Nonuniversality of transverse Coulomb exchange at small x

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Within an explicit scalar QED model we compare, at fixed $x \ll 1$, the leading-twist $K_\perp$-dependent "quark" distribution $f_q(x, K_\perp)$ probed in deep inelastic scattering and Drell-Yan production and show that the model is consistent with the universality of $f_q(x, K_\perp)$. The extension of the model from the aligned jet to the symmetric kinematical regime reveals interesting properties of the physics of Coulomb rescatterings when comparing DIS and DY processes. At small x the transverse momentum $\langle k_\perp^2 \rangle$ induced by multiple scattering on a single center is process dependent, as well as the transverse momentum broadening occurring in collisions on a finite size nuclear target.

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I. INTRODUCTION

A significant modification of the quark and gluon distribution functions in heavy nuclei—as compared to light targets such as a proton or deuterium—is observed in deep inelastic scattering (DIS) experiments on nuclei (for a review, see [1,2]). Although such effects manifest themselves on a wide range in the light-cone momentum fraction x carried by the parton struck in the target, it is useful to discuss separately two relevant kinematical regimes at work. In the target rest frame, the typical lifetime for the hadronic fluctuation of the virtual photon of energy ν is given by the coherence length $l_c = 2\nu/Q^2 \equiv 1/(Mx)$ (where M is the mass of one scattering center in the target, here a nucleon in the nucleus). At large x the coherence length remains small compared to the typical distance d between two centers, $l_c \lesssim d$. This is the *incoherent* regime for which one expects factorization between the hard production process on a given center and the subsequent final state interaction of the hadronic (or partonic) fluctuation. Conversely, at small $x \ll 1$, the fluctuation scatters coherently on several scattering centers. In this regime, rescatterings actually affect the hard process. Taking the average distance $d \simeq 2$ fm between two nucleons in heavy nuclei, one expects the onset of coherence effects such as shadowing below $x \simeq 0.1$. This is indeed seen in the small x measurements of nuclear structure functions performed at CERN and Fermilab by the NMC and E665 experiments, respectively [3]. In addition to DIS, evidence for shadowing corrections also comes from Drell-Yan (DY) data at small $x_2 = x$ measured by the fixed-target experiments E772 and E866/NuSea [4]. More data are expected at smaller x from the relativistic heavy ion collider facility and in a few years from the large hadron collider.

Although the nuclear parton densities probed in DIS or through the DY mechanism appear to be similar within experimental errors, other (intrinsically nonperturbative) observables depend significantly on the considered hard process. In particular, the transverse momentum nuclear broadening of the DY pair [4,5] is found to be a factor of 5 or more[1] smaller than what is measured in dijet photo- or hadro-production [6]. Note that a similar discrepancy between DY pair and heavy quarkonium transverse momentum broadening has also been reported [4,5,7].

In the incoherent regime, one would expect the broadening for large nuclei to be proportional to the path length covered by the parton produced in the hard process, $\langle p_\perp^2 \rangle = \Lambda^2 A^{1/3}$, coming from the diffusion in transverse momentum space due to multiple scattering [8]. Moreover the factorization between the hard process and the rescatterings should make the strength of the nuclear broadening Λ a universal quantity (up to trivial geometrical and color factors). Yet the smallness of nuclear broadening in the DY process seems to contradict universality, which remains so far not fully understood [7,9–11]. Given the fact that most of the data lie at the borderline between the incoherent and coherent regimes, we may wonder whether coherent effects could be at the origin of this observation.

Since rescattering affects the hard process as soon as $l_c \gtrsim d$, there is *a priori* no reason to expect transverse momentum broadening to be universal at small x. However, when comparing two different processes, it is difficult to foresee in which process the broadening will be the largest.

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[1]Given the various incident reaction energies and depending on the precise definition adopted for the nuclear broadening, telling precisely how big the discrepancy is turns out to be somewhat delicate although it is statistically significant.

In this paper those questions are addressed by studying in parallel DIS and DY within an explicit scalar QED (SQED) model in the small x limit. In this limit we will consider two kinematical regimes, namely, the aligned-jet region where the largest component 2ν of the incoming light-cone momentum is mostly transferred to a single final state particle, and the symmetric region where it is transferred to two final state particles. In the aligned-jet region (where *soft* rescatterings contribute to the total cross section to leading twist), the model is shown to be consistent with the universality of the $K_\perp$-dependent distribution $f_{q/T}(x, K_\perp)$ of the target quark participating to the hard subprocess, for an arbitrary number of scattering centers in the target, i.e., $f^{\rm DIS}_{q/T}(x, K_\perp) = f^{\rm DY}_{q/T}(x, K_\perp)$. In the symmetric region (where soft rescatterings contribute to higher twist to the DIS cross section), the distribution in the Coulomb transfer $k_\perp$ to the outgoing two-particle system is different in DIS and DY (for $x \ll 1$). This nonuniversality appears both for a pointlike or finite size target. In particular, in the symmetric kinematics the nuclear $k_\perp$ broadening is driven by monopole rescattering in DY production and by dipole rescattering in DIS.

The paper is organized as follows: In Sec. II we recall the models of Refs. [12,13] for the leading-twist quark distributions probed in DIS and DY production on a pointlike heavy target, which we extend to the case of a finite size target in Sec. II B. Section III is devoted to the extension of the model to the symmetric kinematical regime. A summary of our results is given in Sec. IV.

II. MODEL FOR LEADING-TWIST DIS AND DY QUARK DISTRIBUTIONS: ALIGNED-JET KINEMATICS

A. Single scattering center

1. Perturbative model for DIS

The leading-twist $K_\perp$-dependent quark distributions $f_{q/T}(x, K_\perp)$ in DIS and DY are modeled within a scalar QED model. Let us start by briefly recalling the features of the model for DIS [12]. The contribution to the DIS cross section $\sigma_{\rm DIS}$ (or to the *forward* DIS amplitude) studied in [12] is obtained by squaring the DIS *production* amplitude shown in Fig. 1(a). The target T is chosen to be a scalar "heavy quark" of momentum p and mass M. The incoming virtual photon of momentum q couples to scalar "light quarks" of mass m, which appear with on shell momenta p_1 and p_2 in the final state. The "electromagnetic" charge of the light quarks is denoted by e and the light and heavy quarks interact with "strong" coupling g. We work in a target rest frame where[2] $q = (-Mx_B, q^-, \vec{0}_\perp)$, x_B being fixed and $q^- \equiv 2\nu = Q^2/Mx_B \to \infty$ in the Bjorken limit. In this limit the contribution to the cross section of transverse virtual photons is subleading (in SQED), and we take the photon to be longitudinally polarized in the following.

Studying the effect of Coulomb *soft rescatterings* between the light and heavy quarks *at leading-twist*[3] requires concentrating on the aligned-jet region [2], where most of the photon energy ν is transferred to the struck quark, i.e., $p_1^- \simeq q^- \gg p_2^-$ (see Fig. 1). Working moreover in the $x_B \ll 1$ limit as in [12], our leading-twist kinematics is defined by the following hierarchy of scales,

$$\begin{gathered} Q^2, \nu \to \infty \gg p_2^- \gg M \gg k_{i\perp}, p_{i\perp}, k_i^-, m \gg k_i^+, p_2^+ \sim Mx_B \gg p_1^+ \propto 1/\nu \\ p_2^- \text{ fixed} \Leftrightarrow y = \frac{p_2^-}{2\nu} \to 0. \end{gathered} \tag{1}$$

In the first line the first and last inequalities arise from the Bjorken limit and the aligned-jet kinematics (the latter being stressed in the second line). The other inequalities arise from the limit $x_B \ll 1$. All scales other than ν are soft, i.e., intrinsic to the target system. Note that taking a relatively large target mass M is not essential to our analysis but will simplify the expressions of cross sections.

The kinematics (1) allows the following physical interpretation:

(i) Since the hard scale ν does not flow in the internal propagators of the lower part of Fig. 1(a), the hard vertex $\gamma^* q \to q$ is taken at zeroth order in g. The square of the DIS production amplitude describes the soft dynamics which can be directly interpreted (apart from a trivial factor $\propto e^2 Q^2$) as a contribution to the light quark distribution in the target $f_{q/T}(x, K_\perp)$.

(ii) The momentum K corresponds to the quark probed in the target and reads $K = k - p_2$, where $k = \sum k_i$ is the total Coulomb momentum transfer between the light and heavy quarks. In Fig. 1(a) we have $K = p_1 - q$, and the quark distribution is thus probed at $x = K^+/p^+ \simeq -q^+/p^+ = x_B$. (Since $K^+ > 0$, it is easy to realize, for instance in a light-cone time-ordered formulation, that the hard subprocess in Fig. 1(a) is indeed $\gamma^* q \to q$, as in the infinite momentum frame.)

[2]We use the light-cone variables $k^\pm = k^0 \pm k^z$ to define a momentum $k = (k^+, k^-, \vec{k}_\perp)$.

[3]In Sec. III we will extend the model to a higher-twist kinematical domain, where p_2^- scales with ν, i.e., the ratio $y = p_2^-/(2\nu)$ is fixed.

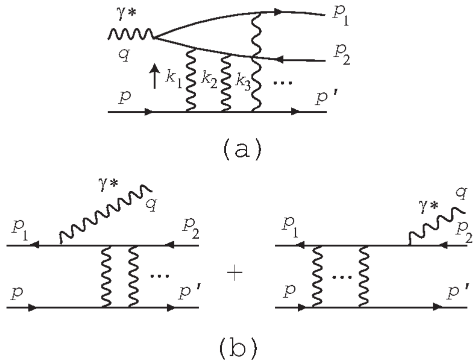

FIG. 1. (a) The DIS production amplitude in the model of Ref. [12]. Coulomb rescatterings are resummed. (b) The DY production amplitude obtained from (a) by crossing [13]. Diagrams where the heavy photon is emitted from an internal quark line are suppressed (in covariant gauges) in the limit $x_B \ll 1$.

(iii) The diagrams contributing to the Q^2 evolution of $f_{q/T}$ being excluded, the model thus describes $f_{q/T}$ at an initial soft scale Q_0.

The DIS amplitude of Fig. 1(a), including any number of Coulomb rescatterings, has been calculated in [12] in the kinematical domain defined by (1). It incorporates leading-twist shadowing effects and reads in transverse coordinate space,

$$\hat{\mathcal{M}}_{\rm DIS}(\vec{r}_\perp, \vec{R}_\perp) = \sqrt{4\pi}\psi(r_\perp)T_{q\bar{q}}(\vec{r}_\perp, \vec{R}_\perp), \tag{2}$$

where we define normalized scattering amplitudes $\hat{\mathcal{M}}$ in terms of Feynman amplitudes $\mathcal{M}$ by

$$\hat{\mathcal{M}} = \frac{1}{4M\nu}\mathcal{M}. \tag{3}$$

In Eq. (2) the factor $\psi(r_\perp)$ denotes the $\gamma^* \to q\bar{q}$ dipole wave function:

$$\psi(r_\perp) = y\sqrt{\alpha}QV(m_{||}r_\perp), \tag{4}$$

where $\alpha = e^2/(4\pi)$, y is the light-cone momentum fraction carried away by the "antiquark" p_2,

$$y \equiv \frac{p_2^-}{2\nu} \ll 1, \tag{5}$$

and the function V is given by

$$V(m_{||}r_\perp) \equiv \int \frac{d^2\vec{p}_\perp}{(2\pi)^2}\frac{e^{i\vec{r}_\perp\cdot\vec{p}_\perp}}{p_\perp^2 + m_{||}^2} = \frac{K_0(m_{||}r_\perp)}{2\pi}. \tag{6}$$

Here $r_\perp$ denotes the dipole size and

$$m_{||}^2 = p_2^- M x_B + m^2 = yQ^2 + m^2. \tag{7}$$

The $q\bar{q}$ dipole scattering amplitude $T_{q\bar{q}}$ appearing in (2) reads

$$T_{q\bar{q}}(\vec{r}_\perp, \vec{R}_\perp) = -i(1 - e^{-ig^2 W(\vec{r}_\perp, \vec{R}_\perp)}), \tag{8}$$

where $\vec{R}_\perp$ is the impact parameter of the outgoing quark and W is the dipole *single* scattering amplitude:

$$W(\vec{r}_\perp, \vec{R}_\perp) \equiv \int \frac{d^2\vec{k}_\perp}{(2\pi)^2}\frac{1 - e^{i\vec{r}_\perp\cdot\vec{k}_\perp}}{k_\perp^2 + \lambda^2}e^{i\vec{R}_\perp\cdot\vec{k}_\perp} = \frac{K_0(\lambda R_\perp) - K_0(\lambda|\vec{R}_\perp + \vec{r}_\perp|)}{2\pi}. \tag{9}$$

We have introduced a finite photon mass λ. Indeed, while the amplitudes W and T are infrared safe in the $\lambda \to 0$ limit,[4] some other quantities are not, such as the dipole scattering cross section $\sigma_{q\bar{q}}(r_\perp)$ defined below in Eq. (11). Since $\sigma_{q\bar{q}}(r_\perp)$ is a basic quantity which will enter our main equations, we choose to use the infrared regulator λ from now on. This provides a mathematically well-defined framework and will moreover allow to compare unambiguously DIS with DY production, for which the infrared sensitivity appears at the amplitude level [see Eq. (16)].

We stress that

$$|T_{q\bar{q}}(\vec{r}_\perp, \vec{R}_\perp)|^2 = -2\,{\rm Im}T_{q\bar{q}}(\vec{r}_\perp, \vec{R}_\perp), \tag{10}$$

a unitarity relation which is satisfied by $T_{q\bar{q}}$ since the latter resums Coulomb rescatterings. For later use, we give the scattering cross section of a $q\bar{q}$ dipole of size $r_\perp$,

$$\sigma_{q\bar{q}}(r_\perp) = \int d^2\vec{R}_\perp |T_{q\bar{q}}(\vec{r}_\perp, \vec{R}_\perp)|^2 = 2i\int d^2\vec{R}_\perp T_{q\bar{q}}(\vec{r}_\perp, \vec{R}_\perp), \tag{11}$$

where the second equality follows from (10) and

$$\int d^2\vec{R}_\perp\, {\rm Re}T_{q\bar{q}}(\vec{r}_\perp, \vec{R}_\perp) = 0. \tag{12}$$

We give the explicit form of the dipole cross section in our model,

$$\sigma_{q\bar{q}}(r_\perp) = 4\int d^2\vec{R}_\perp \sin^2\left[\frac{g^2}{4\pi}(K_0(\lambda R_\perp) - K_0(\lambda|\vec{R}_\perp + \vec{r}_\perp|))\right]. \tag{13}$$

Finally, cross sections will be given in the following by (use $dp_2^z/p_2^0 = dp_2^-/p_2^-$ and $y \ll 1$):

$$\frac{d\sigma}{d\log y} = \frac{1}{4\pi}\int \frac{d^2\vec{p}_{2\perp}}{(2\pi)^2}\frac{d^2\vec{k}_\perp}{(2\pi)^2}|\hat{\mathcal{M}}(\vec{p}_{2\perp}, \vec{k}_\perp)|^2, \tag{14}$$

[4] We have for instance $\lim_{\lambda\to 0} W(\vec{r}_\perp, \vec{R}_\perp) = \frac{1}{2\pi}\log(\frac{|\vec{R}_\perp + \vec{r}_\perp|}{R_\perp})$.

where the Fourier transform is defined as

$$\hat{\mathcal{M}}(\vec{p}_{2\perp}, \vec{k}_\perp) = \int d^2\vec{r}_\perp d^2\vec{R}_\perp \hat{\mathcal{M}}(\vec{r}_\perp, \vec{R}_\perp) e^{-i\vec{r}_\perp\cdot\vec{p}_{2\perp} - i\vec{R}_\perp\cdot\vec{k}_\perp}. \tag{15}$$

2. Model for DY production

In order to obtain a model for the quark distribution probed in DY production, the first step is simply to exchange the virtual photon and the struck quark lines in Fig. 1(a) and to replace $q^2 = -Q^2 < 0$ by $q^2 = Q^2 > 0$. The momentum of the DY pair has now $q^+ > 0$ and reads $q = (Mx_B, q^-, \vec{q}_\perp)$, and the incoming antiquark is chosen with $\vec{p}_{1\perp} = \vec{0}_\perp$. We keep the same notation for k, p_2 [see Fig. 1(a)], and $K = k - p_2$, implying that $\vec{K}_\perp = \vec{q}_\perp$ in DY instead of $\vec{K}_\perp = \vec{p}_{1\perp}$ in DIS. In coordinate space the DY production amplitude pictured in Fig. 1(b) is simply related to the DIS amplitude by a phase factor [13],

$$\hat{\mathcal{M}}_{\rm DY}(\vec{r}_\perp, \vec{R}_\perp) = -e^{ig^2 G(R_\perp)} \hat{\mathcal{M}}_{\rm DIS}(\vec{r}_\perp, \vec{R}_\perp), \tag{16}$$

$$G(R_\perp) = \int \frac{d^2\vec{k}_\perp}{(2\pi)^2} \frac{e^{i\vec{R}_\perp\cdot\vec{k}_\perp}}{k_\perp^2 + \lambda^2} = \frac{K_0(\lambda R_\perp)}{2\pi} \underset{\lambda\to 0}{\simeq} \frac{1}{2\pi}\log\left(\frac{1}{\lambda R_\perp}\right). \tag{17}$$

As already mentioned, in the case of DY production the infrared sensitivity shows up at the amplitude level through the phase shift $g^2 G(R_\perp)$. This is a direct consequence of the fact that the DY production amplitude of Fig. 1(b) involves the scattering of a charge instead of a dipole in DIS.

Since the phase factor in (16) has no effect on the total DY cross section, the present model for DIS and DY is consistent with the universality of the $K_\perp$-integrated quark distribution. In Ref. [13] it was shown that the nontrivial crossing (16) between DIS and DY has nevertheless interesting consequences. In particular the distribution $d\sigma/d^2\vec{k}_\perp$ in the Coulomb transfer $\vec{k}_\perp$ ($\vec{k}_\perp \neq \vec{K}_\perp$) is different in DIS and DY, i.e., nonuniversal, as we will recall in Sec. III B.

This result is not in contradiction with the universality of the quark distribution $f_{q/T}(x, K_\perp)$ since in the aligned-jet kinematics (1) $k_\perp$ is a variable internal to the target structure, integrated out in $f_{q/T}(x, K_\perp)$. Indeed, we show now that $f_{q/T}(x, K_\perp)$ is universal within the model of Fig. 1, in agreement with factorization theorems [14,15]. The differential DY cross section at fixed $K_\perp$ is obtained from (14),

$$(2\pi)^2 \frac{d\sigma_{\rm DY}}{d\log y\, d^2\vec{K}_\perp} = \frac{1}{4\pi} \int \frac{d^2\vec{p}_{2\perp}}{(2\pi)^2} \times |\hat{\mathcal{M}}_{\rm DY}(\vec{p}_{2\perp}, \vec{k}_\perp = \vec{p}_{2\perp} + \vec{K}_\perp)|^2. \tag{18}$$

Going to transverse coordinate space and using (16) we obtain

$$(2\pi)^2 \frac{d\sigma_{\rm DY}}{d\log y\, d^2\vec{K}_\perp} = \frac{1}{4\pi} \int d^2\vec{r}_\perp d^2\vec{R}_\perp d^2\vec{r}'_\perp d^2\vec{R}'_\perp \times \delta^{(2)}(\vec{r}_\perp + \vec{R}_\perp - \vec{r}'_\perp - \vec{R}'_\perp) \times e^{-i(\vec{R}_\perp - \vec{R}'_\perp)\cdot\vec{K}_\perp} e^{ig^2(G(R_\perp) - G(R'_\perp))} \times \hat{\mathcal{M}}_{\rm DIS}(\vec{r}_\perp, \vec{R}_\perp) \hat{\mathcal{M}}^*_{\rm DIS}(\vec{r}'_\perp, \vec{R}'_\perp). \tag{19}$$

Using the constraint from the delta function, we can rewrite the phase difference as

$$\begin{aligned} G(R_\perp) - G(R'_\perp) &= G(R_\perp) - G(|\vec{R}_\perp + \vec{r}_\perp|) - (G(R'_\perp) - G(|\vec{R}'_\perp + \vec{r}'_\perp|)) \\ &= W(\vec{r}_\perp, \vec{R}_\perp) - W(\vec{r}'_\perp, \vec{R}'_\perp), \end{aligned} \tag{20}$$

since $W(\vec{r}_\perp, \vec{R}_\perp) = G(R_\perp) - G(|\vec{R}_\perp + \vec{r}_\perp|)$. It is easy to see that in (19), the phase difference (20) can be absorbed in the expression of the DIS amplitude given by (2) and (8). After the change of variables $r \leftrightarrow r'$, $R \leftrightarrow R'$ we obtain

$$(2\pi)^2 \frac{d\sigma_{\rm DY}}{d\log y\, d^2\vec{K}_\perp} = \frac{1}{4\pi} \int d^2\vec{r}_\perp d^2\vec{R}_\perp d^2\vec{r}'_\perp d^2\vec{R}'_\perp \times \delta^{(2)}(\vec{r}_\perp + \vec{R}_\perp - \vec{r}'_\perp - \vec{R}'_\perp) \times e^{-i(\vec{R}'_\perp - \vec{R}_\perp)\cdot\vec{K}_\perp} \hat{\mathcal{M}}_{\rm DIS}(\vec{r}_\perp, \vec{R}_\perp) \times \hat{\mathcal{M}}^*_{\rm DIS}(\vec{r}'_\perp, \vec{R}'_\perp). \tag{21}$$

Interpreting the differential cross sections as $f_{q/T}(x, \vec{K}_\perp)$ we thus write

$$f^{\rm DY}_{q/T}(x, \vec{K}_\perp) = f^{\rm DIS}_{q/T}(x, -\vec{K}_\perp) = f^{\rm DIS}_{q/T}(x, \vec{K}_\perp), \tag{22}$$

where we used the fact that $f_{q/T}(x, \vec{K}_\perp)$ is a function of $K_\perp \equiv |\vec{K}_\perp|$ only. We stress that the universality found in (22) directly translates into observable quantities. Indeed, in DIS $\vec{K}_\perp = \vec{p}_{1\perp}$ [Fig. 1(a)] and in DY $\vec{K}_\perp = \vec{q}_\perp$ [Fig. 1(b)]. Thus in the present model the leading-twist DIS struck quark $p_{1\perp}$ distribution and the DY $q_\perp$ distribution are identical. We stress that in DY the target quark distribution is probed at $x_2 = K^+/p^+ \simeq q^+/p^+ = x_B$, i.e., $x_2 \ll 1$ in our model. As already mentioned in the introduction, we expect the *coherent* rescattering physics to come into play at $x_2 \lesssim 0.1$, where nuclear shadowing becomes quantitatively important.

One might ask whether the universality (22) would be preserved in a more realistic model for DY production with a *composite* projectile. The role of spectators in our model for DY production is studied in Appendix A. We find that spectator rescatterings do not affect the DY distribution $d\sigma_{\rm DY}/d^2\vec{q}_\perp$ [see the right-hand side of Eq. (A12) which only depends on the DIS amplitude]. The main consequence of using a composite projectile is to replace the infrared cutoff λ (the finite photon mass) by the inverse size δ of the projectile. We also show that the model with a composite projectile is consistent with factorization (see (A14) and Ref. [15]). Since spectator rescatterings do not lead to any breaking of universality between the DIS and DY $K_\perp$ distributions (at least in our SQED model), we will neglect (projectile) spectators in the following and use for DY production the model with a pointlike scalar projectile presented above.

It is worth recalling that the $K_\perp$ distribution[5] can be expressed in terms of the $q\bar{q}$ dipole cross section (see for instance [16]). From (21) and (2) we obtain

$$(2\pi)^2\frac{d\sigma}{d\log y\, d^2\vec{K}_\perp} = \int d^2\vec{r}_\perp d^2\vec{r}\,'_\perp e^{-i(\vec{r}_\perp-\vec{r}\,'_\perp)\cdot\vec{K}_\perp}\psi(r_\perp)\psi(r'_\perp) \times \int d^2\vec{R}_\perp T_{q\bar{q}}(\vec{r}_\perp,\vec{R}_\perp) \times T^*_{q\bar{q}}(\vec{r}\,'_\perp,\vec{R}_\perp+\vec{r}_\perp-\vec{r}\,'_\perp). \tag{23}$$

With the identity

$$T_{q\bar{q}}(\vec{r}_\perp,\vec{R}_\perp)T^*_{q\bar{q}}(\vec{r}\,'_\perp,\vec{R}_\perp+\vec{r}_\perp-\vec{r}\,'_\perp) = iT_{q\bar{q}}(\vec{r}_\perp,\vec{R}_\perp) - iT^*_{q\bar{q}}(\vec{r}\,'_\perp,\vec{R}_\perp+\vec{r}_\perp-\vec{r}\,'_\perp) - iT_{q\bar{q}}(\vec{r}_\perp-\vec{r}\,'_\perp,\vec{R}_\perp), \tag{24}$$

the expression (23) becomes [using (11)]

$$(2\pi)^2\frac{d\sigma}{d\log y\, d^2\vec{K}_\perp} = \int d^2\vec{r}_\perp d^2\vec{r}\,'_\perp e^{-i(\vec{r}_\perp-\vec{r}\,'_\perp)\cdot\vec{K}_\perp}\psi(r_\perp)\psi(r'_\perp) \times\left[\frac{1}{2}\sigma_{q\bar{q}}(r_\perp)+\frac{1}{2}\sigma_{q\bar{q}}(r'_\perp) - \frac{1}{2}\sigma_{q\bar{q}}(|\vec{r}_\perp-\vec{r}\,'_\perp|)\right]. \tag{25}$$

The latter expression can be found in [16] (in the more general case of finite y).

B. Nuclear target

The model for DIS and DY production on a pointlike target described in the previous section can be directly generalized to the case of several scattering centers. Since the DIS and DY amplitudes off a single center have been derived in the limit $x_B \ll 1$, they also should describe the production off a "nuclear target" in the limit where the nuclear radius R_A is kept smaller than the coherence length l_c,

$$R_A \ll l_c = \frac{1}{Mx_B}. \tag{26}$$

In this limit the production amplitudes are simply obtained from (2) and (16) by replacing the scattering potential on a single center $G(R_\perp)$ given in (17) by the scattering potential on A centers located at transverse[6] positions $\vec{x}_{i\perp}$,

$$G(R_\perp) = \int\frac{d^2\vec{k}_\perp}{(2\pi)^2}\frac{e^{i\vec{R}_\perp\cdot\vec{k}_\perp}}{k_\perp^2+\lambda^2} \longrightarrow G_A(\vec{R}_\perp) \equiv \sum_{i=1}^{A} G(|\vec{R}_\perp - \vec{x}_{i\perp}|). \tag{27}$$

Denoting

$$W_A(\vec{r}_\perp,\vec{R}_\perp) = G_A(\vec{R}_\perp) - G_A(\vec{R}_\perp+\vec{r}_\perp), \tag{28}$$

$$T^A_{q\bar{q}}(\vec{r}_\perp,\vec{R}_\perp) = -i(1-e^{-ig^2W_A(\vec{r}_\perp,\vec{R}_\perp)}), \tag{29}$$

we obtain for the DIS and DY amplitudes on the nuclear target A

$$\hat{\mathcal{M}}^A_{\rm DIS}(\vec{r}_\perp,\vec{R}_\perp) = \sqrt{4\pi}\psi(r_\perp)T^A_{q\bar{q}}(\vec{r}_\perp,\vec{R}_\perp), \tag{30}$$

$$\hat{\mathcal{M}}^A_{\rm DY}(\vec{r}_\perp,\vec{R}_\perp) = -e^{ig^2G_A(\vec{R}_\perp)}\hat{\mathcal{M}}^A_{\rm DIS}(\vec{r}_\perp,\vec{R}_\perp). \tag{31}$$

We can repeat the steps leading from (18) to (22) to realize that $\hat{\mathcal{M}}^A_{\rm DIS}$ and $\hat{\mathcal{M}}^A_{\rm DY}$ provide a model for the "nuclear" quark distribution function which is consistent with the universality of the $K_\perp$-dependent distribution

$$f^{\rm DY}_{q/A}(x,\vec{K}_\perp) = f^{\rm DIS}_{q/A}(x,-\vec{K}_\perp). \tag{32}$$

The physical content of $f^{\rm DIS}_{q/A} = f^{\rm DY}_{q/A}$ in terms of Coulomb rescatterings on the A static centers is depicted in Fig. 2.

We now evaluate the universal distribution $d\sigma^A/d^2\vec{K}_\perp$ by averaging over the positions of the scattering centers.

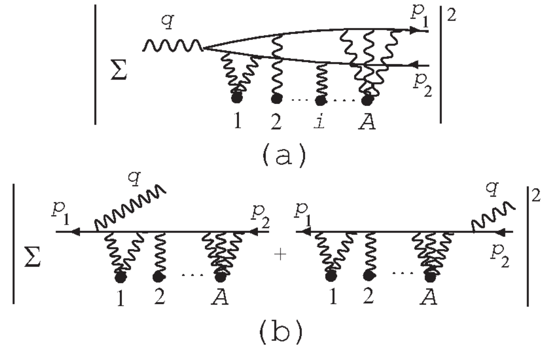

FIG. 2. Model for the DIS (a) and DY (b) nuclear quark distribution functions $f^{\rm DIS}_{q/A}(x,\vec{K}_\perp)$ and $f^{\rm DY}_{q/A}(x,\vec{K}_\perp)$. In DIS we have $\vec{K}_\perp = \vec{k}_\perp - \vec{p}_{2\perp} = \vec{p}_{1\perp}$ and in the DY case $\vec{K}_\perp = \vec{k}_\perp - \vec{p}_{2\perp} = \vec{q}_\perp$. The model takes into account any number of Coulomb rescatterings on every center. In the total coherence limit (26) the A centers only differ by their relative *transverse* positions.

[5]From now on we suppress the subscript DIS or DY for this universal distribution.

[6]In the total coherence limit (26) the longitudinal positions of the centers are irrelevant and only the thickness function T of the target enters (see below).

For our purpose it is sufficient to average with a uniform distribution,

$$\langle\ \rangle_A \equiv \int \prod_{i=1}^{A}\left(\frac{d^2\vec{x}_{i\perp}}{S}\right), \tag{33}$$

where $S = \int d^2\vec{x}$ is the transverse area of the target. Inserting (30) in (21) we find

$$\begin{aligned}(2\pi)^2\frac{d\sigma^A}{d\log y\, d^2\vec{K}_\perp} = &\int d^2\vec{r}_\perp d^2\vec{r}'_\perp e^{-i(\vec{r}_\perp-\vec{r}'_\perp)\cdot\vec{K}_\perp}\psi(r_\perp)\psi(r'_\perp)\\ &\times\int d^2\vec{R}_\perp\langle iT^A_{q\bar{q}}(\vec{r}_\perp,\vec{R}_\perp)\\ &-iT^A_{q\bar{q}}(\vec{r}'_\perp,\vec{R}'_\perp)^*\\ &-iT^A_{q\bar{q}}(\vec{r}_\perp-\vec{r}'_\perp,\vec{R}_\perp)\rangle_{A,\rho}\end{aligned} \tag{34}$$

where $\vec{R}'_\perp = \vec{R}_\perp + \vec{r}_\perp - \vec{r}'_\perp$. Using $W_A(\vec{r}_\perp, \vec{R}_\perp) = \sum_{i=1}^{A} W(\vec{r}_\perp, \vec{R}_\perp - \vec{x}_{i\perp})$ and the approximation[7] $\sigma_{q\bar{q}}(r_\perp) \ll S$ (valid when the nuclear radius is much larger than the interaction range λ^{-1}), we show using (11) that

$$\langle 1 - iT^A_{q\bar{q}}(\vec{r}_\perp, \vec{R}_\perp)\rangle_A = 0 \tag{35}$$

for $\vec{R}_\perp$ outside the nucleus, and

$$\begin{aligned}\langle 1 - iT^A_{q\bar{q}}(\vec{r}_\perp, \vec{R}_\perp)\rangle_A &= \langle e^{-ig^2W_A(\vec{r}_\perp,\vec{R}_\perp)}\rangle_A\\ &= \left[1-\frac{T}{A}\int d^2\vec{x}\, iT_{q\bar{q}}(\vec{r}_\perp, \vec{R}_\perp - \vec{x})\right]^A\\ &\underset{A\to\infty}{\longrightarrow} e^{-T\int d^2\vec{x}\, iT_{q\bar{q}}(\vec{r}_\perp,\vec{R}_\perp-\vec{x})}\\ &= e^{-T\sigma_{q\bar{q}}(r_\perp)/2},\end{aligned} \tag{36}$$

for $\vec{R}_\perp$ inside the nucleus, where $T = A/S$ is the target "thickness." We used the formal A, $S \to \infty$ limit at fixed T.

The average (33) of the $q\bar{q}$ dipole matrix element $S_{q\bar{q}} = 1 - iT_{q\bar{q}}$ resulting in the form (36) allows to interpret the rescattering process as a random walk in the transverse plane (see Ref. [11]).

The Eq. (34) becomes

$$\begin{aligned}(2\pi)^2\frac{d\sigma^A}{d\log y\, d^2\vec{K}_\perp} = S&\int d^2\vec{r}_\perp d^2\vec{r}'_\perp e^{-i(\vec{r}_\perp-\vec{r}'_\perp)\cdot\vec{K}_\perp}\psi(r_\perp)\\ &\times\psi(r'_\perp)\{1 - e^{-T\sigma_{q\bar{q}}(r_\perp)/2}\\ &- e^{-T\sigma_{q\bar{q}}(r'_\perp)/2} + e^{-T\sigma_{q\bar{q}}(|\vec{r}_\perp-\vec{r}'_\perp|)/2}\}.\end{aligned} \tag{37}$$

This corresponds to an "eikonalization" of the distribution off a single scattering center (25) (see also [16]). In the limit of vanishing thickness $T \to 0$, (37) reproduces (25) up to the factor A. Since $d\sigma^A/d^2\vec{K}_\perp$ is universal, we see that the DY $q_\perp$-distribution (and *a fortiori* the total DY cross section) can be expressed in terms of the $q\bar{q}$ dipole scattering cross section, as is the case for DIS [18].

III. EXTENSION OF THE MODEL TO THE SYMMETRIC KINEMATICAL REGIME

A. Symmetric kinematics and interpretation

In the preceding section we have focused on leading-twist contributions to the DIS and DY cross sections, arising from the aligned-jet region defined in (1). We would like to stress however that the validity of our calculations so far is not restricted to the aligned-jet domain. Since we only used $p_2^- \ll \nu$, they remain correct even when p_2^- scales with ν (i.e., in the "symmetric" kinematical region), provided the ratio $y = p_2^-/(2\nu)$ is fixed to a small finite value, i.e., in the region

$$\begin{gathered}p_1^- \sim \nu \to \infty \gg p_2^- \gg M \gg k_{i\perp}, p_{i\perp}, k_i^-, m \gg k_i^+ \sim Mx_B \gg p_1^+, p_2^+ \propto 1/\nu\\ \text{fixed } y = \frac{p_2^-}{2\nu} \ll 1 \Leftrightarrow p_2^- \text{ scales as } \nu.\end{gathered} \tag{38}$$

In fact our results can easily be extended to the domain where y is not small as compared to 1. For simplicity we concentrate on the $y \ll 1$ limit, but expect our following considerations to apply also to situations where $y \sim 1$, such as quarkonium ($y \simeq 1/2$) and dijet leptoproduction.

With the symmetric kinematics (38) the interpretation of the contributions to the DIS and DY cross sections we evaluated is modified.

(i) The antiquark of final momentum p_2 is now part of the hard subprocess, which reads $\gamma^* g \to q\bar{q}$ in DIS [Fig. 1(a)] and $\bar{q}g \to \bar{q}\gamma^*$ in DY [Fig. 1(b)]. The DIS and DY processes are now interpreted, respectively, as dijet leptoproduction and the associated production of a DY pair and a jet.

(ii) The transverse momentum transfer to the hard system now corresponds to the Coulomb $k_\perp$ exchange, i.e., not to $\vec{K}_\perp = \vec{k}_\perp - \vec{p}_{2\perp}$ any longer. In the case of DIS, $\vec{k}_\perp = \vec{p}_{1\perp} + \vec{p}_{2\perp}$ is the momentum imbalance between the jets. In DY, $\vec{k}_\perp = \vec{q}_\perp + \vec{p}_{2\perp}$ is the imbalance between the DY pair and the associated produced jet.

In the following we concentrate, in the region (38), on the DIS and DY $k_\perp$ distributions, and show that those are different.

In Sec. III B we show that the $k_\perp$ distribution on a single scattering center becomes nonuniversal beyond leading

[7] With this approximation and the uniform distribution (33), we can derive Glauber-like expressions without resorting to the full formalism of [17].

order in g^2. In particular higher order corrections to $d\sigma_{\rm DY}/d^2\vec{k}_\perp$ vanish in the $\lambda \to 0$ limit (at fixed $k_\perp$), contrary to DIS. In Sec. III C we use the model for a nuclear target presented in Sec. II B to investigate the target size dependence of $d\sigma^A/d^2\vec{k}_\perp$ and find that the strength of $k_\perp$ broadening depends on the process.

B. Production off a single center: nonuniversality of the $k_\perp$-dependent distribution

In this section we show that in the symmetric kinematics (38) the DIS (dijet) and DY (+ jet) $k_\perp$ distributions are different. We first consider the DIS model presented in Sec. II A. From (14) the differential DIS cross section *at fixed* $k_\perp$ reads

$$(2\pi)^2 \frac{d\sigma_{\rm DIS}}{d\log y\, d^2\vec{k}_\perp} = \frac{1}{4\pi}\int \frac{d^2\vec{p}_{2\perp}}{(2\pi)^2} |\hat{\mathcal{M}}_{\rm DIS}(\vec{p}_{2\perp}, \vec{k}_\perp)|^2 = \frac{1}{4\pi}\int d^2\vec{r}_\perp |\hat{\mathcal{M}}_{\rm DIS}(\vec{r}_\perp, \vec{k}_\perp)|^2, \quad (39)$$

where

$$\hat{\mathcal{M}}_{\rm DIS}(\vec{r}_\perp, \vec{k}_\perp) = \int d^2\vec{R}_\perp e^{-i\vec{R}_\perp\cdot\vec{k}_\perp} \hat{\mathcal{M}}_{\rm DIS}(\vec{r}_\perp, \vec{R}_\perp). \quad (40)$$

From (2) we obtain

$$\frac{d\sigma_{\rm DIS}}{d\log y\, d^2\vec{k}_\perp} = \int d^2\vec{r}_\perp |\psi(r_\perp)|^2 \frac{d\sigma_{q\bar{q}}(\vec{r}_\perp, \vec{k}_\perp)}{d^2\vec{k}_\perp}, \quad (41)$$

where we defined

$$\frac{d\sigma_{q\bar{q}}(\vec{r}_\perp, \vec{k}_\perp)}{d^2\vec{k}_\perp} \equiv \frac{|T_{q\bar{q}}(\vec{r}_\perp, \vec{k}_\perp)|^2}{(2\pi)^2}. \quad (42)$$

Let us turn to the case of the DY process,

$$(2\pi)^2 \frac{d\sigma_{\rm DY}}{d\log y\, d^2\vec{k}_\perp} = \frac{1}{4\pi}\int d^2\vec{r}_\perp |\hat{\mathcal{M}}_{\rm DY}(\vec{r}_\perp, \vec{k}_\perp)|^2, \quad (43)$$

where $\hat{\mathcal{M}}_{\rm DY}(\vec{r}_\perp, \vec{k}_\perp)$ can be shown using (2) and (16) to be proportional to its own Born value,

$$\hat{\mathcal{M}}_{\rm DY}(\vec{r}_\perp, \vec{k}_\perp) = -\int d^2\vec{R}_\perp e^{-i\vec{R}_\perp\cdot\vec{k}_\perp} e^{ig^2 G(R_\perp)} \hat{\mathcal{M}}_{\rm DIS}(\vec{r}_\perp, \vec{R}_\perp) \quad (44)$$

$$= C(k_\perp^2) \hat{\mathcal{M}}^{\rm Born}_{\rm DY}(\vec{r}_\perp, \vec{k}_\perp), \quad (45)$$

where we define

$$C(k_\perp^2) = \frac{k_\perp^2 + \lambda^2}{ig^2}\int d^2\vec{R}_\perp e^{ig^2 G(R_\perp)} e^{-i\vec{R}_\perp\cdot\vec{k}_\perp}, \quad (46)$$

$$\hat{\mathcal{M}}^{\rm Born}_{\rm DY}(\vec{r}_\perp, \vec{k}_\perp) = -\sqrt{4\pi}\psi(r_\perp) T^{\rm Born}_{q\bar{q}}(\vec{r}_\perp, \vec{k}_\perp), \quad (47)$$

$$T^{\rm Born}_{q\bar{q}}(\vec{r}_\perp, \vec{k}_\perp) = -2ig^2 \frac{\sin(\vec{r}_\perp\cdot\vec{k}_\perp/2)}{k_\perp^2 + \lambda^2} e^{i\vec{r}_\perp\cdot\vec{k}_\perp/2}. \quad (48)$$

Using (45) and (47), and (42) we find

$$\frac{d\sigma_{\rm DY}}{d\log y\, d^2\vec{k}_\perp} = |C(k_\perp^2)|^2 \int d^2\vec{r}_\perp |\psi(r_\perp)|^2 \frac{d\sigma^{\rm Born}_{q\bar{q}}(\vec{r}_\perp, \vec{k}_\perp)}{d^2\vec{k}_\perp}. \quad (49)$$

This can be reexpressed as

$$(2\pi)^2 \frac{d\sigma_{\rm DY}}{d\log y\, d^2\vec{k}_\perp} = \int d^2\vec{r}_\perp |\psi(r_\perp)|^2 4\sin^2\left(\frac{\vec{r}_\perp\cdot\vec{k}_\perp}{2}\right) \times \int d^2\vec{b} e^{i\vec{b}\cdot\vec{k}_\perp}\left[-\frac{1}{2}\sigma_{q\bar{q}}(b)\right] \quad (50)$$

by using (46) and the identity

$$\left|\int d^2\vec{R}_\perp e^{ig^2 G(R_\perp)} e^{-i\vec{R}_\perp\cdot\vec{k}_\perp}\right|^2 = \int d^2\vec{b} e^{i\vec{b}\cdot\vec{k}_\perp} \int d^2\vec{R}_\perp (iT^*_{q\bar{q}}(\vec{b}, \vec{R}_\perp) + 1). \quad (51)$$

Using (49) and (50) we also obtain

$$|C(k_\perp^2)|^2 = \frac{\int d^2\vec{b} e^{i\vec{b}\cdot\vec{k}_\perp}\sigma_{q\bar{q}}(b)}{\int d^2\vec{b} e^{i\vec{b}\cdot\vec{k}_\perp}\sigma^{\rm Born}_{q\bar{q}}(b)}. \quad (52)$$

Comparing (41) and (49) it is clear that the $k_\perp$ distribution is process dependent. We can stress this point by noting that for $k_\perp \gg \lambda$, the DY distribution (49) equals the Born distribution, contrary to the DIS distribution. Let us consider the $k_\perp \to \infty$ limit at fixed λ. In the expression of $|C(k_\perp^2)|^2$ given in (52), the phase factor $e^{i\vec{b}\cdot\vec{k}_\perp}$ rapidly oscillates, except if $b \lesssim 1/k_\perp \to 0$. When $b \to 0$ the dipole cross section can be approximated by its Born value [see (13)]. Thus when $k_\perp \gg \lambda$ we have $|C(k_\perp^2)|^2 \simeq 1$, yielding

$$\left.\frac{d\sigma_{\rm DY}}{d\log y\, d^2\vec{k}_\perp}\right|_{k_\perp\gg\lambda} = \int d^2\vec{r}_\perp |\psi(r_\perp)|^2 \frac{d\sigma^{\rm Born}_{q\bar{q}}(\vec{r}_\perp, \vec{k}_\perp)}{d^2\vec{k}_\perp} = \frac{d\sigma^{\rm Born}_{\rm DY}}{d\log y\, d^2\vec{k}_\perp}. \quad (53)$$

We can obtain this latter equation more rigorously by using (49) and calculating $C(k_\perp^2)$ given in (46) for $\lambda \to 0$, the other scales being fixed. Using $G(R_\perp) \simeq -\log(\lambda R_\perp)/(2\pi)$ we get

$$C(k_\perp^2) \underset{\lambda\to 0}{\sim} -\frac{\Gamma(-\frac{ig^2}{4\pi})}{\Gamma(\frac{ig^2}{4\pi})} e^{(ig^2/2\pi)(\log(\frac{k_\perp}{\lambda})-\gamma)}[1 + \mathcal{O}(\lambda)] \Rightarrow |C(k_\perp^2)| = 1. \quad (54)$$

Thus $C(k_\perp^2 \gg \lambda^2)$ is a pure phase factor, leading to (53), which confirms to all orders the result shown in [13] at next-to-leading order in g^2.

Since the $k_\perp$-*integrated* DIS and DY cross sections are identical (and different from their Born value), the result (53) implies that in the $\lambda \to 0$ limit and in the DY case, only vanishing $k_\perp \sim \lambda \to 0$ contributes to $\Delta\sigma \equiv \sigma^{\rm tot} - \sigma^{\rm Born}$. This difference observed between the $k_\perp$-dependent DIS and DY cross sections is similar to what Bethe and Maximon found in the case of high energy pair production and bremsstrahlung [19]. In the present context the effect clearly arises from the infrared divergent Coulomb phase (for $\lambda \to 0$) in the DY production amplitude, and suggests that in collisions on a proton, the $k_\perp$ exchange in DY +jet production might be smaller than in dijet leptoproduction, and thus not ''intrinsic'' to the target.

In a realistic situation, when the projectile and target are composite and neutral, the DY production amplitude is infrared finite. Effectively, the role of the infrared cutoff λ is played by the *largest* infrared momentum cutoff at disposal, given by the inverse size δ of the smallest incoming hadron, projectile, or target. We expect the typical transverse momentum contributing to $\Delta\sigma_{\rm DY}$ to be $k_\perp^2 \sim \delta^2$ instead of $k_\perp^2 \sim \lambda^2$, as discussed in the end of Appendix A.

In the next section, we investigate the nonuniversality of the $k_\perp$ distribution in the case of a nuclear target.

C. Nuclear target: nonuniversality of $k_\perp$ broadening

The nuclear target model of Sec. II B is now used to derive the target size dependence of the DIS and DY $k_\perp$ distributions, where $k_\perp$ is the total transverse Coulomb exchange. We have shown that the distribution in $\vec{K}_\perp = \vec{k}_\perp - \vec{p}_{2\perp}$ is universal [see (32)], and this holds in the two kinematical domains (1) and (38).

On the contrary, we now explicitly show that the distribution in the transverse Coulomb exchange $d\sigma^A/d^2\vec{k}_\perp$ is process dependent. Similarly to the case of a single scattering center, this can easily be guessed from Eq. (31). The DIS and DY amplitudes differ by a pure phase factor in $R_\perp$ space, but there is no reason to expect this to hold in the space of the conjugate variable $k_\perp$.

Inserting (30) in (39) we get

$$\frac{d\sigma^A_{\rm DIS}}{d\log y\, d^2\vec{k}_\perp} = \int d^2\vec{r}_\perp |\psi(r_\perp)|^2 \frac{d\sigma^A_{q\bar{q}}(\vec{r}_\perp, \vec{k}_\perp)}{d^2\vec{k}_\perp}, \quad (55)$$

with

$$\frac{d\sigma^A_{q\bar{q}}}{d^2\vec{k}_\perp} = \frac{1}{(2\pi)^2}\int d^2\vec{R}_\perp d^2\vec{R}'_\perp e^{-i(\vec{R}_\perp - \vec{R}'_\perp)\cdot\vec{k}_\perp} \times \langle 1 - e^{-ig^2 W_A(\vec{r}_\perp,\vec{R}_\perp)} - e^{ig^2 W_A(\vec{r}_\perp,\vec{R}'_\perp)} + e^{-ig^2(W_A(\vec{r}_\perp,\vec{R}_\perp) - W_A(\vec{r}_\perp,\vec{R}'_\perp))}\rangle_A \quad (56)$$

representing the differential elastic scattering cross section of a $q\bar{q}$ dipole of size $r_\perp$ on a nuclear target. The form (55) emphasizes the decoupling between production and rescattering of the $q\bar{q}$ dipole in the process. The average over the positions of the scattering centers leads to

$$\frac{d\sigma^A_{q\bar{q}}}{d^2\vec{k}_\perp} = \frac{S}{(2\pi)^2}\int d^2\vec{b} e^{i\vec{b}\cdot\vec{k}_\perp}\{1 - 2e^{-T\sigma_{q\bar{q}}(r_\perp)/2} + e^{-T\sigma_{q\bar{q}}(r_\perp)} e^{T\int d^2\vec{x} T_{q\bar{q}}(\vec{r}_\perp,\vec{x}) T^*_{q\bar{q}}(\vec{r}_\perp,\vec{x}+\vec{b})}\}. \quad (57)$$

Using (42) we eventually obtain

$$(2\pi)^2 \frac{d\sigma^A_{\rm DIS}}{d\log y\, d^2\vec{k}_\perp} = S\int d^2\vec{r}_\perp |\psi(r_\perp)|^2 \times \int d^2\vec{b} e^{i\vec{b}\cdot\vec{k}_\perp}\{1 - 2e^{-T\sigma_{q\bar{q}}(r_\perp)/2} + e^{-T\int d^2\vec{l}(1-e^{-i\vec{b}\cdot\vec{l}})[d\sigma_{q\bar{q}}(\vec{r}_\perp,\vec{l})/d^2\vec{l}]}\}. \quad (58)$$

A similar calculation for DY production is performed inserting the Fourier transform of (31) into (43),

$$(2\pi)^2 \frac{d\sigma^A_{\rm DY}}{d\log y\, d^2\vec{k}_\perp} = \int d^2\vec{r}_\perp |\psi(r_\perp)|^2 \left\langle \left| \int d^2\vec{R}_\perp (e^{ig^2 G_A(R_\perp)} - e^{ig^2 G_A(|\vec{R}_\perp + \vec{r}_\perp|)}) e^{-i\vec{R}_\perp\cdot\vec{k}_\perp} \right|^2 \right\rangle_A = \int d^2\vec{r}_\perp |\psi(r_\perp)|^2 \left\langle \left| (1 - e^{i\vec{k}_\perp\cdot\vec{r}_\perp}) \int d^2\vec{R}_\perp e^{ig^2 G_A(R_\perp) - i\vec{R}_\perp\cdot\vec{k}_\perp} \right|^2 \right\rangle_A, \quad (59)$$

giving

$$\frac{d\sigma^A_{\rm DY}}{d\log y\, d^2\vec{k}_\perp} = \frac{d\sigma^A_{q}}{d^2\vec{k}_\perp}\int d^2\vec{r}_\perp |\psi(r_\perp)|^2 4\sin^2\left(\frac{\vec{r}_\perp\cdot\vec{k}_\perp}{2}\right), \quad (60)$$

where $d\sigma^A_q/d^2\vec{k}_\perp$ is the differential elastic scattering cross section of a quark on a nuclear target in our model, i.e.:[8]

$$\left.\frac{d\sigma^A_q}{d^2\vec{k}_\perp}\right|_{k_\perp\neq 0} = \frac{1}{(2\pi)^2}\int d^2\vec{R}_\perp d^2\vec{R}'_\perp e^{-i(\vec{R}_\perp - \vec{R}'_\perp)\cdot\vec{k}_\perp} \times \langle e^{-ig^2(G_A(R'_\perp) - G_A(R_\perp))}\rangle_A. \quad (61)$$

We perform the average using $G_A(R'_\perp) - G_A(R_\perp) = W_A(\vec{R}_\perp - \vec{R}'_\perp, \vec{R}'_\perp)$ and (36):

$$\left.\frac{d\sigma^A_q}{d^2\vec{k}_\perp}\right|_{k_\perp\neq 0} = \frac{S}{(2\pi)^2}\int d^2\vec{b} e^{i\vec{b}\cdot\vec{k}_\perp} e^{-T\sigma_{q\bar{q}}(b)/2}, \quad (62)$$

[8]We neglect contributions $\sim\delta^{(2)}(\vec{k}_\perp)$ which do not contribute to $d\sigma^A_{\rm DY}/d^2\vec{k}_\perp$ due to the factor $\sin^2(\vec{r}_\perp\cdot\vec{k}_\perp/2)$ in (60).

finally leading to

$$(2\pi)^2 \frac{d\sigma^A_{\rm DY}}{d\log y\, d^2\vec{k}_\perp} = S \int d^2\vec{r}_\perp |\psi(r_\perp)|^2 4\sin^2\Big(\frac{\vec{r}_\perp \cdot \vec{k}_\perp}{2}\Big) \times \int d^2\vec{b} e^{i\vec{b}\cdot\vec{k}_\perp} e^{-T\sigma_{q\bar{q}}(b)/2}. \tag{63}$$

In line with the above discussion for DIS, (63) exhibits some decoupling between production and rescattering, up to the factor $4\sin^2(\vec{r}_\perp \cdot \vec{k}_\perp/2)$ specific to DY production. Notice that despite the formal appearance of $\sigma_{q\bar{q}}(b)$ in its expression (62), $d\sigma^A_q/d^2\vec{k}_\perp$ represents *monopole* elastic scattering,[9] in contradistinction with the dipole scattering cross section (57) appearing in DIS.

Comparing (55) and (60) we interpret the nonuniversality of $d\sigma^A/d^2\vec{k}_\perp$ in our model as a direct consequence of the type of object which interacts with the nuclear target. In the $r_\perp$ integral giving $d\sigma^A_{\rm DIS}/d^2\vec{k}_\perp$, the dipole wave function ψ selects $r_\perp \lesssim 1/m_{||}$ in $d\sigma^A_{q\bar{q}}/d^2\vec{k}_\perp$. Therefore, the hard scale[10] $m_{||}$ enters the physics of rescattering and is expected to play a major role for $k_\perp$ broadening. Conversely, in (60) the scale $m_{||}$ enters $d\sigma^A_{\rm DY}/d^2\vec{k}_\perp$ only through a target-independent factor and is therefore not expected to govern $k_\perp$ broadening.

In order to display the differences between DIS and DY we investigate the ratio

$$R(k_\perp) = \frac{1}{A}\frac{d\sigma^A}{d^2\vec{k}_\perp}\Big/\frac{d\sigma^p}{d^2\vec{k}_\perp}, \tag{64}$$

where σ^p and σ^A denote the DIS or DY cross sections off a single scattering center and on A centers, to leading order in g^2 and next-to-leading order in the target thickness[11] $T = A/S$. By expanding (58) and (63) we get, for $\lambda \ll k_\perp \ll m_{||}$ and keeping only the leading logarithms:

$$R_{\rm DIS}(k_\perp) = 1 - \frac{4g^4T}{5\pi m_{||}^2}\log\frac{m_{||}}{k_\perp} + \mathcal{O}(T^2), \tag{65}$$

$$R_{\rm DY}(k_\perp) = 1 + \frac{2g^4T}{\pi k_\perp^2}\log\frac{k_\perp}{\lambda} + \mathcal{O}(T^2). \tag{66}$$

The latter results explicitly demonstrate the nonuniversality of the $k_\perp$-dependent distributions which was already apparent when comparing the full expressions (55) and (60). As compared to the production on a single center, and in the region under consideration $\lambda \ll k_\perp \ll m_{||}$, $d\sigma^A/d^2\vec{k}_\perp$ is slightly reduced in DIS and strongly enhanced in DY production. In the DY case, we recall that for a single scattering center (see Sec. III B), the $k_\perp$ distribution for $k_\perp \gg \lambda$ equals that at Born level, i.e., in the absence of rescattering [see Eq. (53)]. This might have suggested $k_\perp$ broadening to be reduced in DY as compared to DIS. The explicit calculation with a finite size target of thickness T shows that this does not happen: the deviation from unity of the ratio $R_{\rm DY}(k_\perp)$ survives the $\lambda \to 0$ limit.

It is simple to realize that the $k_\perp$ broadening defined as

$$\Delta\langle k_\perp^2\rangle \equiv \langle k_\perp^2\rangle_A - \langle k_\perp^2\rangle_p, \tag{67}$$

although different in DIS and DY, scales as g^4T both in DIS and DY. Evaluating $\Delta\langle k_\perp^2\rangle$ from (65) and (66) (and using $d\sigma^p/d^2\vec{k}_\perp \propto 1/k_\perp^2$) puts some light on the difference between DIS and DY. In DIS the small probability $\sim T/m_{||}^2$ for the rescattering of a dipole of size $\sim 1/m_{||}$ is compensated by a large typical momentum transfer $k_\perp^2 \sim m_{||}^2$. In DY the hard scale $m_{||}$ does not enter the expression of $R(k_\perp)$, and the relatively large (monopole) rescattering probability $\sim T/k_\perp^2$ is now compensated by a small typical transfer $k_\perp^2 \ll m_{||}^2$. As already discussed, the nonuniversality of nuclear $k_\perp$ broadening in the coherent limit studied in our model is a natural consequence of the type of object (dipole or monopole) which interacts with the target.

IV. SUMMARY AND OUTLOOK

Within an explicit scalar QED model, we have studied transverse momentum distributions in the coherent limit $x \ll 1$ for DIS and the DY process. In the aligned-jet kinematics, where the leading quark or the DY pair carries most of the projectile momentum, the distribution in the transverse momentum $K_\perp$ of these particles is universal, both for pointlike and extended targets. This is consistent with the universality of the $K_\perp$-dependent target quark distribution. On the contrary, in the symmetric kinematical region, the relevant transverse momentum $k_\perp$ ($k_\perp \neq K_\perp$) is that of the hard subsystem, i.e., of the quark-antiquark pair of the photon fluctuation in the case of DIS, and of the DY virtual photon and the final antiquark in the case of DY. The transverse momentum transfer $k_\perp$ between the target and the hard subsystem is different in DIS and DY already for pointlike targets. The extension to a finite size nuclear target stresses the physical origin of this difference. In DIS the $q\bar{q}$ dipole rescatters with a small probability but undergoes large $k_\perp$ kicks, whereas the DY $k_\perp$ distribution is sensitive to *monopole* rescattering, more likely but involving smaller kicks.

We stress that our nuclear transverse momentum distributions are expressed in terms of the dipole cross section $\sigma_{q\bar{q}}$ and of the thickness function T, and contain factors of

[9] This point is discussed in detail in [11].

[10] In the symmetric kinematics (38) we have $m_{||}^2 = yQ^2 + m^2 \simeq yQ^2$.

[11] Note that the limit of small target thickness $T \propto A^{1/3}$ is consistent with the total coherence limit $R_A \ll l_c$ [see (26)] in which our DIS and DY amplitudes (30) and (31) have been derived.

the type $1 - e^{-T\sigma}$ which are expected for classical scattering. This behavior is a consequence of the statistical average we have performed on the positions of the scattering centers. Thus also in the coherent region rescattering off an extended target turns out to have the nature of stochastic multiple scattering [20]. Coherence is important in that it fixes the nature of the object (dipole or monopole) that rescatters.

The nonuniversality of the $k_\perp$ distribution found in our scalar QED model in the coherent small x limit calls for a systematic study of the nuclear $k_\perp$ broadening measured in fixed-target experiments ($x \sim 0.1$) or at the relativistic heavy ion collider ($x \sim 0.01$). Indeed, at the present stage we cannot give a quantitative answer to the puzzling observation mentioned in the introduction, namely, the smallness of transverse momentum broadening in DY production, as compared for instance to the broadening of the dijet momentum imbalance in dijet photoproduction. We however emphasize that with our notations the observed transverse momentum is $q_\perp = K_\perp$ in DY production instead of $k_\perp$ in dijet photoproduction (analogous to our DIS process in the symmetric kinematics). We have shown that $d\sigma_{\rm DIS}/d^2\vec{k}_\perp \neq d\sigma_{\rm DY}/d^2\vec{k}_\perp$, thus it is even more natural to expect the distributions $d\sigma_{\rm DIS}/d^2\vec{k}_\perp$ and $d\sigma_{\rm DY}/d^2\vec{q}_\perp$ with respect to *distinct* transverse momentum variables to be different in the coherent regime. This actually can be checked explicitly in our SQED model. The universal ratio

$$R(K_\perp) = \frac{1}{A}\frac{d\sigma^A}{d^2\vec{K}_\perp}\Big/\frac{d\sigma^p}{d^2\vec{K}_\perp} \tag{68}$$

can be obtained from (37) and (25), and reads at leading order in g^2 and for $K_\perp \ll m_{||}$:

$$\begin{aligned} R(K_\perp) &= R_{\rm DY}(q_\perp) = R_{\rm DIS}(p_{1\perp}) \\ &= 1 + \frac{2g^4T}{\pi m_{||}^2}\log^2\left(\frac{m_{||}}{\lambda}\right) + \mathcal{O}(T^2). \end{aligned} \tag{69}$$

We see that contrary to the "dijet" leptoproduction ratio $R_{\rm DIS}(k_\perp)$ given in (65), the ratio $R_{\rm DY}(q_\perp \ll m_{||})$ exceeds unity. Small $q_\perp$'s are favored in DY production off a nucleus. Whether the latter result, obtained in our Abelian model, can explain the observed smallness of $q_\perp$ broadening in the hadronic world will be addressed in a future work.

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APPENDIX A: DY PRODUCTION WITH COMPOSITE PROJECTILE

Here we study the role of spectators in DY production. For this purpose we extend the model of Fig. 1(b) to the case of a composite projectile. The corresponding model is depicted in Fig. 3.

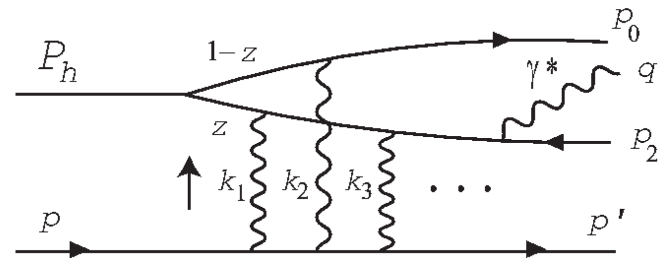

FIG. 3. Model for DY production with a composite projectile. The spectator is produced with final momentum p_0.

The fluctuation of the "hadron" projectile of mass m_h and momentum P_h into the spectator quark (of final momentum p_0) and the active antiquark (of final momentum p_2), is described by a scalar cubic coupling ig_0. The projectile is chosen to carry the electromagnetic charge e but to be neutral with respect to the strong interaction of coupling g. The spectator quark has no electric charge but strong charge g. The large incoming light-cone momentum P_h^- splits into the active antiquark and spectator momenta with finite fractions z and $1 - z$. The different momenta appearing in Fig. 3 read (we choose $m_h < 2m$ in order to forbid the $h \to q\bar{q}$ decay):

$$\begin{aligned} P_h &= \left(\frac{m_h^2}{2\nu}, 2\nu, \vec{0}_\perp\right); \qquad p = (M, M, \vec{0}_\perp), \\ p_0 &= \left(\frac{p_{0\perp}^2 + m^2}{(1-z)2\nu}, (1-z)2\nu, \vec{p}_{0\perp}\right); \\ p_2 &= \left(\frac{p_{2\perp}^2 + m^2}{p_2^-}, p_2^-, \vec{p}_{2\perp}\right), \\ q &= \left(\frac{Q^2 + q_\perp^2}{z2\nu}, z2\nu, \vec{q}_\perp\right) \simeq (Mx_B, z2\nu, \vec{q}_\perp), \end{aligned} \tag{A1}$$

where now $x_B = Q^2/(z2M\nu)$. We will use again the limit $x_B \ll 1$ as well as the kinematics defined in (1).

The DY virtual photon can be radiated either by the projectile or by the active antiquark, both having electric charge e. However, in the kinematics (1), the typical times associated to the fluctuations $h \to q\bar{q}$ and $\bar{q} \to \gamma^* \bar{q}$ are, respectively, of order $\nu/p_{0\perp}^2 \to \infty$ and $1/Mx_B$. Thus the photon is radiated *after* the $h \to q\bar{q}$ fluctuation. The diagrams where the virtual photon is emitted from the projectile are suppressed (in Feynman gauge) by a factor $\sim p_{0\perp}^2/(\nu M x_B) \sim \mathcal{O}(p_2^-/\nu)$ according to (1).

1. Consistency with factorization

The covariant calculation of the DY production amplitude of Fig. 3 is similar to the calculations performed in Refs. [12,13]. The leading-twist contribution is obtained for a virtual photon with longitudinal polarization

$$\epsilon_L = \left(\frac{Q}{z\nu}, -\frac{Q}{z\nu}, \vec{0}_\perp\right). \tag{A2}$$

Going to transverse coordinate space,

$$\mathcal{M}(\vec{r}_\perp, \vec{R}_\perp, \vec{u}_\perp) = \int \frac{d^2\vec{p}_{2\perp} d^2\vec{k}_\perp d^2\vec{p}_{0\perp}}{(2\pi)^6} \mathcal{M}(\vec{p}_{2\perp}, \vec{k}_\perp, \vec{p}_{0\perp}) \times e^{i(\vec{r}_\perp\cdot\vec{p}_{2\perp} + \vec{R}_\perp\cdot\vec{k}_\perp + \vec{u}_\perp\cdot\vec{p}_{0\perp})}, \tag{A3}$$

and resumming Coulomb scatterings yields the result

$$\mathcal{M}_{\rm DY}(\vec{r}_\perp, \vec{R}_\perp, \vec{u}_\perp) = \mathcal{M}_{\rm DY}(\vec{r}_\perp, \vec{R}_\perp) e^{-ig^2 G(|\vec{R}_\perp + \vec{u}_\perp|)} \frac{\phi(z, u_\perp)}{z}. \tag{A4}$$

Here $\mathcal{M}_{\rm DY}(\vec{r}_\perp, \vec{R}_\perp)$ is the DY production amplitude in the absence of spectator obtained from (3) and (16), the function G is defined in (17) and $\phi(z, u_\perp)$ is the $h \to q\bar{q}$ wave function

$$\phi(z, u_\perp) = g_0 z(1-z) V(\delta u_\perp), \tag{A5}$$

$$\delta^2 = m^2 - z(1-z) m_h^2, \tag{A6}$$

which can be represented as

$$\phi(z, u_\perp) = \int \frac{d^2\vec{p}_{0\perp}}{(2\pi)^2} \phi(z, p_{0\perp}) e^{i\vec{u}_\perp\cdot\vec{p}_{0\perp}}, \tag{A7}$$

$$\phi(z, p_{0\perp}) = g_0 \frac{z(1-z)}{p_{0\perp}^2 + \delta^2}. \tag{A8}$$

The phase factor in (A4) arises from Coulomb rescatterings of the spectator quark. Those indeed contribute to the DY production amplitude of Fig. 3, since vanishingly small light-cone energies $k_i^+ \propto 1/\nu$ can be transferred to the spectator without any cost. The finite energy $k^+ = \sum k_i^+ \sim \mathcal{O}(Mx_B)$ is transferred to the active antiquark in order to produce the final state invariant mass $\sim Q^2$. The phase in (A4) is infrared divergent (the spectator carries the charge g) but this divergence compensates that appearing in (16), as expected for dipole rescattering

$$\mathcal{M}_{\rm DY}(\vec{r}_\perp, \vec{R}_\perp, \vec{u}_\perp) = -e^{ig^2 W(\vec{u}_\perp, \vec{R}_\perp)} \frac{\phi(z, u_\perp)}{z} \times \mathcal{M}_{\rm DIS}(\vec{r}_\perp, \vec{R}_\perp), \tag{A9}$$

where we used $G(R_\perp) - G(|\vec{R}_\perp + \vec{u}_\perp|) = W(\vec{u}_\perp, \vec{R}_\perp)$.

We now proceed as in Sec. II [see Eq. (18) and following]. In the presence of the spectator the differential DY cross section is of the form[12]

$$\frac{d\sigma_{\rm DY}}{d^2\vec{q}_\perp} \propto \int \frac{d^2\vec{p}_{2\perp}}{(2\pi)^2} \frac{d^2\vec{p}_{0\perp}}{(2\pi)^2} \times |\mathcal{M}_{\rm DY}(\vec{p}_{2\perp}, \vec{k}_\perp = \vec{p}_{2\perp} + \vec{p}_{0\perp} + \vec{q}_\perp, \vec{p}_{0\perp})|^2. \tag{A10}$$

Going to transverse coordinate space and using (A4) leads to

$$\frac{d\sigma_{\rm DY}}{d^2\vec{q}_\perp} \propto \int d^2\vec{r}_\perp d^2\vec{R}_\perp d^2\vec{u}_\perp d^2\vec{r}'_\perp d^2\vec{R}'_\perp d^2\vec{u}'_\perp \phi(z, u_\perp)\phi^*(z, u'_\perp) \delta^{(2)}(\vec{r}_\perp + \vec{R}_\perp - \vec{r}'_\perp - \vec{R}'_\perp)\delta^{(2)}(\vec{u}_\perp + \vec{R}_\perp - \vec{u}'_\perp - \vec{R}'_\perp) \times e^{ig^2(G(R_\perp) - G(R'_\perp))} e^{-i(G(|\vec{R}_\perp + \vec{u}_\perp|) - G(|\vec{R}'_\perp + \vec{u}'_\perp|))} e^{-i(\vec{R}_\perp - \vec{R}'_\perp)\cdot\vec{q}_\perp} \mathcal{M}_{\rm DIS}(\vec{r}_\perp, \vec{R}_\perp)\mathcal{M}^*_{\rm DIS}(\vec{r}'_\perp, \vec{R}'_\perp). \tag{A11}$$

From the δ constraints the Coulomb phase associated to spectator rescattering cancels out, $G(|\vec{R}_\perp + \vec{u}_\perp|) - G(|\vec{R}'_\perp + \vec{u}'_\perp|) \to 0$, and the remaining phase difference $G(R_\perp) - G(R'_\perp)$ is absorbed in the expression of $\mathcal{M}_{\rm DIS}$ given by (2) and (8), leading to

$$\frac{d\sigma_{\rm DY}}{d^2\vec{q}_\perp} \propto \int d^2\vec{r}_\perp d^2\vec{R}_\perp d^2\vec{r}'_\perp d^2\vec{R}'_\perp \delta^{(2)}(\vec{r}_\perp + \vec{R}_\perp - \vec{r}'_\perp - \vec{R}'_\perp) e^{-i(\vec{R}'_\perp - \vec{R}_\perp)\cdot\vec{q}_\perp} \mathcal{M}_{\rm DIS}(\vec{r}_\perp, \vec{R}_\perp)\mathcal{M}^*_{\rm DIS}(\vec{r}'_\perp, \vec{R}'_\perp) \times \int d^2\vec{u}_\perp \phi(z, u_\perp)\phi^*(z, |\vec{u}_\perp + \vec{R}_\perp - \vec{R}'_\perp|). \tag{A12}$$

Thus spectator Coulomb rescattering does not affect $d\sigma_{\rm DY}/d^2\vec{q}_\perp$ (and *a fortiori* not the total leading-twist DY cross section either), which is consistent with factorization, as we briefly see now.

From (A7) one gets

$$\int d^2\vec{u}_\perp \phi(z, u_\perp)\phi^*(z, |\vec{u}_\perp + \vec{R}_\perp - \vec{R}'_\perp|) = \int \frac{d^2\vec{p}_{0\perp}}{(2\pi)^2} |\phi(z, p_{0\perp})|^2 e^{-i(\vec{R}'_\perp - \vec{R}_\perp)\cdot\vec{p}_{0\perp}}. \tag{A13}$$

[12]For the purposes of the present appendix we do not need to specify the normalization of differential cross sections in the following.

Inserting this into (A12) and identifying $\int d^2\vec{p}_{0\perp}|\phi(z, p_{0\perp})|^2$ with the projectile antiquark distribution $f_{\bar{q}/h}(z, -\vec{p}_{0\perp})$ we obtain

$$\frac{d\sigma_{\mathrm{DY}}}{d^2\vec{q}_\perp} \propto \int d^2\vec{p}_{0\perp} f_{\bar{q}/h}(z, -\vec{p}_{0\perp}) f_{q/T}(x_B, \vec{p}_{0\perp} + \vec{q}_\perp). \tag{A14}$$

The latter equation shows that our DY model with spectator is consistent with factorization theorems involving $K_\perp$-dependent parton distributions [15].

2. Nonuniversality of $k_\perp$ Coulomb exchange

Here we argue that the result found in Sec. III B, namely, that the typical $k_\perp$ contributing to $\Delta\sigma_{\mathrm{DY}} = \sigma_{\mathrm{DY}}^{\mathrm{tot}} - \sigma_{\mathrm{DY}}^{\mathrm{Born}}$ is $k_\perp \sim \lambda \to 0$, naturally translates to $k_\perp \sim \delta$ in the case of a composite projectile of size $R_h \sim 1/\delta$.

The $k_\perp$ distribution reads

$$\begin{aligned}\frac{d\sigma_{\mathrm{DY}}}{d^2\vec{k}_\perp} &\propto \int \frac{d^2\vec{p}_{2\perp}}{(2\pi)^2}\frac{d^2\vec{p}_{0\perp}}{(2\pi)^2} |\mathcal{M}_{\mathrm{DY}}(\vec{p}_{2\perp}, \vec{k}_\perp, \vec{p}_{0\perp})|^2 \\ &\propto \int d^2\vec{u}_\perp |\phi(z, u_\perp)|^2 \int d^2\vec{r}_\perp d^2\vec{R}_\perp d^2\vec{R}'_\perp e^{-i(\vec{R}_\perp - \vec{R}'_\perp)\cdot\vec{k}_\perp} e^{ig^2(W(\vec{u}_\perp,\vec{R}_\perp) - W(\vec{u}_\perp,\vec{R}'_\perp))} \mathcal{M}_{\mathrm{DIS}}(\vec{r}_\perp, \vec{R}_\perp)\mathcal{M}^*_{\mathrm{DIS}}(\vec{r}_\perp, \vec{R}'_\perp),\end{aligned} \tag{A15}$$

where we used (A9). The integrand of (A15) depends on the scales $m_{||}$ and δ, the latter corresponding [see (A5)] to the inverse transverse size of the projectile. When $\delta \to 0$, $u_\perp \sim 1/\delta \to \infty$, and the (finite) Coulomb phase in (A15) becomes

$$W(\vec{u}_\perp, \vec{R}_\perp) - W(\vec{u}_\perp, \vec{R}'_\perp) = G(R_\perp) - G(R'_\perp) + \frac{1}{2\pi}\log\left(\frac{|\vec{u}_\perp + \vec{R}_\perp|}{|\vec{u}_\perp + \vec{R}'_\perp|}\right) \underset{u_\perp\to\infty}{\longrightarrow} G(R_\perp) - G(R'_\perp). \tag{A16}$$

We thus recover, in the $\delta \to 0$ limit, the $k_\perp$ distribution (43) in the DY model without spectator, for which we have shown that $k_\perp \sim \lambda \to 0$. In other words, when the size of the projectile $R_h \sim 1/\delta \to \infty$, the spectator plays no screening role any longer. In practice δ is nonzero, $\delta \sim m$ [but still $\delta \ll m_{||} \simeq \sqrt{y}Q$ in the kinematical region (38)], and the typical $k_\perp$ contributing to $\Delta\sigma_{\mathrm{DY}}$ is of order δ, the largest infrared cutoff at disposal.

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