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Production of $K_0^*(1430)$ and K_1 in B decays

Chuan-Hung Chen,[1,2,*] Chao-Qiang Gengc,[3,†] Yu-Kuo Hsiaoc,[3,‡] and Zheng-Tao Wei[1,2,4,§]

[1]*Department of Physics, National Cheng-Kung University, Tainan, 701 Taiwan*
[2]*National Center for Theoretical Sciences, National Cheng-Kung University, Tainan, 701 Taiwan*
[3]*Department of Physics, National Tsing-Hua University, Hsin-Chu, 300 Taiwan*
[4]*Institute of Physics, Academia Sinica, Taipei, 115 Taiwan*

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We study the productions of p-wave mesons $K_0^*(1430)$, $K_1(1270)$, and $K_1(1400)$ in B decays. By the generalized factorization approach, we find that the branching ratios of $B \to K_0^*(1430)\phi$ are similar to those of $B \to K\phi$ while the branching ratios of $B \to K_1(1270)\phi$ and $B \to K_1(1400)\phi$ are $O(10^{-5})$ and $O(10^{-6})$, respectively. In terms of the observation of $B \to K_1(1270)\gamma$ by BELLE, we can remove the sign ambiguity in the mixing angle for physical states $K_1(1270)$ and $K_1(1400)$. In addition, we analyze annihilation contributions in the decays $B \to K_1\phi$ and we conclude that they could be neglected.

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I. INTRODUCTION

It is known that there have been some anomalies in penguin dominant B decay processes, which cannot be easily explained in the standard model (SM), especially the two puzzles: (a) the large branching ratios (BRs) of $B \to K\eta'$ [1] and (b) the small longitudinal fractions of $B \to K^*\phi$ decays [2]. Note that, at the quark level, both puzzles (a) and (b) belong to the penguin dominant transitions $b \to sq\bar{q}$. Although it is possible that some complicated hadronic effects [3,4] or new physics [5,6] could solve these anomalies, to find out the real causes it is clear that we have to study more processes, in particular, those involving with similar weak interactions. Inspired by the polarization abnormalities in $B \to K^*\phi$, we investigate the decays of $B \to K_1\phi$ in the SM, where K_1, denoting $K_1(1270)$ and $K_1(1400)$, are axial-vector bosons and the mixtures of states K_{3P_1} and K_{1P_1}. Our purpose of this work is to see whether similar anomalies occur when these modes are measured. Similarly, we will also study $B \to K_0^*(1430)\phi$.

As usual, the challenge to study the exclusive decays is the estimations of the transition matrix elements. By the naive factorization (NF), the decay amplitudes can be simplified as $c(\mu)\langle O\rangle_{\rm fact}$, in which $\langle O\rangle_{\rm fact}$ denotes the factorizable part. Using the approach of the NF, we immediately suffer from the problem of the μ-scale dependence on hadronic matrix elements since the μ-dependent Wilson coefficient $c(\mu)$ cannot get compensation from $\langle O\rangle_{\rm fact}$. However, by the QCD factorization (QCDF) [7] or perturbative QCD (PQCD) [8] approaches, we need to know the detailed hadronic spin structures and the associated distribution amplitudes of involving mesons to deal with factorized and nonfactorized effects. For B and ϕ mesons, they have been studied by the heavy quark effective theory (HQET) [9] and QCD sum rules [10], respectively, and at least, their asymptotic behaviors of the leading twist and twist-3 are known clearly. Nevertheless, so far we know nothing about the axial-vector mesons of K_1. In order to reliably estimate the relevant hadronic effects for the p-wave modes, we employee the generalized factorization approach (GFA) [11,12], in which the leading effects are factorized parts and the nonfactorized effects are lumped and characterized by the effective number of colors, denoted by $N_c^{\rm eff}$ [13]. Note that the scale and scheme dependence on effective WCs $C_i^{\rm eff}$ are insensitive.

In addition, we will also analyze the annihilation contributions which are important in $B \to PP$, $VP(PV)$, and VV decays. However, we will demonstrate that the factorized annihilation effects in $B \to \phi K_1$ decays are smaller than those of final sates being pseudoscalars and/or vector bosons.

The paper is organized as follows. In Sec. II, we first show the relevant effective interactions and the parametrization of the form factors. We then give the decay amplitudes in the framework of the generalized factorization approach and define the polarizations for $B \to K_1\phi$ decays. In Sec. III, we present our numerical analysis. We give our conclusions in Sec. IV.

II. FORM FACTORS, DECAY AMPLITUDES AND POLARIZATIONS

At the quark level, the effective interactions for the decays of $B \to K_0^*(1430)\phi$ and $B \to K_1\phi$ are described by $b \to sq\bar{q}$, which are the same as $B \to K^{(*)}\phi$ decays, and given by [14]

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_q \left[C_1(\mu)O_1^{(q)}(\mu) + C_2(\mu)O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right], \tag{1}$$

*Electronic address: phychen@mail.ncku.edu.tw
†Electronic address: geng@phys.nthu.edu.tw
‡Electronic address: ykhsiao@phys.nthu.edu.tw
§Electronic address: weizt@phys.sinica.edu.tw

where $V_q = V_{qs}^* V_{qb}$ are the Cabibbo-Kobayashi-Maskawa (CKM) [15] matrix elements and the operators O_1–O_{10} are defined as

$$\begin{gathered}
O_1^{(q)} = (\bar{s}_\alpha q_\beta)_{V-A}(\bar{q}_\beta b_\alpha)_{V-A}, \qquad O_2^{(q)} = (\bar{s}_\alpha q_\alpha)_{V-A}(\bar{q}_\beta b_\beta)_{V-A}, \qquad O_3 = (\bar{s}_\alpha b_\alpha)_{V-A}\sum_q (\bar{q}_\beta q_\beta)_{V-A}, \\
O_4 = (\bar{s}_\alpha b_\beta)_{V-A}\sum_q (\bar{q}_\beta q_\alpha)_{V-A}, \qquad O_5 = (\bar{s}_\alpha b_\alpha)_{V-A}\sum_q (\bar{q}_\beta q_\beta)_{V+A}, \qquad O_6 = (\bar{s}_\alpha b_\beta)_{V-A}\sum_q (\bar{q}_\beta q_\alpha)_{V+A}, \\
O_7 = \frac{3}{2}(\bar{s}_\alpha b_\alpha)_{V-A}\sum_q e_q(\bar{q}_\beta q_\beta)_{V+A}, \qquad O_8 = \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A}\sum_q e_q(\bar{q}_\beta q_\alpha)_{V+A}, \\
O_9 = \frac{3}{2}(\bar{s}_\alpha b_\alpha)_{V-A}\sum_q e_q(\bar{q}_\beta q_\beta)_{V-A}, \qquad O_{10} = \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A}\sum_q e_q(\bar{q}_\beta q_\alpha)_{V-A},
\end{gathered} \tag{2}$$

with α and β being the color indices. In Eq. (1), O_1–O_2 are from the tree level of weak interactions, O_3–O_6 are the so-called gluon penguin operators and O_7–O_{10} are the electroweak penguin operators, while C_1–C_{10} are the corresponding Wilson coefficients (WCs). Using the unitarity condition, the CKM matrix elements for the penguin operators O_3–O_{10} can also be expressed as $V_u + V_c = -V_t$.

To deal with the hadronic transition matrix elements in the framework of the GFA, we parameterize the relevant form factors to be [16]

$$\begin{aligned}
\langle S(p_2)|A_\mu|\bar{B}(p_B)\rangle &= -i\left[\left(P_\mu - \frac{m_B^2 - m_S^2}{q^2}q_\mu\right)F_1^{BS}(q^2) + \frac{m_B^2 - m_S^2}{q^2}q_\mu F_0^{BS}(q^2)\right], \\
\langle A(p_2, \varepsilon_A)|V_\mu|\bar{B}(p_B)\rangle &= -i\left\{(m_B - m_A)\varepsilon_{A\mu}^* V_1^{BA}(q^2) - \frac{\varepsilon_A^* \cdot p_B}{m_B - m_A}P_\mu V_2^{BA}(q^2) - 2m_A\frac{\varepsilon_A^* \cdot p_B}{q^2}q_\mu[V_3^{BA}(q^2) - V_0^{BA}(q^2)]\right\}, \\
\langle A(p_2, \varepsilon_A)|A_\mu|\bar{B}(p_B)\rangle &= -\frac{A^{BA}(q^2)}{m_B - m_A}\epsilon_{\mu\nu\rho\sigma}\varepsilon_A^{*\nu}P^\rho q^\sigma,
\end{aligned} \tag{3}$$

with

$$V_3^{BA}(q^2) = \frac{m_B - m_A}{2m_A}V_1^{PA}(q^2) - \frac{m_B + m_A}{2m_A}V_2^{BA}(q^2), \qquad V_3^{BA}(0) = V_0^{BA}(0),$$

where S and A denote the scalar and axial-vector mesons, respectively, and ε_A is the polarization vector of the axial-vector meson. In terms of spin, orbital and total angular momenta, they can be described by $2S + 1L_J$ so that $S = {}^3P_0$ and $A = {}^3P_1({}^1P_1)$, $P = p_B + p_2$, $q = p_B - p_2$. We note that the state A is not a physical state. Because of the decaying topology, the transition matrix elements could be further described by

$$\begin{aligned}
X^{(BS(A),\phi)} &= \langle\phi|(\bar{s}s)_{V\pm A}|0\rangle\langle S(A)|(\bar{s}b)_{V-A}|\bar{B}\rangle, \\
Y_1^{(B,\phi S(A))} &= \langle\phi S(A)|(\bar{q}s)_{V-A}|0\rangle\langle 0|(\bar{q}b)_{V-A}|\bar{B}\rangle, \\
Y_2^{(B,\phi S(A))} &= \langle\phi S(A)|(\bar{q}s)_{S+P}|0\rangle\langle 0|(\bar{q}b)_{S-P}|\bar{B}\rangle,
\end{aligned} \tag{4}$$

where $X^{(BS(A),\phi)}$ denote the factorized parts of emission topology and $Y_{1,2}^{(B,\phi S(A))}$ stand for the factorized parts of annihilation topology. Note that the currents associated with $(S + P) \otimes (S - P)$ in Eq. (4) are from the Fierz transformations of $(V - A) \otimes (V + A)$. From Eqs. (1)–(4), the decay amplitudes for $B \to K_0^*(1430)\phi$ can be written as

$$\begin{aligned}
A(\bar{B}_d \to K_0^{*0}(1430)\phi) &= \frac{G_F}{\sqrt{2}}\{-V_{tb}V_{ts}^*[\tilde{a}^{(s)}X^{(BK_0^*,\phi)} \\
&\quad + a_4^{(s)}Y_1^{(B,\phi K_0^*)} - 2a_6^{(s)}Y_2^{(B,\phi K_0^*)}]\}, \\
A(B_u^- \to K_0^{*-}(1430)\phi) &= \frac{G_F}{\sqrt{2}}\{V_{us}V_{ub}^* a_1 Y_1^{(B,\phi K_0^*)} \\
&\quad - V_{tb}V_{ts}^*[\tilde{a}^{(s)}X^{(BK_0^*,\phi)} \\
&\quad + a_4^{(u)}Y_1^{(B,\phi K_0^*)} - 2a_6^{(u)}Y_2^{(B,\phi K_0^*)}]\},
\end{aligned} \tag{5}$$

with $\tilde{a}^{(s)} = a_3^{(s)} + a_4^{(s)} + a_5^{(s)}$. To be more convenient for our analysis, we can redefine the useful WCs by combing gluon and electroweak penguin contributions to be

$$\begin{aligned}
a_1 &= C_2^{\rm eff} + \frac{C_1^{\rm eff}}{N_c^{\rm eff}}, \qquad a_2 = C_1^{\rm eff} + \frac{C_2^{\rm eff}}{N_c^{\rm eff}}, \\
a_3^{(q)} &= C_3^{\rm eff} + \frac{C_4^{\rm eff}}{N_c^{\rm eff}} + \frac{3}{2}e_q\left(C_9^{\rm eff} + \frac{C_{10}^{\rm eff}}{N_c^{\rm eff}}\right), \\
a_4^{(q)} &= C_4^{\rm eff} + \frac{C_3^{\rm eff}}{N_c^{\rm eff}} + \frac{3}{2}e_q\left(C_{10}^{\rm eff} + \frac{C_9^{\rm eff}}{N_c^{\rm eff}}\right), \\
a_5^{(q)} &= C_5^{\rm eff} + \frac{C_6^{\rm eff}}{N_c^{\rm eff}} + \frac{3}{2}e_q\left(C_7^{\rm eff} + \frac{C_8^{\rm eff}}{N_c^{\rm eff}}\right), \\
a_6^{(q)} &= C_6^{\rm eff} + \frac{C_5^{\rm eff}}{N_c^{\rm eff}} + \frac{3}{2}e_q\left(C_8^{\rm eff} + \frac{C_7^{\rm eff}}{N_c^{\rm eff}}\right),
\end{aligned} \tag{6}$$

where the WCs $C_i^{\rm eff}$ have contained vertex corrections for smearing the μ-scale dependence in transition matrix elements [12]. We note that in order to include nonfactorizable effects, the color number $N_c^{\rm eff}$ is regarded as a variable and it may not be equal to 3. Similarly, the decay amplitudes for $B \to A\phi$ are described by

$$\begin{aligned} A(\bar{B}_d \to A\phi) &= \frac{G_F}{\sqrt{2}}\{-V_{tb}V_{ts}^*[\tilde{a}^{(s)}X^{(BA,\phi)} \\ &\quad + a_4^{(s)}Y_1^{(B,\phi A)} - 2a_6^{(s)}Y_2^{(B,\phi A)}]\}, \\ A(B_u^- \to A\phi) &= \frac{G_F}{\sqrt{2}}\{V_{us}V_{ub}^*a_1X_1^{(B,\phi A)} - V_{tb}V_{ts}^*[\tilde{a}^{(s)}X^{(BA,\phi)} \\ &\quad + a_4^{(u)}Y_1^{(B,\phi A)} - 2a_6^{(u)}Y_2^{(B,\phi A)}]\}. \end{aligned} \tag{7}$$

As known that the physical states $K_1(1270)$ and $K_1(1400)$ are the mixtures of states ${}^1P_1$ and ${}^3P_1$, their realtions could be parametrized by [16,17],

$$\begin{aligned} K_1(1270) &= K_{1P_1}\cos\theta + K_{3P_1}\sin\theta, \\ K_1(1400) &= -K_{1P_1}\sin\theta + K_{3P_1}\cos\theta. \end{aligned} \tag{8}$$

Hence, the physical decaying amplitudes are given by

$$\begin{aligned} A(B \to K_1(1270)\phi)_p &= \cos\theta A(B \to K_{1P_1}\phi) \\ &\quad + \sin\theta A(B \to K_{3P_1}\phi), \\ A(B \to K_1(1400)\phi)_p &= -\sin\theta A(B \to K_{1P_1}\phi) \\ &\quad + \cos\theta A(B \to K_{3P_1}\phi). \end{aligned} \tag{9}$$

Since the final sates of $B \to AV$ carry spin degrees of freedom, the decay amplitudes in terms of helicities, like those in the $B \to V_1V_2$ decays, can be generally described by

$$\begin{aligned} \mathcal{M}^{(\lambda)} &= \epsilon_{V\mu}^*(\lambda)\epsilon_{A\nu}^*(\lambda)[ag^{\mu\nu} + bp_B^\mu p_B^\nu \\ &\quad + ic\epsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}]. \end{aligned}$$

Because B is a pseudoscalar, the two outgoing vector mesons A and V have to carry the same helicity. Consequently, the amplitudes with different helicities can be decomposed as

$$\begin{aligned} H_{00} &= \frac{-1}{2m_Vm_A}[(m_B^2 - m_V^2 - m_A^2)a + 2m_B^2p^2b], \\ H_{\pm\pm} &= a \mp m_Bpc, \end{aligned} \tag{10}$$

where p is the magnitude of vector momenta of vector mesons. In addition, we can also write the amplitudes in terms of polarizations as

$$A_L = H_{00}A_{\parallel(\perp)} = \frac{1}{\sqrt{2}}(H_{--} \pm H_{++}). \tag{11}$$

Accordingly, the polarization fractions can be defined to be

$$R_i = \frac{|A_i|^2}{|A_L|^2 + |A_\parallel|^2 + |A_\perp^2|}, \qquad (i = L, \parallel, \perp), \tag{12}$$

representing longitudinal, transverse parallel and transverse perpendicular components, respectively. Note that $\sum_i R_i = 1$. In sum, the decay rate expressed by polarization amplitudes is given by

$$\Gamma = \frac{G_F^2 p}{16\pi m_B^2}(|A_L|^2 + |A_\parallel|^2 + |A_\perp|^2). \tag{13}$$

III. NUMERICAL ANALYSIS

A. The analysis of annihilation contributions on $B \to AV$ decays

It has been believed that the annihilation contributions could significantly reduce the longitudinal polarization of $B \to K^*\phi$ decays. It is interesting to ask whether annihilation effects could also play an important role on the polarization fractions of $B \to K_1\phi$ decays. To answer the question, we start with the analysis on the annihilation contributions in $B \to PP$ and $B \to VV$ decays. For $B \to PP$ decays, the factorized amplitude associated with the $(V - A) \otimes (V - A)$ interaction for annihilated topology can be expressed as

$$\begin{aligned} &\langle P_1P_2|\bar{q}_1\gamma^\mu(1 - \gamma_5)q_2\bar{q}_3\gamma^\mu(1 - \gamma_5)b|\bar{B}\rangle_a \\ &\quad = -if_B(m_1^2 - m_2^2)F_0^{P_1P_2}(m_B^2), \end{aligned} \tag{14}$$

where $m_{1(2)}$ are the masses of outgoing particles and f_B and $F_0^{P_1P_2}(m_B^2)$ correspond to the B decay constant and the timelike form factor, defined by

$$\begin{aligned} \langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}(p_B)\rangle &= if_Bp_B^\mu, \\ \langle P_1(p_1)P_2(p_2)|\bar{q}_1\gamma_\mu q_2|0\rangle &= \left[q_\mu - \frac{m_1^2 - m_2^2}{Q^2}Q_\mu\right]F_1^{P_1P_2}(Q^2) \\ &\quad + \frac{m_1^2 - m_2^2}{Q^2}Q_\mu F_0^{P_1P_2}(Q^2), \end{aligned} \tag{15}$$

respectively, with $q = p_1 - p_2$ and $Q = p_1 + p_2$. From Eq. (14), it is clear that if $m_1 = m_2$, the factorized effects of annihilation topology vanish. However, if the associated interactions are $(S + P) \otimes (S - P)$, by equation of motion, the decay amplitude becomes

$$\begin{aligned} &\langle P_1P_2|\bar{q}_1(1 + \gamma_5)q_2\bar{q}_3(1 - \gamma_5)b|\bar{B}\rangle_a \\ &\quad = if_B\frac{(m_1^2 - m_2^2)m_B^2}{(m_{q_1} - m_{q_2})(m_b + m_{q_3})}F_0^{P_1P_2}(m_B^2). \end{aligned} \tag{16}$$

We see that the subtracted factors appear in the numerator and denominator simultaneously. As a result, the annihilation effects by $(S + P) \otimes (S - P)$ interactions can be sizable due to $(m_1^2 - m_2^2)/(m_{q_1} - m_{q_2}) \propto (m_1 + m_2)$. The suppression only comes from the form factor $F_0^{P_1P_2}(m_B^2) \propto 1/m_B^2$ which can be calculated by PQCD [18]. Similarly, we expect that the same conclusion can be given to the VV modes, i.e., the longitudinal polarization should satisfy

$$\begin{aligned}&\langle V_{1L}V_{2L}|\bar{q}_1\gamma^\mu(1-\gamma_5)q_2\bar{q}_3\gamma^\mu(1-\gamma_5)b|\bar{B}\rangle_a\\&\quad=-if_B(m_1^2-m_2^2)F_{0L}^{V_1V_2}(m_B^2),\\&\langle V_{1L}V_{2L}|\bar{q}_1(1+\gamma_5)q_2\bar{q}_3(1-\gamma_5)b|\bar{B}\rangle_a\\&\quad=if_B\frac{(m_1^2-m_2^2)m_B^2}{(m_{q_1}-m_{q_2})(m_b+m_{q_3})}F_{0L}^{V_1V_2}(m_B^2).\end{aligned}\tag{17}$$

By the helicity analysis, we find the transverse components to be

$$\begin{aligned}&\langle V_{1T}V_{2T}|\bar{q}_1\gamma^\mu(1-\gamma_5)q_2\bar{q}_3\gamma^\mu(1-\gamma_5)b|\bar{B}\rangle_a\\&\quad\propto -if_B(m_1^2-m_2^2)\frac{m_1m_2}{m_B^2}F_{0T}^{V_1V_2}(m_B^2),\\&\langle V_{1T}V_{2T}|\bar{q}_1(1+\gamma_5)q_2\bar{q}_3(1-\gamma_5)b|\bar{B}\rangle_a\\&\quad\propto if_B\frac{(m_1^2-m_2^2)m_B^2}{(m_{q_1}-m_{q_2})(m_b+m_{q_3})}\times\frac{m_1+m_2}{m_B}F_{0T}^{V_1V_2}(m_B^2).\end{aligned}\tag{18}$$

Consequently, for the VV modes, the annihilation effects of the longitudinal polarizations by $(S+P)\otimes(S-P)$ interactions are only suppressed by the corresponding timelike form factor $F_{0L}^{V_1V_2}(m_B^2)$ while those of the transverse parts are suppressed by $(m_1+m_2)/m_B\cdot F_{0T}^{V_1V_2}(m_B^2)$. Hence, the annihilation contributions can be sizable and important on polarizations of $B\to VV$ decays.

We now examine the decays of $B\to AV$ and check if the suppression factor $m_1^2-m_2^2$ of annihilation contributions could be smeared in the decays. Similar to the PP and VV cases, we start by considering the decays of $B\to SP$ with S being the p-wave scalar boson. The decay amplitude associated with $(V-A)\otimes(V-A)$ interactions can be expressed by

$$\begin{aligned}&\langle P_1S_2|\bar{q}_1\gamma^\mu(1-\gamma_5)q_2\bar{q}_3\gamma^\mu(1-\gamma_5)b|\bar{B}\rangle_a\\&\quad=f_B(m_1^2-m_2^2)F_0^{P_1S_2}(m_B^2).\end{aligned}\tag{19}$$

By equation of motion, the decay amplitude associated with $(S+P)\otimes(S-P)$ interactions is found to be

$$\begin{aligned}&\langle P_1S_2|\bar{q}_1(1+\gamma_5)q_2\bar{q}_3(1-\gamma_5)b|\bar{B}\rangle_a\\&\quad=-f_B\frac{(m_1^2-m_2^2)m_B^2}{(m_{q_1}+m_{q_2})(m_b+m_{q_3})}F_0^{P_1S_2}(m_B^2).\end{aligned}\tag{20}$$

Clearly, the suppressed factor by the mass difference only appears in the numerator, i.e. $(m_1^2-m_2^2)/(m_{q_1}+m_{q_2})\propto m_1-m_2$. As a result, we expect that the annihilation effects in $B\to SP$ decays are much smaller than those of $B\to PP$ decays. From Eqs. (17) and (18), we could immediately see that the suppressed factor in $B\to SP$ and $B\to AV$ are the same. In sum, by our analysis, we conjecture that if the final states are composed of a vector (pseudoscalar) boson and an axial-vector (scalar) boson, the annihilation contributions could be ignored.

Unlike $B\to SP(AV)$ decays, there is no extra suppressing factor for the decays of $B\to SV$ except the $1/m_B^2$ suppression. Nevertheless, by comparing to the dominant emission topology, due to the $1/m_B^2$ suppression factor on the timelike form factor, the annihilated effects are still small. Therefore, in our calculations we still neglect the annihilation contributions to the BRs of $B\to K_0^*(1430)\phi$.

B. Branching ratios and polarization fractions

To get the numerical estimations, we use that the decay constant $f_\phi=0.233$ GeV and the CKM matrix elements $V_{tb}V_{ts}^*\approx -A\lambda^2$ with $A=0.83$ and $\lambda=0.224$ [1]. Since the color number is regarded as a variable, the effective WCs for different effective colors are found to be $\tilde{a}^{(s)}(\mu=2.5\text{ GeV})=(-584-97\text{i},-418-73\text{i},-284-55\text{i},84-27\text{i})\times10^{-4}$ and $\tilde{a}^{(s)}(\mu=4.4\text{ GeV})=(-522-107\text{i},-375-81\text{i},-257-61\text{i},-80-29\text{i})\times10^{-4}$ for $N_c^{\text{eff}}=(2,3,5,\infty)$, respectively. The μ-scale dependence could be taken as theoretical uncertainties. According to the results of LFQCD [16], the values of form factors for $B\to K_0^*(1430)$, $B\to K_{3P_1}$, and $B\to K_{1P_1}$ at $q^2=m_\phi^2$ are shown in Table I.

It is interesting to note that all values of form factors are positive except $V_2^{BK_{1P_1}}=-0.0555$. We will discuss the implication of this negative value on the BRs and $R_{L(\perp)}$ for $B\to K_1\phi$. From the definition of form factors for B decaying to axial-vector boson, shown in Eq. (3), we have to know the masses of states K_{3P_1} and K_{1P_1}. To obtain the masses, we adopt the results of Ref. [17] so that $m_{K_{3P_1}}^2=m_{K_1(1270)}^2+m_{K_1(1400)}^2-m_{K_{1P_1}}^2$ and $2m_{K_{1P_1}}^2=m_{b_1(1232)}^2+m_{h_1(1380)}^2$. The remaining unknown parameter is the mixing angle θ. It is known that by the decays $\tau\to\nu_\tau K_1(1270)\times(K_1(1400))$, θ can be determined to be around 37^0 and 58^0 with a twofold ambiguity [19]. Recently, $\text{BR}(B^-\to K_1^-(1270)\gamma)=(4.28\pm0.94\pm0.43)\times10^{-5}$ has been measured by BELLE, in which the errors are statistical and systematical, respectively. Note that there has been no measurement on the $B^-\to K_1^-(1400)\gamma$ decay yet [20]. That is, the BR of $B\to K_1(1400)\gamma$ might be much smaller than that of $B\to K_1(1270)\gamma$. The observation of the decay could remove the sign ambiguity and conform $\theta\approx37^0$ or 58^0 [21].

In terms of Eqs. (5) and (7), the BRs for the different values of the mixing angle θ are displayed in Tables II and III with $\mu=2.5$ and 4.4 GeV, respectively. From the tables, we clearly see that the BRs of $B\to(K_0^*(1430),K_1(1270),K_1(1400))\phi$ are increasing while N_c^{eff} is decreasing. Interestingly, when $N_c^{\text{eff}}=2$, BR($B\to$

TABLE I. The values of form factors for $B\to K_0^*(1430)$, $B\to K_{3P_1}$ and $B\to K_{1P_1}$ at $q^2=m_\phi^2$ calculated by LFQCD [16].

	$F_1^{BK_0^*}$	$V_1^{BK_{3P_1}}$	$V_2^{BK_{3P_1}}$	$A^{BK_{3P_1}}$	$V_1^{BK_{1P_1}}$	$V_2^{BK_{1P_1}}$	$A^{BK_{1P_1}}$
$F(m_\phi^2)$	0.275	0.393	0.177	0.275	0.197	-0.0555	0.118

$K_0^*(1430)\phi) \sim \mathrm{BR}(B \to K\phi) \sim 8 \times 10^{-6}$ [1]. It is worth mentioning that the BRs of $B \to K\phi$ are consistent with the data when $N_c^{\mathrm{eff}} = 2 \sim 3$ by the GFA [12]. We may conjecture that $N_c^{\mathrm{eff}} = 2 \sim 3$ is also applicable for the decay modes with the p-wave mesons. Moreover, from Tables II and III, we find that if $\theta = 37^0$, $\mathrm{BR}(B \to K_1(1270)\phi)$ is about 1 order of magnitude larger than $\mathrm{BR}(B \to K_1(1400)\phi)$. On the other hand, if $\theta = 58^0$, the ratio $\mathrm{BR}(B \to K_1(1270)\phi)$ to $\mathrm{BR}(B \to K_1(1400)\phi)$ is around 2. Following the results, we suggest that one could measure the ratio of $\mathrm{BR}(B \to K_1(1270)\phi)/\mathrm{BR}(B \to K_1(1400)\phi)$ to further determine the angle θ. To be more clear, we present $\mathrm{BR}(B \to K_1\phi)$ with $\mu = 2.5$ GeV as a function of θ in Fig. 1.

As discussed before, since axial-vector and vector bosons carry the spin degrees of freedom, by the angular distribution analysis we can study the various polarizations in $B \to AV$ decays. Hence, from Eq. (7) with neglecting the annihilation contributions, the polarization amplitudes for $B \to A$ decays are given by

$$A_L(B \to A\phi) = -\frac{G_F}{\sqrt{2}}\tilde{a}^{(s)}\frac{f_\phi}{2m_A}\bigg[(m_B^2 - m_\phi^2 - m_A^2) \times (m_B - m_A)V_1^{BA} - \frac{4m_B^2 p^2}{m_B - m_A}V_2^{BA}\bigg],$$

$$A_\parallel(B \to A\phi) = G_F\tilde{a}^{(s)}f_\phi m_\phi(m_B - m_A)V_1^{BA}, \tag{21}$$

$$A_\perp(B \to A\phi) = -G_F\tilde{a}^{(s)}f_\phi m_\phi\frac{2m_B p}{m_B - m_A}A^{BA}.$$

The amplitudes for physical states can be obtained by following Eq. (9). From the polarization amplitudes, it is clear that by the GFA, the polarization fractions depend on the form factors $V_{1(2)}^{BA}$, A^{BF}, and the mixing angle θ but they are independent of the effective WC $\tilde{a}^{(s)}$. From Eq. (12) and Table I, our results for polarization fractions R_L and $R_\perp$ are presented in Table IV for $\theta = 37^0(58^0)$. Note that $R_\parallel$ can be derived by the identity $R_\parallel = 1 - R_L - R_\perp$.

From Table IV, we can see that the polarization fractions are somewhat insensitive to the values of θ in $B \to K_1(1270)\phi$, i.e., $R_L(B \to K_1(1270)\phi) = 91.9\%$ with $\theta = 37^0$ while $R_L(B \to K_1(1270)\phi) = 85.7\%$ with $\theta = 58^0$. However, those for $B \to K_1(1400)\phi$ are more sensitive to θ, i.e. $R_L(B \to K_1(1400)\phi) = 79.2\%$ with $\theta = 37^0$ whereas $R_L(B \to K_1(1400)\phi) = 99.5\%$ with $\theta = 58^0$. In Fig. 2, we show R_L as a function of θ.

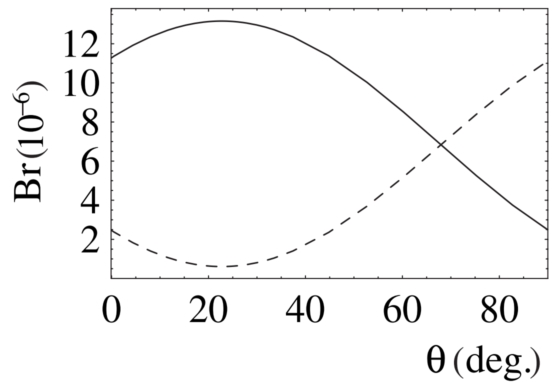

FIG. 1. The branching ratios (in units of 10^{-6}) as a function of the mixing angle θ. The solid and dashed curves correspond to the decays of $\bar{B}^0 \to K_1^0(1270)\phi$ and $\bar{B}^0 \to K_1^0(1400)\phi$, respectively.

TABLE II. The branching ratios (in units of 10^{-6}) of $B \to (K_0^*(1430), K_1(1270), K_1(1400))\phi$ decays for $\theta = 37^0(58^0)$ with $\mu = 2.5$ GeV.

Mode	$N_c^{\mathrm{eff}} = 2$	$N_c^{\mathrm{eff}} = 3$	$N_c^{\mathrm{eff}} = 5$	$N_c^{\mathrm{eff}} = \infty$
$\bar{B}^0 \to K_0^{*0}(1430)\phi$	8.06	4.13	1.93	0.18
$\bar{B}^0 \to K_1^0(1270)\phi$	24.18(16.63)	12.40(8.53)	5.78(3.98)	0.54(0.37)
$\bar{B}^0 \to K_1^0(1400)\phi$	2.66(8.70)	1.36(4.46)	0.64(2.08)	0.06(0.20)
$B^- \to K_0^{*-}(1430)\phi$	8.77	4.50	2.10	0.20
$B^- \to K_1^-(1270)\phi$	25.57(18.09)	13.11(9.28)	6.11(4.33)	0.57(0.40)
$B^- \to K_1^-(1400)\phi$	2.81(9.47)	1.44(4.85)	0.67(2.26)	0.06(0.21)

TABLE III. The Legend is the same as Table II but $\mu = 4.4$ GeV.

Mode	$N_c^{\mathrm{eff}} = 2$	$N_c^{\mathrm{eff}} = 3$	$N_c^{\mathrm{eff}} = 5$	$N_c^{\mathrm{eff}} = \infty$
$\bar{B}^0 \to K_0^{*0}(1430)\phi$	6.58	3.40	1.61	0.17
$\bar{B}^0 \to K_1^0(1270)\phi$	19.60(13.57)	10.14(7.01)	4.80(3.32)	0.50(0.35)
$\bar{B}^0 \to K_1^0(1400)\phi$	2.15(7.10)	1.11(3.67)	0.53(1.74)	0.05(0.18)
$B^- \to K_0^{*-}(1430)\phi$	7.16	3.70	1.75	0.18
$B^- \to K_1^-(1270)\phi$	20.73(14.76)	10.73(7.62)	5.08(3.61)	0.53(0.38)
$B^- \to K_1^-(1400)\phi$	2.28(7.72)	1.18(3.99)	0.56(1.89)	0.06(0.20)

TABLE IV. The polarization fractions (in unit of %) of $B \to (K_1(1270), K_1(1400))\phi$ with the form factors in Table I and $\theta = 37^0(58^0)$.

Mode	R_L	$R_\perp$
$B \to K_1(1270)\phi$	91.9(85.7)	4.2(7.8)
$B \to K_1(1400)\phi$	79.2(99.5)	12.6(0.4)

Finally, we discuss the implication of $V_2^{BK_{1P_1}} = -0.0555$ on BRs and $R_{L(\perp)}$. In fact, if V_2^{BA} is positive, by comparing with $B \to K^*\phi$, $A_L(B \to A\phi)$ could be smaller because the factor of $1/(m_B - m_A)$ enhances the cancellation between the two terms in Eq. (21), whereas the corresponding factor is $1/(m_B + m_{K^*})$ for $B \to K^*\phi$, which suppresses the cancellation. However, as shown in Table IV, $R_L(B \to K_1(1270)\phi)$ for $\theta = 37^0$ still satisfies $1 - 2m_\phi^2/m_B^2 \sim O(1)$, which is the same as the estimation for $B \to K^*\phi$ by only considering the factorized parts. It is clear that the main reason is from the negative form factor of $V_2^{BK_{1P_1}}$. To illustrate the influence, we tune the sign of $V_2^{BK_{1P_1}}$ to be positive artificially and we find that BRs and polarization fractions for $\theta = 37^0(58^0)$, $N_c^{\rm eff} = 2$, and $\mu = 2.5$ GeV are given as follows:

$$\begin{aligned} &\mathrm{BR}\,(B^0 \to K_1^0(1270)\phi) = 7.88(8.06) \times 10^{-6}, \\ &\mathrm{BR}(B^0 \to K_1^0(1400)\phi) = 0.62(0.44) \times 10^{-6}, \\ &R_L(B \to K_1(1270)\phi) = 75(69)\%, \\ &R_\perp(B \to K_1(1270)\phi) = 13(17)\%, \\ &R_L(B \to K_1(1400)\phi) = 10(91)\%, \\ &R_\perp(B \to K_1(1400)\phi) = 54(7)\%. \end{aligned} \tag{22}$$

Since BRs and R_L are reduced significantly, the measurements on BRs and $R_{L(\perp)}$ could also test the sign of $V_2^{BK_{1P_1}}$.

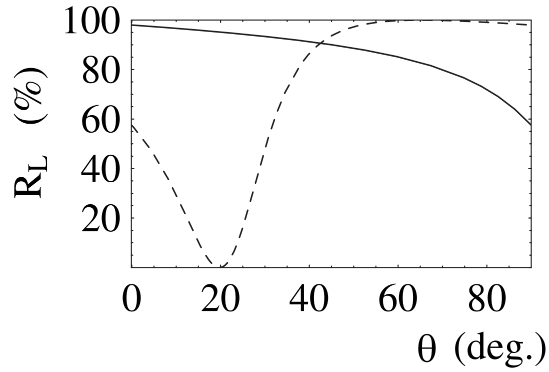

FIG. 2. The longitudinal polarization fractions (in units of %) as a function of the mixing angle θ. The solid and dashed curves correspond to $\bar{B}^0 \to K_1^0(1270)\phi$ and $\bar{B}^0 \to K_1^0(1400)\phi$, respectively.

IV. CONCLUSIONS

We have studied the productions of p-wave mesons $K_0^*(1430)$, $K_1(1270)$, and $K_1(1400)$ in B decays in the framework of the GFA. In terms of form factors calculated by LFQCD, with $N_c^{\rm eff} = 2$ we have found that $\mathrm{BR}(B \to K_0^*(1430)\phi) \sim \mathrm{BR}(B \to K\phi) \sim 8 \times 10^{-6}$. We have also obtained that $\mathrm{BR}(B \to K_1(1270)\phi) \sim O(10^{-5})$ while $\mathrm{BR}(B \to K_1(1400)\phi) \sim O(10^{-6})$. Since the specific values of BRs are sensitive to the mixing angle θ, we can determine the angle by the future measurements on these modes. Moreover, we have shown that $R_L(B \to K_1\phi) \sim 80 - 100\%$ and we have demonstrated that the BRs and polarization fractions are also sensitive to the sign of the form factor $V_2^{BK_{1P_1}}$.

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