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Cosmological constraints on Newton's constant

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We present cosmological constraints on deviations of Newton's constant at large scales without specifying a model, analyzing latest cosmic microwave background (CMB) anisotropies and primordial abundances of light elements synthesized by big bang nucleosynthesis (BBN). BBN limits the possible deviation at typical scales of BBN epoch, say at 10^8–10^{12} m, to lie between -5% and $+1\%$ of the experimental value, and CMB restricts the deviation at larger scales 10^2–10^9 pc to be between -26% and $+66\%$ at the 2σ confidence level. The cosmological constraints are compared with the astronomical one from the evolution of isochrone of globular clusters.

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Newton's law of gravitation has been extensively tested and verified in three length scales: the laboratory scales $r \lesssim 1$ m [1], the geophysical scales $r \approx 100$ m [2], and the astronomical scales $r \approx 10^8$ m [3]. Such measurements nicely agree with the inverse square law within their experimental or observational uncertainty [4,5]. In particular, the first two measurements at the laboratory and geophysical scales succeeded also in determining the experimental value $G_{\rm N}$ of Newton's constant, and the value determined at such terrestrial scales is applied for all phenomena from Planck scale to cosmological scale.

The astronomical measurements [3], mainly through planetary and satellite orbits, yield a strong constraint on the deviation from the inverse square law. However, it should be noted that the measurements cannot give any information about the value of Newton's constant G itself without evaluating masses M of interacting bodies, since constraint is possible only on GM. Therefore, the measurements cannot exclude the possibility of a different value of G at astronomical and cosmological scales, if G is almost constant at limited scales relevant to the measurements. In particular, we have only a poor knowledge at scales larger than the solar system, say $r \gtrsim 1\ \text{pc} \approx 3 \times 10^{16}$ m [5]. Interesting trials to solve this problem were recently reported [6,7], in which the deviation of G at Mpc scales is restricted by the power of the clustering of galaxies.

The possibility that Newton's constant at laboratory scale, $G_{\rm N}$, is different from that at very large scales, G_∞, arises in many contexts. Historically, studies toward the problem of unifying gravity with the other fundamental forces suggested a departure from Newtonian gravity in the range 10–100 m [8]. It is often assumed that such a correction can be represented by the addition of a Yukawa term to the conventional gravitational potential: $V = -\frac{G(r)M}{r}$ for $G(r) = G_\infty(1 + \alpha e^{-r/\lambda})$, where α is the relative weight of the non-Newtonian term. In this expression, at cosmological distances r satisfying $r \gg \lambda$, the exponential term vanishes, so that $G(r) = G_\infty$. On the other hand, for r of experimental scales which satisfies $r \ll \lambda$, the exponential becomes unity and $G(r)$ recovers $G_{\rm N}$, that is, $G_{\rm N} = G_\infty(1 + \alpha)$.

Recently several types of higher-dimensional theories of gravity, motivated by superstring, have been proposed and many researchers pay great attention to the extra dimension scenario. As a characteristic feature, all the theories lead to deviations from the conventional Newton's law [9,10], since the theories allow the graviton to propagate in higher-dimensional spacetime. Among them, an interesting idea was proposed by Dvali, Gabadadze, and Porrati (DGP model). In the model, the present accelerating expansion of the universe is attributed to leaking gravity into an extra dimension [11]. This idea reproduces the present cosmic acceleration without a dark energy component, and consequently predicts the modification of Newton's law at cosmological scales. Another interesting proposal is a braneworld model with a Gauss-Bonnet term, which suggests $G_{\rm N} = \frac{2+\bar{\alpha}}{3\bar{\alpha}} G_\infty$ with a model parameter $\bar{\alpha}$ [12].

These theoretical suggestions indicate that it is quite important to place possible constraints on the value of G at astronomical and cosmological scales. In this paper, we take a simple parametrization of

$$G(r) = \xi G_{\rm N} \tag{1}$$

for a finite region of r relevant to the measurement which we are considering. This parametrization is justified, when the range λ of the Yukawa-type interaction is much smaller than any r belonging to the relevant region. As such a measurement, we consider cosmic microwave background (CMB) anisotropies and primordial abundances created at the big bang nucleosynthesis (BBN) epoch, and assume λ which well satisfies the condition $\lambda \ll r$ for r relevant to these measurements. So we calculate CMB anisotropies and primordial abundances simply with Eq. (1) and put constraints on ξ at two different scales relevant to these observations. The cosmological constraints are compared with an astronomical constraint determined from the isochrone of globular clusters.

The observed primordial light-element abundances constrain the value of G during the BBN epoch from the time of weak-reaction freeze-out ($t \sim 1$ sec, $T \sim 1$ MeV) to the freeze-out of nuclear reactions ($t \sim 10^4$ sec, $T \sim 10$ keV). In this epoch, the length of cosmic horizon varies from 10^8 m to 10^{12} m, and thus BBN can constrain Newton's constant at these scales.

The primordial helium abundance is obtained by measuring extra galactic ionized hydrogen regions. We adopt a range of $Y_p = 0.2452 \pm 0.0015$ [13] for the helium abundance. The primordial deuterium is best determined from its absorption lines in high redshift Lyman α clouds along the lines of sight to background quasars. For deuterium there is a similar possibility for either a high or a low value. For the present discussion, however, we shall adopt the generally accepted low value for the D/H abundance, $\mathrm{D/H} = 2.78^{+0.44}_{-0.38} \times 10^{-5}$ [14].

The increase of Newton's constant causes the increase of the universal expansion rate. This makes the neutron-to-proton ratio larger, because the weak reactions freeze-out at a higher temperature, and also because there is less time for neutrons to decay between the time of weak-reaction freeze-out and the onset of BBN. Consequently, a larger value of G during the BBN epoch yields a larger ^{4}He abundance, since most of the free neutrons are converted into ^{4}He nuclei. D/H also increases largely because the reactions destroying deuterium fall out of nuclear statistical equilibrium while the deuterium abundance is higher. Similarly, there is less time for the destructive reaction $^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$. This causes ^{7}Li to be more abundant for $\eta < 3 \times 10^{-10}$. However, there is also less time for the $^4\mathrm{He}(^3\mathrm{He},\gamma)^7\mathrm{Be}$ reaction to occur. This causes ^{7}Li to be less abundant for $\eta > 3 \times 10^{-10}$ [15]. Figure 1 illustrates the dependence of the nucleosynthesis yields with ξ.

The upper limit of Newton's constant comes from the ^{4}He upper bound and the D/H upper bound. The lower limit comes from the lower bounds. We note that the constraint from ^{7}Li is not consistent with those from ^{4}He and D/H, even when we vary ξ. In the present analysis, however, we omit the constraint from ^{7}Li abundance, since it involves an uncertainty more largely than the other primordial elements do. The shaded region on Fig. 2 shows allowed values of ξ. The BBN constraint thus obtained is $0.95 \leq \xi \leq 1.01$ at 10^8–10^{12} m scales.

The effect of ξ on primordial abundances degenerates with that of the effective number of neutrino species N_ν. A constraint on the cosmological expansion rate, i.e. the effective number, during the BBN epoch is obtained in a model independent way [16]. In our analysis, however, we have fixed N_ν to the canonical value 3.04, since the present model is a simplification of sophisticated models such as the DGP model [11], the braneworld model with Gauss-Bonnet term [12], or a scalar-tensor theory [17] without any extra energy component. The present result on ξ, $0.95 \leq \xi \leq 1.01$, corresponds to $-0.3 \lesssim \Delta N_\nu \lesssim 0.1$. This result naturally includes the canonical value $\Delta N_\nu = 0$ in consequence of taking the latest data on helium and deuterium abundances and neutron lifetime.

One point to be mentioned here is that the canonical value of 3.04 may be changed due to an increase or a decrease of the expansion rate, because the decoupling

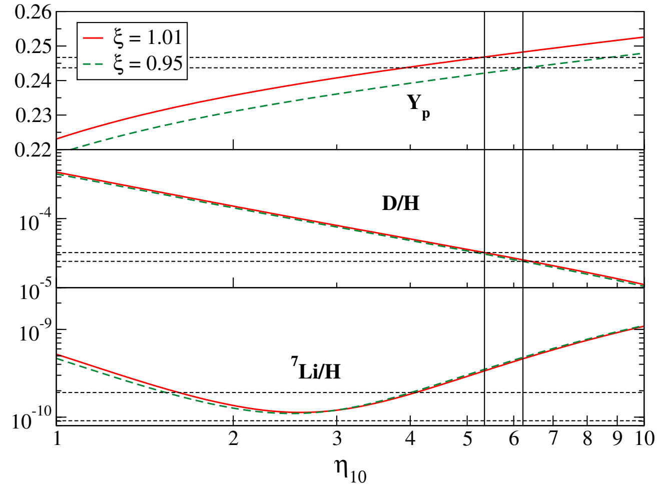

FIG. 1 (color online). Predicted BBN light-element abundances vs the baryon-to-photon ratio η_{10}. ^{4}He, D, and ^{7}Li abundances are shown in the top, center, and bottom panels, respectively. They are compared with the observationally inferred primordial abundances (horizontal lines). Plotted are models with $\xi = 1.01$ (solid lines) and $\xi = 0.95$ (dashed lines). In our analysis, neutron lifetime is taken to be the average value of $\tau_n = 878.5$ sec [27] and $\tau_n = 885.7$ sec [29].

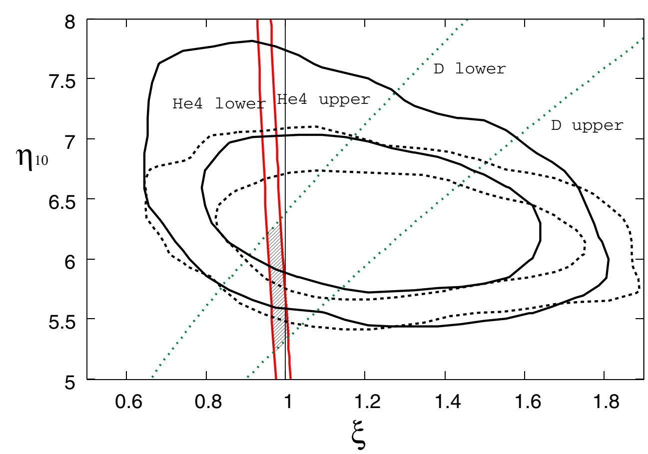

FIG. 2 (color online). Constraints from the primordial abundances (lines) and CMB (contours) on η_{10} and ξ. The shaded region denotes parameters allowed by BBN. The contours show the marginalized 1, 2σ limits on this parameter plane from fits to the CMB power spectrum. The solid and dashed contours correspond to the limits from WMAP data alone, and those from WMAP, CBI, and ACBAR data sets, respectively.

temperature of neutrinos is changed. In this paper, however, we assumed that this effect is small and neglected it.

The temperature fluctuations at recombination observed through the CMB anisotropies contain much information on many kinds of cosmological parameters and evolution of perturbations at a wide range of scales. Typically, the scale which can be explored by CMB observations currently available is from the horizon scale at present ($\sim$ Gpc) to $\sim$10 Mpc in a comoving coordinate. This shows that the scales relevant to CMB are from 10^2 pc to $\sim$Gpc, if we consider the evolution of perturbations from the horizon crossing of each Fourier mode; for example, 10^2 pc is the horizon scale at the time when the mode of $\sim$10 Mpc in a comoving coordinate enters the horizon. Thus it follows that CMB can constrain the value of ξ at scales larger than $\sim 10^2$ pc. Here, in order to calculate CMB anisotropies in a consistent manner, we assume that the scale dependence of Newton's constant is very weak at the relevant scales, which is consistent with a simple parametrization of Eq. (1).

In order to obtain a constraint on ξ from latest CMB anisotropy data sets, we generate CMB angular power spectra C_ℓ in a wide range of ξ by using a Boltzmann code of CMBFAST [18]. It is well known, however, that in addition to ξ there exist many other cosmological parameters relevant to CMB. Thus, we explore the likelihood in seven-dimensional parameter space, i.e., $\Omega_b h^2$ (baryon density), $\Omega_c h^2$ (cold dark matter density), h (Hubble parameter), z_{re} (reionization redshift), n_s (power spectrum index), A_s (overall amplitude), and ξ. We then marginalize over nuisance parameters through the use of the Markov Chain Monte Carlo technique [19].

The most distinguishable effects of changing Newton's constant appear at the amplitude of the acoustic peaks in the CMB power spectrum as shown in Fig. 3. The main reason for this is that, as already found in [20], the visibility function, $g(\tau) = \dot{\kappa}\exp(-\kappa)$, changes with ξ, where κ is the optical depth of the Thomson scattering. More specifically, increasing Newton's constant makes the expansion of the universe faster at a given redshift, and makes it more difficult for a proton and a electron to recombine to form a hydrogen atom. This leads to a larger ionization fraction and a broader visibility function at last scattering epochs, which damp the anisotropies at small scales due to the canceling effect.

Second, in addition to the effect discussed above, we find that increasing Newton's constant suppresses the second and higher acoustic peaks even larger, since for the increase of ξ the diffusion scale for photons to spread through the random walk ($\sim t_{\rm dec}^{1/2} \propto \xi^{-1/4}$) is shifted more largely than the scale of the first acoustic peak ($\sim t_{\rm dec} \propto \xi^{-1/2}$). Thus, the shape of acoustic peaks can be used to constrain the variation of ξ.

Figure 4 shows the marginalized likelihood of ξ. We obtain from the figure that $0.74 \lesssim \xi \lesssim 1.66$ by WMAP data alone [21], $0.75 \lesssim \xi \lesssim 1.74$ by WMAP, CBI, and ACBAR data sets [22,23], at 95% confidence level.

Another constraint on the value of Newton's constant can be obtained by analyzing the age of stars in globular clusters. The key idea is that increasing Newton's constant causes stars to burn faster [24]. Thus, this allows us to constrain ξ at stellar scale $\sim 10^9$ m, as we shall see below, by analyzing the timing of the main sequence turn-off.

Let us assume that the luminosity of the star depends on Newton's constant G and helium abundance Y, approximately as $L \propto y(Y)g(G)$, where y and g are functions of Y and G [25]. Since helium production should be proportional to the luminosity, we have $\frac{dY}{dt} \propto y(Y)g(G)$. A star

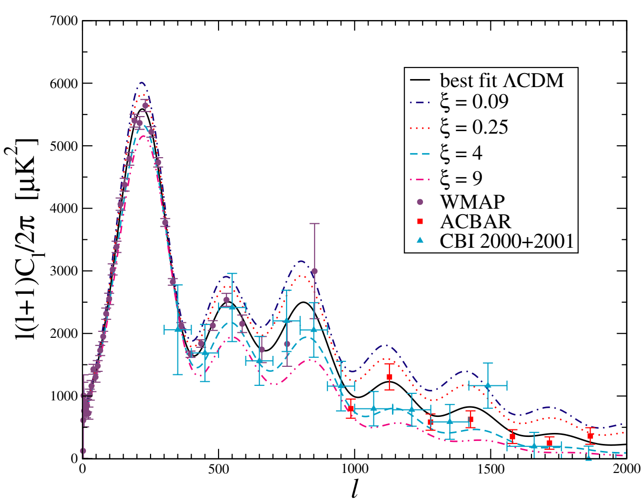

FIG. 3 (color online). CMB power spectrum with and without the variation of ξ. Higher peaks are more severely damped as ξ increases, while the height of the first peak is almost unchanged.

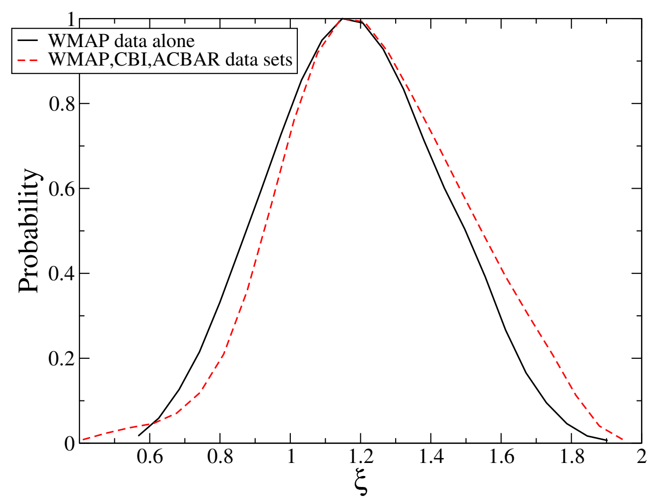

FIG. 4 (color online). Marginalized probability distribution of ξ. The solid and dashed lines correspond to the probability distributions obtained by WMAP data alone and by WMAP, CBI, and ACBAR data sets, respectively.

which departs from the main sequence today (t_0) should be considered to have $Y \approx 1$ at its center so that $\int_{Y_{\rm init}}^{1} \frac{dY}{y(Y)} \propto g(G) \int_{t_{\rm init}}^{t_0} dt$. We further assume that $g(G) \propto G^{\gamma}$, where $\gamma \approx 5.6$ have been obtained from numerical simulation [25]. From the fact that the left-hand side of the above equation does not depend on G and time, we have the relation

$$\tau_* = \xi^{\gamma} \int_{t_0-\tau}^{t_0} dt = \tau \xi^{\gamma}. \tag{2}$$

Here τ_* is the apparent turn-off age, which should be obtained by analyzing the Hertzsprung-Russell diagram of a globular cluster with the standard value of G, and τ is the true age of the globular cluster. Thus, if information on the true age of globular cluster is available, the globular cluster can be used to constrain ξ:

$$\left(\frac{\tau_{\rm max}}{\tau_*}\right)^{-(1/\gamma)} \lesssim \xi \lesssim \left(\frac{\tau_{\rm min}}{\tau_*}\right)^{-(1/\gamma)}. \tag{3}$$

If we take $\tau_{\rm max} = 15.8$ (2σ upper bound on the expansion age of the universe obtained by our CMB analysis discussed above including the variation of ξ), and conservatively assume that $\tau_{\rm min} = 10$ Gyr [25], we then obtain

$$0.93 \lesssim \xi \lesssim 1.09, \tag{4}$$

where we use $\gamma = 5.6$ and $\tau_* = 12.9 \pm 2.9$ Gyr, which is the age of the galactic globular clusters [26].

All the higher-dimensional theories of gravity proposed recently allow Newton's constant to be scale dependent. In this paper, assuming the dependence is weak for horizon scales in BBN epoch (10^8–10^{12} m) and also for those in the CMB epoch (10^2–10^9 pc), we place constraints on $\xi = G/G_{\rm N}$ at the cosmological scales. An important point is that the present analysis yields constraints on the value of G itself, while other astronomical tests of the inverse square law do so only on the value of GM including unknown mass M of interacting bodies.

Increasing Newton's constant enhances the universal expansion rate, and then leads to larger helium and deuterium abundances produced at BBN epoch. We have reexamined this effect including the latest experimental data on the neutron lifetime [27,28]. We found that the experimental value $G_{\rm N}$ ($\xi = 1$) is now quite consistent with the observed abundances of primordial light elements, and the variation of Newton's constant is tightly constrained to $0.95 \lesssim \xi \lesssim 1.01$.

The variation of Newton's constant also affects the power spectrum of CMB anisotropies through the change of the recombination and photon diffusion processes. We found that the difference emerges at smaller scales. Thus, observations at higher multipoles are essential to put a tighter constraint on ξ. However, even when higher multipole data currently available from CBI and ACBAR are included, we found no improvement in constraint on ξ, because of scatters in data at higher multipoles. WMAP data alone place a constraint: $0.74 \lesssim \xi \lesssim 1.66$. If we combine CBI and ACBAR data sets, the constraint becomes $0.75 \lesssim \xi \lesssim 1.74$.

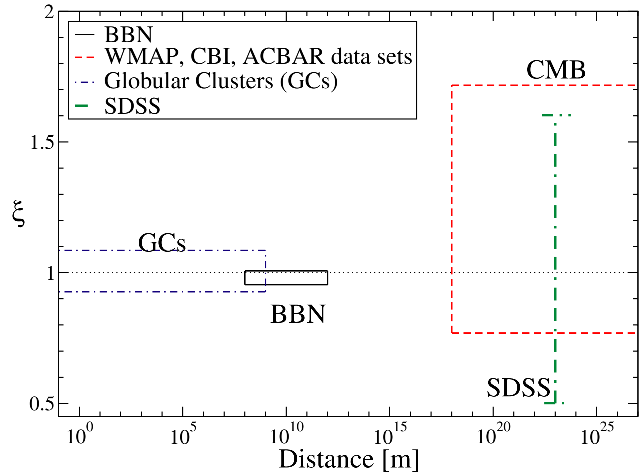

FIG. 5 (color online). Limits on the variation of ξ from various observations. Limit from Sloan Digital Sky Survey is taken from [6].

In Fig. 5, we summarize results of the current work. The value of ξ is fixed to one at laboratory scale $\sim$1 m by direct experiments. We have two possibilities of transition from the short distance regime where $G = G_N$ to the long distance one where $G = \xi G_N$; one is the geophysical scale (i.e., $\sim$1–100 km, where the constraints on the inverse square law are relatively weak), the other is scale beyond the solar system ($\gtrsim 10^{13}$ m), where we have only poor knowledge on G. If we consider the former case, BBN gives the tightest constraint on ξ. The globular cluster also gives a consistent but weaker constraint. On the other hand, if we consider the latter case, CMB anisotropies and galaxy clustering [6,7] are the only observations to put constraints on ξ. Thus, higher precision measurements of CMB anisotropies, particularly in its higher multipoles, are highly expected to determine the value of G at large scales beyond the solar system and then to confirm the necessity of the higher-dimensional theories of gravity.

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