Parametrized post-Newtonian limit of fourth order gravity inspired by scalar-tensor gravity

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Based on the *dynamical* equivalence between higher order gravity and scalar-tensor gravity the parametrized post-Newtonian (PPN) limit of fourth order gravity is discussed. We exploit this analogy developing a fourth order gravity version of the Eddington PPN parameters. As a result, Solar System experiments can be reconciled with higher order gravity, if physical constraints descending from experiments are fulfilled.

I. INTRODUCTION

The recent debate about the origin of the cosmic acceleration, induced by the results of several astrophysical observations [1–3], led to investigate several theoretical approaches capable of providing viable physical mechanisms to the dark energy problem. In this wide discussion, no scheme seems, up to now, to furnish a final answer to this puzzling conundrum. Nevertheless among the different models, ranging from quintessential scenarios [4], which generalize the cosmological constant approach [5], to higher dimensional scenarios [6,7] or the resort to cosmological fluids with exotic equation of state [8,9] and unified approaches considering even dark matter [10,11], an interesting scheme which seems to deserve a major attention is represented by higher order theories of gravity. This approach obtained by the generalization of the Einstein gravity, has led to interesting results both in the metric formulation [12–17] and in the Palatini one [18,19]. Recently some authors have analyzed the PPN limit of such theories both in the metric and in the Palatini approach [20–22] with contrasting results. Thus, it seems interesting to deepen the discussion about the Post Parametrized Newtonian (PPN) behavior of this theory. The purpose is to verify if the cosmological reliability of such a scheme can be drawn even on the Solar System scales and to understand if the hypothesis of a unique fluid working as a two "faces" component (matter and geometry) can be a workable one. In this paper we exploit the strict analogy between the higher order gravity and the scalar-tensor theories to develop a PPN formalism for a general fourth order gravity model in the metric framework, working, in general, for extended theories of gravity. There are strong analogies between these two approaches. The similarity between the nonminimally coupled scalar models and the higher order gravity ones is known since 1983 [23], when it was demonstrated the similarity between a scalar-tensor Lagrangian of Brans-Dicke type and

fourth order gravity. Actually, such an interpretation goes well beyond conformal transformations, since it is a formal analogy without any physical change in the dynamical variables of the system. In this paper, we further discuss the analogy between fourth order gravity and scalar-tensor gravity considering the PPN parametrization descending from such a similarity. As main result, we show, despite some recent studies [21], that Solar System experiments do not exclude the possibility that higher order gravity theories can represent a viable approach even at scales shorter than the cosmological ones. In other words, standard general relativity should be revised both at cosmological and Solar System distances in order to solve several mismatches between the theoretical predictions and the observational results.

II. FOURTH ORDER GRAVITY VS. SCALAR-TENSOR GRAVITY

Let us recall how the analogies between the two schemes arise. As it is well known, scalar-tensor gravity is obtained if a scalar-field-matter Lagrangian is nonminimally coupled with the Hilbert-Einstein Lagrangian. The general action for a such theory is [24]:

$$\mathcal{A} = \int d^4x \sqrt{-g} \left[F(\phi)R + \frac{1}{2} g^{\mu\nu} \phi_{;\mu} \phi_{;\nu} - V(\phi) + \kappa \mathcal{L}_m \right], \tag{1}$$

where $F(\phi)$ is the coupling function, $V(\phi)$ the self-interaction potential, ϕ a scalar field, $\mathcal{L}_m$ the ordinary matter Lagrangian and κ the dimensional coupling. This relation naturally provides Brans-Dicke gravity [25] if it is rearranged through the substitutions: $\varphi = F(\phi)$, $\omega(\varphi) = -F(\phi)/[2F'(\phi)^2]$ [26]; its peculiarity is to account for Mach principle which leads back inertial forces within the background of gravitational interactions. The $f(R)$ gravity action in the metric formalism is the following [12,13,27]

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$$\mathcal{A} = \int d^4x \sqrt{-g}[f(R) + \kappa \mathcal{L}_m], \qquad (2)$$

which depends on the metric $g_{\mu\nu}$ and the matter fields. Again κ defines the dimensional coupling. The energy-momentum tensor of matter is given by the relation $T^m_{\mu\nu} = (-2/\sqrt{-g})(\delta \mathcal{L}_m/\delta g^{\mu\nu})$. From the action (2), we obtain the fourth order field equations:

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} = f'(R)^{;\alpha\beta}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\alpha}g_{\alpha\beta})$$
$$+ \kappa T^m_{\mu\nu}, \qquad (3)$$

which can be recast in a more expressive form as:

$$G_{\mu\nu} = \frac{1}{f'(R)}\left\{ \frac{1}{2}g_{\mu\nu}[f(R) - f'(R)R] + f'(R)_{;\mu\nu} \right.$$
$$\left. + -g_{\mu\nu}\Box f'(R) \right\} + \frac{\kappa}{f'(R)}T^m_{\mu\nu}, \qquad (4)$$

where $G_{\mu\nu}$ is the Einstein tensor and $f'(R) \equiv df/dR$; the two terms $f'(R)_{;\mu\nu}$ and $\Box f'(R)$ imply fourth order derivatives of the metric $g_{\mu\nu}$. On the other side, if $f(R)$ is a linear function of the scalar curvature, $f(R) = a + bR$, the field equations become the ordinary second-order ones. Considering the trace of Eq. (4),

$$3\Box f'(R) + f'(R)R - 2f(R) = \kappa T. \qquad (5)$$

Such an equation can be interpreted as the equation of motion of a self-interacting scalar field, where the self-interaction potential role is played by the quantity $V(R) = f'(R)R - 2f(R)$. This analogy can be developed each time one considers an analytic function of R which can be algebraically inverted so that R reads as $R = R(f')$, in other words it has to be $f''(R) \neq 0$. In fact, defining

$$\phi \equiv f'(R) \qquad (6)$$

$$V(\phi) \equiv R(\phi)f'(R) - f(\phi), \qquad (7)$$

we can write Eqs. (4) and (5) as

$$G_{\mu\nu} = \frac{\kappa}{\phi}T_{\mu\nu} - \frac{V(\phi)}{2\phi}g_{\mu\nu} + \frac{1}{\phi}(\phi_{;\mu\nu} - g_{\mu\nu}\Box\phi) \quad (8)$$

$$3\Box\phi + 2V(\phi) - \phi\frac{dV}{d\phi} = \kappa T, \qquad (9)$$

which can also be obtained from a Brans-Dicke action of the form

$$\mathcal{A}_\phi = \int d^4x \sqrt{-g}[\phi R - V(\phi) + \kappa\mathcal{L}_m]. \qquad (10)$$

This expression is related to the so-called *O'Hanlon Lagrangian*, which belongs to a class of Lagrangians introduced in order to achieve a covariant model for a massive dilaton theory [28]. It is evident that the Lagrangian (10) is very similar to a Brans-Dicke theory,

but is lacking of the kinetic term. The formal analogy between the Brans-Dicke scheme and fourth order gravity schemes is obtained in the particular case $\omega_{BD} = 0$. If we consider the matter term vanishing, Eq. (5) becomes an ordinary Klein-Gordon equation, where $f'(R)$ plays the role of an effective scalar field whose mass is determined by the self-interaction potential.

III. PPN FORMALISM IN SCALAR-TENSOR GRAVITY

Along this paper, we base our discussion on the analogy between scalar-tensor theories of gravity and the higher order ones to analyze the problem of the PPN limit for the fourth order gravity model. Recently the cosmological relevance of higher order gravity has been widely demonstrated. On the other side, the low energy limit of such theories is still not satisfactory investigated, although some results on the galactic scales have been already achieved [29]. A fundamental test to understand the relevance of such a scheme is to check if there is even an accord with Solar System experiments. As outlined in the introduction, some controversial results have been recently proposed [21,22]. To better develop this analysis, we can refer again to the scalar-tensor–higher order gravity analogy, exploiting the PPN results obtained in the scalar-tensor scheme [30]. A satisfactory description of PPN limit for this kind of theories has been developed in [30–32]. In these works, the problem has been treated providing interesting results even in the case of strong gravitational sources like pulsars and neutron stars where the deviations from General Relativity are obtained in a nonperturbative regime [32]. A clear summary of this formalism can be found in the papers [30,33]. The action to describe a scalar-tensor theory can be assumed, in natural units, of the form (1). The matter Lagrangian density is again considered depending only on the metric $g_{\mu\nu}$ and the matter fields. This action can be easily redefined in term of a minimally coupled scalar field model *via* a conformal transformation of the form $g^*_{\mu\nu} = F(\phi)g_{\mu\nu}$. In fact, assuming the transformation rules:

$$\left(\frac{d\psi}{d\phi}\right)^2 = \frac{3}{4}\left(\frac{d\ln F(\phi)}{d\phi}\right)^2 + \frac{1}{2F(\phi)}, \qquad (11)$$

and

$$A(\psi) = F^{-1/2}(\phi), \qquad V(\psi) = V(\phi)F^{-2}(\phi), \qquad (12)$$

$$\mathcal{L}^*_m = \mathcal{L}_m F^{-2}(\phi), \qquad (13)$$

one gets the action

$$\mathcal{A}_* = \int \sqrt{-g_*}\left[R_* + \frac{1}{2}g^{\mu\nu}_*\psi_{,\mu}\psi_{,\nu} - V(\psi) + \mathcal{L}^*_m \right]. \qquad (14)$$

The first consequence of such a transformation is that now

TABLE I.   A schematic resume of recent constraints on the PPN parameters. They are the perihelion shift of Mercury [35], the Lunar Laser Ranging [36], the upper limit coming from the Very Long Baseline Interferometry [37] and the results obtained by the estimate of the Cassini spacecraft delay into the radio waves transmission near the Solar conjunction [38].

Mercury Perihelion Shift	$|2\gamma_0^{\text{PPN}} - \beta_0^{\text{PPN}} - 1| < 3 \times 10^{-3}$
Lunar Laser Ranging	$4\beta_0^{\text{PPN}} - \gamma_0^{\text{PPN}} - 3 = -(0.7 \pm 1) \times 10^{-3}$
Very Long Baseline Interferometry	$|\gamma_0^{\text{PPN}} - 1| = 4 \times 10^{-4}$
Cassini spacecraft	$\gamma_0^{\text{PPN}} - 1 = (2.1 \pm 2.3) \times 10^{-5}$

the nonminimal coupling is transferred on the ordinary matter sector. In fact, the Lagrangian $\mathcal{L}_m^*$ is dependent not only on the conformally transformed metric $g_{\mu\nu}^*$ and the matter field but it is even characterized by the coupling function $A(\psi)^2$. In the same way, the field equations can be recast in the Einstein frame. The energy-momentum tensor is defined as $T_{\mu\nu}^{m*} = (2/\sqrt{-g^*})(\delta \mathcal{L}_m/\delta g_{\mu\nu}^*)$ and it is related to the Jordan expression as $T_{\mu\nu}^{m*} = A(\psi)T_{\mu\nu}^m$. The function:

$$\alpha(\psi) = \frac{d \ln A(\psi)}{d\psi} \qquad (15)$$

establishes a measure of the coupling arising in the Einstein frame between the scalar sector and the matter one as an effect of the conformal transformation (General Relativity is recovered when this quantity vanishes). It is possible even to define a control of the variation of the coupling function through the definition of the parameter $\beta = d\alpha(\psi)/d\psi$. Regarding the effective gravitational constant, it can be expressed in term of the function $A(\psi)$ as $G_{\text{eff}} = \frac{G_N}{F(\phi)} = G_N A^2(\psi)$. It has to be remarked that such a quantity is, in reality, well different by the Newton constant measured in the Cavendish-like terrestrial experiments (see Eq. (21) below). Let us now, concentrate on the scalar-tensor generalization of the local gravitational constraints. Deviations from General Relativity can be characterized through Solar System experiments [34] and binary-pulsar observations which give an experimental estimate of the PPN parameters. These parameters were introduced by Eddington to better determine the deviation from the standard prediction of General Relativity, expanding local metrics as the Schwarzschild one, to higher order terms. The generalization of this quantities to scalar-tensor theories allows the PPN parameters to be expressed in term of nonminimal coupling function $F(\phi)$ or, equivalently, in term of the parameter α defined in Eq. (15), that is:

$$\gamma^{\text{PPN}} - 1 = -\frac{F'(\phi)^2}{F(\phi) + 2F'(\phi)^2} = -2\frac{\alpha^2}{1 + \alpha^2}, \quad (16)$$

$$\beta^{\text{PPN}} - 1 = \frac{1}{4}\frac{F(\phi) \cdot F'(\phi)}{2F(\phi) + 3F'(\phi)^2}\frac{d\gamma^{\text{PPN}}}{d\phi}$$

$$= \frac{1}{2}\frac{\alpha^2}{(1 + \alpha^2)^2}\frac{d\alpha}{d\psi}. \qquad (17)$$

The above definitions imply that the PPN-parameters become dependent on the nonminimal coupling function $F(\phi)$ and its derivatives. They can be directly constrained by the observational data. Actually, Solar System experiments give accurate indications on the ranges of $\gamma_0^{\text{PPN}}, \beta_0^{\text{PPN}}$[1]. Results are summarized in Table I. The experimental results can be substantially resumed into the two limits [33]:

$$|\gamma_0^{\text{PPN}} - 1| \leq 2 \times 10^{-3}, \qquad |\beta_0^{\text{PPN}} - 1| \leq 6 \times 10^{-4},$$

$$(18)$$

which can be converted into constraints on α_0 and β_0. In particular, the Cassini spacecraft value induces the bound $\alpha_0 \sim (F_{0,\phi}/F_0) < 4 \times 10^{-4}$. At first sight, one can deduce that the first derivative of the coupling function $A(\psi)$ has to be very small, which means a very low interaction between matter and the scalar field; conversely the second derivative β_0 can take large values so that the matter sector may be strongly coupled with scalar degrees of freedom [30]. Together with the Solar System experiments, even binary-pulsar tests can be physically significant to characterize the PPN parameters. From this analysis [30–32] descends that the second derivative can be a large number, i.e. $\beta_0 > -4.5$ even for a vanishingly small α_0. This constraint allows to achieve a further limit on the two PPN parameters γ^{PPN} and β^{PPN}, which can be outlined by means of the ratio:

$$\frac{\beta^{\text{PPN}} - 1}{\gamma^{\text{PPN}} - 1} < 1.1. \qquad (19)$$

The singular (0/0) nature of this ratio puts in evidence that it was not possible to get such a limit in the case of weak-field experiments (see for details [30]). For sake of completeness, we cite here even the shift that the scalar-tensor gravity induces on the theoretical predictions for the local value of the gravitational constant as coming from Cavendish-like experiments. This quantity represents the gravitational coupling measured when the Newton force arises between two masses:

$$G_{\text{Cav}} = \frac{F \cdot r^2}{m_1 \cdot m_2}. \qquad (20)$$

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[1]We indicate with the subscript $_0$ the Solar System measured estimates.

In the case of scalar-tensor gravity, the Cavendish coupling reads:

$$G_{\text{Cav}} = \frac{G_N}{F(\phi)}\left[1 + \frac{F'(\phi)^2}{2F(\phi) + 3F'(\phi)^2}\right]$$
$$= G_N A^2(\psi)(1 + \alpha^2). \tag{21}$$

From the limit on α coming from Cassini spacecraft, the difference between G_{Cav} and G_{eff} is not more than the $10^{-3}\%$.

IV. PPN LIMIT OF FOURTH ORDER GRAVITY INSPIRED BY THE SCALAR-TENSOR ANALOGY

In previous section, we discussed the PPN limit in the case of a scalar-tensor gravity. These results can be extended to the case of fourth order exploiting the analogy with scalar-tensor case developed in Sec. II. We have seen that fourth order gravity is equivalent to the introduction of a scalar extra degree of freedom into the dynamics. In particular, from this transformation, it derives a Brans-Dicke type Lagrangian with a vanishing Brans-Dicke parameter $\omega_{\text{BD}} = 0$. Performing the change of variables implied by a conformal transformation, the Brans-Dicke Lagrangian can be furtherly transformed into a Lagrangian where the nonminimal coupling is moved onto the matter side as in (14). The net effect is that, as in the case of a "true" scalar-tensor theory, it is possible to develop an Einstein frame formalism which allows a PPN limit analysis. The basic physical difference between the two descriptions is that the quantities entering the PPN parameters γ^{PPN} and β^{PPN}, or the derivatives of the nonminimal coupling function $A(\psi)$, are now $f'(R)$ and its derivatives with respect to the Ricci scalar R since the nonminimal coupling function in the Jordan frame is $f'(R) \equiv \phi$. Alternatively, to obtain a more versatile equivalence between the two approaches it is possible to write down fourth order gravity by an analytic function of the Ricci scalar considering the identification induced by the field equations, i.e. $\varphi \to R$. In fact, if one takes into account the scalar-tensor Lagrangian:

$$\int d^4\sqrt{-g}[F(\varphi) + (R - \varphi)F'(\varphi) + \kappa\mathcal{L}_m], \tag{22}$$

the variation with respect φ and the metric provide the above identification and a system of field equations which are completely equivalent to the ordinary ones descending from fourth order gravity. The expression (22) can be recast in the form of the O'Hanlon Lagrangian (10) by means of the substitutions:

$$\phi \equiv F'(\varphi), \qquad V(\phi) \equiv \varphi F'(\varphi) - F(\varphi), \tag{23}$$

where, in such a case, the prime means the derivative with respect to φ. It is evident that the new scalar-tensor description implies a nonminimal coupling function through the term

$$F'(\varphi) = \frac{df(R)}{dR}, \tag{24}$$

and the identification $\varphi \to R$ implies that the higher order derivatives can be straightforwardly generalized. At this point, it is immediate to extend the results of the PPN formalism developed for scalar-tensor gravity to the case of a fourth order theory. In fact, it is possible to recast the PPN parameters (16) and (17) in term of the curvature invariants quantities. This means that the nonminimal coupling function role, in the fourth order scenario, is played by the $df(R)/dR$ quantity. As a consequence the PPN parameters (16) and (17) become:

$$\gamma_R^{\text{PPN}} - 1 = -\frac{f''(R)^2}{f'(R) + 2f''(R)^2}, \tag{25}$$

$$\beta_R^{\text{PPN}} - 1 = \frac{1}{4}\frac{f'(R) \cdot f''(R)}{2f'(R) + 3f''(R)^2}\frac{d\gamma_R^{\text{PPN}}}{d\varphi}. \tag{26}$$

These quantities have, now, to fulfill the requirements drawn from the experimental tests resumed in Table I. The immediate consequence of such definitions is that derivatives of fourth order gravity theories have to satisfy constraints in relation to the actual measured values of the Ricci scalar R_0. As a matter of fact, one can check these quantities by the Solar System experimental prescriptions and deduce the compatibility between fourth order gravity and General Relativity. Since the definitions (25) and (26) do not allow to obtain, in general, upper limits on $f(R)$ from the constraints of Table I, one can arbitrarily chose classes of fourth order Lagrangians, in order to check if the approach is working. We shall adopt classes of Lagrangians which are interesting from a cosmological point of view since give viable results to solve the dark energy problem [12–14]. In principle, one can try to obtain some hints on the form of $F(\varphi)$ (or similarly of the $f(R)$) by imposing constraints provided from the Lunar Laser Ranging (LLR) experiments and the Cassini spacecraft measurements which give direct stringent estimates of PPN-parameters. After, one can try to solve these relations and then to verify what is the response to the pulsar upper limit with respect to the ratio $(\beta^{\text{PPN}} - 1)/(\gamma^{\text{PPN}} - 1) < 1.1$. This procedure has shown that generally if the two Solar System relations are verified, the pulsar constraint is well fitted by a modified gravity model. However this result is strictly influenced by the error range of the Cassini and LLR tests. After this remark, one can consider different fourth order Lagrangians with respect to the two Solar System constraints coming from the perihelion shift of Mercury and the Very Long Baseline Interferometry. The results are summarized in Table II. We have listed the fourth order Lagrangians considered in the first column and the limit on the model parameters induced by the Solar System constraints is in the second column. As it is possible to see, the PPN limits induced by the Solar System

TABLE II. Constraints induced by PPN-experimental upper bounds for different cases of fourth order gravity Lagrangians. Solar System experiments are the Mercury Perihelion Shift and the Very Long Baseline Interferometry.

Lagrangian	Parameters constraints
$f_0 R^2$	$R_0 < 0, \frac{R_0}{4996} < f_0 < -\frac{R_0}{5004}$ $R_0 > 0, -\frac{R_0}{5004} < f_0 < \frac{R_0}{4996}$
$f_0 R^3$	$-\frac{1}{30024} < f_0 < \frac{1}{29976}$
$R + aR^2$	$a = 0$ $\{a > 09992a < \frac{1}{a} + 2R_0\}$ $\{a > 0, \frac{1}{a} + 10008a + 2R_0 < 0\}$ $\{a < 0, \frac{1}{a} + 10008a + 2R_0 > 0\}$ $\{a < 09992a > \frac{1}{a} + 2R_0\}$
$A \log[R]$	$A \leq 0, R_0 < 13.5685A^{1/3}$ $R_0 > -13.5757A^{1/3}$ $A > 0, R_0 < -13.5757A^{1/3}$ $R_0 > 13.5685A^{1/3}$

tests can be fulfilled by different kinds of fourth order Lagrangians provided that their parameters remain well defined with respect to the background value of the curvature. These results corroborate evidences for a defined PPN limit which does not exclude higher order gravity. They are in contrast with other recent investigations [21,39], where it has been pointed out that this kind of theories are not excluded by experimental results in the weak field limit and with respect to the PPN prescriptions. Similar results also hold for Lagrangians as $f(R) = f_0 R^n$ and $f(R) = R + \frac{\mu}{R}$ which have shown interesting properties from a cosmological point of view [12–15]. This fact allows to establish a significant link between gravity at local and cosmological scales. Finally a remark is in order. It has to be taken into account that the $f(R) = A \ln[R]$ does not admit a Minkowski background around which to perform the usual post-Newtonian analysis. Because of this fact this model is essentially different from the others in the weak energy limit.

V. CONCLUSIONS

Since the issue of higher order gravity is recently become a very debated matter, we have discussed its low energy limit considering the PPN formalism in the metric framework. The study is based on the analogy between the scalar-tensor gravity and fourth order gravity. Such an investigation is particularly interesting even in relation to the debate about the real meaning of the curvature fluid which could be a natural explanation for dark energy [12–19,21,20,22,40]. The PPN limit indicates that several fourth order Lagrangians could be viable on the Solar System scales. It has to remarked that the Solar System experiments pose rather tight constraints on the values of coupling constants, e.g. f_0 (see Table II). Such a result does not agree with the very recent papers [21] which suggest negative conclusions in this sense, based on questionable theoretical assumptions and extrapolations. It is evident that such discussion does not represent a final answer on this puzzling issue. Nevertheless it is reasonable to affirm that extended gravity theories cannot be ruled out, definitively, by Solar System experiments. Of course, further accurate investigations are needed to achieve some other significant indications in this sense, both from theoretical and experimental points of view. For example the study of higher order gravity PPN limit directly in the Jordan frame could represent an interesting task for forthcoming investigations. An important concluding remark is due at this point. A scalar-tensor theory can be recast in the Einstein frame, *via* a conformal transformation, implying an equivalent framework. Actually, dealing with higher order gravity, there is no more such a conformal transformation able to "equivalently" transform the whole system from the Jordan frame to the Einstein one. Effectively, it is possible to conformally transform a higher order (and, in particular, a fourth order) theory into an Einstein-like with the addiction of some scalar fields as a direct consequence of the equivalence between the higher order framework and the scalar-tensor one at level of the classical field equations. This equivalence addressed, as *dynamical equivalence* [41], does not holds anymore when one considers configurations which do not follow the classical trajectories, for example, in the case of quantum effects. A fundamental result which follows from this considerations is that dealing with the early-time inflationary scenario one can safely perform calculations for the primordial perturbations in the Einstein conformal frame of a scalar-tensor model while it is not possible to develop such calculations in the case of an higher order gravity scenario since the scalar degrees of freedom are no more independent of the gravitational field source. This issue holds, if the effective field is induced from geometrical degrees of freedom. Since the PPN limit is achieved in the semiclassical limit, when the conformal factor turns out to be well defined, deductions about the PPN limit for fourth order gravity models, developed exploiting the analogy with the scalar-tensor scheme, are safe from problems.

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