Fourth standard model family neutrino at future linear colliders

A. K. Ciftci

Physics Department, Faculty of Sciences, Ankara University, 06100 Tandogan, Ankara, Turkey

R. Ciftci and S. Sultansoy*

Physics Department, Faculty of Sciences and Arts, Gazi University, 06500 Teknikokullar, Ankara, Turkey

(Received 23 May 2005; published 14 September 2005)

It is known that flavor democracy favors the existence of the fourth standard model (SM) family. In order to give nonzero masses for the first three-family fermions flavor democracy has to be slightly broken. A parametrization for democracy breaking, which gives the correct values for fundamental fermion masses and, at the same time, predicts quark and lepton Cabibbo-Kobayashi-Maskawa (CKM) matrices in a good agreement with the experimental data, is proposed. The pair productions of the fourth SM family Dirac (ν_4) and Majorana (N_1) neutrinos at future linear colliders with $\sqrt{s} = 500$ GeV, 1 TeV, and 3 TeV are considered. The cross section for the process $e^+e^- \to \nu_4\overline{\nu}_4(N_1N_1)$ and the branching ratios for possible decay modes of the both neutrinos are determined. The decays of the fourth family neutrinos into muon channels ($\nu_4(N_1) \to \mu^\pm W^\mp$) provide cleanest signature at e^+e^- colliders. Meanwhile, in our parametrization this channel is dominant. W bosons produced in decays of the fourth family neutrinos will be seen in detector as either di-jets or isolated leptons. As an example, we consider the production of 200 GeV mass fourth family neutrinos at $\sqrt{s} = 500$ GeV linear colliders by taking into account di-muon plus four jet events as signatures.

DOI: 10.1103/PhysRevD.72.053006 PACS numbers: 12.15.Ff, 13.66.De, 14.60.Pq, 14.80.−j

I. INTRODUCTION

It is well-known that the standard model (SM) does not fix the number of fermion families. There is only one indication that this number is less than 16 coming from the asymptotic freedom of QCD. On the other hand, the flavor democracy (or, in other words, the Democratic Mass Matrix) approach [1–4] favors the existence of the fourth SM family with the nearly degenerate masses in the range of 300–700 GeV [5–8]. Concerning the experimental situation, the LEPI data show that there are three SM families with light neutrinos [9]. However, extra SM families are allowed by the data, as long as the mass of new neutrinos is larger than $M_Z/2$. Furthermore, precision electroweak data do not exclude the fourth SM family; moreover, two and even three extra generations are also allowed if $m_{\nu_4} \sim 45$–50 GeV [10,11]. Experimental constraints [9] on the masses of fundamental SM fermions are presented in Table I.

The fourth SM family quarks will be copiously produced at the LHC [12,13] if their masses are less than 1 TeV. The FNAL Tevatron Run II can observe u_4 and d_4 before the LHC if there is an anomalous interaction with enough strength between the fourth family quarks and known quarks [14]. In addition, evidence for the extra SM families may come from the search for the SM Higgs boson due to an essential enhancement in the production of the Higgs boson via gluon-gluon fusion [15].

The observation of the fourth SM family leptons at hadron colliders is difficult due to a large background. Therefore, the fourth family leptons will be observed at lepton colliders with sufficient center of mass energy. This subject was investigated in [16] for muon colliders and in [17] for e^+e^- and $\gamma\gamma$ colliders. In these papers, the Dirac nature of the fourth SM family neutrino was assumed. Actually, the SM does not prohibit Majorana mass terms for right-handed neutrino. The fourth SM family Majorana neutrino search strategy changes greatly compared to the Dirac case.

In the four-family case seesaw mechanism, in principle, is not required to get light masses for the first three SM family neutrinos [18]. Meanwhile, in the case of Majorana neutrinos, there will be double suppression because of both the democratic mass matrix (DMM) and the seesaw mechanism. The existence of the fourth family neutrinos leads to a number of cosmological consequences [19].

The most important barrier in the front of high-energy electron-positron colliders is synchrotron radiation emitted by charged particles of circular motion. To avoid the resulting energy loss, one needs to build either a ring with a circumference of thousands of kilometers or a linear machine with the length of tens of kilometers. Because of the cost, the only choice for the high-energy colliders is the linear colliders.

The International Linear Collider (ILC), with the center of mass energy of 500 GeV (preferably extendable to 1 TeV) and with 10^{34} cm^{-2} s^{-1} luminosity, is being developed for use by particle physicists. The two technologies for the ILC use different types of cavities to accelerate electrons and positrons. The TESLA technology [20] has involved superconducting cavities operating at 2 K, whereas the technology of the Next Linear Collider (NLC) and Japan Linear Collider (JLC) was based on

*Also at Institute of Physics, Academy of Sciences, H. Cavid Avenue 33, Baku, Azerbaijan.

TABLE I. Masses of fundamental SM fermions in units GeV/c^2 [9].

Neutrinos	Charged leptons	Up quarks	Down quarks
ν_e: $< 3 \times 10^{-9}$	e:$0.510\,998\,90 \times 10^{-3}$	u:$(1.5\text{–}4.0) \times 10^{-3}$	d:$(4\text{–}8) \times 10^{-3}$
ν_μ: $< 0.19 \times 10^{-3}$	μ:0.105 658 357	c:1.15–1.35	s:$(80\text{–}130) \times 10^{-3}$
ν_τ: $< 18.2 \times 10^{-3}$	τ:1.776 99	t:174.3 ± 5.1	b:4.1–4.4
ν_4: > 45 (stable)	l_4: > 102.6 (stable)	u_4: > 200	d_4: > 128 (charged current decay)
ν_4: > 90.3 (unstable)	l_4: > 100.8 (unstable)		d_4: > 199 (neutral current decay)

copper cavities that would be run at room temperature. However, due to the huge cost of the linear collider, the physicists selected only one. Following evaluation of limitations of each cavity type, the International Steering Committee preferred the superconducting approach. Assuming that the design work is completed on time, construction of the $\sqrt{s} = 0.5$ TeV machine could start about 2010. Meanwhile LHC will provide a first glimpse of any new physics at energies up to about 1 TeV. Therefore, depending on LHC results, a machine with higher energy than 1 TeV may be preferred. In this case, the Compact Linear Collider (CLIC) [21] will be the right machine with the center of mass energy of 3 TeV and with $10^{35}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ luminosity. CLIC generates an accelerating gradient of 150 $\mathrm{MV\,m}^{-1}$ with the resulting 20 km of active length. To reach this high accelerating gradient, CLIC uses two beam accelerator technology operating at 30 GHz radio frequency.

In this paper we consider pair production of the fourth SM family neutrinos at future e^+e^- colliders. In Sec. II basic assumptions of the flavor democracy hypothesis are given and the fourth family quark Cabibbo-Kobayashi-Maskawa (CKM) matrix is evaluated. The leptonic sector is analyzed in Sec. III, where a leptonic CKM matrix is reproduced by using the same parametrization for democracy breaking as in the quark sector and possible decay modes of the fourth family Dirac and Majorana neutrinos are discussed. The numerical calculations for the processes $e^+e^- \to \nu_4\overline{\nu_4}$ (Dirac case) and $e^+e^- \to N_1N_1$ (Majorana case) are performed in Sec. IV using the COMPHEP 4.4.3 package. Finally, we give some concluding remarks in Sec. V.

II. FLAVOR DEMOCRACY AND THE FOURTH SM FAMILY

It is useful to consider three different bases:

(i) Standard model basis $\{f^0\}$,

(ii) Mass basis $\{f^m\}$ and

(iii) Weak basis $\{f^w\}$.

According to the three-family SM, before the spontaneous symmetry breaking quarks are grouped into the following $\mathrm{SU}(2) \times \mathrm{U}(1)$ multiplets,

$$\begin{pmatrix} u_L^0 \\ d_L^0 \end{pmatrix}, u_R^0, d_R^0; \qquad \begin{pmatrix} c_L^0 \\ s_L^0 \end{pmatrix}, c_R^0, s_R^0; \qquad \begin{pmatrix} t_L^0 \\ b_L^0 \end{pmatrix}, t_R^0, b_R^0. \quad (1)$$

In the one-family case all bases are equal and, for example, d-quark mass is obtained due to

$$L_Y^{(d)} = a_d(\overline{u}_L \quad \overline{d}_L)(\overline{u}_L \quad \overline{d}_L)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_R + \mathrm{H.c.} \Rightarrow L_m^{(d)} = m_d\overline{d}d, \quad (2)$$

to Yukawa interaction where $m_d = a_d\eta$, $\eta = \langle\varphi^0\rangle \cong$ 249 GeV. In the same manner $m_u = a_u\eta$, $m_e = a_e\eta$, and $m_{\nu_e} = a_{\nu_e}\eta$ (if the neutrino is a Dirac particle). In the n-family case,

$$L_Y^{(d)} = \sum_{i,j=1}^{n} a_{ij}^d\left(\overline{u^0}_{\mathrm{Li}} \quad \overline{d^0}_{\mathrm{Li}}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_{Rj}^0 + \mathrm{H.c.} \Rightarrow L_Y^{(d)} = \sum_{i,j=1}^{n} m_{ij}^d\overline{d^0}_i d_j^0, \quad (3)$$

$$m_{ij}^d = a_{ij}^d\eta,$$

where d_1^0 denotes d^0, d_2^0 denotes s^0, etc. The diagonalization of the mass matrix of each type of fermion, or in other words transition from SM basis to mass basis, is performed by well-known bi-unitary transformation:

$$d_{iL}^m = (U_L^d)_{ij}d_{jL}^0, \qquad d_{iR}^m = (U_R^d)_{ij}d_{jR}^0, \quad (4)$$

similarly,

$$\begin{aligned} u_{iL}^m &= (U_L^u)_{ij}u_{jL}^0, & u_{iR}^m &= (U_R^u)_{ij}u_{jR}^0, \\ l_{iL}^m &= (U_L^l)_{ij}l_{jL}^0, & l_{iR}^m &= (U_R^l)_{ij}l_{jR}^0, \\ \nu_{iL}^m &= (U_L^\nu)_{ij}\nu_{jL}^0, & \nu_{iR}^m &= (U_R^\nu)_{ij}\nu_{jR}^0, \end{aligned} \quad (5)$$

where superscripts 0 and m represent SM and mass bases, respectively. Also, subscripts L and R correspond to left- and right-handed components, respectively. The last expression is valid for the Dirac neutrino, while the situation is more complicated for Majorana neutrinos. In general, there are 6 angles and 10 phases in every transformation matrices.

If one takes only electromagnetic interactions into consideration, one gets

$$J_{em}^0(d) = q_d\sum_i(\overline{d_{iL}^0}\gamma_\mu d_{iL}^0 + \overline{d_{iR}^0}\gamma_\mu d_{iR}^0) \quad (6)$$

in the SM basis. When the transformation from SM basis to mass basis is performed with the use of inverse of Eq. (4), one obtains

$$J^m_{em}(d) = q_d \sum_{i,j,k} (\overline{d^m_{kL}}(U^d_L)_{ki}\gamma_\mu (U^{d^+}_L)_{ij} d^m_{jL} + \overline{d^m_{kR}}(U^d_R)_{ki}\gamma_\mu (U^{d^+}_R)_{ij} d^m_{jR}). \tag{7}$$

Since

$$\sum_i (U^d_L)_{ki}(U^{d^+}_L)_{ij} = (U^d_L U^{d^+}_L)_{kj} = \delta_{kj};$$
$$\sum_i (U^d_R)_{ki}(U^{d^+}_R)_{ij} = (U^d_R U^{d^+}_R)_{kj} = \delta_{kj}, \tag{8}$$

one obtains

$$J^m_{em}(d) = q_d \sum_k (\overline{d^m_{kL}}\gamma_\mu d^m_{kL} + \overline{d^m_{kR}}\gamma_\mu d^m_{kR}). \tag{9}$$

As one can observe, the electromagnetic current is not changed with transformation from SM to mass basis. A similar situation takes place for interactions with the Z boson.

In the case of charged weak current,

$$J^0_W = \frac{g}{\sqrt{2}} \sum_i \overline{u^0_{iL}}\, \gamma_\mu d^0_{iL} \tag{10}$$

in the SM basis. The transformation from SM basis to mass basis leads to

$$J^m_W = \frac{g}{\sqrt{2}} \sum_{i,j,k} \overline{u^m_{kL}}(U^u_L)_{ki}\gamma_\mu (U^{d^+}_L)_{ij} d^m_{jL}, \tag{11}$$

where

$$\sum_i (U^u_L)_{ki}(U^{d^+}_L)_{ij} = (U^u_L U^{d^+}_L)_{jk} \neq \delta_{jk}. \tag{12}$$

In this context the well-known CKM matrix is defined as

$$U_{\mathrm{CKM}} = U^u_L (U^d_L)^+, \tag{13}$$

and contains 3 (6) observable mixing angles and 1 (3) observable CP-violating phases in the case of three (four) SM families. The weak basis is determined by the following transformation:

$$d^w_i = (U_{\mathrm{CKM}})_{ij} d^m_j. \tag{14}$$

In this basis the charged weak current is given by

$$J^w_W = \frac{g}{\sqrt{2}} \sum_i \overline{u^m_{iL}}\, \gamma_\mu d^w_{iL}. \tag{15}$$

Let us turn to flavor democracy hypothesis. Before the spontaneous symmetry breaking, all quarks are massless and there are no differences between d^0, s^0, and b^0. In other words, fermions with the same quantum numbers are indistinguishable. This leads us to the *first assumption* [1–3,22]; namely, Yukawa couplings are equal within each type of fermion:

$$a^d_{ij} \cong a^d, \quad a^u_{ij} \cong a^u, \quad a^l_{ij} \cong a^l, \quad a^\nu_{ij} \cong a^\nu. \tag{16}$$

The first assumption results in $n-1$ massless particles and one massive particle with $m = na^F\eta$ ($F = u, d, l, \nu$) for each type of the SM fermion.

Because there is only one Higgs doublet which gives Dirac masses to all four types of fermions, it seems natural to make the *second assumption* [5,6]; namely, the Yukawa constant for different types of fermions should naturally be equal:

$$a^d \approx a^u \approx a^l \approx a^\nu. \tag{17}$$

Taking into account the mass values for the third generation, the second assumption leads to the statement that, according to the flavor democracy, the fourth SM family should exist.

In terms of the mass matrix, the above arguments mean

$$M^0 = a\eta \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \Rightarrow M^m = 4a\eta \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}. \tag{18}$$

Now, let us make the *third assumption*; namely, a is between $e = g_W \sin(\theta_W) = \sqrt{4\pi\alpha_{em}}$ and $g_Z = g_W/\cos(\theta_W)$. Therefore, the fourth family fermions are almost degenerate, in good agreement with experimental value $\rho = 0.9998 \pm 0.0008$ [23], and their common mass lies between 320 and 730 GeV. The last value is close to the upper limit on heavy quark masses, $m_Q \leq 700$ GeV, which follows from partial-wave unitary at high energies [24]. It is of interest that, with preferable value $a \approx g_W$, flavor democracy predicts $m_4 \approx 8m_W \approx 640$ GeV.

In order to give nonzero masses for the first three SM family fermions, flavor democracy has to be slightly broken [7]. To perform this scheme, one has to consider getting the masses and CKM mixing matrix elements in the correct experimental range. Below we use the following parametrization for democracy breaking (assuming a modification which has a minimum effect on full democracy):

$$M_{(M)} = a\eta \begin{bmatrix} 1 & 1+\gamma & 1+\beta & 1-\beta \\ 1+\gamma & 1+2\gamma & 1+\beta & 1-\beta \\ 1+\beta & 1+\beta & 1+\alpha & 1-\alpha \\ 1-\beta & 1-\beta & 1-\alpha & 1+\alpha+2\beta \end{bmatrix}. \tag{19}$$

At the limit of $\gamma = \beta = 0$ this matrix becomes the one given in [6]. The parametrization given in Eq. (19) is slightly different from the texture of Eq. (2) in [7]. The addition of β in the fourth column and row allows us to improve the description of data in the quark sector as well as to describe correctly leptonic data. Let us remind the reader that according to the flavor democracy parameter a

is the same for all types of fermions, whereas α, β, and γ depend on F, where F denotes type of fermions, namely, up-quarks, down-quarks, charged leptons, and neutrinos. For the sake of simplicity, we do not consider CP-violating phases at this stage.

Current limits [9] on the known quark masses are presented in Table I, where the renormalization scale has been chosen to be $\mu = 2$ GeV for light quarks ($q = u, d, s$) and $\mu = m_q$ for heavy quarks ($q = c, b, t$). At the electroweak scale ($\mu \cong m_Z$), the mass values are converted into the ones given in Table II following the procedure presented in [25]. Eigenvalues of matrix (3) give us masses of corresponding fermions which are used to fix the values of parameters α, β, and γ. In Tables III and IV, we present these values for the up- and down-quark sectors with predicted values of the fourth family quark masses, taking g equal to g_W and e, respectively. The fourth SM family quarks' mass values $m_{q_4}(\mu \cong m_Z) \approx 400$ GeV correspond to $m_{q_4}(\mu = m_{q_4}) \approx 320$ GeV, and $m_{q_4}(\mu \cong m_Z) \approx 800$ GeV to $m_{q_4}(\mu = m_{q_4}) \approx 640$ GeV.

The quark CKM matrix is given as $O_{\rm CKM} = O_u O_d^T$, where O_u and O_d are (real) rotations which diagonalize up- and down-quark mass matrices. (We assume that 3 phase parameters in the quarks' CKM matrix are small enough to be neglected.) With the parameters given in Table III, one obtains

$$O_{\rm CKM} = \begin{bmatrix} 0.9747 & -0.2235 & -0.0028 & -0.0001 \\ 0.2232 & -0.9738 & -0.0439 & -0.0006 \\ -0.0125 & 0.0422 & -0.9990 & -0.0008 \\ -0.0002 & 0.0005 & 0.0008 & -1.0000 \end{bmatrix}. \tag{20}$$

With the parameters given in Table IV, the CKM matrix of quarks takes the form

$$O_{\rm CKM} = \begin{bmatrix} 0.9747 & 0.2236 & -0.0030 & -0.0002 \\ 0.2232 & -0.9738 & -0.0439 & -0.0012 \\ -0.0125 & -0.0422 & 0.9990 & -0.0014 \\ 0.0005 & -0.0011 & -0.0014 & 1.0000 \end{bmatrix}. \tag{21}$$

These matrices should be compared with the experimental one

$$\begin{bmatrix} 0.9730\text{–}0.9746 & 0.2174\text{–}0.2241 & 0.0030\text{–}0.0044 & * \\ 0.213\text{–}0.226 & 0.968\text{–}0.975 & 0.039\text{–}0.044 & * \\ 0\text{–}0.08 & 0\text{–}0.11 & 0.07\text{–}0.9993 & * \\ * & * & * & * \end{bmatrix} \tag{22}$$

taken from the Review of Particle Physics [9]. It is seen that our predictions are in good agreement with experimental data. It is remarkable that in quark sector 6 parameters allow to fit 6 masses and 3 mixing angles. This might be a sign of correctness of parametrization given in Eq. (19).

III. THE FOURTH SM FAMILY NEUTRINO

A. Dirac case

In the leptonic sector, we know masses of charged leptons precisely, whereas experiments give only upper limits for neutrino masses. Therefore, when determining parameters α, β, and γ in the neutrino sector we try to incorporate experimental data on mass square differences coming from neutrino oscillations. In Tables V and VI we present α, β, and γ parameters and corresponding masses for the leptonic sector with predicted mass values of the fourth SM family leptons taking a equal to g_W and e, respectively. As seen from Tables V and VI, the squared-mass differences are $\Delta m^2_{\rm SUN} = \Delta m^2_{21} = 7.8 \times 10^{-5}$ (eV)^2

TABLE II. Masses of known quarks at $\mu = m_Z$ scale.

Up quarks	m_u:(0.92–2.75) MeV	m_c:(545–763) MeV	m_t:184.4 ± 5.4 GeV
Down quarks	m_d:(3.06–5.20) MeV	m_s:(48.9–94.8) MeV	m_b:(2.82–3.17) GeV

TABLE III. Parameters and corresponding mass values for quark sector (at $\mu = m_Z$) taking $a = g_W$.

Up quarks	$\gamma = -0.00024$	$\beta = -0.005424$	$\alpha = 0.464$	
	$m_u = 2.03$ MeV	$m_c = 564.3$ MeV	$m_t = 186.714$ GeV	$m_{u_4} = 799.411$ GeV
Down quarks	$\gamma = 0.0001016$	$\beta = 0.0002152$	$\alpha = 0.0072$	
	$m_d = 4.21$ MeV	$m_s = 48.94$ MeV	$m_b = 2.84$ GeV	$m_{d_4} = 800.042$ GeV

TABLE IV. Parameters and corresponding mass values for quark sector (at $\mu = m_Z$) taking $a = e$.

Up quarks	$\gamma = -0.00048$	$\beta = -0.10848$	$\alpha = 0.928$	
	$m_u = 2.03$ MeV	$m_c = 564.6$ MeV	$m_t = 186.71$ GeV	$m_{u_4} = 399.41$ GeV
Down quarks	$\gamma = 0.0002032$	$\beta = 0.0004304$	$\alpha = 0.0144$	
	$m_d = 4.2$ MeV	$m_s = 48.94$ MeV	$m_b = 2.84$ GeV	$m_{d_4} = 400.042$ GeV

and $\Delta m^2_{\rm ATM} = \Delta m^2_{31} = 2.8\times 10^{-3}\,({\rm eV})^2$ which should be compared with the experimental data (the allowed ranges at 3σ) $7.2\times 10^{-5}\,({\rm eV})^2 < \Delta m^2_{\rm SUN} < 9.1\times 10^{-5}\,({\rm eV})^2$ and $1.4\times 10^{-3}\,({\rm eV})^2 < \Delta m^2_{\rm ATM} < 3.3\times 10^{-3}\,({\rm eV})^2$ [26].

The leptonic CKM matrix is $O^l_{\rm CKM} = O_\nu O_l^T$, where O_ν and O_l are rotations which diagonalize neutrino and charged lepton mass matrices. With the parameters given in Table V, one obtains

$$O^l_{\rm CKM} = \begin{bmatrix} 0.82 & 0.29 & 0.49 & -6.43\times 10^{-6} \\ -0.55 & 0.60 & 0.58 & 1.28\times 10^{-4} \\ 0.12 & 0.74 & -0.66 & 8.14\times 10^{-4} \\ -2.34\times 10^{-5} & 6.81\times 10^{-4} & 4.64\times 10^{-4} & 1.00 \end{bmatrix}. \tag{23}$$

With the parameters given in Table VI, the CKM matrix of leptons takes the form

$$O^l_{\rm CKM} = \begin{bmatrix} 0.82 & 0.29 & 0.49 & -3.59\times 10^{-6} \\ -0.55 & 0.60 & 0.58 & 6.39\times 10^{-5} \\ 0.12 & 0.74 & -0.66 & 4.07\times 10^{-4} \\ -1.17\times 10^{-5} & 3.41\times 10^{-4} & 2.32\times 10^{-4} & 1.00 \end{bmatrix}. \tag{24}$$

These matrices should be compared with the experimental data

$$\begin{bmatrix} 0.70\text{–}0.87 & 0.20\text{–}0.61 & 0.21\text{–}0.63 \\ 0.50\text{–}0.69 & 0.34\text{–}0.73 & 0.36\text{–}0.74 \\ 0.00\text{–}0.16 & 0.60\text{–}0.80 & 0.58\text{–}0.80 \end{bmatrix}, \tag{25}$$

which is the transpose of the matrix given in [27]. The reason for use of the transposed matrix is: the Maki-Nakawaga-Sakata matrix used in [27] links the neutrino weak eigenstates to the mass eigenstates, while leptonic CKM matrices in (23) and (24) relate charged lepton weak eigenstates to the mass eigenstates. As can be seen, our predictions are in good agreement with experimental data. Note that the values in Eq. (25), which are estimated for the three-family case, might be relaxed in the four-family case (as it happens in quark sector [9]).

With predicted fourth family lepton masses, given in Tables V and VI and lepton CKM matrices (23) and (24), one sees that the decay modes of the fourth SM family neutrinos are the following: $Br(\nu_4 \to \mu^- + W^+) \approx 0.68$, $Br(\nu_4 \to \tau^- + W^+) \approx 0.32$, and $Br(\nu_4 \to e^- + W^+) \approx 8\times 10^{-4}$.

B. Majorana case

As mentioned above, the SM does not prohibit Majorana mass terms for right-handed neutrinos. Therefore, the (4×4) mass matrix is replaced by the (8×8) mass matrix:

$$\frac{1}{2}\begin{pmatrix} \overline{\nu^0_{iL}} & \overline{\nu^{0C}_{iR}} \end{pmatrix}\begin{pmatrix} 0 & m^\nu_{ij} \\ m^\nu_{ij} & M_{ij} \end{pmatrix}\begin{pmatrix} \nu^{0C}_{jL} \\ \nu^0_{jR} \end{pmatrix}, \tag{26}$$

where $i, j = 1, 2, 3, 4$ (in this section we follow the notations of Ref. [28]).

According to the flavor democracy $m^\nu_{ij} = a^\nu_{ij}\eta = a^\nu\eta = a\eta$ and $M_{ij} = M$, where M is the Majorana mass scale of right-handed neutrinos. As a result of the transition from the SM basis to the mass basis, we obtain six massless Majorana neutrinos and two massive Majorana neutrinos with $m_1 = 2(\sqrt{4(a\eta)^2 + M^2} - M)$ and $m_2 = 2(\sqrt{4(a\eta)^2 + M^2} + M)$. In this case, while breaking flavor democracy, one should keep contributions to known neutrinos (ν_e, ν_μ, ν_τ) from other neutrino components small enough in order to avoid contradictions with experimental data on the weak charged currents. We have assumed that the light fourth family Majorana neutrino

TABLE V. Parameters and corresponding mass values for lepton sector taking $a = g_W$.

Charged leptons	$\gamma = -7.05\times 10^{-5}$	$\beta = -1.951\times 10^{-3}$	$\alpha = -3.729\times 10^{-3}$	
	$m_e = 0.511$ MeV	$m_\mu = 105.4$ MeV	$m_\tau = 1.777$ GeV	$m_{l_4} = 639.8$ GeV
Neutrinos	$\gamma = -0.732\times 10^{-13}$	$\beta = 0.671\times 10^{-13}$	$\alpha = -1.647\times 10^{-13}$	
	$m_{\nu_1} = 5.24\times 10^{-3}$ eV	$m_{\nu_2} = 1.03\times 10^{-2}$ eV	$m_{\nu_3} = 5.33\times 10^{-2}$ eV	$m_{\nu_4} = 640$ GeV

TABLE VI. Parameters and corresponding mass values for lepton sector taking $a = e$.

Charged leptons	$\gamma = -1.41\times 10^{-4}$	$\beta = -3.902\times 10^{-3}$	$\alpha = -7.458\times 10^{-3}$	
	$m_e = 0.511$ MeV	$m_\mu = 105.3$ MeV	$m_\tau = 1.777$ GeV	$m_{l_4} = 319.8$ GeV
Neutrinos	$\gamma = -1.464\times 10^{-13}$	$\beta = 1.342\times 10^{-13}$	$\alpha = -3.294\times 10^{-13}$	
	$m_{\nu_1} = 5.244\times 10^{-3}$ eV	$m_{\nu_2} = 1.028\times 10^{-2}$ eV	$m_{\nu_3} = 5.333\times 10^{-2}$ eV	$m_{\nu_4} = 320$ GeV

TABLE VII. The estimated values of heavy fourth family Majorana neutrino mass and mixing angle for $a = e$.

m_1, GeV	50	75	100	125	150	175	200
m_2, GeV	2048	1365	1024	819	683	585	512
$\cos\theta$	0.9968	0.9926	0.9861	0.9767	0.9637	0.9456	0.9214

TABLE VIII. The estimated values of heavy fourth family Majorana neutrino mass and mixing angle for $a = g_W$.

m_1, GeV	50	100	150	200	250	300	350	400
m_2, GeV	8192	4096	2731	2048	1638	1365	1170	1024
$\cos\theta$	0.9992	0.9968	0.9926	0.9861	0.9767	0.9636	0.9456	0.9214

mixes with known neutrinos in the same manner as the Dirac case. Mixings and masses of known neutrinos are assumed to be the same as the Dirac case also.

Let us consider the fourth family neutrinos in details. After the diagonalization of the fourth family neutrino mass term we have [29]

$$\nu_{4L} = i\cos\theta N_1 + \sin\theta N_2 \tag{27a}$$

$$(\nu_{4R})^C = -i\sin\theta N_1 + \cos\theta N_2, \tag{27b}$$

where N_1 and N_2 are light and heavy mass eigenstates of the fourth family Majorana neutrinos with corresponding mass eigenvalues,

$$m_{1,2} = 2(\sqrt{M^2 + 4(a\eta)^2} \mp M), \tag{28}$$

where $\tan 2\theta = (a\eta)/M$.

In Tables VII and VIII we present the estimated values of heavy fourth family Majorana neutrino mass and mixing angle for various m_1 values with $a = e$ and $a = g_W$, respectively. Right-handed Majorana mass scale M is assumed to be larger than Dirac case $a\eta$, which corresponds to $\tan 2\theta < 1$. Therefore, upper limits on m_1 are 200 and 400 GeV for $a = e$ and $a = g_W$, respectively.

It is seen that the ILC with $\sqrt{s} = 500$ GeV permits only pair production of N_1, whereas N_1N_2 and N_2N_2 production could be possible at higher center of mass energies. However, corresponding cross sections are suppressed by factors of $\sin^2\theta$ for N_1N_2 and $\sin^4\theta$ for N_2N_2 in addition to kinematical suppression. Moreover, the dominant decay mode of N_2 will be $N_2 \to l_4^{\pm}W^{\mp}$ since $m_{l_4} < m_2$; with subsequent decay $l_4 \to N_1 W$. For these reasons we will focus our attention on the process $e^+e^- \to N_1N_1$.

The part of the interaction Lagrangian responsible for production and decays of N_1 follows:

$$L = -\frac{g_W}{4\cos\theta_W}\cos^2\theta\overline{N_1}\gamma^\mu\gamma^5 N_1 Z_\mu - \left[i\sum_{i=1}^{3}\frac{g_W}{2\sqrt{2}} \times \cos\theta O_{4i}\overline{l_i}\gamma^\mu(1-\gamma^5)NW_\mu^- + \text{H.c.}\right], \tag{29}$$

where $l_1 = e$, $l_2 = \mu$, and $l_3 = \tau$. For numerical calculations we have used the O_{4i} values given in Eqs. (23) and (24). As a result, estimated branching ratios are the following: $Br(N_1 \to \mu^- + W^+) = Br(N_1 \to \mu^+ + W^-) \approx 0.34$, $Br(N_1 \to \tau^- + W^+) = Br(N_1 \to \tau^+ + W^-) \approx 0.16$, and $Br(N_1 \to e^- + W^+) = Br(N_1 \to e^+ + W^-) \approx 4 \times 10^{-4}$.

IV. PAIR PRODUCTION OF THE FOURTH SM FAMILY NEUTRINO AT e^+e^- COLLIDERS

In this section we analyze the processes $e^+e^- \to N_1N_1$ and $e^+e^- \to \nu_4\overline{\nu_4}$. For numerical calculations we have implemented the fourth SM family leptons into the COMPHEP 4.4.3 package [30]. The computed cross sections as a function of neutrino masses at three different center of mass energies, namely $\sqrt{s} = 0.5$ TeV (ILC), $\sqrt{s} = 1$ TeV (ILC or CLIC), and $\sqrt{s} = 3$ TeV (CLIC) are given in Figs. 1–3, respectively. Following arguments given in the previous section, we cut short the mass of m_1 at 200 GeV for $a = e$ and 400 GeV for $a = g_W$. Low value of

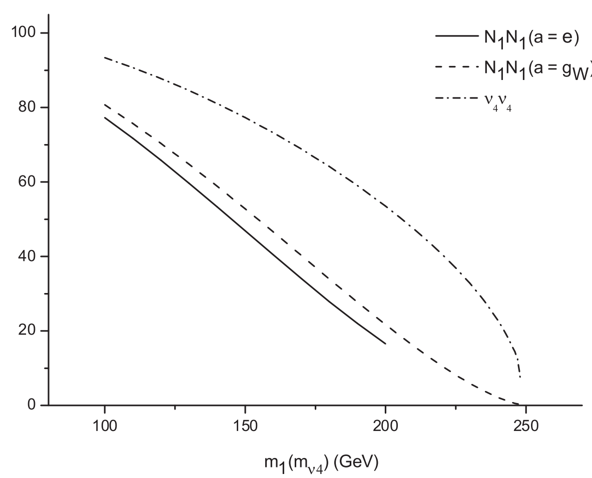

FIG. 1. Cross section for pair production of the fourth SM family neutrinos at $\sqrt{s} = 500$ GeV.

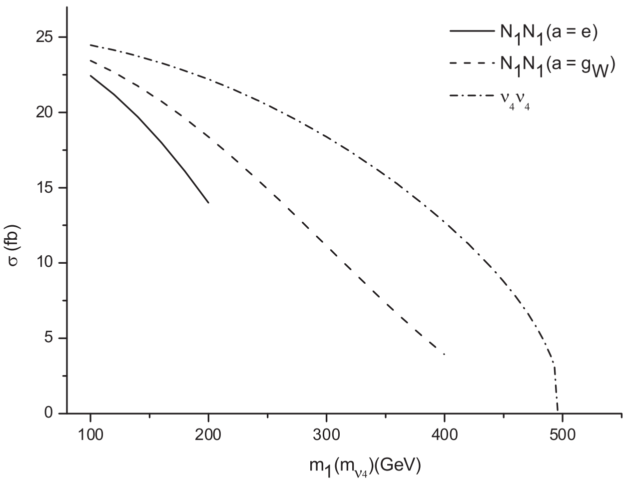

FIG. 2. Cross section for pair production of the fourth SM family neutrinos at $\sqrt{s} = 1$ TeV.

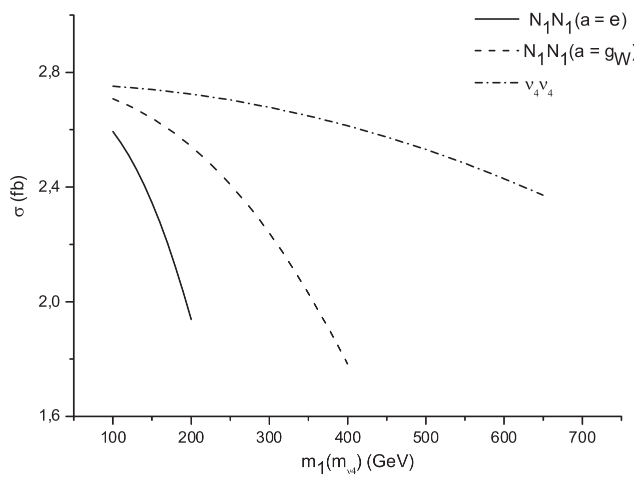

FIG. 3. Cross section for pair production of the fourth SM family neutrinos at $\sqrt{s} = 3$ TeV.

TABLE X. Expected event topologies ($l = e,\ \mu$).

	Branching ratios (%)	
Events	N_1N_1	$\nu_4\overline{\nu_4}$
$\mu^+\mu^+ + 4j$	5.35	⋯
$\mu^-\mu^- + 4j$	5.35	⋯
$\mu^+\mu^- + 4j$	10.7	21.4
$\mu^+\mu^+ + 2j + l + P_T^{\rm miss}$	3.3	⋯
$\mu^-\mu^- + 2j + l + P_T^{\rm miss}$	3.3	⋯
$\mu^+\mu^- + 2j + l + P_T^{\rm miss}$	6.6	13.2

Majorana neutrino production cross section with respect to Dirac neutrino one originates from two points: kinematical suppression [31,32] and mixing angle θ (see Tables VII and VIII). Indeed the ratio of cross sections for Majorana and Dirac neutrinos is given by

$$\frac{\sigma(e^+e^- \to N_1N_1)}{\sigma(e^+e^- \to \nu_4\overline{\nu_4})} = \frac{4\beta^2\cos^4\theta}{3+\beta^2}, \tag{30}$$

where $\beta = [1-(2m/s)^2]^{1/2}$ and $\cos\theta$ is defined in Eqs. (27). The expected event numbers per year for several mass values are presented in Table IX.

The decays of the fourth family neutrinos into muon channels provide the cleanest signature at e^+e^- colliders. Meanwhile, in our parametrization this channel is dominant. In the Majorana case the same-sign di-muon signature does not have any background and the total number of $(\mu^+\mu^+W^-W^-)$ and $(\mu^-\mu^-W^+W^+)$ events is 23% of the values given in Table IX. In the Dirac case the di-muon channel results in $(\mu^+\mu^-W^+W^-)$ events and their number is 46% of the values given in Table IX. The background from SM with three families computed by using COMPHEP leads to 830 events for $\sqrt{s} = 0.5$ TeV with 100 fb^{-1}, 1600 events for $\sqrt{s} = 1$ TeV with 300 fb^{-1} and 2000 events for $\sqrt{s} = 3$ TeV with 1000 fb^{-1}.

TABLE IX. Numbers of produced neutrino pairs in a working year for different center of mass energies.

	$\sqrt{s} = 0.5$ TeV, $L = 100$ fb^{-1}			$\sqrt{s} = 1$ TeV, $L = 300$ fb^{-1}			$\sqrt{s} = 3$ TeV, $L = 1000$ fb^{-1}		
	N_1N_1			N_1N_1			N_1N_1		
m (GeV)	$a = e$	$a = g_W$	$\nu_4\overline{\nu}_4$	$a = e$	$a = g_W$	$\nu_4\overline{\nu}_4$	$a = e$	$a = g_W$	$\nu_4\overline{\nu}_4$
100	7700	8000	9300	6700	7000	7300	2600	2700	2750
150	4700	5300	7700	5600	6400	7000	2300	2600	2740
200	1700	2200	5300	4200	5500	6700	1900	2500	2720
250	⋯	⋯	⋯	⋯	4500	6100	⋯	2400	2700
300	⋯	⋯	⋯	⋯	3300	5500	⋯	2200	2680
350	⋯	⋯	⋯	⋯	2200	4700	⋯	2000	2650
400	⋯	⋯	⋯	⋯	1200	3800	⋯	1800	2600

W bosons produced in decays of the fourth family neutrinos will be seen in the detector as either di-jets or isolated leptons. Keeping in mind the reconstruction of the fourth family neutrino mass, we assume that at least one W boson is decaying into di-jet. The expected event topologies are presented in Table X. As seen from the table, the events with a clean signature constitute about 35% of the total number of events given in Table IX.

As an example we would like to consider production of 200 GeV mass fourth family neutrinos at $\sqrt{s} = 500$ GeV linear collider by taking into account di-muon plus four jet events as signatures. In the Dirac case 1130 signal and 380 background events are expected. Concerning the fourth family Majorana neutrino, the events with the same-sign di-muons topology do not have significant SM background. In this case we expect 180 (230) signal events for $a = e$ ($a = g_W$).

V. CONCLUSION

Future lepton colliders will give a clear answer to the question of which nature the fourth family neutrino has: either Dirac or Majorana. The clearest signature for the Majorana neutrino case will be provided by same-sign dileptons accompanying either four jets or $2j + l + P_T^{\mathrm{miss}}$. In the Dirac case channel with the opposite-sign dileptons accompanying four jets seems to be the preferable one. The number of signal events with these topologies are sufficiently high to investigate the fourth family neutrino properties in detail.

ACKNOWLEDGMENTS

This work is supported in part by Turkish Planning Organization (DPT) under Grant No. 2002K120250 and Turkish Atomic Energy Authority.

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