Neutrinoless double β-decay and neutrino mass hierarchies

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(Received 29 July 2005; published 30 September 2005)

In the framework of the seesaw mechanism the normal hierarchy is favorable for the neutrino mass spectrum. For this spectrum we present a detailed calculation of the half-lives of neutrinoless double β-decay for several nuclei of experimental interest. The half-lives are evaluated by considering the most comprehensive nuclear matrix elements, which were obtained within the renormalized quasiparticle random phase approximation by the Bratislava–Caltech–Tuebingen group. The dependence of the half-lives on $\sin^2\theta_{13}$ and the lightest neutrino mass is studied. We present also the results of the calculations of the half-lives of neutrinoless double β-decay in the case of the inverted hierarchy of neutrino masses.

I. INTRODUCTION

The discovery of neutrino oscillations in the atmospheric Super-Kamiokande experiment [1], in the solar SNO experiment [2], in the reactor KamLAND experiment , in the accelerator K2K experiment [4], and other neutrino experiments [5–8] is one of the most compelling evidence in favor of new physics beyond the standard model. All existing neutrino oscillation data, with the exception of the data of the short baseline accelerator experiment LSND [9] (the LSND result will be checked by the running MiniBooNE experiment [10] soon) are described by the three-neutrino mixing scheme

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL}(x); \quad l = e, \mu, \tau. \tag{1}$$

Here, $\nu_i(x)$ is the field of the neutrino with mass m_i ($i = 1, 2, 3$) and $\nu_{lL}(x)$ is a flavor neutrino field which enters into the standard charged and neutral currents

$$\begin{aligned}
j_\alpha^{CC}(x) &= 2\sum_l \bar\nu_{lL}(x)\gamma_\alpha l_L(x), \\
j_\alpha^{NC}(x) &= \sum_l \bar\nu_{lL}(x)\gamma_\alpha \nu_{lL}(x),
\end{aligned} \tag{2}$$

U is the unitary Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [11,12] mixing matrix. For massive Dirac neutrinos the PMNS matrix U^D in the standard parametrization has the form

$$U^D = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{3}$$

Here $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, θ_{ij} ($i < j$) is the neutrino mixing angle and δ is the CP violating phase.

From the analysis of the Super-Kamiokande atmospheric neutrino data for the neutrino mass-squared difference Δm_{23}^2 and the parameter $\sin^2 2\theta_{23}$ it was obtained [1]:

$$\begin{aligned}
\text{best fit:} \quad &\Delta m_{23}^2 = 2.1 10^{-3}\ \text{eV}^2, \qquad \sin^2 2\theta_{23} = 1.00, \\
90\%\text{C.L.:} \quad &1.5 10^{-3}\ \text{eV}^2 \le \Delta m_{23}^2 \le 3.4 10^{-3}\ \text{eV}^2, \qquad \sin^2 2\theta_{23} > 0.92.
\end{aligned} \tag{4}$$

The global analysis of the data of the solar neutrino experiments and KamLAND experiment yields the following best fit values and 90% C.L. ranges of the relevant neutrino oscillation parameters [3]:

$$\begin{aligned}
\text{best fit:} \quad &\Delta m_{12}^2 = 7.9 10^{-5}\ \text{eV}^2, \qquad \tan^2\theta_{12} = 0.40, \\
90\%\text{C.L.:} \quad &7.4 10^{-5}\ \text{eV}^2 \le \Delta m_{12}^2 \le 8.5 10^{-5}\ \text{eV}^2, \qquad 0.33 \le \tan^2\theta_{12} \le 0.50.
\end{aligned} \tag{5}$$

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Notice that neutrino mass-squared difference is determined as $\Delta m_{ik}^2 = m_k^2 - m_i^2$. For the angle θ_{13} only the upper bound is known. From the exclusion plot obtained from the data of the reactor experiment CHOOZ [13,14] we have

$$\sin^2\theta_{13} \leq 5 10^{-2} \quad (90\% \text{C.L.}). \tag{6}$$

The CP-violating phase δ remains undetermined. A recent global analysis of the oscillation data leads to the following bound: $\sin^2\theta_{13} \leq 0.9^{+2.3}_{-0.9}10^{-2}$ (95% C.L.) [15].

At present the structure of the neutrino mass spectrum is not known as well. Two types of spectra are possible:
 (1) Normal spectrum:

$$m_1 < m_2 < m_3; \qquad \Delta m_{12}^2 \ll \Delta m_{23}^2. \tag{7}$$

 (2) Inverted spectrum:

$$m_3 < m_1 < m_2; \qquad \Delta m_{12}^2 \ll |\Delta m_{13}^2| \tag{8}$$

We note that it is common to label neutrino masses differently in the case of the normal and the inverted spectra. For both spectra we have $m_2 > m_1$. But in the case of the normal spectrum m_3 is the mass of the heaviest neutrino and in the case of the inverted hierarchy m_3 is the mass of the lightest neutrino. This convention allows us to keep the same notation of the mixing angles for both spectra. Existing oscillation data are compatible both with normal and the inverted spectra.

The lightest neutrino mass $m_0 = m_1(m_3)$, which determines the absolute values of neutrino masses, is currently also unknown. From an analysis of the data of the Mainz [16] and Troitsk [17] tritium experiments it was found

$$m_0 \leq 2.3 \text{ eV}. \tag{9}$$

A more stringent bound on the sum of neutrino masses can be found from the measurement of the matter power spectrum $P(k)$. Depending on the data which were taken into account, the cosmological upper bound on the sum of neutrino masses was obtained as (see [18,19] and references therein)

$$\sum_i m_i \leq (0.5\text{–}1.7) \text{ eV}. \tag{10}$$

An important evidence that masses and mixing of neutrinos are of a nature beyond the standard model (SM) would be that massive neutrinos are Majorana particles. If ν_i are Majorana particles
 (1) Neutrino fields $\nu_i(x)$ satisfy the Majorana conditions

$$\nu_i^c(x) = \nu_i(x), \tag{11}$$

  where $\nu_i^c(x) = C\bar{\nu}_i^T(x)$ is the conjugated field (C is the charge conjugation matrix).
 (2) The neutrino mixing matrix has the form [20]

$$U = U^D S(\alpha) \tag{12}$$

where $S(\alpha)$ is a diagonal phase matrix. In the case of three-neutrino mixing the matrix $S(\alpha)$ is characterized by two Majorana CP-violating phases. The matrix $S(\alpha)$ can be presented in the form

$$S_{ik} = e^{i\alpha_i}\delta_{ik}; \alpha_3 = 0. \tag{13}$$

The unitary matrix U^D, which is characterized by the three mixing angles θ_{12}, θ_{23}, θ_{13} and one phase δ, was already introduced in Eq. (3).

If in the lepton sector CP invariance holds, for the Majorana mixing matrix we have [21]

$$U_{li} = U_{li}^*\eta_i, \tag{14}$$

where $\eta_i = \pm i$ is the CP parity of the Majorana neutrino ν_i. The condition (14) can be presented in the form

$$U_{li}^2 = |U_{li}|^2 e^{i(\pi/2)\rho_i}, \tag{15}$$

where $\rho_i = \pm 1$.

Investigations of neutrino oscillations in a vacuum and in matter do not allow us to distinguish massive Dirac from massive Majorana neutrinos [20,22,23]. In order to reveal the Majorana nature of ν_i it is necessary to study processes in which the total lepton number is violated. Because the standard electroweak interaction conserves helicity, the probabilities of such processes are proportional to the squares of the neutrino masses, and, consequently, they are strongly suppressed. The best sensitivity on small Majorana neutrino masses can be reached in the investigation of neutrinoless double β-decay ($0\nu\beta\beta$) of some even-even nuclei.

II. NEUTRINOLESS DOUBLE β-DECAY

In the case of Majorana neutrino mixing and the standard electroweak CC interaction the $0\nu\beta\beta$-decay,

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-, \tag{16}$$

is a second order process in the Fermi constant G_F with virtual neutrinos. The half-life of the process is given by the following general expression [24]

$$\frac{1}{T_{1/2}^{0\nu}(A, Z)} = |m_{\beta\beta}|^2 |M^{0\nu}(A, Z)|^2 G^{0\nu}(E_0, Z). \tag{17}$$

Here

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i \tag{18}$$

is the effective Majorana mass, $M^{0\nu}(A, Z)$ is the nuclear matrix element (NME) and $G^{0\nu}(E_0, Z)$ is a known phase-space factor (E_0 is the energy release). Let us stress that the NME is determined only by nuclear properties (its dependence on the small neutrino masses can be safely neglected).

After the discovery of neutrino oscillations, the search for neutrinoless double beta-decay ($0\nu\beta\beta$-decay) became one of the most fundamental problems of neutrino physics. Observation of this process would be the proof that massive neutrinos ν_i are Majorana particles [25]. Furthermore the observation of $0\nu\beta\beta$-decay will allow us to reveal the type of the neutrino mass spectrum, to determine the mass of the lightest neutrino, and, possibly, Majorana *CP* phases.

The most stringent lower bounds on the half-life of $0\nu\beta\beta$-decay were obtained in the Heidelberg-Moscow ^{76}Ge [26] and CUORICINO ^{130}Te [27] experiments:

$$T^{0\nu}_{1/2}(^{76}\text{Ge}) \geq 1.9 \times 10^{25} \text{ years,}$$
$$T^{0\nu}_{1/2}(^{130}\text{Te}) \geq 1.8 \times 10^{24} \text{ years.} \quad (19)$$

Using recently calculated nuclear matrix elements with significantly reduced theoretical uncertainties [28,29] from these data the following upper bounds for the effective Majorana mass can be inferred

$$|m_{\beta\beta}| \leq 0.55 \text{ eV} \quad \text{(Heidelberg-Moscow)}$$
$$|\text{m}_{\beta\beta}| \leq 1.1 \text{ eV(CUORICINO)}. \quad (20)$$

The Heidelberg group, which includes a few authors of the Heidelberg-Moscow collaboration, recently claimed [30] evidence for the $0\nu\beta\beta$-decay of ^{76}Ge with $T^{0\nu}_{1/2} = (0.69\text{–}4.18) \times 10^{25}$ years at the 4.2σ confidence level. Using the NME obtained in Ref. [28,29], from this data one finds for the effective Majorana mass the range $0.37 \text{ eV} \leq |m_{\beta\beta}| \leq 0.91 \text{ eV}$. The claim made in [30] was reexamined and criticized by different authors [31] and in particular by the Moscow participants of the Heidelberg-Moscow collaboration [32]. The GERDA I experiment [33], now at preparation at Gran Sasso, will be able to check relatively soon the claim made in [30].

The effective Majorana mass given by Eq. (18) is determined by the values of the neutrino masses m_i, which for the case of the normal neutrino mass spectrum are given by

$$m_2 = \sqrt{m_1^2 + \Delta m_{12}^2}, \qquad m_3 = \sqrt{m_1^2 + \Delta m_{12}^2 + \Delta m_{23}^2}, \quad (21)$$

and by the matrix elements U_{ei}^2, which in the standard parametrization take the form

$$U_{e1}^2 = \cos^2\theta_{13}\cos^2\theta_{12}e^{2i\alpha_1},$$
$$U_{e2}^2 = \cos^2\theta_{13}\sin^2\theta_{12}e^{2i\alpha_2}, \qquad U_{e3}^2 = \sin^2\theta_{13}e^{2i\alpha_3}, \quad (22)$$

where α_i are Majorana phases.

The values of the neutrino masses depend on the lightest neutrino mass $m_0 = m_1(m_3)$, on the neutrino mass spectrum and the neutrino mass-squared differences Δm_{12}^2 and

Δm_{23}^2 ($|\Delta m_{13}^2|$), which are known from neutrino oscillation data [see (4) and (5)]. The value of the parameter $\sin^2\theta_{12}$ is deduced from the analysis of the solar and KamLAND data [see (5)]. The lightest neutrino mass m_0 and the *CP* Majorana phases α_i are unknown and will be considered as free parameters.

In the case of the normal mass hierarchy,

$$m_1 \ll m_2 \ll m_3, \quad (23)$$

the lowest two neutrino masses and the effective Majorana mass $|m_{\beta\beta}|$ have the minimal values. If we neglect the contribution of m_1 to (18), for the upper bound of $|m_{\beta\beta}|$ we get

$$|m_{\beta\beta}| \leq (\sin^2\theta_{12}\sqrt{\Delta m_{12}^2} + \sin^2\theta_{13}\sqrt{\Delta m_{23}^2}). \quad (24)$$

The contribution of the first term to $|m_{\beta\beta}|$ is small because of the smallness of $\sqrt{\Delta m_{12}^2}$. The contribution proportional to the "large" $\sqrt{\Delta m_{23}^2}$ is suppressed by the smallness of the parameter $\sin^2\theta_{13}$. With (5), (6), and (24) for the upper bound of the effective Majorana mass we find

$$|m_{\beta\beta}| \lesssim 6.4 \times 10^{-3} \text{ eV.} \quad (25)$$

In the case of the inverted hierarchy

$$m_3 \ll m_1 < m_2, \quad (26)$$

we can safely neglect the contribution of the lightest mass m_3 to the effective Majorana mass. For $|m_{\beta\beta}|$ we have the following expression

$$|m_{\beta\beta}| \simeq \sqrt{|\Delta m_{13}^2|}(1 - \sin^2 2\theta_{12}\sin^2\alpha_{21})^{1/2}, \quad (27)$$

where $\alpha_{21} = \alpha_2 - \alpha_1$. The only unknown parameter in (27) is $\sin^2\alpha_{12}$. From (27) we have

$$\sqrt{|\Delta m_{13}^2|}\cos 2\theta_{12} \leq |m_{\beta\beta}| \leq \sqrt{|\Delta m_{13}^2|}. \quad (28)$$

The bounds in (28) correspond to the case of the *CP* invariance in the lepton sector (the upper bound corresponds to $\rho_1 = \rho_2$ and the lower bound corresponds to $\rho_1 = -\rho_2$.) From (4), (5), and (28) we find that in the case of the inverted hierarchy the value of $|m_{\beta\beta}|$ must lie in the range

$$1.0 \times 10^{-2} \leq |m_{\beta\beta}| \leq 5.5 \times 10^{-2} \text{ eV,} \quad (29)$$

In the case of the quasidegenerate spectrum of neutrino masses,

$$m_1 < m_2 < m_3; \qquad m_i \simeq m_0 \gg \sqrt{\Delta m_{23}^2}, \quad (30)$$

the effective Majorana mass is given by Eq. (27) in which the replacement $\sqrt{|\Delta m_{13}^2|} \rightarrow m_0$ must be performed.

Thus, in the case of the quasidegenerate spectrum the effective Majorana mass depends on two parameters: m_0 and $\sin^2 \alpha_{21}$. For the common neutrino mass we have

$$|m_{\beta\beta}| \leq m_0 \leq \frac{|m_{\beta\beta}|}{\cos 2\theta_{12}}. \tag{31}$$

The current bound on m_0, obtained from the measurement of the high energy part of the β-spectrum of tritium, is given in (9). The future KATRIN tritium experiment [34] will be sensitive to a value of $m_0 \simeq 2 \times 10^{-1}$ eV.

There are many models of neutrino masses (see e.g. Refs. [35–37]). The seesaw mechanism of neutrino mass generation [38] is considered to be the most plausible one. This mechanism is based on the assumption that violation of the total lepton number is at a large scale and connects the smallness of the Majorana neutrino masses with heavy right-handed Majorana particles. The existence of such particles provide a natural framework for the explanation of the baryon asymmetry of the Universe (see Ref. [39]).

In the framework of the seesaw mechanism the degenerate neutrino mass spectrum requires a fine-tuning which includes the Dirac mass matrix and the right-handed Majorana mass matrix.

The inverted hierarchy of neutrino masses requires a specific lepton symmetry like a global gauge symmetry which provides conservation of $L_e - L_\mu - L_\tau$. In order to explain existing data this symmetry must be broken. In such a framework it is difficult to reconcile the large θ_{23} with the not so maximal mixing angle θ_{12} and the small mixing angle θ_{13} [35,36]. Neutrino mass hierarchy is a natural spectrum in the case of the seesaw mechanism. Such a spectrum is realized in the case of SO(10) [40] and another grand unified theory (GUT) models which connect quark and lepton sectors. The problems of the CP phases

and the renormalization group effects in the SO(10) GUT were discussed in [41].

In this paper we calculate the half-lives of the $0\nu\beta\beta$-decay in the case of the normal and inverted hierarchies of neutrino masses. Neutrino masses for such spectra are very small. For the case of the normal hierarchy we have

$$m_2 \simeq 9 \times 10^{-3} \text{ eV}; \qquad m_3 \simeq 5 \times 10^{-2} \text{ eV};$$
$$m_1 \ll m_2 \tag{32}$$

For the case of the inverted hierarchy we obtain

$$m_1 \simeq m_2 \simeq 5 \times 10^{-2} \text{ eV}; \qquad m_3 \ll m_1. \tag{33}$$

Effects of such small neutrino masses cannot be observed in tritium and other β-decay experiments in the foreseeable future. Future data on the distribution of clusters of galaxies and gravitational lensing data, however, will be sensitive to the following value of the sum of neutrino masses [19,42]

$$\sum_i m_i \simeq 3 \times 10^{-2} \text{ eV}. \tag{34}$$

Thus, apparently, future cosmological measurements can probe neutrino mass hierarchies.

Several future experiments on the search for $0\nu\beta\beta$-decay will be sensitive to values of the effective Majorana mass in the range (29), which corresponds to the inverted mass hierarchy (see Table I).

Taking into account the theoretical plausibility of the normal hierarchy of neutrino masses we believe that it is worthwhile to consider the $0\nu\beta\beta$-decay in detail for this type of neutrino mass spectrum. Recently, important progress in the evaluation of the $0\nu\beta\beta$-decay NME's for the ground state transitions of nuclei of experimental interest was achieved [28,29]. We shall use these new results

TABLE I. Sensitivities of future $0\nu\beta\beta$-decay experiments to the effective Majorana neutrino mass $|m_{\beta\beta}|$ calculated with the RQRPA nuclear matrix elements $M^{0\nu}(A, Z)$ of Ref. [29]. For the axial coupling constant g_A the value $g_A = 1.25$ was assumed. $T_{1/2}^{0\nu-\exp}$ is the maximal half-life, which can be reached in the experiment and $|m_{\beta\beta}|$ is the corresponding upper limit of the effective Majorana neutrino mass.

Nucleus	Experiment	Source	$T_{1/2}^{0\nu-\exp}$ [yr]	Ref.	$M^{0\nu}(A, Z)$	$|m_{\beta\beta}|$ [eV]
^{76}Ge	GERDA(I)	15 kg of $^{\text{enr}}$Ge	3×10^{25}	[33]	2.40	0.44
	GERDA(II)	100 kg of $^{\text{enr}}$Ge	2×10^{26}	[33]	2.40	0.17
	Majorana	0.5 t of $^{\text{enr}}$Ge	4×10^{27}	[43]	2.40	0.038
^{82}Se	SuperNEMO	100 kg of $^{\text{enr}}$Se	2×10^{26}	[44]	2.12	0.091
^{100}Mo	MOON	3.4 t of $^{\text{nat}}$Mo	1×10^{27}	[45]	1.16	0.058
^{116}Cd	CAMEO	1 t of CdWO$_4$ crystals	$\approx 10^{26}$	[45]	1.43	0.14
^{130}Te	CUORE	750 kg of TeO$_2$	$\approx 10^{27}$	[46]	1.47	0.047
^{136}Xe	XMASS	10 t of liq. Xe	3×10^{26}	[45]	0.98	0.12
					0.73	0.17
	EXO	1 t $^{\text{enr}}$Xe	2×10^{27}	[47]	0.98	0.048
					0.73	0.064

to calculate expected half-lives of $0\nu\beta\beta$-decay for both neutrino mass hierarchies.

III. NUCLEAR MATRIX ELEMENTS

From the measurement of the half-life of the $0\nu\beta\beta$-decay only the product $|m_{\beta\beta}||M^{0\nu}(A, Z)|$ can be determined. Thus, without accurate calculation of nuclear matrix elements, it is not possible to reach qualitative conclusions about neutrino masses and the type of neutrino mass spectrum [48–53].

The calculation of the $0\nu\beta\beta$-decay matrix elements is a difficult problem because ground and many excited states of open-shell nuclei with complicated nuclear structure have to be considered. In the calculation of the $0\nu\beta\beta$-decay NME's the nuclear shell model (NSM) and the proton-neutron quasiparticle random phase approximation (pn-QRPA) or extensions to it [54] are used.

These two approaches are significantly different. The NSM is limited to a set of single-particle states in the vicinity of the Fermi level. Thus, configurations with only small excitations are considered. These excitations are correlated in all possible ways. The open problem is the effect of single-particle states further away from the Fermi level, which is neglected. The large NSM spaces face the problem of diagonalization of large matrices and the construction of good effective interaction.

From the available shell-model calculations of the NME of the $0\nu\beta\beta$-decay, the most advanced are those of the Strasbourg group [55], which appeared about ten years ago. From that time, in spite of significant progress in computer speed and memory no large-scale NSM calculations of the $0\nu\beta\beta$-decay NME have been published.

The pn-QRPA method takes into account many single-particle states including states relatively far from the Fermi surface but with correlations which are of the specific simple type. Many extensions of the pn-QRPA have been proposed with the aim to improve the many-body approximation scheme. The so-called renormalized pn-QRPA (pn-RQRPA) scheme takes into account the Pauli exclusion principle by improving the quasiboson approximation [56]. In this method for the evaluation of the commutators of bifermion operators the QRPA ground state is used (instead of the uncorrelated BCS ground state). This refinement of QRPA approach allows to reduce significantly the sensitivity of the results to the renormalization of the particle-particle interaction of the nuclear Hamiltonian. The fact that pn-RQRPA is an improvement of the pn-QRPA approach has been confirmed by the studies performed within a schematic model [57].

The pn-QRPA and its extensions remain a popular technique of the calculation of $0\nu\beta\beta$-decay NME's. However, various implementations of the QRPA introduced by different authors have produced a spread of results with a factor of 2–3 and even more difference in the NME [58]. Some authors simplified this problem by assuming that the pub-lished range of calculated NME's defines a plausible approximation to the uncertainty in our knowledge of the matrix elements [58,59]. We do not share this position. We believe that it is not appropriate to consider all calculated $0\nu\beta\beta$-decay NME's at the same level. The correct procedure is to understand the difference between various QRPA-like calculations and the origin of contradictions between different results. In Ref. [29] a list of main reasons leading to a spread of the pn-QRPA and the pn-RQRPA nuclear matrix elements was presented.

One of the most important factors of the QRPA calculation of the NME's is the way how the particle-particle strength of the nuclear Hamiltonian g_{pp} is fixed. When the early QRPA calculations were performed only limited information on half-lives of the $2\nu\beta\beta$-decay was available. At that time g_{pp} was fitted to existing data on single β^--transitions or alternatively g_{pp} was chosen to be equal to unity. Nowadays, the half-lives of $2\nu\beta\beta$-decay of ten nuclei has been measured. Recently, it has been shown that by adjusting g_{pp} to the $2\nu\beta\beta$-decay rates, we can significantly eliminate uncertainties associated with variations in QRPA calculations of decay rates [28,29]. In particular, the results obtained in this way are essentially independent of the size of the basis, the form of different realistic nucleon-nucleon potentials, or on whether QRPA or RQRPA is used. Furthermore, the matrix elements are shown to be also rather stable with respect to the possible quenching of the axial vector constant g_A.

The procedure proposed in [28,29] was critically analyzed in [60]. The author's conclusion was that fitting of g_{pp} to β^+ [or electron capture (EC)] and single β^--decay of the ground state of the intermediate nucleus is a more meaningful procedure. This criticism has been refuted in Ref. [29]. In particular, it was shown that there is no reason to give preference to the lowest state of the intermediate nucleus. The β^- and β^+/EC matrix elements move with g_{pp} in opposite directions, which makes it difficult to adjust g_{pp} by choosing one of them. It is preferable to use the sum of the products of amplitudes, i.e., the $2\nu\beta\beta$-decay half-life. It was also noticed that practically for all multipolarities a significant amount of strength is concentrated up to 10–15 MeV and that the contributions of the 1^+ multipole to the $2\nu\beta\beta$- and $0\nu\beta\beta$-decay matrix elements are correlated. Thus, there is no reason to choose any one particular state or transition for adjustment. It is worth also mentioning that only for three double β-decay nuclear systems of interest ($A = 100, 116, 128$) the ground state of the intermediate nucleus is just the 1^+ state. Thus, the procedure of how to fix g_{pp} proposed in Ref. [60] is not only disfavored but also strongly limited. These and other arguments of Ref. [29] clearly favor the procedure of the $2\nu\beta\beta$-decay fitting rather than the procedure of the beta-decay fitting.

A discussion concerning the previous QRPA and RQRPA calculations of the $0\nu\beta\beta$-decay NME's, in which

different assumptions and approximations were made, can be found in [29]. It was shown that in most, albeit not all, cases the differences among them can be understood. Attention was also paid to the fact that the results of [61] differ significantly from those of [28,29] in spite of the fact that the same procedure of adjustment of g_{pp} was used (for the most important 1^+ channel). Contrary to the NME of [28,29] the NME calculated in [61] strongly depend on the size of the model space. In particular the levels lying far from the Fermi surface severely influence the decay rate. Actually, there is no explanation of this fact and nobody else reported such a strong effect.

The calculations performed in Ref. [29] can be considered as the most reliable QRPA and RQRPA calculations of the NME of the $0\nu\beta\beta$-decay. We stress that the NME calculations in which the dependence of the results on the particular choice made is not discussed could not be considered on the same footing as those where these points are carefully explained (see e.g. Ref. [29]).

There are no doubts that further progress in the calculation of the $0\nu\beta\beta$-decay NME's is needed. The nuclei of experimental interest for the investigation of $0\nu\beta\beta$-decay have been extensively studied by assuming spherical symmetry. However, many of the nuclei undergoing double beta decay are deformed [62] and it is important to study the effect of deformation on the $0\nu\beta\beta$-decay. Recently a new suppression mechanism of the $2\nu\beta\beta$-decay matrix elements based on the relative deformation of the initial and final nuclei has been found [63]. The effect of deformation is large, if there is a significant difference in the deformations of the parent and daughter nuclei. The effect of deformations could also be large in the case of the $0\nu\beta\beta$-decay matrix elements.

For further progress in the field it is also important to apply the methods of the calculation of the $0\nu\beta\beta$-decay matrix elements to the calculation of the matrix elements of related processes like charge-exchange reactions [64], muon capture [65], and charged current (anti)neutrino-nucleus reactions [66]. The observation of these processes might probe forbidden transitions which contribute to the $0\nu\beta\beta$-decay half-lives. Thus, it is essential to understand how the methods applied for the calculation of the NME of the $0\nu\beta\beta$-decay can reproduce the observables related to these processes. For the improvement of many-body approaches it is also important to test them in the cases of exact solutions of solvable models which are as realistic as possible.

The improvement of the calculation of the nuclear matrix elements is a very important and challenging problem. The nuclear matrix elements of the $0\nu\beta\beta$-decay cannot be related exactly to other observables. Complementary experimental information from related processes is highly required, but it cannot fully solve the problem of the uncertainties in the $0\nu\beta\beta$-decay matrix elements. This problem might be solved only by observation of the

$0\nu\beta\beta$-decay of at least three different nuclei [53,67]. This would be a model independent test of the theoretically calculated NME's.

IV. THE EXPECTED $0\nu\beta\beta$-DECAY HALF-LIVES

Here, we present the results of the calculation of the half-lives of the $0\nu\beta\beta$-decay of the following nuclei ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, and ^{136}Xe, many of which presumably will be investigated in future experiments. We will use nuclear matrix elements $M^{0\nu}(A, Z)$ obtained in Ref. [29].

First, we discuss the case of the normal hierarchy of neutrino masses. Taking into account all terms for $|m_{\beta\beta}|$
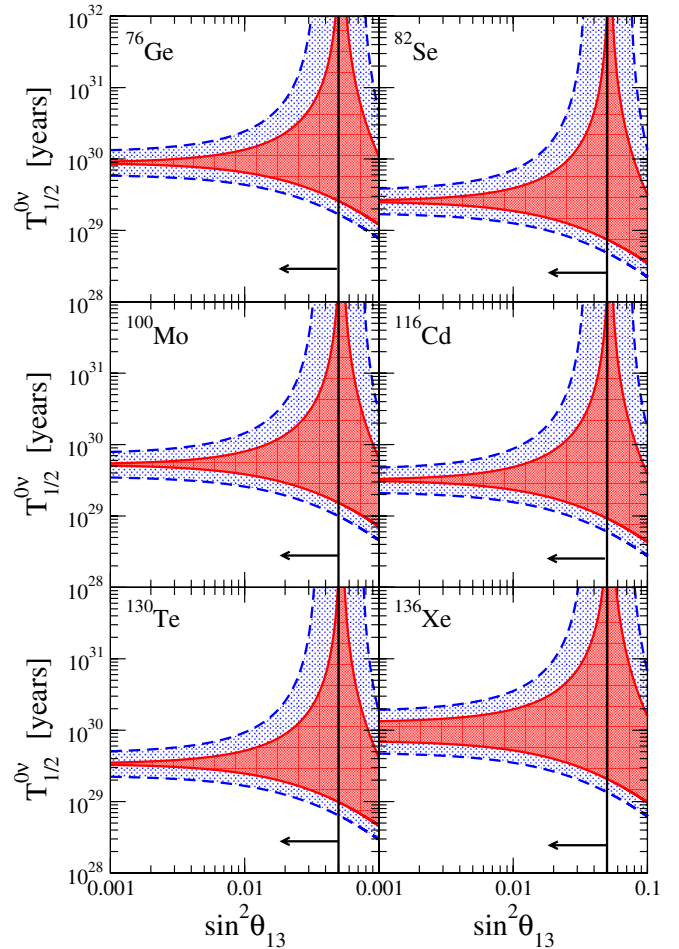

FIG. 1 (color online). The neutrinoless double beta decay half-life $T_{1/2}^{0\nu}$ for nuclei of experimental interest as a function of the parameter $\sin^2\theta_{13}$. The case of the normal hierarchy of neutrino masses is considered and the lightest neutrino mass is assumed to be negligibly small. The region with solid line (dashed line) boundaries corresponds to the best fit (90% C.L.) of the neutrino oscillation parameters [1,3]. The calculations were performed by using recently evaluated nuclear matrix elements with significantly reduced theoretical uncertainty [29]. The vertical line indicates the current upper limit on $\sin^2\theta_{13}$ set by the CHOOZ experiment.

we have the following expression

$$|m_{\beta\beta}| = |\cos^2\theta_{12}m_1 + \sin^2\theta_{12}\sqrt{\Delta m^2_{12}}e^{2i\alpha_{12}}$$

$$+ \sin^2\theta_{13}\sqrt{\Delta m^2_{23}}e^{2i\alpha_{13}}|. \qquad (35)$$

There are four unknown parameters in (35), namely m_1, $\sin^2\theta_{13}$, and two CP phase differences α_{12} and α_{13}. First we shall assume that the lightest mass m_1 is very small and we can neglect its contribution to $|m_{\beta\beta}|$. In this case the effective Majorana mass $m_{\beta\beta}$ depends on two free parameters: $\sin^2\theta_{13}$ and α_{23}.

The current 90% C.L. CHOOZ bound on the parameter $\sin^2\theta_{13}$ is given in (6). A significant improvement on the value of this parameter is planned to be achieved in future experiments. The expected sensitivity of the Double-CHOOZ reactor experiment [68] is $\sin^2\theta_{13} \simeq 7.5 \times 10^{-3}$. In the accelerator T2K $\nu_\mu \to \nu_e$ experiment [69] a sensitivity of $\sin^2\theta_{13} \simeq 1.5 \times 10^{-3}$ is planned to be achieved. Thus, we will consider for the parameter $\sin^2\theta_{13}$ the range $(10^{-3}$–$5 \times 10^{-2})$.

In Fig. 1 we present the half-life $T^{0\nu}_{1/2}(A, Z)$ as a function of $\sin^2\theta_{13}$ for the nuclei ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, and ^{136}Xe. The regions with solid line boundaries (dashed line boundaries) were calculated with the best fit values (90% C.L. values) of the neutrino oscillation parameters Δm^2_{23}, Δm^2_{12}, and $\sin^2\theta_{12}$ [see Eqs. (4) and (5)]. The boundaries of these regions correspond to the case of CP-conservation. The lower (upper) boundaries correspond to $\cos 2\alpha_{23} = 1$ ($\cos 2\alpha_{23} = -1$), i.e., to the case of same (opposite) CP parities of the second and third neutrino. For the case of the $0\nu\beta\beta$-decay of ^{136}Xe the allowed regions in Fig. 1 are larger than for other nuclei. This is connected to the fact that the corresponding NME is constrained to a range of values since the $2\nu\beta\beta$-decay of this isotope has not been measured yet [29].

From Fig. 1 it follows that for $\sin^2\theta_{13} \leq 10^{-2}$ the calculated half-lives practically do not depend on $\sin^2\theta_{13}$. For such small values of $\sin^2\theta_{13}$ the dominant contribution to the effective Majorana mass $m_{\beta\beta}$ comes from the solar term [the second term on the right-hand side of Eq. (35)]. From Fig. 1 we also deduce that if the limit on $\sin^2\theta_{13}$ is be pushed further down by the Double-CHOOZ and T2K experiments, the expected half-lives of the $0\nu\beta\beta$-decay of ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, and ^{136}Xe will be about 10^{29}–10^{30} years.

In Table II we present the minimal values of $T^{0\nu}_{1/2}(A, Z)$ in the case of the neutrino mass hierarchy with a negligibly small lightest mass. They are given for three possible values of the parameter $\sin^2\theta_{13}$: 0.05, 0.01, 0.001. These values are expected to be slightly increased if the limit on the parameter $\sin^2\theta_{13}$ will further decrease (see Fig. 1). These values can be confronted with the sensitivities on half-lives of future $0\nu\beta\beta$-decay experiments given in Table I. From the comparison of the calculated half-lives

with the experimental sensitivities, we conclude that further experimental efforts will be required to reach lower limits comparable to the predicted values in the case of the normal hierarchy of neutrino masses.

Up to now we neglected the contribution of the lightest neutrino mass $m_0 = m_1$ to $|m_{\beta\beta}|$. From (32) it follows that $m_2 \simeq 0.2m_3$. If we (arbitrarily) assume that $m_1 \simeq 0.2m_2$ it follows that the modulus of the first term in (35) is about half of the modulus of the second solar term. Thus, the contribution of the m_1-term to $|m_{\beta\beta}|$ can be sizable. In Fig. 2 we present three-dimensional plots for the half-life of the $0\nu\beta\beta$-decay of ^{76}Ge as a function of m_1 and $\sin^2\theta_{13}$ under different assumptions for the relative CP Majorana phases. In the case of other isotopes similar results are expected. From Fig. 2 it follows that for small values of $\sin^2\theta_{13} \leq 0.01$, the pronounced hill region with relatively large values of the half-lives is not more accessible.

In the case of the inverted hierarchy of neutrino masses the upper and lower bounds on the effective Majorana mass $|m_{\beta\beta}|$ are well determined by the known oscillation parameters [see (28)]. In Table III we give the corresponding ranges for the half-lives of relevant nuclei calculated with the nuclear matrix elements of [29] by taking into account

TABLE II. Normal hierarchy of neutrino masses: The neutrinoless double beta decay half-life $T^{0\nu}_{1/2}(A, Z)$ of ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te ^{136}Xe, and ^{150}Nd The results are presented for three values of θ_{13} from the allowed range $\sin^2\theta_{13} \leq 0.05$ of CHOOZ [14]. The best fit and 90% C.L. values of $T^{0\nu}_{1/2}$ were calculated by assuming equal values of Majorana CP-violating phases, i.e., the lowest allowed values for $T^{0\nu}_{1/2}(A, Z)$ are given. The $0\nu\beta\beta$-decay nuclear matrix elements of Ref. [29] are used.

| Nuclear transition	Parameter set	Normal hierarchy: $T^{0\nu}_{1/2}(A, Z)$ [years] $\sin^2\theta_{13}$		
		0.05	0.01	0.001
^{76}Ge $\to$ ^{76}Se	best fit	2.6×10^{29}	6.5×10^{29}	8.6×10^{29}
	90% C.L.	1.7×10^{29}	4.4×10^{29}	5.8×10^{29}
^{82}Se $\to$ ^{82}Kr	best fit	7.5×10^{28}	1.9×10^{29}	2.5×10^{29}
	90% C.L.	4.8×10^{28}	1.3×10^{29}	1.7×10^{29}
^{96}Zr $\to$ ^{96}Mo	best fit	1.7×10^{30}	4.2×10^{30}	5.5×10^{30}
	90% C.L.	1.1×10^{30}	2.8×10^{30}	3.8×10^{30}
^{100}Mo $\to$ ^{100}Ru	best fit	1.5×10^{29}	3.8×10^{29}	5.1×10^{29}
	90% C.L.	9.9×10^{28}	2.6×10^{29}	3.5×10^{29}
^{116}Cd $\to$ ^{116}Sn	best fit	9.3×10^{28}	2.3×10^{29}	3.1×10^{29}
	90% C.L.	6.0×10^{28}	1.6×10^{29}	2.1×10^{29}
^{128}Te $\to$ ^{128}Xe	best fit	2.1×10^{30}	5.2×10^{30}	6.9×10^{30}
	90% C.L.	1.4×10^{30}	3.5×10^{30}	4.7×10^{30}
^{130}Te $\to$ ^{130}Xe	best fit	9.9×10^{28}	2.5×10^{29}	3.3×10^{29}
	90% C.L.	6.4×10^{28}	1.7×10^{29}	2.2×10^{29}
^{136}Xe $\to$ ^{136}Ba	best fit	2.1×10^{29}	5.2×10^{29}	6.9×10^{29}
	90% C.L.	1.4×10^{29}	3.5×10^{29}	4.7×10^{29}
^{150}Nd $\to$ ^{150}Sm	best fit	1.0×10^{28}	2.6×10^{28}	3.5×10^{28}
	90% C.L.	6.8×10^{27}	1.8×10^{28}	2.4×10^{28}
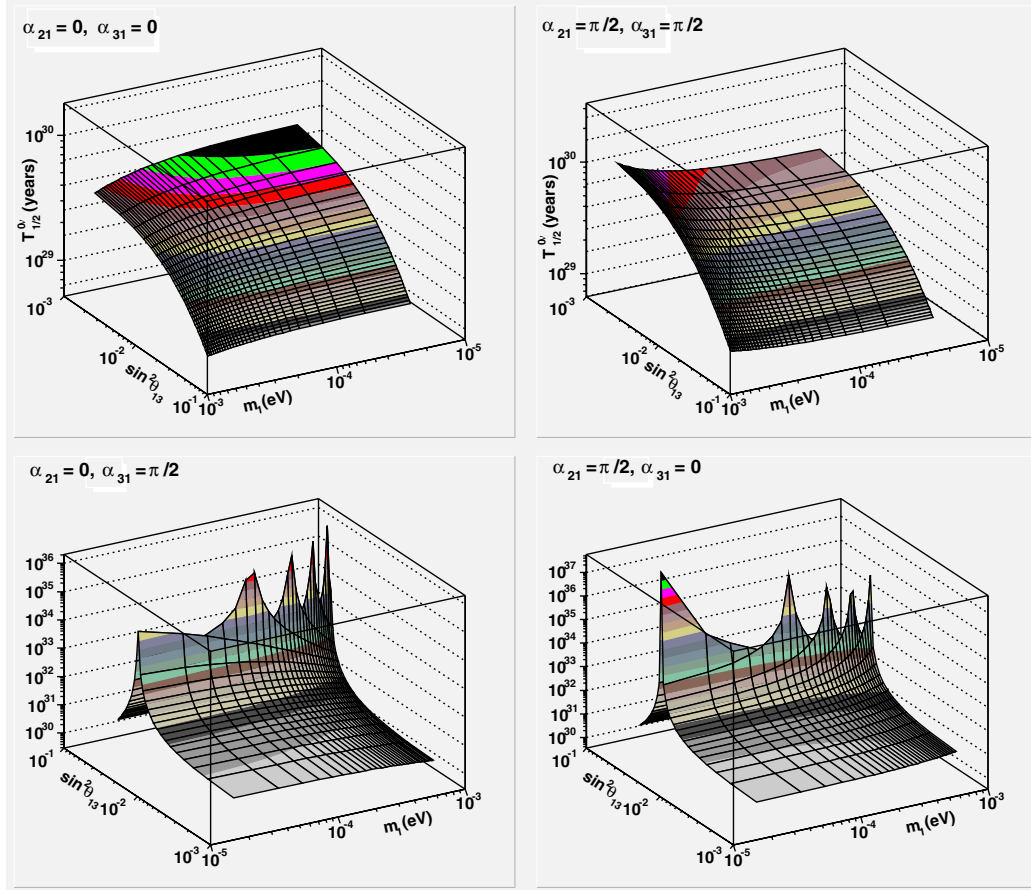
FIG. 2 (color online). The neutrinoless double beta half-life $T_{1/2}^{0\nu}$ of ^{76}Ge as a function of the parameter $\sin^2\theta_{13}$ and the lightest neutrino mass m_1. The normal hierarchy of neutrino masses is assumed and the best fit values of neutrino oscillation parameters are taken into account. The relative CP Majorana phases of neutrinos are denoted by α_{21} and α_{31}.

TABLE III. Inverted hierarchy of neutrino masses: The neutrinoless double beta decay half-life $T_{1/2}^{0\nu}(A, Z)$ calculated in RQRPA for ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te ^{136}Xe, and ^{150}Nd. $T_{1/2}^{0\nu}$ was calculated for best fit values and 90% C.L. ranges of the neutrino oscillation parameters [see Eqs. (4)–(6)]. The lightest neutrino mass is assumed to be negligibly small. The $0\nu\beta\beta$-decay nuclear matrix elements of Ref. [29] are used. The each pair of numbers in brackets refers to a range of allowed values of the half-life. The current experimental lower limits on the $0\nu\beta\beta$-decay half-lives ($T_{1/2}^{0\nu-\exp}$) are from experiments with saturated sensitivity except those denoted by the symbol ∗, which indicate experiments still taking data. H-M means Heidelberg-Moscow.

| Nucleus | Inverted hierarchy: $T_{1/2}^{0\nu}(A, Z)$ [years] | | $T_{1/2}^{0\nu-\exp}(A, Z)$ [years] | |
| | Parameter set | | Current limit | Planed sensitivity of future exper. |
	90% C.L.	best fit		
^{76}Ge	$(1.7 \times 10^{27}, 4.1 \times 10^{28})$	$(2.8 \times 10^{27}, 1.8 \times 10^{28})$	1.9×10^{25} H-M [26]	3×10^{25} GERDA I [33]
			1.6×10^{25} IGEX [70]	2×10^{26} GERDA II [33]
				4×10^{27} Majorana [43]
^{82}Se	$(4.9 \times 10^{26}, 1.2 \times 10^{28})$	$(8.1 \times 10^{26}, 5.3 \times 10^{27})$	1.0×10^{23} NEMO 3∗[71]	2×10^{26} SuperNEMO [44]
^{96}Zr	$(1.1 \times 10^{28}, 2.6 \times 10^{29})$	$(1.8 \times 10^{28}, 1.2 \times 10^{29})$	1×10^{21} NEMO 2 [72]	
^{100}Mo	$(1.0 \times 10^{27}, 2.4 \times 10^{28})$	$(1.7 \times 10^{27}, 1.1 \times 10^{28})$	4.6×10^{23} NEMO 3∗[71]	1×10^{27} MOON [45]
^{116}Cd	$(6.1 \times 10^{26}, 1.5 \times 10^{28})$	$(1.0 \times 10^{27}, 6.5 \times 10^{27})$	1.7×10^{23} [73]	$\approx 10^{26}$ CAMEO [44]
^{128}Te	$(1.4 \times 10^{28}, 3.3 \times 10^{29})$	$(2.3 \times 10^{28}, 1.5 \times 10^{29})$	2×10^{24} [74]	
^{130}Te	$(6.5 \times 10^{26}, 1.6 \times 10^{28})$	$(1.1 \times 10^{27}, 7.0 \times 10^{27})$	1.8×10^{24} CUORICINO∗[27]	$\approx 10^{27}$ CUORE [46]
^{136}Xe	$(1.4 \times 10^{27}, 5.9 \times 10^{28})$	$(2.3 \times 10^{27}, 2.7 \times 10^{28})$	1.2×10^{24} DAMA [75]	2×10^{27} EXO [47]
				3×10^{26} [45]
^{150}Nd	$(6.9 \times 10^{25}, 1.7 \times 10^{27})$	$(1.1 \times 10^{26}, 7.4 \times 10^{26})$	1.2×10^{21} [76]	
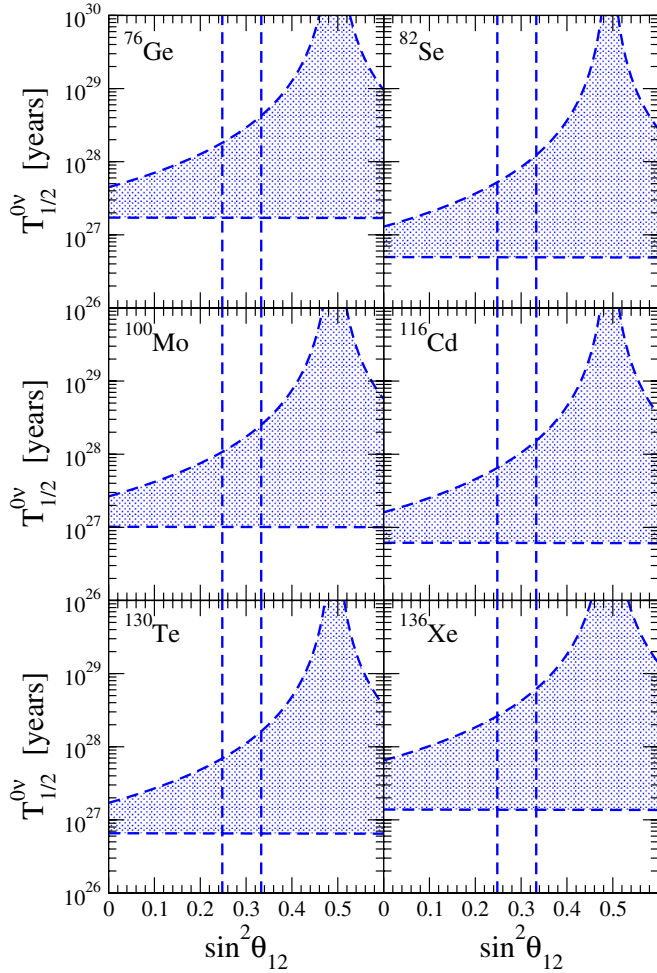
FIG. 3 (color online).   The neutrinoless double beta decay half-life $T_{1/2}^{0\nu}$ for nuclei of experimental interest as a function of the parameter $\sin^2\theta_{12}$. The case of the inverted hierarchy of neutrino masses is assumed. Conventions are the same as in Fig. 1. The vertical dashed lines correspond to the boundaries set by the 90% C.L. values of θ_{12} [3]. These results are not sensitive to m_3 for values below 10^{-2} eV [53].

the best fit and 90% C.L. values of the parameters $\sin^2\theta_{12}$ and $|\Delta m_{13}^2|$. In the same table the predicted ranges of the half-lives are compared to the current experimental lower limits on the $0\nu\beta\beta$-decay half-life from experiments with saturated sensitivity and from two running experiments (NEMO 3 [71] and CUORICINO [27]) together with half-life sensitivities of proposed future experiments. We note that the designed future $0\nu\beta\beta$-decay experiments allow different strategies. Some of them plan to proceed with smaller steps forward and some of them prefer to make large steps. The time-scale for these experiments except the GERDA I [33], whose main task is to confirm or rule out the recent claim of evidence for the

$0\nu\beta\beta$-decay of ^{76}Ge [30], is not determined yet. From Table III we conclude that the most ambitious future $0\nu\beta\beta$-decay experiments could probe the inverted hierarchy of neutrino masses. Finally, in Fig. 3 we present the expected lifetimes of the $0\nu\beta\beta$-decay of ^{76}Ge, ^{130}Te, ^{136}Xe, and other nuclei calculated for 90% C.L.-allowed values of the parameters. The dependence of the $0\nu\beta\beta$-decay half-life on $\sin^2\theta_{12}$ is outlined.

V. CONCLUSION

The effective Majorana neutrino mass, which determines the half-life of neutrinoless double β-decay, crucially depends on the character of the neutrino mass spectrum. All possible physical neutrino mass spectra (hierarchy, inverted hierarchy, and quasidegenerate) are at present viable. However, in the framework of the seesaw mechanism, which is a plausible explanation of the smallness of neutrino masses, hierarchy is a favorable spectrum (see, [35,36]). Generically, the neutrino mass hierarchy naturally appears in GUT models like SO(10) which unify quarks, leptons, and neutrinos.

Having in mind that the normal hierarchy of neutrino masses is a plausible neutrino mass spectrum, in this case we performed detailed calculations of half-lives of the $0\nu\beta\beta$-decay of several nuclei of experimental interest. We used nuclear matrix elements which were obtained in the recent most updated and most comprehensive RQRPA calculations [29]. We studied the dependence of the half-lives of $0\nu\beta\beta$-decay on $\sin^2\theta_{13}$ and the lightest neutrino mass m_1. The current [13] limit and future [68,69] sensitivities to the value of the parameter $\sin^2\theta_{13}$ were considered. The calculated lower limits on the half-lives of nuclei considered are listed in Table II. As it is seen they are in the range of 10^{28}–10^{30} years. The expected half-life sensitivities of the next generation of the $0\nu\beta\beta$-decay experiments are significantly lower.

The most ambitious future $0\nu\beta\beta$-decay experiments aim to probe the inverted hierarchy of neutrino masses which requires a symmetry in the neutrino mass matrix. Using the updated RQRPA calculations of NME [29], we have calculated half-lives of $0\nu\beta\beta$-decay of several nuclei in this case.

ACKNOWLEDGMENTS

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