

Cosmological evolution of cosmic strings with time-dependent tension

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We discuss the cosmological evolution of cosmic strings with time-dependent tension. We show that, in the case that the tension changes as a power of time, the cosmic string network obeys the scaling solution: the characteristic scale of the string network grows with the time. But due to the time dependence of the tension, the ratio of the energy density of infinite strings to that of the background universe is *not* necessarily constant.

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I. INTRODUCTION

Cosmic strings are produced in field theories, through various thermal [1] or nonthermal [2] phase transitions, as a result of spontaneous symmetry breaking. Recently, cosmic superstrings have also received much attention, partly because they are produced after brane inflation [3].

Cosmic strings have fruitful implications for cosmology. They can generate density fluctuations, gravitational waves, gravitational lensing effects, and so on (for review, see [4,5]). Particularly, they were investigated as possible seeds of galaxies and large scale structure. Unfortunately, recent observations of the cosmic microwave background radiation (CMBR) anisotropy by the Wilkinson Microwave Anisotropy Probe (WMAP) revealed that cosmic strings cannot become a dominant source of primordial density fluctuations [6]. Only a 10% contribution by cosmic strings to the CMBR angular power spectrum is allowed on large scales [7–9]. However, they can still play an important role in large scale structure formation because the density fluctuations generated by cosmic strings are peculiar, that is, incoherent, nonadiabatic, and highly non-Gaussian. In fact, cosmic strings may cause the early star formation and early reionization observed by WMAP [7,10]. Furthermore, the contribution from cosmic strings to the galaxy power spectrum may loosen the bounds on the sum of the neutrino masses, and this possibility is discussed [11].

The cosmological evolution of cosmic strings has been studied for a long time. The key property is scaling, when the typical length of the cosmic string network grows with the horizon scale. Then, the number of infinite strings per horizon volume is a constant irrespective of time and hence the ratio of the energy density of infinite strings to that of the background universe is a constant. Such a scaling solution is confirmed both analytically [1] and numerically [12–14] by use of the Nambu-Goto action [15], which is an effective action obtained after integrating out heavy modes of particles and neglecting high curvature of the geometry.¹

¹This scaling property is confirmed not only for cosmic (local) strings but also for global strings [16] and global monopoles [17].

Based on this property, many cosmological implications have been discussed, as stated above.

In the past the constancy of the tension of cosmic strings has been assumed in order to derive the scaling solution. However, in cosmological situations the tension of the strings can depend on the cosmic time. Let us consider a complex scalar field ϕ , whose effective potential depends on its interactions and can be described by

$$V(\phi) = \frac{\lambda}{4} (|\phi|^2 - \eta^2)^2, \quad (1)$$

where η is the effective expectation value of $|\phi|$. Then, the typical radius of a string δ_s is given by $\delta_s \simeq 1/(\sqrt{\lambda}\eta)$ and the tension μ is given by $\mu \simeq \eta^2$. Note that the tension μ does not depend on the coupling constant but on the expectation value alone. For example, assume that ϕ has a wine-bottle potential at zero temperature. The finite temperature effective potential has the above form with $\eta = \eta(T) = \sqrt{(T_c^2 - T^2)/6}$.² Here T is the cosmic temperature and T_c is the critical temperature given by the coupling constants and the zero temperature breaking scale. Thus, η depends on the temperature T , and hence on the cosmic time t , $\eta = \eta(t)$.³ This implies that the tension μ also depends on the cosmic time, $\mu = \mu(t)$. As a more interesting example, we consider a real scalar field χ oscillating around the origin with a mass and a quartic self coupling, which may be the cold dark matter or the self-interacting cold dark matter [19]. We also assume that χ field has also a coupling to the ϕ field of the form of $|\phi|^2\chi^2$. That is, the potential is given by

$$V(\phi, \chi) = \frac{\lambda}{4} (|\phi|^2 - \chi^2)^2 + \frac{1}{2} m_\chi^2 \chi^2 + \frac{g}{4} \chi^4 \quad (2)$$

If a coupling constant λ is small enough, the backreaction

²In this case, the change of the tension is too mild. But, for example, the change of the tension for an embedded domain wall after the core phase transition is significant due to thermal effects.[18].

³Strictly speaking, the presence of cosmic strings has effects on the finite temperature effective potential so that η can differ at points inside and outside strings. Here, we assume that such backreaction effects are sufficiently small.

to the oscillation of χ is negligible and η in Eq. (1) is given by the root mean square of the expectation value of χ . Thus, for example, in case that the oscillation of χ is dominated by the mass term, $\eta^2 \propto a^{-3}$ (a : the scale factor), which is proportional to $t^{-3/2}$ in the radiation domination and t^{-2} in the matter domination. Hence, the tension μ depends on the power of the cosmic time. Furthermore, in the warped geometry, the expectation value η , that is, the tension $\mu \simeq \eta^2$ depends on the position of the brane in the bulk, which can depend on the cosmic time before the radion is fixed completely. Thus, the case often takes place in which the tension η of cosmic strings associated with the effective potential Eq. (1) changes with the cosmic time.

Then, we wonder whether the scaling solution also applies to cosmic strings with time-dependent tension. In this paper we discuss this topic in detail. In the next section, we derive the effective action for such cosmic strings, which corresponds to the Nambu-Goto action for strings with constant tension. In section III, we discuss the cosmological evolution of such strings by considering the equation of motion in an expanding universe. In the final section, we give our conclusions and discussion.

II. EFFECTIVE ACTION FOR COSMIC STRINGS WITH TIME-DEPENDENT TENSION

Cosmic strings appear in the Abelian-Higgs model with a broken $U(1)$ symmetry, whose Lagrangian density is given by

$$\mathcal{L} = |D_\mu \phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4} (|\phi|^2 - \eta^2)^2, \quad (3)$$

where ϕ is a complex scalar field, A_μ are gauge fields, $F_{\mu\nu}$ are field strengths, and $D^\mu \phi \equiv (\partial^\mu - ieA^\mu)\phi$. The expectation value η is now assumed to be constant, and e and λ are coupling constants. Then, there exists a static cylindrically-symmetric solution (ϕ_s, A_s^μ) of the form [20,21],

$$\phi_s(r, \theta) = \eta e^{in\theta} f(r), \quad A_s^\theta(r, \theta) = \frac{n\alpha(r)}{er}, \quad (4)$$

where n is the winding number, and $f(r)$ and $\alpha(r)$ satisfy the following equations,

$$\begin{aligned} \frac{d^2 f}{dr^2} + \frac{1}{r} \frac{df}{dr} - \frac{n^2 f}{r^2} (\alpha - 1)^2 - \frac{\lambda}{2} \eta^2 f (f^2 - 1) &= 0, \\ \frac{d^2 \alpha}{dr^2} - \frac{1}{r} \frac{d\alpha}{dr} - 2e^2 \eta^2 f^2 (\alpha - 1) &= 0 \end{aligned} \quad (5)$$

with the boundary conditions $f(0) = \alpha(0) = 0$ and $f(\infty) = \alpha(\infty) = 1$. Asymptotically ($r \rightarrow \infty$), $f(r)$ and $\alpha(r)$ can be approximated as

$$\begin{aligned} f(r) &\sim 1 - \mathcal{O}(\exp(-r/\delta_s)), \\ \alpha(r) &\sim 1 - \mathcal{O}(\sqrt{r\eta} \exp(-r/\delta_v)) \end{aligned} \quad (6)$$

for $\lambda/(2e^2) \lesssim 4$. Here $\delta_s \equiv (\sqrt{\lambda}\eta)^{-1}$ and $\delta_v \equiv (\sqrt{2}e\eta)^{-1}$ are typical radii of cosmic string cores.

Now we consider the Abelian-Higgs model with the potential Eq. (1) in an expanding universe, whose action is given by

$$S = \int d^4 y \sqrt{-g} \left[|D_\mu \phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi) \right]. \quad (7)$$

Here, note that the expectation value η is now assumed to be time dependent, $\eta = \eta(t)$. The metric is taken to be that of the spatially flat expanding universe,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) dx^2 = a^2(\tau) (d\tau^2 - dx^2) \quad (8)$$

with $d\tau = dt/a(t)$ and $a(t)$ the cosmic scale factor. The worldsheet swept by a cosmic string can be characterized by two parameters ζ^a ($a = 0, 1$) with

$$x^\mu = x^\mu(\zeta^a). \quad (9)$$

We take the timelike coordinate ζ^0 to be the conformal time τ and the spacelike coordinate ζ^1 to be l , which parametrizes the string at a fixed time. Then, the metric of the spacetime can be written as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \gamma_{ab} d\zeta^a d\zeta^b, \quad (10)$$

where $\gamma_{ab} \equiv g_{\mu\nu} x_{,a}^\mu x_{,b}^\nu$ and γ^{ab} is defined such that $\gamma_{ab} \gamma^{bc} = \delta_a^c$. The comma represents the partial derivative. Since we are interested in only the transverse motion of cosmic strings, without generality, we can require the metric to satisfy

$$\dot{x} \cdot x' = 0 \iff \gamma_{01} = \gamma_{10} = 0, \quad (11)$$

where dots and primes represent derivatives with respect to conformal time τ and the spacelike parameter l , respectively.

Now we derive an effective action for the above strings [4,22]. By introducing two spacelike vectors n_μ^A , ($A = 1, 2$) with $g^{\mu\nu} n_\mu^A n_\nu^B = -\delta^{AB}$ and $n_\mu^A x_{,a}^\mu = 0$, any point y^μ around the worldsheet can be reparametrized as

$$y^\mu(\xi) = x^\mu(\zeta) + \rho^a n_a^\mu(\zeta), \quad (12)$$

where ρ^A are two radial coordinates and $\xi^\mu = (\zeta^a, \rho^A)$. The new coordinates ξ^μ are determined uniquely if the point y^μ is inside the string's curvature radius. Then, the string configuration can be approximated as

$$\phi(y(\xi)) = \phi_s(r), \quad A^\mu(y(\xi)) = n_B^\mu A_{sB}(r), \quad (13)$$

where $r^2 \equiv (\rho^1)^2 + (\rho^2)^2$ and (ϕ_s, A_s^μ) are a cylindrically-symmetric solution with $\eta = \eta(\tau)$.⁴ Here, we assume that

⁴For a moving string, we use its Lorentz-boosted version. Strictly speaking, time-dependent tension breaks Lorentz invariance, which causes ambiguity for tension. But, the change of the tension is not so rapid in our paper that the error associated with the ambiguity is small and that we can safely use the tension with cosmic conformal time τ .

the tension of a cosmic string does not change so rapidly that its configuration can readjust with the change of its tension, that is, $\delta_s, \delta_v \ll \eta/(d\eta/dt)$. The Jacobian associated with the coordinate transformation from y^μ to ξ^μ becomes $\sqrt{-\gamma}$ with $\gamma \equiv \det(\gamma_{ab})$ up to curvature terms $\mathcal{O}(\delta_s/R), \mathcal{O}(\delta_v/R)$ with R the typical curvature of the string. On the other hand, integration of Eq. (7) over the coordinates ρ^A yields the time-dependent tension $\mu(t) \sim \eta(t)^2$ up to curvature terms.⁵ Thus, neglecting higher-order curvature terms, we obtain the following effective action for a cosmic string with time-dependent tension,

$$S_{\text{eff}} = - \int d^2 \zeta \sqrt{-\gamma} \mu(\tau). \quad (14)$$

This is quite similar to the Nambu-Goto action with an additional factor for the time-dependent tension. Note that S_{eff} is invariant *neither* under the general coordinate transformation of x^μ *nor* under the reparametrization of ζ^a . However, in the cosmological situations, such an effective action can naturally appear as a result of a cosmic string with time-dependent tension.

III. COSMOLOGICAL EVOLUTION OF COSMIC STRINGS WITH TIME-DEPENDENT TENSION

In this section, we will discuss the cosmological evolution of cosmic strings with time-dependent tension. The Euler-Lagrange equation for the effective action is given by

$$\mu x_{,a}^{\mu;a} + \mu^a x_{,a}^{\mu} + \mu \Gamma_{\nu\sigma}^{\mu} \gamma^{ab} x_{,a}^{\sigma} x_{,b}^{\nu} = 0, \quad (15)$$

where the semicolon represents the covariant derivative. $\Gamma_{\nu\sigma}^{\mu}$ is the four-dimensional Christoffel symbol and is given by

$$\Gamma_{\nu\sigma}^{\mu} = \frac{1}{2} g^{\mu\tau} (g_{\nu\tau,\sigma} + g_{\tau\sigma,\nu} - g_{\nu\sigma,\tau}). \quad (16)$$

This equation can be rewritten as

$$\frac{\mu}{\sqrt{-\gamma}} \partial_a (\sqrt{-\gamma} \gamma^{ab} x_b^{\mu}) + \mu^a x_{,a}^{\mu} + \mu \Gamma_{\nu\sigma}^{\mu} \gamma^{ab} x_{,a}^{\sigma} x_{,b}^{\nu} = 0. \quad (17)$$

The $\mu = 0$ component of the equation of motion is given by

$$\dot{\epsilon} + \frac{\dot{\mu}}{\mu} \epsilon + 2 \frac{\dot{a}}{a} \epsilon x^2 = 0 \quad (18)$$

where $\epsilon \equiv \sqrt{x/(1-x^2)}/2$. The $\mu = i$ component is given

⁵In the case that typical curvature R of the string is the horizon scale, which is expected in the scaling regime, curvature terms are higher-order effects than that of the variation of the tension. This is mainly because its typical time scale of the variation is the Hubble scale when $\mu \propto t^q$ as adopted later.

by

$$\ddot{x} + 2 \frac{\dot{a}}{a} (1 - \dot{x}^2) \dot{x} - \frac{1}{\epsilon} (\epsilon^{-1} \dot{x}') = 0. \quad (19)$$

The energy E of a cosmic string in an expanding universe can be defined as

$$E = a(\tau) \mu(\tau) \int dl \epsilon. \quad (20)$$

Defining the energy density as $\rho \equiv E/V$ with V some relevant volume, the rate of change of ρ is given by

$$\dot{\rho} = -2 \frac{\dot{a}}{a} (1 + \langle v^2 \rangle) \rho, \quad (21)$$

where $\langle v^2 \rangle$ is the average velocity squared of a cosmic string defined as

$$\langle v^2 \rangle \equiv \frac{\int dl \epsilon \dot{x}^2}{\int dl \epsilon}. \quad (22)$$

Note that multiplying the rate equation of ρ by $d\tau/dt$ gives an equation with the same form but a derivative taken with respect to the cosmic time t . Since it is more convenient to deal with t , hereafter the dot represents the derivative with respect to the cosmic time.

In order to investigate the evolution for such a string, we define a characteristic scale L of the string network as

$$\rho_{\infty} = \frac{\mu(t)}{L^2}, \quad (23)$$

where ρ_{∞} is the energy density of a string whose length is larger than the horizon scale (called infinite strings). Furthermore, in order to incorporate intercommutation effects,⁶ we assume that the rate of energy transfer from infinite strings to loops is given by

$$\dot{\rho}_{\infty \rightarrow \text{loops}} = c \frac{\rho_{\infty}}{L}, \quad (24)$$

where c is a constant which parametrizes the efficiency of energy transfer. Then, the rate equation for the energy density of infinite strings becomes

$$\dot{\rho}_{\infty} = -2 \frac{\dot{a}}{a} (1 + \langle v^2 \rangle) \rho_{\infty} - c \frac{\rho_{\infty}}{L}. \quad (25)$$

Inserting Eq. (23) into this equation yields

$$-2 \frac{\dot{L}}{L} + \frac{\dot{\mu}}{\mu} = -2 \frac{\dot{a}}{a} (1 + \langle v^2 \rangle) - \frac{c}{L}. \quad (26)$$

Here we have assumed the constancy of $\langle v^2 \rangle$. In the scaling regime, it seems reasonable just as the case with the constant tension. But, strictly speaking, it must be checked

⁶Intercommutation is a local process near the string core. Since $\delta_s, \delta_v \ll \eta/(d\eta/dt)$, the time dependence of the tension has little effect on the intercommutation.

by numerical calculations, which will be done in near future.

In order to investigate the time development of L , we define γ as $L = \gamma t$ and assume that $\mu \propto t^q$. Then, in a radiation-dominated universe, the above equation can be recast into

$$\frac{\dot{\gamma}}{\gamma} = -\frac{1}{2t} \left[1 - q - \langle v^2 \rangle - \frac{c}{\gamma} \right]. \quad (27)$$

Hence, the stable fixed point γ_r is given by

$$\gamma_r = \frac{c_r}{1 - q - \langle v_r^2 \rangle}. \quad (28)$$

In the same way, the stable fixed point in a matter-dominated universe γ_m is given by

$$\gamma_m = \frac{3c_m}{2 - 3q - 4\langle v_m^2 \rangle}. \quad (29)$$

Therefore, the characteristic scale L of a string network also scales with time for strings with time-dependent tension. That is, the number of infinite strings per horizon volume is a constant irrespective of time. But it should be noted that, due to the time dependence of the tension, the ratio of the energy density of infinite strings to that of the background universe is *not* a constant.

IV. CONCLUSIONS AND DISCUSSION

In this paper, we have discussed the cosmological evolution of cosmic strings with time-dependent tension. In all

works about cosmic strings which have been done thus far the tension of the strings has been constant. However, strings with time-dependent tension often appear in cosmological situations. In this paper, first we have derived the effective action for such strings which, with an additional factor for the time-dependent tension, is the Nambu-Goto action. Next, we gave the equation of motion in an expanding universe and investigated the evolution of such strings. We confirmed that, in the case where the tension changes as the power q of time, the string network obeys the scaling solution—implying that the characteristic scale of the string network grows with cosmic time. However, the ratio of the energy density of infinite strings to that of the background universe is *not* necessarily constant due to the time dependence of the tension. Interestingly, the scaling may not be realized if q is larger than the critical value $q_r = 1 - \langle v_r^2 \rangle$ in a radiation-dominated universe, or $q_m = 2(1 - 2\langle v_m^2 \rangle)/3$ in a matter-dominated one. But if this is the case, the change in the tension is rapid enough that the effective action S_{eff} may not be a good approximation. We will investigate this problem in the future. Furthermore, numerical simulations are necessary to confirm our results and constrain several parameters.

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