1$, i.e., Σ quantum numbers.

SU(3) CGC are indicated by the notation

$$\left(\begin{array}{cc|c} R_1 & R_2 & R_\gamma \\ I_1, I_{1z}, Y_1 & I_2, I_{2z}, Y_2 & I, I_z, Y \end{array}\right) = \left(\begin{array}{cc\|c} R_1 & R_2 & R_\gamma \\ I_1, Y_1 & I_2, Y_2 & I, Y \end{array}\right)\left(\begin{array}{cc|c} I_1 & I_2 & I \\ I_{1z} & I_{2z} & I_z \end{array}\right), \quad (4)$$

where the last factor is an ordinary SU(2) isospin CGC. The quantities containing a double bar, which do not depend upon the additive I_z quantum numbers, are called SU(3) isoscalar factors and, like the full CGC, form orthogonal matrices. We may refer to the SU(3) isoscalar factors themselves as CGC if no ambiguity arises. The subscript γ indicates possible distinct copies of a particular irrep R within the product $R_1 \otimes R_2$.

The SU(3) products phenomenologically useful in meson-baryon scattering are

$$\begin{aligned} \text{“}\mathbf{8}\text{”} \otimes \mathbf{8} &= \text{“}\mathbf{27}\text{”} \oplus \text{“}\mathbf{10}\text{”} \oplus \text{“}\overline{\mathbf{10}}\text{”} \oplus \text{“}\mathbf{8}_1\text{”} \oplus \text{“}\mathbf{8}_2\text{”} \oplus \text{“}\mathbf{1}\text{”} \oplus \text{“}\mathbf{S}\text{”}, \\ \text{“}\mathbf{10}\text{”} \otimes \mathbf{8} &= \text{“}\mathbf{35}\text{”} \oplus \text{“}\mathbf{27}\text{”} \oplus \text{“}\mathbf{10}_1\text{”} \oplus \text{“}\mathbf{10}_2\text{”} \oplus \text{“}\mathbf{8}\text{”} \oplus \text{“}\mathbf{S}\text{”} \\ &\quad \oplus [5, (N_c - 5)/2] \oplus [4, (N_c - 7)/2]. \end{aligned} \quad (5)$$

The final two irreps need not be considered further, not only because they are absent for $N_c = 3$, but also (unlike "**S**") do not contain any isomultiplets with $N_c = 3$ quantum numbers. In Ref. [8], $\mathbf{10}_1$ is defined as the unique product **10** irrep whose "**10**" ⊗ **8** CGC all vanish with powers of $N_c - 3$ (which occurs because only one **10** appears in the $N_c = 3$ product **10** ⊗ **8**). The CGC for $\mathbf{10}_1$ and "**S**" are not needed for strict $N_c = 3$ phenomenology and therefore were not compiled in Ref. [8], but are useful for formal large N_c results requiring unitarity at arbitrary N_c, as is employed in the following analysis.

A similar notation may also be extended to SU(6) spin-flavor multiplets. As shown long ago, the requirement of order-by-order unitarity in powers of $1/N_c$ in meson-baryon scattering requires that the $J^P = \frac{1}{2}^+$ "**8**" and the $J^P = \frac{3}{2}^+$ "**10**" belong to a single spin-flavor multiplet [11] whose members differ in mass only at $O(1/N_c)$ [17,18], the completely symmetric SU(6) "**56**" ≡ $(N_c, 0, 0, 0, 0)$. For $N_c > 3$ the "**56**" also contains $J^P = \frac{5}{2}^+, \ldots, \frac{N_c}{2}^+$ flavor multiplets. Since the physical $N_c = 3$ **8** baryons are stable against strong decay, the same is true for the full "**56**" when N_c is sufficiently large; hence the "**56**" is labeled the "ground-state" band.

In the SU(6) quark model, the first excited multiplet consists of states symmetric on all except one of the quarks, the **70**-plet. The exceptional quark is then combined with the symmetric "core" as an $\ell = 1$ orbital excitation. We denote the analogue for arbitrary N_c as "**70**" ≡ $(N_c - 2, 1, 0, 0, 0)$. Its decomposition into SU(3) × SU(2) [the total spin SU(2) factor including not only quark spin but the orbital angular momentum as well] gives numerous spin-flavor multiplets [3], but only those multiplets containing states with N, Δ, Λ, Σ, Ξ, and Ω quantum numbers and spins possible with 3 quarks are phenomenologically relevant. These multiplets are

$$\begin{aligned} &2\left(\text{“}\mathbf{8}\text{”}, \frac{1}{2}\right) \oplus 2\left(\text{“}\mathbf{8}\text{”}, \frac{3}{2}\right) \oplus \left(\text{“}\mathbf{8}\text{”}, \frac{5}{2}\right) \oplus 2[1]\left(\text{“}\mathbf{10}\text{”}, \frac{1}{2}\right) \oplus 3[1]\left(\text{“}\mathbf{10}\text{”}, \frac{3}{2}\right) \oplus 2[0]\left(\text{“}\mathbf{10}\text{”}, \frac{5}{2}\right) \oplus 1[0]\left(\text{“}\mathbf{10}\text{”}, \frac{7}{2}\right) \oplus \left(\text{“}\mathbf{1}\text{”}, \frac{1}{2}\right) \\ &\oplus \left(\text{“}\mathbf{1}\text{”}, \frac{3}{2}\right) \oplus 2[0]\left(\text{“}\mathbf{S}\text{”}, \frac{1}{2}\right) \oplus 2[0]\left(\text{“}\mathbf{S}\text{”}, \frac{3}{2}\right) \oplus 1[0]\left(\text{“}\mathbf{S}\text{”}, \frac{5}{2}\right), \end{aligned} \quad (6)$$

where the coefficients indicate multiplicities for N_c large (and for $N_c = 3$ in brackets, if different). For $N_c > 3$ the "**8**" and "**10**" contain no additional states with $N_c = 3$ quantum numbers, but "**1**" gains a Ξ, and "**S**" has Σ, Ξ, and Ω states. One of our results below is that in the absence of SU(3) breaking the 20 [SU(3),SU(2)] multiplets have only 5 distinct masses split at $O(N_c^0)$, meaning that the large N_c SU(6) × O(3)("**70**", 1^-) is actually reducible to 5 distinct multiplets.

III. A PROPERTY OF SU(3) CGC

Much of the power of the analysis rests on an observation that holds for all arbitrary-N_c SU(3) CGC thus far computed, which includes every coupling relevant to $N_c = 3$ phenomenology. We do not prove this result exhaustively as a theorem, but rather show below by direct mathematical construction how it arises. But first, we state the property:

Let $R_B = (2S_B, \frac{N_c}{2} - S_B)$ denote an SU(3) irrep [corresponding to baryons in the ground-state SU(6) "**56**" with spin S_B, for which the top (nonstrange) row in the weight diagram has isospin $I_{B,\text{top}} = S_B$ and $Y_{B,\max} = \frac{N_c}{3}$], let $R_\phi = (p_\phi, q_\phi)$ be an SU(3) (meson) irrep with weights p_ϕ, $q_\phi = O(N_c^0)$, and let $R_s \gamma_s \subset R_B \otimes R_\phi$, where $Y_{s,\max} = \frac{N_c}{3} + r$ and $R_s = (2I_{s,\text{top}}, \frac{N_c}{2} + \frac{3r}{2} - I_{s,\text{top}})$, $r = O(N_c^0)$. Then the SU(3) CGC satisfy

$$\left(\begin{array}{cc\|c} R_B & R_\phi & R_s \gamma_s \\ I_B, \frac{N_c}{3} - m & I_\phi, Y_\phi & I_s, \frac{N_c}{3} + Y_\phi - m \end{array}\right) \leq O(N_c^{-|Y_\phi - r|/2}), \quad (7)$$

for all allowed $O(N_c^0)$ values of m, saturation of the inequality occurring for almost all CGC. One may observe this remarkable fact in the tables of Ref. [8].

This interesting property indicates that baryon resonances in various SU(3) irreps preferentially couple to mesons with a unique value of hypercharge. In particular, those with $Y_{\max} = \frac{N_c}{3} + 1$ ("$\overline{\mathbf{10}}$", "**27**", and "**35**") decay via a K^+ or K^0, those with $Y_{\max} = \frac{N_c}{3}$ ("**8**" and "**10**") decay via π or η, and those with $Y_{\max} = \frac{N_c}{3} - 1$ ("**1**" and "**S**") decay via $\overline{K^0}$ or K^-.

The property Eq. (7) results from a combination of unitarity and completeness of the SU(3) CGC, in addition to the U-spin and V-spin values of the states in question. Unitarity and completeness require that, for every choice of R_B, I_B, Y_B and R_ϕ, I_ϕ, Y_ϕ with total coupled isospin I_s and hypercharge Y_s, there must exist at least one product irrep $R_s\gamma_s$ whose corresponding CGC assumes the largest allowed magnitude, $O(N_c^0)$. One may therefore begin with "stretched" quantum numbers, for which precisely one R_s is allowed and the corresponding CGC are therefore guaranteed to be $O(N_c^0)$—indeed, unity in magnitude. For example, the state in the product "**8**" ⊗ **8** ⊂ "$\overline{\mathbf{10}}$" with $I_s = 0$, $Y = \frac{N_c}{3} + 1$ is the only one in the product of $I_B = \frac{1}{2}$, $Y_B = \frac{N_c}{3}$ and $I_\phi = \frac{1}{2}$, $Y_\phi = 1$, and therefore its CGC is $O(N_c^0)$ (− 1, in fact).

Now note that the $U_\pm$ and $V_\pm$ SU(3) ladder operators assume a very useful form [19]. For example,

$$V_+|(p,q)II_zY\rangle = +g[(p,q),I,I_z,Y]|(p,q),I+1/2,I_z+1/2,Y+1\rangle + g[(p,q),-(I+1),I_z,Y]|(p,q),I-1/2,I_z+1/2,Y+1\rangle, \tag{8}$$

where the function g is given by [8]

$$g[(p,q)II_zY] = \left\{\frac{(I+I_z+1)[\frac{1}{3}(p-q)+I+\frac{Y}{2}+1][\frac{1}{3}(p+2q)+I+\frac{Y}{2}+2][\frac{1}{3}(2p+q)-I-\frac{Y}{2}]}{(2I+1)(2I+2)}\right\}^{1/2}, \tag{9}$$

and is the analogue to the familiar SU(2) functions $[(I \mp I_z)(I \pm I_z + 1)]^{1/2}$ that appear with the operators $I_\pm$. SU(3) CGC are then derived by the same coupling approach as for SU(2) (e.g., $V_{s,\pm} = V_{B,\pm} + V_{\phi,\pm}$). As seen from Eq. (8), these ladder operators generally produce two states, and therefore the SU(3) recursion relations generally involve six CGC [8,19]. We decline to present these cumbersome expressions here [e.g., Eq. (2.5) in [8]], but merely indicate features important to the current analysis.

Since the meson irrep R_ϕ does not scale with N_c, the functions g appearing from $U_{\phi,\pm}$ or $V_{\phi,\pm}$, which have $|\Delta Y_\phi| = 1$, do not induce any N_c factors. However, for the baryon irreps R_B and R_s, the quantities q and Y appearing in $U_{B,\pm}$, $V_{B,\pm}$, $U_{s,\pm}$, and $V_{s,\pm}$ (all of which have $\Delta Y_\phi = 0$) both scale as N_c. Interestingly, two of the three factors in g containing q and Y appear in the combination $\frac{q}{3} - \frac{Y}{2}$, whose $O(N_c)$ term cancels, while the $O(N_c)$ term in the third (in the combination $\frac{2q}{3} + \frac{Y}{2}$) does not, making the corresponding g factors $O(N_c^{1/2})$. This factor can also be seen from the fact that the given states, lying near $Y_{\max}$, are linear combinations of eigenstates carrying large values of U- and V-spin and near-maximal values of U_3 and V_3. Since the g-factors are simply $[(U \mp U_3)(U \pm U_3 + 1)]^{1/2}$ and $[(V \mp V_3)(V \pm V_3 + 1)]^{1/2}$ in disguise, each one has but a single $O(N_c^{1/2})$ factor. Dividing through by this $N_c^{1/2}$, the $U_\pm$, $V_\pm$ CGC recursion relations assume the form

$$(4\text{ CGC with } \Delta Y_\phi = 0) + \frac{1}{\sqrt{N_c}}(2\text{ CGC with } |\Delta Y_\phi| = 1) = 0. \tag{10}$$

This result indicates that all CGC with $\Delta Y_\phi = 0$ tend to appear at the same order in N_c, barring a fortuitous cancellation. However, since the 6-CGC recursion relations also include ordinary SU(2) isospin CGC [again, see Eq. (2.5) in [8] for an example], and the same SU(3) CGC appear for several independent charge states, such cancellations are comparatively rare.

In practice, one begins with the stretched states for all $R_s\gamma_s \subset R_B \otimes R_\phi$ that have the largest allowed $Y_{s,\max}$ value, which therefore have $O(N_c^0)$ CGC, and uses the $U_\pm$, $V_\pm$ recursion relations to obtain all other $O(N_c^0)$ CGC for the given $R_s\gamma_s$, all of which [by Eq. (10)] have the same value of $Y_\phi = Y_{s,\max} - Y_{B,\max}$, where $Y_{B,\max} = \frac{N_c}{3}$. For example, in "**8**" ⊗ **8** the largest $Y_{s,\max}$ value $\frac{N_c}{3} + 1$ is obtained for R_s = "**27**" and "$\overline{\mathbf{10}}$", and the $O(N_c^0)$ CGC for these two product irreps all have $Y_\phi = 1$. Order-by-order unitarity in N_c and the completeness of SU(3) CGC with a given fixed value of Y_ϕ then imply that the $O(N_c^0)$ CGC thus obtained are the only ones carrying the given Y_ϕ value. But then, Eq. (10) and unitarity imply that changing the value of Y_ϕ by one unit (call it $Y_{\phi'}$) for the same $R_s\gamma_s$ produces CGC that are generically a factor $N_c^{-1/2}$ smaller. By completeness, other irreps $R_{s'}\gamma_{s'}$ must step in to provide the $O(N_c^0)$ CGC for the new value $Y_{\phi'}$, and by noting again that the stretched cases (those carrying $Y_{s',\max}$) must have $O(N_c^0)$ CGC, we see from $Y_{\phi'} = Y_{s',\max} - Y_{B,\max}$ that the value of $Y_{s',\max}$ must also change by one unit. In the "**8**" ⊗ **8** example, the $Y_\phi = 0$ CGC with R_s = "**27**" and "$\overline{\mathbf{10}}$" are at most $O(N_c^{-1/2})$, meaning that the remaining $R_s\gamma_s$ with $Y_{s,\max} = \frac{N_c}{3}$ states ("$\mathbf{8}_\gamma$" and **10**) must provide the $O(N_c^0)$ CGC. Continuing the construction in this fashion establishes Eq. (7).

IV. REDUCING THE 3-FLAVOR CASE TO THE 2-FLAVOR CASE

The master amplitude expression for a 3-flavor meson-baryon scattering process $\phi(S_\phi, R_\phi, I_\phi, Y_\phi) + B(S_B, R_B, I_B, Y_B) \to \phi'(S_{\phi'}, R_{\phi'}, I_{\phi'}, Y_{\phi'}) + B'(S_{B'}, R_{B'}, I_{B'}, Y_{B'})$, where S is particle spin, was originally obtained in Ref. [20], and generalized to include $O(N_c)$ quantum numbers in Ref. [9]. The master expression for such scattering amplitudes in the large N_c limit then reads

$$S_{LL'SS'J_sR_s\gamma_s\gamma'_sI_sY_s} = (-1)^{S_B-S_{B'}}([R_B][R_{B'}][S][S'])^{1/2}/[R_s] \sum_{\substack{I\in R_\phi, I'\in R_{\phi'},\\ I''\in R_s, Y\in R_\phi\cap R_{\phi'}}} (-1)^{I+I'+Y}[I''] \begin{pmatrix} R_B & R_\phi & \| & R_s\gamma_s \\ S_B\frac{N_c}{3} & IY & \| & I''Y+\frac{N_c}{3} \end{pmatrix} \times \begin{pmatrix} R_B & R_\phi & \| & R_s\gamma_s \\ I_BY_B & I_\phi Y_\phi & \| & I_sY_s \end{pmatrix} \begin{pmatrix} R_{B'} & R_{\phi'} & \| & R_s\gamma'_s \\ S_{B'}\frac{N_c}{3} & I'Y & \| & I''Y+\frac{N_c}{3} \end{pmatrix} \begin{pmatrix} R_{B'} & R_{\phi'} & \| & R_s\gamma'_s \\ I_{B'}Y_{B'} & I_{\phi'}Y_{\phi'} & \| & I_sY_s \end{pmatrix} \times \sum_{K,\tilde{K},\tilde{K}'} [K]([\tilde{K}][\tilde{K}'])^{1/2} \begin{Bmatrix} L & I & \tilde{K} \\ S & S_B & S_\phi \\ J_s & I'' & K \end{Bmatrix} \begin{Bmatrix} L' & I' & \tilde{K}' \\ S' & S_{B'} & S_{\phi'} \\ J_s & I'' & K \end{Bmatrix} \tau^{\{II'Y\}}_{K\tilde{K}\tilde{K}'LL'}. \tag{11}$$

S and S' indicate the total hadron spin angular momentum (i.e., not including orbital angular momentum). The quantities in braces are ordinary SU(2) $9j$ symbols. The key quantum number describing the dynamics of the reduced scattering amplitudes τ is K, which in chiral soliton models represents the grand spin $\mathbf{K} = \mathbf{I} + \mathbf{J}$.

In light of the results Eqs. (1) and (7), Eq. (11) may be simplified considerably. In particular, Eq. (7) requires that the leading $[O(N_c^0)]$ SU(3) CGC in Eq. (11) have $Y = Y_\phi = Y_{\phi'} = r$, where $Y_{s,\max} = \frac{N_c}{3} + r$. We immediately see that the leading-order processes in $1/N_c$ require $Y_B = Y_{B'}$, i.e., no strangeness change in the scattered baryon, a fact that was used in Ref. [7]. Also, Eq. (1) eliminates the SU(3) multiplet dimensions:

$$([R_B][R_{B'}])^{1/2}/[R_s] \to ([S_B][S_{B'}])^{1/2}/[I_{s,\text{top}}]. \tag{12}$$

Moreover, the product degeneracy factors $\gamma_{s,s'}$ cannot be discerned in any physical process and therefore must also be summed over coherently in the full physical amplitude.

Specializing now to the case of nonstrange scattered baryons ($I_{B,B'} = S_{B,B'}$, $Y_{B,B'} = \frac{N_c}{3}$), one first notes that only intermediate states with $Y_s = Y_{s,\max}$ appear at $O(N_c^0)$. In order to recover the 2-flavor result, one must also note that implicit in Eq. (11) is a factor $\delta_{R_sR'_s}$, and that the R_s factor in the last two SU(3) CGC actually start as R'_s. Then one sums over the intermediate SU(3) irreps R_s and R'_s. Employing the well-known SU(3) CGC completeness relation [21]

$$\sum_{R\gamma Y} \begin{pmatrix} R_1 & R_2 & \| & R\gamma \\ I_1Y_1 & I_2Y_2 & \| & IY \end{pmatrix} \begin{pmatrix} R_1 & R_2 & \| & R\gamma \\ I'_1Y'_1 & I'_2Y'_2 & \| & IY \end{pmatrix} = \delta_{I_1I'_1}\delta_{I_2I'_2}\delta_{Y_1Y'_1}\delta_{Y_2Y'_2}, \tag{13}$$

in the current case removes all SU(3) CGC and imposes $\delta_{II_\phi}\delta_{I'I_{\phi'}}\delta_{I''I_s}$. Noting that $I_\phi + Y_\phi/2$ and $I_{\phi'} + Y_{\phi'}/2$ are integers for mesons, one is left with the 2-flavor result [3,22],

$$S_{LL'SS'I_sJ_s} = \sum_{K,\tilde{K},\tilde{K}'} [K]([S_B][S_{B'}][S][S'][\tilde{K}][\tilde{K}'])^{1/2} \times \begin{Bmatrix} L & I_\phi & \tilde{K} \\ S & S_B & S_\phi \\ J_s & I_s & K \end{Bmatrix} \times \begin{Bmatrix} L' & I_{\phi'} & \tilde{K}' \\ S' & S_{B'} & S_{\phi'} \\ J_s & I_s & K \end{Bmatrix} \tau_{K\tilde{K}\tilde{K}'LL'}, \tag{14}$$

where $\tau_{K\tilde{K}\tilde{K}'LL'} \equiv (-1)^{I_B-I_{B'}+I_\phi-I_{\phi'}} \tau^{\{I_\phi I_{\phi'} Y_\phi\}}_{K\tilde{K}\tilde{K}'LL'}$.

The phenomenologically most useful special case of Eq. (11) occurs for mesons in the 0^- octet, $S_\phi = S_{\phi'} = 0$. Then the $9j$ symbols collapse to $6j$ symbols:

$$\begin{Bmatrix} L & I & \tilde{K} \\ S & S_B & 0 \\ J_s & I'' & K \end{Bmatrix} = (-1)^{L+I''+K+S_B}([S][\tilde{K}])^{-1/2} \times \delta_{SS_B}\delta_{\tilde{K}K} \begin{Bmatrix} K & I'' & J_s \\ S_B & L & I \end{Bmatrix}, \tag{15}$$

and similarly for the other $9j$ symbol. Then Eq. (11) simplifies by losing the sums on $\tilde{K}$ and $\tilde{K}'$ and the $([S][S'] \times [\tilde{K}][\tilde{K}'])^{1/2}$ factors, as well as the phase $(-1)^{S_B-S_{B'}}$ [These phases in Eqs. (2) and (6) of Ref. [9] are incorrect]. For reference, the spinless meson expression reads

$$S_{LL'S_BS_{B'}J_sR_s\gamma_s\gamma'_sI_sY_s} = (-1)^{L-L'}([R_B][R'_B])^{1/2}/[R_s] \sum_{\substack{I,I',Y\in 8,\\ I''\in R_s}} (-1)^{I+I'+Y}[I''] \begin{pmatrix} R_B & 8 & \Big\| & R_s\gamma_s \\ S_B\frac{N_c}{3} & IY & \Big\| & I''Y+\frac{N_c}{3} \end{pmatrix} \begin{pmatrix} R_B & 8 & \Big\| & R_s\gamma_s \\ I_BY_B & I_\phi Y_\phi & \Big\| & I_sY_s \end{pmatrix}$$
$$\times \begin{pmatrix} R_{B'} & 8 & \Big\| & R_s\gamma'_s \\ S_{B'}\frac{N_c}{3} & I'Y & \Big\| & I''Y+\frac{N_c}{3} \end{pmatrix} \begin{pmatrix} R_{B'} & 8 & \Big\| & R_s\gamma'_s \\ I_{B'}Y_{B'} & I_{\phi'}Y_{\phi'} & \Big\| & I_sY_s \end{pmatrix} \sum_K [K] \begin{Bmatrix} K & I'' & J_s \\ S_B & L & I \end{Bmatrix} \begin{Bmatrix} K & I'' & J_s \\ S_{B'} & L' & I' \end{Bmatrix} \tau^{\{II'Y\}}_{KKKLL'}. \quad (16)$$

Note that, although we restrict to spinless mesons in this special case, they are not necessarily pseudo-Goldstone bosons. Chiral symmetry is not imposed in any way and would provide additional constraints.

The reduction of Eq. (16) to its nonstrange equivalent works in precisely the same way as the reduction of Eq. (11) to Eq. (14). One finds [2,23]

$$S^{\phi\phi'}_{LL'S_BS_{B'}I_sJ_s} = (-1)^{S_{B'}-S_B}([S_B][S_{B'}])^{1/2}\sum_K [K] \times \begin{Bmatrix} K & I_s & J_s \\ S_{B'} & L' & I_{\phi'} \end{Bmatrix} \begin{Bmatrix} K & I_s & J_s \\ S_B & L & I_\phi \end{Bmatrix} s^{\phi\phi'}_{KL'L}, \quad (17)$$

where $s^{\phi\phi'}_{KL'L} \equiv (-1)^{L-L'}\tau_{KKKLL'} = (-1)^{L-L'+I_B-I_{B'}+I_\phi-I_{\phi'}}\tau^{\{I_\phi I_{\phi'}Y_\phi\}}_{KKKLL'}$; for example, $s^{\pi}_{KL'L}$ in Ref. [2] means $\phi = \phi' = \pi$.

The notation for the full amplitudes is admittedly cumbersome due to the numerous indices required for their unambiguous characterization. The standard notation in the literature uses $L_{2I_s2J_s}$ for resonances with half-integer isospin, $L_{I_s2J_s}$ otherwise. Of course, in reality almost all experiments scatter 0^- initial mesons (π's and K's) on nucleons ($J^P = \frac{1}{2}^+$), which combined with parity conservation forces $L = L'$ to be an even integer. In the following we are more interested in uncovering the full pole structure than in presenting only expressions for $N_c = 3$ physical amplitudes; we allow amplitudes with, for example, an $\eta\Sigma^*$ initial state. Nevertheless, as discussed below we restrict to $B = B'$, $\phi = \phi'$, $L = L'$. A sufficient notation for the amplitudes we present, compatible with that used in Refs. [2–7], is therefore $L^{\phi B}_{(2)I_s2J_s}$. The SU(3) labels $R_s\gamma_s$ can also be made explicit if one is discussing a particular resonant channel, but of course in real data these channels are summed coherently.

V. THE FIVE MULTIPLETS

In Ref. [9] we explained how Eq. (16) can be used to uncover degenerate SU(3) multiplets of resonances. In short, one begins with a single established resonance with given I_s, J_s quantum numbers, finds which reduced amplitude $\tau^{\{II'Y\}}_{KKKLL'}$ contains the pole, notes that the quantum numbers LL' and $II'Y$ refer only to the details of the formation of the resonance and not the resonance itself, and concludes that resonance poles are labeled solely by K [in the SU(3) limit].

TABLE I. Partial-wave amplitudes containing resonances with quantum numbers corresponding to states in the large N_c quark-picture SU(6) × O(3) ("**70**", 1^-). Expansions are given in terms of K-amplitudes, according to Eq. (16).

State	"**70**" pole masses	Partial wave, K-amplitudes
$\Lambda_{1/2}$("**8**")	m_0, m_1	$S^{\pi\Sigma}_{01} = \tau^{\{110\}}_{11100}$ $S^{\eta\Lambda}_{01} = \tau^{\{000\}}_{00000}$
$\Lambda_{3/2}$("**8**")	m_1, m_2	$D^{\pi\Sigma}_{03} = \frac{1}{2}(\tau^{\{110\}}_{11122} + \tau^{\{110\}}_{22222})$ $D^{\eta\Lambda}_{03} = \tau^{\{000\}}_{22222}$
$\Lambda_{5/2}$("**8**")	m_2	$D^{\pi\Sigma}_{05} = \frac{1}{9}(2\tau^{\{110\}}_{22222} + 7\tau^{\{110\}}_{33322})$ $D^{\eta\Lambda}_{05} = \tau^{\{000\}}_{22222}$
$\Lambda_{1/2}$("**1**")	$m_{\frac{1}{2}}$	$S^{\bar{K}N}_{01} = \tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{1}{2}\frac{1}{2}\frac{1}{2}00}$
$\Lambda_{3/2}$("**1**")	$m_{\frac{3}{2}}$	$D^{\bar{K}N}_{03} = \tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}$
$\Sigma_{1/2}$("**8**")	m_0, m_1	$S^{\pi\Lambda}_{11} = \frac{1}{3}\tau^{\{110\}}_{11100}$ $S^{\pi\Sigma}_{11} = \frac{2}{3}\tau^{\{110\}}_{11100}$ $S^{\eta\Sigma}_{11} = \tau^{\{000\}}_{00000}$
$\Sigma_{3/2}$("**8**")	m_1, m_2	$D^{\pi\Lambda}_{13} = \frac{1}{6}(\tau^{\{110\}}_{11122} + \tau^{\{110\}}_{22222})$ $D^{\pi\Sigma}_{13} = \frac{1}{3}(\tau^{\{110\}}_{11122} + \tau^{\{110\}}_{22222})$ $D^{\eta\Sigma}_{13} = \tau^{\{000\}}_{22222}$
$\Sigma_{5/2}$("**8**")	m_2	$D^{\pi\Lambda}_{15} = \frac{1}{27}(2\tau^{\{110\}}_{22222} + 7\tau^{\{110\}}_{33322})$ $D^{\pi\Sigma}_{15} = \frac{2}{27}(2\tau^{\{110\}}_{22222} + 7\tau^{\{110\}}_{33322})$ $D^{\eta\Sigma}_{15} = \tau^{\{000\}}_{22222}$
$\Sigma_{1/2}$("**S**")	$m_{\frac{1}{2}}$, $m_{\frac{3}{2}}$	$S^{\bar{K}N}_{11} = \tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{1}{2}\frac{1}{2}\frac{1}{2}00}$ $D^{\bar{K}\Delta}_{11} = \tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}$
$\Sigma_{3/2}$("**S**")	$m_{\frac{1}{2}}$, $m_{\frac{3}{2}}$	$D^{\bar{K}N}_{13} = \frac{1}{5}(\tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22} + 4\tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22})$ $S^{\bar{K}\Delta}_{13} = \tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{1}{2}\frac{1}{2}\frac{1}{2}00}$ $D^{\bar{K}\Delta}_{13} = \frac{1}{5}(4\tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22} + \tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22})$
$\Sigma_{5/2}$("**S**")	$m_{\frac{3}{2}}$	$D^{\bar{K}N}_{15} = \frac{1}{15}(8\tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22} + 7\tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22})$ $D^{\bar{K}\Delta}_{15} = \frac{1}{15}(7\tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22} + 8\tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22})$ $G^{\bar{K}\Delta}_{15} = \tau^{\{\frac{1}{2}\frac{1}{2}-1\}}_{\frac{7}{2}\frac{7}{2}\frac{7}{2}44}$

We now show that the collection of resonance states with the quantum numbers of the SU(6) × O(3) (“**70**”, 1^-) multiplet (the parity entering through allowed values of L, L') is accommodated by 5 poles, one in each of 5 reduced amplitudes with $K = 0, \frac{1}{2}, 1, \frac{3}{2}$, and 2.

An exhaustive demonstration of this point would require the tabulation of a huge set of amplitudes, including scattering with not only the “**8**” and “**10**”, but also the stable baryons in the “**56**” with $J^P = \frac{5}{2}^+, \ldots, \frac{N_c}{2}^+$, as well as with members of the “**8**” and “**10**” carrying quantum numbers not appearing for $N_c = 3$ (such as an isospin-$\frac{3}{2}$ Ξ), or between states with $B \neq B'$, $\phi \neq \phi'$, or $L \neq L'$. (Note however, that parity conservation in “**8**” $\frac{1}{2}^+ \to \frac{1}{2}^+$ scattering with 0^- mesons dictates $L = L'$.) While many of these processes are physically interesting (e.g., $\pi N \to \eta N$), for our purposes it is equally convincing to demonstrate the pole structure by restricting the tabulation to a much smaller set: All quantum numbers are chosen diagonal ($B = B'$, $\phi = \phi'$, $L = L'$), and only “**8**” → “**8**” transitions allowed for $N_c = 3$ are exhibited, except in the few circumstances where “**8**” scattering does not access all the poles, in which case “**10**” → “**10**” scattering is also exhibited.

The results of this analysis appear in Tables I, II, III, and IV. Note that the sign for the $\eta\Xi$(“**8**”) → Ξ(“$\mathbf{8}_2$”) CGC in Ref. [8] (relevant to Table II) is incorrect; it should begin with $+(3N_c - 19)$. Also, the symbol Ω' (Table IV) indicates the $I = 1$ partner to the Ω for “**10**” with $N_c > 3$, introduced to show that all expected resonance poles indeed occur.

TABLE II. First continuation of Table I.

State	“**70**” pole masses	Partial wave, K-amplitudes
$\Sigma_{1/2}$(“**10**”)	m_1, m_2	$S_{11}^{\pi\Lambda} = \frac{2}{3}\tau_{11100}^{\{110\}}$ $S_{11}^{\pi\Sigma} = \frac{1}{3}\tau_{11100}^{\{110\}}$ $D_{11}^{\eta\Sigma^*} = \tau_{22222}^{\{000\}}$
$\Sigma_{3/2}$(“**10**”)	m_0, m_1, m_2	$D_{13}^{\pi\Lambda} = \frac{1}{30}(\tau_{11122}^{\{110\}} + 5\tau_{22222}^{\{110\}} + 14\tau_{33322}^{\{110\}})$ $D_{13}^{\pi\Sigma} = \frac{1}{60}(\tau_{11122}^{\{110\}} + 5\tau_{22222}^{\{110\}} + 14\tau_{33322}^{\{110\}})$ $S_{13}^{\eta\Sigma^*} = \tau_{00000}^{\{000\}}$ $D_{13}^{\eta\Sigma^*} = \tau_{22222}^{\{000\}}$
$\Sigma_{5/2}$(“**10**”)	m_1, m_2	$D_{15}^{\pi\Lambda} = \frac{1}{135}(27\tau_{11122}^{\{110\}} + 35\tau_{22222}^{\{110\}} + 28\tau_{33322}^{\{110\}})$ $D_{15}^{\pi\Sigma} = \frac{1}{270}(27\tau_{11122}^{\{110\}} + 35\tau_{22222}^{\{110\}} + 28\tau_{33322}^{\{110\}})$ $D_{15}^{\eta\Sigma^*} = \tau_{22222}^{\{000\}}$ $G_{15}^{\eta\Sigma^*} = \tau_{44444}^{\{000\}}$
$\Sigma_{7/2}$(“**10**”)	m_2	$G_{17}^{\pi\Lambda} = \frac{1}{108}(7\tau_{33344}^{\{110\}} + 21\tau_{44444}^{\{110\}} + 44\tau_{55544}^{\{110\}})$ $G_{17}^{\pi\Sigma} = \frac{1}{216}(7\tau_{33344}^{\{110\}} + 21\tau_{44444}^{\{110\}} + 44\tau_{55544}^{\{110\}})$ $D_{17}^{\eta\Sigma^*} = \tau_{22222}^{\{000\}}$ $G_{17}^{\eta\Sigma^*} = \tau_{44444}^{\{000\}}$
$\Xi_{1/2}$(“**8**”)	m_0, m_1	$S_{11}^{\pi\Xi} = \frac{1}{9}\tau_{11100}^{\{110\}}$ $S_{11}^{\eta\Xi} = \tau_{00000}^{\{000\}}$
$\Xi_{3/2}$(“**8**”)	m_1, m_2	$D_{13}^{\pi\Xi} = \frac{1}{18}(\tau_{11122}^{\{110\}} + \tau_{22222}^{\{110\}})$ $D_{13}^{\eta\Xi} = \tau_{22222}^{\{000\}}$
$\Xi_{5/2}$(“**8**”)	m_2	$D_{15}^{\pi\Xi} = \frac{1}{81}(2\tau_{22222}^{\{110\}} + 7\tau_{33322}^{\{110\}})$ $D_{15}^{\eta\Xi} = \tau_{22222}^{\{000\}}$
$\Xi_{1/2}$(“**1**”)	$m_{\frac{1}{2}}$	$S_{11}^{\bar{K}\Lambda} = \frac{1}{4}\tau_{\frac{1}{2}\frac{1}{2}\frac{1}{2}00}^{\{\frac{1}{2}\frac{1}{2}-1\}}$ $S_{11}^{\bar{K}\Sigma} = \frac{3}{4}\tau_{\frac{1}{2}\frac{1}{2}\frac{1}{2}00}^{\{\frac{1}{2}\frac{1}{2}-1\}}$
$\Xi_{3/2}$(“**1**”)	$m_{\frac{3}{2}}$	$D_{13}^{\bar{K}\Lambda} = \frac{1}{4}\tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}}$ $D_{13}^{\bar{K}\Sigma} = \frac{3}{4}\tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}}$

TABLE III. Second continuation of Table I.

State	"70" pole masses	Partial wave, K-amplitudes
$\Xi_{1/2}$("**10**")	m_1, m_2	$S_{11}^{\pi\Xi} = \frac{8}{9}\tau_{11100}^{\{110\}}$
		$D_{11}^{\eta\Xi^*} = \tau_{22222}^{\{000\}}$
$\Xi_{3/2}$("**10**")	m_0, m_1, m_2	$D_{13}^{\pi\Xi} = \frac{2}{45}(\tau_{11122}^{\{110\}} + 5\tau_{22222}^{\{110\}} + 14\tau_{33322}^{\{110\}})$
		$S_{13}^{\eta\Xi^*} = \tau_{00000}^{\{000\}}$
		$D_{13}^{\eta\Xi^*} = \tau_{22222}^{\{000\}}$
$\Xi_{5/2}$("**10**")	m_1, m_2	$D_{15}^{\pi\Xi} = \frac{4}{405}(27\tau_{11122}^{\{110\}} + 35\tau_{22222}^{\{110\}} + 28\tau_{33322}^{\{110\}})$
		$D_{15}^{\eta\Xi^*} = \tau_{22222}^{\{000\}}$
		$G_{15}^{\eta\Xi^*} = \tau_{44444}^{\{000\}}$
$\Xi_{7/2}$("**10**")	m_2	$G_{17}^{\pi\Xi} = \frac{1}{81}(7\tau_{33344}^{\{110\}} + 21\tau_{44444}^{\{110\}} + 44\tau_{55544}^{\{110\}})$
		$D_{17}^{\eta\Xi^*} = \tau_{22222}^{\{000\}}$
		$G_{17}^{\eta\Xi^*} = \tau_{44444}^{\{000\}}$
$\Xi_{1/2}$("**S**")	$m_{\frac{1}{2}}$, $m_{\frac{3}{2}}$	$S_{11}^{\bar{K}\Lambda} = \frac{3}{4}\tau_{\frac{1}{2}\frac{1}{2}\frac{1}{2}00}^{\{\frac{1}{2}\frac{1}{2}-1\}}$
		$S_{11}^{\bar{K}\Sigma} = \frac{1}{4}\tau_{\frac{1}{2}\frac{1}{2}\frac{1}{2}00}^{\{\frac{1}{2}\frac{1}{2}-1\}}$
		$D_{11}^{\bar{K}\Sigma^*} = \tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}}$
$\Xi_{3/2}$("**S**")	$m_{\frac{1}{2}}$, $m_{\frac{3}{2}}$	$D_{13}^{\bar{K}\Lambda} = \frac{3}{20}(\tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}} + 4\tau_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}})$
		$D_{13}^{\bar{K}\Sigma} = \frac{1}{20}(\tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}} + 4\tau_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}})$
		$S_{13}^{\bar{K}\Sigma^*} = \tau_{\frac{1}{2}\frac{1}{2}\frac{1}{2}00}^{\{\frac{1}{2}\frac{1}{2}-1\}}$
		$D_{13}^{\bar{K}\Sigma^*} = \frac{1}{5}(4\tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}} + \tau_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}})$
$\Xi_{5/2}$("**S**")	$m_{\frac{3}{2}}$	$D_{15}^{\bar{K}\Lambda} = \frac{1}{20}(8\tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}} + 7\tau_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}})$
		$D_{15}^{\bar{K}\Sigma} = \frac{1}{60}(8\tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}} + 7\tau_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}})$
		$D_{15}^{\bar{K}\Sigma^*} = \frac{1}{15}(7\tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}} + 8\tau_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}})$
		$G_{15}^{\bar{K}\Sigma^*} = \tau_{\frac{7}{2}\frac{7}{2}\frac{7}{2}44}^{\{\frac{1}{2}\frac{1}{2}-1\}}$

In light of the fact that Eq. (17) is a special case of the full result Eq. (11), the results appearing in Table I of Ref. [2] still hold, demonstrating that all nonstrange resonances in the "**70**" reduce to 3 poles with $K = 0, 1,$ and 2. Then, since Eq. (11) is an SU(3)-symmetric expression, the same pole that produces a given nonstrange resonance must also produce all its SU(3)-multiplet partners. For example, the $N_{1/2}$ state corresponding to the $K = 0$ pole is but the nonstrange member of an "**8**" corresponding to the same pole. This point is also apparent in the tables.

One concludes from studying Tables I, II, III, and IV that the 20 SU(3) multiplets of the ("**70**", 1^-) listed in Eq. (6) actually collect into 5 irreps labeled by K:

$$\begin{aligned}
&K = 0{:}\left(\text{“}\mathbf{8}\text{”}, \frac{1}{2}\right) \oplus \left(\text{“}\mathbf{10}\text{”}, \frac{3}{2}\right), \qquad K = \frac{1}{2}{:}\left(\text{“}\mathbf{1}\text{”}, \frac{1}{2}\right) \oplus \left(\text{“}\mathbf{S}\text{”}, \frac{1}{2}\right) \oplus \left(\text{“}\mathbf{S}\text{”}, \frac{3}{2}\right),\\
&K = 1{:}\left(\text{“}\mathbf{8}\text{”}, \frac{1}{2}\right) \oplus \left(\text{“}\mathbf{8}\text{”}, \frac{3}{2}\right) \oplus \left(\text{“}\mathbf{10}\text{”}, \frac{1}{2}\right) \oplus \left(\text{“}\mathbf{10}\text{”}, \frac{3}{2}\right) \oplus \left(\text{“}\mathbf{10}\text{”}, \frac{5}{2}\right), \qquad K = \frac{3}{2}{:}\left(\text{“}\mathbf{1}\text{”}, \frac{3}{2}\right) \oplus \left(\text{“}\mathbf{S}\text{”}, \frac{1}{2}\right) \oplus \left(\text{“}\mathbf{S}\text{”}, \frac{3}{2}\right) \oplus \left(\text{“}\mathbf{S}\text{”}, \frac{5}{2}\right),\\
&K = 2{:}\left(\text{“}\mathbf{8}\text{”}, \frac{3}{2}\right) \oplus \left(\text{“}\mathbf{8}\text{”}, \frac{5}{2}\right) \oplus \left(\text{“}\mathbf{10}\text{”}, \frac{1}{2}\right) \oplus \left(\text{“}\mathbf{10}\text{”}, \frac{3}{2}\right) \oplus \left(\text{“}\mathbf{10}\text{”}, \frac{5}{2}\right) \oplus \left(\text{“}\mathbf{10}\text{”}, \frac{7}{2}\right).
\end{aligned} \tag{18}$$

That the large N_c quark model SU(6) × O(3) ("**70**", 1^-) multiplet actually contains 5 independent mass eigenvalues split by $O(N_c^0)$ can also be seen by referring to the Hamiltonian operator basis used in Refs. [14,15]. This analysis extends that performed in Ref. [2] for the nonstrange case, in which one finds 3 operators with linearly independent matrix elements up to $O(N_c^0)$, and only 3 distinct mass eigenvalues. By direct construction, one finds a single operator, $O_1 = \mathbb{1}$, whose matrix elements on all baryons is precisely N_c, and 4 operators with $O(N_c^0)$ matrix elements:

$$O_2 = \ell s, \qquad O_3 = \frac{1}{N_c}\ell^{(2)} g G_c,$$
$$O_4 = \ell s + \frac{4}{N_c+1}\ell t G_c, \qquad O_5 = \frac{1}{N_c}\left(t T_c - \frac{1}{12}\mathbb{1}\right). \tag{19}$$

Here, ℓ is the orbital excitation operator, while $\ell^{(2)}$ is the $\Delta\ell = 2$ tensor operator ($\ell^i\ell^j - \frac{1}{3}\delta^{ij}\ell^2$). Lowercase indicates operators acting upon the excited quark, and uppercase (with subscript c) indicates operators acting upon the core. S, T, and G denote operators with spin, flavor, and both spin and flavor indices, respectively, summed over all relevant quarks, and all spin and flavor indices implied by the component operators in Eq. (19) are summed in the unique nontrivial manner and then suppressed (e.g., $\ell t G_c \equiv \ell^i t^a G_c^{ia}$). The operators are equivalent to those at $O(N_c^0)$ in Ref. [15], except for the addition of O_5, which was omitted in that work [24]. O_4 and O_5 appear in more complicated forms whose matrix elements vanish for all states in multiplets with $Y_{\rm max} = \frac{N_c}{3}$ (which includes all nonstrange states in the "**70**"); in Ref. [14] they were termed "demotable," For the $Y_{\rm max} = \frac{N_c}{3} - 1$ multiplets ("**1**" and "**S**"), the matrix elements of O_5 at $O(N_c^0)$ are found to be $-\frac{1}{4}$.

Using this notation, one finds that each mass eigenstate in the "**70**" assumes one of only 5 distinct eigenvalues. Those in the SU(3) multiplets with $Y = \frac{N_c}{3}$ assume the values

$$\begin{aligned} m_0 &\equiv c_1 N_c - \left(c_2 + \frac{5}{24}c_3\right), \\ m_1 &\equiv c_1 N_c - \frac{1}{2}\left(c_2 - \frac{5}{24}c_3\right), \\ m_2 &\equiv c_1 N_c + \frac{1}{2}\left(c_2 - \frac{1}{24}c_3\right), \end{aligned} \tag{20}$$

which are the same expressions as in Ref. [2]. For the $Y_{\rm max} = \frac{N_c}{3} - 1$ multiplets, one additionally finds only the eigenvalues

TABLE IV. Third continuation of Table I. Ω' is the $I = 1$ partner of the Ω in "**10**" for $N_c > 3$.

State	"**70**" pole masses	Partial wave, K-amplitudes
$\Omega_{1/2}$("**10**")	m_1, m_2	$S_{01}^{\pi\Omega'} = \tau_{11100}^{\{110\}}$ $D_{01}^{\eta\Omega} = \tau_{22222}^{\{000\}}$
$\Omega_{3/2}$("**10**")	m_0, m_1, m_2	$D_{03}^{\pi\Omega'} = \frac{1}{20}(\tau_{11122}^{\{110\}} + 5\tau_{22222}^{\{110\}} + 14\tau_{33322}^{\{110\}})$ $S_{03}^{\eta\Omega} = \tau_{00000}^{\{000\}}$ $D_{03}^{\eta\Omega} = \tau_{22222}^{\{000\}}$
$\Omega_{5/2}$("**10**")	m_1, m_2	$D_{05}^{\pi\Omega'} = \frac{1}{90}(27\tau_{11122}^{\{110\}} + 35\tau_{22222}^{\{110\}} + 28\tau_{33322}^{\{110\}})$ $D_{05}^{\eta\Omega} = \tau_{22222}^{\{000\}}$ $G_{05}^{\eta\Omega} = \tau_{44444}^{\{000\}}$
$\Omega_{7/2}$("**10**")	m_2	$G_{07}^{\pi\Omega'} = \frac{1}{72}(7\tau_{33344}^{\{110\}} + 21\tau_{44444}^{\{110\}} + 44\tau_{55544}^{\{110\}})$ $D_{07}^{\eta\Omega} = \tau_{22222}^{\{000\}}$ $G_{07}^{\eta\Omega} = \tau_{44444}^{\{000\}}$
$\Omega_{1/2}$("**S**")	$m_{\frac{1}{2}}$, $m_{\frac{3}{2}}$	$S_{01}^{\bar{K}\Xi} = \tau_{\frac{1}{2}\frac{1}{2}\frac{1}{2}00}^{\{\frac{1}{2}\frac{1}{2}-1\}}$ $D_{01}^{\bar{K}\Xi^*} = \tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}}$
$\Omega_{3/2}$("**S**")	$m_{\frac{1}{2}}$, $m_{\frac{3}{2}}$	$D_{03}^{\bar{K}\Xi} = \frac{1}{5}(\tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}} + 4\tau_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}})$ $S_{03}^{\bar{K}\Xi^*} = \tau_{\frac{1}{2}\frac{1}{2}\frac{1}{2}00}^{\{\frac{1}{2}\frac{1}{2}-1\}}$ $D_{03}^{\bar{K}\Xi^*} = \frac{1}{5}(4\tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}} + \tau_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}})$
$\Omega_{5/2}$("**S**")	$m_{\frac{3}{2}}$	$D_{05}^{\bar{K}\Xi} = \frac{1}{15}(8\tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}} + 7\tau_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}})$ $D_{05}^{\bar{K}\Xi^*} = \frac{1}{15}(7\tau_{\frac{3}{2}\frac{3}{2}\frac{3}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}} + 8\tau_{\frac{5}{2}\frac{5}{2}\frac{5}{2}22}^{\{\frac{1}{2}\frac{1}{2}-1\}})$ $G_{05}^{\bar{K}\Xi^*} = \tau_{\frac{7}{2}\frac{7}{2}\frac{7}{2}44}^{\{\frac{1}{2}\frac{1}{2}-1\}}$

$$m_{1/2} \equiv c_1 N_c - (c_2 + c_4) - \frac{1}{4} c_5,$$
$$m_{3/2} \equiv c_1 N_c + \frac{1}{2}(c_2 + c_4) - \frac{1}{4} c_5. \tag{21}$$

Again, one sees that the ("**70**", 1^-) is actually a reducible collection of 5 multiplets. The mass eigenvalues, labeled by m_K, are listed in Tables I, II, III, and IV. While the old SU(6) symmetry does not hold at $O(N_c^0)$, the remaining level of degeneracy remains remarkable; for example, the multiplets listed in Eq. (18) mean that 5 eigenvalues [in the SU(3) limit] span 71 distinct isomultiplets, 30 for $N_c = 3$. And even when SU(3) symmetry is arbitrarily broken (i.e., reduced amplitudes τ with the same K but different Y are taken to be distinct), isomultiplets with the same value of Y and K but in different SU(3) irreps in Eq. (18) remain degenerate.

VI. PHENOMENOLOGICAL RESULTS

In this section we combine our qualitative results with experimental extractions of BR to determine phenomenologically the SU(3) and K irreducible representations (irreps) of various excited baryons. This is useful for two reasons. First, it gives insight into the nature of these resonant states in a framework independent of the quark model. Second, the extent to which the decays fall into patterns consistent with large N_c predictions provides a check on the applicability of the large N_c approach to excited states for the real world of $N_c = 3$.

Before discussing individual states in detail a few comments are in order. As noted in the introduction, the extraction of BR necessarily involves some modeling. In some cases the model dependence is small, and robust extractions of BR are possible. However, in many cases either the model dependence is large or the experimental data is insufficient, and the BR are not known well. Often the ranges for BR quoted by the Particle Data Group are quite broad [25]. Indeed, they are often so large that it is impossible to make even qualitative assessments of the dominant mode of decay. Accordingly, we focus our attention on those cases where the BR are relatively well established.

Another issue that should be kept in mind in this discussion is that the analysis presented so far is based on exact SU(3) flavor symmetry. Of course, in the real world SU(3) flavor is broken. The analysis is useful provided that SU(3) flavor violations are relatively modest (which they usually are). Similarly, the analysis is based on large N_c and implicitly assumes that $1/N_c$ corrections are small. However, in one obvious case both SU(3) violations and $1/N_c$ corrections can be expected to be greatly enhanced: resonant states not far above thresholds. In such regions the phase space is a very sensitive function of the masses, and a relatively small mass change can lead to dramatic shifts in the phase space. This near-threshold behavior is particularly critical in high L partial waves, where the partial decay rate scales as p^{2L+1}, p being the 3-momentum of either outgoing particle in the center-of-momentum frame. Accordingly, we focus on large N_c predictions for the coupling in various decays rather than on the partial widths or BR, since the couplings are far less sensitive to threshold effects.

The most striking result of this work has already been described in Sec. III and particularly by the constraint Eq. (7): In the large N_c limit, baryon resonances couple only to mesons with a hypercharge equal to the amount by which the hypercharge of the top row of its SU(3) multiplet exceeds that of the stable baryons ($Y_{\max} = \frac{N_c}{3}$). This result provides a means by which the singlet and octet Λ may be distinguished: The former prefers $\bar{K}N$ to $\pi\Sigma$ decays with a coupling $O(N_c)$ larger than predicted by phase space alone, and vice versa.

One sees this effect clearly in some of the Λ resonances. The first state for which it is apparent [25] is the $\Lambda(1520)D_{03}$ [note that the $\Lambda(1405)S_{01}$ lies below the $\bar{K}N$ threshold]. This state is traditionally assigned to be an SU(3) singlet. The phase space ($\propto p^1$) for decay into $\bar{K}N$ is only about 3/4 of that for $\pi\Lambda$, but the BR for the former is actually slightly larger than for the latter. Note however that the decay is a d wave, so that the partial width goes as the p to the *fifth* power. Thus the $\bar{K}N$ decay is kinematically suppressed by a factor of ~4–5 compared to the $\pi\Lambda$, so that the coupling is ~4–5 times larger. The dominance of the $\bar{K}N$ coupling as what one expects at large N_c if the state is a singlet.

Virtually all of the low-lying Λ resonances have substantial $\bar{K}N$ BR, again suggesting a sizable **1** component in most Λ resonances. However, for most of these states the BR are not determined with sufficient certainty to make definitive statements. Many of these states appear to have substantial BR to both $\bar{K}N$ and $\pi\Lambda$. To the extent that these results are reliable, one has evidence for important effects of SU(3) breaking, indicating the mixing of SU(3) irreps. The $\Lambda(1830)D_{05}$ is a notable exception: Its BR to $\bar{K}N$ is less than 10%, strongly suggesting that it is predominantly octet.

The situation with Σ resonances is intriguing. Since the only SU(3) irreps available at $N_c = 3$ are **8** and **10**, the large N_c selection rule suggests small $\bar{K}N$ BR. While this is true for most of these resonances, a few [notably the $\Sigma(1775)D_{15}$] have substantial $\bar{K}N$ couplings. Nevertheless, such effects may well be $1/N_c$ corrections of a type relatively easy to understand. For any $N_c \geq 5$, the "**S**" irrep would contain Σ resonances with large $\bar{K}N$ couplings; in the final step of setting $N_c = 3$, by unitarity some part of these couplings must spill over into the SU(3) **8** and **10** irreps. It is very tempting to study these $1/N_c$ effects simply by retaining the full arbitrary-N_c CGC in Eq. (11), but this is only one source of $1/N_c$ corrections;

Eq. (11) in its current form only includes amplitudes that survive the large N_c limit.

Unfortunately, too little is known about Ξ and Ω resonances [25] to perform an interesting analysis of this sort.

The next result of phenomenological interest to the $N_c = 3$ universe is that the $K = 0$ multiplet, ("**8**", $1/2$) $\oplus$ ("**10**", $3/2$), couples to η but not π, while the other ("**8**", $1/2$) has $K = 1$ and couples to π but not η. More generally, the $K = 1$ pole appears only in channels coupled to π. These results are exact in the large N_c limit. In the spin-3/2 case large N_c provides "**10**"s, with $K = 0$, 1, and 2; however, for $N_c = 3$ only one remains, and so in that case it is not obvious how to identify physical states with the large N_c multiplets. Nevertheless, both $N_c = 3$ and larger N_c provide exactly 2 ("**8**", $1/2$) multiplets, making the coupling prediction testable. Indeed, the fact that one of these physical resonances, $N(1535)$, is η-philic and π-phobic, while the other, $N(1650)$, is the reverse, was the original phenomenological evidence [2] offered in support of this type of analysis.

This effect appears in the state $\Lambda(1670)S_{01}$, which lies a mere 5 MeV above the $\eta\Lambda$ threshold (a phase space about 6 times smaller than that for $\pi\Sigma$), and yet has a BR to this channel of 10%–25%. This suggests that the state is predominantly an η-philic $K = 0$ state. Likewise, the $\Sigma(1750)S_{11}$ lies only a few MeV above the $\eta\Sigma$ threshold but has a substantial (15%–55%) BR to that channel, and therefore is also predominantly $K = 0$. On the other hand, $\Lambda(1800)S_{01}$ has no detected $\eta\Lambda$ coupling, and therefore appears to be the $K = 1$ state.

One more interesting result of this analysis is a method of distinguishing **8** and **10** resonances based upon their decay modes. One such category arises from SU(3) CGC that are smaller than the saturation of the bound given in Eq. (7); in the cases considered here, this occurs for $\eta\Sigma \to \Sigma$("**10**"), for $\eta\Xi \to \Xi$("**10**"), for $\eta\Sigma^* \to \Sigma$("**8**"), and for $\eta\Xi^* \to \Xi$("**8**"), all of which are $O(1/N_c)$ smaller than naively expected. One then concludes, for example, that a Σ resonance with a large $\eta\Sigma^*$ coupling (none such yet observed) is mostly **10**. Another category arises from the interesting property that Eq. (11) applied to $\pi\Lambda$ and $\pi\Sigma$ external states differs only by the isospin quantum number in the external-state CGC. In particular, using the CGC in Ref. [8] one finds the amplitude ratios

$$r_{\mathbf{8}} \equiv \frac{\mathcal{A}[\Sigma\pi \to \Sigma(\text{``}\mathbf{8}\text{''})]}{\mathcal{A}[\Lambda\pi \to \Sigma(\text{``}\mathbf{8}\text{''})]} = \frac{N_c(N_c+7)}{N_c+6}\sqrt{\frac{2}{(N_c+3)(N_c-1)}},$$
$$r_{\mathbf{10}} \equiv \frac{\mathcal{A}[\Sigma\pi \to \Sigma(\text{``}\mathbf{10}\text{''})]}{\mathcal{A}[\Lambda\pi \to \Sigma(\text{``}\mathbf{10}\text{''})]} = -\frac{N_c+1}{\sqrt{2(N_c+3)(N_c-1)}}. \tag{22}$$

The calculation of $r_{\mathbf{8}}$ requires one to sum coherently over the "$\mathbf{8}_\gamma$" irreps. One finds $r_{\mathbf{8}}(\infty) = +\sqrt{2}$ and $r_{\mathbf{10}}(\infty) = -1/\sqrt{2}$, which explains why (as seen in Tables I and II) scattering amplitudes for Σ("**8**") prefer $\pi\Sigma$ to $\pi\Lambda$ couplings by a 2:1 ratio, and those for Σ("**10**") are the reverse. The function $r_{\mathbf{8}}(N_c)/r_{\mathbf{8}}(\infty)$ equals $5/(3\sqrt{3}) \approx 0.96$ for $N_c = 3$ and rises monotonically to 1 (for odd integers N_c) as N_c increases, while $r_{\mathbf{10}}(N_c)/r_{\mathbf{10}}(\infty)$ equals $2/\sqrt{3} \approx 1.15$ for $N_c = 3$ and drops monotonically to 1 as N_c increases. While elucidating but one source of $1/N_c$ breaking in the full amplitudes, this exercise gives an indication of how well one might expect the large N_c predictions to work.

VII. CONCLUSIONS

The $1/N_c$ expansion applied to the baryon resonance sector continues to provide surprises, both in terms of the organization of states into multiplets and the implications for couplings to asymptotic meson-baryon states, which enter into production and decay processes. We have shown that a remnant of the old quark-picture ("**70**", 1^-) of SU(6) $\times$ O(3) survives as a consequence of the fundamental emergent SU(6) contracted spin-flavor symmetry at large N_c: The ("**70**", 1^-) is a reducible multiplet whose remaining undetermined index, K, is the same one that distinguishes the nonstrange multiplets. In the 3-flavor case, K assumes the 5 values $0, \frac{1}{2}, 1, \frac{3}{2}, 2$, and distinct SU(3) multiplets with the same K value are degenerate in mass and width up to $O(1/N_c)$ corrections.

We showed furthermore that both the SU(3) group theory and the spin-flavor symmetry produce phenomenologically interesting predictions that appear to be borne out where data is available. The former predicts that resonances in the **8** and **10** representations of SU(3) prefer to decay via nonstrange mesons, while those in the **1** prefer to decay via $\bar{K}$'s. The latter predicts that, of the two spin-$\frac{1}{2}$ **8**'s in ("**70**", 1^-), one decays via η and one via π.

Thus far, this analysis remains descriptive and exploratory. Improvements require advances in both data measurement and partial-wave analysis, as well as the theoretical method. Anyone who has examined the hyperon resonance section of the *Review of Particle Physics* [25] will agree that improvements on published data and methods of analysis will prove extremely useful in understanding the physical baryon resonance sector. From the theoretical point of view, the most significant improvements required both fall into the category of $1/N_c$ corrections. For the 2-flavor system, it is known how to incorporate $1/N_c$-suppressed amplitudes [5]. The leading-order 2-flavor amplitudes in $1/N_c$ all assume an extremely simple behavior when expressed in the t channel: $I_t = J_t$, while amplitudes with $|I_t - J_t| = n$ are suppressed by $O(1/N_c^n)$. The generalization of this rule to three flavors is one of the next problems to tackle. The other $1/N_c$ effect that must be mastered is the nature of decoupling of the spurious states that only occur for $N_c > 3$, such as isospin-$\frac{3}{2}$ Ξ's. Once these effects are fully understood, the $1/N_c$ expansion will be fully available to the 3-flavor baryon resonance sector in the same way that chiral perturbation theory describes soft mesons.

ACKNOWLEDGMENTS

T. D. C. was supported by the D. O. E. through Grant No. DE-FGO2-93ER-40762; R. F. L. was supported by the N. S. F. through Grant No. PHY-0140362.

[1] Two recent reviews are D. B. Leinweber *et al.*, presented at the 2nd Cairns Topical Workshop on Lattice Hadron Physics, nucl-th/0406032; F. X. Lee and C. Bennhold, in *Proceedings of the Eighth International Conference on Hypernuclear and Strange Particle Physics* [Nucl. Phys. **A754**, 248 (2005)].
[2] T. D. Cohen and R. F. Lebed, Phys. Rev. Lett. **91**, 012001 (2003); Phys. Rev. D **67**, 096008 (2003).
[3] T. D. Cohen and R. F. Lebed, Phys. Rev. D **68**, 056003 (2003).
[4] T. D. Cohen, D. C. Dakin, A. Nellore, and R. F. Lebed, Phys. Rev. D **69**, 056001 (2004).
[5] T. D. Cohen, D. C. Dakin, A. Nellore, and R. F. Lebed, Phys. Rev. D **70**, 056004 (2004).
[6] T. D. Cohen, D. C. Dakin, R. F. Lebed, and D. R. Martin, Phys. Rev. D **71**, 076010 (2005).
[7] T. D. Cohen and R. F. Lebed, Phys. Lett. B **578**, 150 (2004).
[8] T. D. Cohen and R. F. Lebed, Phys. Rev. D **70**, 096015 (2004).
[9] T. D. Cohen and R. F. Lebed, Phys. Lett. B **619**, 115 (2005).
[10] T. D. Cohen, Nucl. Phys. **A755**, 40 (2005); R. F. Lebed, hep-ph/0501021.
[11] R. Dashen and A. V. Manohar, Phys. Lett. B **315**, 425 (1993); R. F. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. D **49**, 4713 (1994).
[12] Z. Dulinski and M. Praszałowicz, Acta Phys. Pol. B **18**, 1157 (1988).
[13] C. D. Carone, H. Georgi, L. Kaplan, and D. Morin, Phys. Rev. D **50**, 5793 (1994); J. L. Goity, Phys. Lett. B **414**, 140 (1997); D. Pirjol and T.-M. Yan, Phys. Rev. D **57**, 1449 (1998); **57**, 5434 (1998); C. E. Carlson and C. D. Carone, Phys. Lett. B **441**, 363 (1998); Phys. Rev. D **58**, 053005 (1998); Phys. Lett. B **484**, 260 (2000); D. Pirjol and C. L. Schat, Phys. Rev. D **67**, 096009 (2003); **71**, 036004 (2005); M. Wessling, Phys. Lett. B **603**, 152 (2004); N. Matagne and Fl. Stancu, Phys. Rev. D **71**, 014010 (2005); J. L. Goity, C. Schat, and N. N. Scoccola, Phys. Lett. B **564**, 83 (2003); Phys. Rev. D **71**, 034016 (2005); J. L. Goity and N. N. Scoccola, Phys. Rev. D **72**, 034024 (2005); J. L. Goity, hep-ph/0504121; N. N. Scoccola, hep-ph/050450.
[14] C. E. Carlson, C. D. Carone, J. L. Goity, and R. F. Lebed, Phys. Lett. B **438**, 327 (1998); Phys. Rev. D **59**, 114008 (1999).
[15] C. L. Schat, J. L. Goity, and N. N. Scoccola, Phys. Rev. Lett. **88**, 102002 (2002); Phys. Rev. D **66**, 114014 (2002).
[16] E. Guadagnini, Nucl. Phys. **B236**, 35 (1984); P. O. Mazur, M. A. Nowak, and M. Praszałowicz, Phys. Lett. **147B**, 137 (1984); A. V. Manohar, Nucl. Phys. **B248**, 19 (1984); M. Chemtob, Nucl. Phys. **B256**, 600 (1985); S. Jain and S. R. Wadia, Nucl. Phys. **B258**, 713 (1985).
[17] E. Jenkins, Phys. Lett. B **315**, 441 (1993).
[18] R. F. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. D **51**, 3697 (1995); E. Jenkins and R. F. Lebed, Phys. Rev. D **52**, 282 (1995).
[19] L. C. Biedenharn, Phys. Lett. **3**, 69 (1962); in *Lectures in Theoretical Physics, Vol. 5* (Lectures delivered at the Summer Institute for Theoretical Physics, University of Colorado), edited by W. E. Brittin and J. Downs (Interscience Publishers, New York, 1962); K. T. Hecht, Nucl. Phys. **B62**, 1 (1965).
[20] M. P. Mattis and M. Mukerjee, Phys. Rev. D **39**, 2058 (1989).
[21] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).
[22] M. P. Mattis and M. Mukerjee, Phys. Rev. Lett. **61**, 1344 (1988).
[23] M. P. Mattis and M. E. Peskin, Phys. Rev. D **32**, 58 (1985); M. P. Mattis, Phys. Rev. Lett. **56**, 1103 (1986); **63**, 1455 (1989); Phys. Rev. D **39**, 994 (1989).
[24] We thank Carlos Schat, José Goity, and Norberto Scoccola for discussions on this point.
[25] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).