PHYSICAL REVIEW D **72,** 045006 (2005)

Semiclassical approximation to supersymmetric quantum gravity

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(Received 1 June 2005; published 8 August 2005)

We develop a semiclassical approximation scheme for the constraint equations of supersymmetric canonical quantum gravity. This is achieved by a Born-Oppenheimer type of expansion, in analogy to the case of the usual Wheeler-DeWitt equation. The formalism is only consistent if the states at each order depend on the gravitino field. We recover at consecutive orders the Hamilton-Jacobi equation, the functional Schrödinger equation, and quantum gravitational correction terms to this Schrödinger equation. In particular, the following consequences are found: (i) the Hamilton-Jacobi equation and therefore the background spacetime must involve the gravitino, (ii) a (many-fingered) local time parameter has to be present on *super Riem* Σ (the space of all possible tetrad and gravitino fields), (iii) quantum supersymmetric gravitational corrections affect the evolution of the very early Universe. The physical meaning of these equations and results, in particular, the similarities to and differences from the pure bosonic case, are discussed.

DOI: 10.1103/PhysRevD.72.045006 PACS numbers: 04.65.+e, 04.60.−m, 11.15.Kc

I. INTRODUCTION

The consistent accommodation of the gravitational interaction into the framework of a quantum theory remains to be completed. Among the major approaches are string theory and canonical quantum gravity, cf. [1–3] and the references therein. It is generally assumed that supersymmetry [4] is a major ingredient of string theory. This is also one of the motivations for the study of a supersymmetric version of the canonical quantization of gravity, independent of the search for a unified theory of all interactions. In addition, the implementation of supersymmetry (SUSY) into canonical quantum gravity may simplify the formalism in a specific aspect: If the quantum constraint algebra closes, the (more complicated) Hamiltonian constraint is automatically fulfilled once the (simpler) SUSY constraints hold. The formalism is especially simplified if suitable choices are made regarding the canonical conjugate momenta, leading to simplifications in the SUSY constraints and therefore the Hamiltonian constraint. This has led to a detailed study of supersymmetric canonical quantum gravity, see [5,6] and the references therein for an introduction and review. Pertinent applications include black-hole physics [7] and quantum cosmology [5,6,8].

Research in supersymmetric quantum cosmology (SQC) provides the means to investigate some relevant problems concerning the evolution of the very early Universe, namely, (a) relating exact solutions found for spatially isotropic and anisotropic cosmologies with those obtained from the use of specific boundary conditions in usual quantum cosmology [9], (b) probing how the symmetry properties of dualities in superstring theory can be induced into the quantum states [10] and (c) analyzing the possibility of inflation occurrence and structure formation [11–13]. The essential feature is that SQC subscribes to the idea that treating both quantum gravity and SUSY effects as dominant would give an improved description of the early Universe. Such an approach can bring profound consequences for the wave function of the Universe: the quantum state can be written as an expansion in linearly independent fermionic sectors, each associated with a specific bosonic functional (of the same type as those satisfying the Wheeler-DeWitt equation). Besides the pertinent question of how to interpret the meaning of such quantum states [14], investigating whether conserved currents and a positive probability density can be obtained in this setting [15] must be performed by taking into account the enlarged structure for the wave function. Moreover, any cosmological evolution determined within SQC must eventually be consistent with a mechanism for SUSY breaking [11,16]. Nevertheless, in spite of all the progress achieved so far, further efforts are required to find *new* states determining a consistent dynamical path from a supersymmetric quantum cosmological to a classical cosmological stage [6].

It is in the context of the above description that the purpose of the present paper can be seen: investigate the semiclassical approximation of supersymmetric canonical quantum gravity. The viability of such a scheme is crucial for the framework of quantum gravity in the presence of

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supersymmetry. In the bosonic case, a formal Born-Oppenheimer type of approximation scheme has been successfully applied to the Wheeler-DeWitt equation [1]. This has led, in particular, to the derivation of quantum gravitational corrections terms which modify the limit of quantum theory on a fixed background spacetime [17–19]. Here we extend this formalism to the supersymmetric case. We restrict ourselves to the theory of $N = 1$ supergravity (SUGRA) in four spacetime dimensions.[1] Compared with the case of the bosonic Wheeler-DeWitt equation, this leads to equations of the same type, bearing similarities as well as various important differences arising from the presence of fermions (via SUSY). The framework and results presented herewith may constitute an efficient means to study the influence of SUSY in the physics of the very early Universe.

Our article is hence organized as follows. In Sec. II the Hamilton-Jacobi equation for supergravity is recovered. It is shown that the presence of the gravitino field is mandatory at each order of the approximation. The gravitino thus has to appear already at the order of the classical background spacetime. Section III presents the derivation of the functional Schrödinger equation for nongravitational fields, that is, the limit of quantum field theory on a given background. In Sec. IV we then derive supersymmetric quantum gravitational correction terms to this equation. Section V conveys a summary and discussion of our work and results, together with an outlook of subsequent future research to be followed. In order to assist the reader, some appendixes have been included. In Appendix A we review the formalism of supersymmetric canonical quantum gravity. The Born-Oppenheimer scheme for the bosonic case is briefly reviewed in Appendix B. Technical calculations referring to Secs. II, III, and IV are relegated to Appendixes C and D.

II. RECOVERY OF THE HAMILTON-JACOBI EQUATION

The purpose of this section is the application of the semiclassical approximation scheme, previously developed for the nonsupersymmetric case, to canonical quantum gravity with SUSY. Readers who are not familiar with semiclassical gravity, or with the canonical formalism for SUSY quantum gravity, may wish to consult Appendixes A, B, and C, see also [20].

We start by mentioning a well known general feature of all supersymmetric theories, namely, that the commutator of a primed and an unprimed SUSY transformation yields a coordinate transformation in spacetime. This translates into the anticommutator expression

$$[S_A(x), \bar{S}_{A'}(y)]_+ = 4\pi G\hbar \mathcal{H}_{AA'}(x)\delta(x, y), \tag{1}$$

where $S_A(x)$ and $\bar{S}_{A'}(y)$ are the constraints corresponding to the SUSY transformations. For consistency, then, $\mathcal{H}_{AA'}$ should also vanish as a constraint. This constraint is in fact related to the generators of spacetime transformations. We shall make use of the decomposition

$$\mathcal{H}_{AA'} = -n_{AA'}\mathcal{H}_\perp + e^i_{AA'}\mathcal{H}_i, \tag{2}$$

where $n_{AA'}$ and $e^i_{AA'}$ are the spinorial versions of the normal vector and the dreibein, respectively, cf. Appendix A; $\mathcal{H}_i$ and $\mathcal{H}_\perp$ denote, respectively, the gravitational momentum and Hamiltonian constraints. In particular, $\mathcal{H}_\perp$ is the normal projection of the constraint $\mathcal{H}_{AA'}$. We obtain it from (2) after multiplication and contraction with $-n^{AA'}$. Explicitly we have (in a quantum mechanical representation; the quantities are introduced and explained in Appendix A),

$$\begin{aligned}\mathcal{H}_{AA'} = {}& 4\pi G i\hbar^2 \psi^B_i \frac{\delta}{\delta e^{AB'}_j}\left[\epsilon^{ilm} D_B{}^{B'}{}_{mj} D^C{}_{A'kl}\frac{\delta}{\delta\psi^C_k}\right] \\ & - 4\pi G i\hbar^2 \frac{\delta}{\delta e^{AB'}_j}\left[D^{BB'}_{ij}\frac{\delta}{\delta e^{BA'}_i}\right] \\ & - \frac{i\hbar}{2}\epsilon^{ijk}\left[({}^{3s}\mathcal{D}_j\psi_{Ak})D^B{}_{A'li}\frac{\delta}{\delta\psi^B_l} \right. \\ & \left. + \psi_{Ai}\left({}^{3s}\mathcal{D}_j D^B{}_{A'lk}\frac{\delta}{\delta\psi^B_l}\right)\right] \\ & - i\hbar {}^{3s}\mathcal{D}_i\left(\frac{\delta}{\delta e^{AA'}_i} + \frac{1}{2}\epsilon^{ijk}\psi_{Aj}D^B{}_{A'lk}\frac{\delta}{\delta\psi^B_l}\right) \\ & + n_{AA'}\frac{1}{G}V[e],\end{aligned} \tag{3}$$

where $V[e] = \sqrt{h}{}^{3s}R/16\pi$. For the semiclassical approximation developed here we employ this version (1)–(3) of the constraints instead of those extracted directly from the action, cf. [5,6]. This has the advantage that the (formal) closure of the algebra, cf. also Eq. (A21), is automatically implemented. Expression (3) can be written in a less symmetric, but somewhat simplified form. For the fermionic part of the fifth line in (3), one can write

$$\begin{aligned}-\frac{1}{2}i\hbar^{3s}\mathcal{D}_i\left(\epsilon^{ijk}\psi_{Aj}D^B{}_{A'lk}\frac{\delta}{\delta\psi^B_l}\right) &= \frac{1}{2}\epsilon^{ijk}\,{}^{3s}\mathcal{D}_i(\psi_{Aj}\bar{\psi}_{A'k}) \\ &= \frac{1}{2}\epsilon^{ijk}[({}^{3s}\mathcal{D}_i\psi_{Aj})\bar{\psi}_{A'k} + \psi_{Aj}({}^{3s}\mathcal{D}_i\bar{\psi}_{A'k})] \\ &= \frac{1}{2}\epsilon^{ijk}[({}^{3s}\mathcal{D}_j\psi_{Ak})\bar{\psi}_{A'i} - \psi_{Ai}({}^{3s}\mathcal{D}_j\bar{\psi}_{A'k})].\end{aligned}$$

Comparing this with the third and fourth lines of (3),

$$\begin{aligned}&-\frac{i\hbar}{2}\epsilon^{ijk}\left[({}^{3s}\mathcal{D}_j\psi_{Ak})D^B{}_{A'li}\frac{\delta}{\delta\psi^B{}_l} + \psi_{Ai}\left({}^{3s}\mathcal{D}_j D^B{}_{A'lk}\frac{\delta}{\delta\psi^B_l}\right)\right] \\ &\quad = \frac{1}{2}\epsilon^{ijk}[({}^{3s}\mathcal{D}_j\psi_{Ak})\bar{\psi}_{A'i} + \psi_{Ai}({}^{3s}\mathcal{D}_j\bar{\psi}_{A'k})],\end{aligned}$$

[1] $N = 1$ SUGRA in four spacetime dimensions is the simplest SUSY extension of general relativity. It is related to the theory of $N = 1$ SUGRA in 11 spacetime dimensions, to which superstrings are associated in the context of M-theory.

we find that the terms containing ${}^{3s}\mathcal{D}_j(\bar{\psi}_{A'k})$ cancel out. The normal projection of the remaining term containing ${}^{3s}\mathcal{D}_j\psi^A_k$ is given by, cf. (A15) and (A16),

$$\begin{aligned} n^{AA'} i\hbar\epsilon^{ijk}({}^{3s}\mathcal{D}_j\psi_{Ak})D^B{}_{A'li}\frac{\delta}{\delta\psi^B_l} &= \frac{2\hbar}{\sqrt{h}} n^{AA'} e^{BC'}_i e_{CC'l} n^C{}_{A'} \epsilon^{ijk}({}^{3s}\mathcal{D}_j\psi_{Ak})\frac{\delta}{\delta\psi^B_l} \\ &= \frac{\hbar}{\sqrt{h}}\epsilon^{ijk} e^{BC'}_i e^A{}_{C'l}({}^{3s}\mathcal{D}_j\psi_{Ak})\frac{\delta}{\delta\psi^B_l}. \end{aligned}$$

For later use we introduce for the normal projection of the expression $\epsilon^{ilm}D_B{}^{B'}{}_{mj}D^C{}_{A'kl}$ the definition,

$$\begin{aligned} E^{CAB'i}_{Bjk} &\equiv n^{AA'}\epsilon^{ilm}D_B{}^{B'}{}_{mj}D^C{}_{A'kl} \\ &= \frac{-2i}{\sqrt{h}}\epsilon^{ilm}D_B{}^{B'}{}_{mj}e^{CD'}_l e_{DD'k} n^D_{A'} n^{AA'} \\ &= \frac{i}{\sqrt{h}}\epsilon^{ilm}D_B{}^{B'}{}_{mj}e^{CD'}_l e^A_{D'k}. \end{aligned} \tag{4}$$

We then get the following expression from (3):

$$\begin{aligned} \mathcal{H}_\perp &= -n^{AA'}\mathcal{H}_{AA'} \\ &= \underbrace{-4\pi i G\hbar^2 n^{AA'}\psi^B_i\frac{\delta}{\delta e^{AB'}_j}\left[\epsilon^{ilm}D_B{}^{B'}{}_{mj}D^C{}_{A'kl}\frac{\delta}{\delta\psi^C_k}\right]}_{\text{(i)}} \\ &\quad \underbrace{+ 4\pi G i\hbar^2 n^{AA'}\frac{\delta}{\delta e^{AB'}_j}\left[D^{BB'}_{ij}\frac{\delta}{\delta e^{BA'}_i}\right]}_{\text{(ii)}} \\ &\quad \underbrace{+\frac{\hbar}{\sqrt{h}}\epsilon^{ijk}e^{BC'}_i e^A{}_{C'l}({}^{3s}\mathcal{D}_j\psi_{Ak})\frac{\delta}{\delta\psi^B_l}}_{\text{(iii)}} \\ &\quad \underbrace{+ i\hbar n^{AA'\,3s}\mathcal{D}_i\frac{\delta}{\delta e^{AA'}_i}}_{\text{(iv)}} \underbrace{-\frac{1}{G}V[e]}_{\text{(v)}}. \end{aligned} \tag{5}$$

Since in the parts (i) and (ii) of (5) the functional derivative $\delta/\delta e^{AB'}_j$ can also act on $D_B{}^{B'}{}_{mj}D^C{}_{A'kl}$ and $D^{BB'}_{ij}$, we first calculate these derivatives (see Appendix C). We find

$$n^{AA'}\epsilon^{ilm}\frac{\delta}{\delta e^{AB'}_j}D_B{}^{B'}{}_{mj}D^C{}_{A'kl} = \left(\frac{-3i}{\sqrt{h}}\delta^i_k\epsilon_B{}^C - 2h^{ij}\epsilon_{jkl}n^{CB'}e^l_{BB'}\right)\delta(0), \tag{6}$$

and

$$n^{AA'}\frac{\delta}{\delta e^{AB'}_j}D^{BB'}_{ij} = \frac{i}{\sqrt{h}}e^{BA'}_i\delta(0). \tag{7}$$

The "divergence" $\delta(0)$ arises from the functional derivative at the same space point. It has to be regularized in a rigorous way, which is beyond the scope of this article. For the semiclassical approximation, addressing this issue is of less relevance, since it just corresponds to a factor ordering ambiguity. In the rest of this article we shall suppress $\delta(0)$.

The SUSY version of the Wheeler-DeWitt equation is then found to read

$$(\mathcal{H}_\perp + \mathcal{H}^{\mathrm{m}}_\perp)\Psi = 0, \tag{8}$$

where $\mathcal{H}_\perp$ (see discussion above) denotes the gravitational SUSY contribution to the Hamiltonian constraint, and $\mathcal{H}^{\mathrm{m}}_\perp$ is the contribution from nongravitational ("matter") fields. For definiteness we shall take for $\mathcal{H}^{\mathrm{m}}_\perp$ the Hamiltonian density of a minimally coupled scalar field Φ,

$$\mathcal{H}^{\mathrm{m}}_\perp = \frac{1}{2}\left(-\frac{\hbar^2}{\sqrt{h}}\frac{\delta^2}{\delta\Phi^2} + \sqrt{h}h^{ij}\Phi_{,i}\Phi_{,j} + \sqrt{h}(m^2\Phi^2 + U(\Phi))\right), \tag{9}$$

where h_{ij} denotes the three-metric, h its determinant, and the self-coupling potential $U(\Phi)$ is left unspecified. The more realistic (and more complicated) case of supermatter should be treated in a future work, cf. [5,6]. The state Ψ of SUSY quantum gravity is a wave functional defined on the space of all tetrad and gravitino fields (plus possible other fields) on a spatial hypersurface Σ. We shall call this space *super Riem* Σ, extending the notion *Riem* Σ for the space of all three-metrics in canonical quantum gravity [1].

Similarly to the bosonic case, we shall use an ansatz of the form (see Appendix B)

$$\Psi[e,\psi,\Phi] = \exp\left(\frac{i}{\hbar}S[e,\psi,\Phi]\right), \tag{10}$$

and expand S into a power series with respect to G,

$$S[e,\psi,\Phi] = \sum_{n=0}^{\infty} S_n[e,\psi,\Phi]G^{n-1}. \tag{11}$$

By means of this procedure, we then investigate the expansion of (8) in powers of G. The lowest order is G^{-2}. As in the bosonic case, this yields the independence of S_0 on the matter field Φ, that is, $S_0 \equiv S_0[e,\psi]$, cf. Appendix B.

At order G^{-1} we find contributions which determine the Hamilton-Jacobi equation of supersymmetric quantum gravity. It reads

$$\begin{aligned} 0 = 4\pi i\left(\psi^B_i\frac{\delta S_0}{\delta e^{AB'}_j}E^{CAB'i}_{Bjk}\frac{\delta S_0}{\delta\psi^C_k} - n^{AA'}D^{BB'}_{ij}\frac{\delta S_0}{\delta e^{AB'}_j}\frac{\delta S_0}{\delta e^{BA'}_i}\right) \\ + \frac{i}{\sqrt{h}}\epsilon^{ijk}e^{BC'}_i e^A{}_{C'l}({}^{3s}\mathcal{D}_j\psi_{Ak})\frac{\delta S_0}{\delta\psi^B_l} - n^{AA'\,3s}\mathcal{D}_i\frac{\delta S_0}{\delta e^{AA'}_i} \\ - V. \end{aligned} \tag{12}$$

Let us begin by investigating the question whether the Hamilton-Jacobi equation,

$$\frac{1}{2}G_{ijkl}\frac{\delta S_0}{\delta h_{ij}}\frac{\delta S_0}{\delta h_{kl}} + V^{\rm g} = 0, \tag{13}$$

of the bosonic case is contained in (12). For this purpose we ignore all terms involving the gravitino and reformulate the remaining part in terms of the three-metric h_{ij}.

If we assume that $S_0[e]$ can be rewritten as $S_0[h_{ij}]$, we can use Eq. (C29) to transform the functional derivatives. In addition, we have to take into account that the expansion parameter used in Appendix B (where the conventions of the bosonic case are used) differs by a factor 32π from the one we use here. Moreover, we have the following relations:

$$V[e] = -\frac{1}{32\pi}V^{\rm bos}[h_{ij}], \tag{14}$$

and

$$S_0 = \frac{1}{32\pi}S_0^{\rm bos}. \tag{15}$$

This, then, leads to

$$\frac{1}{2}G_{ijkl}\frac{\delta S_0^{\rm bos}}{\delta h_{ij}}\frac{\delta S_0^{\rm bos}}{\delta h_{kl}} + 64\pi n^{AA'}\partial_i e_{AA'j}\frac{\delta S_0^{\rm bos}}{\delta h_{ij}} + V^{\rm bos} = 0. \tag{16}$$

A comparison with (13) shows that there is almost a total equivalence. Only the second term in (16) has no counterpart there. Its presence can be traced back to the following observation.

As described in detail in [5,6], the constraints of supersymmetric quantum gravity can be established either (i) directly from the $N = 1$ SUGRA action by means of a variational principle or (ii) by further simplifying those mathematical expressions through a sensible choice of, for example, the canonical conjugate momenta in their (quantum) operator representation; this leads to much more tractable expressions. It is the anticommutator of the SUSY constraints within (ii) that produces the much simpler form of $\mathcal{H}_{AA'}$ and therefore of $\mathcal{H}_\perp$ and $\mathcal{H}_i$ [see Eqs. (1)–(3)]. If we had directly applied the approximation scheme to the quantum version of $\mathcal{H}_\perp$ within (i), the second term in (16) would be absent. One can interpret this term as originally belonging to the momentum constraints $\mathcal{H}_i$.

In the bosonic version of the semiclassical approximation one starts with the constraints as they are derived from the action. This is why there a term analogously to term (iv) in (5) is absent from the very beginning. Therefore, to facilitate the comparison with the bosonic case and to concentrate on the intrinsic differences, we shall not take into account term (iv),

$$i\hbar n^{AA'3s}\mathcal{D}_i\frac{\delta}{\delta e_i^{AA'}}, \tag{17}$$

in the following. Nevertheless, if one wanted, one could carry term (iv) through all the following expressions; this would, however, not have any consequences for the main results.

Since the Hamilton-Jacobi equation of pure general relativity, Eq. (13), can be recovered in this way, one may try to decompose the full Eq. (12) into a part depending only on the tetrad and a mixed part. We impose in addition the requirement that we must find the standard classical spacetime background in our approximation. Therefore we make the ansatz,

$$S_0[e,\psi] = B_0[e] + F_0[e,\psi], \tag{18}$$

for the lowest order in the expansion (11). On the level of the WKB wave functional, this corresponds to the factorization

$$\Psi[e,\psi] = \exp\left(\frac{i}{\hbar}B_0G^{-1}\right)\exp\left(\frac{i}{\hbar}(F_0G^{-1} + S_1 + \ldots)\right).$$

The pure bosonic part B_0 can be *chosen* such that

$$4\pi i n^{AA'}D_{ij}^{BB'}\frac{\delta B_0}{\delta e_j^{AB'}}\frac{\delta B_0}{\delta e_i^{BA'}} + V = 0, \tag{19}$$

corresponding to the Hamilton-Jacobi equation (13). A solution B_0 then determines the condition for the part F_0,

$$\begin{aligned} 0 = 4\pi i\Bigg(&\psi_i^B\frac{\delta F_0}{\delta e_j^{AB'}}E_{Bjk}^{CAB'i}\frac{\delta F_0}{\delta \psi_k^C} + \psi_i^B\frac{\delta B_0}{\delta e_j^{AB'}}E_{Bjk}^{CAB'i}\frac{\delta F_0}{\delta \psi_k^C} \\ &- n^{AA'}D_{ij}^{BB'}\frac{\delta F_0}{\delta e_j^{AB'}}\frac{\delta F_0}{\delta e_i^{BA'}} - n^{AA'}D_{ij}^{BB'}\frac{\delta F_0}{\delta e_j^{AB'}}\frac{\delta B_0}{\delta e_i^{BA'}} \\ &- n^{AA'}D_{ij}^{BB'}\frac{\delta B_0}{\delta e_j^{AB'}}\frac{\delta F_0}{\delta e_i^{BA'}}\Bigg) \\ &+ \frac{i}{\sqrt{h}}\epsilon^{ijk}e_i^{BC'}e^A{}_{C'l}({}^{3s}\mathcal{D}_j\psi_{Ak})\frac{\delta F_0}{\delta \psi_l^B}, \end{aligned} \tag{20}$$

which is automatically fulfilled if S_0 is a solution of (12)—without the omitted term (17)—and $F_0 = S_0 - B_0$.

It is now appropriate to interpret the solutions of the Hamilton-Jacobi equation (12). A particular aspect, distinguishing this equation from its bosonic analogue (see Appendix B) is the presence of the gravitino in the first and third terms. This means that it will generically be present in S_0 (or in F_0)—see Eq. (18) above. Moreover, one can indirectly prove that S_0 *must* depend on the gravitino. The argument goes as follows.

It is known from the full theory that a pure bosonic solution, $\Psi[e]$, to the full set of constraints cannot exist [21]. In fact, this argument can be extended in a straightforward way to each term in the semiclassical expansion, as we shall show now. In analogy to the full theory we act with the Hermitian conjugated SUSY constraint on Ψ [see Appendix A and, in particular, Eq. (A19)] and multiply it with $[\Psi]^{-1}$. With the ansatz $\Psi = \exp(i[S_0 G^{-1} + S_1 + S_2 G + \ldots]/\hbar)$ we obtain in the lowest order G^0:

$$[\Psi]^{-1}\bar{S}_{A'}\Psi \overset{\mathcal{O}(G^0)}{=} \epsilon^{ijk} e_{AA'i}{}^{3s}\mathcal{D}_j \psi_k^A + 4\pi i \psi_i^A \frac{\delta S_0}{\delta e^{AA'}{}_i} = 0. \tag{21}$$

Similar to the full theory this must hold for arbitrary fields ψ_i^A and $e_i^{AA'}$. First we find that (21) does not allow trivial solutions, that is, S_0 must at least depend on $e_i^{AA'}$. Otherwise we would get the condition

$$\epsilon^{ijk} e_{AA'i}{}^{3s}\mathcal{D}_j \psi_k^A = 0,$$

which cannot hold for all fields. Let us now assume that S_0 does not depend on the gravitino field ψ_i^A. Integrating (21) with an arbitrary continuous spinorial test function $\bar{\epsilon}^{A'}(x)$ over space leads to

$$I_0 \equiv \int d^3x \bar{\epsilon}^{A'} \left(\epsilon^{ijk} e_{AA'i}{}^{3s}\mathcal{D}_j \psi_k^A + 4\pi i \psi_i^A \frac{\delta S_0}{\delta e_i^{AA'}} \right) = 0.$$

Using as in [21] the replacement $\psi_i^A \mapsto \psi_i^A \exp(\phi(x))$ and $\bar{\epsilon}^{A'}(x) \mapsto \bar{\epsilon}^{A'}(x)\exp(-\phi(x))$, respectively, this yields a new integral,

$$\begin{aligned} I_0' \equiv \int d^3x \bar{\epsilon}^{A'} \exp(-\phi) \Big(& \epsilon^{ijk} e_{AA'i}{}^{3s}\mathcal{D}_j (\exp(\phi)\psi_k^A) \\ & + 4\pi i \exp(\phi) \psi_i^A \frac{\delta S_0}{\delta e_i^{AA'}} \Big) \\ = 0. & \end{aligned}$$

From $\Delta I_0 = I_0 - I_0' = 0$ one gets the same contradiction as in the full theory, since

$$\Delta I_0 = \int d^3x \epsilon^{ijk} e_i^{AA'}(x) \bar{\epsilon}^{A'}(x) \psi_{Ak}(x) \partial_j \phi(x) = 0$$

cannot hold for all fields. In higher orders, the calculation turns out to be simpler than in the lowest order. For $n \geq 1$ we obtain

$$[\Psi]^{-1}\bar{S}_{A'}\Psi \overset{\mathcal{O}(G^n)}{=} -4\pi i G^n \psi_i^A \frac{\delta S_n}{\delta e^{AA'}{}_i} = 0. \tag{22}$$

There are two possible conclusions. The first possibility is to assume that S_n does not depend on the bosonic field $e_i^{AA'}$. This would be very restrictive and, moreover, no proper bosonic limit would exist. We therefore dismiss this option as irrelevant for the semiclassical approximation. The second possibility to satisfy (22) is to introduce a dependence on the gravitino field at each order. Hence we must have $S^n \equiv S^n[e, \psi]$ for all n. The consequence is that the Hamilton-Jacobi equation—and therefore the now retrieved "background spacetime"—must necessarily involve the gravitino, a conclusion identical to what followed from the SUSY Hamilton-Jacobi equation (12), or (19) and (20).

Let us now elaborate more on this important feature. As shown in [22,23] (cf. also [1] and Appendix B) for the pure bosonic case (that is, for pure general relativity), a solution of the Hamilton-Jacobi equation conveys a classical spacetime which can serve as the appropriate background for the higher orders. This is due to the fact that such a solution is equivalent to the field equations originating from the Einstein-Hilbert action. This constitutes DeWitt's interpretation [22]: Every solution S_0 describes a family of solutions to the classical field equations. For every three-geometry there is one member of this family with a spacelike hypersurface being equal to this three-geometry. But, as mentioned, the situation in the semiclassical approximation of supersymmetric quantum gravity has a particular difference: *the presence of the gravitino.*

In order to address this feature it may prove relevant to mention the following. The use of strictly bosonic backgrounds constitutes the sole procedure in general relativity and has also been the norm when dealing with classical black-hole solutions in SUGRA and superstring theories [3,4]. Being more specific, it is required that those backgrounds, while satisfying the equations of motion, be invariant under SUSY transformations. This leads to conditions, namely, that the parameters of the SUSY transformation must satisfy a Killing spinor equation. Nevertheless, there have been some notable exceptions, see, for example, [24–26] and in particular [27]. In [24] an exact, asymptotically flat, stationary solution of the field equations of a SUGRA theory was found, constituting a supersymmetric generalization of a black-hole geometry. Subsequently, another type of solutions describing superpartners to the bosonic configurations was presented in [25]. In these solutions, the role of the classical configuration (e.g., the black hole) is played by a solution with certain fermionic (i.e., gravitino) field excitations. The full metric solution consists of a supermultiplet, formed by supertranslated partners to the purely bosonic configuration (see [25] for more details).

It is in this context that we can interpret our results for the SUSY Hamilton-Jacobi equation (12), inducing a spacetime background with both tetrad (graviton) and fermionic (gravitino) terms. Being more concrete, such a supersymmetric configuration will be a solution of the equations of motion of the theory, with a metric being written as

$$g = g_{\mathrm{B}} + g_{\mathrm{S}}, \tag{23}$$

where the term g_{B} denotes the "body" and g_{S} the "soul," adopting the definitions and nomenclature introduced by DeWitt in [27]. It should be noticed that g_{B} and g_{S} correspond, respectively, to the purely bosonic and fermionic sectors. In other words, a spacetime configuration induced from a solution of (12) will constitute a Grassmann-algebra-valued field that can be decomposed into the body which takes values in the domain of real or complex numbers and a soul which is nilpotent [27]. Moreover, we take the point of view that the body of the Grassmann-valued field must be given an operational interpretation and identified with a standard classical bosonic configuration. This overall description corresponds to the scenario of supermanifolds (and therefore superRiemannian geometries) thoroughly described in [27].[2]

A solution of the SUSY Hamilton-Jacobi equation (12) will thus correspond to a spacetime (yielding an appropriate background for the higher orders), whose metric includes the standard classical bosonic sector plus corrections in the form of gravitino terms. Therefore, DeWitt's interpretation could again be employed: Every such solution S_0 describes a family of solutions to the classical field equations. For every three-geometry there is one member of this family with an appropriate spacelike hypersurface. The important additional feature is that we will then be dealing with a configuration defined on the space of all possible spatial tetrads and gravitino fields, *super Riem* Σ. The standard classical background for Eq. (12) and the expansion (18) can be interpreted as follows: B_0 together with (19) and a condition obtained from the expansion of the other constraints yields a standard classical spacetime without gravitino. The part F_0 and (20) would then provide corrections to this. Such an interpretation, however, does not close the discussion on the issue and further analysis is certainly required. Finally, and although perhaps surprising at first glance, the presence of the gravitino (even at higher orders of approximation) is not necessarily in conflict with observation. Long ago, Pauli has performed a WKB approximation for a Dirac electron, which has some similarities to the present scheme [28]: The semiclassical approximation of the Dirac equation leads, in the leading order, to a Hamilton-Jacobi equation for a spinless classical relativistic particle (where the mass is given by the electron mass). Only the next order (order $\hbar$) contains information about the electron spin. In the same way one might expect that the spin-3/2 nature of the gravitino does not play a role at the leading order of our semiclassical expansion scheme, that is, at the order of the Hamilton-Jacobi equation, but that it comes into play only at the following orders.

III. RECOVERY OF THE FUNCTIONAL SCHRÖDINGER EQUATION

We shall now proceed with the semiclassical expansion of (5) and (8). At order G^0 we expect to recover the functional Schrödinger equation. Neglecting the contribution of term (iv) in (5), we find

$$\begin{aligned} 0 = {}& \frac{1}{2}\Big(\frac{1}{\sqrt{h}}\frac{\delta^2 S_1}{\delta\Phi^2} - \frac{i\hbar}{\sqrt{h}}\frac{\delta S_1}{\delta\Phi}\frac{\delta S_1}{\delta\Phi} + \sqrt{h}h^{ij}\Phi_{,i}\Phi_{,j} \\ &+ \sqrt{h}(m^2\Phi^2 + U(\Phi))\Big) + 4\pi\Big[2i\psi_i^B\frac{\delta S_{(0}}{\delta e_j^{AB'}}E_{Bjk}^{CAB'i}\frac{\delta S_{1)}}{\delta\psi_k^C} \\ &- \hbar\psi_i^B E_{Bjk}^{CAB'i}\frac{\delta^2 S_0}{\delta e_j^{AB'}\delta\psi_k^C} - \hbar\Big(\frac{3i}{\sqrt{h}}\psi_k^C \\ &+ \psi^{Bj}\epsilon_{jkl}n^{CB'}e^l_{BB'}\Big)\frac{\delta S_1}{\delta\psi_k^C} - 2in^{AA'}\frac{\delta S_{(0}}{\delta e_j^{AB'}}D_{ij}^{BB'}\frac{\delta S_{1)}}{\delta e_i^{BA'}} \\ &- \frac{i\hbar}{\sqrt{h}}e_i^{BA'}\frac{\delta S_0}{\delta e_i^{BA'}} - \hbar n^{AA'}D_{ij}^{BB'}\frac{\delta^2 S_0}{\delta e_j^{AB'}\delta e_i^{BA'}}\Big] \\ &+ \frac{i}{\sqrt{h}}\epsilon^{ijk}e_i^{BC'}e^A{}_{C'l}({}^{3s}\mathcal{D}_j\psi_{Ak})\frac{\delta S_1}{\delta\psi_l^B}. \end{aligned} \tag{24}$$

In analogy to Appendix B we simplify this equation by introducing the wave functional

$$\chi = W[e,\psi]\exp\Big(\frac{i}{\hbar}S_1[e,\psi,\Phi]\Big), \tag{25}$$

cf. Eq. (B7), by demanding the following condition for the WKB prefactor W:

$$\begin{aligned} 0 = {}& \psi_i^B\frac{\delta S_0}{\delta e_j^{AB'}}E_{Bjk}^{CAB'i}\frac{\delta W}{\delta\psi_k^C} + \psi_i^B\frac{\delta W}{\delta e_j^{AB'}}E_{Bjk}^{CAB'i}\frac{\delta S_0}{\delta\psi_k^C} \\ &- n^{AA'}\frac{\delta S_0}{\delta e_j^{AB'}}D_{ij}^{BB'}\frac{\delta W}{\delta e_i^{BA'}} - n^{AA'}\frac{\delta W}{\delta e_j^{AB'}}D_{ij}^{BB'}\frac{\delta S_0}{\delta e_i^{BA'}} \\ &+ \frac{1}{4\pi\sqrt{h}}\epsilon^{ijk}e_i^{BC'}e^A{}_{C'l}({}^{3s}\mathcal{D}_j\psi_{Ak})\frac{\delta W}{\delta\psi_l^B} \\ &- \Big[\psi_i^B E_{Bjk}^{CAB'i}\frac{\delta^2 S_0}{\delta e_j^{AB'}\delta\psi_k^C} + \Big(\frac{3i}{\sqrt{h}}\psi_k^C \\ &+ \psi^{Bj}\epsilon_{jkl}n^{CB'}e^l_{BB'}\Big)\frac{\delta S_0}{\delta\psi_k^C} + n^{AA'}D_{ij}^{BB'}\frac{\delta^2 S_0}{\delta e_j^{AB'}\delta e_i^{BA'}} \\ &+ \frac{i}{\sqrt{h}}e_i^{BA'}\frac{\delta S_0}{\delta e_i^{BA'}}\Big]W. \end{aligned} \tag{26}$$

We can then rewrite this condition in the form

$$\begin{aligned} &n^{AA'}\frac{\delta}{\delta e_j^{AB'}}\Big(D_{ij}^{BB'}\frac{\delta S_0}{\delta e_i^{BA'}}W - \psi_i^B\epsilon^{ilm}D_B{}^{B'}{}_{mj}D^C{}_{A'kl}\frac{\delta S_0}{\delta\psi_k^C}W\Big) \\ &= n^{AA'}\frac{\delta S_0}{\delta e_j^{AB'}}D_{ij}^{BB'}\frac{\delta W}{\delta e_i^{BA'}} - \psi_i^B\frac{\delta S_0}{\delta e_j^{AB'}}E_{Bjk}^{CAB'i}\frac{\delta W}{\delta\psi_k^C} \\ &\quad - \Big(\frac{3i}{\sqrt{h}}\psi_k^C + \psi^{Bj}\epsilon_{jkl}n^{CB'}e^l_{BB'}\Big)\frac{\delta S_0}{\delta\psi_k^C}. \end{aligned} \tag{27}$$

[2]A related discussion is made in [26], with the introduction of a line element $ds^2 = g_{AB}dz^A dz^B$, where $z^A = (x^\mu, \theta^\alpha)$, and θ^α being Grassmannian coordinates. The metric can then be divided in sectors such as, e.g., Bose-Bose $g_{\mu\nu}$, and Fermi-Fermi $g_{\alpha\beta}$.

We recognize that the right-hand side of Eq. (27) prevents it to be interpreted as a conservation law [see Eq. (B8)] in the context of *super Riem* Σ. As expected, only in the very special case of a vanishing dependence of S_0 and W on the gravitino can a conservation law be formulated. By assuming that $S_0[e]$ and $W[e]$ can be rewritten as $S_0[h_{ij}]$ and $W[h_{ij}]$, we then obtain a simpler expression from (27), in the form of a conservation equation,

$$n^{AA'}\frac{\delta}{\delta e_j^{AB'}}\left(D_{ij}^{BB'}\frac{\delta S_0}{\delta e_i^{BA'}}W^{-2}\right)=0. \tag{28}$$

However, as we have seen above, S_0 *must* depend on the gravitino.

Inserting (25) and (26) into (24), we find the Tomonaga-Schwinger equation,

$$\begin{aligned}4\pi i(i\hbar)\bigg[&\psi_i^B\frac{\delta S_0}{\delta e_j^{AB'}}E_{Bjk}^{CAB'i}\frac{\delta}{\delta\psi_k^C}+\psi_i^B E_{Bjk}^{CAB'i}\frac{\delta S_0}{\delta\psi_k^C}\frac{\delta}{\delta e_j^{AB'}}\\&-n^{AA'}\frac{\delta S_0}{\delta e_j^{AB'}}D_{ij}^{BB'}\frac{\delta}{\delta e_i^{BA'}}-n^{AA'}\frac{\delta S_0}{\delta e_i^{BA'}}D_{ij}^{BB'}\frac{\delta}{\delta e_j^{AB'}}\\&+\frac{1}{4\pi\sqrt{h}}\epsilon^{ijk}e_i^{BC'}e^{A}{}_{C'l}({}^{3s}\mathcal{D}_j\psi_{Ak})\frac{\delta}{\delta\psi_l^B}\bigg]\chi\\&\equiv i\hbar\frac{\delta\chi}{\delta\tau}=\mathcal{H}_\perp^{\rm m}\chi.\end{aligned} \tag{29}$$

The time functional $\tau(x;e,\psi]$ is defined by

$$\begin{aligned}4\pi\bigg[&\psi_i^B\frac{\delta S_0}{\delta e_j^{AB'}(y)}E_{Bjk}^{CAB'i}\frac{\delta}{\delta\psi_k^C(y)}+\psi_i^B E_{Bjk}^{CAB'i}\frac{\delta S_0}{\delta\psi_k^C(y)}\\&\times\frac{\delta}{\delta e_j^{AB'}(y)}-n^{AA'}\frac{\delta S_0}{\delta e_j^{AB'}(y)}D_{ij}^{BB'}\frac{\delta}{\delta e_i^{BA'}(y)}\\&-n^{AA'}\frac{\delta S_0}{\delta e_i^{BA'}(y)}D_{ij}^{BB'}\frac{\delta}{\delta e_j^{AB'}(y)}\\&+\frac{1}{4\pi\sqrt{h}}\epsilon^{ijk}e_i^{BC'}e^{A}{}_{C'l}({}^{3s}\mathcal{D}_j\psi_{Ak})\frac{\delta}{\delta\psi_l^B(y)}\bigg]\tau(x;e,\psi]\\&=\delta(x-y).\end{aligned} \tag{30}$$

For clarity we show the arguments (y) of the functional derivatives on the left-hand side. Note that, of course, all quantities involving the tetrad or the gravitino on this side depend on y. The functional Schrödinger equation is found from (29) after integration over space.

One may wish to separate (29) into a bosonic and a fermionic part, in analogy to the treatment of (12). In addition to the already decomposed S_0 we try the ansatz,

$$S_1=B_1[e,\Phi]+F_1[e,\psi,\Phi] \tag{31}$$

and assume a product ansatz for the WKB prefactor,

$$W[e,\psi]=W^b[e]W^f[\psi].$$

The wave functional χ can be factorized as

$$\chi=\tilde{\chi}\xi, \tag{32}$$

where

$$\tilde{\chi}=W^b\exp\left(\frac{i}{\hbar}B_1\right),\qquad \xi=W^f\left(\frac{i}{\hbar}F_1\right). \tag{33}$$

Now we see that an expansion of (29) with decomposed S_0 and S_1 contains a part of the form

$$\begin{aligned}4\pi i(i\hbar)\bigg[&\psi_i^B E_{Bjk}^{CAB'i}\frac{\delta F_0}{\delta\psi_k^C}\frac{\delta}{\delta e_j^{AB'}}-n^{AA'}\frac{\delta F_0}{\delta e_j^{AB'}}D_{ij}^{BB'}\frac{\delta}{\delta e_i^{BA'}}\\&-n^{AA'}\frac{\delta F_0}{\delta e_i^{BA'}}D_{ij}^{BB'}\frac{\delta}{\delta e_j^{AB'}}\bigg]\tilde{\chi}.\end{aligned} \tag{34}$$

To demand that these terms vanish is *not* possible, since F_0 is already determined by the Hamilton-Jacobi equation (12) and $\tilde{\chi}$ should be a solution of the reduced local Schrödinger equation (35), see below. This suggests that the requirement for a local Schrödinger equation that does not depend on the gravitino is impossible to achieve. We could of course simply demand that

$$\begin{aligned}-4\pi(i\hbar)i\bigg[&\frac{\delta S_0}{\delta e_j^{AB'}}n^{AA'}D_{ij}^{BB'}\frac{\delta}{\delta e_i^{BA'}}+\\&n^{AA'}D_{ij}^{BB'}\frac{\delta S_0}{\delta e_i^{BA'}}\frac{\delta}{\delta e_j^{AB'}}\bigg]\tilde{\chi}\equiv i\hbar\frac{\delta\tilde{\chi}}{\delta\tilde{\tau}}=\mathcal{H}_\perp^{\rm m}\tilde{\chi}\end{aligned} \tag{35}$$

holds with a redefined time functional $\tilde{\tau}$. But in order to obtain this, we have to impose various additional conditions, namely, for the factors W^b and W^f, as well as for the part F_1 that depends on the solution $\tilde{\chi}$ of (35). In particular, the part F_1 would be determined by the matter field, since (34) does not vanish in general. This should be carefully analyzed in view of the ansatz (31). Furthermore, additional conditions are hard to justify and may be without any physical meaning. The lesson learned from this is that the presence of the gravitino is mandatory for the definition of the time functional as well as for the Schrödinger equation.

Nevertheless, the interpretation of the time functional should be similar to the one given in Appendix B: It defines a local (''many-fingered'') time parameter. However, this should now be on the space of all possible spatial tetrad and gravitino fields, *super Riem* Σ. The question of how to interpret a classical background containing the gravitino, inducing this type of time functional, would require the discussion and proposed interpretation presented in the last section.

Finally, let us indicate that the functional Schrödinger equation can only be recovered in this way if a real solution S_0 to the Hamilton-Jacobi equation is chosen. One would not have been able to derive it from, for example, a superposition $\propto(\exp(iS_0)+\exp(-iS_0))$. This problem arises, of course, already in the nonsupersymmetric case where it was shown that the components in such a superposition become effectively independent due to decoherence by additional degrees of freedom [29]. The same is expected to hold here. Decoherence should be efficient during the greatest part of the evolution of the Universe. In some regions (such as the Planck regime or the region corresponding to a classical turning point) the various semiclassical components may interfere with each other and thereby spoil the validity of the approximation scheme presented here [30].

IV. CORRECTIONS TO THE SCHRÖDINGER EQUATION

We shall now continue with the semiclassical expansion scheme. At the order G^1 we find the following equation:

$$\Psi^{-1}(H_\perp + \mathcal{H}_\perp^{\mathrm{m}})\Psi \overset{\mathcal{O}(G^1)}{=} 4\pi G\Big[\underbrace{i\psi_i^B \frac{\delta S_1}{\delta e_j^{AB'}} E_{Bjk}^{CAB'i}\frac{\delta S_1}{\delta \psi_k^C}}_{\text{(i)}} + \underbrace{\hbar\psi_i^B E_{Bjk}^{CAB'i}\frac{\delta^2 S_1}{\delta e_j^{AB'}\delta\psi_k^C}}_{\text{(ii)}} - \underbrace{\hbar\Big(\frac{3i}{\sqrt{h}}\psi_k^C + \psi^{Bj}\epsilon_{jkl}n^{CB'}e_{BB'}^l\Big)\frac{\delta S_1}{\delta\psi_k^C}}_{\text{(iii)}} + \underbrace{2i\psi_i^B\frac{\delta S_{(0}}{\delta e_j^{AB'}}E_{Bjk}^{CAB'i}\frac{\delta S_{2)}}{\delta\psi_k^C}}_{\text{(iv)}} - \underbrace{2in^{AA'}\frac{\delta S_{(0}}{\delta e_j^{AB'}}D_{ij}^{BB'}\frac{\delta S_{2)}}{\delta e_i^{BA'}}}_{\text{(v)}} - \underbrace{in^{AA'}\frac{\delta S_1}{\delta e_j^{AB'}}D_{ij}^{BB'}\frac{\delta S_1}{\delta e_i^{BA'}}}_{\text{(vi)}} - \underbrace{\hbar n^{AA'}D_{ij}^{BB'}\frac{\delta^2 S_1}{\delta e_j^{AB'}\delta e_i^{BA'}}}_{\text{(vii)}} - \underbrace{\frac{i\hbar}{\sqrt{h}}e_i^{BA'}\frac{\delta S_1}{\delta e_i^{BA'}}}_{\text{(viii)}}\Big] + \underbrace{\frac{iG}{\sqrt{h}}\epsilon^{ijk}e_i^{BC'}e^A{}_{C'l}({}^{3s}\mathcal{D}_j\psi_{Ak})\frac{\delta S_2}{\delta\psi_l^B}}_{\text{(ix)}} + \underbrace{\frac{G}{2\sqrt{h}}\Big[2\frac{\delta S_1}{\delta\Phi}\frac{\delta S_2}{\delta\Phi} - i\hbar\frac{\delta^2 S_2}{\delta\Phi^2}\Big]}_{\text{(x)}} = 0. \tag{36}$$

In order to obtain the Schrödinger equation with corrections of order G, we must perform various steps on a formal level, which are very similar to those applied in [17]. The derivation is straightforward but lengthy. We therefore relegate some of the calculations to Appendix D and present here only the results and their physical discussion.

First we use Eqs. (D1)–(D8) to rewrite the expressions containing S_1 in (36) in terms of χ and W. We also use the definition (30) of the time functional $\tau(x; e, \psi]$ and make the decomposition

$$S_2[e, \psi, \Phi] = \sigma_2[e, \psi] + \eta[e, \psi, \Phi] \tag{37}$$

in order to separate the pure gravitational parts of (36) from those containing the matter field [17]. By demanding the following condition for $\sigma_2[e, \psi]$:

$$\frac{\delta\sigma_2}{\delta\tau} = -\frac{4\pi\hbar^2}{iW}\Big[\psi_i^B E_{Bjk}^{CAB'i}\Big(\frac{2}{W}\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta W}{\delta\psi_k^C} - \frac{\delta^2 W}{\delta e_j^{AB'}\delta\psi_k^C}\Big) + \Big(\frac{3i}{\sqrt{h}}\psi_k^C + \psi^{Bj}\epsilon_{jkl}n^{CB'}e_{BB'}^l\Big)\frac{\delta W}{\delta\psi_k^C} - \frac{2}{W}n^{AA'}D_{ij}^{BB'}\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta W}{\delta e_i^{BA'}} + n^{AA'}D_{ij}^{BB'}\frac{\delta^2 W}{\delta e_j^{AB'}\delta e_i^{BA'}} - \frac{i}{\sqrt{h}}e_i^{BA'}\frac{\delta W}{\delta e_i^{BA'}}\Big], \tag{38}$$

which is motivated by the analogous step in the standard quantum mechanical WKB expansion [31], we can rewrite (36) as

$$\frac{\delta\eta}{\delta\tau} = -\frac{4\pi\hbar^2}{i\chi}\Big[\psi_i^B E_{Bjk}^{CAB'i}\Big(\frac{\delta^2\chi}{\delta e_j^{AB'}\delta\psi_k^C} - \frac{1}{W}\Big\{\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta\chi}{\delta\psi_k^C} + \frac{\delta\chi}{\delta e_j^{AB'}}\frac{\delta W}{\delta\psi_k^C}\Big\}\Big) - \Big(\frac{3i}{\sqrt{h}}\psi_k^C + \psi^{Bj}\epsilon_{jkl}n^{CB'}e_{BB'}^l\Big)\times\frac{\delta\chi}{\delta\psi_k^C}\Big] + \frac{4\pi\hbar^2}{i\chi}\Big[n^{AA'}D_{ij}^{BB'}\frac{\delta^2\chi}{\delta e_j^{AB'}\delta e_i^{BA'}} - \frac{1}{W}n^{AA'}D_{ij}^{BB'}\Big\{\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta\chi}{\delta e_i^{BA'}} + \frac{\delta\chi}{\delta e_j^{AB'}}\frac{\delta W}{\delta e_i^{BA'}}\Big\} + \frac{i}{\sqrt{h}}e_i^{BA'}\frac{\delta\chi}{\delta e_i^{BA'}}\Big] + \frac{iG\hbar}{2\sqrt{h}}\Big[\frac{2}{\chi}\frac{\delta\chi}{\delta\Phi}\frac{\delta\eta}{\delta\Phi} + \frac{\delta^2\eta}{\delta\Phi^2}\Big]. \tag{39}$$

Up to the current order the wave functional has assumed the form

$$\Psi = \exp\Big(\frac{i}{\hbar}[S_0G^{-1} + S_1 + S_2G]\Big) = \frac{1}{W}\exp\Big(\frac{i}{\hbar}[S_0G^{-1} + \sigma_2 G]\Big)\chi\exp\Big(\frac{i}{\hbar}\eta G\Big). \tag{40}$$

Since we have already fixed the pure gravitational phase σ_2 in (38), and we are mainly interested in the matter part, we can restrict our attention to

$$\Theta \equiv \chi\exp\Big(\frac{i}{\hbar}\eta G\Big), \tag{41}$$

which contains the functional χ and the not yet determined part η of S_2. Now we multiply the uncorrected Eq. (29)

with $\exp(i\eta G/\hbar)$ and add it to Eq. (39) multiplied by $-G\chi\exp(i\eta G/\hbar)$. Using (D9) and (D10), we can perform the next steps and obtain the local Schrödinger equation with corrections up to the order G^1:

$$\begin{aligned} i\hbar\frac{\delta\Theta}{\delta\tau} &= \mathcal{H}^{\rm m}_{\perp}\Theta + \frac{4\pi G\hbar^2}{i\chi}\bigg[-\frac{1}{W}\psi^B_i E^{CAB'i}_{Bjk}\bigg(\frac{\delta W}{\delta e^{AB'}_j}\frac{\delta\chi}{\delta\psi^C_k} \\ &\quad + \frac{\delta\chi}{\delta e^{AB'}_j}\frac{\delta W}{\delta\psi^C_k}\bigg) + \psi^B_i E^{CAB'i}_{Bjk}\frac{\delta^2\chi}{\delta e^{AB'}_j\delta\psi^C_k} \\ &\quad - \bigg(\frac{3i}{\sqrt{h}}\psi^C_k + \psi^{Bj}\epsilon_{jkl}n^{CB'}e^l_{BB'}\bigg)\frac{\delta\chi}{\delta\psi^C_k}\bigg]\Theta \\ &\quad - \frac{4\pi G\hbar^2}{\chi}\bigg[n^{AA'}D^{BB'}_{ij}\frac{\delta^2\chi}{\delta e^{AB'}_j\delta e^{BA'}_i} \\ &\quad - \frac{1}{W}n^{AA'}D^{BB'}_{ij}\bigg(\frac{\delta W}{\delta e^{AB'}_j}\frac{\delta\chi}{\delta e^{BA'}_i} + \frac{\delta\chi}{\delta e^{AB'}_j}\frac{\delta W}{\delta e^{BA'}_i}\bigg) \\ &\quad + \frac{i}{\sqrt{h}}e^{BA'}_i\frac{\delta\chi}{\delta e^{BA'}_i}\bigg]\Theta. \end{aligned} \tag{42}$$

On a formal level, the terms in the fourth and fifth lines have the same structure as those which already appeared in the expansion of the Wheeler-DeWitt equation [17].

For further treating (42), we apply the idea of the decomposition into a "normal" and a "tangential" part [17], where normal means normal to hypersurfaces $S_0 =$ constant (thus being directed along the classical spacetimes defined by S_0), and tangential means tangential to $S_0 =$ constant, see below. For this purpose it is appropriate to introduce a metric $\mathcal{G}$ on the space *super Riem* Σ, which we shall call "super-DeWitt metric" and whose properties still have to be investigated. This metric should be the supersymmetric analogue of the DeWitt metric G_{ijkl}. The main difference to Appendix B is that we must consider now *super Riem* Σ, the direct sum of the tetrad space and the gravitino space. It contains vectors of the form $(e^{AA'i}, \psi^B_j) \equiv q_a$. Henceforth, the Latin indices starting with a are "condensed" superindices which run through all bosonic and fermionic degrees of freedom.

The metric $\mathcal{G}$ reads in block form

$$\mathcal{G}_{ab} = \begin{pmatrix} \mathcal{B} & S_1 \\ S_2 & \mathcal{F} \end{pmatrix}. \tag{43}$$

The blocks $\mathcal{B}$ and $\mathcal{F}$ denote the pure bosonic and pure fermionic part, respectively; S_1 and S_2 are the mixed off-diagonal parts. We determine the blocks by the requirement that the metric applied to the vectors $\delta S_0/\delta q_a \equiv (\delta S_0/\delta e^{AA'}_i, \delta S_0/\delta\psi^B_j)$ and $\delta\chi/\delta q_b$ yields all terms containing two derivatives in the local Schrödinger equation (29). The explicit form of the blocks can be read off immediately. For $\mathcal{B}$ one gets

$$\mathcal{B} = -4\pi i(n^{AA'}D^{BB'}_{ij} + n^{BB'}D^{AA'}_{ji}). \tag{44}$$

Since (29) contains no terms with a double derivative with respect to ψ^A_i, the lower diagonal block $\mathcal{F}$ vanishes,

$$\mathcal{F} = 0.$$

For S_1 and S_2 we get

$$\begin{aligned} S_1 &= 4\pi i n^{BB'}\psi^C_j\epsilon^{jkm}D_C{}^{A'}{}_{mi}D^D{}_{B'lk}, \\ S_2 &= 4\pi i n^{AA'}\psi^B_i\epsilon^{ilm}D_B{}^{B'}{}_{mj}D^C{}_{A'kl}. \end{aligned} \tag{45}$$

With arbitrary vectors

$$v^a = \begin{pmatrix} B^j_{AB'} \\ F^l_D \end{pmatrix} \quad\text{and}\quad \tilde{v}^b = \begin{pmatrix} \tilde{B}^i_{BA'} \\ \tilde{F}^k_C \end{pmatrix}, \tag{46}$$

we then obtain

$$\begin{aligned} \mathcal{G}_{ab}v^a\tilde{v}^b &= -4\pi i(n^{AA'}D^{BB'}_{ij} + n^{BB'}D^{AA'}_{ji})B^j_{AB'}\tilde{B}^i_{BA'} \\ &\quad + 4\pi i n^{BB'}\psi^C_j\epsilon^{jkm}D_C{}^{A'}{}_{mi}D^D{}_{B'lk}F^l_D\tilde{B}^i_{BA'} \\ &\quad + 4\pi i n^{AA'}\psi^B_i\epsilon^{ilm}D_B{}^{B'}{}_{mj}D^C{}_{A'kl}B^j_{AB'}\tilde{F}^k_C. \end{aligned} \tag{47}$$

For reasons of consistency, the upper diagonal part should contain the DeWitt metric. Of course, it cannot be exactly the DeWitt metric due to the change of the fundamental bosonic field from h_{ij} to $e^{AA'}_i$. In some sense it is a tetrad version of it, as we can easily see. We just have to apply the transformation (C29). Let $a[e]$ and $b[e]$ be two arbitrary functionals that can also be written as $a[h_{ij}]$ and $b[h_{ij}]$. We then have

$$\begin{aligned} &4\pi i\bigg(n^{AA'}D^{BB'}_{ij}\frac{\delta a}{\delta e^{AB'}_j}\frac{\delta b}{\delta e^{BA'}_i} + n^{BB'}D^{AA'}_{ji}\frac{\delta a}{\delta e^{AB'}_j}\frac{\delta b}{\delta e^{BA'}_i}\bigg) \\ &\quad = -32\pi G_{iljk}\frac{\delta a}{\delta h_{jk}}\frac{\delta b}{\delta h_{il}}. \end{aligned} \tag{48}$$

Therefore, we see that for quantities on superspace that can be written in terms of the three-metric h_{ij} and the gravitino the block $\mathcal{B}$ *is* the DeWitt metric.[3]

For further abbreviation and a clearer notation we introduce the operator

$$A := \frac{i}{\sqrt{h}}\epsilon^{ijk}e^{BC'}_i e^A{}_{C'l}({}^{3s}\mathcal{D}_j\psi_{Ak})\frac{\delta}{\delta\psi^B_l}. \tag{49}$$

Using these definitions, the Hamilton-Jacobi equation (12) without the omitted term (17) assumes the condensed form

$$\frac{1}{2}\mathcal{G}_{ab}\frac{\delta S_0}{\delta q_a}\frac{\delta S_0}{\delta q_b} + A(S_0) - V = 0. \tag{50}$$

We also obtain a short form of the local Schrödinger

[3] Note that the factor 32π arises due to the choice of our expansion parameter. It can be removed by a simple rescaling as in Sec. II. The minus sign appears due to our convention for the definition of (43).

equation (29),

$$i\hbar \mathcal{G}_{ab}\frac{\delta S_0}{\delta q_a}\frac{\delta \chi}{\delta q_b} + i\hbar A\chi = i\hbar\frac{\delta \chi}{\delta \tau} = \mathcal{H}_\perp^{\mathrm{m}}\chi. \tag{51}$$

The corrected local Schrödinger equation (42) then reads

$$\begin{aligned} i\hbar\frac{\delta\Theta}{\delta\tau} = \mathcal{H}_\perp^{\mathrm{m}}\Theta + \frac{G\hbar^2}{\chi}\bigg[&\frac{1}{W}\mathcal{G}_{ab}\frac{\delta\chi}{\delta q_a}\frac{\delta W}{\delta q_b} - \frac{1}{2}\mathcal{G}_{ab}\frac{\delta^2\chi}{\delta q_a\delta q_b} \\ &+ 4\pi i\Big(\frac{3i}{\sqrt{h}}\psi_k^C + \psi^{Bj}\epsilon_{jkl}n^{CB'}e^l_{BB'}\Big)\frac{\delta\chi}{\delta\psi_k^C} \\ &- \frac{4\pi}{\sqrt{h}}e_i^{BA'}\frac{\delta\chi}{\delta e_i^{BA'}}\bigg]\Theta. \end{aligned} \tag{52}$$

If the potential term V vanished, this would be the mathematical expression with which we would have to work. In our case of nonvanishing V it makes sense to decompose the correction terms in a normal and a tangential part, as mentioned above [17]. The directions are defined with respect to hypersurfaces *super Riem* Σ in which $S_0 =$ constant holds. The normal part is given by a vector parallel to $\delta S_0/\delta q_a$ and the tangential part by a vector orthogonal to $\delta S_0/\delta q_a$. In other words, we consider a trajectory of a classical spacetime[4] in configuration space and split it into a part in the direction of the evolution and a part transverse to it (see Appendix B and references therein for more details).

For the decomposition of the first correction term in (52), $\mathcal{G}_{ab}\delta\chi/\delta q_a \delta W/\delta q_b$, we make the ansatz

$$\mathcal{G}_{ab}\frac{\delta\chi}{\delta q_a} = \gamma\mathcal{G}_{ab}\frac{\delta S_0}{\delta q_a} + T_b. \tag{53}$$

Therein, γ denotes a factor which we determine as follows. For the tangential part T_b,

$$T_b\frac{\delta S_0}{\delta q_b} = 0 \tag{54}$$

holds.

Multiplication of (53) by $\delta S_0/\delta q_b$ yields

$$\mathcal{G}_{ab}\frac{\delta\chi}{\delta q_a}\frac{\delta S_0}{\delta q_b} = \gamma\mathcal{G}_{ab}\frac{\delta S_0}{\delta q_a}\frac{\delta S_0}{\delta q_b}. \tag{55}$$

Making use of (50) and (51) we get

$$\gamma = \frac{(\mathcal{H}_\perp^{\mathrm{m}} - i\hbar A)\chi}{2i\hbar\tilde{V}}, \tag{56}$$

where $\tilde{V} = (V - AS_0)$ denotes a modified potential.

[4] Let us remind the reader that we use a notion of "classical spacetime," as an element of *super Riem* Σ, which for our configuration involves the gravitino (see Sec. II and discussion at the end). More precisely, our background spacetime could be interpreted as a "classical spacetime with correction terms involving gravitinos."

The second-derivative terms in (52) can be decomposed by differentiating (53) with respect to q_b,

$$\begin{aligned} \mathcal{G}_{ab}\frac{\delta^2\chi}{\delta q_a\delta q_b} &= -\frac{\delta\mathcal{G}_{ab}}{\delta q_b}\frac{\delta\chi}{\delta q_a} + \frac{\delta T_a}{\delta q_a} + \frac{\delta\gamma}{\delta q_a}\mathcal{G}_{ab}\frac{\delta S_0}{\delta q_a} \\ &\quad + \gamma\mathcal{G}_{ab}\frac{\delta^2 S_0}{\delta q_a\delta q_b} + \gamma\frac{\delta\mathcal{G}_{ab}}{\delta q_b}\frac{\delta S_0}{\delta q_a} \\ &= \frac{\delta\gamma}{\delta q_a}\mathcal{G}_{ab}\frac{\delta S_0}{\delta q_a} + \gamma\mathcal{G}_{ab}\frac{\delta^2 S_0}{\delta q_a\delta q_b} + \tilde{T}, \end{aligned} \tag{57}$$

where $\tilde{T}$ denotes the sum of the tangential parts. We now rewrite the condition (26) in terms of the metric $\mathcal{G}_{ab}$ and the operator A,

$$\begin{aligned} 0 = \mathcal{G}_{ab}\frac{\delta S_0}{\delta q_a}\frac{\delta W}{\delta q_b} - A(W) - \frac{W}{2}\mathcal{G}_{ab}\frac{\delta^2 S_0}{\delta q_a\delta q_b} \\ - 4\pi iW\bigg(\frac{i}{\sqrt{h}}e_i^{BA'}\frac{\delta S_0}{\delta e_i^{BA'}} + \Big(\frac{3i}{\sqrt{h}}\psi_k^C \\ + \psi^{Bj}\epsilon_{jkl}n^{CB'}e^l_{BB'}\Big)\frac{\delta S_0}{\delta\psi_k^C}\bigg). \end{aligned} \tag{58}$$

Using (53), (57), and (58), we can write the correction terms in the form

$$\begin{aligned} &\frac{G\hbar^2}{\chi}\bigg[\frac{1}{W}\mathcal{G}_{ab}\frac{\delta\chi}{\delta q_a}\frac{\delta W}{\delta q_b} - \frac{1}{2}\mathcal{G}_{ab}\frac{\delta^2\chi}{\delta q_a\delta q_b} - \frac{4\pi}{\sqrt{h}}e_i^{BA'}\frac{\delta\chi}{\delta e_i^{BA'}} \\ &\quad + 4\pi i\Big(\frac{3i}{\sqrt{h}}\psi_k^C + \psi^{Bj}\epsilon_{jkl}n^{CB'}e^l_{BB'}\Big)\frac{\delta\chi}{\delta\psi_k^C}\bigg]\Theta \\ &\quad = C_n + C_t. \end{aligned} \tag{59}$$

We do not consider the tangential part C_t any further. It was discussed in the nonsupersymmetric case in [19], where technical and physical interpretations can be found. Because of the complicated formalism of supergravity, we restrict ourselves to the normal part C_n, which in analogy to the bosonic case is expected anyway to contain the dominating terms. To obtain an explicit form of it, we need a decomposition of the third and fourth term on the left-hand side of (59). It is obtained by defining

$$w_a := \Big(\frac{i}{\sqrt{h}}e_i^{BA'}, \frac{3i}{\sqrt{h}}\psi_k^C + \psi^{Bj}\epsilon_{jkl}n^{CB'}e^l_{BB'}\Big) \tag{60}$$

and writing

$$w_a\frac{\delta\chi}{\delta q_a} = \gamma w_a\frac{\delta S_0}{\delta q_a} + w_a\tilde{T}^a, \tag{61}$$

where $\tilde{T}^a$ is the tangential part. Making use of all preparations, we find that in the normal part many terms cancel out, and we obtain the form

$$
\begin{aligned}
C_n &= \frac{G\hbar^2}{\chi}\left[-\gamma\frac{(AW)}{W} - \frac{1}{2}\mathcal{G}_{ab}\frac{\delta S_0}{\delta q_a}\frac{\delta\gamma}{\delta q_b}\right] \\
&= \frac{G}{4\tilde{V}\chi}\left[(\mathcal{H}_\perp^{\mathrm{m}})^2 + i\hbar\frac{\delta(\mathcal{H}_\perp^{\mathrm{m}} - i\hbar A)}{\delta\tau}\right. \\
&\quad - \frac{i\hbar}{\tilde{V}}\left(\frac{\delta\tilde{V}}{\delta\tau} - (A\tilde{V})\right)(\mathcal{H}_\perp^{\mathrm{m}} - i\hbar A) \\
&\quad \left. - \frac{2(AW)}{W}(\mathcal{H}_\perp^{\mathrm{m}} - i\hbar A) + \frac{3A\mathcal{H}_\perp^{\mathrm{m}}}{i\hbar} - 2A^2\right]\chi.
\end{aligned} \tag{62}
$$

The definition (41) of the wave functional Θ leads to the following relation for arbitrary derivatives:

$$\frac{\delta\Theta}{\delta q} = \frac{\delta\chi}{\delta q}\exp\left(\frac{i}{\hbar}\eta G\right) + \mathcal{O}(G) = \frac{\delta\chi}{\delta q}\frac{\Theta}{\chi} + \mathcal{O}(G). \tag{63}$$

Therefore, the same relation holds for all higher derivatives:

$$\frac{\delta^n\Theta}{\delta q^n} = \frac{\delta^n\chi}{\delta q^n}\frac{\Theta}{\chi} + \mathcal{O}(G). \tag{64}$$

This enables us to rewrite all expressions containing χ in (62) in terms of Θ. We then obtain the final result for the normal part of the corrected local Schrödinger equation,

$$
\begin{aligned}
i\hbar\frac{\delta\Theta}{\delta\tau} &= \mathcal{H}_\perp^{\mathrm{m}}\Theta + \frac{G}{4\tilde{V}\chi}\left[(\mathcal{H}_\perp^{\mathrm{m}})^2 + i\hbar\frac{\delta(\mathcal{H}_\perp^{\mathrm{m}} - i\hbar A)}{\delta\tau}\right. \\
&\quad - \frac{i\hbar}{\tilde{V}}\left(\frac{\delta\tilde{V}}{\delta\tau} - (A\tilde{V})\right)(\mathcal{H}_\perp^{\mathrm{m}} - i\hbar A) \\
&\quad \left. - \frac{2(AW)}{W}(\mathcal{H}_\perp^{\mathrm{m}} - i\hbar A) + \frac{3A\mathcal{H}_\perp^{\mathrm{m}}}{i\hbar} - 2A^2\right]\Theta.
\end{aligned} \tag{65}
$$

It would yield a considerable simplification if we had a vanishing operator A. In particular, the term containing W would be absent. For a negligible A one would reduce the previous expression to

$$
\begin{aligned}
i\hbar\frac{\delta\Theta}{\delta\tau} &= \mathcal{H}_\perp^{\mathrm{m}}\Theta + \frac{4\pi G}{\sqrt{h}^{3s}R}\left[(\mathcal{H}_\perp^{\mathrm{m}})^2 + i\hbar\frac{\delta\mathcal{H}_\perp^{\mathrm{m}}}{\delta\tau} - \frac{i\hbar}{\sqrt{h}^{3s}R}\right. \\
&\quad \left. \times\frac{\delta(\sqrt{h}^{3s}R)}{\delta\tau}\mathcal{H}_\perp^{\mathrm{m}}\right]\Theta.
\end{aligned} \tag{66}
$$

On a formal level this is exactly the result that has been obtained from the expansion of the Wheeler-DeWitt equation. However, there is a difference: The definition of the time functional is different due to the involvement of the gravitino. But it can be seen that a vanishing gravitino would yield exactly the same time functional as in the pure bosonic case. In addition, using the definition (49), this would lead to a vanishing operator A. Therefore, the "bosonic limit" of supersymmetric quantum gravity yields up to the first order of correction terms bosonic canonical quantum gravity. This is a strong argument for the overall consistency of the supersymmetric theory.

As in [17], the presence of $\mathcal{H}_\perp^{\mathrm{m}}$ in the above corrections allows one to estimate their importance. For a Friedmann universe with scale factor a we can roughly estimate the ratio of the second (and third) to the first correction term in (66),

$$\frac{\hbar}{(\mathcal{H}_\perp^{\mathrm{m}})^2}\frac{\delta\mathcal{H}_\perp^{\mathrm{m}}}{\delta\tau} \sim \frac{\hbar\dot{a}}{(\mathcal{H}_\perp^{\mathrm{m}})^2}\frac{d\mathcal{H}_\perp^{\mathrm{m}}}{da} \sim \frac{\hbar H_0}{E}, \tag{67}$$

where E is a typical energy associated with the matter field. For $E = 700$ GeV and $H_0 = 70$ km/(s Mpc) we obtain approximately 10^{-44}. Therefore the quadratic matter Hamiltonian is usually the most important correction [17,19]. Interesting exceptions could be very light particles.

Let us add that a violation of unitarity due to the purely imaginary terms cannot be immediately concluded. This would require an inner product that we have not defined here, cf. [1]. Equation (65) is not independent of the chosen factor ordering. The explicit representation (49) depends on the factor ordering, since a commutation of ψ_i^A with its derivative changes this term. Future investigations will deal with the application of (65) in the context of quantum cosmology and structure formation.

V. DISCUSSION AND OUTLOOK

The purpose of this was paper was to establish a semiclassical approximation scheme for supersymmetric quantum gravity. This has been achieved by extending the Born-Oppenheimer method from the bosonic to the supersymmetric case. We have considered $N = 1$ SUGRA in four spacetime dimensions [4] and performed an expansion of the Hamiltonian constraint in powers of the gravitational constant by employing its quantum mechanical operator representation acting on a wave functional of the form (10) and (11). We have derived, at consecutive orders, the Hamilton-Jacobi equation, the functional Schrödinger equation, and quantum gravitational correction terms to this Schrödinger equation.

Within such a framework some relevant features have emerged. We have obtained explicit formulas to compute the quantum supersymmetric gravitational corrections that affect the evolution of the very early Universe during a phase where SUSY plays a crucial role. This would be of particular relevance for the quantum-to-classical transition and the ensuing structure formation [13]. We have also found that (i) the Hamilton-Jacobi equation and therefore the background spacetime must involve the gravitino, and (ii) a (many-fingered) local time parameter is present on *super Riem* Σ, the space of all tetrad and gravitino fields (plus possible other fields) on a spatial hypersurface Σ.

A possible interpretation for that was introduced and extensively discussed at the end of Sec. II. Summarizing it, the SUSY Hamilton-Jacobi equation (12) induces a spacetime background with both tetrad (graviton) and fermionic (gravitino) terms. It corresponds to a spacetime metric that will be a solution of the equations of motion of the theory, constituting a Grassmann-algebra-valued field that can be decomposed into the body which takes values in the domain of real or complex numbers and a soul which is nilpotent [24–27]. This description was introduced by DeWitt in the context of supermanifold configurations and is thoroughly described in [27]. Hence, a solution of the SUSY Hamilton-Jacobi equation (12) will correspond to a (classical) spacetime, in the sense of a classical spacetime with fermionic (gravitino) corrections, leading to a spacetime which can serve as the appropriate background for the higher orders.

Nevertheless, the proper interpretation of these issues require more study. A detailed investigation would, perhaps, require us to follow and extend the work of Gerlach [23]. More precisely, we should proceed to consider functionals of the form $\Psi \sim e^{iS/\hbar}$ and aim to derive the complete set of the equations of motion of $N = 1$ SUGRA in four spacetime dimensions, with S being a solution of the SUSY Hamilton-Jacobi equation (12). A directly obtained set of equations should be the Hamiltonian equations of motion with the presence of Tomonaga's local (many-fingered) time parameter. Integrating these equations on some special hypersurface should give the usual SUGRA equations of motion. The overall procedure should thus be checked with respect to the limiting case without gravitinos (and torsion), that is, with respect to general relativity. This would provide us with a better understanding of how and what type of spacetime background with fermionic corrections emerges, elucidating on the physical meaning of these deviations with respect to the case of canonical general relativity [1,22,23]. We intend to address this issue in a future research work.

Somewhat related with the above, there are two additional lines of work to be considered. In Sec. IV we have derived the quantum gravitational corrections to the Schrödinger equation, namely, normal and tangential correction components. Regarding the former, it would be of interest to investigate it further, applying it to illustrative minisuperspace case studies, and aiming to determine which type of effective quantum field theory and vacuum state are obtained as corrections regarding the general relativity case [17,18], in particular, to analyze if any shift in expectation values of, for example, energy levels in a matter Hamiltonian can be produced through a SUSY quantum gravitational origin. This would constitute a definite prediction from SQC, that is, the SUSY Wheeler-DeWitt equation. Even without addressing the issue of regularization, such correction terms could lead to quantum gravitational induced shifts, observable in principle in the spectrum of the cosmic background radiation. Concerning the tangential correction component (which was not studied in this paper), it would be of interest to check if and how it would reflect a breakdown of the classical background picture [17], probing the superspace environment near a classical solution of the SUGRA equations. Moreover, and following the footsteps of [19], perhaps the use of all constraints, interconnected by their constraint algebra, together with these component corrections, would allow one to generate a Feynman diagrammatic technique involving graviton and gravitino loops and vertices, revealing explicitly the backreaction effects. It could point as well to a correspondence between the framework of canonical and covariant SUGRA in a semiclassical limit. This is surely a rather ambitious line to investigate but we think it will provide most elucidating features for quantum gravity in general.

Another pertinent issue to address in the sequence of the framework present in this paper is the validity of minisuperspace approximation in SQC [6]. Different attempts in standard quantum cosmology can be found in [32,33]. In particular, it was pointed out that the minisuperspace approximation in quantum cosmology is valid only if the production of gravitons is negligible [33]. Hence, it would be fairly interesting to establish if the presence of fermions (gravitinos) and SUSY can either bring additional restrictive features on the validity of minisuperspace approximation or enlarge the range (through some regularization feature) where it can be employed. We intend to report on this issue in a future publication.

Finally, the introduction of the super-DeWitt metric in Sec. IV suggests the following possible work. In [34], a connection between the sign of the Wheeler-DeWitt metric and the attractivity of gravity was studied. The structure of *super Riem* Σ and its projection down to the true configurations space was studied for the bosonic case in [35]. It would be of interest to investigate what consequences the extra fermion (gravitino) correction terms would bring into this context.

ACKNOWLEDGMENTS

This work was supported by DAAD-GRICES/2004/2005-D/03/40416, CRUP-AI-A-21/2004 as well as Grants No. POCTI(FEDER) P-FIS-57547/2004 and No. CERN P-FIS-49529/2003. P. V. M. is supported by Grant No. FCT (FEDER) SFRH-BSAB 396/2003. He also wishes to thank Queen Mary College, London, for kind hospitality during his sabbatical and expresses his warmest thanks to the Institut für Theoretische Physik, Universität zu Köln, for kind hospitality during his visits. C. K. and T. L. acknowledge kind hospitality at the Universidade de Beira Interior, Covilhã.

APPENDIX A: CANONICAL QUANTIZATION OF $N = 1$ SUGRA

The canonical quantization scheme of general relativity starts with the 3 + 1 decomposition of spacetime and the reformulation of the classical action in terms of three-metric and extrinsic curvature. The central role is played by constraints which reflect the invariances of the classical theory. Upon quantization these constraints lead to restrictions on the allowed wave functionals [1]. In quantum geometrodynamics the central equations are the quantum Hamiltonian constraint or Wheeler-DeWitt equation and the diffeomorphism (or momentum) constraints.

In the following we shall summarize the canonical formulation of $N = 1$ SUGRA and its quantization. Details can be found in [5,6] and the references therein.

Dealing with general relativity in the presence of fermions requires that one has to work with a tetrad formalism instead of the metric. This formalism is therefore also needed for SUGRA where bosons and fermions are treated symmetrically: for the $N = 1$ case we shall have the gravitino as the fermionic partner to the graviton. More specifically, at every point of the spacetime manifold we introduce a pseudo-orthonormal basis e_a^μ of the tangential space and the corresponding basis e_μ^a of the cotangential space, where a is the flat index of the tetrad and runs from 0 to 3. Indices $a, b, c, \ldots$ are raised and lowered with η^{ab} and η_{ab}, respectively, where η^{ab} has the signature $(-, +, +, +)$. Spacetime indices are raised and lowered with $g^{\mu\nu}$ and $g_{\mu\nu}$, respectively. The connection between the spacetime metric and the internal metric is given by

$$g_{\mu\nu} = \eta_{ab} e_\mu^a e_\nu^b \tag{A1}$$

and

$$\eta^{ab} = g^{\mu\nu} e_\mu^a e_\nu^b, \tag{A2}$$

respectively.

In order to treat the bosonic and fermionic variables as similar as possible, it is appropriate to introduce a spinorial representation of the tetrad. This is possible since we can associate in flat space a spinor to any vector by the Infeld-van der Waerden symbols $\sigma_a^{AA'}$ which are given by

$$\sigma_0 = -\frac{1}{\sqrt{2}}\mathbb{I}, \qquad \sigma_i = \frac{1}{\sqrt{2}}\Sigma_i. \tag{A3}$$

Here, $\mathbb{I}$ denotes the unit matrix and Σ_i are the three Pauli matrices. The unprimed spinor indices $A, B, C, \ldots$ run from 1 to 2 and the primed spinor indices $A', B', C', \ldots$ take the values $1'$ and $2'$. The Latin indices starting with $i, j, k, \ldots$ assume the values 1, 2, and 3. Hence, the spinorial version of the tetrad reads

$$e_\mu^{AA'} = e_\mu^a \sigma_a^{AA'}. \tag{A4}$$

To raise and lower the spinor indices the different representations of the antisymmetric spinorial metric ϵ^{AB}, ϵ_{AB}, $\epsilon^{A'B'}$ and $\epsilon_{A'B'}$ are used. Each of them can be written as the same matrix given by

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Details of this formalism can be found, for example, in [4,36]. In curved spacetime, every tensor can be associated with a spinor using the spinorial tetrad, $T^{AA'} = e_\mu^{AA'} T^\mu$, with the inverse relation given by $T^\mu = -e^\mu_{AA'} T^{AA'}$.

For the foliation of spacetime into spatial hypersurfaces we need the future pointing unit normal vector n^μ, whose spinorial version is

$$n^{AA'} = e_\mu^{AA'} n^\mu. \tag{A5}$$

The tetrad is decomposed into the timelike and the spatial components $e_0^{AA'}$ and $e_i^{AA'}$. With the relation (A1) we find the three-metric

$$h_{ij} = -e_{AA'i} e_j^{AA'} = g_{ij}. \tag{A6}$$

This metric and its inverse are used to lower and raise the spatial indices $i, j, k, \ldots$. From the definition of $n^{AA'}$ as a future pointing unit normal to the spatial hypersurfaces Σ we obtain the relations

$$n_{AA'} e_i^{AA'} = 0 \quad \text{and} \quad n_{AA'} n^{AA'} = 1, \tag{A7}$$

which allow one to express $n^{AA'}$ in terms of $e_i^{AA'}$. An explicit representation is

$$n^{AA'} = \frac{i}{3\sqrt{h}} \epsilon^{ijk} e_i^{AB'} e_{BB'j} e_k^{BA'}, \tag{A8}$$

where $h \equiv \det h_{ij}$. Using the lapse function, N, and the shift vector, N^i, the timelike component of the tetrad can be decomposed according to

$$e_0^{AA'} = N n^{AA'} + N^i e_i^{AA'}. \tag{A9}$$

Further relations are collected in Appendix C.

The starting point of the formalism is the action of $N = 1$ SUGRA in four spacetime dimensions [4,37],[5]

$$S[e, \psi] = \int d^4x \bigg(\frac{1}{16\pi G} \det(e_\mu^a) R + \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} (\bar{\psi}_\mu^{A'} e_{AA'\nu} \mathcal{D}_\rho \psi_\sigma^A + \mathcal{D}_\rho \bar{\psi}_\sigma^{A'} e_{AA'\nu} \psi_\mu^A) \bigg), \tag{A10}$$

which includes the Einstein-Hilbert sector (with $\Lambda = 0$) and the Rarita-Schwinger component for the gravitino field ψ_μ^A with spin 3/2. The factor $\det(e_\mu^a)$ equals the square root of the determinant, $\sqrt{-g}$. The covariant derivative $\mathcal{D}_\rho$ acts only on the spinor indices and is defined via the spin connection forms $\omega^A{}_{B\rho}$ and $\bar{\omega}^{A'}{}_{B'\rho}$. Their explicit form can be found in [5]. The action (A10) is invariant under the following local transformations of the basic fields $e_\mu^{AA'}$ and ψ_μ^A: Local SUSY transformations, local Lorentz transfor-

[5]If the fields $e_i^{AA'}$ and ψ_i^A appear in the argument of a functional, the indices are often omitted for simplicity. For example, we write $S[e, \psi]$ instead of $S[e_i^{AA'}, \psi_i^A]$.

mations, and local coordinate transformations (diffeomorphisms).

The canonical fields for the Hamiltonian formulation of $N = 1$ SUGRA are the spatial components of the tetrad $e_i^{AA'}$ and the gravitino ψ_i^A and $\bar{\psi}_i^{A'}$. The momentum conjugate to the tetrad is defined by

$$p^i_{AA'} = \frac{\delta S}{\delta \dot{e}_i^{AA'}}, \tag{A11}$$

where the dot denotes the partial derivative with respect to the timelike direction. Often a symmetrized version is used,

$$\pi^{ij} \equiv -\tfrac{1}{2}p^{(ij)}, \qquad p^{ij} = -e^{AA'j}p^i_{AA'}. \tag{A12}$$

The momenta conjugate to the gravitino read

$$\begin{aligned} \pi^i_A &= \frac{\delta S}{\delta \dot{\psi}_i^A} = -\frac{1}{2}\epsilon^{ijk}\bar{\psi}_b^{A'}e_{AA'k}, \\ \tilde{\pi}^i_{A'} &= \frac{\delta S}{\delta \dot{\bar{\psi}}_i^{A'}} = \frac{1}{2}\epsilon^{ijk}\psi_b^A e_{AA'k}. \end{aligned} \tag{A13}$$

We denote the momentum conjugate to $\bar{\psi}_i^{A'}$ by $\tilde{\pi}^i_{A'}$ since it is *minus* the Hermitian conjugate of π^i_A. Since no time derivatives occur here, these are constraints which turn out to be of second class (since their algebra does not close). We thus have to formulate Dirac brackets instead of Poisson brackets [5]. They read

$$\begin{aligned} &[e_i^{AA'}(x), e_j^{BB'}(x)]_* = 0, \\ &[e_i^{AA'}(x), p^j_{BB'}(y)]_* = \epsilon^A{}_B\epsilon^{A'}{}_{B'}\delta_i^j\delta(x-y), \\ &[p^i_{AA'}(x), p^j_{BB'}(y)]_* = \tfrac{1}{4}(\epsilon^{jln}\psi_{Bn}D_{AB'kl}\epsilon^{ikm}\bar{\psi}_{A'm} \\ &\qquad + \epsilon^{jln}\psi_{Am}D_{BA'lk}\epsilon^{ikm}\bar{\psi}_{B'n})\delta(x-y), \\ &[\psi_i^A(x), \psi_j^B(y)]_* = 0, \\ &[\psi_i^A(x), \bar{\psi}_j^{A'}(y)]_* = -D_{ij}^{AA'}\delta(x-y), \\ &[e_i^{AA'}(x), \psi_j^B(y)]_* = 0, \\ &[p^i_{AA'}(x), \psi_j^B(y)]_* = \tfrac{1}{2}\epsilon^{ikl}\psi_{Al}D^B{}_{A'jk}\delta(x-y), \end{aligned} \tag{A14}$$

where

$$D_{ik}^{AB'} = \frac{-2i}{\sqrt{h}}e_k^{AC'}e_{CC'i}n^{CB'}. \tag{A15}$$

The remaining brackets are obtained by conjugating the relations containing the field ψ_i^A.

Because the action (A10) is invariant under local Lorentz, SUSY, and coordinate transformations, the canonical fields are subject to constraints. Regarding their quantum representation the following has to be included. As usual, the classical brackets (here: the Dirac brackets) are replaced by $-i/\hbar$ times the commutator or anticommutator of the corresponding field operators. For the Dirac brackets (A14) this can be achieved by choosing the following operator representation of the fundamental fields and momenta[6]:

$$\begin{aligned} \bar{\psi}^{A'}_i &= -i\hbar D_{ji}^{AA'}\frac{\delta}{\delta\psi_j^A}, \\ p^i_{AA'} &= -i\hbar\frac{\delta}{\delta e_i^{AA'}} - \frac{1}{2}i\hbar\epsilon^{ijk}\psi_{Aj}D^B{}_{A'lk}\frac{\delta}{\delta\psi_l^B}. \end{aligned} \tag{A16}$$

This is, of course, not the only possible choice. We can also represent ψ_i^A by a derivative with respect to $\bar{\psi}_i^{A'}$ if we choose a basis consisting of eigenstates of $\bar{\psi}_i^{A'}$. But since the Dirac bracket between $\bar{\psi}_i^{A'}$ and $\bar{\psi}_i^{A'}$ does not vanish, it is not possible to choose a basis of eigenstates with respect to both of them.

Upon quantization one encounters the usual factor ordering problems. This is of crucial relevance for the construction of the full theory, but of less relevance for the present issue of semiclassical approximation. We shall follow here Ref. [5] and do not consider other possibilities. The quantized Lorentz constraints read

$$J_{AB} = -\frac{i\hbar}{2}\left(e^{A'}_{Ba}\frac{\delta}{\delta e_a^{AA'}} + e^{A'}_{Aa}\frac{\delta}{\delta e_a^{BA'}} + \psi_{Ba}\frac{\delta}{\delta\psi^A{}_a} + \psi_{Aa}\frac{\delta}{\delta\psi_a^B}\right), \tag{A17}$$

$$\bar{J}_{A'B'} = -\frac{i\hbar}{2}\left(e^A_{B'a}\frac{\delta}{\delta e_a^{AA'}} + e^A_{A'a}\frac{\delta}{\delta e_a^{AB'}}\right), \tag{A18}$$

and the quantized SUSY constraints are given by

$$\bar{S}_{A'} = \epsilon^{ijk}e_{AA'i}{}^{3s}\mathcal{D}_j\psi_k^A + 4\pi G\hbar\psi_i^A\frac{\delta}{\delta e_i^{AA'}}, \tag{A19}$$

$$S_A = i\hbar^{3s}\mathcal{D}_i\left(\frac{\delta}{\delta\psi_i^A}\right) + 4\pi iG\hbar\frac{\delta}{\delta e_i^{AA'}}\left(D_{ji}^{BA'}\frac{\delta}{\delta\psi_j^B}\right). \tag{A20}$$

Calculating the anticommutator between the SUSY constraints yields

$$[S_A(x), \bar{S}_{A'}(y)]_+ = 4\pi G\hbar\mathcal{H}_{AA'}(x)\delta(x,y), \tag{A21}$$

with

[6]This form for the representation of the momenta allows the algebra of the constraints to have a simpler form; cf. Refs. [5,6] for further details.

$$\mathcal{H}_{AA'} = 4\pi G i\hbar^2 \psi_i^B \frac{\delta}{\delta e_j^{AB'}}\left[\epsilon^{ilm} D_B{}^{B'}{}_{mj} D^C{}_{A'kl} \frac{\delta}{\delta \psi_k^C}\right] - 4\pi G i\hbar^2 \frac{\delta}{\delta e_j^{AB'}}\left[D_{ij}^{BB'} \frac{\delta}{\delta e_i^{BA'}}\right] - \frac{i\hbar}{2}\epsilon^{ijk}\left[({}^{3s}\mathcal{D}_j \psi_{Ak}) D^B{}_{A'li} \frac{\delta}{\delta \psi_l^B} + \psi_{Ai}\left({}^{3s}\mathcal{D}_j D^B{}_{A'lk} \frac{\delta}{\delta \psi_l^B}\right)\right] - i\hbar\, {}^{3s}\mathcal{D}_i\left(\frac{\delta}{\delta e_i^{AA'}} + \frac{1}{2}\epsilon^{ijk}\psi_{Aj} D^B{}_{A'lk}\frac{\delta}{\delta \psi_l^B}\right) + n_{AA'}\frac{1}{G}V[e], \qquad \text{(A22)}$$

where $V[e] = \sqrt{h}{}^{3s}R/16\pi$. Note that from ${}^3\mathcal{D}_j$ (denoting a spatial covariant derivative acting on the spinor indices),

$${}^3\mathcal{D}_j T^{AA'} = \partial_j T^{AA'} + {}^3\omega_B^A T^{BA'} + {}^3\bar{\omega}_{B'}^{A'} T^{AB'}, \qquad \text{(A23)}$$

where ${}^3\omega_B^A$ and ${}^3\bar{\omega}_{B'}^{A'}$ are the two parts of the spin connection, see (C15), we obtain, by decomposing the three-dimensional spin connection ${}^3\omega_i^{AA'BB'}$ contained in the covariant derivative ${}^3\mathcal{D}_j$ into a pure bosonic part and the contorsion (C13),

$${}^3\omega_i^{AA'BB'} = {}^{3s}\omega_i^{AA'BB'} + {}^3\kappa_i^{AA'BB'}. \qquad \text{(A24)}$$

The torsion-free derivative is denoted by ${}^{3s}\mathcal{D}_j$. This also leads to simpler versions of the SUSY constraints [5], where $\bar{S}_{A'}$ is the Hermitian conjugate of S_A. They guarantee the invariance of the action under left- and right-handed SUSY transformations, respectively. Note that no torsion terms appear there. Moreover, ${}^3R$ is the three-dimensional scalar curvature (C17).

The calculation leading to (A21) shows that this factor ordering does not lead to quantum anomalies, at least not on a formal level. The expression of the right-hand side of (A21) can be interpreted as a combination of the Hamiltonian and momentum constraints obtained from the action of $N = 1$ SUGRA through variational methods plus combinations of the Lorentz constraints. A solution of the above quantum SUSY constraints must thus automatically obey the other constraints. It is an unsolved issue whether the full quantum algebra of constraints is free of anomalies. Calculations in [38] seem to indicate that anomalies may occur in the commutators of the SUSY constraints with $\mathcal{H}_{AA'}(x)$. A definite statement can, however, only be made after a rigorous regularization scheme has been employed. We assume in this paper that anomalies are absent. The question of anomalies is an open issue in all approaches of canonical quantum gravity [39].

APPENDIX B: THE SEMICLASSICAL APPROXIMATION SCHEME FOR CANONICAL QUANTUM GRAVITY

This appendix contains a brief review of the semiclassical approximation scheme, as it has been applied on a formal level to quantum geometrodynamics [1]. This will enable us, in particular, to make a comparison with the SUSY case discussed herein this article.

Our starting point is the full Wheeler-DeWitt equation and the momentum constraints,

$$\left(-16\pi G\hbar^2 G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} - \frac{1}{16\pi G}\sqrt{h}{}^3R + \mathcal{H}_\perp^{\mathrm{m}}\right)\Psi[h_{ij},\Phi] = 0, \qquad \text{(B1)}$$

$$\left(-\frac{2i}{\hbar}{}^3\nabla_j h_{ik}\frac{\delta}{\delta h_{jk}} + \mathcal{H}_i^{\mathrm{m}}\right)\Psi[h_{ij},\Phi] = 0, \qquad \text{(B2)}$$

where Φ denotes here a general nongravitational field. It is convenient to introduce the parameter

$$M \equiv \frac{1}{32\pi G}$$

and perform an expansion with respect to M. Although M does not have the dimension of a mass (it is proportional to the Planck mass squared), it brings the Wheeler-DeWitt equation into a form similar to the Schrödinger equation in quantum mechanics and thus allows the (formal) application of the Born-Oppenheimer scheme [1,18].[7] More generally, the approximation scheme starts with a division into "slow" and "fast" degrees of freedom. An expansion with respect to M is the simplest way to implement this idea, in that the gravitational variables are slow and the remaining (matter) variables (whose Hamiltonian is denoted by $\mathcal{H}_\perp^{\mathrm{m}}$) are fast. Equation (B1) then becomes

$$\left(-\frac{\hbar^2}{2M}G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{jk}} + MV^{\mathrm{g}} + \mathcal{H}_\perp^{\mathrm{m}}\right)\Psi = 0, \qquad \text{(B3)}$$

with $V^{\mathrm{g}} = -2\sqrt{h}{}^3R$. For the matter Hamiltonian density $\mathcal{H}_\perp^{\mathrm{m}}$ we assume for simplicity a minimally coupled scalar field Φ.

Making for the wave functional the ansatz,

$$\Psi[h_{ij},\Phi] = \exp\left(\frac{i}{\hbar}S[h_{ij},\Phi]\right) \qquad \text{(B4)}$$

and expanding

$$S[h_{ij},\Phi] = \sum_{n=0}^{\infty} S_n[h_{ij},\Phi]M^{-n+1},$$

[7] In quantum electrodynamics, one can perform an expansion with respect to the electric charge [40].

we find from (B3) several relevant equations at consecutive orders of M.

The highest order (M^2) expresses the independence of S_0 on the matter field Φ, that is, $S_0 \equiv S_0[h_{ij}]$. The next order (M^1) yields the Hamilton-Jacobi equation for the gravitational field,

$$\frac{1}{2}G_{ijkl}\frac{\delta S_0}{\delta h_{ij}}\frac{\delta S_0}{\delta h_{kl}} + V^{\mathrm{g}} = 0. \tag{B5}$$

Actually, (B5) represents an infinite number of equations, one at every point of space. In addition we have to expand the momentum constraints (B2) and obtain

$$h_{ij}{}^3\nabla_k\left(\frac{\delta S_0}{\delta h_{ik}}\right) = 0. \tag{B6}$$

Every solution of (B5) determines a family of solutions of the classical field equations. Equations (B5) and (B6) are equivalent to Einstein's field equations [22,23].

The next order (M^0) can be simplified by defining the wave functional

$$\chi = D[h_{ij}]\exp\left(\frac{iS_1[h_{ij}, \Phi]}{\hbar}\right). \tag{B7}$$

Choosing for the ''WKB prefactor'' D the ''conservation law'' (which in quantum mechanics would just express the conservation of probability)

$$G_{ijkl}\frac{\delta}{\delta h_{ij}}\left(\frac{1}{D^2}\frac{\delta S_0}{\delta h_{kl}}\right) = 0, \tag{B8}$$

the equation at this order becomes the ''Tomonaga-Schwinger equation'' or ''local Schrödinger equation''

$$i\hbar G_{ijkl}\frac{\delta S_0}{\delta h_{ij}}\frac{\delta \chi}{\delta h_{kl}} \equiv i\hbar\frac{\delta\chi}{\delta\tau} = \mathcal{H}_\perp^{\mathrm{m}}\chi, \tag{B9}$$

where the time functional τ is implicitly defined by

$$G_{ijkl}(x)\frac{\delta S_0}{\delta h_{ij}(x)}\frac{\delta\tau(y; h_{ij}]}{\delta h_{kl}(x)} = \delta(x-y). \tag{B10}$$

''Time'' is thus defined through the chosen solution S_0 of the Hamilton-Jacobi equation. In fact, τ is not a spacetime scalar, but the semiclassical scheme can nevertheless be consistently defined [41]. The (functional) Schrödinger equation is found upon integrating (B10) over three-dimensional space.

The next order (M^{-1}) yields quantum gravitational correction terms to (B9). In [17] only those correction terms were considered that act along the chosen classical spacetime; those terms appear to be the dominating one. In [19] all correction terms were treated in great detail. In the present case we followed the treatment in [17] in order to show the essential features of the semiclassical approximation scheme.

APPENDIX C: FORMULAS USED FOR THE CALCULATION OF SUPERSYMMETRIC EXPRESSIONS

1. General formulas

In Appendix A, we have chosen the signature of the four-metric $g_{\mu\nu}$ as $(-, +, +, +)$. Therefore, the metric h_{ij} on the spacelike hypersurfaces has the signature $(+, +, +)$ which gives a positive determinant. Thus the three-dimensional total antisymmetric tensor density can be defined by $\epsilon^{123} = \epsilon_{123} = +1$. Using this definition, we have for the timelike normal vector $n_{AA'}$ and the tetrad $e_i^{AA'}$ the relations

$$n_{AA'}n^{AB'} = \tfrac{1}{2}\epsilon_{A'}{}^{B'}, \tag{C1}$$

$$n_{AA'}n^{BA'} = \tfrac{1}{2}\epsilon_A{}^B, \tag{C2}$$

$$e_{AA'i}e_j^{AB'} = -\tfrac{1}{2}h_{ij}\epsilon_{A'}{}^{B'} - i\sqrt{h}\epsilon_{ijk}n_{AA'}e^{AB'k}, \tag{C3}$$

$$e_{AA'i}e_j^{BA'} = -\frac{1}{2}h_{ij}\epsilon_A{}^B - i\frac{1}{\sqrt{h}}\epsilon_{ijk}n_{AA'}e^{BA'k}, \tag{C4}$$

$$e_{AA'i}e^i_{BB'} = n_{AA'}n_{BB'} - \epsilon_{AB}\epsilon_{A'B'}. \tag{C5}$$

From Eqs. (C3) and (C4) we obtain by contracting with ϵ^{ijl},

$$n_{AA'}e^{AB'l} = -n^{AB'}e^k_{AA'} = \frac{i}{2\sqrt{h}}\epsilon^{ijl}e_{AA'i}e_j^{AB'}, \tag{C6}$$

$$n_{AA'}e^{BA'l} = -n^{BA'}e^k_{AA'} = -\frac{i}{2\sqrt{h}}\epsilon^{ijl}e_{AA'i}e_j^{BA'}. \tag{C7}$$

The three-dimensional torsion-free spin connection ${}^{3\mathrm{s}}\omega_i^{AA'BB'}$ can be expressed in terms of $n^{AA'}$ and $e_i^{AA'}$ [5],

$$\begin{aligned} {}^{3\mathrm{s}}\omega_i^{AA'BB'} = {}& e^{BB'j}\partial_{[j}e^{AA'}_{i]} - \tfrac{1}{2}(e^{AA'j}e^{BB'k}e_i^{CC'}\partial_j e_{CC'k} \\ &+ e^{AA'j}n^{BB'}n^{CC'}\partial_j e_{CC'i} + n^{AA'}\partial_i n^{BB'}) \\ &- e^{AA'j}\partial_{[j}e^{BB'}_{i]} + \tfrac{1}{2}(e^{BB'j}e^{BB'k}e_i^{CC'}\partial_j e_{CC'k} \\ &+ e^{BB'j}n^{AA'}n^{CC'}\partial_j e_{CC'i} + n^{BB'}\partial_i n^{AA'}). \end{aligned} \tag{C8}$$

The four-dimensional torsion is given by

$$S^{AA'}_{\mu\nu} = -4\pi i G\bar{\psi}^{A'}_{[\mu}\psi^A_{\nu]}, \tag{C9}$$

and its tensorial version reads

$$S^\rho{}_{\mu\nu} = -e^\rho_{AA'}S^{AA'}_{\mu\nu}. \tag{C10}$$

The contorsion tensor κ is defined by

$$\kappa_{\mu\nu\rho} = S_{\nu\mu\rho} + S_{\rho\nu\mu} + S_{\mu\nu\rho}. \tag{C11}$$

The three-dimensional contorsion is simply obtained by restriction of the four-dimensional quantity,

$$^3\kappa_{ijk} = \kappa_{ijk}. \tag{C12}$$

With the spinorial contorsion $^3\kappa^{AA'BB'i} = e^{AA'j} e_k^{BB'3}\kappa_{jki} = -^3\kappa^{BB'AA'i}$, the spin connection reads

$$^3\omega_i^{AA'BB'} = {}^{3s}\omega_i^{AA'BB'} + {}^3\kappa_i^{AA'BB'}. \tag{C13}$$

It can be decomposed into a primed and an unprimed part:

$$^3\omega_i^{AA'BB'} = {}^3\omega_i^{AB}\bar{\epsilon}^{A'B'} + {}^3\bar{\omega}_i^{A'B'}\epsilon^{AB}. \tag{C14}$$

Using the antisymmetry $^3\omega_i^{AA'BB'} = -^3\omega_i^{BB'AA'}$, we obtain the symmetries $^3\omega_i^{AB} = {}^3\omega_i^{BA}$ and $^3\bar{\omega}_i^{A'B'} = {}^3\bar{\omega}_i^{B'A'}$ and the explicit representations

$$^3\omega_i^{AB} = \tfrac{1}{2}{}^3\omega^A{}_{B'i}{}^{BB'}, \qquad {}^3\bar{\omega}_i^{A'B'} = \tfrac{1}{2}{}^3\omega_{Bi}{}^{A'BB'}. \tag{C15}$$

Analogous relations hold for $^{3s}\omega_i^{AA'BB'}$ and $^3\kappa_i^{AA'BB'}$. The components of the three-dimensional curvature in terms of the spin connection read

$$\begin{aligned} ^3R_{ij}^{AB} &= 2(\partial_{[i}{}^3\omega_{j]}^{AB} + {}^3\omega_{C[i}^A{}^3\omega_{j]}^{CB}), \\ ^3\bar{R}_{ij}^{A'B'} &= 2(\partial_{[i}{}^3\bar{\omega}_{j]}^{A'B'} + {}^3\bar{\omega}_{C'[i}^{A'}{}^3\bar{\omega}_{j]}^{C'B'}). \end{aligned} \tag{C16}$$

Because of the symmetry of $^3\omega_i^{[AB]} = 0$ and $^3\bar{\omega}_i^{[A'B']} = 0$, the chosen notation $^3\omega_{Bi}^A$ and $^3\bar{\omega}_{B'i}^{A'}$ is unambiguous. The horizontal position of the indices does not need to be fixed. The scalar curvature is given by

$$^3R = e_{AA'}^i e_{BB'}^j ({}^3R_{ij}^{AB}\bar{\epsilon}^{A'B'} + {}^3\bar{R}^{A'B'}\epsilon^{AB}). \tag{C17}$$

The same procedure performed on $^{3s}\omega_i^{AA'BB'}$ leads to the torsion-free scalar curvature,

$$\begin{aligned} ^{3s}R_{ij}^{AB} &= 2(\partial_{[i}{}^{3s}\omega_{j]}^{AB} + {}^{3s}\omega_{C[i}^A{}^{3s}\omega_{j]}^{CB}), \\ ^{3s}\bar{R}_{ij}^{A'B'} &= 2(\partial_{[i}{}^{3s}\bar{\omega}_{j]}^{A'B'} + {}^{3s}\bar{\omega}_{C'[i}^{A'}{}^{3s}\bar{\omega}_{j]}^{C'B'}), \end{aligned} \tag{C18}$$

and

$$^{3s}R = e_{AA'}^i e_{BB'}^j ({}^{3s}R_{ij}^{AB}\epsilon^{A'B'} + {}^{3s}\bar{R}^{A'B'}\epsilon^{AB}). \tag{C19}$$

2. Equations used in Sec. II

In Sec. II we need the explicit form of the expressions

$$\epsilon^{ilm} n^{AA'} \frac{\delta}{\delta e_j^{AB'}} (D_B{}^{B'}{}_{mj} D^C{}_{A'kl}) \tag{C20}$$

and

$$n^{AA'} \frac{\delta}{\delta e_j^{AB'}} D_{ij}^{BB'}. \tag{C21}$$

To evaluate these terms we first need an explicit form of $\delta n^{AA'}/\delta e_j^{BB'}$. Of course, for this purpose relation (A8), which expresses $n^{AA'}$ in terms of the tetrad, can be used, but it is more convenient to start from $n^{AA'} e_{AA'i} = 0$:

$$\begin{aligned} 0 &= e^{CC'i} \frac{\delta n^{AA'} e_{AA'} i}{\delta e_j^{BB'}} \\ &= n^{CC'} n_{AA'} \frac{\delta n^{AA'}}{\delta e_j^{BB'}} - \epsilon_A{}^C \epsilon_{A'}{}^{C'} \frac{\delta n^{AA'}}{\delta e_j^{BB'}} + e^{CC'j} n_{BB'}. \end{aligned} \tag{C22}$$

In addition we have

$$\frac{\delta n^{AA'}}{\delta e_j^{BB'}} = \frac{\delta n^{CC'} n_{CC'} n^{AA'}}{\delta e_j^{BB'}} = 2n_{CC'} n^{AA'} \frac{\delta n^{AA'}}{\delta e_j^{BB'}} + \frac{\delta n^{AA'}}{\delta e_j^{BB'}} \tag{C23}$$

and obtain

$$\frac{\delta n^{AA'}}{\delta e_j^{BB'}} = e^{AA'j} n_{BB'}. \tag{C24}$$

We often need the derivative of the determinant h of the three-metric,

$$\frac{\partial h}{\partial h_{ij}} = h^{ij} h. \tag{C25}$$

Therefore we get

$$\frac{\delta h}{\delta e_i^{AA'}} = -2h e_{AA'}^i. \tag{C26}$$

Using this as well as (C1)–(C5), we are able to calculate expressions (6) and (7):

$$\begin{aligned} & n^{AA'} \epsilon^{ilm} \frac{\delta}{\delta e_j^{AB'}} (D_B{}^{B'}{}_{mj} D^C{}_{A'kl}) \\ &= -4n^{AA'} \epsilon^{ilm} \frac{\delta}{\delta e_j^{AB'}} \bigg(\frac{1}{h} e_{Bj}{}^{D'} e_{DD'm} n^{DB'} e_l^{CE'} e_{EE'k} n^E{}_{A'} \bigg) \\ &= \epsilon_B{}^C \delta_k^i \frac{i}{\sqrt{h}} \bigg(1 - 1 + \frac{1}{2} - \frac{1}{2} \bigg) + \frac{2i}{\sqrt{h}} (2e^{CB'i} e_{BB'k} \\ &\quad + e_{BB'}^i e_k^{CB'}) \\ &= \frac{-3i}{\sqrt{h}} \delta_k^i \epsilon_B{}^C - 2h^{ij} \epsilon_{jkl} n^{CB'} e_{BB'}^l, \end{aligned} \tag{C27}$$

$$\begin{aligned} n^{AA'} \frac{\delta}{\delta e_j^{AB'}} D_{ij}^{BB'} &= -2in^{AA'} \frac{\delta}{\delta e_j^{AB'}} \bigg(\frac{1}{\sqrt{h}} e_j^{BC'} e_{CC'i} n^{CB'} \bigg) \\ &= -\frac{2i}{\sqrt{h}} n^{AA'} n^{BC'} e_{AC'i}. \end{aligned} \tag{C28}$$

In order to compare the results in Secs. II, III, and IV with those in Appendix B, we need some rules for the transformation of formulas in terms of the tetrad $e_i^{AA'}$ into formulas in terms of the three-metric h_{ij}. Let $\mathcal{F}[e]$ be a functional depending on the tetrad. Indeed, h_{ij} can be expressed in terms of the tetrad, since we have the relation $h_{ij} = -e_i^{AA'} e_{AA'_j}$, but an inverse relation does of course not exist. We have therefore to restrict the functional $\mathcal{F}$. We must demand that it can be written in the form $\mathcal{F}[h_{ij}]$. Then we find for the transformation of the functional derivatives, by using the chain rule,

$$\begin{aligned}\frac{\delta\mathcal{F}}{\delta e_i^{AA'}} &= \frac{\delta\mathcal{F}}{\delta h_{jk}}\frac{\delta h_{jk}}{\delta e_i^{AA'}} = -\frac{\delta\mathcal{F}}{\delta h_{jk}}\epsilon_{BC}\epsilon_{B'C'}\frac{\delta e_j^{BB'}e_k^{CC'}}{\delta e_i^{AA'}}\\ &= -\frac{\delta\mathcal{F}}{\delta h_{ik}}\epsilon_{AC}\epsilon_{A'C'}e_k^{CC'} - \frac{\delta\mathcal{F}}{\delta h_{ji}}\epsilon_{BA}\epsilon_{B'A'}e_j^{BB'}\\ &= -2\frac{\delta\mathcal{F}}{\delta h_{ij}}e_{AA'j}.\end{aligned} \tag{C29}$$

Using $e^{AA'i}e_{AA'j} = -\delta^i_j$, the inverse relation can be read off immediately. It holds for an arbitrary functional $\mathcal{G}[h_{ij}]$ without any restrictions, since it is always possible to rewrite $\mathcal{G}[h_{ij}]$ in the form $\mathcal{G}[e]$,

$$\frac{\delta\mathcal{G}}{\delta h_{ij}} = \frac{1}{2}e^{AA'j}\frac{\delta\mathcal{G}}{\delta e_i^{AA'}}. \tag{C30}$$

APPENDIX D: RELATIONS USED FOR THE CORRECTIONS OF THE SCHRÖDINGER EQUATION

In Sec. IV we have calculated the corrections of the Schrödinger equation at order G^1. To obtain the explicit form of the correction terms, the following relations are used. For the treatment of terms (i), (ii), (iii), and (iv) in (36) we need

$$\begin{aligned}\frac{1}{\chi}\frac{\delta^2\chi}{\delta e_j^{AB'}\delta\psi_k^C} &= \frac{1}{W}\frac{\delta^2 W}{\delta e_j^{AB'}\delta\psi_k^C} + \frac{i}{\hbar W}\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta S_1}{\delta\psi_k^C} + \frac{i}{\hbar W}\\ &\quad\times\frac{\delta S_1}{\delta e_j^{AB'}}\frac{\delta W}{\delta\psi_k^C} + \frac{i}{\hbar}\frac{\delta^2 S_1}{\delta e_j^{AB'}\delta\psi_k^C} - \frac{1}{\hbar^2}\frac{\delta S_1}{\delta e_j^{AB'}}\\ &\quad\times\frac{\delta S_1}{\delta\psi_k^C},\end{aligned} \tag{D1}$$

$$\frac{1}{\chi}\frac{\delta\chi}{\delta\psi_k^C} = \frac{1}{W}\frac{\delta W}{\delta\psi_k^C} + \frac{i}{\hbar}\frac{\delta S_1}{\delta\psi_k^C}, \tag{D2}$$

$$\frac{1}{\chi}\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta\chi}{\delta\psi_k^C} = \frac{1}{W}\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta W}{\delta\psi_k^C} + \frac{i}{\hbar}\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta S_1}{\delta\psi_k^C}, \tag{D3}$$

and

$$\frac{1}{\chi}\frac{\delta\chi}{\delta e_j^{AB'}}\frac{\delta W}{\delta\psi_k^C} = \frac{1}{W}\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta W}{\delta\psi_k^C} + \frac{i}{\hbar}\frac{\delta S_1}{\delta e_j^{AB'}}\frac{\delta W}{\delta\psi_k^C}. \tag{D4}$$

For parts (v)–(viii) of (39), we use

$$\begin{aligned}\frac{1}{\chi}\frac{\delta^2\chi}{\delta e_j^{AB'}\delta e_i^{BA'}} &= \frac{1}{W}\frac{\delta^2 W}{\delta e_j^{AB'}\delta e_i^{BA'}} + \frac{i}{\hbar W}\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta S_1}{\delta e_i^{BA'}} + \frac{i}{\hbar W}\\ &\quad\times\frac{\delta S_1}{\delta e_j^{AB'}}\frac{\delta W}{\delta e_i^{BA'}} + \frac{i}{\hbar}\frac{\delta^2 S_1}{\delta e_j^{AB'}\delta e_i^{BA'}} - \frac{1}{\hbar}\\ &\quad\times\frac{\delta S_1}{\delta e_j^{AB'}}\frac{\delta S_1}{\delta e_i^{BA'}}\end{aligned} \tag{D5}$$

and

$$\frac{1}{\chi}\frac{\delta\chi}{\delta e_i^{BA'}} = \frac{1}{W}\frac{\delta W}{\delta e_i^{BA'}} + \frac{i}{\hbar}e_j^{AB'}\frac{\delta S_1}{\delta e_i^{BA'}}, \tag{D6}$$

$$\frac{1}{\chi}\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta\chi}{\delta e_i^{BA'}} = \frac{1}{W}\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta W}{\delta e_i^{BA'}} + \frac{i}{\hbar}\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta S_1}{\delta e_i^{BA'}}, \tag{D7}$$

$$\frac{1}{\chi}\frac{\delta\chi}{\delta e_j^{AB'}}\frac{\delta W}{\delta e_i^{BA'}} = \frac{1}{W}\frac{\delta W}{\delta e_j^{AB'}}\frac{\delta W}{\delta e_i^{BA'}} + \frac{i}{\hbar}\frac{\delta S_1}{\delta e_j^{AB'}}\frac{\delta W}{\delta e_i^{BA'}}. \tag{D8}$$

To perform the next step we need

$$i\hbar\frac{\delta\Theta}{\delta\tau} = i\hbar\exp\left(\frac{i}{\hbar}\eta G\right)\frac{\delta\chi}{\delta\tau} - G\chi\exp\left(\frac{i}{\hbar}\eta G\right)\frac{\delta\eta}{\delta\tau}. \tag{D9}$$

The last term in (39) is part of the expression

$$\begin{aligned}\mathcal{H}^{\mathrm{m}}_{\perp}\Theta &= \exp\left(\frac{i}{\hbar}\eta G\right)\mathcal{H}^{\mathrm{m}}_{\perp}\chi - \frac{i\hbar G}{2\sqrt{h}}\left(\frac{2}{\chi}\frac{\delta\chi}{\delta\Phi}\frac{\delta\eta}{\delta\Phi}\right.\\ &\quad\left.+\frac{\delta^2\eta}{\delta\Phi^2}\right)\Theta + \mathcal{O}(G^2).\end{aligned} \tag{D10}$$

Since we are only interested in corrections of order G, we neglect terms of order G^2.

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