Improving the ultraviolet behavior in baryon chiral perturbation theory

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We introduce a new formulation of baryon chiral perturbation theory which improves the ultraviolet behavior of propagators and can be interpreted as a smooth cutoff regularization scheme. It is equivalent to the standard approach, preserves all symmetries, and therefore satisfies the Ward identities. Our formulation is equally well defined in the vacuum, one-nucleon, and few-nucleon sectors of the theory. The equations (Bethe-Salpeter, Lippmann-Schwinger, etc.) for the scattering amplitudes of the few-nucleon sector are free of divergences in the new approach. Unlike the usual cutoff regularization, our ''cutoffs'' are parameters of the Lagrangian and do not have to be removed.

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I. INTRODUCTION

Weinberg's work in 1979 [1] originated effective field theory (EFT) as one of the most important theoretical tools for investigating strong-interaction processes in the low-energy regime. The key progress due to Weinberg's approach was the development of a perturbative scheme not in terms of a coupling constant, but rather in terms of external momenta and the pion mass [1]. In the traditional sense, effective field theories are nonrenormalizable theories. However, as long as one includes all of the infinite number of interactions allowed by symmetries, from the point of view of removing divergences there is no difference between the so-called nonrenormalizable theories and renormalizable theories [2]. Infinities encountered in the calculation of loop diagrams are removed by a renormalization of fields and the infinite number of free parameters of the most general effective Lagrangian.

The ideas of Weinberg were further developed and comprehensively applied to the vacuum sector of QCD by Gasser and Leutwyler in Refs. [3,4]. Chiral perturbation theory (ChPT) in the mesonic sector has been successfully applied to calculations of various physical quantities (for a recent review see, e.g., Ref. [5]). Processes involving one nucleon in the initial and final states were first considered by Gasser, Sainio, and Švarc [6]. They observed that higher-loop diagrams can contribute to terms as low as $\mathcal{O}(q^2)$, where q generically denotes a small expansion parameter such as, e.g., the pion mass. This problem has widely been interpreted as the absence of a systematic power counting in the manifestly Lorentz-invariant formulation of baryon chiral perturbation theory (BChPT). As an alternative the heavy-baryon formulation (HBChPT) was suggested [7,8]. Most of the calculations in the one-baryon sector have been performed in this framework using dimensional regularization in combination with the modified minimal subtraction scheme ($\widetilde{\mathrm{MS}}$) of ChPT (for an overview see, e.g., Refs. [5,9]). The advantage of this approach is that it leads to a straightforward power counting. Meanwhile it has been realized that, choosing an appropriate renormalization condition, one can restore the power counting in the original manifestly Lorentz-invariant formulation of BChPT [10–19].

A generalization to the few-nucleon sector was suggested in Weinberg's papers on constructing nuclear forces from effective field theory [20,21]. For processes involving $N > 1$ nucleons, Weinberg proposed applying the power counting to the potential, which is defined as the sum of all N-nucleon-irreducible diagrams. The scattering amplitudes are then calculated by solving the Lippmann-Schwinger (LS) or Schrödinger equation.

The application of these ideas has encountered various problems. They originate from the renormalization of the LS equation with nonrenormalizable potentials (i.e. the iteration of the potential generates divergent terms with structures which are not included in the original potential). A consistent subtractive renormalization requires the inclusion of the contributions of an infinite number of counterterms which, in most cases, turns out to be technically unfeasible. As a practical solution of the problem, one can perform the calculations in cutoff EFT. This approach reproduces the results of the subtractively renormalized theory to a given order, provided that the value of the cutoff parameter is suitably chosen [22–29]. While this approach has been successful in various applications [29–33], the applied cutoff regularization scheme breaks certain symmetries of the theory and therefore special care has to be taken. The application of cutoff regularization schemes to effective theories has been of interest for a long time [34–42]. A symmetry-preserving lattice regularization of ChPT in the presence of at most a single baryon has been considered in Ref. [43]. Although this regularization could, in principle, also be applied in the few-nucleon sector, to the best of our knowledge the question of preserving symmetries in calculations of few-nucleon processes still remains open.[1] Therefore, the construction of a symmetry-

[1]So far, dimensional regularization has only been used for a very restricted number of cases when the equations are exactly solvable.

preserving formulation of BChPT which renders equations free of divergences is of great interest.

In this work we use an old idea by Slavnov [44], who introduced chirally invariant terms with higher derivatives as a regulator of the nonlinear sigma model. We include symmetry-preserving higher-derivative terms in the effective Lagrangian of baryon chiral perturbation theory which modify the ultraviolet behavior of the pion and baryon propagators. To regularize the still remaining infinite number of primitively divergent diagrams [44], we apply dimensional regularization. This ensures that *all* loop diagrams are regulated. The advantage of this approach is that it can be applied to individual Feynman diagrams as well as to equations of the few-nucleon sector.

Our work is organized as follows. In Sec. II we provide the terms which we add to the standard effective Lagrangian. The nucleon mass is calculated within our new approach in Sec. III. In Sec. IV we demonstrate that the new scheme satisfies the U(1) Ward identity, while in Sec. V it is shown that in HBChPT, analogously to the manifestly Lorentz-invariant formulation, the existence of a consistent power counting depends on the applied renormalization condition. Section VI considers an application to simple examples of the nucleon-nucleon scattering problem. A summary is given in Sec. VII, while the appendix contains the expressions for the required loop integrals.

II. THE MODIFIED EFFECTIVE LAGRANGIAN

The standard effective Lagrangian consists of the sum of the purely mesonic and the πN, NN, etc. Lagrangians, respectively,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \cdots. \tag{1}$$

The terms in Eq. (1) are organized in a (chiral) derivative and quark-mass expansion [1,3,4,6,45–49]. Counting the quark-mass term as $\mathcal{O}(q^2)$ [3,50], the mesonic Lagrangian contains only even powers, whereas the baryonic Lagrangian involves both even and odd powers due to the additional spin degree of freedom. We choose to not show the counterterms explicitly. Instead we accompany the Feynman rules with the subtraction rules within a fixed renormalization condition. In particular, we use the extended on-mass-shell (EOMS) renormalization of Ref. [16].

The lowest-order mesonic Lagrangian reads [3]

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{Tr}[D_\mu U (D^\mu U)^\dagger] + \frac{F^2}{4}\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger), \tag{2}$$

where U is a unimodular unitary (2×2) matrix containing the Goldstone boson fields. The covariant derivative is defined as

$$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu,$$

where

$$r_\mu = v_\mu + a_\mu, \qquad l_\mu = v_\mu - a_\mu,$$
$$\chi = 2B(s + ip).$$

Here, v_μ, a_μ, s, and p are external vector, axial-vector, scalar, and pseudoscalar sources, respectively. In Eq. (2), F denotes the pion-decay constant in the chiral limit: $F_\pi = F[1 + \mathcal{O}(\hat{m})] = 92.4$ MeV. We work in the isospin-symmetric limit $m_u = m_d = \hat{m}$, and the lowest-order expression for the squared pion mass is $M^2 = 2B\hat{m}$, where B is related to the quark condensate $\langle \bar{q}q\rangle_0$ in the chiral limit [3].

In the nucleon sector, let

$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$$

denote the nucleon field with two four-component Dirac fields, p and n, describing the proton and neutron, respectively. The most general πN Lagrangian is bilinear in $\bar{\Psi}(x)$ and $\Psi(x)$ and involves the quantities u, u_μ, Γ_μ, $v_\mu^{(s)}$, and $\chi_\pm$ (and their derivatives), which are defined as

$$u^2 = U, \qquad u_\mu = i u^\dagger D_\mu U u^\dagger,$$
$$\Gamma_\mu = \frac{1}{2}[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger - i(u^\dagger r_\mu u + u l_\mu u^\dagger)],$$
$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u.$$

In terms of these building blocks, the lowest-order Lagrangian reads [5,6]

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\gamma_\mu D^\mu - m + \frac{1}{2}\mathring{g}_A \gamma_\mu \gamma_5 u^\mu \right)\Psi, \tag{3}$$

where $D_\mu \Psi = (\partial_\mu + \Gamma_\mu - i v_\mu^{(s)})\Psi$ denotes the covariant derivative. (In the definition of the covariant derivative we follow Ref. [51], where Γ_μ only contains traceless external fields and the coupling to the isosinglet vector field $v_\mu^{(s)}$ is considered separately.) In Eq. (3), m and $\mathring{g}_A$ refer to the chiral limit of the physical nucleon mass and the axial-vector coupling constant, respectively.

Below we will calculate the nucleon self-energy to third order. For that purpose, we will need one of the seven structures of the Lagrangian at $\mathcal{O}(q^2)$ [6,47],

$$\mathcal{L}_{\pi N}^{(2)} = c_1 \,\mathrm{Tr}(\chi_+)\bar{\Psi}\Psi + \cdots. \tag{4}$$

The Lagrangian $\mathcal{L}_{\pi N}^{(3)}$ does not contribute to the nucleon mass at the given order.

To improve the ultraviolet behavior of the propagators generated by the Lagrangian of Eq. (1) we introduce additional terms into the Lagrangian which modify the propagators of the pion and the nucleon. In particular, we consider the modified pion propagator

$$\Delta_\pi^\Lambda(p) = \frac{1}{p^2 - M^2 + i0^+}\prod_{j=1}^{N_\pi}\frac{\Lambda_{\pi j}^2}{\Lambda_{\pi j}^2 - p^2 - i0^+} \tag{5}$$

and the modified nucleon propagator

$$S_N^\Lambda(p) = \frac{1}{(\not{p} - m + i0^+)} \prod_{j=1}^{N_\Psi} \frac{\Lambda_{\Psi j}^2}{\Lambda_{\Psi j}^2 + m^2 - p^2 - i0^+}. \quad (6)$$

Here, $\Lambda_{\pi i}$ and $\Lambda_{\Psi j}$ are (independent) parameters.[2] For simplicity we use the standard prescription for dealing with poles in the Λ-dependent factors of the modified propagators. For sufficiently large values of the parameters Λ any other prescription leads to the same results for low-energy physical quantities. The propagators above can be generated by a Lagrangian which, in addition to the standard BChPT Lagrangian of Eq. (1), contains additional *symmetry-preserving* terms. These terms vanish in the limit $\Lambda_{\pi i} \to \infty$, $\Lambda_{\Psi j} \to \infty$.

The choice of the additional terms of the Lagrangian is not unique. Furthermore these terms not only generate the above propagators, but also result in additional interaction terms. Our choice is motivated by the simplicity of calculations. For the pion sector we choose

$$\mathcal{L}_{\pi\pi}^{\text{reg}} = \sum_{n=1}^{N_\pi} \frac{X_n}{4} \frac{F_0^2}{4} \text{Tr}(\{(D^2)^n U U^\dagger - U[(D^2)^n U]^\dagger\} \times [D^2 U U^\dagger - U(D^2 U)^\dagger - \chi U^\dagger + U\chi^\dagger]),$$

where $D^2 U = D_\alpha D^\alpha U$ and X_n are functions of $\Lambda_{\pi i}$. For example, in order to generate the modified propagator

$$\Delta_\pi^\Lambda(p) = \frac{1}{p^2 - M^2 + i0^+} \prod_{j=1}^{3} \frac{\Lambda_{\pi j}^2}{\Lambda_{\pi j}^2 - p^2 - i0^+}, \quad (7)$$

we need to take $N_\pi = 3$ and

$$X_1 = \frac{1}{\Lambda_{\pi 1}^2} + \frac{1}{\Lambda_{\pi 2}^2} + \frac{1}{\Lambda_{\pi 3}^2}, \qquad X_2 = \frac{\Lambda_{\pi 1}^2 + \Lambda_{\pi 2}^2 + \Lambda_{\pi 3}^2}{\Lambda_{\pi 1}^2 \Lambda_{\pi 2}^2 \Lambda_{\pi 3}^2},$$
$$X_3 = \frac{1}{\Lambda_{\pi 1}^2 \Lambda_{\pi 2}^2 \Lambda_{\pi 3}^2}. \quad (8)$$

For the additional terms of the Lagrangian of the nucleon sector we choose

$$\mathcal{L}_{\pi N}^{\text{reg}} = \sum_{n=1}^{N_\Psi} \frac{Y_n}{2} [\bar{\Psi}(i\gamma_\mu D^\mu - m)(D^2 + m^2)^n \Psi + \text{H.c.}], \quad (9)$$

where Y_n are functions of $\Lambda_{\Psi j}$. For example, for the modified nucleon propagator

$$S_N^\Lambda(p) = \frac{\Lambda_\Psi^2}{(\not{p} - m + i0^+)(\Lambda_\Psi^2 + m^2 - p^2 - i0^+)}, \quad (10)$$

we have $N_\Psi = 1$ and $Y_1 = 1/\Lambda_\Psi^2$.

Depending on the order of the performed calculations, we choose the modified propagators, i.e. fix N_π and N_Ψ, such that all loop diagrams (except some of the primitively divergent diagrams) contributing to the given order converge. To obtain the modified Lagrangian for lower order calculations, one needs to take $\Lambda_{\pi i} \to \infty$, $\Lambda_{\Psi j} \to \infty$ for some of the parameters Λ in the modified Lagrangian used in higher-order calculations.

Analogously to the nonlinear sigma model [44], the additional terms do not render all loop diagrams finite. There still remain an infinite number of primitively divergent diagrams in the mesonic sector as well as divergences in diagrams with fermion loops. These diagrams can be regularized in a symmetry-preserving way by introducing additional auxiliary fields analogously to the case of Yang-Mills theory [52]. However, in practical calculations such a technique is rather difficult to apply. Instead it is possible (and much more convenient) to use standard dimensional regularization. This is due to the fact that the remaining divergent diagrams contribute either in physical quantities of the vacuum (purely mesonic) and the one-nucleon sectors, or they appear as subdiagrams in the potentials of the few-nucleon sector. In both cases the calculations are perturbative, i.e., to any given order in the chiral expansion one needs to calculate a finite number of diagrams. Therefore, divergences which show up as the $1/(n-4)$ poles (where n denotes the number of space-time dimensions) can be explicitly subtracted (i.e. absorbed in the redefinition of the parameters of the effective Lagrangian).

To summarize, our scheme consists of adding symmetry-preserving additional terms in the standard effective Lagrangian and applying dimensional regularization to the resulting effective theory. All symmetries are preserved in the regularized theory, i.e., regularized quantities satisfy all relevant Ward identities. We expand the regularized diagrams in powers of $n-4$ and subtract $1/(n-4)$ pole-terms observing that there is a finite number of them to any given (finite) order in the chiral expansion of physical quantities in the vacuum and one-nucleon sectors, and the potentials in the few-nucleon sector. No further divergences occur (for finite parameters Λ) neither in the vacuum and one-nucleon sector nor in the equations of the few-nucleon sector. Therefore, for the equations of the few-nucleon sector we can take $n = 4$.

Using a field transformation, the additional higher-derivative terms which we introduced in the effective Lagrangian can be reexpressed in a canonical form, i.e. a form with a minimal number of independent terms [53,54]. This clearly shows that any Λ dependence of the physical quantities can systematically be absorbed in the redefinition of the parameters of the standard canonical effective Lagrangian.

[2] In the following we let Λ collectively represent the $\Lambda_{\pi i}$ and $\Lambda_{\Psi j}$.

III. NUCLEON SELF-ENERGY

As an example of the application of our approach, we calculate the nucleon self-energy to order $\mathcal{O}(q^3)$ in this section. For this calculation it is sufficient to take $N_\pi = N_\Psi = 1$. We parametrize the complete nucleon propagator as

$$S_N(p) = \frac{1}{(\not{p} - m + i0^+)[1 - (p^2 - m^2)/\Lambda_\Psi^2] - \Sigma(\not{p})}, \tag{11}$$

where m is the nucleon pole mass in the chiral limit and the nucleon self-energy $-i\Sigma(\not{p})$ represents the sum of all one-particle-irreducible perturbative contributions to the two-point function. The physical nucleon mass is defined through the pole of the full propagator at $\not{p} = m_N$,

$$(m_N - m)\left(1 - \frac{m_N^2 - m^2}{\Lambda_\Psi^2}\right) - \Sigma(m_N) = 0. \tag{12}$$

At $\mathcal{O}(q^3)$, the self-energy receives contact contributions from $\mathcal{L}_{\pi N}^{(2)}$ as well as the one-loop contribution of Fig. 1,

$$\Sigma = \Sigma_{\text{contact}} + \Sigma_{\text{loop}}, \tag{13}$$

where $\Sigma_{\text{contact}} = -4c_1M^2$. Applying Feynman rules, we obtain for the one-loop contribution

$$\begin{aligned}\Sigma_{\text{loop}} = &-\frac{3\mathring{g}_A^2\Lambda_\pi^2\Lambda_\Psi^2}{4F^2}\{(\not{p} + m)I(1011) \\ &+ M^2(\not{p} + m)I(1111) + (p^2 - m^2)\not{p}I^{(p)}(1111) \\ &- \not{p}I^{(p)}(1110)\},\end{aligned} \tag{14}$$

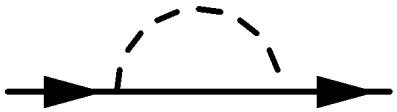

FIG. 1. One-loop contribution to the nucleon self-energy at $\mathcal{O}(q^3)$.

where

$$\{I(abcd),\, p^\mu I^{(p)}(abcd)\} = i\int \frac{d^4k}{(2\pi)^4}\frac{\{1, k^\mu\}}{A^aB^bC^cD^d}, \tag{15}$$

with

$$\begin{aligned}&A = k^2 - \Lambda_\pi^2 + i0^+, \qquad B = k^2 - M^2 + i0^+, \\ &C = (p + k)^2 - m^2 - \Lambda_\Psi^2 + i0^+, \\ &D = (p + k)^2 - m^2 + i0^+.\end{aligned}$$

To further simplify the calculation we take $\Lambda_\pi = \Lambda_\Psi$. The parameter n of dimensional regularization has been put $n = 4$, as the diagram is finite for finite Λ. We perform the renormalization by applying the extended on mass-shell (EOMS) scheme of Ref. [16]. First we substitute the expressions for the loop integrals from the appendix and expand Eq. (14) in a power series in Λ (around $\Lambda = \infty$). We then subtract all positive powers of Λ and $\ln(\Lambda/m)$.[3] Next we expand the remaining expression in powers of small quantities, i.e., M, $p^2 - m^2$ and $\not{p} - m$ and subtract all terms of zeroth, first, and second order in this expansion, so that the renormalized expression is indeed of order q^3 as mandated by the power counting. The resulting expression for the subtraction terms reads

$$\begin{aligned}\Sigma^{\text{sub}} = &-\frac{3\mathring{g}_A^2\Lambda^2(4m + 5\not{p})}{256\pi^2F^2} + \frac{\mathring{g}_A^2}{256\pi^2mF^2}\Big[-12m^4 + 3m^2M^2 + 8m^2p^2 - 6(p^2)^2 - 10m^3\not{p} + mp^2\not{p} \\ &+ 12m(2m^3 + 4mM^2 + 3m^2\not{p} - p^2\not{p})\ln\left(\frac{\Lambda}{m}\right)\Big] \\ &- \frac{\mathring{g}_A^2 m}{2560\pi^2F^2\Lambda^2}\Big[-48m^4 - 10(p^2)^2 + 6m^2(15M^2 + 41p^2) + 63m^3\not{p} + 130mp^2\not{p} \\ &- 240m^2(m^2 + 2M^2 + p^2 + 2m\not{p})\ln\left(\frac{\Lambda}{m}\right)\Big].\end{aligned} \tag{16}$$

Subtracting Eq. (16) from Eq. (14) and taking $\not{p} = m_N$, we obtain for the renormalized on-mass-shell self-energy to order q^3

$$\Sigma^R|_{\not{p}=m_N} = -\frac{3\mathring{g}_A^2M^3}{32\pi F^2} + \mathcal{O}\left(\frac{1}{\Lambda^4}\right).$$

Using Eq. (12), the nucleon mass to order q^3 follows as

$$m_N = m - 4c_1M^2 - \frac{3\mathring{g}_A^2}{32\pi F^2}M^3 + \mathcal{O}\left(\frac{1}{\Lambda^4}\right), \tag{17}$$

which agrees with the standard BChPT result [12,16,55,56].

[3]Note that, since our scheme respects all symmetries of the theory, the Ward identities are satisfied separately in each order of the expansion in powers of Λ.

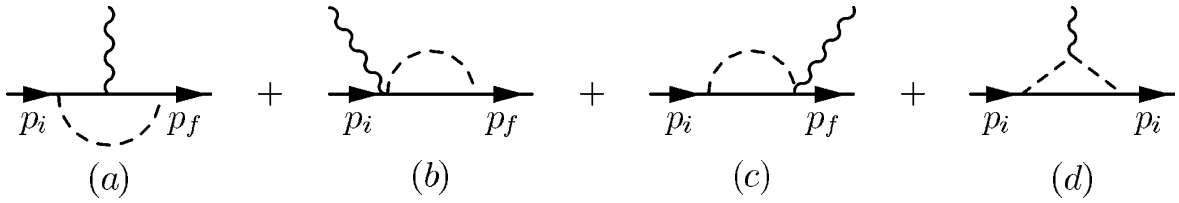

FIG. 2. One-loop contributions to the electromagnetic vertex.

IV. ELECTROMAGNETIC WARD IDENTITY

To demonstrate that the new formulation indeed respects the symmetries of the theory, we analyze the electromagnetic Ward identity for the nucleon which, in units of the elementary charge, reads

$$(p_f - p_i)_\mu \Gamma^\mu_N(p_f, p_i) = \frac{1+\tau_3}{2}[S_N^{-1}(p_f) - S_N^{-1}(p_i)]. \tag{18}$$

Here,

$$\Gamma^\mu_N(p_f, p_i) = \Gamma^\mu_{N0}(p_f, p_i) + \Lambda^\mu_N(p_f, p_i) \tag{19}$$

is the one-particle-irreducible three-point function $(\Psi J^\mu \bar{\Psi})$ with J^μ the electromagnetic current operator in units of the elementary charge. $\Gamma^\mu_{N0}(p_f, p_i)$ corresponds to the tree-order contribution and $\Lambda^\mu_N(p_f, p_i)$ consists of loop corrections. In order to determine Γ^μ_N, we consider the coupling to an external electromagnetic field $\mathcal{A}_\mu$ and insert for the external fields in Eq. (3)

$$r_\mu = l_\mu = -e\frac{\tau_3}{2}\mathcal{A}_\mu, \qquad v^{(s)}_\mu = -\frac{e}{2}\mathcal{A}_\mu.$$

For the purpose of this section it is sufficient to take $N_\pi = N_\Psi = 1$. From our modified Lagrangian we obtain

$$\begin{aligned}\Gamma^\mu_{N0}(p_f, p_i) = {} & \frac{1+\tau_3}{2}\gamma^\mu \\ & - \frac{1+\tau_3}{2}\frac{1}{2\Lambda^2}[\gamma^\mu(p_f^2 + p_i^2 - 2m^2) \\ & + (p_f + p_i)^\mu(\not{p}_f + \not{p}_i - 2m)],\end{aligned} \tag{20}$$

where $\Gamma^\mu_{N0}(p_f, p_i)$ and the free propagator of Eq. (10) satisfy the relation

$$(p_f - p_i)_\mu \Gamma^\mu_{N0}(p_f, p_i) = \frac{1+\tau_3}{2}[S_N^{\Lambda -1}(p_f) - S_N^{\Lambda -1}(p_i)]. \tag{21}$$

Of course, this result is not surprising, because the coupling to an external electromagnetic field in the Lagrangian of Eq. (9) proceeds via covariant derivatives which essentially amount to a minimal coupling. At tree level this automatically results in contributions satisfying the Ward identity.[4]

For the one-loop corrections to the nucleon self-energy (of Fig. 1) and the vertex (diagrams of Fig. 2), we obtain

$$\Sigma_{\text{loop}}(\not{p}) = \frac{3\mathring{g}_A^2}{4F^2} i \int \frac{d^n k}{(2\pi)^n} \not{k}\gamma_5 S_N^\Lambda(p+k)\not{k}\gamma_5 \Delta_\pi^\Lambda(k), \tag{22}$$

$$\begin{aligned}\Lambda^\mu_{Na}(p_f, p_i) = {} & \frac{\mathring{g}_A^2}{4F^2} i \int \frac{d^n k}{(2\pi)^n} \not{k}\gamma_5 \tau^a S_N^\Lambda(p_f + k) \\ & \times \Gamma^\mu_{N0}(p_f + k, p_i + k) S_N^\Lambda(p_i + k) \\ & \times \not{k}\gamma_5 \tau^a \Delta_\pi^\Lambda(k),\end{aligned} \tag{23}$$

$$\Lambda^\mu_{Nb}(p_f, p_i) = 2\tau_3 \frac{\mathring{g}_A^2}{4F^2} i \int \frac{d^n k}{(2\pi)^n} \not{k}\gamma_5 S_N^\Lambda(p_f + k)\gamma^\mu\gamma_5 \Delta_\pi^\Lambda(k), \tag{24}$$

$$\Lambda^\mu_{Nc}(p_f, p_i) = 2\tau_3 \frac{\mathring{g}_A^2}{4F^2} i \int \frac{d^n k}{(2\pi)^n} \gamma^\mu\gamma_5 S_N^\Lambda(p_i + k)\not{k}\gamma_5 \Delta_\pi^\Lambda(k), \tag{25}$$

$$\begin{aligned}\Lambda^\mu_{Nd}(p_f, p_i) = {} & 2\tau_3 \frac{\mathring{g}_A^2}{4F^2} i \int \frac{d^n k}{(2\pi)^n} (\not{p}_i + \not{k} - \not{p}_f) \\ & \times \gamma_5 S_N^\Lambda(p_i + k)\Gamma^\mu_{\pi 0}(p_f - p_i - k, -k) \\ & \times \not{k}\gamma_5 \Delta_\pi^\Lambda(k + p_i - p_f)\Delta_\pi^\Lambda(k),\end{aligned} \tag{26}$$

where

$$\Gamma^\mu_{\pi 0}(p', p) = (p' + p)^\mu \left(1 - \frac{p'^2 + p^2 - M^2}{\Lambda_\pi^2}\right) \tag{27}$$

is the leading tree-order contribution in Γ^μ_π, which is related to the one-particle-irreducible three-point function $(\pi_j J^\mu \pi_i)$ by the relation

[4] In the context of EFT the use of minimal-substitution terms alone is not sufficient to generate a consistent framework, because the *most general* effective Lagrangian also contains terms involving field-strength tensors such as, e.g., the l_5 and l_6 terms of $\mathcal{L}_4$ [3]. In general, the presence of these terms is also necessary for the purposes of renormalization (see Ref. [57] for a critical discussion of this issue).

$$\Gamma^{\mu}_{\pi ji}(p', p) = i\epsilon_{3ij}\Gamma^{\mu}_{\pi}(p', p). \tag{28}$$

To check the Ward identity for the above loop diagrams, we multiply $\Lambda^{\mu}_{N} = \Lambda^{\mu}_{Na} + \Lambda^{\mu}_{Nb} + \Lambda^{\mu}_{Nc} + \Lambda^{\mu}_{Nd}$ with $(p_f - p_i)_{\mu}$, use Eq. (21) and the Ward identity for pions (at leading tree order)

$$(p' - p)_{\mu}\Gamma^{\mu}_{\pi 0}(p', p) = \Delta^{\Lambda -1}_{\pi}(p') - \Delta^{\Lambda -1}_{\pi}(p), \tag{29}$$

and obtain after a straightforward calculation

$$(p_f - p_i)_{\mu}\Lambda^{\mu}_{N}(p_f, p_i) = \frac{1 + \tau_3}{2}[\Sigma_{\rm loop}(\not{p}_i) - \Sigma_{\rm loop}(\not{p}_f)], \tag{30}$$

which verifies the Ward identity of Eq. (18).

V. NUCLEON SELF-ENERGY DIAGRAM IN HBCHPT

It is common practice to assume the existence of a consistent power counting in HBChPT without specifying the renormalization scheme used. In HBChPT, as in any quantum field theory, one has the freedom to choose a renormalization condition. Dimensional regularization in combination with the $\widetilde{\rm MS}$ scheme, which is commonly used in HBChPT, is only one among an infinite number of possibilities. In this section we apply our higher-derivative formulation to the nucleon self-energy diagram of Fig. 1 in order to show that, analogously to the manifestly Lorentz-invariant formulation, the existence of a consistent power counting in HBChPT depends on the choice of the renormalization condition.

Using the pion propagator of Eq. (5) for $N_{\pi} = 2$ and $\Lambda_{\pi 1} = \Lambda_{\pi 2} = \Lambda$, we obtain (see, e.g., Sec. 5.5.9 and Appendix C.2 of Ref. [5] for a detailed calculation in HBChPT)

$$\Sigma^{(3)}_{\rm loop}(p) = 3\frac{\mathring{g}^2_A\Lambda^4}{F^2}S^{\nu}_{\mu}S^{\nu}_{\nu}J^{\mu\nu}_{\pi N}(121; \omega), \tag{31}$$

where $\omega = (p \cdot v - m)$ and

$$J^{\mu\nu}_{\pi N}(abc; \omega) = i\int\frac{d^4k}{(2\pi)^4}\frac{k^{\mu}k^{\nu}}{[k^2 - M^2 + i0^+]^a[k^2 - \Lambda^2 + i0^+]^b[v \cdot k + \omega + i0^+]^c}. \tag{32}$$

One can parametrize $J^{\mu\nu}_{\pi N}(121; \omega)$ as

$$J^{\mu\nu}_{\pi N}(121; \omega) = c_1 g^{\mu\nu} + c_2 v^{\mu}v^{\nu}. \tag{33}$$

Since $S_v \cdot v = 0$, c_2 does not contribute to the self-energy. For c_1 we find

$$c_1 = \frac{1}{3}[(M^2 - \omega^2)J_{\pi N}(121; \omega) + J_{\pi N}(021; \omega) + \omega J_{\pi N}(120; \omega)], \tag{34}$$

where

$$J_{\pi N}(abc; \omega) = i\int\frac{d^4k}{(2\pi)^4}\frac{1}{[k^2 - M^2 + i0^+]^a[k^2 - \Lambda^2 + i0^+]^b[v \cdot k + \omega + i0^+]^c}. \tag{35}$$

Standard power counting of HBChPT assigns the order $\mathcal{O}(q^3)$ to the diagram of Fig. 1. Calculating the loop integrals of Eq. (35) (see Appendix B), we obtain

$$\Sigma^{(3)}_{\rm loop}(p) = -\frac{3\mathring{g}^2_A}{64\pi^2F^2}[\pi\Lambda^3 + 3\omega\Lambda^2] + \mathcal{O}(\Lambda). \tag{36}$$

Both terms inside the square brackets (as well as the term proportional to Λ which, for the sake of brevity, we have not displayed) violate the power counting. They are analytic in momenta and can be absorbed in the renormalization of the nucleon mass and the nucleon field. Note that the corresponding mass counterterm $\delta m\bar{N}_v N_v$, canceling the Λ^3 term in Eq. (36), is equal to zero in the standard formulation of HBChPT with dimensional regularization and is, therefore, usually not indicated in the effective Lagrangian of HBChPT.[5] Choosing the renormalization scheme appropriately, one can subtract *all* terms in Eq. (36) which violate the power counting so that the renormalized diagram is of order $\mathcal{O}(q^3)$.

VI. *NN* SECTOR

A. Contact interaction

In this section we consider a demonstrating example of the application of our approach to the *NN* problem in the

[5]This is analogous to the case of the pion tadpole self-energy in cutoff regularization, where one needs a counterterm of order p^2 for the pion mass [34,58].

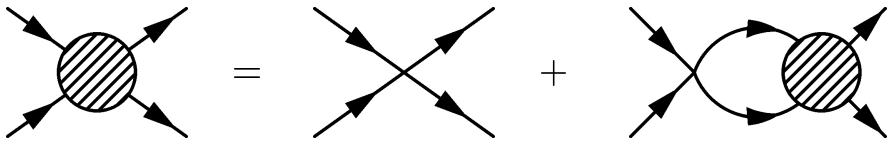

FIG. 3. Graphical illustration of the equation for the NN scattering amplitude.

manifestly Lorentz-invariant formulation of baryon chiral perturbation theory.

Let us consider the simplest possible $(\bar{\Psi}\Psi)^2$ contact interaction term. The corresponding equation for the lowest-order amplitude can be solved analytically; therefore, one can apply standard dimensional regularization to this problem. The interaction term in the Lagrangian reads

$$\mathcal{L}_{NNNN} = C\bar{\Psi}\Psi\bar{\Psi}\Psi. \tag{37}$$

In the following, we will consider the scattering amplitude in the center-of-mass frame. Let $P = p_1 + p_2$ denote the total four-momentum of the scattered nucleons, where $p_1^\mu = (\sqrt{m^2 + p^2}, \vec{p})$, $p_2^\mu = (\sqrt{m^2 + p^2}, -\vec{p})$ with $p = |\vec{p}|$ and $P^2 = 4m^2 + 4p^2$.

The interaction Lagrangian of Eq. (37) generates the following NN vertex (two-nucleon-irreducible contribution in the NN scattering amplitude):

$$iV_{\lambda\sigma,\mu\nu} = 2iC(\delta_{\lambda\nu}\delta_{\sigma\mu} - \delta_{\lambda\mu}\delta_{\sigma\nu}). \tag{38}$$

According to Weinberg's approach, to find the corresponding lowest-order NN scattering amplitude, we need to solve the equation[6]

$$T_{\lambda\sigma,\mu\nu}(P) = V_{\lambda\sigma,\mu\nu} + i\int \frac{d^4k}{(2\pi)^4} V_{\lambda\sigma,\alpha\gamma} G_{\alpha\gamma,\beta\delta}(P,k) T_{\beta\delta,\mu\nu}(P), \tag{39}$$

schematically shown in Fig. 3, where

$$G^{\rm dr}_{\alpha\gamma,\beta\delta}(P,k) = -\frac{(k_\mu\gamma^\mu_{\alpha\beta} + m\delta_{\alpha\beta})[(P_\nu - k_\nu)\gamma^\nu_{\gamma\delta} + m\delta_{\gamma\delta}]}{[k^2 - m^2 + i0^+][(P-k)^2 - m^2 + i0^+]},$$

$$G^{\rm hd}_{\alpha\gamma,\beta\delta}(P,k) = \frac{\Lambda_\Psi^4}{[k^2 - m^2 - \Lambda_\Psi^2 + i0^+][(P-k)^2 - m^2 - \Lambda_\Psi^2 + i0^+]} G^{\rm dr}_{\alpha\gamma,\beta\delta}(P,k)$$

are the (two-nucleon) propagators to be used in standard dimensional regularization and higher-derivative formulation, respectively. Integrating Eq. (39) over k, we obtain

$$T_{\lambda\sigma,\mu\nu}(P) = V_{\lambda\sigma,\mu\nu} + iV_{\lambda\sigma,\alpha\gamma}\mathcal{G}_{\alpha\gamma,\beta\delta}(P)T_{\beta\delta,\mu\nu}(P), \tag{40}$$

where

$$\mathcal{G}_{\alpha\gamma,\beta\delta}(P) = i\{m\delta_{\alpha\beta}(\gamma^\mu_{\gamma\delta}P_\mu + m\delta_{\gamma\delta})I_{NN} + [\gamma^\mu_{\alpha\beta}(\gamma^\nu_{\gamma\delta}P_\nu + m\delta_{\gamma\delta}) - m\delta_{\alpha\beta}\gamma^\mu_{\gamma\delta}]I^{(P)}_{NN}P_\mu - \gamma^\mu_{\alpha\beta}\gamma^\nu_{\gamma\delta}I_{NN,\mu\nu}\}, \tag{41}$$

with

$$\{I_{NN}, P^\mu I^{(P)}_{NN}, I^{\mu\nu}_{NN}\}^{\rm dr} = i\int \frac{d^nk}{(2\pi)^n} \frac{\{1, k^\mu, k^\mu k^\nu\}}{[k^2 - m^2 + i0^+][(P-k)^2 - m^2 + i0^+]} \tag{42}$$

in standard dimensional regularization and

$$\{I_{NN}, P^\mu I^{(P)}_{NN}, I^{\mu\nu}_{NN}\}^{\rm hd} = i\int \frac{d^4k}{(2\pi)^4} \frac{\{1, k^\mu, k^\mu k^\nu\}}{[k^2 - m^2 + i0^+][(P-k)^2 - m^2 + i0^+]} \times \frac{\Lambda_\Psi^4}{[k^2 - m^2 - \Lambda_\Psi^2 + i0^+][(P-k)^2 - m^2 - \Lambda_\Psi^2 + i0^+]} \tag{43}$$

in higher-derivative formulation, respectively.

We renormalize Eq. (40) by subtracting the contributions of loop integrals at $P^2 = 4m^2$. Next we expand the subtracted loop integrals ($I^R = I - I|_{P^2=4m^2}$) in p and retain terms to order $\mathcal{O}(p)$. The resulting equation reads

$$T^R_{\lambda\sigma,\mu\nu}(P) = V_{\lambda\sigma,\mu\nu} + iV_{\lambda\sigma,\alpha\gamma}\mathcal{G}^R_{\alpha\gamma,\beta\delta}T^R_{\beta\delta,\mu\nu}(P), \tag{44}$$

where

[6]It is understood that $T_{\lambda\sigma,\mu\nu}$ needs to be multiplied with the corresponding Dirac spinors.

$$\mathcal{G}^R_{\alpha\gamma,\beta\delta} = \frac{p}{16\pi m}\Big\{m\delta_{\alpha\beta}(\gamma^\mu_{\gamma\delta}P_\mu + m\delta_{\gamma\delta}) + \frac{P_\mu}{2} \times [\gamma^\mu_{\alpha\beta}(\gamma^\nu_{\gamma\delta}P_\nu + m\delta_{\gamma\delta}) - m\delta_{\alpha\beta}\gamma^\mu_{\gamma\delta}] - \gamma^\mu_{\alpha\beta}\gamma^\nu_{\gamma\delta}\frac{P^\mu P^\nu}{4}\Big\}. \quad (45)$$

Comparing Eq. (44) for standard dimensional regularization and higher-derivative formulation [remembering that terms of order $\mathcal{O}(p^2)$ have been neglected] we obtain identical results.[7]

Note that, in contrast to the example considered above, it is not clear how to apply standard dimensional regularization to equations involving potentials derived from BChPT. The advantage of our higher-derivative formulation is that it is also applicable in these cases while preserving the symmetries of the theory.

B. Inclusion of one-pion exchange potential

Below we consider some conceptual issues of renormalization and the Λ dependence of the leading-order NN scattering amplitude in a nonrelativistic formalism of BChPT. This amplitude is obtained by solving the Lippmann-Schwinger equation with a contact interaction plus the one-pion exchange potential.[8]

Using an appropriate field redefinition and the standard heavy-baryon reduction with $\upsilon = (1, 0, 0, 0)$, we obtain the leading-order NN potential (for the choice $N_\Psi = 1$, $N_\pi = 0$)

$$V(\vec{p}', \vec{p}) = C_S + C_T\vec{\sigma}_1\cdot\vec{\sigma}_2 - \left(\frac{\mathring{g}_A}{2F}\right)^2(\vec{\tau}_1\cdot\vec{\tau}_2) \times \frac{\vec{\sigma}_1\cdot(\vec{p}' - \vec{p})\vec{\sigma}_2\cdot(\vec{p}' - \vec{p})}{(\vec{p}' - \vec{p})^2 + M_\pi^2}, \quad (46)$$

where C_S and C_T are the coupling constants of the four-nucleon contact interaction Lagrangian at leading order. The scattering amplitude satisfies the equation

$$T(\vec{p}', \vec{p}) = V(\vec{p}', \vec{p}) + m\int\frac{d^3\vec{k}}{(2\pi)^3}V(\vec{p}', \vec{k}) \times \frac{\Lambda^2}{(\Lambda^2 + \vec{k}^2)(mE - \vec{k}^2 + i0^+)}T(\vec{k}, \vec{p}), \quad (47)$$

where $E = \vec{p}^2/m$ is the energy of two nucleons in the center-of-mass system.

We fix the free parameters C_S and C_T as functions of Λ by demanding that the solution of Eq. (47) reproduces two physical quantities for fixed kinematics. Following Ref. [21], we choose the renormalization points of the order of small external momenta. This exactly corresponds to the following general renormalization program (see, e.g., [59]). First, one calculates the quantities of physical interest in terms of bare parameters in the regularized theory. Once a sufficient number of physical quantities are determined as functions of bare parameters, one inverts the results and expresses the bare quantities in terms of physical quantities. These expressions are then used to eliminate the bare parameters in all other quantities of physical interest. This procedure preserves all symmetries provided that the applied regularization scheme respects them. If the considered theory is renormalizable in the standard sense, the above procedure removes all divergences.

For definiteness, let us take the zero kinematics as renormalization points.[9] This corresponds to the subtraction of loop diagrams at zero kinematics. In the above case, expressing C_S and C_T from two physical quantities[10] and substituting them into other quantities we only eliminate some of the terms that diverge in the limit $\Lambda \to \infty$. This is due to the nonrenormalizability of BChPT in the traditional sense. However, note that Λ is a parameter of the Lagrangian and we do not have to take it to infinity. The remaining Λ dependence of the amplitude is of higher order in the small-parameter expansion (pion mass, small external momenta). As the potential of Eq. (46) is nonrenormalizable in the traditional sense, the perturbative expansion of the renormalized amplitude contains negative as well as positive powers of Λ (and/or positive powers of $\ln\Lambda$). These contributions contain terms of the form

$$\sim\frac{q^i}{\Lambda^j} \quad \text{as well as} \quad \sim\frac{q^i\Lambda^j}{Q^{i+j}}, \qquad \text{with } i > 0, \qquad j > 0, \quad (48)$$

where q denotes small external momenta or the pion mass and Q stands for $4\pi F$ and/or the large scale parameter hidden in renormalized contact interaction constants. To keep these formally higher-order contributions indeed suppressed numerically, one should take $\Lambda \sim Q$. The existence of such an optimal value of the parameter Λ depends on the validity of the assumption of Weinberg's approach that the renormalized coupling constants are natural for renormalization points of the order of or less than small external momenta. The validity of this assumption has to be checked at each order of calculations. While one cannot

[7] Although Eq. (44) can be solved exactly, it is beyond the scope of this paper to perform this straightforward but rather cumbersome calculation.

[8] A detailed discussion of the heavy-baryon reduction of our new Lagrangian (including a numerical analysis in the few-body sector) will be given in a forthcoming publication.

[9] This would not be a good choice if we took the $\Lambda \to \infty$ limit in the end. We would be faced with the problem of very poor convergence [60].

[10] We could take as "quantities of physical interest" the scattering lengths of the ${}^1S_0$ and ${}^3S_1$ NN scattering.

take the existence of the optimal value of Λ for granted, the reasonable success of cutoff EFT suggests that it should exist. A detailed analysis of this issue in our symmetry-preserving approach is in progress.

The complete Λ dependence of physical quantities can be absorbed in the redefinition of couplings constants of the canonical Lagrangian.[11] For the above example, this means that the contributions of the form of Eq. (48) can be absorbed in the redefinition of higher-order coupling constants. We could take any value for the parameter Λ provided that the compensating contributions of higher-order terms (an infinite number of them) are also included, but that does not seem to be feasible. Note that Λ is *not* a cutoff regularization parameter and does not need to be taken to infinity. The above specified optimal choice of the *free parameter of the Lagrangian*, Λ, ensures that, to the accuracy of the given calculations, physical quantities do not depend on higher-order terms which we introduced in the Lagrangian (Λ independence).

As one cannot solve equations exactly, one carries out the above renormalization program numerically by fixing coupling constants as functions of Λ so that the given particular physical quantities at the renormalization points (i.e. the fixed Λ-independent values of them) are reproduced. The reliability of this numerical renormalization procedure in comparison with the explicit analytic renormalization depends only on the accuracy of the numerical approximation, i.e. the two approaches are conceptually equivalent.

In the approach suggested in this work, Ward identities are satisfied order by order in the loop expansion as well as in the chiral expansion of physical quantities [61]. In the few-nucleon sector the physical quantities, at any finite order in the chiral expansion, contain an infinite number of terms in the loop expansion. On the other hand, to any specified order q^n, for a process involving A nucleons, there is a *finite* number of A-nucleon irreducible diagrams. The sum of these diagrams is defined as the effective potential. An infinite number of diagrams contributing in the scattering amplitude (at given order q^n) is summed up by solving the corresponding equations with given nth order effective potential. That is, substituting the q^n-order potential in the Lippmann-Schwinger equation and performing the renormalization properly (as specified above) corresponds to the summation of all *renormalized* diagrams up to order q^n. The solution of the Lippmann-Schwinger equation also contains some, but not all, of the higher-order contributions, and the result is reliable only to order q^n, the error being of order $O(q^{n+1})$. As the Ward identities are satisfied order by order in the chiral expansion, and the Lippmann-Schwinger equation resums *all* contributions to order q^n, the contributions in physical quantities which can violate the identities are of order $\sim O(q^{n+1})$, i.e. beyond the accuracy of the given calculations.

VII. SUMMARY

We have discussed a new formulation of BChPT, which preserves all symmetries of the theory. The main idea is to use some of the structures of the most general effective Lagrangian to improve the ultraviolet behavior of propagators. The coefficients of these terms depend on parameters (with dimension of mass) which serve as smooth cutoffs of the theory. For practical applications it is convenient to choose these parameters to be equal.

We have explicitly applied our new approach to a calculation of the nucleon mass to order $\mathcal{O}(q^3)$. We have also explicitly verified that the electromagnetic Ward identities are satisfied by (strong) one-loop-order corrections. The application of this scheme to the one-nucleon sector of HBChPT demonstrates that the existence of a consistent power counting scheme in HBChPT actually depends on the applied renormalization scheme.

The considerable advantage of the new formulation in comparison with standard dimensional regularization is that, while preserving all symmetries of the effective theory, it leads to equations in the few-nucleon (NN, NN, etc.) sector which are free of divergences. We have explicitly considered examples of the contact interaction and one-pion exchange potentials in the NN scattering problem and have discussed issues of renormalization and consistency.

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APPENDIX A: NUCLEON SELF-ENERGY

The explicit expressions for the loop integrals of Eq. (15) contributing in the calculation of the nucleon self-energy up to and including order $\mathcal{O}(\frac{1}{\Lambda^2})$ read

$$\Lambda^4 I(1111) = I^f_{\pi N} + \frac{1}{16\pi^2} + \frac{1}{8\pi^2}\ln\left(\frac{m}{\Lambda}\right) + \frac{1}{\Lambda^2}\left[M^2 I^f_{\pi N} + \frac{5p^2 - 3m^2 + 6M^2}{96\pi^2} + \frac{(m^2+2M^2)\ln(\frac{m}{\Lambda})}{8\pi^2} + \frac{M^2\ln(\frac{M}{m})}{8\pi^2}\right], \quad \text{(A1)}$$

[11] The original coupling constants c_i are written as $c_i = c_i^r + \delta c_i$, where the c_i^r are redefined coupling constants and the loop expansion of the δc_i part exactly cancels the corresponding Λ-dependent parts of loop diagrams. The c_i^r are independent of momenta, i.e., local interaction terms of the effective Lagrangian remain local.

$$\Lambda^4 I^{(p)}(1110) = -\frac{\Lambda^2}{64\pi^2} + \frac{m^2}{96\pi^2} + \frac{M^2}{64\pi^2} - \frac{p^2}{192\pi^2} + \frac{1}{\Lambda^2}\Big[\frac{3M^4 + M^2p^2}{64\pi^2} - \frac{m^4}{128\pi^2} - \frac{m^2M^2}{48\pi^2} + \frac{7m^2p^2 - 2(p^2)^2}{960\pi^2} + \frac{M^4}{16\pi^2}\ln\Big(\frac{M}{\Lambda}\Big)\Big], \tag{A2}$$

$$\Lambda^4 I^{(p)}(1111) = \Big(-\frac{1}{2} + \frac{m^2}{2p^2} - \frac{M^2}{2p^2}\Big) I^f_{\pi N} - \frac{1}{32\pi^2} - \frac{1}{16\pi^2}\ln\Big(\frac{m}{\Lambda}\Big) + \frac{M^2}{16p^2\pi^2}\ln\Big(\frac{M}{m}\Big) + \frac{1}{\Lambda^2}\Big[I^f_{\pi N}\Big(-\frac{M^2}{2} + \frac{m^2M^2}{2p^2} - \frac{M^4}{2p^2}\Big) - \frac{m^2}{48\pi^2} - \frac{5p^2}{192\pi^2} - \frac{2m^2 + M^2}{16\pi^2}\ln\Big(\frac{m}{\Lambda}\Big) + \frac{M^4}{16\pi^2p^2}\ln\Big(\frac{M}{m}\Big)\Big], \tag{A3}$$

$$\Lambda^4 I(1011) = \frac{\Lambda^2}{16\pi^2} + \frac{m^2}{32\pi^2} + \frac{p^2}{48\pi^2} + \frac{m^2}{8\pi^2}\ln\Big(\frac{m}{\Lambda}\Big) + \frac{1}{\Lambda^2}\Big[-\frac{m^4}{96\pi^2} + \frac{19m^2p^2}{192\pi^2} + \frac{3(p^2)^2}{320\pi^2} + \frac{m^2(m^2 + p^2)}{8\pi^2}\ln\Big(\frac{m}{\Lambda}\Big)\Big], \tag{A4}$$

with

$$I^f_{\pi N} = \frac{1}{16\pi^2}\Big[-1 + \frac{p^2 - m^2 + M^2}{p^2}\ln\Big(\frac{M}{m}\Big) + \frac{2mM}{p^2}F(\Omega)\Big], \tag{A5}$$

where

$$F(\Omega) = \begin{cases} \sqrt{\Omega^2 - 1}\ln(-\Omega - \sqrt{\Omega^2 - 1}), & \Omega \le -1, \\ \sqrt{1 - \Omega^2}\arccos(-\Omega), & -1 \le \Omega \le 1, \\ \sqrt{\Omega^2 - 1}\ln(\Omega + \sqrt{\Omega^2 - 1}) - i\pi\sqrt{\Omega^2 - 1}, & 1 \le \Omega, \end{cases}$$

and

$$\Omega = \frac{p^2 - m^2 - M^2}{2mM}.$$

APPENDIX B: HBCHPT

The expansions of the considered heavy-baryon integrals of Eq. (35) around $\Lambda = \infty$ are given by

$$J_{\pi N}(121;\omega) = \mathcal{O}(\Lambda), \tag{B1}$$

$$J_{\pi N}(021;\omega) = \frac{\Lambda^3}{16\pi} + \frac{\Lambda^2}{8\pi^2}\omega + \mathcal{O}(\Lambda), \tag{B2}$$

$$J_{\pi N}(120;\omega) = \frac{\Lambda^2}{16\pi^2} + \frac{M^2[1 + 2\ln(\frac{M}{\Lambda})]}{16\pi^2} + \mathcal{O}\Big(\frac{1}{\Lambda^2}\Big). \tag{B3}$$

APPENDIX C: *NN* SECTOR

1. Standard dimensional regularization

The explicit expression for the loop integral I_{NN} in dimensional regularization is given by

$$I_{NN} = 2\bar{\lambda} + I^f_{NN}, \tag{C1}$$

with

$$\bar{\lambda} = \frac{m^{n-4}}{16\pi^2}\Big\{\frac{1}{n-4} - \frac{1}{2}[\ln(4\pi) + \Gamma'(1) + 1]\Big\} \tag{C2}$$

and

$$I^f_{NN} = -\frac{1}{16\pi^2}\Big[1 + \sqrt{1 - \frac{4m^2}{P^2}}\ln\Big(\frac{1 - \sqrt{1 - \frac{4m^2}{P^2}}}{1 + \sqrt{1 - \frac{4m^2}{P^2}}}\Big) + i\pi\sqrt{1 - \frac{4m^2}{P^2}}\Big] = -\frac{1}{16\pi^2} - \frac{ip}{16\pi m} + \mathcal{O}(p^2). \tag{C3}$$

The subtracted loop integral I^R_{NN} reads

$$I^R_{NN} = I_{NN} - I_{NN}|_{P^2=4m^2} = -\frac{ip}{16\pi m} + \mathcal{O}(p^2). \tag{C4}$$

For the vector integral $I^{(P)}_{NN}$ we obtain

$$I^{(P)}_{NN} = \bar{\lambda} + \frac{1}{2}I^f_{NN}, \tag{C5}$$

and the subtracted loop integral $I^{(P)R}_{NN}$ is given by

$$I^{(P)R}_{NN} = I^{(P)}_{NN} - I^{(P)}_{NN}|_{P^2=4m^2} = -\frac{ip}{32\pi m} + \mathcal{O}(p^2). \tag{C6}$$

The tensor integral is given by

$$I^{\mu\nu}_{NN} = g^{\mu\nu}\left[\frac{6m^2 - P^2}{6}\bar{\lambda} + \frac{P^2 - 6m^2}{288\pi^2}\right] + P^\mu P^\nu\left[\frac{6m^2 - P^2}{288\pi^2 P^2} + \frac{2}{3}\bar{\lambda}\right] + \left[\frac{g^{\mu\nu}(4m^2 - P^2)}{12} + \frac{P^\mu P^\nu}{3}\left(1 - \frac{m^2}{P^2}\right)\right] I^f_{NN}, \quad \text{(C7)}$$

which, after subtraction, reads

$$I^{\mu\nu R}_{NN} = I^{\mu\nu}_{NN} - I^{\mu\nu}_{NN}|_{P^2=4m^2} = -\frac{ip}{64\pi m} P^\mu P^\nu + \mathcal{O}(p^2). \quad \text{(C8)}$$

2. Higher-derivative formulation

In the following, the explicit expressions for the loop integrals in higher-derivative formulation up to and including order $\mathcal{O}(\frac{1}{\Lambda^2})$ are given. The scalar integral reads

$$I_{NN} = I^f_{NN} + \frac{1}{16\pi^2} + \frac{1}{8\pi^2}\ln\left(\frac{m}{\Lambda}\right) + \frac{1}{\Lambda^2}\left[-\frac{m^2}{16\pi^2} + \frac{5P^2}{96\pi^2} + \frac{m^2}{4\pi^2}\ln\left(\frac{m}{\Lambda}\right)\right] + \mathcal{O}\left(\frac{1}{\Lambda^4}\right), \quad \text{(C9)}$$

so that

$$I^R_{NN} = I_{NN} - I_{NN}|_{P^2=4m^2} = -\frac{ip}{16\pi m} + \mathcal{O}(p^2). \quad \text{(C10)}$$

The vector integral is given by

$$I^{(P)}_{NN} = \frac{1}{2} I^f_{NN} + \frac{1}{32\pi^2} + \frac{1}{16\pi^2}\ln\left(\frac{m}{\Lambda}\right) + \frac{1}{\Lambda^2}\left[-\frac{m^2}{32\pi^2} + \frac{5P^2}{192\pi^2} + \frac{m^2}{8\pi^2}\ln\left(\frac{m}{\Lambda}\right)\right], \quad \text{(C11)}$$

and, after subtraction, one obtains

$$I^{(P)R}_{NN} = I^{(P)}_{NN} - I^{(P)}_{NN}|_{P^2=4m^2} = -\frac{ip}{32\pi m} + \mathcal{O}(p^2). \quad \text{(C12)}$$

The expression for the tensor integral reads

$$I^{\mu\nu}_{NN} = \frac{g^{\mu\nu}}{64\pi^2}\Lambda^2 + \frac{g^{\mu\nu}[m^2 + (P^2 - 6m^2)\ln(\frac{\Lambda}{m})]}{96\pi^2} + \frac{P^\mu P^\nu[1 + \frac{m^2}{p^2} - 2\ln(\frac{\Lambda}{m})]}{48\pi^2} + \left[\frac{g^{\mu\nu}(4m^2 - P^2)}{12} + \frac{P^\mu P^\nu}{3}\left(1 - \frac{m^2}{P^2}\right)\right] I^f_{NN} + \frac{1}{\pi^2\Lambda^2}\left\{\left[-\frac{m^4}{32} + \frac{5m^2P^2}{192} - \frac{(P^2)^2}{480} - \frac{m^4}{16}\ln\left(\frac{\Lambda}{m}\right)\right] g^{\mu\nu} + P^\mu P^\nu\left[\frac{m^2}{48} + \frac{17P^2}{960} - \frac{m^2}{8}\ln\left(\frac{\Lambda}{m}\right)\right]\right\}, \quad \text{(C13)}$$

and

$$I^{\mu\nu R}_{NN} = I^{\mu\nu}_{NN} - I^{\mu\nu}_{NN}|_{P^2=4m^2} = -\frac{ip}{64\pi m} P^\mu P^\nu + \mathcal{O}(p^2). \quad \text{(C14)}$$

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