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Yukawa enhanced electroweak corrections at high energy in the minimal supersymmetric standard model

M. Beccaria[1,2] and E. Mirabella[1]

[1]*Dipartimento di Fisica, Università di Lecce, Via Arnesano, 73100 Lecce, Italy*

[2]*INFN, Sezione di Lecce*

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We consider the electroweak radiative corrections in the minimal supersymmetric standard model and study the purely Yukawa contributions $\mathcal{O}(\alpha^L y_t^{2p} y_b^{2q})$, $p + q = L$, where $y_{t,b}$ are the top and bottom quark Yukawa couplings. We show that these corrections can be computed in a gaugeless limit of the minimal supersymmetric standard model where they are under renormalization group control. As an application, we present explicit results for various international linear collider and LHC processes valid at all orders in the loop expansion and at leading order in the large logarithms that arise at high energy.

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I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) [1] is one of the major theoretical laboratories where physics beyond the standard model can be tested at quantitative level. Of course, the large number of free parameters of the MSSM remains a main practical problem. The comparison between theory and experiment is difficult to perform in a model independent way and with no specific assumptions.

Some simplification in the treatment of radiative corrections could be quite useful to disentangle individual and different physical mechanisms. High-energy expansions of virtual effects are a natural candidate in such direction [2]. The Sudakov type approximation is relevant as long as the available collider energy is much larger than the typical sypersymmetry scale.

High-energy expansions are based on the simple observation that asymptotic scattering amplitudes are dominated by large radiative corrections growing with energy. For brevity, we shall indicate collectively all these contributions as *Sudakov logarithms* (SL), although only a part of them deserves this terminology. A detailed quantitative analysis of the Sudakov effects is by now very well understood both in the standard model [3] and in the MSSM [4].

At one loop and in $2 \to 2$ processes, SL can be classified in four very different classes [5]. We concentrate on the electroweak sector and discuss the four categories emphasizing, in particular, the status of their theoretical control. The detailed application of this formalism to realistic physical problems can be found in [6] for ILC processes and in [7,8] for LHC processes.

The first class is that of *universal SL* which are infrared-like contributions. The $W^\pm$ and Z^0 gauge boson masses act as infrared regulators in the loop diagrams, and we can observe enhanced corrections at high energy with leading form at one loop $\sim\alpha\log^2(s/M_W^2)$, where $\sqrt{s}$ is the center of mass energy and $\alpha = e^2/4\pi$. These mass singularities are called universal because they receive a contribution from each external particle (initial or final) independently of the diffusion details. Also, the weight of the squared logarithm is a simple combination of the gauge group quantum number of the external particles. At L loops, the leading contributions of this kind are $\sim\alpha^L\log^{2L}(s/M_W^2)$ with subleading corrections involving all powers smaller than $2L$. The universal SL are the most relevant from the numerical point of view. For this reason there have been large efforts to compute them at higher orders or (working at fixed order in α) to extend the calculation to additional subleading terms. The theoretical control of the universal SL is coded in the hard evolution equations. Detailed applications at the two-loop level in the standard model can be found in [9]. Explicit diagrammatic calculations have also been accomplished [10].

In the MSSM, the existing calculations are at the one-loop level, with the exception of a certain number of resummation proposals valid at all orders in the perturbative expansion and at next-to-leading order in the logarithms [11].

The next class we consider is that of *renormalization group* (RG) SL. These are complementary to the universal ones. Indeed, RG-SL are effects with an ultraviolet origin and arise as a consequence of the running of the coupling constants. Being a short distance effect they are expected to be independent on the long distance physics. At L loops they have the leading behavior $\sim\alpha^L\log^L(s/M_W^2)$. Since there is only one logarithm per loop and not two as in the universal case, the RG Sudakov contributions are NLO at one loop, next-to-next-to-leading order NNLO at two loop and so on. Thus, although they are completely under control from the theoretical point of view, they are not expected to be the dominant contribution in any reasonable limit.

The third class is composed of the *angular* SL. At one loop, they are contributions coming from standard model box diagrams and with asymptotic behavior $\sim\alpha\log(s/M_W^2)\log|x/s|$ where $x = t$ or u—the Mandelstam variables involving the scattering angle. The standard model contributions have been investigated at two loops [10]. They do not receive supersymmetric corrections and we shall regard them as known without further discussion.

The fourth and last class is the most interesting for our purposes and consists of the *Yukawa* SL. These contributions are present only when heavy quarks or their supersymmetry partners are produced in the final state (or belong to the initial state, at LHC). More precisely, Yukawa SL are effects with characteristic factors $m_{t,b}/M_W$ where $m_{t,b}$ are the top and bottom quark masses [12]. These factors are of course Yukawa couplings in disguise. Again, at leading order, we have one logarithm for each loop. In the standard model, Yukawa SL are numerically nonnegligible, but not so interesting for the same reasons we mentioned about RG-SL. However, in the MSSM the Yukawa couplings can be enhanced in certain regions of the parameter space and Yukawa SL permit interesting phenomenological analyses. These regions are characterized by large values of the mixing parameter $\tan\beta$. This important parameter is singled out in the Yukawa SL weights, and it is possible to fix bounds on it from the Sudakov expansion of the observables of several processes [12].

The theoretical control over Yukawa SL is not very satisfactory as it stands. In the standard model, they have been often neglected by working in processes involving only light fermions [9]. In the MSSM, where the top quark physics program is very promising, the existing calculations are limited to the one-loop level.

On the other hand, the one-loop analysis suggests that a more detailed investigation should be possible because of some remarkable features [12,13]. First, the Yukawa SL turn out to be correction factors associated to the external particles and not to the specific details of each process. This is similar to what happened with the universal SL. Second, there is only one logarithm per loop and this seems to be the marker of an ultraviolet effect. *A posteriori*, it is strange that there are RG-SL related to the gauge coupling running but nothing analogous in the Yukawa sector where additional independent couplings are present.

In this paper, we fill these gaps and explain the above features. We show that Yukawa SL are indeed a short distance effect which is governed by renormalization group equations in the gaugeless limit of the MSSM, i.e. in the Yukawa sector. Contrary to the RG-SL, we have motivations to pursue a calculation of Yukawa SL at higher order. We anticipated the reason and we repeat it here to emphasize its importance. There are regions in the MSSM parameter space where the Yukawa couplings can be enhanced. If we are in such a region, the Yukawa SL can give large corrections at one loop, comparable to universal SL, although they are subleading in a formal logarithmic expansion. For the purposes of a high precision measurement, the determination of the next perturbative SL contributions acquire therefore a substantial relevance.

II. YUKAWA ENHANCED ELECTROWEAK CORRECTIONS IN THE MSSM AT HIGH ENERGY

The detailed structure of Yukawa enhanced Sudakov corrections in the MSSM is illustrated by two sample processes shown in Fig. 1. The first example, $e^+e^- \to t\bar{t}$, is a typical s-channel neutral process and describes top quark pair production at ILC. The second example is instead a charged process, $u\bar{d} \to t\bar{b}$, and is one of the three partonic processes which permit single top quark production at LHC [14]. In the Feynman diagrams we have drawn the final vertex by means of a dashed circle standing for a particular set of radiative corrections. These are precisely the electroweak virtual exchanges that give rise to purely Yukawa SL, i.e. all contributions that at the L loop level and at high energy correct the tree-level amplitude by factors that take the form

$$\alpha^L\left(\frac{m_t^2}{M_W^2}\right)^p\left(\frac{m_b^2}{M_W^2}\right)^q\log^r\frac{s}{M^2}, \qquad p+q=L, \tag{1}$$

where $m_{t,b}$ are the top and bottom masses, s is the squared c.m. energy, and M is a typical process mass scale. We emphasize that there are also SL with factors like above, but with $p+q<L$. We shall not discuss these contributions which mix the infrared and ultraviolet structure of the MSSM. In particular, the papers [11] resum SL at NLO order (that is including terms $\sim\log^{2L-1}s$) and determine at $L>1$ the contribution of the form Eq. (1) with $p+q=1$ and $r=2L-1$. Our calculation extends their result by independent additional terms. We deal with the corrections having $p+q=L$ which we can prove to have a clean ultraviolet origin. As a consequence, the exponent r turns out to be bounded as $r\le L$ in this case. Our study will also be further limited by the condition $r=L$, i.e. we shall calculate the leading contributions in the logarithmic expansion.

Since the combined power of factors $m_{t,b}$ is maximal in Eq. (1), it is impossible to generate Yukawa SL in other 1PI parts. For instance, gauge boson self-energies will always have too many gauge couplings. Box diagrams will have suppressed Yukawa couplings to initial state light fermions. As in the above examples, similar diagrams occur in other processes like, in particular, those related to Fig. 1 by supersymmetry, as discussed in [13].

Because of the counting of couplings, the Yukawa effects in Eq. (1) can be computed in an essentially gaugeless limit of the MSSM. In particular, the radiative corrections shown in Fig. 1 require the calculation of a vertex diagram with two on shell chiral and antichiral fields and an external classical vector field, neutral or charged according to the process.

In this gaugeless limit, the only relevant piece of the MSSM Lagrangian is the Yukawa superpotential

$$\mathcal{W} = y_t\bar{t}(tH_u^0 - bH_u^+) + y_b\bar{b}(tH_d^- - bH_d^0). \tag{2}$$

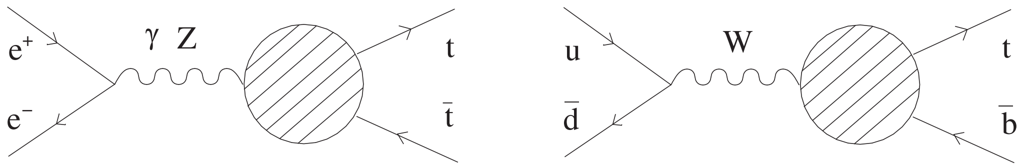

FIG. 1. Two sample processes relevant for ILC and LHC, respectively. In both processes the leading Yukawa SL are confined in the dashed circles representing the final dressed interaction vertex.

The chiral fields $\bar{t}$, $\bar{b}$ contain the right-handed antitop and antibottom, the SU(2) doublet $Q = (t, b)$ is composed of the chiral fields t and b containing the left-handed top and bottom, the SU(2) doublets $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^0, H_d^-)$ contain the various Higgs and Higgsino fields of the MSSM. The couplings $y_{t,b}$ have the tree-level value [g is the SU(2) gauge coupling and M_W is the W boson mass]

$$y_t = \frac{g}{\sqrt{2}} \frac{m_t}{M_W} \frac{1}{\sin\beta}, \qquad y_b = \frac{g}{\sqrt{2}} \frac{m_b}{M_W} \frac{1}{\cos\beta}, \quad (3)$$

in terms of the conventional vacuum alignment angle β.

About soft breaking terms, we shall see in a moment that the corrections Eq. (1) arise as a short distance effect and are thus quite independent on the soft breaking Lagrangian.

After these preliminary discussion and definitions, we lift the discussion from the examples of Fig. 1 to the specific universal source of Yukawa enhanced logarithms. This is the basic vertex diagram $V \to \Phi_i \overline{\Phi}_j$ shown in Fig. 2, where V is a charged or neutral gauge superfield, depending on the specific final state $\Phi_i \overline{\Phi}_j$. Let us denote by g the relevant gauge coupling. We want to compute the corrections $\mathcal{O}(g y_t^{2p} y_b^{2q} \hbar^L)$, $p + q = L$, to the one particle irreducible (1PI) vertex in Fig. 2. We denote the subset of such corrections as $\Gamma_{ij}^{(V)}(Q)$ where Q is a large momentum entering on the gauge field line and where the two matter fields are on shell. Notice that we shall keep working in manifest supersymmetric formalism by employing superfields. Indeed, perturbation theory will be trivial at the leading logarithmic accuracy with all nontrivial effects being encoded in the renormalization group equations.

The fact that $\Gamma_{ij}^{(V)}(Q)$ can be computed in a gaugeless limit with only external classical gauge fields and not internal quantum ones is very important and has deep consequences. Indeed, under these conditions, the large momentum behavior of $\Gamma_{ij}^{(V)}(Q)$ is governed by RG evolution equations [15,16] and does not require the so-called hard evolution equations [9]. Technically, in the Callan-Symanzik equation for $\Gamma_{ij}^{(V)}(Q)$ the mass insertion term is irrelevant because there are no infrared singularities. As a consequence, one is left with the much simpler renormalization group equation which we can solve. This means that the large logarithms of Yukawa origin are actually a genuine short distance effect that can be addressed in the deep Euclidean region. In particular, they are expected to be independent at leading order on the various mass terms appearing in $\mathcal{L}_{\text{soft}}$.

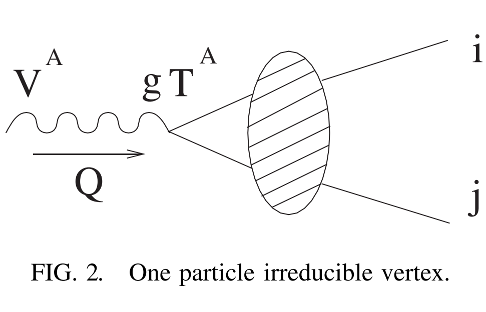

FIG. 2. One particle irreducible vertex.

The RG evolution equation for $\Gamma_{ij}^{(V)}$ is

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_t(y_t, y_b)\frac{\partial}{\partial y_t} + \beta_b(y_t, y_b)\frac{\partial}{\partial y_b} - \gamma_{ij}(y_t, y_b)\right) \times \Gamma_{ij}^{(V)}\left(\frac{Q}{\mu}, y_t, y_b\right) = 0, \quad (4)$$

where $\beta_{t,b}$ are the Yukawa β functions, $\gamma_{ij} = \gamma_i + \gamma_j$, and γ_i is the anomalous dimension of Φ_i. All RG functions are computed in the gaugeless limit $g \to 0$. The scale μ is the RG subtraction mass. In deriving Eq. (4) we neglect systematically all nonleading terms in the $y_{t,b}$ expansion like gauge boson self-energy corrections.

In the next section, we shall analyze the consequences of Eq. (4) at the leading logarithmic accuracy.

III. RENORMALIZATION GROUP ANALYSIS

As is well known, the general solution of the evolution equation Eq. (4) is

$$\Gamma_{ij}^{(V)}\left(\frac{Q}{\mu}, y_t, y_b\right) = \exp\left\{-\int_\mu^Q \gamma_{ij}\left[y_t\left(\frac{\lambda}{\mu}\right), y_b\left(\frac{\lambda}{\mu}\right)\right]\frac{d\lambda}{\lambda}\right\} \times \Gamma_{ij}^{(V)}\left(1, y_t\left(\frac{Q}{\mu}\right), y_b\left(\frac{Q}{\mu}\right)\right), \quad (5)$$

where $y_{t,b}(Q/\mu)$ are the running Yukawa couplings according to the MSSM Yukawa β functions. Equation (5) expresses the renormalized vertex at the scale Q in terms of its value at the initial scale μ. In principle, we should need the perturbative expansion of $\Gamma_{ij}^{(V)}(1, y)$. However, if we are only interested in the leading logarithms $\mathcal{O}(\hbar^L \log^L \frac{s}{\mu^2})$ we can argue from Eq. (5) that they appear in a single coefficient multiplying the tree-level value of the vertex

$$\Gamma_{ij}^{(V)}\left(\frac{Q}{\mu}\right) \overset{LL}{=} c_{ij}\left(\frac{Q}{\mu}\right)\Gamma_{ij}^{(V),\mathrm{Born}}, \tag{6}$$

where c_{ij} depends only on the one-loop terms in the anomalous dimensions and β functions.

To give an explicit expression for c_{ij} it is convenient to define the coefficients $\gamma_i^{(1)t}$, $\gamma_i^{(1)b}$ appearing at one loop in the anomalous dimensions $[\alpha_{t,b} = y_{t,b}^2/(4\pi)]$

$$\gamma_i = \gamma_i^{(1)t}\frac{\alpha_t}{4\pi} + \gamma_i^{(1)b}\frac{\alpha_b}{4\pi} + \cdots, \tag{7}$$

as well as the following sums over the external i, j fields

$$\gamma_{ij}^t = \gamma_i^{(1)t} + \gamma_j^{(1)t}, \qquad \gamma_{ij}^b = \gamma_i^{(1)b} + \gamma_j^{(1)b}. \tag{8}$$

After some algebra, the coefficient c_{ij} turns out to be given by the following compact formula

$$c_{ij} = \left[\frac{\alpha_t(\mu^2)}{\alpha_t(Q^2)}\right]^{\eta_{ij}^t}\left[\frac{\alpha_b(\mu^2)}{\alpha_b(Q^2)}\right]^{\eta_{ij}^b}, \quad \text{with} \begin{cases} \eta_{ij}^t = \frac{1}{70}(6\gamma_{ij}^t - \gamma_{ij}^b), \\ \eta_{ij}^b = \frac{1}{70}(6\gamma_{ij}^b - \gamma_{ij}^t), \end{cases} \tag{9}$$

where the running couplings $\alpha_{t,b}(Q^2)$ are consistently evaluated at leading logarithmic accuracy (i.e. solving the one-loop RG equations).

The practical evaluation of Eq. (9) requires (a) the various anomalous dimensions at one loop, (b) the solution of the one-loop RG evolution equations for the running Yukawa couplings. We discuss separately each issue and also provide a perturbative expansion of Eq. (9) whose validity we shall discuss in the applications.

A. Renormalization group functions

The RG functions (anomalous dimensions and β functions) can be found in the literature [17] for general $N = 1$ gauge models. In the gaugeless limit they reduce at the one-loop level to the following simple results that we present according to the field numbering $(\Phi_1, \ldots, \Phi_8) \equiv (\bar{t}_R, t_L, H_u^0, b_L, H_u^+, \bar{b}_R, H_d^-, H_d^0)$. The first three anomalous dimensions are

$$\gamma_1 = 2\left(\frac{y_t}{4\pi}\right)^2, \qquad \gamma_2 = \frac{1}{(4\pi)^2}(y_t^2 + y_b^2), \qquad \gamma_3 = \frac{N_C}{(4\pi)^2}y_t^2. \tag{10}$$

The other ones are $\gamma_4 = \gamma_2$, $\gamma_5 = \gamma_3$. Also, γ_6 and γ_7 are obtained from γ_1 and γ_3 with the replacement $y_t \leftrightarrow y_b$. Finally $\gamma_8 = \gamma_7$.

The Yukawa coupling β functions for the two couplings y_t and y_b are

$$\beta_t = \frac{y_t}{(4\pi)^2}[(N_C + 3)y_t^2 + y_b^2], \qquad \beta_b = \frac{y_b}{(4\pi)^2}[(N_C + 3)y_b^2 + y_t^2]. \tag{11}$$

B. Running $\alpha_{t,b}(Q^2)$

From the one-loop β functions we determine the running Yukawa couplings. In terms of $s = Q^2$ and setting $N_C = 3$ the evolution equations read

$$\frac{d\alpha_t}{d\log s} = \frac{\alpha_t}{4\pi}(6\alpha_t + \alpha_b), \qquad \frac{d\alpha_b}{d\log s} = \frac{\alpha_b}{4\pi}(6\alpha_b + \alpha_t), \tag{12}$$

with given initial conditions at the scale μ. These equations can be solved analytically in implicit form but the exact solution is not particularly enlightening. In the appendix, we provide full details about the solution and its numerical evaluation. In the following, we shall denote by SAA (semianalytic approximation) the solution described in the appendix. Besides the SAA solution, it is interesting to have also its perturbative expansion permitting to discuss the convergence of the loop expansion. At second order, we have

$$\frac{\alpha_t(Q^2)}{\alpha_t(\mu^2)} = 1 + \frac{1}{4\pi}(6\alpha_t + \alpha_b)\log\frac{Q^2}{\mu^2} + \frac{1}{(4\pi)^2}\left(36\alpha_t^2 + \frac{19}{2}\alpha_t\alpha_b + \frac{7}{2}\alpha_b^2\right)\log^2\frac{Q^2}{\mu^2}, \tag{13}$$

where in the right-hand side we denote $\alpha_{t,b} \equiv \alpha_{t,b}(\mu^2)$. The result for $\alpha_b(Q^2)/\alpha_b(\mu^2)$ is obtained by exchanging $\alpha_t \leftrightarrow \alpha_b$. The expansion Eq. (13) can be substituted in Eq. (9). Expanding again in powers of the logarithms we obtain the leading logarithm approximation for the Sudakov correction c_{ij} (no sum over repeated indices)

$$\begin{aligned} c_{ij} = 1 &- \frac{1}{2(4\pi)}\log\frac{Q^2}{\mu^2}(\gamma_{ij}^t\alpha_t + \gamma_{ij}^b\alpha_b) \\ &+ \frac{1}{8(4\pi)^2}\log^2\frac{Q^2}{\mu^2}[\alpha_t^2\gamma_{ij}^t(\gamma_{ij}^t - 12) \\ &+ \alpha_b^2\gamma_{ij}^b(\gamma_{ij}^b - 12) + 2\alpha_t\alpha_b(\gamma_{ij}^t\gamma_{ij}^b - (\gamma_{ij}^t + \gamma_{ij}^b))] \\ &+ \mathcal{O}(\hbar^3), \end{aligned} \tag{14}$$

Equations (9) and its two-loop expansion Eq. (14) are the main result of this paper. The one-loop approximation (the first line) is in perfect agreement with the results already published in the literature [6–8] and coming from explicit diagram calculations in component fields followed by high-energy expansions. In the next section, we shall discuss in details the consequences of our results by considering specific applications.

IV. APPLICATIONS AND DISCUSSION

A. Preliminary perturbative analysis

We begin with a discussion of the perturbative result at two-loop order Eq. (14). This will be useful in comparing the one-loop approximation with higher order corrections. We denote $L_{t,b} = \frac{\alpha_{t,b}}{4\pi}\log\frac{Q^2}{\mu^2}$. From Eq. (14), we can obtain the following list of specific cases

$$\begin{aligned} t_R t_R^* &: c_{11} = 1 - 2L_t - 4L_t^2 - L_t L_b, \\ t_L t_L^* &: c_{22} = 1 - L_t - L_b - \frac{5}{2}(L_t^2 + L_b^2), \\ b_L b_L^* &: c_{44} = c_{22}, \\ t_L b_L^* &: c_{24} = c_{22}, \\ b_R b_R^* &: c_{55} = 1 - 2L_b - 4L_b^2 - L_t L_b, \\ H_u^+ H_u^- &: c_{55} = 1 - 3L_t - \frac{9}{2}L_t^2 - \frac{3}{2}L_t L_b, \\ H_d^+ H_d^- &: c_{77} = 1 - 3L_b - \frac{9}{2}L_b^2 - \frac{3}{2}L_t L_b. \end{aligned} \tag{15}$$

The meaning of Eq. (15) is as follows: Let us consider for instance the first line which reports the expression of c_{11}. This is the correction factor for the vertex which has as a final state the chiral field (and its conjugate) whose fermionic component is the right-handed top quark. By supersymmetry, the same correction factor is also obtained if we consider in the final state various combinations of the scalar partners, for instance $\tilde{t}_R \tilde{t}_R^*$. In such a diagonal case, the vector V is necessarily a neutral one, γ or Z. Also, by supersymmetry, from the real superfield V we also can take its gaugino component.

Notice that the physical charged Higgs boson is a mixture of the charged components of H_u and H_d. A proper two-loop result for the vertex with this field would require a one-loop treatment of the mixing that we defer to later work. The vertices with final quark superfields (describing production of quark pairs, squark pairs, or quark-squark combinations) are correct as they stand, since mixing is irrelevant when it appears only in the internal lines.

The above logarithmic expansions Eqs. (15) can be resummed in closed form at least in the single Yukawa coupling limit. For instance, if we are in a point of MSSM parameters where we can make the approximation $y_b \ll y_t$, then it is possible to repeat the derivation leading to c_{ij} and exploit the exact solution for the running $\alpha_t(Q^2)$. The final result is remarkably simple

$$y_b \equiv 0: \qquad c_{ij} = (1 - 6L_t)^{\gamma^t_{ij}/12}. \tag{16}$$

As an example, in the case of final states $t_R\bar{t}_R$ or $t_L\bar{t}_L$, we find [in agreement with the L_t terms in Eq. (15)]

$$\begin{aligned} t_R t_R^*&: c_{11} = (1 - 6L_t)^{1/3} = 1 - 2L_t - 4L_t^2 + \cdots, \\ t_L t_L^*&: c_{22} = (1 - 6L_t)^{1/6} = 1 - L_t - \frac{5}{2}L_t^2 + \cdots. \end{aligned} \tag{17}$$

B. Discussion of the full result

Now, we turn to consider the numerical relevance of the computed effects according to Eq. (9) with both Yukawa couplings being active. We concentrate on the same processes shown in Fig. 1, i.e.

$$\text{ILC} \quad \mathrm{e^+e^-} \to \mathrm{f}_\alpha \bar{\mathrm{f}}_\alpha, \quad \mathrm{f} = \mathrm{t, b} \qquad \text{LHC} \quad \mathrm{u\bar{d}} \to \mathrm{t_L \bar{b}_L}. \tag{18}$$

In both cases the polarization of the initial state affects only the tree-level amplitude and is factored in the correction. The chirality index α determines the polarization of the final state in the ILC process. For brevity, we consider only a final state with asymptotic vanishing helicity. The final state of the LHC process has $\alpha = L$ because of W boson exchange. In both cases, the correction factor in the cross section is simply $(c_{ij})^2$.

The explicit values of the exponents $\eta^{t,b}_{ij}$ required for the evaluation of Eq. (9) are summarized in the following table where, for clarity, we replace the pair (i, j) with the fermion in the associated chiral field. As we remarked, by supersymmetry the same corrections are obtained with the sfermion components.

i, j	η^t	η^b
$t_R\bar{t}_R$	12/35	−2/35
$t_L\bar{t}_L$	1/7	1/7
$b_L\bar{b}_L$	1/7	1/7
$b_R\bar{b}_R$	−2/35	12/35

The initial condition for $\alpha_t(Q^2)$, $\alpha_b(Q^2)$ is fixed as follows: We choose a scale μ and define at that scale

$$\begin{aligned} \left(\frac{y_t}{4\pi}\right)^2 &= \frac{\alpha}{8\pi s_W^2}\frac{m_t^2}{M_W^2}(1 + \cot^2\beta_{\text{eff}}), \\ \left(\frac{y_b}{4\pi}\right)^2 &= \frac{\alpha}{8\pi s_W^2}\frac{m_b^2}{M_W^2}(1 + \tan^2\beta_{\text{eff}}), \end{aligned} \tag{19}$$

where the parameter $\tan\beta_{\text{eff}}$ is a scale-dependent effective mixing angle. At tree level, this is the conventional mixing angle. Beyond tree level, it is just a convenient parametrization of the initial values of the top and bottom Yukawa couplings. For simplicity, we shall denote in the following discussion $\beta \equiv \beta_{\text{eff}}$.

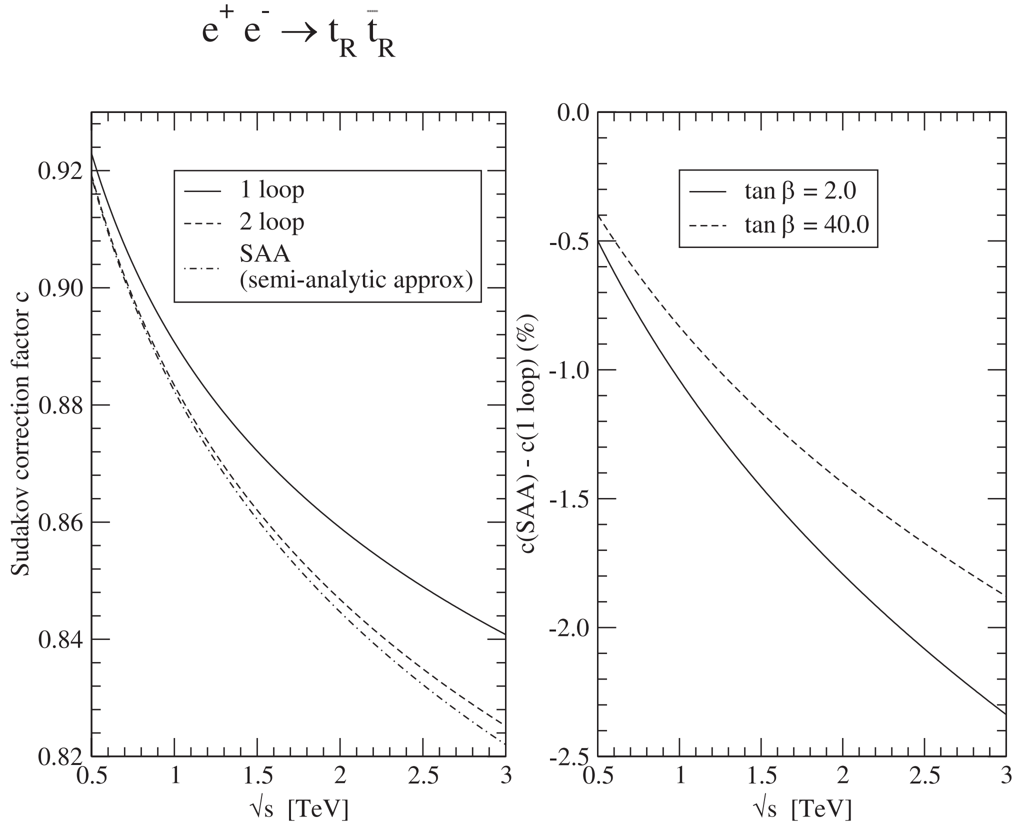

FIG. 3. Energy and $\tan\beta$ dependence of the Yukawa Sudakov correction factor c in the process $e^+e^- \to t_R\bar{t}_R$. The left panel shows the Yukawa Sudakov correction to the cross section evaluated at one- and two-loop level. The line labeled SAA shows the all order result Eq. (9). The initial condition of the running Yukawa couplings is obtained from Eq. (19) with $\tan\beta = 40$. The renormalization group scale μ is fixed at $\mu = 100$ GeV, an intermediate value between m_b and m_t. The dependence on μ is shown in Fig. 6. The difference between the one-loop and SAA approximations are appreciable. In the right panel, we show this difference as a function of $\sqrt{s}$ for two reference values of $\tan\beta = 2, 40$.

Our results are shown in the three Figs. 3–5 for the ILC processes. The left-hand side of each figure shows the (Yukawa) Sudakov correction to the cross section at one-loop and two-loop level. We also show a line with the label SAA standing for the evaluation of the all order Eq. (9). The initial condition is obtained from Eq. (19) with $\tan\beta = 40$. The curves are shown as functions of $\sqrt{s}$ with the RG scale fixed at the arbitrary value $\mu = 100$ GeV, somewhat between m_b and m_t. As usual, the choice of the scale μ cannot be fixed at the leading order in the logarithmic expansion.

In all the considered cases the two-loop correction is practically equivalent to the SAA result. Instead, the correction to the one-loop approximation is appreciable. In the right-hand side of the three figures, we show this difference as a function of $\sqrt{s}$ for two reference values of $\tan\beta = 2, 40$.

The value of the extra correction is in all cases at the level of 1%–2%, which can be visible at ILC. The only exception is the process $e^+e^- \to b_R\bar{b}_R$ at small $\tan\beta$ due to the very small value of y_b in that case.

About the considered LHC process, the equality $c_{24} = c_{22}$ implies that the correction factor is identical to that occurring in $e^+e^- \to t_L\bar{t}_L$. However, there are important differences deserving a couple of comments. First, the available energy for the partonic process will be in general smaller than at ILC reducing the overall effect. Second, it will be nontrivial to identify it with the invariant mass of the final state $t_L\bar{b}_L$ due to various systematic corrections (for a realistic analysis of these effects in $t\bar{t}$ production, see the second reference in [7]).

For completeness, we report in the following table the percentual correction at one loop to the total cross section due to the various Sudakov components, i.e. those of universal, angular-dependent, RG, and Yukawa type. More details on the ILC processes can be found in [18]. We remark that the Yukawa contribution is sizable, in particular, at high $\tan\beta$ values. More important, its variation with $\tan\beta$ is also large offering a possible way of discriminating low from high values. For each process, $\delta\sigma$ is the percentual effect relative to the Born value of the cross section:

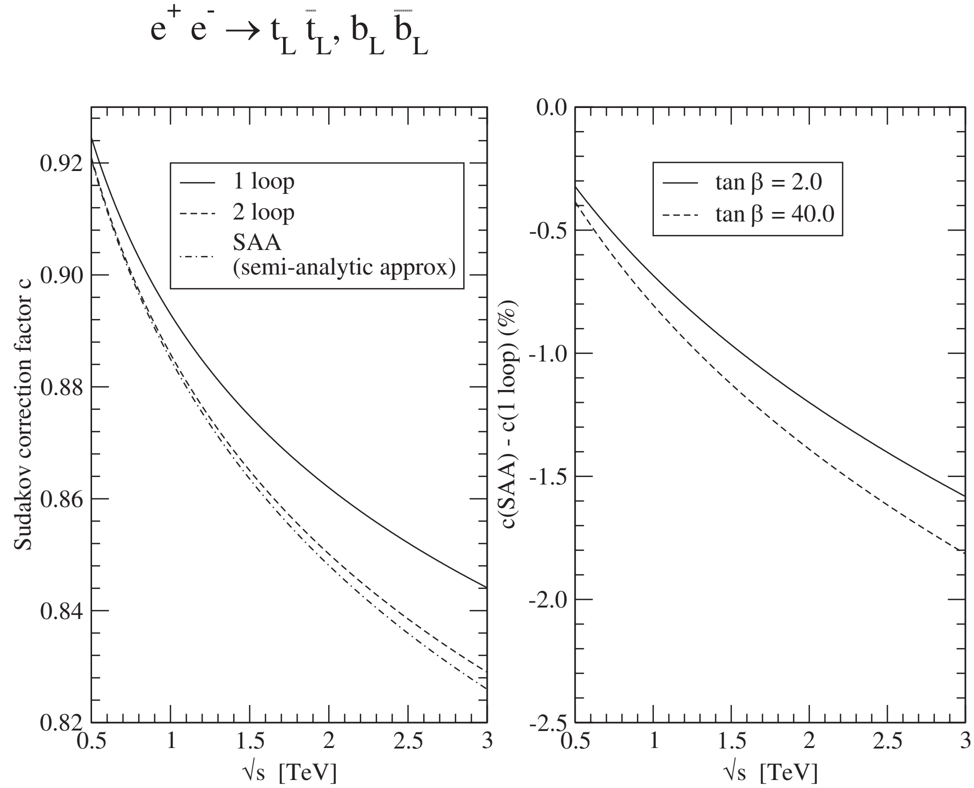

FIG. 4. Energy and $\tan\beta$ dependence of the Yukawa Sudakov correction factor c in the process $e^+e^- \to t_L\bar{t}_L$ and $e^+e^- \to b_L\bar{b}_L$. The figure is analogous to Fig. 3 and it is drawn with the same parameters.

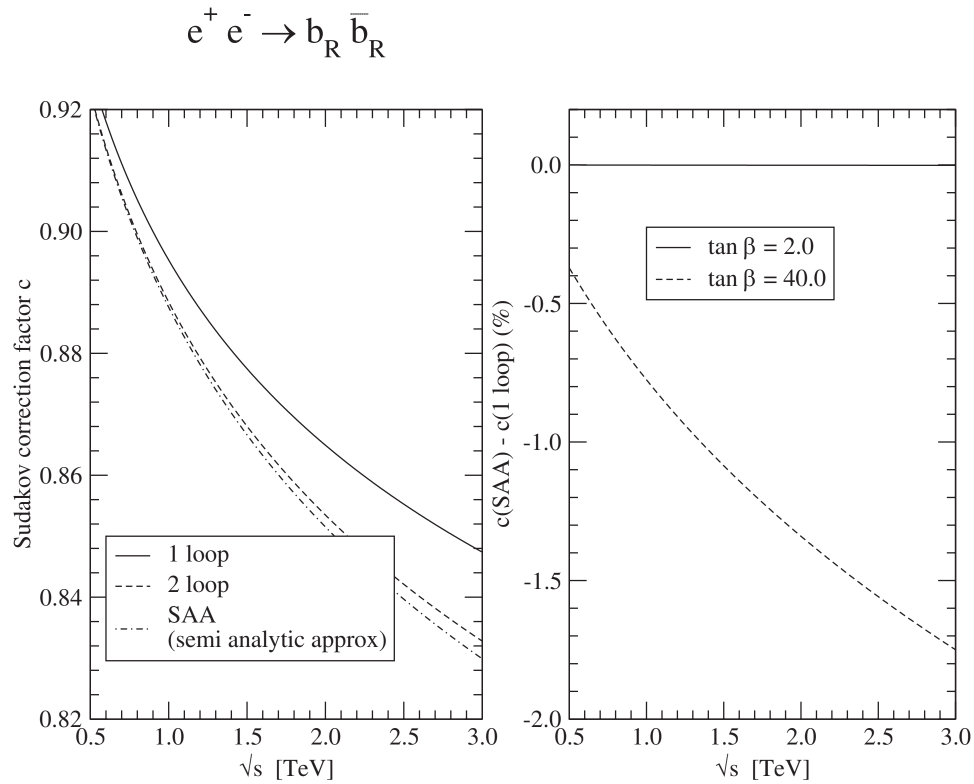

FIG. 5. Energy and $\tan\beta$ dependence of the Yukawa Sudakov correction factor c in the process $e^+e^- \to b_R\overline{b}_R$. The figure is analogous to Fig. 3 and it is drawn with the same parameters.

	$\delta\sigma(e^+e^- \to t\bar{t})$ (%)	$\delta\sigma(e^+e^- \to b\bar{b})$ (%)	$\delta\sigma(u\bar{d} \to t\bar{b})$ (%)
RG	5.1	3.4	2.8
Universal	−9.4	−11	−13
Angular	−1.5	19	5.2
Yukawa, $\tan\beta = 5$	−3.7	−2.3	−6.9
Yukawa, $\tan\beta = 50$	−6.3	−7.2	−16

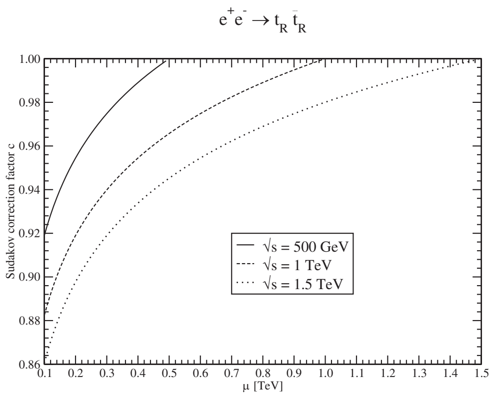

FIG. 6. Dependence of the Yukawa Sudakov correction factor c on the renormalization group scale μ at the three different energies 500, 1000, and 1500 GeV. The process is $e^+e^- \to t_R\bar{t}_R$. The parameter $\tan\beta$ is fixed at 40. The curves show the SAA value of the coefficient c.

As a final point, we would like to discuss the dependence on the μ scale. In our leading logarithmic approximation, the Sudakov correction factor c is a function of $\log(s/\mu^2)$. Thus the values of c for different μ can be obtained by suitably rescaling $\sqrt{s}$. For the reader's convenience, we have shown in Fig. 6 what is obtained in the process $e^+e^- \to t_R\bar{t}_R$ at $\tan\beta = 40$. The figure shows three curves displaying the correction factor c as a function of μ at three reference energies 500, 1000, and 1500 GeV. The curves are drawn for $\mu \leq \sqrt{s}$. Of course the correction is $c = 1$ at the trivial point $\mu = \sqrt{s}$. For $\mu < \sqrt{s}$ the correction amounts to a reduction ($c < 1$) which is larger as the difference between the RG scale and the process energy increases.

V. SUMMARY AND CONCLUSIONS

This paper has been devoted to the analysis of a specific subset of the radiative corrections affecting physical processes in the MSSM. In the framework of a high-energy expansion we have concentrated on large logarithmic corrections to the invariant amplitudes $\mathcal{A}$ that take the following asymptotic form at L loops

$$\mathcal{A} = \mathcal{A}^{\text{tree}}\left(1 + \sum_{L\geq 1} c_L^{\mathcal{A}} \alpha^L \log^L \frac{s}{\mu^2}\right), \tag{20}$$

$$c_L^{\mathcal{A}} = \sum_{p=0}^{L} d_{L,p}^{\mathcal{A}} \left(\frac{m_t^2}{M_W^2}\right)^p \left(\frac{m_b^2}{M_W^2}\right)^{L-p}. \tag{21}$$

We have shown how to compute the coefficients $\{d_{L,p}^{\mathcal{A}}\}$ for various specific processes at ILC or LHC by solving the renormalization group equation governing the relevant 1PI parts from which the above correction originates. The final recipe is quite simple and universal. It explains various features of the above correction already noted in the literature as a byproduct of explicit component calculations.

In the standard model, the coefficients $\{d_{L,p}^{\mathcal{A}}\}$ are fixed numerical constants. Instead, in the MSSM, they can vary by large amounts as the MSSM parameter space is explored. In benchmark scenarios where one of the two relevant Yukawa couplings is large, we can obtain sizable corrections with relevant two-loop contributions. The next-order corrections are rather small in all the presented examples.

We remark that such two-loop contributions are difficult to obtain in a straightforward approach that would consist in evaluating the relevant Feynman diagrams at generic kinematical points, followed by analytic high-energy expansions. For instance the two-loop contribution is NNLO in the logarithmic expansion whereas the very powerful tools presented recently in [19] work only at NLO as they stand. Instead, the calculation of $\{d_{L,p}^{\mathcal{A}}\}$ is quite easy within the framework of the renormalization group.

The main point of our paper has been precisely to emphasize that the Yukawa Sudakov corrections actually can be computed in a gaugeless limit where they arise not as mass singularities but as ultraviolet effects. This observation relies on old results about the asymptotic properties of quantum field theory form factors and combines them with the modern phenomenological interest toward large Yukawa coupling regions of the vast MSSM parameter space. A typical example of such scenario is the conventional SPS4 point [20]. Of course, this kind of strategy is well known for static quantities where large logarithms also appear. A nice example is the recent calculation [21] of gluino lifetime and branching ratios in split supersymmetric models where the large corrections involve $\log(\tilde{m}/m_{\tilde{g}})$, where $\tilde{m}$ is the squark and slepton scale and $m_{\tilde{g}}$ is the gluino mass. In that case, the large logarithms controlled by the top quark Yukawa coupling are also resummed separately.

The aim of our analysis has been that of showing that similar simple RG treatments are possible in the context of high-energy expansions and, in particular, for the Yukawa SL whose existence is somewhat independent of the more involved genuinely Sudakov double logarithms that require more sophisticated tools. From a numerical point of view, our analysis has shown that the higher order SL effect could be of the percent size at ILC, in which case it would become visible. Less optimistic conclusions would apply to the considered LHC processes for which the effect appears to be definitely small. Note, however, that this fact represents in any case a valuable information, telling us that in this sector the available one-loop expansions are certainly sufficient.

Various extensions of the present analysis are possible. For instance, two physically interesting examples at LHC are $t\bar{t}$ pair production via the partonic processes $gg \to t\bar{t}$, $q\bar{q} \to t\bar{t}$ and the single top quark production mechanisms that we have not discussed, i.e. $bu \to td$ and $bg \to tW$. They admit similar corrections localized in specific three point vertices [7,8] to which the present formalism can be applied to obtain higher order contributions.

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APPENDIX: SOLUTION OF THE COUPLED RG EQUATIONS

The equations to be solved are

$$\dot{\alpha}_t(t) = \alpha_t(6\alpha_t + \alpha_b), \tag{A1}$$

$$\dot{\alpha}_b(t) = \alpha_b(6\alpha_b + \alpha_t), \tag{A2}$$

where we have introduced the variable $t = \frac{1}{4\pi}\log(s/s_0)$ that we shall call *time* in the following. This dynamical system admits the family of integral curves

$$(\alpha_t - \alpha_b)^7 = C(\alpha_t\alpha_b)^6, \tag{A3}$$

where C is a constant that can be computed at the initial point $t = t_0$. Exploiting Eq. (A3), we can write $\alpha_{t,b}$ parametrically in terms of the ratio $R = \alpha_t/\alpha_b$. It is necessary to treat separately the two cases $R(t_0) > 1$ and $R(t_0) < 1$. If $R(t_0) = 1$, then $C = 0$ and $R = 1$ at all times. In this case we have simply the single coupling RG solution $\alpha_t = \alpha_b = \alpha_t(0)/(1 - 7t)$. It is enough to work out the case $R(t_0) > 1$, since the other is obtained by the swap $\alpha_t \leftrightarrow \alpha_b$. The parametric equations are

$$\begin{aligned} \alpha_t &= C^{-1/5}R^{-1/5}(R-1)^{7/5}, \\ \alpha_b &= C^{-1/5}R^{-6/5}(R-1)^{7/5}, \end{aligned} \tag{A4}$$

where the ratio R satisfies

$$\dot{R}(t) = 5C^{-1/5}R^{-1/5}(R-1)^{12/5}. \tag{A5}$$

We immediately see that R increases as t increases. In practice there is an exploding time at which $R \to \infty$. This happens at unphysical large energies for realistic initial values. Solving the separable differential equation (A5), we find the (implicit) solution for the ratio $R \equiv R(t)$ in terms of its initial value $R_0 \equiv R(t_0)$

$$F(R) - F(R_0) = C^{-1/5}(t - t_0), \tag{A6}$$

where the monotonically increasing function $F(R)$ is

$$F(R) = -\frac{1}{14}\left[\frac{R^{1/5}(R+1)}{(R-1)^{7/5}} + \frac{1}{3}(R-1)^{3/5}{}_2F_1\left(\frac{3}{5}, \frac{4}{5}, \frac{8}{5}, 1 - R\right)\right], \tag{A7}$$

where ${}_2F_1$ is Gauss's hypergeometric function [22].

Thus, an accurate and quite simple recipe to solve the initial system is the following three steps algorithm: (i) compute C from the initial values of $\alpha_{t,b}$, (ii) at each time find numerically the unique zero of Eq. (A6), (iii) replace the root $R(t)$ in Eqs. (A4). This semianalytic procedure is superior to any discrete step integration scheme, like for instance Runge-Kutta methods. Indeed, from Eq. (A6) we derive immediately exact properties of the solution. An example is the (unphysical) exploding time which is predicted by Eq. (A6) to be

$$\begin{aligned} t_{\exp} &= t_0 + C^{1/5}(F_\infty - F(R_0)), \\ F_\infty &= -\frac{1}{42}\frac{\Gamma(1/5)\Gamma(8/5)}{\Gamma(4/5)}. \end{aligned} \tag{A8}$$

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