

Probing flavor structure in supersymmetric theories

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 (Received 8 August 2005; published 23 September 2005)

We analyze the possibility of probing the supersymmetric flavor structure through the constraints of the K and B meson systems and those of the electric dipole moments. We show that combining these constraints would favor SUSY models with large flavor mixing either in $LR(RL)$ or LL but with a very small RR and intermediate/large $\tan\beta$. Large LR mixing requires specific patterns for trilinear A terms, while LL mixing seems quite natural and easier to obtain. We present an example for this class of models and show how it can accommodate the current CP asymmetries experimental results.

DOI: [10.1103/PhysRevD.72.055020](https://doi.org/10.1103/PhysRevD.72.055020)

PACS numbers: 12.60.Jv, 11.30.Er, 13.25.Hw

I. INTRODUCTION

Current data from B factories on the branching ratios and the CP asymmetries of $B \rightarrow \phi K$, $B \rightarrow \eta' K$ and $B \rightarrow K\pi$ suggest new sources of flavor and/or CP violation beyond the standard model (SM). An attractive possibility for these new sources can be found in supersymmetric (SUSY) models. These new flavor and CP violation have significant implications and can modify the SM predictions in flavor changing rare processes and CP -violating phenomena. However, experimental bounds on the electric dipole moment (EDM) of the neutron, electron and mercury atom usually impose stringent constraints on mixings and phases in the adopted models. Therefore it is a challenge for SUSY models to give a new source of flavor and CP that can explain the possible discrepancy between CP asymmetry measurements and the expected SM results, while at the same time avoiding the production of EDMs.

The most recent results of $BABAR$ and Belle collaborations [1,2] on the mixing-induced asymmetries of $B \rightarrow \phi K$ and $B \rightarrow \eta' K$ are given as follows: The Belle experimental values of these asymmetries are

$$S_{\phi K} = 0.44 \pm 0.27 \pm 0.05, \quad (1)$$

$$S_{\eta' K} = 0.62 \pm 0.12 \pm 0.04. \quad (2)$$

The $BABAR$ experimental results are

$$S_{\phi K} = 0.50 \pm 0.25^{+0.07}_{-0.04}, \quad (3)$$

$$S_{\eta' K} = 0.30 \pm 0.14 \pm 0.02. \quad (4)$$

Comparison with the world average CP asymmetry of $B \rightarrow J/\psi K$, which is now given by $S_{J/\psi K} = 0.685 \pm 0.032$, shows that the average $S_{\phi K_s} = 0.47 \pm 0.19$ displays about 1σ deviation from SM prediction, while the average $S_{\eta' K_s} = 0.48 \pm 0.09$ displays 2.5σ discrepancy.

On the other hand the latest experimental results for the direct CP violation in $\bar{B}^0 \rightarrow K^- \pi^+$ and $B^- \rightarrow K^- \pi^0$ are given by [3]

$$A_{K^- \pi^+}^{CP} = -0.113 \pm 0.019 \quad (5)$$

$$A_{K^- \pi^0}^{CP} = 0.04 \pm 0.04. \quad (6)$$

The result of $A_{K^- \pi^+}^{CP}$ corresponds to a 4.2σ deviation from zero, while the measured value of $A_{K^- \pi^0}^{CP}$, which may also exhibit a large asymmetry, is quite small. These observations have been considered as possible signals to new physics [4,5].

It is now clear that in order to accommodate the CP asymmetries of different B decays, SUSY models with flavor nonuniversal soft breaking terms are favored. In this class of models, nontrivial flavor structures in the squark mass matrices are obtained, and as a result new flavor mixing and CP violation effects are expected beyond those in the Yukawa couplings. However there is an open debate about the type of the new flavor that one needs to accommodate the current B physics experimental results. The squark mixings can be classified, according to the chiralities of their quark superpartners, into left-handed or right-handed (L or R) squark mixing. The LL and RR mixings represent the chirality conserving transitions in the left- and right-handed squarks and are given by the mass insertions $(\delta_{LL}^{u,d})_{ij}$ and $(\delta_{RR}^{u,d})_{ij}$ respectively. The LR and RL refer to the chirality flipping transitions and are given by the mass insertions $(\delta_{LR}^{u,d})_{ij}$ and $(\delta_{RL}^{u,d})_{ij}$.

In the minimal flavor SUSY models, i.e., SUSY models with universal soft breaking terms, the L and R sectors of the up and down squark matrices remain diagonal at the electroweak scale to a very good approximation. Hence, this class of models can not give any genuine contribution to the CP -violating and flavor changing processes in K and B systems [6]. The situation is drastically changed within the nonminimal flavor SUSY models. Depending on the type of soft SUSY breaking, a large mixing can be generated in these sectors. However, each sector is severely constrained by flavor and/or CP violation experimental limits. For instance, the mass insertions in the LR and RL are constrained by the EDMs, ε'/ε and $BR(b \rightarrow s\gamma)$

results, while the corresponding ones in the LL and RR are constrained by ΔM_K , ΔM_{B_d} , and ε_k [7].

A salient feature of these constraints is that they are generically more stringent on the LR (RL) mass insertions than the LL (RR) mass insertions. Also, the transitions between first and second generations in each sector are severely constrained compared to those between first or second and third generations. This gives the hope that SUSY contributions to the B system could be significant and may constitute an important factor in explaining the current experimental results which show some discrepancies from the SM predictions.

In this paper we pursue the discussion on the type of the SUSY flavor which may contribute significantly to the CP asymmetries of various B decays without conflicting with the EDMs or any other experimental results. We show that the scenario with large $(\delta_{LR}^d)_{23}$ and $(\delta_{RL}^d)_{23}$ is consistent and can give a solution to the CP asymmetry results. However, it requires specific patterns for the nonuniversal trilinear A terms in order to avoid the stringent EDM constraint. One can get another possible consistent solution through a large $(\delta_{LL}^d)_{23}$, but with a very small $(\delta_{RR}^d)_{23}$ and intermediate or large $\tan\beta$. This type of models seems natural and can be obtained by a minimal relaxation for the universality assumption of the minimal supersymmetric standard model (MSSM). Moreover, large $\tan\beta$ is also favored by other experimental results like the branching ratio of $B \rightarrow \mu^+ \mu^-$ [8].

The paper is organized as follows. In Sec. II we make a critical comparison between the two scenarios of large $(\delta_{LR}^d)_{23}$ and large $(\delta_{LL}^d)_{23}$. In Sec. III we present an example for nonminimal flavor SUSY models, where the scalar mass of the first two generations is different from the scalar mass of the third generation. We also show that this model can successfully pass the test of FCNC constraints come from the kaon system. Section IV is devoted to the results of this model for the CP asymmetries of B processes, in particular, the $B \rightarrow K\phi$, $B \rightarrow K\eta'$ and $B \rightarrow K\pi$. Our conclusions are given in Sec. V.

II. SQUARK MIXING: LL VERSUS LR MIXING

As mentioned in the introduction, in SUSY extension of the SM there are new sources of CP violating phases and flavor structure that may be essential for resolving possible discrepancy among the observed CP asymmetries in B meson decays. The mass insertion approximation (MIA) provides a model independent analysis of the flavor changing process in general SUSY models. In this approximation, one adopts a basis where the couplings of the fermion and sfermion to neutral gaugino fields are flavor diagonal, leaving all the sources of flavor violation inside the off-diagonal terms of the sfermion mass matrix. These terms are denoted by $(\Delta_{AB}^q)_{ij}$, where $A, B = (L, R)$ and $q = u, d$, i.e., $(M_{\tilde{q}^2})_{ij} = \tilde{m}^2 \delta_{ij} + (\Delta_{AB}^q)_{ij}$, where \tilde{m} is the average squark mass. Assuming that $(\Delta_{AB}^q) \ll \tilde{m}$, the propagator

can be expanded in powers of $(\delta_{AB}^d)_{ij} \equiv (\Delta_{AB}^q)_{ij}/\tilde{m}^2$. It is important to note that this approximation is valid as long as $(\delta_{AB}^d)_{ij} \lesssim 1$.

It has been recently demonstrated that the EDM constraints severely restrict the LL and RR contributions to the CP asymmetries of $B \rightarrow \phi K$ and $B \rightarrow \eta' K$ [9–11]. It was also pointed out that SUSY models with dominant LR and RL mixing through the nonuniversal A terms may be the most favorite scenario to accommodate the apparent deviation of the CP asymmetries from those expected in the SM without contradicting the experimental limits of EDMs [9]. It is important to note that these conclusions are based on the assumption of considering a single mass insertion. The effect of large $(\delta_{LR}^d)_{23}$ on the CP asymmetries of B decays, particularly $B \rightarrow \phi K$, $B \rightarrow \eta' K$ and $B \rightarrow K\pi$ has been systematically analyzed [4,12–15] and it was emphasized that it could naturally explain the observed CP asymmetry results.

It is worth remembering that in the usual SUSY models, it is rather difficult to arrange for a large mass insertion $(\delta_{LR}^d)_{23} \sim \mathcal{O}(10^{-2})$ while maintaining the mass insertion $(\delta_{LR}^d)_{12}$ small to satisfy the constraints of ΔM_K and ε'/ε :

$$\text{Re}(\delta_{LR}^d)_{12} \lesssim \mathcal{O}(10^{-4}) \quad \& \quad \text{Im}(\delta_{LR}^d)_{12} \lesssim \mathcal{O}(10^{-5}). \quad (7)$$

Since the mass insertions $(\delta_{LR}^d)_{ij}$ are given by

$$(\delta_{LR}^d)_{ij} \simeq [V_L^{d\dagger} \cdot (Y^d A^d) \cdot V_R^d]_{ij} \quad (\text{for } i \neq j), \quad (8)$$

where $V_{L,R}^d$ are the diagonalization of the down quark mass matrix, all off-diagonal mass insertions would be, in principle, of the same order unless one assumes a very specific flavor structure for the A terms. In fact the factorizable A term that has been considered in Ref. [16,17] is an example of this type of pattern that may lead to such a hierarchy between $(\delta_{LR}^d)_{23}$ and $(\delta_{LR}^d)_{12}$. Moreover, one needs to assume nonhierarchical Yukawa textures to avoid a possible suppression for the off-diagonal entries of the mass insertions which, as can be seen from Eq. (8), depend on the corresponding Yukawa couplings. Therefore, it is not an easy task to get $(\delta_{LR}^d)_{23}$ of order 10^{-2} .

However, it was realized that with intermediate/large $\tan\beta$, the double mass insertions could be quite relevant and may lead to an effective $(\delta_{LR}^d)_{23}$ of the required order even with universal A terms [18]. This can be seen from the explicit dependence of $(\delta_{LR(RL)}^d)_{23}$ on the $LL(RR)$ mixing, which is give by

$$(\delta_{LR}^d)_{23\text{eff}} = (\delta_{LR}^d)_{23} + (\delta_{LL}^d)_{23}(\delta_{LR}^d)_{33}, \quad (9)$$

where $(\delta_{LR}^d)_{33} \simeq (m_b(A_b - \mu \tan\beta))/\tilde{m}^2$. Thus if the mass insertion $(\delta_{LR}^d)_{23}$ is negligible one finds

$$(\delta_{LR}^d)_{23\text{eff}} \simeq (\delta_{LL}^d)_{23} \frac{m_b}{\tilde{m}} \tan\beta. \quad (10)$$

Here we assumed that $\mu \sim \tilde{m}$ and the phase of μ set to zero to overcome the EDM constraints. It is clear that with

$(\delta_{LL}^d)_{23} \approx 10^{-2}$ one can easily get $(\delta_{LR}^d)_{23\text{eff}}$ of order $10^{-3} - 10^{-2}$, depending on the value of $\tan\beta$. Similarly, one can generate an effective $(\delta_{RL}^d)_{23}$ of the right order through large $(\delta_{RR}^d)_{23}$.

In Ref. [18], this contribution has been considered as an LL contribution to the CP asymmetry of B decay. This identification was given to indicate the type of large mixing in the squark mass matrix. Nevertheless we should be aware that the main effect of SUSY contribution is still due to the Wilson coefficient C_{8g} of the chromo-magnetic operator, which is enhanced by the chirality flipped factor $m_{\tilde{g}}/m_b$. It is also worth mentioning that it is quite natural in SUSY models to achieve LL mixing between the second and third families of order 10^{-2} . Although this size of mixing is not enough to explain the measured values of the CP asymmetries of B decays, yet it could induce an effective LR mixing that accounts for these results.

Having said that though, one should be very careful with the EDM constraints. The mass insertion $(\delta_{LR}^d)_{22}$, which is severely constrained by the experimental limit on the mercury EDM [19]:

$$\text{Im}(\delta_{LR}^d)_{22} < 5.6 \times 10^{-6}$$

can be overproduced and thus may violate this bound. As explained in Ref. [9], the effective mass insertion $(\delta_{LR}^d)_{22\text{eff}}$ can be expressed as

$$\begin{aligned}
 (\delta_{LR}^d)_{22\text{eff}} \approx & 10^{-2} \tan\beta [(\delta_{LL}^d)_{23}(\delta_{RR}^d)_{23}^* + ((\delta_{RR}^d)_{23} \\
 & \times (\delta_{LL}^d)_{23}^*)^*]. \quad (11)
 \end{aligned}$$

Hence, in this scenario it is necessary to have either $(\delta_{LL}^d)_{23}$ or $(\delta_{RR}^d)_{23}$ less than 10^{-3} . For instance with $\tan\beta = 10$, one should have $(\delta_{LL(RR)}^d)_{23} \approx \mathcal{O}(10^{-1})$ so that $(\delta_{LR}^d)_{23\text{eff}} \approx \mathcal{O}(10^{-2})$ to accommodate the CP asymmetries and $(\delta_{RR(LL)}^d)_{23} < 10^{-4}$ to avoid the mercury EDM constraint. It is known that in MSSM with universal boundary condition, the mass insertion $(\delta_{LL}^d)_{23}$ is of order 10^{-3} . This value can be considered as a lower limit to the $(\delta_{LL}^d)_{23}$, therefore it is clear that models with large RR mixing would be disfavored by the EDM constraints [9–11].

Another argument which also motivates the class of SUSY models with large LL mixing is the fact that both this mixing and the intermediate/large values of $\tan\beta$ are essential requirements for enhancing the chargino contributions which play a crucial role in explaining the experimental results of $B \rightarrow K\pi$ branching ratio and CP asymmetries [4,15]. Note that due to the $SU(2)$ gauge invariance the soft scalar masses M_Q^2 is the same for the up and down sectors. Hence, the up and down mass insertions are related as follows:

$$(\delta_{LL}^d)_{ij} = [V_{\text{CKM}}^+(\delta_{LL}^u)V_{\text{CKM}}]_{ij}, \quad (12)$$

i.e.,

$$(\delta_{LL}^d)_{23} = (\delta_{LL}^u)_{23} + \lambda(\delta_{LL}^u)_{13} + \mathcal{O}(\lambda^2), \quad (13)$$

with $\lambda = 0.22$. Therefore, a nonuniversal M_Q^2 can lead to large $(\delta_{LL}^d)_{23}$ and $(\delta_{LL}^u)_{23}$. In this respect, this scenario is very economical in that it can explain many results with quite few assumptions.

III. SUGGESTED SUPERSYMMETRIC FLAVOR MODEL

As advocated above, the nonuniversal soft breaking terms are crucial ingredients to have a new flavor structure beyond the usual Yukawa couplings and to enhance the effect of the SM phase δ_{CKM} . Moreover, general supergravity models and most of string and D -brane inspired models naturally lead to nonuniversal soft SUSY breaking parameters [20]. The soft scalar masses of the first two generations are generally assumed degenerate in order to avoid the flavor changing neutral current (FCNC) constraints, especially the ΔM_K and ε_K which impose very strong constraints on (12) mixings. As an example, we consider here a SUSY model with the following soft breaking terms at the GUT scale

$$\begin{aligned}
 M_1 = M_2 = M_3 = M_{1/2} \\
 \text{(universal gaugino mass),} \quad (14)
 \end{aligned}$$

$$A^u = A^d = A_0 \quad \text{(universal } A \text{ term),} \quad (15)$$

$$\begin{aligned}
 M_U^2 = M_D^2 = m_0^2 \quad \text{(universal mass for the squark singlets),} \\
 \quad (16)
 \end{aligned}$$

$$m_{H_1}^2 = m_{H_2}^2 = m_0^2 \quad \text{(universal Higgs masses).} \quad (17)$$

The masses of the squark doublets are given by

$$M_Q^2 = \begin{pmatrix} m_0^2 & & \\ & m_0^2 & \\ & & a^2 m_0^2 \end{pmatrix}. \quad (18)$$

The parameter a measures the deviation between the masses of the third and the first two generations. This model is a special case of texture (C) that has been recently studied in Ref. [21].

Given the above boundary condition for the soft terms at the GUT scale, we determine the evolution of the various couplings according to their renormalization group equations. At the weak scale, we impose the electroweak symmetry breaking conditions and calculate the Higgsino mass μ (up to a sign) and the bilinear parameter B . This imposes a constraint on the parameter a . We will assume through the paper the following values: $\tan\beta = 15$ and $m_0 = M_{1/2} = A_0 = 250$ GeV. For these values a has an upper bound $a \leq 5$. The sparticle spectrum is explicitly computed at the weak scale in terms of the parameters: $M_{1/2}$, m_0 , A_0 , a , and $\tan\beta$. With nonuniversal soft SUSY breaking terms, the Yukawa textures play an important role in the CP and flavour supersymmetric results and one has

to specify the type of the Yukawa couplings in order to completely determine the model. Here we will use the following simple Yukawa textures given in terms of the quark masses and CKM mixing matrix:

$$Y^u = \frac{1}{v \sin\beta} \text{diag}(m_u, m_c, m_t), \quad (19)$$

$$Y^d = \frac{1}{v \sin\beta} V_{\text{CKM}}^\dagger \cdot \text{diag}(m_d, m_s, m_b) \cdot V_{\text{CKM}}. \quad (20)$$

This type of Yukawa texture is hierarchical, so it is not the best choice since it dilutes the effect of the SUSY flavor. However, as we will show, this texture gives good results for flavor mixing between the second and third generation in the squark mass matrices.

Although, a very useful tool for analyzing SUSY contributions to FCNC processes is provided by the mass insertion approximation, one should be careful in models with nonuniversal soft terms. In our model, with $a \neq 1$, we get a highly nondegenerate spectrum which violates one of the assumptions of the mass insertion approximation. Therefore, in our analysis we will use the full loop computation. Nevertheless, it may be still useful to consider the mass insertions just to understand the main features of this model and how it differs from the other models with nonuniversal A terms. The LL down mass insertions are defined in the super-CKM basis, as

$$(\delta_{LL}^d)_{ij} = \frac{1}{\tilde{m}^2} [V_L^{d\dagger} (\mathcal{M}^d)_{LL}^2 V_L^d]_{ij}, \quad (21)$$

where $(\mathcal{M}^d)_{LL}^2$ is the LL down squark at the electroweak scale, \tilde{m} is the average of the squark mass, and V_L^d is the rotation matrix that diagonalizes the down quark mass matrix. Thus, for the soft scalar masses M_Q^2 given in Eq. (18) and $a = 5$, one finds

$$(\delta_{LL}^d)_{23} \approx 0.08 e^{0.4i}. \quad (22)$$

Although we are using a hierarchical Yukawa texture, the result looks very promising. It is clear that with such value of $(\delta_{LL}^d)_{23}$, one can easily get $(\delta_{LR}^d)_{23, \text{eff}} \approx \mathcal{O}(10^{-2} - 10^{-3})$. Recall that the corresponding single LR mass insertion is negligible due to the degeneracy of the A terms. Finally, we also find that the $(\delta_{LL}^d)_{12}$ is given by

$$(\delta_{LL}^d)_{12} \approx 0.0002 + 0.0002i. \quad (23)$$

This result satisfies the strongest constraints coming from the kaon physics: $\sqrt{|\text{Re}(\delta_{LL}^d)_{12}^2|} \leq 4 \times 10^{-2}$ which is imposed by the measured value of ΔM_K and $\sqrt{|\text{Im}(\delta_{LL}^d)_{12}^2|} \leq 4 \times 10^{-3}$ from ε_K . Since $(\delta_{LR}^d)_{22} \approx 4 \times 10^{-3}$, the imaginary part of the effective mass insertion $(\delta_{LR}^d)_{12, \text{eff}}$ is given by

$$\text{Im}[(\delta_{LR}^d)_{12, \text{eff}}] \approx 10^{-6}, \quad (24)$$

which satisfies the bound imposed by ε'/ε : $|\text{Im}(\delta_{LR}^d)_{12}| \leq 2 \times 10^{-5}$. Note that in this case both of ΔM_K , ε_K and ε'/ε should be saturated by the SM contribution. However, it is quite possible to enhance the SUSY contribution, if necessary, by considering more nonhierarchical Yukawa texture.

Before we proceed and determine the SUSY contributions to the CP asymmetries of B processes, one important remark is in order. This model, like the constrained MSSM, has in general two independent phases: ϕ_A and ϕ_μ . However, these two phases are strongly constrained by the EDM. Therefore, we set them to zero and assume that the SUSY breaking mechanism is preserving the CP violation. Hence, the only source of CP violation here is the SM phases δ_{CKM} . In the spirit of Ref. [17], we will show that the new source of SUSY flavor with δ_{CKM} is sufficient to accommodate the current experimental results.

IV. CONTRIBUTION TO THE CP ASYMMETRY OF B PROCESSES

As mentioned, the recent experimental measurements lead to the average $S_{\phi K_S} = 0.47 \pm 0.19$ which is about 1σ deviation from SM prediction and the average $S_{\eta/K_S} = 0.48 \pm 0.09$ which is 2.5σ discrepancy. Also the result of $A_{K^-\pi^+}^{CP}$ corresponds to a 4.2σ deviation from zero while $A_{K^-\pi^0}^{CP}$ is quite small. The SM contribution to $A_{K^-\pi^+}^{CP}$ and $A_{K^-\pi^0}^{CP}$ are of the same order. For instance, for the QCD factorization parameters $\rho_{A,H}, \phi_{A,H} \approx 1$, one finds $A_{K^-\pi^+}^{CP} = 0.057$ and $A_{K^-\pi^0}^{CP} = 0.063$ [4]. These correlation among these direct CP asymmetries are inconsistent with the recent experimental data. In this section we will study the contribution of our SUSY model to these CP violating asymmetries.

A. Contributions to $S_{\phi K}$ and $S_{\eta/K}$

As can be seen from Eqs. (2)–(4), it seems that the CP asymmetry $S_{\phi K}$ is consistent with the SM result and SUSY contributions should be within the experimental errors. The situation of $S_{\eta/K}$ is not yet clear for the following two reasons. First Belle and BABAR still give quite different results. Second, it is commonly believed that η' is a more complicated particle than ϕ and its CP asymmetry could be different due to some peculiar dynamics for this particle. In any case, we will consider here $S_{\phi K}$ as a constraint and will study the possible prediction of our SUSY models for $S_{\eta/K}$ and also for the direct CP asymmetries of $B \rightarrow K\pi$ decays.

As emphasized in Refs. [13], the dominant gluino contributions are due to the QCD penguin diagrams and chromo-magnetic dipole operators. The gluino contributions to the corresponding Wilson coefficients at the SUSY scale can be found in Ref. [22]. The LR contributions only enter the Wilson coefficients $C_{7\gamma}$ and C_{8g} of the magnetic and chromo-magnetic operators:

$$\begin{aligned}
 C_{7\gamma}^{\tilde{g}} &= \frac{\alpha_s \pi}{m_{\tilde{g}}^2} \left[\sum_{AB} \Gamma_{sA}^{R*} \Gamma_{bA}^R \left(\frac{-4}{9} D_1(x_A) \right) \right. \\
 &\quad \left. + \frac{m_{\tilde{g}}}{m_b} \sum_A \Gamma_{sA}^{R*} \Gamma_{sA}^L \left(-\frac{4}{9} D_2(x_A) \right) \right], \\
 C_{8g}^{\tilde{g}} &= \frac{\alpha_s \pi}{m_{\tilde{g}}^2} \left[\sum_{AB} \Gamma_{sA}^{R*} \Gamma_{bA}^R \left(\frac{-1}{6} D_1(x_A) + \frac{3}{2} D_3(x_A) \right) \right. \\
 &\quad \left. + \frac{m_{\tilde{g}}}{m_b} \sum_A \Gamma_{sA}^{R*} \Gamma_{sA}^L \left(-\frac{1}{6} D_2(x_A) + \frac{3}{2} D_4(x_A) \right) \right],
 \end{aligned} \tag{25}$$

where $x_A = \tilde{m}_A^2/m_{\tilde{g}}^2$ and the loop functions are given in Ref. [22]. In our numerical analysis, we include Wilson coefficients of all the relevant operators and the ones obtained from these operators by the chirality exchange. In our discussion we will focus on $C_{7\gamma}^{\tilde{g}}$ and $C_{8g}^{\tilde{g}}$ which give the dominant contribution due to the large enhancement factor $m_{\tilde{g}}/m_b$ in front of the term proportional to the LR mixing.

We will apply the QCD factorization which allows to estimate the hadronic matrix elements of the involved operators. In this case, the SUSY contribution to the decay amplitude of $B \rightarrow \phi K$ is given by [13]

$$A(B \rightarrow \phi K) \simeq -i \frac{G_F}{\sqrt{2}} m_B^2 F_+^{B \rightarrow K} f_\phi H_{8g} (C_{8g} + \tilde{C}_{8g}). \tag{26}$$

Here $m_\phi = 1.02$ GeV is the ϕ meson mass, $F_+^{B \rightarrow K} = 0.35 \pm 0.05$ is the transition form factor evaluated at transferred momentum of order m_ϕ , and $f_\phi = 0.233$ GeV is the ϕ meson form factor. The coefficient H_{8g} is given by $H_{8g} = 0.047$ [13]. Note that $H_{7\gamma}$ is 2 order of magnitude smaller than H_{8g} , therefore we neglect the magnetic moment dipole contribution. Since the hard scattering and weak annihilation contributions to Q_{8g} have not been calculated, the coefficient H_{8g} has no strong phase dependence. It is expected that this contribution has an undetermined strong phase. This will increase the theoretical uncertainty since Q_{8g} is giving the dominant contribution in SUSY model. Here, we assume that the matrix element of Q_{8g} induces a strong phase δ_ϕ to the SUSY contribution to the $B \rightarrow \phi K$ amplitude. Thus, the ratio of the SUSY and SM amplitudes can be written as

$$\left(\frac{A^{\text{SUSY}}}{A^{\text{SM}}} \right)_{\phi K} = R_\phi e^{i\theta_\phi} e^{i\delta_\phi}, \tag{27}$$

$$S_{\phi(\eta')K} = \frac{\sin 2\beta + 2R_{\phi(\eta')} \cos \delta_{\phi(\eta')} \sin(\theta_{\phi(\eta')} + 2\beta) + R_{\phi(\eta')}^2 \sin(2\theta_{\phi(\eta')} + 2\beta)}{1 + 2R_{\phi(\eta')} \cos \delta_{\phi(\eta')} \cos \theta_{\phi(\eta')} + R_{\phi(\eta')}^2}. \tag{30}$$

In Fig. 1 we present the CP asymmetries $S_{\phi K}$ and $S_{\eta' K}$ as function of the nonuniversality parameter a for $m_0 = M_{1/2} = A_0 = 250$ GeV and $\tan\beta = 15$. Also the strong phases are fixed as $\delta_\phi \simeq 2\pi/3$ while $\delta_{\eta'} = 0$.

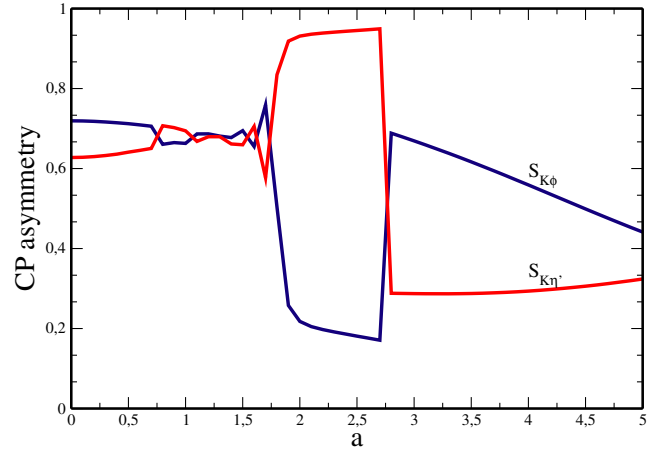


FIG. 1 (color online). CP asymmetries of $B \rightarrow \phi K$ and $B \rightarrow \eta' K$ as function of the squark nonuniversality parameter a for $\tan\beta = 15$, $m_0 = M_{1/2} = A_0 = 250$ GeV, $\delta_\phi \sim 2/3\pi$, and $\delta_{\eta'} = 0$.

where R_ϕ stands for $|(A^{\text{SUSY}}/A^{\text{SM}})_{\phi K}|$ and θ_ϕ for the $\text{Arg}[C_{8g}]$ since \tilde{C}_{8g} is negligible with respect to C_{8g} in this class of model. Similarly, the SUSY contribution to the decay amplitude of $B \rightarrow \eta' K$ is given by [13]

$$A(B \rightarrow \eta' K) \simeq -i \frac{G_F}{\sqrt{2}} m_B^2 F_+^{B \rightarrow K} f_{\eta'} H'_{8g} (C_{8g} - \tilde{C}_{8g}), \tag{28}$$

and the ratio of the SUSY and SM amplitudes can be written as

$$\left(\frac{A^{\text{SUSY}}}{A^{\text{SM}}} \right)_{\eta' K} = R_{\eta'} e^{i\theta_{\eta'}} e^{i\delta_{\eta'}}, \tag{29}$$

where $R_{\eta'}$ refers to $|(A^{\text{SUSY}}/A^{\text{SM}})_{\eta' K}|$, $\theta_{\eta'} \simeq \theta_\phi \simeq \text{Arg}[C_{8g}]$, and $H'_{8g} = -0.89$.

As a result of small RR mixing in the class of models that we consider, the sign difference between C_i and \tilde{C}_i in $B \rightarrow \eta' K$ transition [14], can not be used to create a significant difference between $A_{\phi K}^{\text{SUSY}}$ and $A_{\eta' K}^{\text{SUSY}}$. However, as we will show, due to the fact that the strong phases in $B \rightarrow \phi K$ and $B \rightarrow \eta' K$ are in general different, one can get the required deviation between $S_{\phi K}$ and $S_{\eta' K}$. Using the above parametrization of the SM and SUSY amplitudes, the mixing CP asymmetries $S_{\phi(\eta')K}$ can be written as

As can be seen from this figure, by applying the 1σ constraints on $S_{\eta' K}$, one can set a stringent lower bound on the nonuniversal parameter a , namely $a \gtrsim 3$. Also in this range with a large a , it is quite possible to account

simultaneously for the experimental results of $S_{K\phi}$ and $S_{K\eta'}$.

B. Contributions to $B \rightarrow K\pi$

Now let us turn to the gluino contribution to $B \rightarrow K^- \pi^+$ and $B \rightarrow K^- \pi^0$. As emphasized in Ref. [4], the direct CP asymmetries of $B \rightarrow K\pi$ decays can be approximately given by

$$A_{K^- \pi^+}^{CP} \simeq 2r_T \sin \delta_T \sin(\theta_P + \gamma) + 2r_{EW}^C \sin \delta_{EW}^C \sin(\theta_P - \theta_{EW}^C), \quad (31)$$

$$A_{K^- \pi^0}^{CP} \simeq 2r_T \sin \delta_T \sin(\theta_P + \gamma) - 2r_{EW} \sin \delta_{EW} \sin(\theta_P - \theta_{EW}). \quad (32)$$

The parameters $\theta_P, \theta_{EW}, \theta_{EW}^C$ and $\delta_T, \delta_{EW}, \delta_{EW}^C$ are the CP violating and CP conserving (strong) phases, respectively. The parameters r_T measures the relative size of the tree and QCD penguin contributions. While r_{EW}, r_{EW}^C measure the relative size of the electroweak and QCD contributions. By assuming the same strong phases for SM and SUSY contribution, we can write [4,15]

$$P e^{i\theta_P} = P^{\text{SM}}(1 + k e^{i\theta'_P}), \quad (33)$$

$$r_{EW} e^{i\delta_{EW}} e^{i\theta_{EW}} = (r_{EW})^{\text{SM}} e^{i\delta_{EW}} (1 + l e^{i\theta'_{EW}}), \quad (34)$$

$$r_{EW}^C e^{i\delta_{EW}^C} e^{i\theta_{EW}^C} = (r_{EW}^C)^{\text{SM}} e^{i\delta_{EW}^C} (1 + m e^{i\theta'^C_{EW}}), \quad (35)$$

$$r_T e^{i\delta_T} = \frac{(r_T e^{i\delta_T})_{\text{SM}}}{|1 + k e^{i\theta'_P}|} \quad (36)$$

where k, l, m are given in terms of the $(\delta_{LR}^d)_{23}$ through gluino contributions and $(\delta_{LL}^u)_{32}$ and $(\delta_{LR}^u)_{32}$ through chargino contributions. For gluino mass of order 500 GeV, $m_{\tilde{q}} = 500$ GeV, $m_{\tilde{t}_R} = 150$ GeV, $M_2 = 200$ GeV and $\mu = 400$ GeV, one finds [4,15]

$$k e^{i\theta_P} = -0.0019 \tan \beta (\delta_{LL}^u)_{32} - 35.0 (\delta_{LR}^d)_{23} + 0.061 (\delta_{LR}^u)_{32} \quad (37)$$

$$l e^{i\theta_{EW}} = 0.0528 \tan \beta (\delta_{LL}^u)_{32} - 2.78 (\delta_{LR}^d)_{23} + 1.11 (\delta_{LR}^u)_{32} \quad (38)$$

$$m e^{i\theta_{EW}^C} = 0.134 \tan \beta (\delta_{LL}^u)_{32} + 26.4 (\delta_{LR}^d)_{23} + 1.62 (\delta_{LR}^u)_{32}. \quad (39)$$

Since we have assumed a diagonal up-Yukawa couplings,

the flavor mixing among the up squarks is very small. The typical values of the mass insertion $(\delta_{LL}^u)_{32}$ and $(\delta_{RL}^u)_{32}$ are of order 10^{-3} , so that the chargino contribution is negligible. On the other hand with $a = 5$ and $m_0 = M_{1/2} = A_0 = 250$ GeV, the mass insertion $(\delta_{LR}^d)_{32}$ is give by $(\delta_{LR}^d)_{32} \simeq 0.006 \times e^{-2.7i}$. Therefore one finds

$$k \simeq 0.2 \quad l \simeq 0.009 \quad m \simeq 0.16 \quad (40)$$

From Eqs. (34)–(36), it is clear that in this example, r_{EW} and r_{EW}^C are given, to a good approximation, by the SM values: $r_{EW}^{\text{SM}} \simeq 0.13$ and $(r_{EW}^C)^{\text{SM}} \simeq 0.012$, while r_T is reduced from $r_T^{\text{SM}} \simeq 0.2$ to $r_T \simeq 0.16$. As explained in Ref. [4], in this case with $r_T, r_{EW} \gg r_{EW}^C$, the CP asymmetry $A_{K^- \pi^+}^{CP}$ is given by the first term in Eq. (31) which can easily be of order -0.113 . However, the CP asymmetry $A_{K^- \pi^0}^{CP}$ receives contributions from both terms of Eq. (32). With $r_T \sim r_{EW}$, the possibility of having cancellation between these two terms is quite large and one obtains $A_{K^- \pi^0}^{CP} < A_{K^- \pi^+}^{CP}$, as required by the current experimental results.

V. CONCLUSIONS

In this paper we have studied the possibility of probing the supersymmetric flavor structure. We have used the experimental constraints from the CP asymmetries of K and B meson systems and also from the electric dipole moments. We have shown that these constraints would lead together to a specific SUSY flavor structure. One possibility is to have a large flavor mixing in LR and/or RL sector. The second possibility is to have a large mixing in LL combined with a very small mixing in the RR sector and also intermediate or large $\tan \beta$. The scenario of large LR mixing requires a specific pattern for trilinear A terms, like, factorizable or Hermitian A terms for instance. On the other hand LL mixing scenario seems quite natural and can be obtained by a nonuniversality between the squark masses. As an example, we considered a SUSY model with a minimal relaxation for the universality assumption of the MSSM, where the masses of the left squarks of the first two generations and the third generation are different. We have shown that in this class of models, one can get effective mass insertion $(\delta_{LR}^d)_{23}$ that leads to a significant SUSY contribution to the CP asymmetry of B decays. In particular, we have emphasized that the new results of $S_{\phi k}$ and $S_{\eta' K}$ can be accommodated. Also the model can account for the observed correlation between $A_{K^- \pi^+}^{CP}$ and $A_{K^- \pi^0}^{CP}$.

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