

Cosmic string lensing and closed timelike curves

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In an analysis of the gravitational lensing by two relativistic cosmic strings, we argue that the formation of closed timelike curves proposed by Gott is unstable in the presence of particles (e.g. the cosmic microwave background radiation). Because of the attractorlike behavior of the closed timelike curve, we argue that this instability is very generic. A single graviton or photon in the vicinity, no matter how soft, is sufficient to bend the strings and prevent the formation of closed timelike curves. We also show that the gravitational lensing due to a moving cosmic string is enhanced by its motion, not suppressed.

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I. INTRODUCTION

Although cosmic strings as the seed of structure formation [1] has been ruled out by observations, their presence at a lower level is still possible. Indeed, cosmic strings are generically present in brane inflation in superstring theory, and their properties are close to, but within all observational bounds [2–7]. This is exciting for many reasons since current and near future cosmological experiments/observations will be able to confirm or rule out this explicit stringy prediction. If detected, the rich properties of cosmic strings as well as their inflationary signatures provide a window to both the superstring theory and our preinflationary universe. The cosmic strings are expected to evolve to a scaling string network with a spectrum of tensions. Roughly speaking, the physics and the cosmological implications are entirely dictated by the ground state cosmic string tension μ , or the dimensionless number $G\mu$, where G is the Newton's constant [1]. The present observational bound is around $G\mu \lesssim 6 \times 10^{-7}$. Recently, it was shown [7] that the cosmic string tension can easily saturate this bound in the simplest scenario in string theory, namely, the realistic $D3 - \bar{D}3$ -brane inflationary scenario [4]. This means gravitational lensing by such cosmic strings, providing image separation of order of an arc second or less, is an excellent way to search for cosmic strings. Generic features of cosmic strings include a conical “deficit angle” geometry, so a straight string provides the very distinct signature of an undistorted double image. Cosmic string lensing has been extensively studied [8–11].

In the string network, some segments of cosmic strings will move at relativistic speeds. It is therefore reasonable to consider the gravitational lensing by highly relativistic cosmic strings. We also confront another interesting feature of cosmic strings, first realized by Gott [12]: the possible appearance of closed timelike curves (CTCs) from two parallel cosmic strings moving relativistically past each other. As the strings approach each other fast enough in Minkowski spacetime, the path encircling the

strings in the sense opposite to their motion becomes a CTC. This is sometimes called the Gott spacetime or Gott time machine.

Although there is no proof that a time machine cannot exist in our world [13], their puzzling causal nature leads many physicists to believe that CTCs cannot be formed. This skepticism has been encoded in Hawking's chronology protection conjecture (CPC) [14]. However, CPC as proposed is not very precise; even if we assume CPC is correct, it is not clear exactly what law of physics will prevent the specific Gott spacetime. There are a number of interesting and insightful studies attempting to apply CPC against the Gott's spacetime:

- (a) Recall that the original Chronology Protection Conjecture [14] is motivated by two results. The more general of the two is the semiclassical divergence of the renormalized stress-energy tensor near the “chronology horizon,” or Cauchy surface separating the regions of spacetime containing CTCs from those without. These (vacuum) “polarized” hypersurfaces led Hawking to conjecture that *any* classical spacetime containing a chronology horizon will be excluded from the quantum theory of gravity. However, a recent paper [15] found an example where this chronology horizon is well defined in the background of a string theory (to all orders in α'). So, in superstring theory, the CPC is not generically true: the Taub-NUT geometry receives corrections that preserve the traversibility of the chronology horizon. Unlike classical Taub-NUT, stringy Taub-Nut contains timelike singularities, although they are far from the regions containing CTCs. The second hint that CPC is true is the theorem proved by Hawking and Tipler [14,16] that spacetimes obeying the weak energy condition with regular initial data and whose chronology horizon is compactly generated cannot exist. This theorem depends crucially on the absence of a singularity, and so Hawking's claim that finite lengths of cosmic string cannot produce CTCs is only true if one rejects the possibility of a singularity being present somewhere on the chronology horizon [17].

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- (b) When we consider possible formation of CTCs coming from cosmic string loops (though this is not necessary for our general argument), one may make use of the null energy condition and smoothness to argue against CTCs. The null energy condition can be satisfied even when one smooths out the conical singularity (with a field theory model) at the core of the cosmic strings. However, in superstring theory, such a procedure is not permitted. The cosmic strings are either D1-strings or fundamental strings. Classically, the core is a δ -function with no internal structure (e.g., energy distribution) in the string cross-sections, so the string has only transverse excitation modes. Suppose we smooth out the δ -function. Then one can rotate the string around its axis and endow it with longitudinal modes. The presence of such longitudinal modes violate the unitarity property of the superstring theory. (In fact, in the presence of such longitudinal modes, general relativity is no longer assured in superstring theory.) In this sense, the string geometry is not differentiable, and one must generalize the appropriate theorems before they may be applied.
- (c) Cutler [18] has shown that the Gott spacetime contains regions free of CTCs, that the chronology horizon is classically well defined, and that Gott spacetime contains no closed (as opposed to just self-intersecting) null geodesics. Hawking points out that this last feature must be discarded for bounded versions of Gott (and similar) spacetimes. This means that Gott spacetime undergoes qualitative changes when embedded in asymptotically flat spacetime. Cutler found a global picture of the Gott spacetime very much in agreement with general arguments made by Hawking regarding the instability of Cauchy horizons, specifically the blueshifting of particles in CTCs. Here, we find a concrete example of this phenomenon using a lensing perspective.
- (d) As parallel strings move relativistically past each other to create CTCs, a black hole may be formed by the strings before the CTC appears, thus preventing CTCs. If this happens, one can consider the formation of a black hole as a realization of CPC. However, it is easy to see that, using Thorne's hoop conjecture, there is a range of string speed where the CTCs appear, but no black hole is formed. The results of Tipler and Hawking suggest that the strings are slowed to prevent CTCs, or a singularity forms somewhere else in the geometry. This singularity does not need to disrupt or isolate the region containing CTCs.
- (e) Tipler [16] proved that whenever a CTC is produced in a finite region of spacetime, a singularity must necessarily accompany the CTC. This singularity does not represent a no-go theorem, since the CTC and its sources need not encounter the singularity. In fact Tipler's physical argument against the creation of CTCs is the unfeasibility of creating singularities. However, it is well-known that singularities such as orbifold fixed points and conifolds are perfectly fine in superstring theory, where Einstein gravity is recovered as a low energy effective theory. Furthermore, under the appropriate circumstances, topology changes are perfectly sensible. This consistency is due to the extended nature of string modes.
- (f) One may consider the Gott spacetime in $2 + 1$ dimensions. The $2 + 1$ dimensional gravity relevant for the problem has been studied by Deser, Jackiw and 't Hooft [19]. For a closed universe, 't Hooft [20] argues that the universe will shrink to zero volume before any CTCs can be formed. For an open universe, Carroll, Farhi, Guth, and Olum (CFG0) [17] show that it will take infinite energy to reach Gott's two-particle system from strings at rest. Holonomy calculations show that this system has spacelike total momentum, much like a tachyon. For this reason one might consider Gott spacetime to be unphysical, although there are differences between the global properties (including holonomy) of tachyons and Gott's cosmic strings. In $3+1$ dimensions, the physicality of Gott spacetime is less clear.
- If none of the above arguments against CTCs are fully applicable to the Gott $3 + 1$ spacetime, does this mean CPC fails and Gott spacetime can be realized in the real universe? Or are there other mechanisms which prevent the formation of CTCs?
- In this paper, we use a lensing framework to demonstrate the classical instability near the Cauchy horizon which we argue will prevent the formation of cosmic string CTCs in any realistic situation. To be specific, our argument goes as follows:
- A particle or a photon gets a positive kick in its momentum in the plane orthogonal to the strings each time it goes around a CTC [17,21,22].
 - Once inside the chronology horizon, such a particle is generically attracted to a CTC; that is, a worldline in the vicinity of the CTC will coalesce with the CTC. This is our main observation.
 - The particle will go through the CTC numerous times (actually an arbitrarily large number of times) instantaneously; that is, the particle will be instantaneously blueshifted by an amount unbounded from above.
 - It follows that the backreaction must be important; conservation of angular momentum and energy implies that the cosmic strings will slow down, or, more likely, bend; this in turn prevents the formation of CTCs. Note that this backreaction must disrupt the closed timelike curve, otherwise the infinite blueshift can not be prevented. So, a single particle,

say a graviton or photon, no matter how soft, will bend the cosmic strings so that CTC cannot be formed. The following picture seems reasonable: as the two segments of cosmic strings move toward each other, they are bent and so radiate gravitationally. This slows them down to below the critical value for CTC formation. We expect no singularity/divergence to appear.

- (e) Since there is a cosmic microwave background radiation in our universe, these photons preclude the existence of CTCs. Of course, the cosmic microwave background radiation is not the only wrench in the machine. Gravitons or some other particles can be emitted by the moving strings, either classically or via quantum fluctuation. In particular, gravitons must be present in spacetimes of dimensions $3 + 1$ or greater. A single graviton, no matter how soft, will lead to the above effect. We argue this is how the chronology protection conjecture works in the Gott spacetime.

This result is not too surprising in light of the likely (blueshift) instability of Cauchy horizons discussed by Hawking and others, although a counter example was found by Li and Gott [23] while analyzing possible vacua of Misner space, whose Cauchy horizon can be free of instability. As in our example, a divergence occurs only in the presence of particles, although we find that blueshifting (and not particle number) is the cause.

The blueshift instability is well studied in the literature [24]. The strong cosmic censorship conjecture predicts that a Cauchy horizon is, in general unstable (e.g. that of a Reisner-Nordström black hole in asymptotically flat spacetime), and that this instability is the result of the infinite blueshift of in falling perturbations. This must be similar to the instability we describe, but our picture is resolved differently. We find that the CTC never forms because surrounding particles scatter off the cosmic strings, bending and slowing them. Hence no Cauchy horizon (stable or not) ever forms.

't Hooft [25] argues that, since the local equations of motion for a cosmic string are well-defined, one should be able to list the Cauchy data at any particular time, and demand the Laws of Nature to be applied in a strictly causal order. If one phrases the logic this way, there are no CTCs by construction, in agreement with the chronology protection conjecture and strong cosmic censorship. So the question is: what is wrong with Gott spacetime? His answer to this question is that the Cauchy planes become unstable: in terms of these, the Universe shrinks to a line in $3 + 1$ dimensions. The moment a disturbance from any tiny particle is added somewhere in the past, it generates so much curvature that the inhabitants of this universe are killed by it. In our scenario, we give a specific mechanism with more details: a single graviton or photon, even a very soft one, will suffice. An infinitely blueshifted photon (or any particle) will cause so much curvature that 't Hooft's

collapsing scenario occurs. Here, we agree with the chronology protection principle and 't Hooft that a CTC is not formed. However, we believe that, due to the energy-momentum-angular momentum conservations, the back-reaction will bend the cosmic strings and induce gravitational radiation so that the CTC is never formed. Neither the curvature nor the energy of the photon blows up. Note that the bending of the strings can not happen in $2 + 1$ dimensions.

In this paper, the motion of a photon/graviton around the cosmic strings is a crucial ingredient of the analysis. We shall start with a review of the gravitational lensing by a straight moving cosmic string. Here we correct a mistake lensing formula mistake in the literature (see Appendix A). Next we review the Gott spacetime. Finally we show that the CTCs encircling the two strings are attractors for particles. Our analysis ends here. Supplemented with plausible reasonings, we argue that the above mechanism is a way to prevent CTCs in the real universe. Throughout, we shall assume $\delta_0 = 8\pi G\mu$ to be very small (say, less than 10^{-5}).

II. COSMIC STRING LENSING

One may calculate the observational signatures of rapidly moving cosmic strings (straight and loops), in particular, their lensing effects on distant galaxies and the CMB. The simplest case of a straight, nearly static cosmic string has the distinctive signature of producing two identical images, each being undistorted and equidistant from the observer.

In Fig. 1 below is pictured a cosmic string moving to the right across a distant galaxy. We call the angular separation of the images $\delta\varphi$, and the photon deflection angle δ , as in Fig. 2 below. Although the double images on the left picture of Fig. 1 may be due to two almost identical galaxies (a rare but not impossible scenario), the picture on the right will be a much cleaner signature of cosmic string lensing. If one sees a double image candidate [26], one expects to see other candidates nearby. Searching for incomplete images will be important.

This leads to the well-known result $\delta\varphi = \frac{D_{s,cs}}{D_{s,o}} \delta$. The spacetime around a static cosmic string is Minkowski space with the identification of two semi-infinite hyperplanes whose intersection is the cosmic string world sheet. This is equivalent to identifying every event s in space time with a dual s' where the relation between s and s' with a static cosmic string located at r_{cs} is

$$s' = R_{\delta_0}(s - r_{cs}) + r_{cs}. \tag{1}$$

Where R_{δ_0} is a pure rotation (counter clockwise). (Notice r_{cs} can be any point on the cosmic string world sheet.) It should be noted that s is visible only when it appears to the right of the cosmic string, and s' is visible only when to the left.

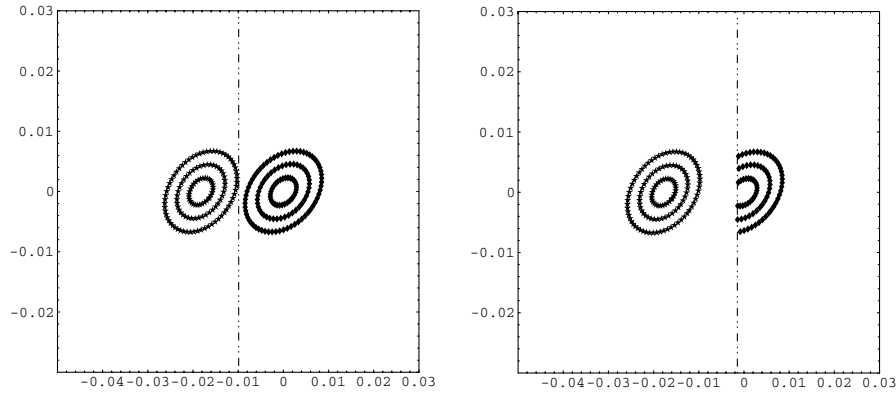


FIG. 1. Gravitational lensing by a straight cosmic string. The two images are identical. Note that they are not mirror reflections of each other, but one is a translational displacement of the other. As the source moves to the left (or equivalently, the cosmic string moves to the right), part of the right image is cut off, eventually leaving only a single image.

The general case involves a cosmic string moving at some four-velocity v_{cs} . We will always take v_{cs} to be perpendicular to the cosmic string world sheet, since any parallel component is unphysical (assuming a pure tension string). Interestingly, this velocity is only well defined in combination with a “branch cut.” This is related to the fact that a passing cosmic string will induce a relative velocity between originally static points in space, so a constant velocity field will not be everywhere single valued. More physically, parallel geodesics moving past a cosmic string will be bent toward each other, provided they pass the string on opposite sides. Specifying a branch cut enables the conical geometry to be mapped to Minkowski space (minus a wedge), where things are simpler. The pure rotation identification is valid only in the center-of-mass frame of the cosmic string, so in general

$$s' = \Lambda_{v_{cs}} R_{\delta_0} \Lambda_{-v_{cs}} (s - r_{cs}) + r_{cs}. \quad (2)$$

Where $\Lambda_{v_{cs}}$ is a pure boost such that $\Lambda_{-v_{cs}} v_{cs} = (1, 0, 0, 0)$. We have simply boosted into the strings reference frame and then performed the rotation-identification. Then we boost back.

Here we investigate the lensing due to nearly straight segments moving at arbitrarily relativistic speeds. We consider the interaction of a photon with a cosmic string’s deficit angle. In Fig. 3, a photon is crossing the deficit angle

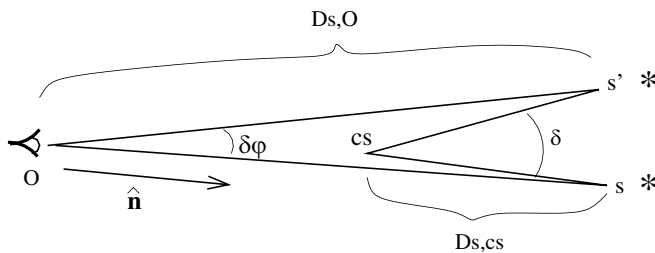


FIG. 2. A straight cosmic string introduces a deficit angle δ . Here, the sources s and s' are to be identified.

at a distance r from the string vertex. The photon is redirected by an angle δ and makes a spacial and temporal jump. This jump ($s' - s$) is found using the coordinate identification in Eq. (2), where $s = A$ is the photon striking the deficit angle and $s' = B$ is its emergence from the deficit angle. If we choose the deficit angle to be perpendicular to v_{cs} , the spacial jump is parallel to v_{cs} , and is given by

$$\Delta x = 2r \tan(\delta_0/2) \gamma_{cs} \quad \Delta t = -2r \tan(\delta_0/2) \gamma_{cs} v_{cs}. \quad (3)$$

where $\gamma_{cs} = 1/\sqrt{1 - v_{cs}^2}$. In Fig. 3, events A and B are identified, while events A and C are simultaneous. If the photon strikes the leading edge of the deficit angle, the jump is behind the cosmic string and backward in time. If the photon strikes the trailing edge, the jump is in front of the cosmic string and forward in time. For ultrarelativistic cosmic strings, only photons traveling almost exactly par-

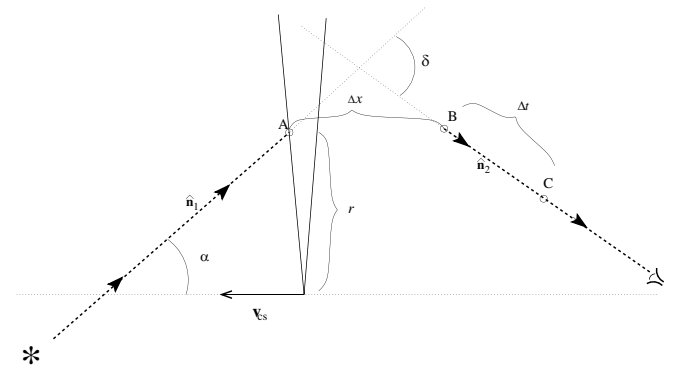


FIG. 3. The path of a photon coming from the source at the lower left and reaching the observer at the right. The straight cosmic string is moving with speed v_{cs} to the left. The photon’s initial velocity \hat{n}_1 makes an angle α with respect to $-v_{cs}$. At A the photon strikes the leading edge of the deficit wedge (a distance r from the position of the cosmic string).

allel to the string's velocity will strike the trailing edge of the deficit angle.

We will calculate the change in momentum of a particle interacting with the cosmic string. In the string rest frame, we know that

$$k_{\text{final}} = R_{\delta_0} k_{\text{initial}} \quad (4)$$

so we simply boost the above equation into the frame

where the string is moving at velocity v_{cs} . Then

$$k_{\text{final}} = \Lambda_{v_{cs}} R_{\delta_0} \Lambda_{-v_{cs}} k_{\text{initial}} \quad (5)$$

To calculate the directional change in a photon's velocity, we take the above formula with $k^2 = -m^2 = 0$ and for simplicity we take the photon to travel in a plane perpendicular to the cosmic string. Then we find (dropping the c s subscript)

$$\cos(\delta) = \frac{((A - B) \cos(\alpha) + \sin(\alpha)(v \sin(\delta_0) - \cos(\delta_0) \sin(\alpha)/\gamma))}{\sqrt{(\sin(\delta_0)(v + \cos(\alpha)) - \cos(\delta_0) \sin(\alpha)/\gamma)^2 + (B - A + \sin(\alpha) \sin(\delta_0))^2}}, \quad (6)$$

where

$$A = \gamma \cos(\alpha)(v^2 - \cos(\delta_0)),$$

$$B = \gamma(v \cos(\delta_0) - 1).$$

The blueshift can be calculated as well, yielding

$$\frac{\omega_{\text{final}}}{\omega_{\text{initial}}} = \gamma \sin(\alpha) \sin(\delta_0) + \gamma^2(1 + v \cos(\alpha) - v \cos(\alpha) \cos(\delta_0) - v^2 \cos(\delta_0)). \quad (7)$$

Straight portions of a cosmic string moving at ultrarelativistic speeds produce photon deflection of order π in conjunction with severe blueshift. Slower moving cosmic strings will obey the Kaiser-Stebbins formula [21] and cause the sky behind the moving string to be blueshifted relative to the sky in front of the string.

With the exception of loops, we expect cosmic strings to move moderately relativistically, but with $\gamma \delta_0 \ll 1$. In this limit, the above formula reduces to

$$\delta = \delta_0 \gamma(1 + v \cos(\alpha)) \rightarrow$$

$$\delta \varphi = \frac{D_{s,cs}}{D_{s,o}} \times \frac{8\pi G \mu}{\sqrt{1 - v^2}} \times (1 + \hat{\mathbf{n}} \cdot \mathbf{v}). \quad (8)$$

The first factor $D_{s,cs}/D_{s,o}$ is a plane-geometric coefficient for $8\pi G \mu \gamma$, which is the relativistic energy of the string. The third term is the result of the finite travel time of light, and does not represent the coordinate locations of the images in the observer's frame. Notice that a moving string (except one moving toward the observer) has a stronger lensing effect. This result disagrees with that given in Ref. [10], which has the γ factor in the denominator. To see this difference more clearly, we give in Appendix A the simple derivation of Ref. [10] and point out where the error occurs. Recall that the typical speed of the cosmic strings in the network is rather large, $v \sim 2/3$ [27]. There will be segments of strings that have $\gamma \gg 1$ and they have the best chance to be detected via lensing.

The above formula only applies to cosmic strings perpendicular to the line of sight. For the most general lensing due to straight cosmic strings, see Ref. [28].

III. THE POSSIBILITY OF RAPID COSMIC STRING FORMATION

Naively, ultrarelativistic straight strings are rather unlikely, and two parallel ultrarelativistic straight strings passing each other with such a large kinetic energy density must take some arrangement. It is along this line of reasoning that CFGO [17] argues against the formation of Gott spacetime in $2 + 1$ dimensions. The situation is no more likely in $3 + 1$ dimensions, although one cannot rule out the possibility of ultrarelativistic cosmic string formation. Recent brane inflation models have proposed having the inflationary branes located in the tip of a Klebanov-Strassler throat of a Calabi-Yau 3-fold [4]. In some scenarios, the standard model branes are in a different throat. One feature of this construction is that cosmic strings produced after inflation are meta stable, and will not be able to decay via open string interaction with the standard model branes [5]. It should be pointed out that extra dimensions have no effect on Gott's construction of CTCs, provided that the radii of the extra dimensions are small compared to the radius of the CTC. This can be seen as follows. A cosmic string in our universe is an object extended in one non-compact direction, and zero or more compact directions. The four dimensional effective theory will always have a conical singularity on the location of the string, and any corrections to this will be due to massive axion and (KK) modes of the metric—particles whose range is limited to sizes of order the radii of the extra dimensions. Thus as long as we do not probe distances so near the string that these corrections are significant, the conical geometry is valid and Gott's construction is meaningful. In fact, the CTCs in Gott's construction exist at large radii from the cosmic strings, and so one never needs to probe the near-string geometry.

IV. GOTT’S CONSTRUCTION OF CTCs

Here we give a brief review of Gott’s original construction. The key feature is the conical deficit angle ($\delta_0 = 8\pi G\mu$) around a cosmic string. This results in a “cosmic shortcut,” since two geodesics passing on opposite sides of the string will differ in length. This shortcut, like a wormhole, leads to apparent superluminal travel. Under boost, “superluminal” travel becomes “instantaneous” travel. Gott realized that this instantaneous travel could be performed in one direction, and then back again when two cosmic strings approach each other at very high speed. The actual trajectory is timelike, i.e. performed by a massive body, and resembles an orbit around the center-of-mass of the cosmic string system (in a direction with opposite angular momentum as the cosmic strings). Below we illustrate the geometry with two strings at rest.

In Fig. 4, there are three (geodesic) paths from A to B . The central path is not necessarily the shortest, in fact it can be seen that

$$w = x \cos\left(\frac{\delta_0}{2}\right) + d \sin\left(\frac{\delta_0}{2}\right). \tag{9}$$

Thus although $A \rightarrow B$ is traversed by a particle on a timelike trajectory above or below the cosmic strings, the departure and arrival events may have spacelike separation in the $y = 0$ hyperplane which extends between the two strings. In this hyperplane, the average velocity of this particle can be calculated as

$$v \leq \frac{2x}{2w} = \frac{1}{\cos(\delta_0/2) + \frac{d}{x} \sin(\delta_0/2)}. \tag{10}$$

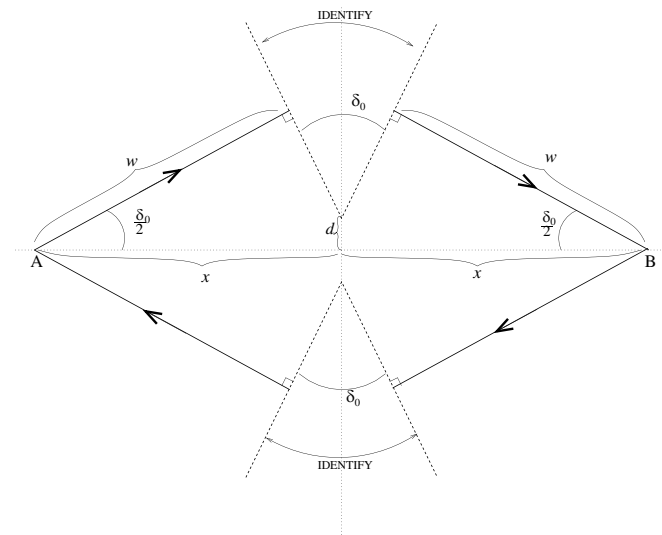


FIG. 4. The Gott spacetime, before the strings are moving. This spacetime will be severed at the $y = 0$ hyperplane (line AB), boosted, and then smoothly glued together again. A and B are separated by a distance of $2x$, while the cosmic strings are separated by a distance of $2d$.

(In our analysis, we focus on lightlike motion since timelike motion is, in a sense bounded by this case.) For large enough x , this velocity is greater than that of light and so we may boost to a frame where the departure and arrival events are simultaneous. This is true for any d . In the above picture, we will sever the spacetime at the ($y = 0$)-hyperplane and boost such that the top string is moving to the left at speed $v_{cs} > \cos(\delta_0/2)$ and the bottom string is moving to the right at the same speed. This means that we can take $A \rightarrow B$ on the upper path and $B \rightarrow A$ on the lower path, and in both directions the elapsed time is zero. This is possible when

$$2 < 2\gamma \sin(\delta_0/2) \approx \gamma \delta_0. \tag{11}$$

It is sufficient that the $y = 0$ hyperplane has vanishing intrinsic and extrinsic curvature for us to consistently sew the two halves together. Notice that the limiting case of $\gamma \delta_0 = 2$ corresponds to closed lightlike curves. (We will assume that $\frac{d}{x} \rightarrow 0$ for simplicity.)

The (two-fold) boosted version of the above setup is pictured in Fig. 5 below. The events are numbered in “proper” chronological order (that is, the order in which they occur on the particle worldline), but the center-of-mass coordinate frame chronological order needs to be explained. Event 1 is the light ray initially traveling up to meet the rapidly moving deficit angle, which happens at event 2. This meeting is identified (under Eq. (2)) with event 3, although in center-of-mass coordinates event 3 happens before the previous events occur. Events 1, 4, and 7 are cm-simultaneous at $t = 0$ while events 2 and 5 occur

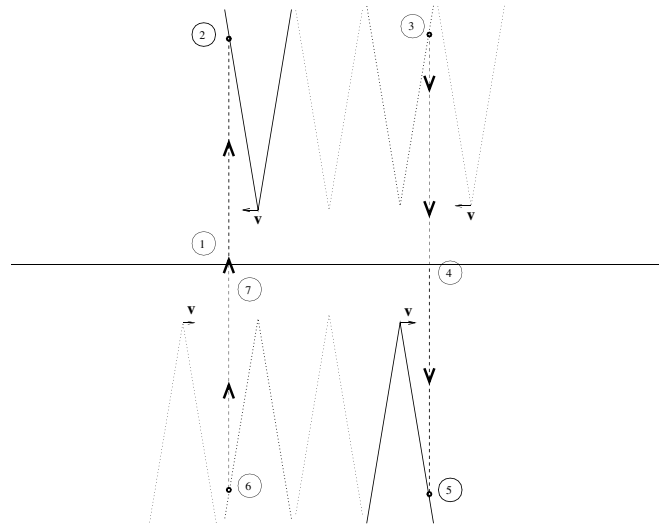


FIG. 5. This is the critical case where $\gamma \delta_0 = 2$. The deflection angle is calculated using Eq. (6) and the discontinuity in the world line using Eq. (3). We have drawn $d > 0$ for clarity. The particle travels along the path labeled 1 to 7 and back to 1, arriving at the same point in space and time, i.e., a closed timelike curve. In this case, the particle is neither blueshifted nor redshifted.

at $t_{cm} = +1$, and events 3 and 6 at $t_{cm} = -1$. Notice that $\delta = \pi$.

We would like to apply our understanding of ultrarelativistic cosmic strings to Gott spacetime. We can use the jumps in location, time and direction (Δt , Δx , and δ) to construct all photon paths around a cosmic string system and determine the complete lensing behavior. Below is a graph of δ for several values of $\gamma\delta_0$, as given by Eq. (6).

Of importance is where the graphs take the value π . Because the two cosmic string velocities differ in direction by π , $\delta = \pi$ is a fixed point of photon direction. As Cutler showed, a crossing with positive slope is stable and blueshifted, while the one with negative slope is unstable and redshifted. The stable blueshifted fixed point will always exist for $\gamma\delta_0 > 2$ (supercritical case) and thus represents a catastrophic divergence. This is because a particle in the presence of this geometry will fall into a stable orbit with exponentially diverging energy. The above graph is accurate for massless particles, but massive particles (equivalently: particles with nonzero momentum along the strings) will behave similarly once they become blueshifted. In total, we find

- (a) Particle trajectories in the vicinity of a $\gamma\delta_0 > 2$ pair of cosmic strings will be attracted to the stable orientation $\delta(\alpha) = \pi$, $\delta'(\alpha) > 0$ (see Fig. 6). This is because the two cosmic strings velocities are equal and opposite, that is, when a particle incident at angle α_n is deflected by an angle δ_n , $\alpha_{n+1} = \alpha_n + \pi - \delta_n$. Thus not only is $\alpha = \pi$ a “fixed point” of incidence angle, but it is a stable one if $\delta'(\alpha) > 0$, since then a slight increase in α will

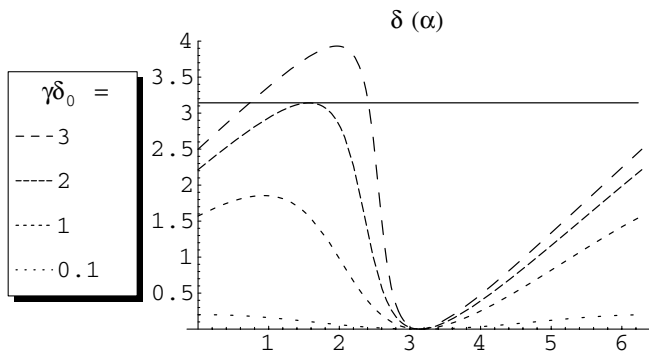


FIG. 6. This plot derives entirely from Eq. (6). Here, $\delta(\alpha)$ is the deflection angle of the photon as a function of the angle α between the photon direction and (minus) the cosmic string velocity (see Fig. 3). $\gamma\delta_0 = 2$ is the critical case, when the peak of $\delta(\alpha)$ touches the value π (the solid horizontal line). A $\gamma\delta_0 > 2$ curve crosses $\delta(\alpha) = \pi$ at 2 points. Notable is where $\delta(\alpha)$ crosses π (see e.g. the $\gamma\delta_0 = 3$ curve) with positive slope at α_s . A positive slope crossing implies a stable fixed point. Photons with different initial α not far from α_s will follow closed timelike paths and approach $\alpha = \alpha_s$. Photons are blueshifted at all positive slope crossings. The amount of blueshift each time is given by Eq. (7).

cause a slight increase in δ . A slight increase in δ will then decrease the next α , and return the system to equilibrium. (This assumes $\delta'(\alpha) < 1$, which is always the case.) Fig. 7 illustrates the attractor in action.

- (b) Photon momentum in the plane perpendicular to the strings will be blueshifted as given by Eq. (7). This formula applies to any relativistic particle. The blueshift occurs twice for each revolution, once from each string;
- (c) Since momenta along the string lengths are not blueshifted, nonrelativistic particles and particles with velocities along the length of the cosmic strings will be attracted to relativistic trajectories perpendicular to the string length;
- (d) In the limit of small d the average distance to the core of the CTC will not vary. This means that the orbits will close, rather than shrink;
- (e) The energy of each particle in the CTC will diverge exponentially as a function of number of cycles

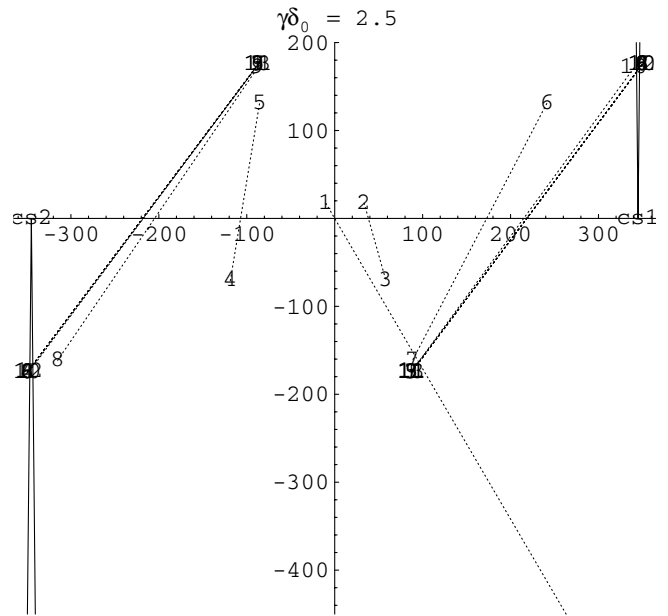


FIG. 7. The supercritical case. Cosmic string #1 is moving to the left and cosmic string #2 is moving to the right. The particle enters from the bottom right and reaches cs1’s upper deficit angle at 1. Its world line appears to jump to the right to 2, now directed down and slightly to the right. This process continues indefinitely, 3, 4, \dots , ∞ , the world line spiraling clockwise outward as it falls into a stable orbit (the two long, outermost parallel segments). The coordinate discontinuity seen here does not reflect any actual discontinuity. The numbers on the world line satisfy $2n + 1 \equiv 2n + 2$ where the equivalence is identification via Eq. (2). The coordinate discontinuities plotted here are calculated with Eq. (3), and the trajectory angle is determined by Eq. (6). The initial conditions of the above trajectory are not tuned, but rather generic. Approximately half of all initial particle trajectories will end up in a CTC.

taken; since it takes no time to travel any number of cycles, unbounded blueshift takes place;

- (f) Therefore, $\gamma\delta_0 > 2$ implies a catastrophic divergence in the presence of even a single particle.

It should be noted that the exponential divergence in energy is kinematic, and has nothing to do with particle number. Below is pictured the trajectory of a photon in a supercritical Gott space. The photon enters from the lower right, and then spirals clockwise outward into a stable, blueshifted orbit. The upper and lower deficit angle wedges are moving to the left and right, respectively.

We thus may conclude that although closed lightlike curves may exist ($\gamma\delta_0 = 2$), a purely classical divergence destroys the Gott solution of closed timelike curves ($\gamma\delta_0 > 2$) in the presence of a dynamical field (e.g. gravitons or photons). Cosmic strings cannot produce closed timelike curves. This is in agreement with Li and Gott [23] which finds Misner space (and by implication Grant space and Gott space) to suffer from a classical instability similar to the one found here. It should be noted that the hoop conjecture is not involved in this breakdown. Even the 2 + 1 dimensional Gott space, where the hoop conjecture is not applicable, is unstable in the presence of dynamical fields.

V. COMMENTS

A. General discussions

Semiclassical gravity raises an objection to the existence of CTCs, or at least to spacetimes that contain both regions with CTCs and regions out of causal contact with them. The boundary between such regions is called the “chronology horizon,” and in known examples this horizon coincides with a divergence of the renormalized energy-momentum tensor. This led Hawking to pose the “chronology protection conjecture,” in which he proposes that any classical examples of spacetimes containing CTCs will be excluded by a quantum theory of gravity. Thorne and Kim disagreed [29] on the grounds that the divergence of $\langle T_{\mu\nu} \rangle_{ren}$ is so weak that a full quantum gravity will remedy the semiclassical pathology. Grant found that the divergence on a set of polarized hypersurfaces is much larger than that on the chronology horizon. String theory can directly address the conjectures made using semiclassical arguments. Recent papers have evoked an Enhancement-like mechanism as a stringy method to forbid the formation of CTCs in some spacetimes [30]. On the other hand, a recent paper by Svendsen and Johnson demonstrated the existence of a fully string theoretic background (exact to all orders in α') containing CTCs and a chronology horizon, namely, the Taub-NUT spacetime. It is not clear if g_s corrections destroy this result once matter is introduced, but the empty background is an exact result. This seems to provide a counterexample to the chronology protection conjecture.

It is easy to see that a cosmic string with only the lowest mode will start as a circle and collapse to a point. Before it

reaches that limit, a black hole is formed. An elliptic or rectangular loop can be made to collapse to parallel, relativistic segments without forming a black hole. Theorems by Tippler and Hawking suggest that such a loop cannot form CTCs without creating a singularity, although we argue that potential CTCs will be disrupted by the presence of photons (or any other mode). It is also possible that the huge blueshift will cause the formation of a black hole, resulting in a black hole with cosmic string loops protruding. In this case, the presence of CTCs inside the black hole is acceptable, since they are not observable. More analysis is needed to fully address this issue.

B. Specific discussions on 2 + 1 dimensions

A simpler scenario with CTCs was found by van Stockum [31] and Deser, Jackiw, and 't Hooft (DJtH) [19] whereby a single stationary cosmic string is given angular momentum about its axis. The resulting background is given by

$$ds^2 = -(dt + Jd\theta)^2 + dz^2 + dr^2 + (1 - 4G\mu)^2 r^2 d\theta^2. \quad (12)$$

It seems unlikely for such a cosmic string to exist in string theory (at least as fundamental objects[32]), since the cosmic strings in superstring theory lack the internal degree of freedom “spin.” DJtH noticed an unusual feature of the Gott spacetime. They classified the energy momentum of Gott’s solution in terms of the Lorentz transformations encountered under parallel transport (PT) around the strings (holonomy). It is well known that the PT transformations around a single cosmic string is rotationlike, i.e. can be expressed as

$$\Xi = \Lambda_\beta R_{\delta_0} \Lambda_{-\beta}, \quad (13)$$

where Λ_β is a pure boost with rapidity β and R_{δ_0} is a pure rotation about the angle δ_0 . One may calculate the holonomy of a Gott pair via

$$\Xi = \Lambda_\beta R_{\delta_0} \Lambda_{-\beta} \Lambda_{-\beta} R_{\delta_0} \Lambda_\beta \quad (14)$$

and it is found that Ξ is boostlike, i.e. $\Xi = \Lambda_\beta \Lambda_\zeta \Lambda_{-\xi}$. DJtH regarded the energy momentum of a Gott pair to be unphysical on the grounds that its holonomy matches that of a tachyon (boostlike). It is an unusual feature of spacetimes that are not asymptotically flat that $T^{\mu\nu}$ can be spacelike (tachyonic) despite the fact that it is made up of terms that are timelike.

Headrick and Gott [33] refuted this criticism by showing that the Gott pair geometry was quite unlike the tachyon geometry both because a tachyon does not yield CTCs and because the holonomy definition of $T^{\mu\nu}$ was incomplete. Later, Carroll, Farhi, Guth, and Olum (CFG0) [17] and 't Hooft [20] gave more convincing arguments against CTCs. CFG0, Gott, and Headrick found that the PT transformation of a spinor distinguished between tachyon and Gott

pair geometries. A more complete description of geometry would include not just the PT transformation, but the homotopy class of the PT transformation as well. Equivalently, one should extend $SO(2, 1)$ to its universal covering group.

One may interpret the “boostlike” holonomy as boost identification, as was done by Grant [34]. This makes the Gott spacetime akin to a generalized Misner space. Grant was able to show that the Gott/Misner spacetime suffers from large quantum mechanical divergences on an infinite family of polarized lightlike hypersurfaces. This divergence is stronger than that at the chronology horizon [34].

Regardless of whether the Gott spacetime is physical or not, it can be shown that the Gott spacetime in $2 + 1$ dimensions cannot evolve from cosmic strings initially at rest [17]. This is quite different from the $3 + 1$ dimensional case.

C. The instability in $3 + 1$ dimensions

Recent realization of the inflationary scenario in superstring theory strongly suggests that cosmic superstrings were indeed produced toward the end of the inflationary epoch. With this possibility, the issue has renewed urgency. In the above discussions, we argue that the reasoning against the Gott spacetime in $2 + 1$ dimensions does not necessarily apply to the $3 + 1$ dimensional case. In short, the Gott spacetime is entirely possible in an ideal classical situation. However, we argue that instability is manifest if there is a quanta/particle nearby. The particle will be attracted to the closed timelike curve and is arbitrarily blueshifted instantly. Of course, backreaction takes place before this happens. This backreaction must disrupt the closed timelike curve, otherwise the infinite blueshift will not be prevented. In an ideal situation where there is no quanta nearby, one still expects particles like (very soft) gravitons/photons can emerge due to quantum fluctuation. In fact, quantum fluctuation of the cosmic strings themselves as they move rapidly toward each other will produce graviton radiations. One graviton, no matter how soft, is sufficient to cause the instability. So we believe that the Gott spacetime is unstable under tiny perturbations and so cannot be formed in any realistic situation.

VI. CONCLUSION

Recent implementation of the inflationary scenario into superstring theory led to the possibility that cosmic strings were produced toward the end of brane inflation in the brane world. This possibility leads us to reexamine the Gott spacetime, where closed timelike curves appear as two cosmic strings move ultrarelativistically pass each other. In an ideal situation, the Gott spacetime is an exact solution to Einstein equation, with a well-defined chronology horizon. It does not collapse into a black hole and can be readily reached. So it seems perfectly sensible that such a spacetime can be present in a universe that contains cosmic

strings. In this paper, we find that nearby photons and gravitons will be attracted to the closed timelike curves, resulting in an instantaneous infinite blueshift. This implies backreaction must be large enough to disrupt the formation of such closed timelike curves. We interpret this as a realization of the chronology protection conjecture in the case of the Gott spacetime.

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APPENDIX A: LENSING BY A MOVING COSMIC STRING

In the text, the lensing due to a moving cosmic string is too general for normal application. Here we reproduce a simple elegant argument due to Vilenkin [10] to calculate the angular separation of images produced by a moving cosmic string in the case when the cosmic string tension as well as the lensing effect are small. We point out where the (small) error in Ref. [10] is. Vilenkin argues that the angular deflection of light by a cosmic string may be calculated by appealing to Lorentz invariance. For a string at rest, the angular separation is given by

$$\delta\varphi = \frac{D_{s,cs}}{D_{O,cs} + D_{s,cs}} \delta_0 \sin(\theta) = \frac{D_{s,cs}}{D_{s,o}} \delta_0, \quad (\text{A1})$$

where θ is the relative angle between the cosmic string and the line of sight, and $D_{x,cs}$ is the *normal* distance between x and the cosmic string. We may consider two light waves, one from each image, k and k' . Their scalar product is given by

$$k^\mu k'_\mu = \omega\omega'(1 - \cos(\delta\varphi)) \approx \frac{1}{2} \omega\omega'(\delta\varphi)^2. \quad (\text{A2})$$

We may assume that the two light waves have the same frequency: $\omega = \omega'$. (This can be true in all reference frames since we are expanding to first order in δ_0 .) We can then relate the angular separation in any two reference frames by the frequency of the light waves:

$$\omega_0 \delta\varphi_0 = \omega \delta\varphi \quad (\text{A3})$$

i.e., the higher the observed frequency, the lower the observed angular separation. The frequency in a reference frame where the string moves at velocity \mathbf{v} relative to the observer is given by

$$\omega = \frac{\omega_0}{\gamma(1 + \hat{\mathbf{n}} \cdot \mathbf{v})}, \quad (\text{A4})$$

where $\hat{\mathbf{n}}$ is the direction from the observer to the source (and thus string), and hence

$$\delta\varphi = \gamma(1 + \hat{\mathbf{n}} \cdot \mathbf{v})\delta\varphi_0. \quad (\text{A5})$$

The roles of ω and ω_0 are erroneously swapped in Ref. [10], which thus agrees with ours only for $\hat{\mathbf{n}} \propto \mathbf{v}$. Physically, a traveler moving transverse to the light we observe should measure a higher frequency than we do (i.e. $\omega < \omega_0$ for $\hat{\mathbf{n}} \cdot \mathbf{v} = 0$). It should also be pointed out that a string moving across the sky will blueshift the CMB behind it by an amount

$$\frac{\omega_{\text{back}}}{\omega_{\text{front}}} = 1 + |\mathbf{v} \times \hat{\mathbf{n}}| \gamma \delta_0 \quad (\text{A6})$$

that is, the sky becomes hotter after a cosmic string passes.

APPENDIX B: BRANONIUM

The analysis of relativistic bound states of D-Branes has been found to be quite simple in the probe brane approximation (no radiation) [35]. We extend this to the case of codimension two branes (cosmic strings) using a similar analysis. The action for a codimension two projectile is written as a DBI-like term and a Wess-Zumino like term as follows:

$$S = -\mu \int \sqrt{g} - q \int C_2 \quad (\text{B1})$$

with

$$C_2 = \log(r/r_0) dz \wedge dt. \quad (\text{B2})$$

We can now write down the canonical momenta associated to θ and r , as well as the conserved Hamiltonian. They are, respectively

$$\ell = \frac{r^2 \dot{\theta}}{\sqrt{1-v^2}}, \quad p_r = \frac{\dot{r}}{\sqrt{1-v^2}}, \quad (\text{B3})$$

$$H = \mu \sqrt{1 + p_r^2 + \frac{\ell^2}{r^2}} + q \log\left(\frac{r}{r_0}\right),$$

where

$$v^2 = \dot{r}^2 + r^2 \dot{\theta}^2 \quad \Rightarrow \quad \frac{1}{\sqrt{1-v^2}} = \sqrt{1 + p_r^2 + \ell^2/r^2}. \quad (\text{B4})$$

The effective potential energy is plotted below. Notable features are the existence of stable orbits for generic initial conditions, and exponentially long orbital periods as a function of energy.

Notice that the total energy of the system contains an arbitrary additive constant. This should be fixed by the thickness of the strings, but it will have no effect on the dynamics. We assume that the tension of the strings dominates the potential energy of the long range interaction. In this approximation, the geometry is flat except for a conical singularity at the location of each string. For simplicity, we define the additive constant using the perihelion distance (making it trajectory dependent): $r_0 = d$. Then we may relate the energy with the angular momentum as follows.

$$\epsilon = \sqrt{1 + \frac{\ell^2}{d^2}} \quad (\text{B5})$$

A closed form solution for any trajectory can be found by making the following substitution.

$$u := 1/r \quad u' := \frac{du}{d\theta} \\ \rightarrow u' = -\frac{p_r}{\mu\ell} = \frac{1}{\ell} \sqrt{\left(\frac{q}{\mu} \log(ud) + \epsilon\right)^2 - 1 - u^2 \ell^2}, \quad (\text{B6})$$

where we have used

$$\frac{dr}{d\theta} = \frac{r^2 p_r}{\mu\ell}$$

and

$$H = \mu \sqrt{1 + \ell^2(u'^2 + u^2)} - q \log(ud) = \mu\epsilon.$$

Now we may solve for θ via

$$\theta = \int d\theta = \ell \int_{1/r_i}^{1/r_f} \frac{du}{\sqrt{\left(\frac{q}{\mu} \log(ud) + \epsilon\right)^2 - 1 - u^2 \ell^2}}. \quad (\text{B7})$$

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